Probability and Mathematical Statistics: Homework #6

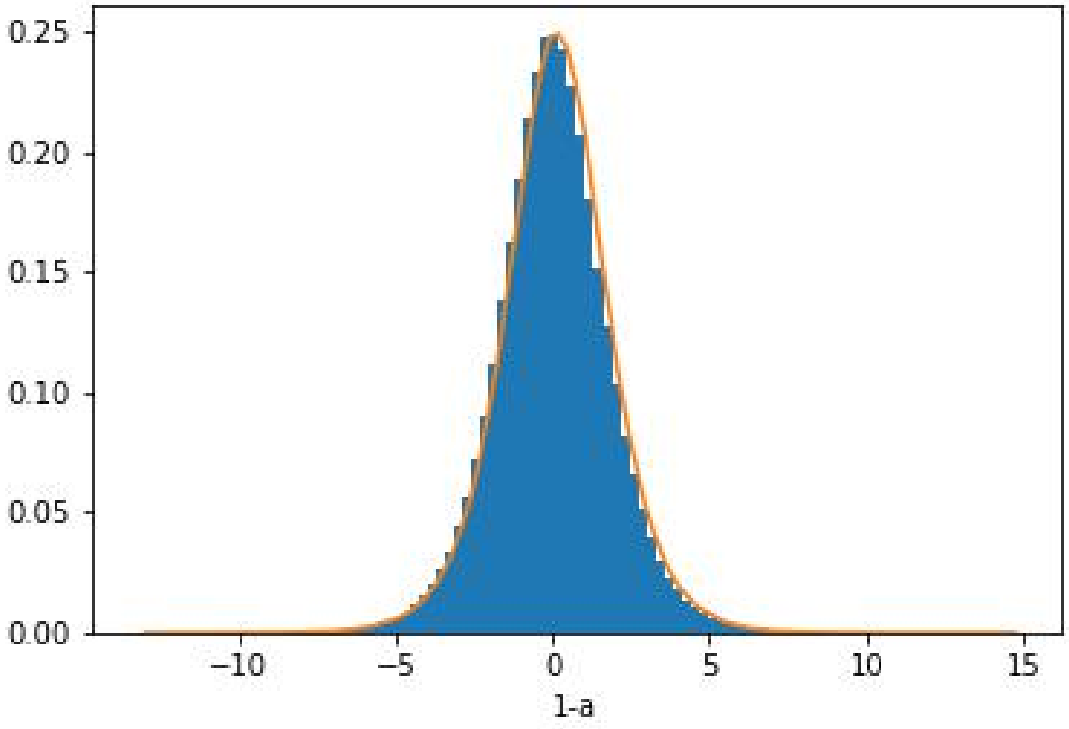
Due on September 18, 2022 at 11:59am

⓪

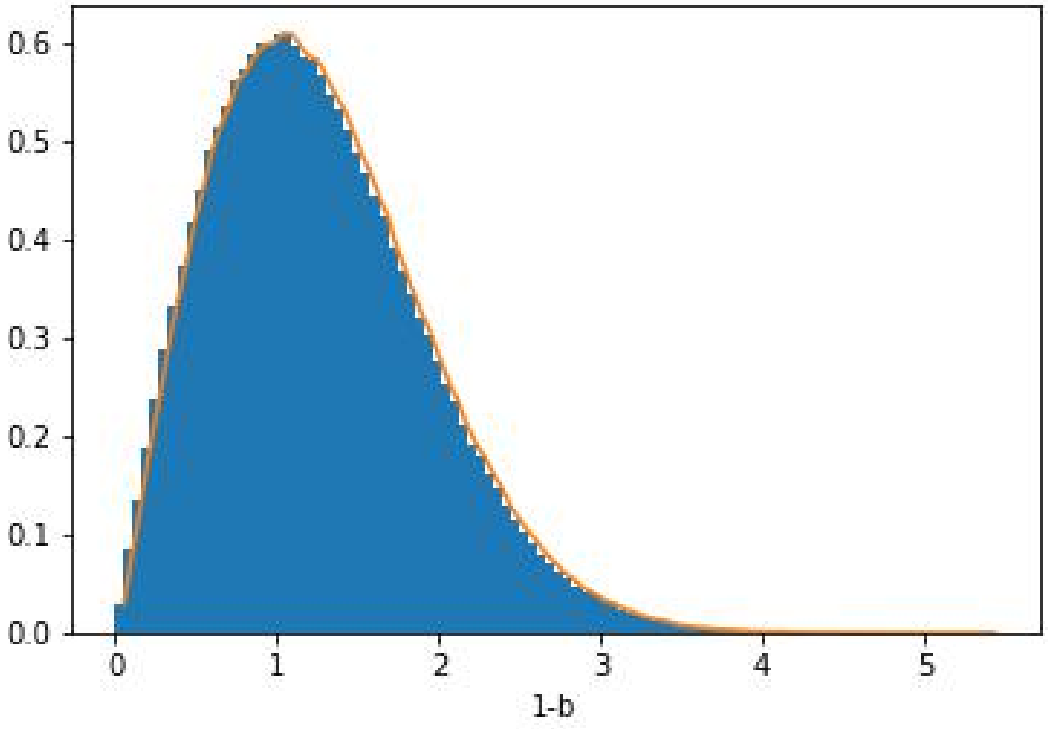
Name: **Zhu Zhelin**
Student ID: 2021533077

Problem 1 (mention the source of question, *e.g.*, BH CH0 #1)

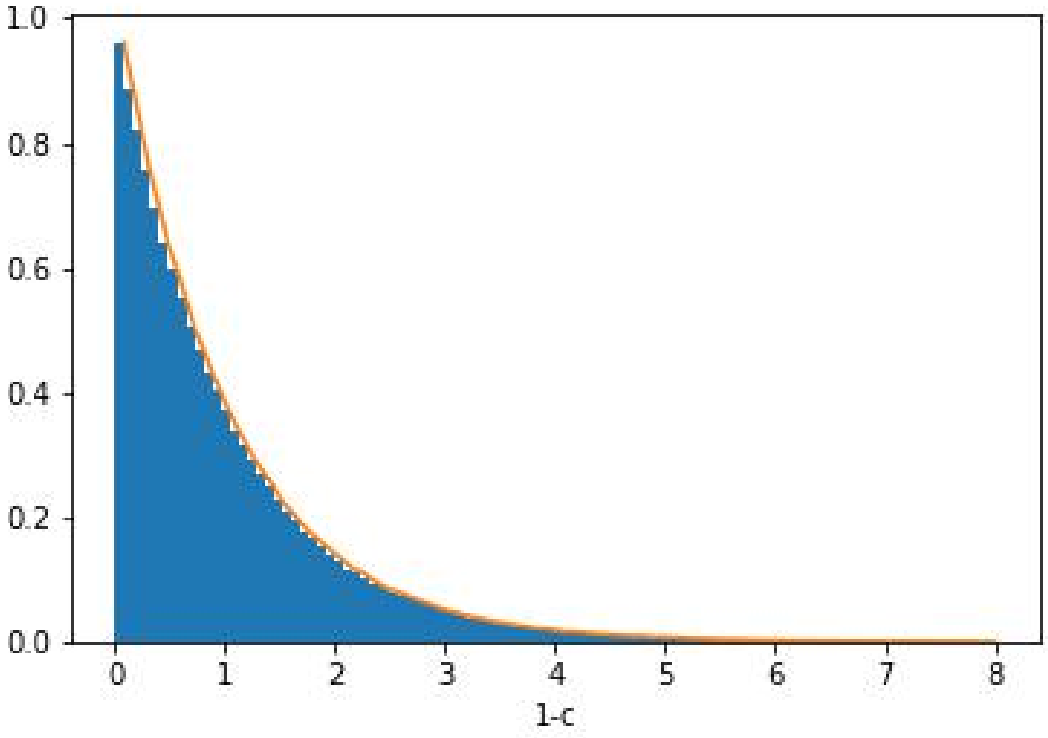
(a) using the inverse transform sampling I can get $F^{-1}(u) = \ln\left(\frac{u}{1-u}\right)$(samples:1000000)



(b) $F^{-1}(u) = \sqrt{-2\ln(1-u)}$(samples:1000000)

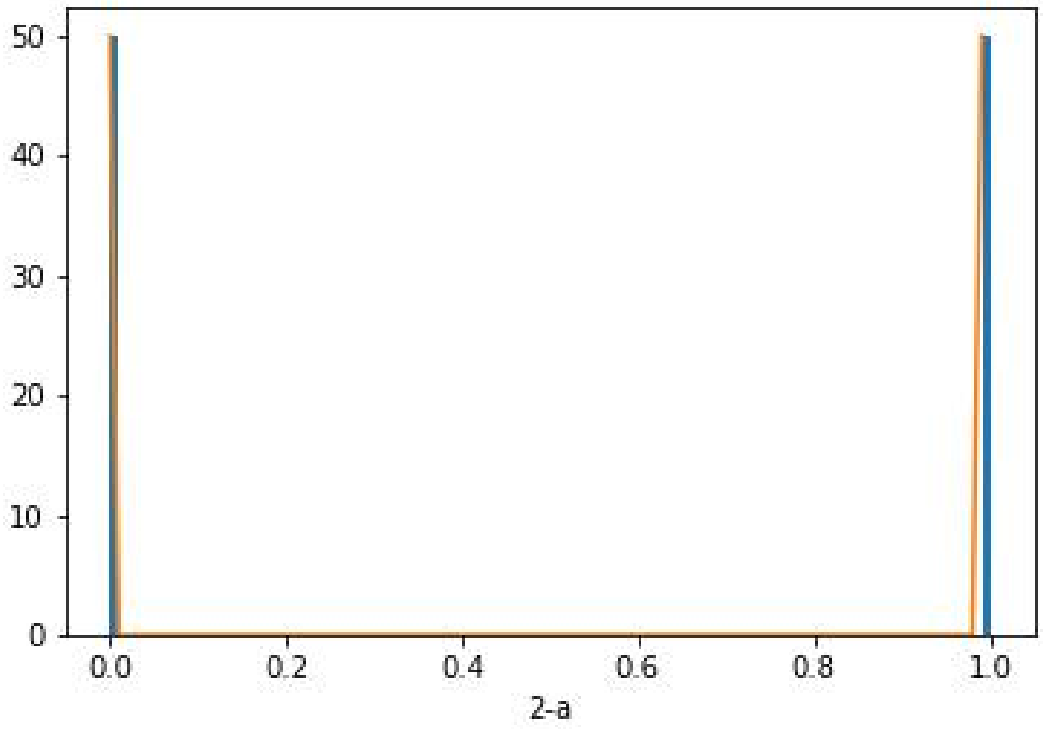


(c) $F^{-1}(u) = -\ln(1-u)$(samples:1000000)

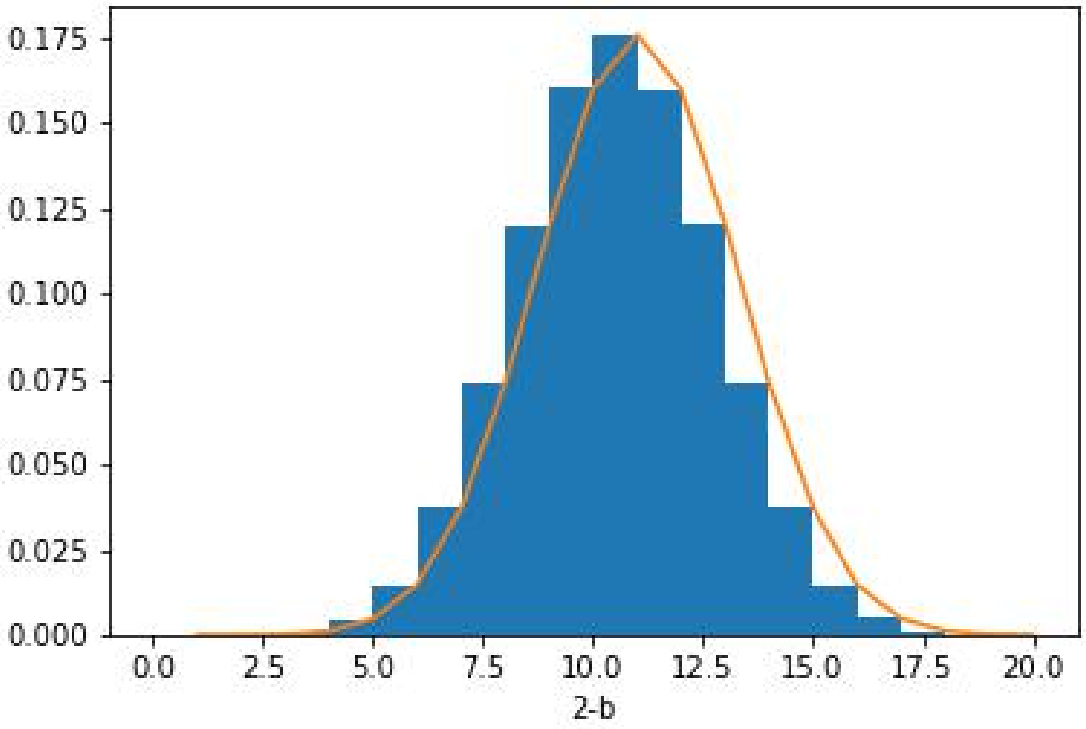


Problem 2 (BH CH0 #2)

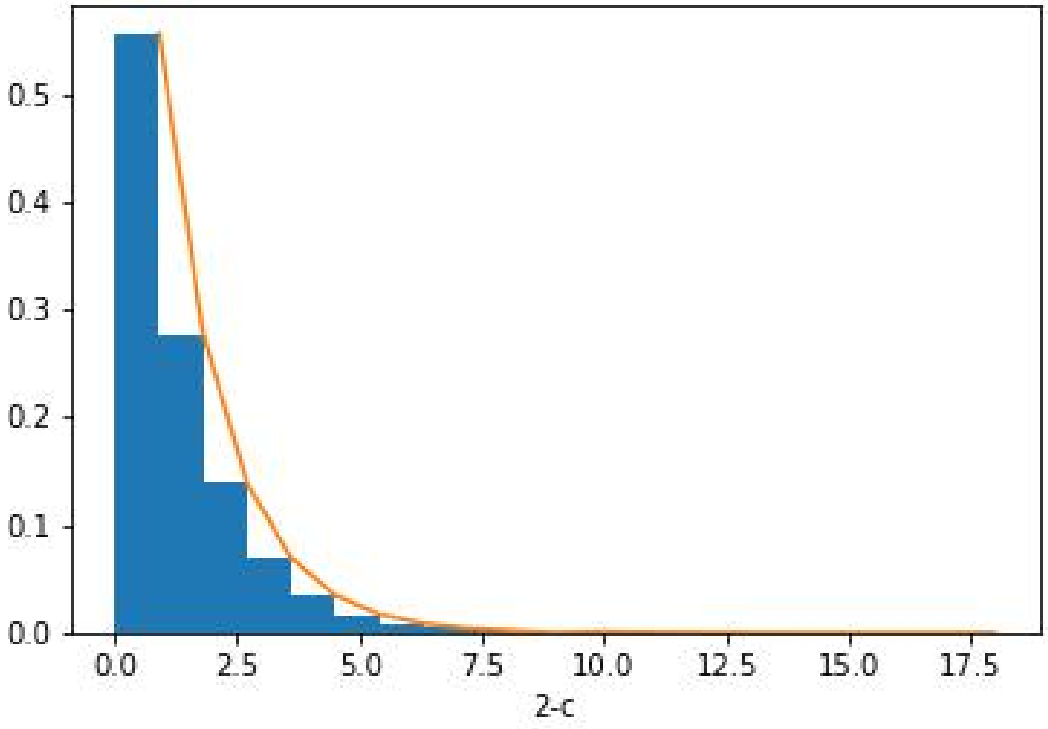
(a) I treat it as the trial to toss the coin,using the uniform random variables,larger than 0.5,then is success,otherwise it fails



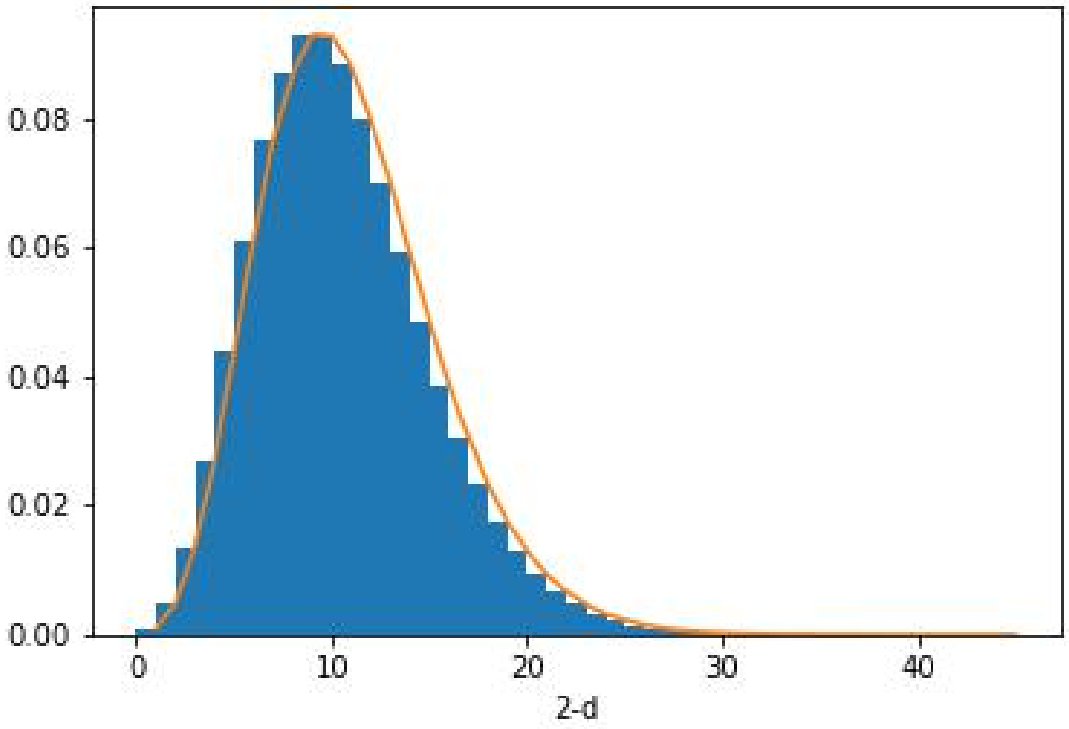
(b) I divide this into many bernolli trials,when trials success ,count+1,so i get the X as the number of success



(c) I will treat it as many bernoli trials,when fail,it will continue,otherwise,record the faiure



(d) I also treat it as mant bernoli trials,when success equal to 10,I will stop,and record the number of failure



Problem 3 (BH CH0 #3)

6 I do the trials by every time multiple a new random variable,if it doesn't satify the condition,then break,count how many variables it multiple,and record count=0,count=1,count=2 to calculate

(a) I calculate mean by adding total and divide sample number then I can get E(N)=1.0042

(b) I calculate it by adding the sum of squares of (N-E(N)),then divide by sample number I get Var(N)=0.9928

(c) $P(N=0)=0.3614, P(N=1)=0.3752, P(N=3)=0.185$

(d) since the mean and var is same,we can get N$\sim Pois(1)$

Problem 4 (BH CH0 #4)

1. in the sample(1000000),if you switch ,you will win 666058 times,if you don't switch you will win 333942 times,so switch better

2. in the sample(1000000),when n=4,if you choose strategy 1,you will win 24950($\approx \frac{1}{4}$),if you choose strategy 2,you will win 62481($\frac{2}{3}$ wins),if you choose strategy 3,you will win 75050 times($\frac{3}{4}$ wins),so strategy 3 are best,then 2,then 1, when n=100,samples=100000,strategy 3 is best(98953(0.98953 possibility win)),then strategy 2(63020(0.63020 win)),then strategy 1(1047(0.01047 possibility win))

Problem 5 (BH CH0 #5)

I perform the simulation,initially set the grid $n \times n$ by randomly set an unopened site as opened(labeled as one),and check if there is a path from top to bottom(using the depth first search) ,if there exist , record the number of opened sites,and add to the total count,finally divide by samplenumber and calculate the p

1. when n=20,I can get the p$\approx$ 0.5935
2. when n=50,I can get the p$\approx$ 0.5928
3. when n=100,I can get the p $\approx$ 0.594