pstat_lab3_zejie

2023-10-20

1. Consider the AR(2) model below:

$$X_t = 0.8X_{t-1} - 0.12X_{t-2} + Z_t, \quad \text{where} \quad Z_t \text{ iid } \sim N(0,1)$$

(a) Express the model in terms of the back shift operator, B.

$$\left(1 - 0.8B + 0.12B^2\right) X_t = Z_t$$

(b) Determine whether the model is causal and/or invertible. (Hint: use polyroot()).

```
polyroot(c(1,-0.8,0.12))
```

```
## [1] 1.666667+0i 5.000000+0i
```

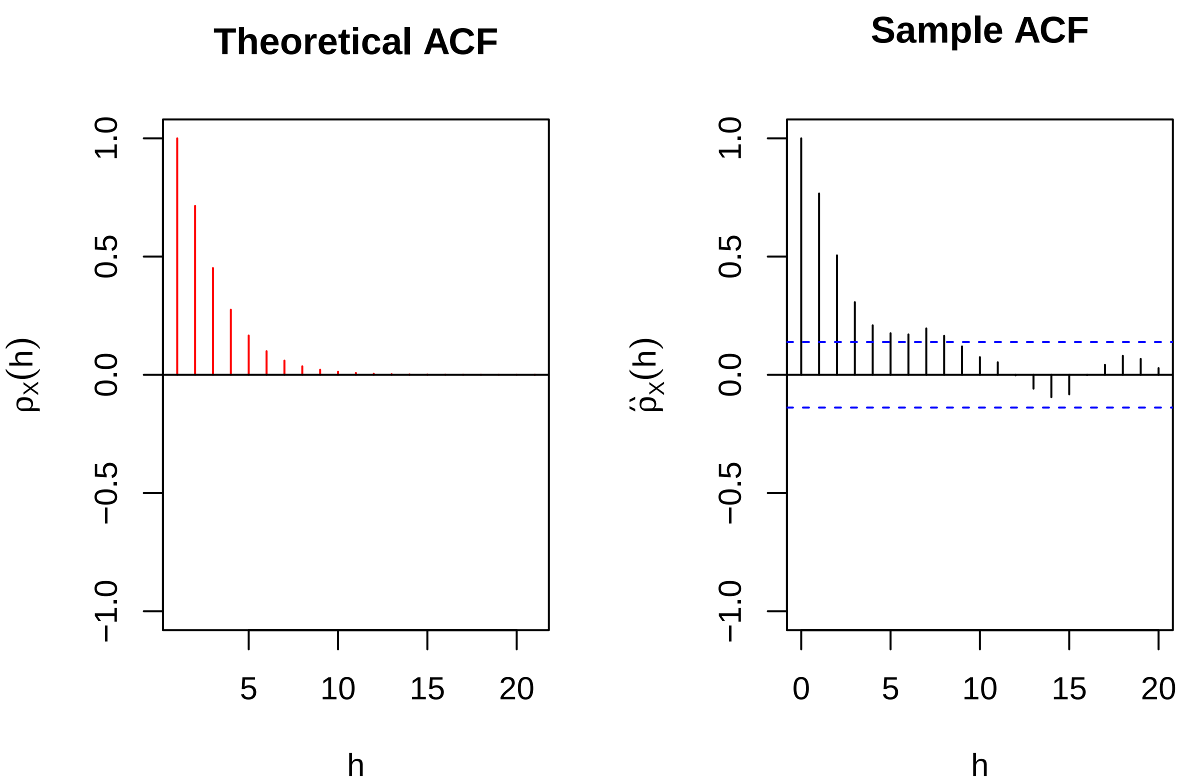
In autoregressive models, invertibility is inherent. For the given process, the AR polynomial is given by $\phi(z) = 1 - 0.8z + 0.12z^2 = (1 - 0.6z)(1 - 0.2z)$ and is therefore zero at z = 1.67 and z = 5. Since both of these values are outside the unit circle, we deduce that X_t represents a causal AR(2) process."

(c) We simulate 200 observations from this AR(2) model with the following code:

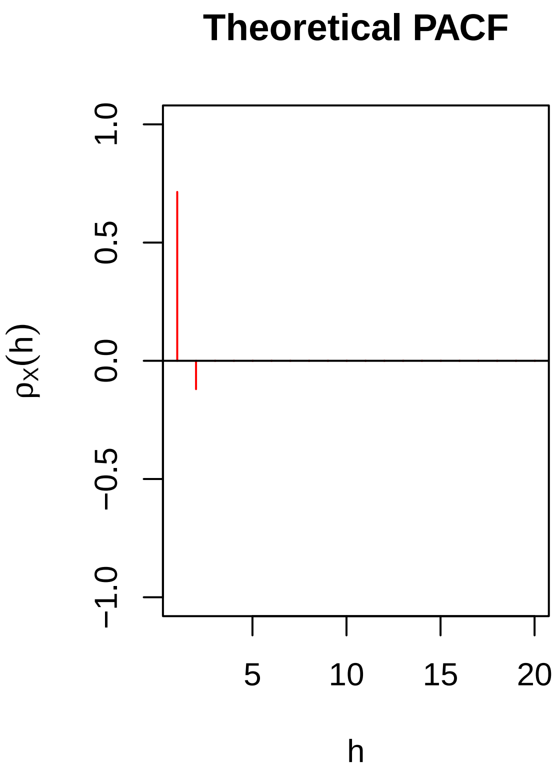
```
set.seed(1234)
ar2 <- arima.sim(model = list(ar = c(0.8,-0.12),sd = 1),n = 200)
```

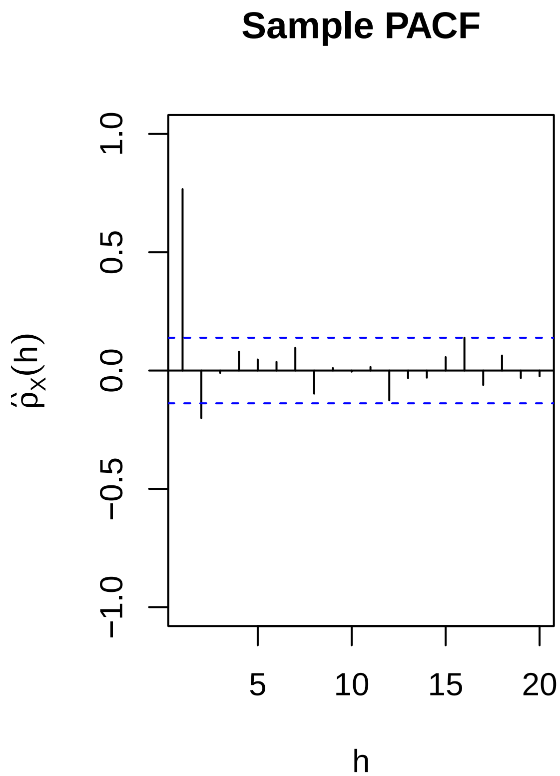
Plot the sample as well as theoretical ACF and PACF.

```
theo_acf <- ARMAacf(ar = c(0.8,-0.12),lag.max = 20, pacf = FALSE)
op <- par(mfrow = c(1,2))
# Theoretical ACF
plot(theo_acf,type = "h",ylim = c(-1,1),
main = "Theoretical ACF",
col = "red",
ylab = expression(rho[X](h)), xlab = "h")
abline(h = 0) # Add horizontal line
# Sample ACF
acf(ar2,lag.max = 20,
main = "Sample ACF",
ylim = c(-1,1),
xlab = "h",
ylab = expression(hat(rho)[X](h)))
```




```r
theo_pacf <- ARMAacf(ar = c(0.8,-0.12),lag.max = 20, pacf = TRUE)
op <- par(mfrow = c(1,2))
plot(theo_pacf,type = "h",ylim = c(-1,1),
main = "Theoretical PACF",
col = "red",
ylab = expression(rho[X](h)), xlab = "h")
abline(h = 0) # Add horizontal line
# Sample PACF
pacf(ar2,lag.max = 20,
main = "Sample PACF",
ylim = c(-1,1),
xlab = "h",
ylab = expression(hat(rho)[X](h)))
```





(d) Compare ACF and PACF plots in (c). AR models are characterized by PACF=0 after lag p but ACFs do not become zeros. In the AR(2) model, theoretical PACFs cut off after lag k=2.