Report of Question 3

Zekun Zhang (zz364)

zekunzhang.1996@gmail.com

a)

1. Logically:

Just as the Question (a) described, we've already known that there must be a goat in one of these three places A, B or C. Moreover, we can define three events In A, In B and In C which means the goat is in location A, B or C. So, we can make a conclusion that the logical equation should be like that:

$$In\ A + In\ B + In\ C = \ U$$

Also, we can have more equations like:

$$if\ \ In\ A\ is\ True, In\ B\ and\ In\ C\ will\ be\ False:$$

$$In\ A = In\ A \land \neg In\ B \land \neg In\ C$$

$$if\ \ In\ B\ is\ True, In\ A\ and\ In\ C\ will\ be\ False:$$

$$InB = In\ B \land \neg In\ A \land \neg In\ C$$

$$if\ \ In\ C\ is\ True, In\ A\ and\ In\ B\ will\ be\ False:$$

$$In\ C = In\ C \land \neg In\ A \land \neg In\ B$$

2. Probabilistically:

As we've analyzed in the logical part, we can make a conclusion and give a probabilistic equation like:

$$P(In\ A) + (In\ B) + P(In\ C) = 1$$

Also, we can use the logical equations given before to transform our probabilistic equation. For instance, we can assume that the goat is just in the location A and we can calculate $P(In\ A)$ like that:

$$P(In\ A) = P(In\ A, \neg In\ B, \neg In\ C) = P(In\ A|\neg In\ B, \neg In\ C) \times \ P(\neg In\ B|\neg In\ C)\ \times\ P(\neg In\ C)$$

Hence, we can include three more events $G_A, G_B\ and\ \ G_C$, and they represent we guess specific place as the location where contains the goat. Then we would have equation like:

$$P(Success) = P(In\ A|G_A) \vee P(In\ B|G_B) \vee P(In\ C|G_C)$$

Based on the equations we've got, we can make sure that at this point when there's no any other additional information, the possibility for each location is just the same and then we would have the equation like:

$$P(In\ A) = P(In\ B) = P(In\ C) = \frac{1}{3}$$

So, for now, if we want to make a decision on which location we are going to search, we can just make this decision randomly because the same probability.

b)

Under the logical formulation, the easiest way is to enumerate all of the situation logically just as we've mentioned in the first part. Since we don't have any other additional information, the success cases are $In\ A$ based on $G_A,\ In\ B$ based on $G_B\ and\ In\ C$ based on G_C. So we can use X to represent the success cases as $In\ X$ based on G_X, which can be represented like:

$$Success = In\ X\ \ \wedge G_X$$

Then we can summarize that there's no difference what we are going to choose without any additional information under the logical formulation.

c)

Under the probabilistic formulation, as we've mentioned at the end of part (a), if we choose A as target, $P(G_A)$ would be 1 and $P(G_B), P(G_C)$ would be 0, then we can transform the equation $P(Success)$ with Bayesian theory into:

$$P(success) = \ P(In\ A|G_A) = \frac{P(G_A|In\ A)\ P(In\ A)}{P(G_A)}$$

Because we've already known that $P(In\ A)$ is equals to $\frac{1}{3}$, then we can have an equation like:

$$P(Success|G_A) = \ P(Success|G_B) = \ P(Success|G_C) = \frac{1}{3}$$

Based on the equations, we can make conclusion there's no difference what we are going to choose without any additional information under the probabilistic formulation.

d)

In this part, we can have new events N_X, which represent the goat is not in location X. Because CBMHBot will only look at the two locations we didn't pick, in another word which means the event N_X is triggered after we've already made a decision. So, we couldn't simply remove the case $In\ X$ and then we would just have equation like:

$$Success = (In\ A\ \wedge G_A \wedge (N_B \vee N_C)) \vee (In\ B\ \wedge G_B \wedge (N_A \vee N_C)) \vee (In\ C\ \wedge G_C \wedge (N_B \vee N_A))$$

e)

When we are going to update our probabilistic functions, we can assume a new function that named $P(Success_A)$ which means the possibility for success when we choose A. Then we could have a function like:

$$P(Success_A) = P(In\ A|G_A, N_B) + P(In\ A|G_A, N_C)$$

Moreover, we still draw a picture (Figure 1) of example to explain that the case when choose A and have N_B as additional information. It would help to understand the function above, and we would talk about it later when we are considering about re-selection.

f)

Under the logical formulation, we would like to introduce new events RG_A, RG_B and RG_C represented as re-select A, B or C. Firstly we assume that we choose A as the initial choice and the helper shows that there's no goat in B. Then we can modify our old equations like:

$$Success_A = In\ A\ \wedge G_A \wedge (N_B\ \vee N_C) \wedge RG_A$$

$$Success_B = In\ B\ \wedge G_A \wedge N_C \wedge RG_B$$

$$Success_C = In\ C\ \wedge G_A \wedge N_B \wedge RG_C$$

Then we can sum these equations up because $In\ A$ is equal to $In\ B$ and $In\ C$. And we know N_B and $Success_B$ would be False. So, we can give a new equation like:

$$Success = In\ A\ \wedge G_A \wedge N_B \wedge (RG_A + RG_C)$$

From the equation above, we would have a same chance to win no matter we choose A or C and we would compare this result with the probabilistic ones.

g)

Under the probabilistic formulation, there's some little difference from the logical formulation. And we use a figure below (Figure 1) to explain an example (If we first choose A and then the helper shows that there's no goat in B). Then we can give equations based on the previous equations:

$$P(In\ A) = P(In\ B) = P(In\ C) = \frac{1}{3}$$

$$P(In\ A|G_A, N_B) = P(In\ A|G_A, N_C) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$P(In\ B|G_A) = \frac{1}{3}$$

$$P(In\ C|G_A) = \frac{1}{3}$$

After we've already got the additional information N_B, we are going to change the $P(In\ A|G_A, N_C)$ to 0 and $P(In\ B|G_A)$ to 0. Then:

$$P(In\ A|G_A, N_B) + P(In\ C|G_A, N_B\ ) = 1$$

We've already know the original value of $P(In\ A|G_A, N_B)\ and\ P(In\ C|G_A, N_B\ )$ which are $\frac{1}{6}$ and $\frac{1}{3}$. Then we can give a new equation that:

$$P(Success_A) = P(In\ A|G_A, N_B) = \frac{1}{3}$$

$$P(Success_C) = P(In\ C|G_A, N_B\ ) = \frac{2}{3}$$

From the result above, we may know that for now (after choose A and know there's no goat in B), the better choice is to choose C as a re-selection. Due to the possibility change, the result becomes different to the logical ones.

h)

Under the logical formulation, as we've discussed previously, there's almost no difference between A and C after we choose A and know not B. And we can use the equations we've presented to prove that there's no difference to choose A or C. So, in the logical part, it doesn't matter what we are going to choose.

i)

Under the probabilistic formulation, as we've presented, we give the functions for the success possibility of choosing A which is $\frac{1}{3}$ and the success possibility of choosing C which is $\frac{2}{3}$. It's really easy to make a decision that choose the one different from your initial choice, which would double your success rate.

j)

Of course, the ProbabilisticGoatDiscoveryBot is much better. It really gives us a completely new way to make a rational decision. Human brains always try to work in the simplest way, but the real world always has complicated issues which need us to do calculation or some deep thinking. And this method could make things much easier to model by the probabilistic way.

Bonus:

For the biased one:

$$P(G_A, N_B | In\ A) = p$$

$$P(G_A, N_B) = P(G_A, N_B | In\ A) P(In\ A)$$

$$+P(G_A, N_B | In\ B) P(In\ B)$$

$$+P(G_A, N_B | In\ C) P(In\ C) = \frac{p+1}{3}$$

And for the previous function we would give that:

$$P(In\ A | G_A, N_B) + P(In\ C | G_A, N_B) = 1$$

So, we can make a conclusion that:

$$P(In\ A | G_A, N_B) = \frac{P(G_A, N_B | In\ A) P(In\ A)}{P(G_A, N_B)} = \frac{p}{p+1}$$

$$P(In\ C | G_A, N_B) = \frac{P(G_A, N_B | In\ C) P(In\ C)}{P(G_A, N_B)} = \frac{1}{p+1}$$

Because we know that p, which is a possibility for telling you there's no goat in B, is less than 1, it's really easy for us to figure out that $\frac{p}{p+1} < \frac{1}{p+1}$. Then absolutely, we should try to go on to choose C.

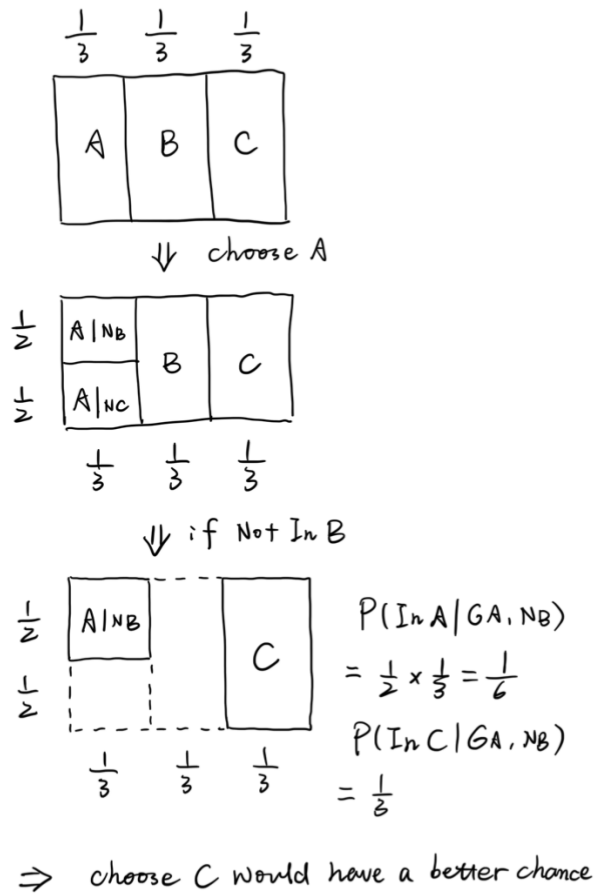

Figure 1 Example when choose A and have a N_B as additional information