5.8: Direct design method of Ragazzini

- An interesting design method *computes* $D(z)$ directly.
- Note: The closed-loop transfer function is

$$\begin{aligned} T(z) &= \frac{D(z)G(z)}{1+D(z)G(z)} \\ (1+D(z)G(z))T(z) &= D(z)G(z) \\ D(z)G(z)(T(z)-1) &= -T(z) \\ D(z) &= \frac{1}{G(z)}\frac{T(z)}{1-T(z)}. \end{aligned}$$

- Can control design be this simple?
- The problem is that this technique may ask for the impossible: (non-causal, unstable . . .).

Causality:

- If $D(z)$ is causal, then it has no poles at ∞. $D(\infty)$ must be finite or zero. Therefore, $T(z)$ must have enough zeros at ∞ to cancel out poles at ∞ from $\dfrac{1}{G(z)}$.
  - $T(z)$ must have a zero at infinity of the same order as the order of the zero of $G(z)$ at infinity.
- Put another way, the delay in $T(z)$ must be at least as long as the delay in $G(z)$.

Stability:

- If $G(z)$ has unstable poles, they cannot be canceled directly by $D(z)$ or there will be trouble!

- The characteristic equation of the closed-loop system is

$$1 + D(z)G(z) = 0$$

Let $D(z) = \dfrac{c(z)}{d(z)}$ and $G(z) = \dfrac{b(z)}{a(z)}$. Then

$$1 + \frac{c(z)}{d(z)}\frac{b(z)}{a(z)} = 0.$$

- Let the unstable pole in $G(z)$ be at α, so $a(z) = (z-\alpha)\overline{a}(z)$. To cancel it, $c(z) = (z-\alpha)\overline{c}(z)$, and

$$(z-\alpha)\overline{a}(z)d(z) + (z-\alpha)\overline{c}(z)b(z) = 0$$

$$(z-\alpha)[\overline{a}(z)d(z) + \overline{c}(z)b(z)] = 0.$$

- The unstable root is still a factor of the characteristic equation! (oops).
- Unstable poles must be canceled via the feedback mechanism. This imposes constraints on $T(z)$.
  - $[1 - T(z)]$ must contain as zeros all the poles of $G(z)$ outside the unit circle.
  - $T(z)$ must contain as zeros all the zeros of $G(z)$ outside the unit circle.

Steady-state accuracy:

- Assume that the system is to be of type-I with velocity constant K_v.
- Note:

$$E(z) = [1 - T(z)]R(z).$$

- We must have zero steady-state error to a step. Therefore

$$\lim_{z\to 1}(z-1)[1-T(z)]\frac{z}{(z-1)} = 0$$

or $T(1) = 1$.

- Must have $1/K_v$ error to a unit ramp. Therefore

$$\lim_{z\to 1}(z-1)[1-T(z)]\frac{Tz}{(z-1)^2}=\frac{1}{K_v}.$$

- Use l'hôpital's rule to evaluate:

$$-T\left.\frac{\mathrm{d}T(z)}{\mathrm{d}z}\right|_{z=1}=\frac{1}{K_v}.$$

EXAMPLE: Consider $T=1$ s, $G(z)=0.0484\left[\dfrac{z+0.9672}{(z-1)(z-0.9048)}\right]$.

- Want $T(z)$ to approximate $s^2+s+1=0$, or, converting to z-plane,

$$z^2-0.7859z+0.3679=0.$$

- So,

$$T(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots}{1-0.7859z^{-1}+0.3679z^{-2}}.$$

- Causality requires $T(z)|_{z=\infty}=0$ because $G(\infty)=0$, so $b_0=0$.
- Note that $G(z)$ has no poles *outside* the unit circle, so don't need to worry about that.
- Zero steady-state error to a step requires

$$T(1)=\frac{b_1+b_2+\cdots}{1-0.7859+0.3679}=1.$$

  Therefore,

$$b_1+b_2+\cdots=0.5820.$$

- Steady-state error of $1/K_v$ to a unit ramp (let $K_v=1$)

$$\frac{1}{K_v}=-\left.\frac{\mathrm{d}T(z)}{\mathrm{d}z}\right|_{z=1}$$

$$1=+\left.\frac{\mathrm{d}T(z)}{\mathrm{d}z^{-1}}\right|_{z=1}$$

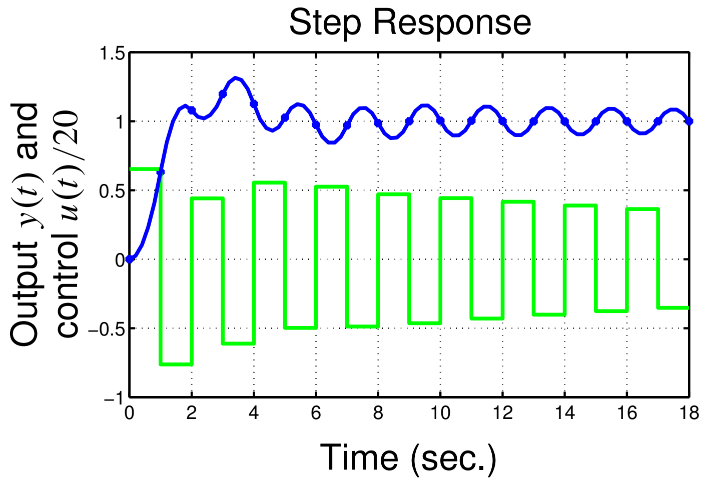
$$= \frac{(0.5820)[b_1 + 2b_2 + 3b_3 + \cdots] - (0.5820)[-0.7859 + 0.3679(2)]}{(0.5820)^2},$$

or, $b_1 + 2b_2 + 3b_3 + \cdots = 0.5318$.

- So, we have two constraints. We can satisfy these constraints with just b_1 and b_2:

$$b_1 = 0.6321$$

$$b_2 = -0.0501.$$



- Then,

$$T(z) = \frac{0.6321z - 0.0501}{z^2 - 0.7859z + 0.3679}.$$

and

$$D(z) = \frac{1}{G(z)} \frac{T(z)}{1 - T(z)} = 13.07 \frac{(z - 0.9048)}{(z + 0.9672)} \frac{(z - 0.07932)}{(z - 0.418)}.$$

- Note that $T(z)$ has two well damped poles. Why the oscillation?

$$\frac{U(z)}{R(z)} = \frac{D(z)}{1 + D(z)G(z)} = \frac{T(z)}{G(z)}$$

$$= 13.07 \frac{(z - 0.0793)}{(z^2 - 0.7859z + 0.3679)} \frac{(z - 1)(z - 0.9048)}{(z + 0.9672)}.$$

- This has a poorly damped pole at -0.9672. Aha! This corresponds to the zero of $G(z)$ at -0.9672.

- A solution: Use a b_3 term in $T(z)$ and add the constraint that $T(z)|_{z=-0.9672} = 0$.