PSTATW 174/274, Fall 2023: Homework # 4.

Note: In all problems, $\{Z_t\} \sim WN(0, \sigma_Z^2)$ denotes white noise, and B denotes the backshift operator $BX_t = X_{t-1}$.

1. Determine which of the following indicates that a nonstationary time series can be represented as a random walk:
I. A plot of the series detects a linear trend and increasing variability.
II. The differenced series follows a white noise model.
III. The standard deviation of the original series is greater than the standard deviation of the differenced series.

2. Identify the following time series model as a specific ARIMA or SARIMA model:
2.a $X_t = 1.5X_{t-1} - 0.5X_{t-2} + Z_t - 0.1Z_{t-1}$
2.b $X_t = 0.5X_{t-1} + X_{t-4} - 0.5X_{t-5} + Z_t - 0.3Z_{t-1}$

3. (a) For the following SARIMA$(p, d, q) \times (P, D, Q)_s$ models, specify parameters p, d, q, P, D, Q and s, and write corresponding equations: (i) SARIMA $(2, 1, 1) \times (0, 1, 1)_6$, (ii) SARIMA$(1, 1, 2) \times (2, 0, 1)_4$.

Hint: It is helpful to first write equations in the form $\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D X_t = \theta(B)\Theta(B^s)Z_t$, $Z_t \sim WN(0, \sigma_Z^2)$, *then proceed to rewrite the equations in the form that eliminates use of the backshift operator* B.

(b) For the following processes $\{X_t\}$, identify SARIMA $(p, d, q) \times (P, D, Q)_s$ model: (i) $(1-B^6)^2X_t = (1-0.3B)Z_t$; (ii) $X_t = 0.3X_{t-12} + Z_t$;

(c) You are given a time series model where PACF is zero except for lags 12 and 24. Which model will have this pattern?

4. Write the form of the model equation for a SARIMA $(0, 0, 1)(0, 0, 1)_{12}$ model.
A. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$.
B. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \theta_1\theta_2 Z_{t-3}$.
C. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \theta_1\theta_2 Z_{t-12}$.
D. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-11} + \theta_1\theta_2 Z_{t-12}$.
E. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-12} + \theta_1\theta_2 Z_{t-13}$.

5. You are given PACF for two stationary processes:
(a) For time series $\{X_t, t = 1, 2, \ldots\}$: $\phi_{11} = 0$, $\phi_{22} = 0.36$, $\phi_{kk} = 0$ for $k \geq 3$.
(b) For time series $\{Y_t, t = 1, 2, \ldots\}$: $\phi_{11} = 0.7$, $\phi_{kk} = 0$ for $k \geq 2$.
In each case, write an appropriate equation for the corresponding stationary process.

6. For the processes $X_t = 0.4X_{t-1} + Z_t - 0.7Z_{t-1}$,
(i) Simulate and plot 100 values of the processes;
(ii) Compute and graph their theoretical ACF and PACF using R.
(iii) Compute and graph their sample ACF and PACF using R. How do sample functions compare to their theoretical counterparts?
(iv) Analyze smoothness of the simulated processes using their ACF's.

Please include the code with clear comments explaining the meaning of the code. Make sure to label the graphs.

The following problems are for students enrolled in PSTAT 274 ONLY

GE 1. Let $\{Y_t\}$ be the AR(1) plus noise time series defined by $Y_t = X_t + W_t$, where $\{W_t\} \sim WN(0, \sigma_W^2)$, $\{X_t\}$ is the AR(1) process $X_t - \phi X_{t-1} = Z_t$, $\{Z_t\} \sim WN(0, \sigma_Z^2)$, $|\phi| < 1$, and $E(W_s Z_t) = 0$ for all t, s.
(a) Show that $\{Y_t\}$ is stationary and find its autocovariance function.
(b) Show that the time series $U_t := Y_t - \phi Y_{t-1}$ is 1-correlated (i.e., $\gamma_U(h) = 0$ for $h > 1$) and hence is MA(1) process.
(c) Conclude from (b) that $\{Y_t\}$ is an ARMA(1,1) process and express the three parameters of this model in terms of ϕ, σ_W^2 and σ_Z^2.

GE 2. Find the ACVF, ACF and PACF for $\{X_t\}$ when $X_t = \Phi X_{t-4} + Z_t$, $|\Phi| < 1$.