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A robust approach for a green periodic competitive VRP under uncertainty: DE and PSO algorithms

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Abstract. The purpose of this paper is to examine and evaluate a new mathematical model of vehicle routing problem in order to optimize fuel consumption and maximize commercial profitability under the conditions of uncertainty of distributor service to customers using robust approach under scenario. According to the real world, distribution companies are interested in minimizing consumption of fuel in the distribution of goods for two reasons: the first reason is that reducing the consumption of fuel will reduce the current costs of distribution companies and ultimately increase their profits. The second reason is that reducing fuel consumption will reduce the harmful effects of greenhouse gases and air pollution. Other words, distribution companies operate in a competitive environment that has more than one distributor in the distribution network, and start time for serving customers has a significant impact on the profitability of the distributors. To calculate the efficiency of the proposed model, we used differential evolution (DE) algorithm and Particle swarm optimization (PSO), and the results were compared in small and medium scales with the results of the exact solution method. To verify proceeds of proposed algorithms in large scales, a number of sample problems were created in large scales and the figures were evaluated. The computational results indicate that DE algorithm has a better computational function, but the PSO algorithm has better computational time.

Keywords: Green periodic VRP, robust optimization, Fuel consumption, Particle swarm optimization algorithm

1. Introduction

Increasing transportation of goods and constraints in the capacity of the transportation network plays a significant role in increasing traffic, fuel consumption, and ultimately, increasing transportation costs. This increases service time to customers and causes the customer dissatisfaction. On the other hand, an increase of fuel consumption creates significant negative impacts on greenhouse gas (GHG) emissions that cause social costs. Social costs are the costs that estimate the destructive or harmful pollutants effects or activities on agricultural products, ecosystems, materials and human health, and often this cost may not be considered at the final cost. Therefore, the development of green vehicle routing problems (GVRPs) is inevitable to reduce fuel consumption, air contamination and GHG emissions. For this purpose, the main objectives of the GVRP model are to reduce fuel consumption and select the best route with regard to the

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speed of vehicles [1, 2]. In the classical VRP, it is assumed that reaching time to customers has no effect on the distributer's profit. According to real-world cases, there is a competition among competitors in a business environment on earning more liquidity and lack of attention to this issue reduces the efficiency of the paths in earning more liquidity. If customer service is conducted later than competitors, market share will be reduced. For this reason, distribution companies try to reach to customers to get the most market share [3]. However in the real world, competitors do not use a certain daily visit pattern. Variations in the sequencing choice for customers on different days by the competitor makes the time period of the competitor's visit of customers unstable. Thus, there are several time periods for customers.

In this paper, a new mathematical model is presented using a scenario-based robust optimization approach to evaluate a competitive periodic vehicle routing problem with time windows (VRPTW) in a competitive environment considering the reduce fuel consumption. Considering a real-world situation, several distributors of goods compete with each other to reach customers a head of other competitors, and to achieve the liquidity profit comparing to them. However, considering variations in customers' demands during different days, traffic, weather, etc., the sequence selected by competitors as for rendering service to customers during various days is uncertain. Thus, it is supposed that the rivals beginig time of service is uncertain, and scenario-based robust optimization methods have been used to make it certain. The model aims at maximizing obtainable profit stemmed from sooner begining time of service comparing to other competitors, and finding a robust response. The advantage of using a scenario-based robust approach in comparison to a certain approach is that the lost profit will be reduced despite decreasing in the obtained profit and achieving the optimized responses will be possible.

The reminder of this paper is as follows. In Section 2, the literature review of the considered problem is presented. Section 3 explains a robust optimization approach based on the scenario. Section 4 presents the mathematical model of the scenario-based robust approach of the green VRPTW. In Section 5, the differential evolution (DE) is first used to solve the model, and then particle swarm optimization (PSO) algorithm showing the quality of solutions is presented. Section 6 solves a number of test problems in small and large sizes and represents the computational results. In Section 6, the results of the proposed algorithms are presented by solving the considered model, and finally some suggestions for future research are given.

2. Literature review

Considering the issue arisen in the proposed problem, this section first presents the research done in the field of vehicle routing problems in a competitive environment, and then the articles in the field of green vehicle routing problems are reviewed.

2.1. Competitive VRP

In the VRP Customers' demands are provided by only one vehicle and there is no competition among distributers [4]. The VRP with time windows is most significant version of VRPs, in which vehicles should meet customer in the internal $[lo_i, up_i]$; where lo_i and up_i are indicates as the earliest and latest times that service to customer should occur. The VRPTW is divided into two sections: hard VRP time window (VRPHTW), in which violation from upper and lower time limits is not allowable and the VRP with soft time windows (VRPSTW), in which violation from time windows is acceptable with a penalty. The VRPTW is used in different scopes, such as just-in-time manufacturing, dangerous materials transportation, groceries transportation, and raw materials transportation [5]. level of customers' scattering and customer demand changes, The VRPTW is expanded, in which providing customers' demand was done in few days [6, 7].

The periodic VRP is a kind of the capacitated VRP that takes into account several scheduled days with customers (i.e., nodes) that need service on several days during the scheduled period. On the other hand, with attention of customer's levels dispersion and changes in their demands, the PVRPTW is expanded [8]. In periodic vehicle routing problem, each customer $m_i \in M$ indicates a set of Γ_i combination, in which a combination of days to be serviced becomes clear. For example, if in a 8-day period, if customer n_i has three servicing combinations $\{5,3,6\}$, $\{1,4,7\}$ and $\{8,2\}$. In this case, if a distributer selects the first combination vehicle, the distributer should service the customer in the fifth, third and sixth days. In the second combination, the distributer should meet the customer in the first, fourth and seventh days. Finally, in the third case, the distributer should service the customer in the eight and second days [9]. In the literature review, there are several variants of the PVRP are considered. Optimizing of the distance and servicing time as well as the cost of travelling can proposed as part of optimization function. Constraints can be categorized in three sectors: (i) the planning of customers servicing (ii) the kind of demand (constant or variable, we return to this issue later) and (iii) the vehicles. The PVRP has some applications, such as collecting and processing the product and distributing the spare parts [10–12].

According to the literature review in the PVRPTW, it is assumed that there is a monopoly in a product distributor's company and the quantity of distributed products has no effect on the driver's net income. Furthermore, the impact of the reply rate on the customers' demand is not considered. Whereas providing the customers' require earlier than other rivals by using rival vehicle routing has a great impact on gaining liquidity more than other distribution companies. Tavakkoli-Moghaddam et al. [13] presented a competitive vehicle routing problem model for the first time that optimizes the revenue of earning liquidity and optimizes the transportation costs at the same time. Norouzi et al. [14] presented a new model for competitive VRP. Their objective functions were to reduce the transportation cost and optimizing the total achieved benefit. They solved the problem using MOPSO and NSGA-II. Alinaghian et al. [15] developed the competitive periodic VRP. They analyzed the effect of considering competitive situation on a PVRP. According to the NP hardness of the problem, a meta-heuristic algorithm was developed based on MOPSO and the results were compared by exact solutions.

However, in most VRPs, parameters are either deterministic or non-deterministic (e.g., a structure of the road network must meet constraints on the road capacity, road speed limit and permanent shelter capacity). A probabilistic optimization approach is used to estimate the probable distribution of the parameters [16–18]. The key challenge of this approach is the impossibility of estimating the probable distribution of parameters [19, 20].

The variations in the sequencing choice for customers on different days by the competitor makes the time period of the competitor's visit of the customers unstable, and thus, there are several time periods for customers that is not considered in the previous researches.

In this paper, a robust optimization approach is used to estimate the customer service time, which is another innovation in the research that presented by Mulvey et al. [21]. In this model, contrary to the pessimistic approaches of robust optimization, the goal is to create a balance between the cost of robustness and other objectives of the optimization problem, so that the optimal solution is justified and for most possible scenarios, the uncertain parameters are close to optimal [22].

2.2. Green VRP

Research has shown that by considering a reduction of fuel consumption, the VRP has attracted many researchers in recent years. Madene et al. [23] considered VRP with time window constraints, whose speed was dependent on the journey. They also developed a meta-heuristic method to solve the problem, which achieved a seven percent savings in carbon dioxide generated in a case study in the UK. Jabali et al. [24] proposed a similar optimization model to Madene et al. [23], in which the amount of pollutant production was estimated by linear optimization of vehicle speed. They proposed an analysis to find optimal speed considering the amount of pollutants and used an iterative tabu search (TS) algorithm to solve typical VRP issues. In this regard, Tavares et al. [25] considered the gradient of the effect of the road and the vehicle's load on fuel consumed in the waste collection problem and did not consider the relationship between fuel consumption and load levels. Moreover considering the effect of the load on the vehicle's fuel consumption, Suzuki [26] put an indication of the impact of the vehicle waiting time at the start of serving customers on fuel consumption. For better understanding the green VRP and research done in this topic, the interested reader refers to [27]. Due to considerable effect of fuel consumption in reducing the cost of operation and service time of competitor's on profit of distributers in a real competitive environment, distributors must revise the design of fleet route with attention of the scenarios of rivals to optimal the profit. A new category of the VRPTW with competitive time windows under uncertainty is considered in this paper that can be noted as a special kind of the VRPTW.

According to the author's knowledge, there is no research on a competitive VRP and its variant considering the instability of the arrival time of competitors in the problem. Also, considering the fuel cost on the competitive VRP is not considered in the previous studies.

Innovation implemented in the paper may be explained below. According to the surveys conducted so far, a green VRP in a competitive state under uncertainty conditions of the time when competitors begin to serve customers in terms of robust optimization methods under the scenario is not presented. Therefore, one of the innovations in this paper is to model a competitive green VRP using a robust optimization approach under this scenario in order to find a consistent response to the uncertainty situations of the competitors' starting time. Additionally, the aim of the mathematical model is to reduce the fuel consumption by distributors, the operational cost of the fleet, the harmful effects of greenhouse gases (especially, carbon and carbon dioxide) and the social costs of the environmental pollution. Furthermore, using DE and PSO algorithms to find optimal solutions is another contribution of this paper.

3. Robust optimization

The linear programming model including random parameters is presented by:

$$\min N\tau + M\vartheta \tag{1}$$

s.t

$$Fx \geq J \tag{2}$$

$$A\tau + W\vartheta \geq K \tag{3}$$

$$\tau, \vartheta \geq 0 \tag{4}$$

Where τ is the decision variables vector and ϑ is indicated as vector of control variables. In addition, F, A, W, N, and M are the model parameters; while, J and K are defined as values vector which represented on the right side. Suppose a limited set of scenarios $\Omega = \{1, 2, ..., s\}$. The uncertain parameter of the model for each scenario ($s \in \Omega$) is defined the subset of $\{A_s, W_s, M_s, K_s\}$, which is named a scenario, and defined by s and dependent probability Ps. Moreover, the control variables vector (ϑ) when the scenario occurs, ϑ_s can be indicated in all of scenario. The model uncertain parameters cannot be feasible for some scenarios. For this purpose, in each scenario, ϕ_s demonstrates the model infeasibility. When the proposed model is infeasible, ϕ_s will be greater than zero. Otherwise, it will be equal to zero. A robust model is demonstrated by [28]:

$$Min\ g(\tau, \vartheta_1, \vartheta_2, ..., \vartheta_s) + \gamma\rho(\phi_1, \phi_2, ..., \phi_s) \tag{5}$$

s.t.

$$F\tau \geq J \tag{6}$$

$$A_s\tau + W_s\vartheta - \phi_s \geq K_s \quad \forall s \in \Omega \tag{7}$$

$$\tau \geq 0 \quad \vartheta s \geq 0 \quad \phi_s \geq 0 \quad \forall s \in \Omega \tag{8}$$

The first term of Equation (5) is solution robustness, the second term indicates robustness weighted by γ, that contains solutions with a fine and does not meet the customer need in a scenario or contravene other limit (e.g., capacity). $g(\tau, \vartheta)$ is denoted for demonstrating the cost and objective function. For each scenario, $\mu_s = g(\tau, \vartheta)$.

To represent $f(\tau, \vartheta)$, which is a profit or cost function $\mu_s = f(\tau, \vartheta_s)$ for scenario s. $\mu_s = g(\tau, \vartheta_s)$ is variance indicates the amount of risk in each decision. A non-linear quadratic term is demonstrated by [21]:

$$obj = \sum_{s\in\Omega} p_s\mu_s + \alpha \sum_{s\in\Omega} p_s \left(\mu_s - \sum_{s'\in\Omega} p_{s'}\mu_{s'}\right)^2 \tag{9}$$

Where obj objective is the function and α is a nonnegative value. Yu and Li [29] To adjust the quadratic term to the absolute deviation, proposed the following equation.

$$obj = \sum\nolimits_{s\in\Omega} p_s\mu_s + \alpha \sum_{s\in\Omega} p_s |\mu_s - \sum_{s'\in\Omega} p'_s\mu'_s| \tag{10}$$

To linearize Equation (10), Leung and Chen [30] indicated two variables according to the additional and basic restrictions minimized by:

$$Min = \sum_{s\in\Omega} p_s\mu_s + \alpha \sum_{s\in\Omega} p_s \left[\left(\mu_s - \sum_{s'\in\Omega} p_{s'}\mu_{s'}\right) + 2\beta_s\right] \tag{11}$$

s.t.

$$\mu_s - \sum_{s\in\Omega} p_s\mu_s + \beta_s \geq 0 \quad \forall s \in \Omega \tag{12}$$

$$\beta_s \geq 0 \quad \forall_s \in \Omega \tag{13}$$

If μ_s is greater than $\sum_{s\in\Omega} p_s\mu_s$, then $\beta_s = 0$ and if $\sum_{s\in\Omega} p_s\mu_s$ is greater than μ_s, then $\beta_s = \sum_{s\in\Omega} p_s\mu_s - \mu_s$.

$\rho(\phi_1, \phi_2, ..., \phi_s)$ in the second function of Equation (5) is used to fine the infeasibility of the model. Base on these arguments, the objective function is represented by:

$$Min = \sum_{s\in\Omega} p_s\mu_s + \alpha \sum_{s\in\Omega} p_s \left[\left(\mu_s - \sum_{s'\in\Omega} p_{s'}\mu_{s'}\right) + 2\beta_s\right] + \gamma \sum_{s\in\Omega} p_s\phi_s \quad (14)$$

4. Problem definition and modeling

4.1. Problem assumptions

This research shows that distribution costs depend on many criteria and can be separated into two broad categories. The first category includes the load, speed, road status, fuel consumption rate (in any distance) and fuel price that are directly related to the scheduling. The second category includes vehicle depreciation, maintenance and repair costs, driver's wages and taxes [31]. In other words, the first group factors are directly related to fuel consumption, for example, a truck with a capacity of 20 tons when its capacity is completed is equal to the cost of fuel consumed in the course of 100 km with 6% of the total cost. On the other hand, minimizing fuel consumption plays a significant role in reducing greenhouse gas emissions. Therefore, by considering other parameters constant, fuel consumption will depend mainly on the distance traveled and the load [32].

Based on the identified aspects of review studies, it is worthwhile to look at the start-up time for competitors. According to the real world, the period of the competitor's visit to customers is not fixed on different days and there are several periods for customers. Among the reasons for the changes in the visit time and the existence of several time intervals that change the pattern of competitor's visit on different days are:

(1) Change in the customer demand in different days that leads to a change in the sequence of customer visits by the competitor (the different composition of customers in different days causes the demand for goods to change on a daily basis; therefore, the daily sequence of service serving customers differs from the competitor, and the time of visiting customers varies from day to day).
(2) Emergency cases in routing (emergencies such as traffic accidents and traffic on some routes cause delays when competitors arrive at their customers, and in some cases, it will cause the competitor to change the sequence of customer visits).

According to the above patterns, the restrictions about a green VRP with a time window are as follows:

- The competitor's arrival time follows several scenarios that are likely to occur in each scenario.
- In each scenario, the reach time of the competitor is clear to customers.
- Each customer is only served by one vehicle.
- Vehicles capacity is limited and the same.
- Customer demand depends on the time of service. If the company begins to distribute a service to a customer later than another provider, the part of customer demand that is dependent on the time of service is lost.
- The number of vehicles is fixed and available in every day.
- Customer service time does not depend on the type of vehicle.

4.2. Mathematical model

4.2.1. Parameters and indices

In this sub-section, the parameters are denoted by

Ω	Set of scenarios, whose indices are s and s'
p_s	Probability of scenario s
N	Set of customers (i and j are the indices of customers)
nv	Set of vehicles (v is the index of the vehicle)
Q	Maximum capacity of vehicle
D	Set of days in the time horizon
Θ_i	Set of customer i's combination (ζ is its index)
$t_{i\tau}$	Arriving time to customer i on day τ
UT	Upper bound of travel time of vehicles
t_{ij}	Required time to travel from node i to node j
ts_i	Time service of the customer
M	Arbitrary large number (say $M >> \infty$).
$tu_{\tau is}$	Upper limit of competitor's arrival time to node i under scenario s on day τ
$tl_{\tau is}$	Lower limit of competitor's arrival time to node i under scenario s on day τ
d_i	Total demand of the i-th customer in each day of combination so that $d_i = d_{ti} + d_{ini}$;
d_{ti}	Time dependent demand of customer i.
d_{ini}	Time independent demand of customer i
c_{rr}	Coefficient of slip friction that is different for different road vehicles
D_d	Coefficient of air resistance
A	Front of the vehicle
p	Air density

G Earth's gravitational force
θ_{ij} Average gradient of the road from nodes i to j
a_{ij} Average acceleration of vehicle in meters per second squared from nodes i to j
w_j^v Load of vehicle v in node j
w Empty weight of vehicle v
v_{ij} Average Speed of vehicles from nodes i to j
d_{ij} Distance between nodes i and j

4.2.2. Variables

$x_{\tau ij}^v$ 1 if vehicle v travels through route (i, j) in day τ; 0, otherwise
$o_{i\zeta}$ 1 if ζ combination of customer i is selected; 0, otherwise
$H_{i\zeta i}$ 1 if day τ is in combination ζ of customer i; 0, otherwise
$E_{\tau i}$ 1 if vehicle service customer i in day τ; 0, otherwise
$\Phi_{s\tau i}$ 1 if the distributer company in scenario s, in day τ reaches the customer more quickly than the lower bound of the competitor; 0, otherwise
$\Gamma_{s\tau i}$ 1 if the distributer company in scenario s, in day τ reaches the customer during the competitors' time period; 0, otherwise
$\wedge_{s\tau i}$ Free if the distributer in scenario s, in day τ begins customer service after competitors' lower bound; 0, otherwise
$\phi_{sv\tau}$ Amount of violated Load from vehicle v capacity at day τ in scenario s
$Z_{s\tau i}$ 1 if the distributer company in scenario s, in day τ begins customer service after competitors' upper bound; 0, otherwise

4.3. Objective function of the model

Calculating expected value of obtained profit in this model, given the probable assumption that the competitor is not reaching the customer is explained in this section. If the distributor company begins to serve the customer in the scenario s before the earliest arrival time $tl_{\tau is}$ begins to serve the customer, the maximum profitable earnings include independent demand d_{ini} and time-dependent demand d_{ti}. Assuming that the competitor's service time in the scenario s is assumed to occur at the time interval $(tl_{\tau is}, tu_{\tau is})$ if the vehicle reaches the customer within the time frame discussed, the expected value is calculated to be faster than reaching the customer relative to the competitor in the scenario s and the amount of time-dependent demand according to relation (15).

$$\left(\frac{tu_{\tau is} - t_{i\tau}}{tu_{\tau is} - tl_{\tau is}}\right) d_{ti} \tag{15}$$

Eventually, if $tu_{\tau is}$ starts serving after the rival companies' shortest start-ups in the scenario s, they will not get any benefit from time-consuming demand and will only benefit from the supply of independent demandd_{ini}. The total value of the expected value of obtained profit $E(S)$ is calculated due to time-dependent demand in scenario s if the distributor at time t_i reaches customer i

$$E(s) = \sum_{i\in N}\sum_{\tau\in D}\sum_{s\in\Omega}\left(\Phi_{s\tau i} d_{ti} + \Gamma_{s\tau i}\left(\frac{tu_{\tau is} - t_{i\tau}}{tu_{\tau is} - tl_{\tau is}}\right) d_{ti}\right) \tag{16}$$

In Equation (16), $\Phi_{s\tau i}$ and $\Gamma_{s\tau i}$, in the scenario s get the values of zero and one according to the arrival time of the vehicle. Moreover, for proper value of these variables the following constraints have been added to the model for the proper placement of these variables.

$$(tu_{\tau is} - t_{i\tau}) + M(1 - \Gamma_{s\tau i}) \geq 0 \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{17}$$

$$(tl_{\tau is} - t_{i\tau}) + M(1 - \Phi_{s\tau i}) \geq 0 \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{18}$$

$$-(tl_{\tau is} - t_{i\tau}) + M\wedge_{s\tau i} \geq 0 \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{19}$$

$$(tu_{\tau is} - t_{i\tau}) + M(1 - Z_{s\tau i}) \geq 0 \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{20}$$

$$\Gamma_{s\tau i} + \wedge_{s\tau i} = 1 \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{21}$$

$$\Phi_{s\tau i} \leq E_{\tau i}, \Gamma_{s\tau i} \leq E_{\tau i} \quad \forall\tau \in D \quad i \in N \quad s \in \Omega \tag{22}$$

As the model specifies, the vehicle will reach the customer after the competitor's service interval. The variables $\Phi_{s\tau i}$ and $\Gamma_{s\tau i}$ are set to zero, and these values in the objective function will have expected value of obtained profit. If the customer's arrival time is within the range, $\Gamma_{s\tau i}$ value will be free, but as objective function is maximized, $\Gamma_{s\tau i}$ variable takes value one. In addition, at this time interval $\Phi_{s\tau i}$ is zero. If the arrival time of the vehicle to the customer is earlier than $tl_{\tau is}$, it will earn all profitable earnings, and at that time interval, $\Phi_{s\tau i}$ is free in value that take value one.

4.4. Mathematical model

The proposed mathematical model is defined as follows:

$$\begin{aligned}&Max \quad Z_1=\sum_{s\in\Omega}p_s\Bigg[\sum_{i\in N}\sum_{\tau\in D}\Bigg(\phi_{s\tau i}d_{ti}\\&+\Gamma_{s\tau i}\left(\frac{tu_{\tau is}-t_{i\tau}}{t}u_{\tau is}-tl_{\tau is}\right)d_{ti}\Bigg)\Bigg]+\\&\alpha\sum_{s\in\Omega}p_s\Bigg[\Bigg(\sum_{s'\in\Omega}p_{s'}(\sum_{i\in N}\sum_{\tau\in D}\Bigg(\phi_{s'\tau i}d_{ti}\\&+\Gamma_{s'\tau i}\left(\frac{tu_{\tau is'}-t_{i\tau}}{tu_{\tau is'}-tl_{\tau is'}}\right)d_{ti}\Bigg)-\Bigg[\sum_{i\in N}\sum_{\tau\in D}\Bigg(\Phi_{s\tau i}d_{ti}\\&+\Gamma_{s\tau i}\left(\frac{tu_{\tau is}-t_{i\tau}}{tu_{\tau is}-tl_{\tau is}}\right)d_{ti}\Bigg)\Bigg]\Bigg)+2\beta_s\Bigg]\\&-\gamma\sum_{s\in\Omega}\sum_{\tau\in D}\sum_{v\in nv}p_s\phi_{sv\tau}\end{aligned}\tag{23}$$

$$\begin{aligned}&Min \quad Z_2=\sum_{v\in nv}\sum_{j\in N}\sum_{i\in N}(a_{ij}\\&+g\sin\theta_{ij}+gc_{rr}\cos\theta_{ij})(W_v+W_j^v)d_{ij}x_{ij}^v\\&+\sum_{v\in nv}\sum_{j\in N}\sum_{i\in N}0.5c_dA\rho v_{ij}^2d_{ij}x_{ij}^v\end{aligned}$$

s.t:

$$\sum\zeta\in\Theta_i o_{i\zeta}=1 \quad i=1,\dots,N \tag{24}$$

$$E_{\tau i}=\sum_{\zeta\in\Theta_i}o_{i\zeta}.H_{\tau\zeta i} \quad i=1,\dots,N; \quad \tau=1,\dots,D \tag{25}$$

$$E_{\tau o}=1 \quad \tau=1,\dots,D \tag{26}$$

$$\begin{aligned}&\sum\nolimits_{v\in nv}x_{ij}^v\le\frac{E_{\tau i}+E_{\tau j}}{2}\\&\tau=1,\dots,D \quad i.j=0,1,\dots,N\end{aligned}\tag{27}$$

$$\begin{aligned}&\sum_{j\in N}x_{\tau ji}^v-\sum_{j\in N}x_{\tau ji}^v=0\tau=1,\dots,D\\&i=0,1,\dots,N \;\; v=1,\dots,nv\end{aligned}\tag{28}$$

$$\sum_{v\in nv}\sum_{i\in N}x_{\tau ji}^v-E_{\tau j}=0 \tag{29}$$

$$\sum_{i\in N}x_{\tau i0}^v=1 \tag{30}$$

$$\begin{aligned}&\sum_{i\in N}(d_i-d_{\tau i}Z_{s\tau i}\sum_{j\in N}x_{\tau ij}^v-\phi_{sv\tau}\le k_{nv}\tau=1,\dots,\\&D\,v=1,2,\dots,nv\,\forall s\in\Omega\end{aligned}\tag{31}$$

$$\begin{aligned}&\sum_{i\in N}ts_i\sum_{j\in N}x_{\tau ij}^v+\sum_{i\in N}\\&\sum_{j\in N}t_{ij}x_{\tau ij}^v\le UT\tau=1,\dots,D \;\; v=1,\dots,nv\end{aligned}\tag{32}$$

$$\begin{aligned}&t_{j\tau}=\sum\nolimits_{i\in N}t_{i\tau}\sum\nolimits_{v\in nv}x_{\tau ij}^v+\sum\nolimits_{i\in N}\\&\sum\nolimits_{v\in nv}(ts_i+t_{ij})x_{\tau ij}^v\tau=1,\dots,D \;\; j=1,\dots,n\end{aligned}\tag{33}$$

$$\begin{aligned}&\sum_{v\in nv}\sum_{i\in N}\sum_{\tau=D}x_{\tau ij}^v=0\tau=1,\dots,D\\&i=1,2,\dots,N \;\forall s\in\Omega\end{aligned}\tag{34}$$

$$\begin{aligned}&(tu_{\tau is}-t_{i\tau})+M(1-\Gamma_{s\tau i})\ge0\tau=1,\dots,D\\&i=1,2,\dots,N \;\forall s\in\Omega\end{aligned}\tag{35}$$

$$(tl_{\tau is}-t_{i\tau})+M(1-\Phi_{s\tau i})\ge0\forall\tau\in D \; i\in N \;\; s\in\Omega \tag{36}$$

$$-(tl_{\tau is}-t_{i\tau})+M(1-\Lambda_{s\tau i})\ge0\forall\tau\in D \; i\in N\; s\in\Omega \tag{37}$$

$$-(tu_{\tau is}-t_{i\tau})+M(1-Z_{s\tau i})\ge0\forall\tau\in D\, i\in N \;\; s\in\Omega \tag{38}$$

$$\Gamma_{s\tau i}+\Lambda_{s\tau i}=1\tau=1,\dots,D \;\; i=1,2,\dots,N \;\; s\in\Omega \tag{39}$$

$$\begin{aligned}&\Phi_{s\tau i}\le E_{\tau i},\tau_{s\tau i}\le E_{\tau i}\\&\tau=1,\dots,D \;\; i=1,2,\dots,N \;\; s\in\Omega\end{aligned}\tag{40}$$

$$\Bigg[\Bigg(\sum_{s'\in\Omega} p_{s'}\Bigg(\sum_{i\in N}\sum_{\tau\in D}\Bigg(\phi_{s'\tau i}\Gamma d_{ri} + \Gamma_{s'\tau i}$$

$$\left(\frac{t\mu_{\tau is'} - t_{i\tau}}{t}\mu_{\tau is'} - tl_{\tau is'} \geq 0\right)d_{ri}$$

$$-\Bigg[\sum_{i\in N}\sum_{\tau\in D}\Bigg(\sum_{\tau\in D}\Bigg(\Phi_{s\tau i}\Gamma d_{ri} + \Gamma_{s\tau i}$$

$$\left(\frac{t\mu_{\tau is} - t_{i\tau}}{t}\mu_{\tau is} - tl_{\tau is}\right)d_{ri}\Bigg)\Bigg]\Bigg) + \beta_s\Bigg] \forall s \in \Omega \quad (41)$$

$$\sum_{i,j\in Q}^{N} x^{v}_{\tau ij} \leq |Q| - 1$$

$$\forall Q \subseteq N \leq 2Q\ \tau = 1,..,D\ v = 1,\ldots,nv \quad (42)$$

$$x^{v}_{\tau ij}, o_{\phi i}, H_{\tau\phi i}, E_{\tau i}, \Phi_{s\tau i}, \Gamma_{s\tau i}\wedge_{s\tau i}, Z_{s\tau i},$$

$$\zeta_{s\tau i} \in [0,1]\tau = 1,\ldots,D \quad i,j = 1,2,\ldots,N \quad (43)$$

$$\forall s \in \Omega\ v = 1,\ldots,nv\ \phi = 1,\ldots,\phi$$

Equation (23) shows the objective functions of the problem, which consists of two parts, the first part is related to the hypothetical optimization of the profit earned by each scenario in terms of uncertainty, and the second part concerns the minimization of fuel consumption and carbon dioxide emissions in considering air resistance, road parameters, vehicle weight and vehicle load. Restriction (24) means each node should be selected in only one combination. Restriction (25) chooses the elected combination days for providing goods the nodes. Restriction (26) indicates that depot is accessible always. Restriction (27) indicate that only when in day τ of scenario s in customer's selected combinations i and j, route i–j has the selection possibility. Restriction (28) indicate that if vehicle v in day τ service costumer i, it should leave that day node i. Restriction (29) indicate providing demands of customers in selected combination days. Restriction (30) indicate that all the distributers always began servicing from depot and come back to depot. Restriction (31) in scenario s indicates the maximum capacity of the distributer. Restriction (32) indicates upper bond of time of distributer's accessibility. Restriction (33) shows required time for servicing vehicle v in day τ to node i. Constraint (34) removes the ring, and Constraints (35) to (40) optimize the profit. Constraint (41) indicates deviation between obtained profit in scenario s and expected value of obtained profit for all scenarios. Restriction (42) indicates the sub-tours, and restriction (43) indicates variables.

5. Problem-solving approach

According to the NP hardness of the VRP [33], in recent years, different meta-heuristic methods were developed in order to solve a VRP, like tabu search [34], simulated annealing (SA) [35] and particle swarm optimization [36]. Since the use of exact methods cannot be used for large-scale problems, for this purpose, for evaluating large-scale solutions by DE algorithm, PSO algorithm will be used and the results will be analyzed.

The DE and PSO algorithms are proposed for solving a problem and presented in Sub-sections 5.1 and 5.2, respectively. Finally in Sub-section 5.3, a creative solution representation is presented.

5.1. Differential evolution

In this sub-section, the DE is presented. It is a method for optimization problems proposed initially by Storn and Price [37]. DE uses operators (consisting of mutation, crossover and selection) to find the best solution from a randomly generated population. Though there are several likenesses among DE and other evolutionary algorithms, the search process in DE especially in direction and distance of the vectors in the present population to generate the next generation is significantly different. Because of their simplicity and suitable computational method, DE is one of the applicable algorithm in continues optimization problems [37, 38]. For better explanation, the phases of the DE algorithm are shown in Fig. 1.

5.2. Particle swarm optimization algorithm

At PSO, the best particle (solution) is obtained in collaboration with all the particles (solutions). Over the years, PSO has been used extensively in solving VRP problems. PSO algorithm is developed based on the rules governing the movement of bird and fish groups.

In this method, each particle tries to create a specific gap relative to the other particles and to improve it over time. To this end, each particle has a memory in which it has stored a history of its successful and successful past moves, and uses it to improve the problem's solutions. If d is the dimension of the

g is a generation counter and set $g = 0$
$c(g)$ is defined as a member of generation G
Initialize γ and p_r randomly as the control parameters;
$y_{ij}(g)$ Denotes to j-th $j \in \{1, \ldots, n_x\}$ gen of chromosome
Set of crossover points are represented as φ.
Initialize $y_i(g) \epsilon C(g)$ randomly as individual of $c(g = 0)$
For each individual, $y_i(g) \epsilon C(g)$ **do**
Calculate, $f(y_i(g))$
For each individual use mutation operator
Select $y_{i_2}(g)$ and $y_{i_3}(g)$ randomly such that $i, i_1, i_2 and\ i_3$ not equal.
Calculate $v_i(g) = y_i(t) + \gamma\ (y_{i_2}(g) - y_{i_2}(g))$
End (Mutation Operator)
For each individual, use the crossover operator to calculate an child, $y'_{ij}(g)$;
$$y'_{ij}(g) = \begin{cases} v_{ij}(g) & \text{if } j \in \varphi \\ y_{ij}(g) & \text{otherwise} \end{cases}$$
Select $\varphi \sim U(1, n_s)$
For *each* gen j of chromosome, $y_i(g)$
If $p_r > U(0,1)$ then
$\varphi \leftarrow \varphi \cup \{j\}$
End
End
End (Crossover Operator)
If $f(y_i(t))$ is less than $f(y'_i(g))$ **then** $y'_i(g) \in c(g + 1)$;
Else $y_i(g) \in c(g + 1)$
End (for).

Fig. 1. Pseudo code of the DE.

search space, then each particle in optimized PSO algorithm consists of three d-dimensional vectors. For particles, such as i-th, these three vectors are P_{id} as the current position of the particle, v_{id} as the particle velocity and p_{best} as the best position that particle has ever experienced.

Z-best is defined as the best place ever found by adjacent particles. PSO algorithm is something more than a particle set. None of the particles has the powers of solving any problem alone, and can solve a problem only when they interact with each other. In fact, in the literature of PSO algorithm, problem-solving process is considered as a social concept in which an optimal solution is obtained from the behavior of individual particles and their interactions.

Accordingly, if objective function of the problem is defined in the PSO as $p_{id}, v_{id}, p_{best}, z_{best}$ are updated in each step based on the following equations:

Create and initialize an N-dimensional particle and v_{id} in first iteration is zero;
Repeat
For each particle p_{id} , $i = 1, \ldots, N$ do
Calculate $f(p_{id})$
If $f(p_{id})$ is better than $f(p_{best,id})$ in history
Set current p_{id} as new $p_{best,id}$
End (if)
Select $z_{best,id}$ as the global best fitness value
If $f(p_{id})$ is better than $f(z_{best,id})$
Set current p_{id} as new $z_{best,id}$
End (if)
End (For)
For each particle p_{id} , $i = 1, \ldots, N$ do
Calculate v_{id} according equation (44)
Upgrade p_{id} according equation (45)
End (For)
Until the maximum iteration or stopping condition is not false
End

Fig. 2. Pseudo code of the PSO algorithm.

$$v_{id}(k+1) = qv_{id}(k) + l_1 rand_1 \left(p_{best,id} - p_{id}(k)\right) + l_2 rand_2 \left(z_{best,id} - p_{id}(k)\right) \tag{44}$$

where k represents the number of iterations in the algorithm, l_1 and l_2 are fixed numbers that are considered as particle accelerators. $rand_1$ and $rand_2$ are stochastic numbers in the interval zero and one. Q is defined as a weighting factor for controlling convergence in the PSO algorithm. $p_{best,id}$ Represents the vector of the best position (the best solution) experienced by the i-th particle in comparison with all the positions (solution) of the total iterations and Z-*best*, vector of the best particle (solution) among all particles are in the current iteration. According to the description, the vector of the new position of the particle is determined from the vector of the current position of the particle and its velocity vector based on the following relation.

$$p_{id}(k+1) = v_{id}(k+1) + p_{id}(k) \tag{45}$$

For each particle ($i = 1, \ldots, N$), velocity and position vectors in the next generation are separately updated by the two above equations. The steps to implement the PSO algorithm are shown in Fig. 2.

5.3. Solutions display

Applying the proposed algorithms to solve the proposed VRP is not very simple. Optimization of integer variables of the VRP by the proposed algorithms is

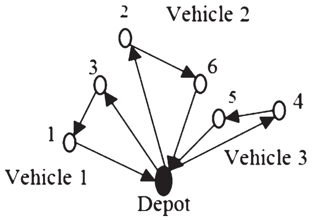

Fig. 3. Problem demonstration.

the most important problem. The customer variation based encoding scheme has been widely used for the VRP. Because of the continuous nature of the DE algorithm, the normal encoding of DE cannot be assumed for the VRP [37]. To utilize the DE algorithm to the proposed model is to get an appropriate mapping among nodes sequence and continuous vectors in DE.

In order to display solutions in the VRP continuously to solve the proposed model by the proposed algorithms, describe vector $y_i(g)$ at the dimension of n_x gene in a way that in which n_x represents customers number. For each gene $j \in \{1, ..., n_x\}$ random integer in interval $(0, nv + 1 - \varepsilon]$ represents the number of particular distributer to customer j and it will be specified if only all vehicles' capacity restrictions, servicing time and other issues are followed. Otherwise, less than amount one is specified to customer j. In order to define sequencing of customers passing vehicles for each gene in vector $y_i(g)$ in integer a decimal figure is added. By sorting the decimal figure aligned in ascending order the sequencing of routing vehicle of tour is considered. Figure 3 depicts a solution for instance

6. Computational results

To compare the performance of the DE with PSO samples will be examined in small and large sizes. In the first section, the problems in small sizes by DE and PSO will be solved and the obtained results will be compared with optimal answers of an exact solution. In the second section, large-scale problems will be solved by the proposed DE and PSO and performance and time computational will be examined.

To transform the objective functions to one section, a weight factor is used. That is, the weight objectives is considered 0.5 and as maximizing the objective functions. In order to do these experiments, Matlab software is used and to execute the programs, a core i7 computer with 2/3 GHZ ability and internal memory 2

Table 1
Sources of the nominal data

Parameter	Corresponding random distribution
N	~Uniform (6,18)
nv	~Uniform (2,4)
t_{ij}	~Uniform (1,9)
D_i	~Uniform (1,15)
k_{nv}	~Uniform (10,15)
Ω	~Uniform (2,5)
t_{uis}	~Uniform (15,60)
tl_{is}	~Uniform (10,40)

GB is used. By a trial and error method, $\gamma = 0.5$, $n_s = 200$ and $p_r = 0.6$ are calculated. In PSO, parameters $l_1 = l_2 = 1.52$ and $q = 0.78$ give the best values. The trial and error method is considered to ascertain the number of iterations in DE, and results present that 100 iterations have suitable performance to solve the proposed model.

6.1. Examining performance of DE and PSO in small sizes

In order to examine the DE performance, 10 sample tests are designed in small sizes and their obtained result are compared with the obtained result of solving model with the CPLEX method using GAMS 23.6 software. The required numerical data to create test in small scale has been considered as uniformly distributed that can be seen in Table 1.

For each test problem in small sizes, the test is run by the proposed DE and PSO algorithms. After 100 iterations, the obtained results are compared to the CPLEX in GAMS software that is shown in Table 2.

All the sample tests in PSO and except one in DE can analyze the solution without the gap and only the value of the gap in sample test 9 is 0.2%, in which DE cannot calculate the optimal solution. As it is known, on average, the time needed to solve the problem in the dimensions of the truncation for the exact resolution method is about 328 seconds and for solving the problem by the proposed algorithms is 19 seconds this information represents the computational velocity of the proposed algorithms. The maximum iterations for DE and PSO algorithms for problems 9 and 10, respectively, is 100 and 94 iterations.

6.2. Examining performance of proposed algorithm in large sizes

In order to evaluate the DE performance in large-scale problems, Solomon's test [39] cases

Table 2
Comparison of results in the CPLEX and DE and PSO algorithm in small tests

No. of sample	CPLEX		DE				PSO			
	(s) Time	OFV	Time (s)	OFV	I/N	Gap%	Time (S)	OFV	I/N	Gap%
1	125	7453	12	7453	26	0.0	11	7453	29	0.0
2	129	12589.3	13	12589.3	39	0.0	9	12589.3	42	0.0
3	178	16896.3	12	16896.3	25	0.0	15	16896.3	26	0.0
4	189	24789.8	18	24789.8	32	0.0	16	24789.8	35	0.0
5	202	26847.6	17	26847.6	13	0.0	16	26847.6	12	0.0
6	207	29568.5	19	29568.5	25	0.0	18	29568.5	25	0.0
7	369	27895.6	18	27895.6	27	0.0	21	27895.6	16	0.0
8	398	29586.2	24	29586.2	13	0.0	23	29586.2	28	0.0
9	589	32695.3	25	32609.8	100	0.2	25	32695.3	92	0.0
10	895	33587.6	26	33587.6	95	0.0	29	33587.6	94	0.0

OFV: Objective Value Function. I/N: Iteration Number.

Table 3
Comparison of the performance of the proposed DE and PSO

No. of problems	PSO			DE		
	CPU time (S)	OFV	Gap (%)	CPU time (Sec.)	OFV	Gap (%)
$C1$	695.3	1,254,575.4	0.83	849.50	1,234,536.6	1.63
$C2$	496.5	899,138.90	3.10	635.70	900,825.6	1.86
$R1$	764.2	1,643,955.20	3.37	894.50	1,653,387.7	2.78
$R2$	908.3	1,751,962.30	3.62	1002.1	1,802,149.6	0.73
$RC1$	842.7	2,099,435.70	2.88	1001.8	2,106,754.6	2.52
$RC2$	1079.9	1,995,428.60	4.81	1206.4	2,012,365.3	3.93
Mean	797.8	1,607,416.02	3.25	931.6	1,618,336.6	2.24

OFV: Objective Value Function.

are used containing 56 test samples. These test data are categorized into six groups, including $R1$, $R2$, $C1$, $C2$, $RC1 and RC2$.. In each category, there are 100 customers that are distributed uniformly in interval [0, 100]. In categories $R1$, $R2$ customers' place, servicing time and demands are selected randomly.

While in sample tests $C1$, $C2$ is represented to customers' places and time and customers' demands as a classified set, in sample tests $RC1$ and $RC2$ combination of data tests from uniform distribution and classified set is presented. Also because of the differences in the required data in the presented model, several assumptions in order to comply this model with sample tests are added. The number of required vehicles for each sample test is considered from the best calculated vehicles in subject. Upper limitt_{uis}, lower limit, tl_{is}, distributers' servicing time to customers are considered as a uniform function (420, 10) and uniform function (460, 30). Start time for all vehicles from a depot is eight in the morning and available time for all vehicles is 460 minutes.

In Table 3, the performance of the proposed strategies in large samples is shown in large sizes. The first and second columns indicate an average of the objective function and test run time by the DE and PSO.

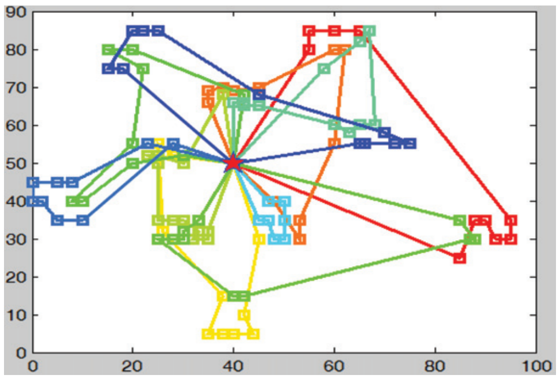

Fig. 4. Schematic routes for Problem RC9 solved by DE.

The third column represents the amount of test exact solution error with the obtained optimal solution in each sample by the DE algorithm.

The average gaps of the proposed PSO and DE algorithms for sets of problems are 3.25% and 2.24%, respectively. The maximum and minimum gaps are 4.81% and 0.73% in test samples RC2 and $R2$, respectively.

The average run times considered for sets of problems in PSO and DE iterations are 797.8, 931.6 seconds, respectably. It proves the capability of the proposed PSO in large-sized problems in terms of computational time. To solve medium and large-sized problems, the running time to find an optimal solution by exact methods due to the increasing complexity of the problem increases significantly. Because of this, the DE algorithm is used to find a near-optimal solution with the least gap. If the number of considered repetitions is large enough, the gap calculation error is nearly zero; however, calculation complexities will result in increasing run time by the DE algorithm. A test solution in the algorithm or test RC9 with DE is shown in Fig. 4.

7. Conclusion

In this paper, a scenario-based robust approach has been presented for a rival vehicle routing problem (VRP) to maximize the obtainable profit via rendering service ahead of other competitors and minimize the fuel consumption, the effects of greenhouse gas emissions and air pollution. The parameters of the starting service time of competing distributors have been considered to be uncertain. Therefore, the VRP with time windows was not capable of solving this kind of the problem under uncertainty. Then, considering the uncertain parameters, an uncertain model has been provided. Finally, to achieve a robust model by a scenario-based robust optimization method, a certain model has been presented. In order to achieve the optimal solution, the differential evolution (DE) and particle swarm optimization (PSO) algorithms have been used. Then, to evaluate the efficiency of the proposed algorithms, a number of sample problems have first been compared in small-sized problems and the results have been compared with an exact solution alg*orithm*. The results have indicated the proper performance of the proposed algorithms. To evaluate the algorithms performance for solving large-sized problems, a number of problems have been compared with the results of the DE algorithm with the PSO algorithm. The results have shown that the computational response of IDE with more computational time has been more suitable than PSO.

For future studies, considering the service time of competitors and the level of customer demands is suggested under uncertainty. In addition, considering the competition in other types of the VRP may be an attractive topic for the future study. Finally, solving the given problem in medium sizes by any exact method and comparing the results with those achived by the proposed algorithms can be suggested.

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