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A Modified Waveform Design for Radar-Communication Integration Based on

LFM-CPM 连续相位调制

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//涂: 设备小型化; 频谱利用率高

Abstract—The integration of radar and communication has attracted general interest with its advantage of equipment miniaturizing and high efficiency of spectrum, especially in intelligent transportation systems because of the need for transmitting and detecting simultaneously and the lack of room for various devices. Among various integration schemes, the integrated waveform which realizes transmitting and detection at the same time is a real integration with high efficiency and no interference. In this paper, we proposed a modified three-section integrated waveform based on linear frequency modulation and continuous phase modulation (LFM-CPM). Short Time Fourier Transform STFT) is used for time-frequency analysis and the modification of the mapping codebook of the communication symbols restrains the spectrum of the integrated waveform within the original bandwidth of the radar system. Simulation results show that the modified waveform holds good bit error rate (BER) performance even when there is strong interference beside the original bandwidth of the radar system, while the waveform without modification can hardly complete the transmission with high BER.修正,改进 突出伏勒: BERJI.

I. INTRODUCTION

The integration of radar and communication has attracted widespread interest in recent years [1]-[5], with the advantage of reducing the interference of radar and communication systems saving resources of hardware and promoting spectrum efficiency of the whole system, especially in intelligent transportation systems where the detecting and tracking of cars and barriers and the transmitting of information among the devices in the same network are needed at the same time. Sharing hardware [2] is a simple way of integration, that is, transmitting radar and communication signals in different time slots, subarrays of the antenna, or in neighboring frequency channels from the same device. However, interference of radar and communication signals can hardly be avoided in these schemes and still double resources for radar and communication respectively is needed. Thus integrated waveform which can realize detection and information transmission at the same time is now a focus of research. In these researches, orthogonal frequency division multiplexing (OFDM) signal which has been widely used for communication is

first used as an integration waveform [3]–[5]. However, the peak average power ratio (PAPR) of OFDM is too high that serious distortion is inevitable with the nonlinear region of the high-power amplifier for radar's detection and tracking. Under this consideration, a waveform with constant envelope will be more appropriate, for example the continuous phase modulation (CPM) waveform. As for radar's detection and tracking, the linear frequency modulation (LFM) is widely used and many sophisticated signal processing schemes are proposed and realized [6]. As a result, integrated waveform based on LFM-CPM can both make use of the radars signal processing schemes and avoid the distortion caused by the nonlinearity of the amplifier.

However, when loaded with communication signals, the spectrum of the integrated signal will be widened and therefore exceed the bandwidth of the original radar. As we know, the spectrum resource is of great shortage, and the spectrum outside the original bandwidth may already be dealt out to other users. Meanwhile, with the electromagnetism environment getting more and more complicate, especially in city road with a huge amount of cars and electronic devices, there may exist serious interference outside the original bandwidth. As a result, both the performance of communication and detection will be degraded. Therefore, we must take steps to restrict the spectrum of the integrated signal so that the interference outside the original bandwidth can be handled with the existing methods like adaptive filtering and so on.

There have been detailed research of the spectrum of CPM signals for communication [7]. However, for integrated waveform based on LFM-CPM, there is few systematic analysis and design. In [8], the author proposed an integrated waveform based on LFM-MSK, a particular case of LFM-CPM, and worked out a modification to avoid the extension of spectrum. However, the low spectrum efficiency of MSK can't meet the growing need for data transmission, and the partial response(correlation length L > 1) provides an opportunity to promote the BER performance in potential. In addition, for CPM in general with various parameters, the spectrum will

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be more complicated. So a general study for LFM-CPM is in urgent need.

In the following of this paper firstly in section II, we take CPM with rectangular shaping pulse for example and give a description of the integrated LFM-CPM waveform. Then in section III, a general analysis of the spectrum of the integrated waveform is showed and a modified three-section waveform in general and in advance are proposed to restrain the spectrum within the original bandwidth of LFM through a limit of the mapping codebook at the beginning and the end of the communication symbol sequence. Finally in section IV, simulation results show that with serious interference outside the original bandwidth, the bit error rate (BER) of the unmodified integrated waveform remains at around 10⁻¹ and can't complete the transmission of information, while the BER performance of the modified waveform we proposed is almost the same as the one without interference.

II. SYSTEM MODEL

A. Continuous Phase Modulation CPM,连续相位调制

CPM is widely used in communication with high spectrum efficiency [7], and its baseband modulator is equivalent to a convolutional encoder, so that decoding algorithm like Viterbi can be used to improve BER performance [9]. For a bipolar modulation sequence $a = [a_0, a_1, a_2, \ldots, a_{N-1}]$, where $a_i \in \{-(M-1), -(M-3), \ldots, (M-3), (M-1)\}$ and M is the modulation order. The modulated CPM sequence is,

$$\begin{array}{cccc}
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\text{(2)} \\
\text{(2)} \\
\text{(2)} \\
\text{(2)} \\
\text{(3)} \\
\text{(4)} \\
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\text{(7)} \\
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\text{(8)} \\
\text$$

To facilitate the following discussion, here we let A=1.

 $\phi(t,a)$ is the continuous phase coding sequence, and q(t) is decided by the integral of the shaping pulse q(t),

$$q(t) = \int_{-\infty}^{t} g(\tau)d\tau$$

There are various g(t) used for CPM, for example, the rectangular shaping pulse, the raised cosine, the Gaussian, and so on. The pulse lasts for LT, where T is the period of a communication symbol, and L is the correlation length. The spectrum of the CPM with different parameters varies a lot. In this paper, we take the rectangular shaping pulse for example. So g(t) is,

$$g(t) = \begin{cases} \frac{1}{2LT} & 0 \le t \le LT\\ 0 & else \end{cases} \tag{4}$$

Now, for $(k-1)T \le t \le kT$, the phase coding is,

$$\phi_{k}(t,a) = \pi h \left(\sum_{i=k-L+1}^{k} a_{i} \frac{t}{LT} + \sum_{i=0}^{k-L} a_{i} \right)$$

$$(5)$$

$$f(b)$$

$$\sum_{i=0}^{k} (i) Q(t-i) T = Q(b-i) T =$$

B. Linear Frequency Modulation 线性频率调制

LFM is widely used in radar [6]. It can be expressed as,

$$S_{\text{LFM}} = A \cdot \exp\left(j2\pi \left(f_c + \frac{\mu}{2}t\right)t\right)$$
 (6)

Similarly, let A=1 to facilitate the following discussion. For a LFM pulse lasts for T_p , we define its bandwidth $B_0=\mu T_p$, starting frequency f_c , where almost all the spectrum component is included in $[f_c,f_o+B_0]$, and therefore its bandwidth-delay product is $D=B_0T_p$.

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C. Integrated Waveform

The integrated waveform S(t) can be built by loading CPM signal on the LFM carrier.

$$S(t) = \sum_{k=0}^{N-1} rect\left(\frac{t-kT}{T}\right) \exp\left(j2\pi\left(f_ct + \frac{\mu}{2}t^2\right)\right) \cdot \exp\left(j\pi h\left(\sum_{i=k-L+1}^{k} a_i \frac{t}{LT} + \sum_{i=0}^{k-L} a_i\right)\right)$$
(7)
$$rect(x) = \begin{cases} 1, & 0 < x < 1\\ 0, & else \end{cases}$$
 (8)

where N is the number of the communication symbols in one pulse, and T is the period of a communication symbol. So that the pulse width is $T_p = NT$

There are several papers [8], [10] studied the signal processing schemes for similar waveforms like LFM-MSK. For radars detection and tracking, since the transmitter and the receiver is at the same terminal, so the correlation between the transmitted signal and the echo signal gives the image of the target objects. For communication, we can get the baseband signal by multiplexing the conjugate LFM signal. Then the signal processing, like synchronization, equalization, decoding, can be carried out like the regular CPM signal.

III. MODIFIED INTEGRATION WAVEFORM

A. Spectrum Analysis

The short-time Fourier transform (STFT) is widely used for time-frequency analysis of a non-stationary signal by dividing it into several small slices with a certain window function [11]. Apply STFT to LFM-CPM waveform,

$$STFT(t,f) = \int_{-\infty}^{+\infty} S(\tau) \underbrace{g_w^*(\tau - t)}_{exp(-j2\pi f\tau)} exp(-j2\pi f\tau) d\tau \quad (9)$$

where $g_w(t)$ is the window function for STFT. Different $g_w(t)$ leads to different distinguishability of time and frequency, respectively. In this paper, we use Gaussian function, that is,

$$g_w(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{10}$$

) 9 (bit) = (0, R < 0)

(0, 14)分成产部分: (0, 12-1), (12-11), (12-11), (12-11)

$$STFT(t,f) = A_N \Phi_N(t,f) \cdot \tag{11}$$

$$\sum_{k=0}^{N-1} rect\left(\frac{t-kT}{T}\right) A_k(t,f) \Phi_k(t,f)$$

$$A_N = \frac{2\sigma^2 \sqrt{\pi}}{\sqrt{(2\pi\sigma^2\mu)^2 + 1}} \tag{12}$$

$$\Phi_N(t,f) = \exp\left(j\pi\left(2\left(f_c - f\right)t + \mu t^2\right)\right) \tag{13}$$

$$A_k(t, f) = \exp(B_k(t, f))$$
 (14)
 $B_k(t, f) = -\frac{2\pi^2 \sigma^2}{(2\pi\mu\sigma^2)^2 + 1}$ (15)

$$B_{k}(t,f) = -\frac{2\pi \sigma^{2}}{(2\pi\mu\sigma^{2})^{2} + 1} \cdot (15)$$

$$\left(f - f_{c} - \mu t - \frac{h}{2LT} \sum_{i=k-L+1}^{k} a_{i}\right)^{2}$$

$$\Phi_{k}(t,f) = \exp\left(j\pi \frac{h}{LT} \sum_{i=k-L+1}^{k} a_{i} \frac{t - iT}{T}\right) \cdot (16)$$

$$\exp\left(2\pi \sum_{i=k-L+1}^{k-L} a_{i} - \mu B_{k}\right)$$

And the power spectrum is,

$$\begin{pmatrix} S_{P}(t,f) = |STFT(t,f)|^{2} \\ = |A_{N}|^{2} \left| \sum_{k=0}^{N-1} rect\left(\frac{t-kT}{T}\right) A_{k}(t,f) \right|^{2} \end{pmatrix}$$
(17)

For a certain t, there is only one k satisfies $rect\left(\frac{t-kT}{T}\right) \neq 0$, so equation (17) can be simplified,

$$S_P(t,f) = |A_N|^2 \exp\left(-\frac{(2\pi\sigma)^2 (f - f_0(t))^2}{(2\pi\mu\sigma^2)^2 + 1}\right)$$
 (18)

$$f_0(t) = f_c + \mu t + \frac{h}{2LT} \sum_{k=0}^{N-1} rect\left(\frac{t - kT}{T}\right) \sum_{i=k-L+1}^{k} a_i$$
 (19)

From equation (18) we can see that the power spectrum follows a negative square exponential distribution with the maximum at $f_0(t)$, which means for a certain t, most power will converge around $f_0(t)$. As a result, in this paper, we take $f_0(t)$ as the main major component of the power spectrum for further discussion.

B. (Three-section integrated waveform in general

To restrain the spectrum within $[f_c,f_c+B]$, firstly, $f_0\left(t\right)\geq f_c$ should always hold for any $k\in[0,1,...,N-1]$. As a result, equation (20) should always hold,

$$\mu kT + \frac{h}{2LT} \sum_{i=k-L+1}^{k} a_i \ge 0$$
(20)

Assume $\mu > 0$ here and in the following discussion, because if $\mu < 0$, symmetrical conclusion can be obtained. In this condition, When $a_i \in \{1,3,...,(M-3),(M-1)\}$, it is obviously that equation (20) holds. However, the communication $\{(M-1), (M-1)\}$

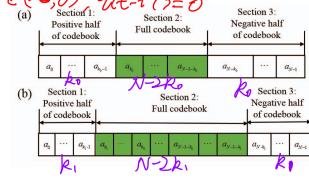


Fig. 1. (a). The modified three-section integrated waveform in general (modified k_0). (b). The modified three-section integrated waveform in advance (modified k_1). 提前,预先

symbols are in random, so we should take the worst case into consideration that is, $a_i = -(M-1)$, so equation (20) is,

$$\mu kT - \frac{h}{2LT}L(M-1) \ge 0$$
(21)

We get,

$$k \ge \frac{h}{2\mu T^2} (M - 1) = \frac{h(M - 1)}{2} \frac{N}{BT_P} N$$
 (22)

Let $k_0 = \left\lceil \frac{h}{2\mu T^2} \left(M - 1 \right) \right\rceil$, where $\lceil x \rceil$ means the minimum integer that is no less than x. In a word, when $k \ge k_0$, equation (20) can always hold, while when $k < k_0$, a_i can only fetch its values from $\{1, 3, ..., (M-3), (M-1)\}$, which means the scale of the codebook for symbol mapping should be halved.

With similar analysis we can know that, when $k \leq N - 1 - k_0$, $f_0(t) \leq f_c + B$ will always hold, while when $k \geq N - k_0, a_i$ should be limited in $\{-(M-1), -(M-3), ..., -1\}$.

Until now, we obtain a modified three-section waveform, whose major component of the power spectrum is limited within the original LFM waveform used for radar. We name it 'modified k_0 ' for convenience in the following discussion. A schematic diagram is showed in fig. 1 (a).

C. Three-section integrated waveform in advance

As we know, one of the advantage of CPM is that for CPM with partial response (correlation length L>1), the coding system has a memory of the preceding L-1 symbols. When the parameters of the waveform satisfy some condition, the length of k_0 can be further reduced. In another word, in certain conditions, there exists $k_1=k_0-L+1$, when $k_1\leq k\leq N-1-k_1$, the whole codebook can be used for symbol mapping, while when $k\leq k_1-1$ and $k\geq N-k_1$, only the positive half and the negative half of the codebook can be used. In this way, the efficiency of communication can be promoted.

Firstly, we will state that the length of k_0 can be reduced to $k_1 = k_0 - L + 1$ at most. If the modified length $k_2 < k_0 - L + 1$, then for $(k_0 - 1)$ $T \le t < k_0 T$, all the symbols in $\sum_{i=k-L+1}^k a_i$ will fetch their values in the whole codebook. In the worst

 $\label{thm:condition} TABLE\ I$ The parameters of the LFM-CPM waveform in simulation

Parameter	Value
Bandwidth	B ₀ =1MHz
Bandwidth-delay product	$B_0 \cdot T = 1024$ (= Lawrence
Number of communication symbols	N=1024 75= N7
in one pulse	N=1024
CPM modulation order M	M=4
CPM modulation index h	h=1/4
CPM shaping pulse	REC
correlation length L	L=4
interference strength	-3dB
bandwidth of the interference	$[f_c + 1.05B_0, f_c + 1.25B_0]$

case, $a_i = -(M-1)$ for any $k_0 - L \le i \le k_0 - 1$. From equation (19) we know that $f_0(t) < 0$, which means the spectrum will exceed the lower border f_c .

Then, we discuss the conditions under which the length of k_0 can be reduced to $k_1=k_0-L+1$. In this case, we should guarantee that for $(k_0-x)\,T\le t<(k_0-x+1)\,T$, where x is an integer and $1\le x\le L-1$, there is $f_0\,(t)\ge 0$ always hold. The key point is to discuss $\sum\limits_{i=k-L+1}^k a_i$. For each x, there are x symbols in section 1 and can only be mapped with the positive half of the codebook, of which the worst case is +1. The other L-x symbols are in section 2, and are mapped with the whole codebook, the worst case of which is -(M-1). That is,

$$\mu(k_0 - x)T + \frac{h}{2LT}(1 \cdot x - (M - 1)(L - x)) \ge 0$$
 (23)

Equation (23) always hold for any $1 \le x \le L - 1$. And we get,

$$\frac{h}{2L}\frac{N}{BT_P}N \ge 1 \tag{24}$$

Under this condition, symmetrically deduction will show that $f_0(t) \leq B$ will also hold.

Usually, h/2L is no less than 1/100 in magnitude, the bandwidth-delay product BT_P is usually from tens to several hundreds, or even thousands [6]. The number of communication symbols N is designed by the user, for example, let N equal to BT_P in magnitude. That is to say, the condition proposed in equation (24) is easy to satisfy in most cases, and k_0 can be reduced to $k_1 = k_0 - L + 1$ to improve the communication efficiency. We name this new three-section waveform 'modified k_1 '. A schematic diagram is showed in fig. 1 (b).

IV. SIMULATION AND ANALYSIS

The parameters in simulation is listed in Table. I, and the spectrum of LFM and the integrated waveforms is showed in fig. 2. From the figure we can see that the spectrum of the simple LFM-CPM integrated waveform has extend to about $2B_0$, while the spectrum of the modified waveform is still restrained within B_0 .

When there is strong and random interference outside the original bandwidth, bandstop filtering is an effective way of anti-jamming. The interference in this simulation is white 抗干扰

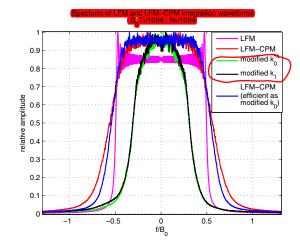


Fig. 2. The spectrum of LFM and the integrated waveforms.

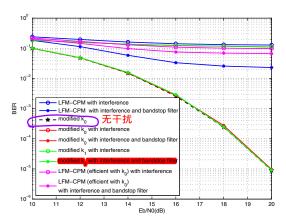


Fig. 3. BER performance of the integrated LFM-CPM waveforms with and without modification with strong interference outside the original bandwidth.

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Gaussian noise whose power spectral density is about -3dB of the peak of the power spectral density of the integrated waveform. The integrated signal goes through an AWGN channel with no frequency offset and perfect synchronization. The Viterbi algorithm is used in decoding. The BER performance of the waveforms with and without bandstop filtering is showed in fig. 3, from where we can see that, the interference seriously influences the BER performance, making the BER of all the waveforms around 10^{-1} . With bandstop filtering, the modified k_0 and k_1 waveforms can hold a good BER performance almost the same as the one without inteference, while the waveform without modification fails because of the expansion of spectrum.

The modified waveform sacrifices some efficiency of communication. Before the modification, the number of bits transmitted by one pulse is,

$$N_b = N \log_2 M \tag{25}$$

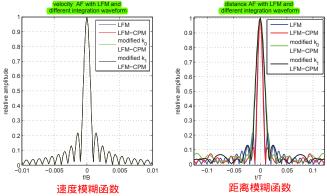


Fig. 4. (a). The velocity ambiguity function of LFM and the integrated LFM-CPM waveforms. (b). The distance ambiguity function of LFM and the integrated LFM-CPM waveforms.

And after the modification,
$$N_{b0} = (N - 2k_0) \log_2 M + 2k_0 (\log_2 M)/2 \\
= (N - k_0) \log_2 M \\
N_{b1} = (N - 2k_1) \log_2 M + 2k_1 (\log_2 M)/2 \\
= (N - k_1) \log_2 M$$
(27)

Therefore the ratio of the sacrificed efficiency of the communication is.

$$r_{b0} = \frac{N_b - N_{b0}}{N_b} = \frac{k_0}{N} r_{b1} = \frac{N_b - N_{b1}}{N_b} = \frac{k_1}{N}$$
 (28)

To further explain the reduction of communication efficiency is deserved, a simulation of power spectrum and BER of the unmodified waveform with same efficiency as the modified waveform is showed in fig. 2 and fig. 3. With the reduction of the symbol rate of CPM, the widening of the spectrum of the integrated waveform is wakened but not eliminated. So with the same interference outside the original LFM bandwidth, the BER performance of the modified waveform is still far better than the integrated waveform without modification.

Finally, the velocity and distance ambiguity function of LFM and the integrated waveforms are showed in fig. 4. From the figure we can see that the velocity ambiguity function of the modified k_0 and k_1 is almost the same as that of the unmodified LFM-CPM and the LFM waveform, while for the distance ambiguity function, the modified ones have larger beam-width of the main lobe and there are more fluctuations in the side lobe. In general, both the velocity and the distance ambiguity function hold the basic properties of LFM with a single peak and low side lobes. So the modified integrated waveform is competent at tracking and detection for radar.

V. CONCLUSION

In this paper, we studied the integrated radar-communication waveform based on LFM-CPM. The spectrum of LFM-CPM is analyzed in detail using STFT and a modified three-section waveform is proposed. For rectangular shaped LFM-CPM with any parameters, the spectrum can be limited by a three-section mapping sequence of the communication symbols, that is, the mapping codebook of the first $k_0 = \left\lceil \frac{h}{2\mu T^2} \left(M - 1 \right) \right\rceil$ symbols

and the last k_0 symbols is the positive and the negative half of the original codebook, respectively. When k_0 can be reduced to $k_1 = k_0 - L + 1$. The simulation results show that the spectrum of the modified waveform is mainly restrained within the bandwidth of the original **LFM** and the ambiguity function holds the basic properties of LFM. When there is a strong interference outside the original bandwidth, bandstop filtering helps the modified waveform holds good BER performance almost the same as the one without interference, while the original integrated waveform can hardly accomplish the transmission assignment with high BER at around 10^{-1} no matter with or without bandstop filter. The modification proposed in this paper partly reduces the communication efficiency, but even for the unmodified integrated waveform with the same efficiency of the modified k_0 , there is still a great loss in BER performance. So the modification is necessary and valid. In a word, the modified three-section waveform based on LFM-CPM proposed in this paper is significant for the integration of radar-communication in intelligent transportation systems with strong interference beside the original bandwidth of the radar.

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