Probability and Mathematical Statistics: Homework #2

Due on September 2, 1999 at 11:59am

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Problem 1 (Bertrands Box Paradox, #1)

Solution

according to the law of total probability that

$$P(B|E) = \sum_{i=1}^{n} P(B|A_i, E)P(A_i|E)$$

,then we can define that

1. A_1:(a) box A_2:(b) box A_3:(c)box
2. E:Happens to be a gold coin
3. B:the probability of the next coin drawn from the same box also being a gold coin.

first,using Bayes's rule,I can calculate$P(A_i|E) = \frac{P(E|A_i)\cdot P(A_i)}{P(E)}$,seperately,they are$\frac{2}{3}, 0, \frac{1}{3}$ so we can get the solution$1 \cdot \frac{2}{3} = \frac{2}{3} \approx 0.6777$

Problem 2 (BH CH0 #2)

Solution

(a) using Bayes's rule we first define:A:Alice actually send a 1 B:Bob receives a 1 A^c :Alice actually send a 0 then we use Bayers'rule,it is equal to calculate the probability of $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$,according to the law of total probability$P(B) = P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^C) = 0.475$,then $P(A|B) = \frac{18}{19} \approx 0.9474$

(b) we define the $A_i$in an increasing number order,A_1 :the event of receiving 000,A_2 :receiving 001,A_3 :receiving 010,A_4 :receiving 011,A_5 :receiving 100,A_6 :receiving 101,A_7:receiving 110,A_8 :receiving 111,B:Alice intened to send a 1,B^c :Bob intended to send a 0 I want to calculate $P(B|A_7) = \frac{P(A_7|B) \cdot P(B)}{P(A_7)}$ while $P(A_7) = P(A_7|B) \cdot P(B) + P(A_7|B^c) \cdot P(B^c) = 0.0416875$ so the result is $\approx$0.97151424287856

Problem 3 (#3)

Solution

(a) since those conditions are independent This is equal to calculate that $P(L|M_j)$ using Bayer's rule we can get that $= \frac{P(M_j|L)\cdot P(L)}{P(M_j)}$ then we use the law of probability to calculate $P(M_j) = P(M_j|L)\cdot P(L) + P(M_j|L^c)\cdot P(L^C)$ since $P(M_j^c|L^c) + P(M_j|L^c) = 1$,so I can get that$P(M_j|L^c) = 0.1$ so $P(M_j) = 0.9\cdot 0.1 + 0.1\cdot 0.9 = 0.18$ so the solution is $\frac{0.9\cdot 0.1}{0.18} = 0.5$.

(b) this condition is equal to calculate that

$$P(L|M_1, M_2) = \frac{P(M_1, M_2|L)\cdot P(L)}{P(M_1, M_2)} = \frac{0.9\cdot 0.9\cdot 0.1}{P(M_1, M_2|L)\cdot P(L) + P(M_1, M_2|L^c)\cdot P(L^c)}$$

$$= \frac{0.081}{0.081 + 0.01\cdot 0.9} = 0.9$$

(c) Yes,according to the conherence of the Bayer's rule,it isn't related with the order of condition given,first update M_1 and in the condition of M_1 happens then M_2 is the same as update the condition $M_1 \cap M_2$ at the same time.

Problem 4 (#4)

(a) the students who admit can be divided into three parts

(i) good at math,good at baseball

(ii) good at math,bad at baseball

(iii) bad at math,good at baseball

conditioning on students good at math get the iii out ,who are good at baseball,it is the same as kick out people who good at baseball from the total,decrease the percentage who good at baseball among students,
so the probability decrease,thus proved

(b) first to consider$P(A|B,C)$,according to the given condition that$A \cup B = C$and A ,B are independent,so $P(A|B,C) = P(A|B) = P(A)$ then considering($P(A|C) = \frac{P(A)}{P(C)}$)since P(C)¡1,so $P(A|C) > P(A) = P(A|B,C)$thus,proved.

Problem 5 (#5)

Solution

according to Bayer's rule ,we can easily get that

$$P(spam|W_1^c, W_2^c, \cdots, W_{22}^c, W_{23}, W_{24}^c, \cdots, W_{63}^c, W_{64}, W_{65}, W_{66}^c \cdots W_{100}^c)$$
$$= \frac{P(W_1^c, W_2^c, \cdots, W_{22}^c, W_{23}, W_{24}^c, W_{63}^c, W_{64}, W_{65}, W_{66}^c \cdots W_{100}^c|spam) \cdot P(spam)}{P(W_1^c, W_2^c, \cdots, W_{22}^c, W_{23}, W_{24}^c, \cdots, W_{63}^c, W_{64}, W_{65}, W_{66}^c \cdots W_{100}^c)}$$
$$= \frac{(1-p_1)(1-p_2)\cdots(1-p_{22})(p_{23})(1-p_{24})\cdots(1-p_{63})(p_{64})(p_{65})(1-p_{66})\cdots(1-p_{100}) \cdot p}{P(W_1^c, W_2^c, \cdots, W_{22}^c, W_{23}, W_{24}^c, W_{63}^c, W_{64}, W_{65}, W_{66}^c \cdots W_{100}^c|spam) \cdot P(spam) + P(W_1^c, W_2^c, \cdots |notspam)P(notspam)}$$

as the equation is to long we define that

1. $a = (1-p_1)(1-p_2)\cdots(1-p_{22})(p_{23})(1-p_{24})\cdots(1-p_{63})(p_{64})(p_{65})(1-p_{66})\cdots(1-p_{100})$
2. $b = (1-r_1)(1-r_2)\cdots(1-r_{22})(r_{23})(1-r_{24})\cdots(1-r_{63})(r_{64})(r_{65})(1-r_{66})\cdots(1-r_{100})$

so the answer is= $\frac{a \cdot p}{a \cdot p + b \cdot (1-p)}$