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A quick convergence Active Constellation Extension Projection onto Convex Sets algorithm for reducing the PAPR of OFDM system

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Abstract— **High Peak to Average Power Ratio, commonly abbreviated as PAPR, represents the main concern in the implementations of all multicarrier modulation schemes such as Orthogonal Frequency Division Multiplexing. Many PAPR reduction techniques were proposed to solve this problem. Amongst these schemes is the Active Constellation Extension (ACE) which is recognized as a promising method that provides higher gains with neither Bit Error Rate (BER) degradation nor receiver side information transmission. In this investigation, we focus on this scheme in its Projection onto Convex Sets (POCS) variant. We propose a quick POCS-ACE convergence scheme that achieves good performances from the first iteration without degradation in BER.**

Keywords—PAPR; OFDM; ACE; POCS; Fast convergence

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is the widely used multicarrier modulation scheme (MCM) for wireline and wireless communication applications. By appending a cyclic prefix (CP), OFDM proves its ability to avoid both intersymbol and intercarrier interference (ISI and ICI), and so its robustness to multipath fading effects. As an MCM, OFDM suffers from high Peak to Average Power Ratio (PAPR) which reflects the envelope fluctuation. This high PAPR becomes annoying in the presence of a nonlinear High Power Amplifier (HPA).

In order to overcome this envelope fluctuation and HPA non linearity, many PAPR reduction schemes were suggested in literature [1] for OFDM. These methods can be classified into three categories: distortion techniques like clipping [2] and companding [3], distortion-free techniques like Selective Mapping (SLM)[4,5,6,7] and Partial Transmit Sequence (PTS) [8], and average-power-increasing techniques like Active Constellation Extension (ACE)[9,10,11,12] and Tone Reservation (TR)[13,14,15].

Amongst the abovementioned schemes is the ACE which is recognized as an efficient PAPR reduction scheme that not only provides a remarkable gain when compared with other schemes but also shows transparence to the receiver since there is no need for side information transmission to help recover data on the side of the receiver. ACE reduces the peak signals amplitude by extending subcarriers located at exterior constellations to an outer region, thus keeping minimum distance between symbols. In its Projection onto Convex Sets (POCS) variant, ACE provides good performances but it converges slowly, exhibiting a big number of iterations. In order to accelerate the convergence speed, the Smart Gradient Project algorithm (SGP) [10] is generally introduced to ACE algorithm but it still requires some iterations.

In this investigation, we put forward a new scheme based on the conventional POCS-ACE, namely: LSA-POCS-ACE. The novel scheme is characterized by faster convergence and better performance when compared with the two variants of ACE algorithm (i.e., conventional POCS and SGP). Our proposed scheme improves the POCS-ACE algorithm by using the Least Square Approximation (LSA) being used in [14] to calculate the portion of correctional or peak cancelling signal that helps reduce the high amplitude peaks. This can be fulfilled though calculating an optimization factor which will be multiplied by the peak cancelling signal in order to approximate the amplitude of the original clipping noise. With one single iteration, the PAPR is noticeably reduced.

This paper is organized as follows. Section II describes the OFDM system. The conventional POCS-ACE technique and the proposed LSA-POCS-ACE are described in section III. Section IV reportes the simulation results to compare the proposed algorithm with the conventional one.The conclusions and some prospective are provided in in Section V.

II. THE OFDM SIGNAL MODEL

In OFDM systems, the transmitted signal consists of a set of orthogonal subcarriers that carry parallel data . The baseband samples of an OFDM symbol can be written as indicated by equation(1) :

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j2\pi\frac{kn}{N}} \,,\ 0 \le k \le N-1 \quad (1)$$

Where N refers to the number of subcarrier of an OFDM system, X_k is the QAM modulated data carried by the k^{th} subcarrier and x(n) is the n^{th} sample of a time-domain symbol. Actually, the equation above can be realized conveniently by using the Inverse Fast Fourier Transform (IFFT).

All MCMs suffer from large PAPR which usually causes inefficient use of the HPA and introduces a decrease in the system performance. A significant metric to measure the envelope fluctuation of any MCM scheme is the PAPR defined as the ratio between the peak power or the maximum of signal amplitude and the average power of the signal. For the OFDM signal, the PAPR is defined as in equation (2).

$$PAPRs[n] = \frac{max_{n\in\{0,\dots,N-1\}}\{|x[n]|^2\}}{E\{|x[n]|^2\}} \quad (2)$$

To analyze the behavior of the PAPR which is a random variable, the Complementary Cumulative Distribution Function (CCDF) is an appropriate metric which is used instead of the Cumulative Distribution Function (CDF). The CCDF is defined in literature as the probability that the PAPR of the discrete time signal exceeds a given and predefined threshold denoted here by (τ). Thus, the CCDF can be evaluated as in equation (3).

$$\mathrm{CCDF} = \Pr\,(\mathrm{PAPR}(x[n]) > \tau) \qquad (3)$$

III. THE NOVEL POCS-ACE SCHEME FOR OFDM

A. Conventional POCS-ACE Scheme

There is a huge number of methods which have been designed to reduce the peak power of the OFDM signal. Among these methods for PAPR reduction is the Active Constellation Extension (ACE) proposed by Krongold and Jones in [10]. In its POCS variant, ACE is claimed to be a simple and refined method with promising results. Such scheme is similar to Tone Injection method (TI) [15].

The POCS-ACE method requires both time-domain and frequency-domain signal processing. In this method, the time domain signal is iteratively clipped and filtered. The clipping and filtering noise moves the constellation points. The extension of each symbol must be within the allowable extension regions. If otherwise, the point is moved to its original position. These procedures are performed iteratively to achieve the target PAPR. The ACE principle is presented in the Fig. 1.

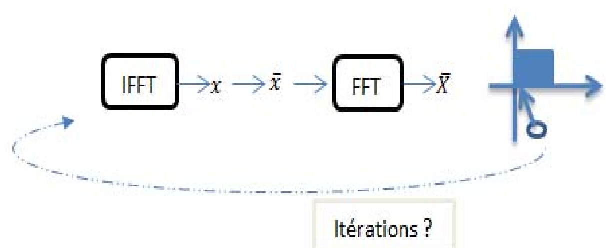

Fig. 1 POCS- Active Constellation Extension principle

As illustrated in Fig. 2, ACE tolerates the corner QAM constellation points to be moved within the quarter planes outside their nominal values (shaded regions on the figure). The inner points are kept unchanged in order to preserve the minimum distance between the constellation symbols. The other border points are allowed to be moved along rays pointing towards the exterior of the constellation. The constellation extension is represented in Fig. 2 for the 64-QAM constellation where we can distinguish the possible extension for the corner and the other border points.

The main advantage of the ACE technique consists in the important reduction gain of PAPR without loss in data rate. Furthermore, ACE has the advantage that no side information is required at the receiver end in order to recover the transmitted data, thus this PAPR reduction scheme is transparent to the receiver.

The extension of some points of the constellation requires additional power, thus, the major issue related to the ACE scheme is the power increase at the transmitter level.

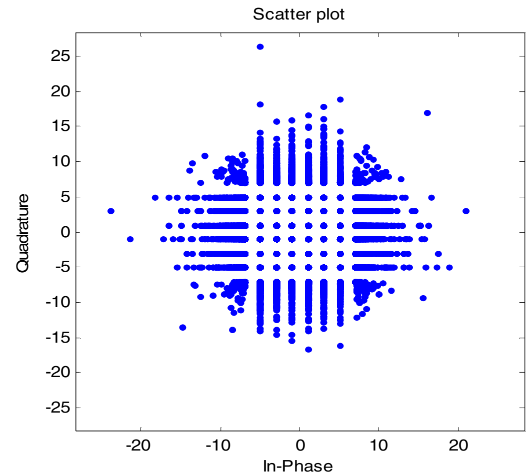

Fig. 2 Illustration of ACE scheme with 64QAM encoding. The shaded regions represent the corner-point extension regions

In this investigation, we focus on the POCS-ACE variant whose algorithm is described in [10]. Such scheme is based on both frequency and time-domain signal processing. An iterative clipping and filtering followed by the ACE decision, based on the position of the constellation points, can be a summary of the POCS-ACE algorithm.

The conventional POCS-ACE algorithm, described in [10], is summarized as follows:

1) Start with the data signal X in frequency domain and use the OFDM modulator to obtain the baseband modulated signal $x[n]$.

2) Clip any $|x[n]| \geq A$ in magnitude to obtain :

$$\bar{x}[n] = \begin{cases} x[n], & |x[n]| \leq A, \\ Ae^{j\theta[n]}, & |x[n]| > A \end{cases} \qquad (4)$$

Where A is the clipping level and:

$$x[n] = |x[n]|e^{j\theta[n]}\ , n \in [1\ ,2, \ldots N] \quad (5)$$

Another way to look at step 2 is to consider $\bar{x}[n]$ as follows:

$$\bar{x}[n] = x[n] + c_{clip}\,[n] \qquad (6)$$

Where c_{clip} refers to the original clipping noise and is calculated as in equation (7).

$$c_{clip}\,[n] \begin{cases} 0, & |x[n]| \leq A \\ A - |x[n]|e^{j\theta[n]}, & |x[n]| > A \end{cases} \quad (7)$$

3) Demodulate c_{clip} to obtain C_{clip} .

4) Enforce all ACE constraints on C_{clip} by projecting exterior points into the region of increased margin while setting all remaining directions to zeros. The frequency-domain resultant signal is termed C_{ext} .

5) Modulate $\mathrm{C}_{\mathrm{ext}}$ to get c, the peak cancelling signal.

6) Compute $x[n] = x[n] + c\,[n]$ (8)

7) Return to step 1 and iterate the algorithm until no points are clipped or the PAR is essentially minimized.

With this variant, ACE-POCS converges slowly and expresses higher complexity. To accelerate the convergence rate of the POCS–ACE scheme, we suggest a new scheme termed LSA-POCS-ACE based on the LSA approximation. The proposed scheme reduces the complexity of the conventional POCS–ACE since it reduces the number of required iterations.

B. LSA-POCS-ACE PAPR Reduction scheme

In order to (i) accelerate the convergence of the POCS-ACE scheme characterized by an extremely smaller amplitude of the peak-cancelling signal during the first iterations when compared with the original clipping noise, and (ii) reduce the complexity of the POCS-ACE method due to the high number of iterations, we propose here a fast convergence POCS-ACE based on the LSA algorithm called LSA-POCS-ACE. The proposed LSA-POCS-ACE algorithm calculates the optimization factor using the LSA approximation.

Our proposed LSA-POCS-ACE scheme uses the LSA approximation to calculate an optimization factor that we denote as μ. The correctional signal is then multiplied by μ and added to the original OFDM signal. The main goal is to rapidly make the amplitude of the peak cancelling signal close to that of the original clipping noise.

The proposed scheme converges in a just one iteration and performs a good PAPR reduction. The PAPR is hence markedly reduced. Based on the definition of the POCS-ACE scheme mentioned above, the objective of the optimization problem is to minimize the difference between the amplitude of the peak cancelling signal and the amplitude of the original clipping noise. It can be formulated as follows:

$$min\{\sum_{n\epsilon p}[\mu.|c(n)|-|c_{clip}(n)|]^2\} \tag{9}$$

If we define $q(\mu)=\sum_{n\epsilon p}[\mu.|c(n)|-|c_{clip}(n)|]^2$, then we have;

$$\frac{\partial \mathrm{q}(\mu)}{\partial \mu}=\frac{\partial(\sum_{n\epsilon p}[\mu.|c(n)|-|c_{clip}(n)|]^2)}{\partial p} \tag{10}$$

$$\frac{\partial q(\mu)}{\partial \mu}$$

$$=\frac{\partial(\sum_{n\epsilon p}[\mu^2.|c(n)|^2+|c_{clip}(n)|]^2-2\mu|c(n)||c_{clip}(n)|)}{\partial \mu}$$

$$=2\mu\sum_{n\epsilon p}|c(n)|^2-2\sum_{n\epsilon p}|c(n)||c_{clip}(n)| \tag{11}$$

When $\frac{\partial q(\mu)}{\partial \mu}=0$, the optimal value of μ correspond to:

$$\hat{\mu}=\frac{\sum_{n\epsilon p}|c(n)||c_{clip}(n)|}{\sum_{n\epsilon p}|c(n)|^2} \tag{12}$$

Therefore, the anticipated LSA-POCS-ACE algorithm proceeds as follows:

1) Starting with the data signal X, in frequency domain, use the OFDM modulator to obtain the baseband modulated signal $x[n]$.
2) Clip any $|x[n]|\geq A$ in magnitude to obtain:

$$\bar{x}[n]=\begin{Bmatrix} x[n], & |x[n]|\leq A, \\ Ae^{j\theta[n],} & |x[n]|>A \end{Bmatrix} \tag{13}$$

Where A is the clipping level. Consider only the clipping noise c_{clip} as indicated by equation (14):

$$c_{clip}[n]\begin{Bmatrix} 0, & |x[n]|\leq A \\ A-|x[n]|e^{j\theta[n],} & |x[n]|>A \end{Bmatrix} \tag{14}$$

3) Apply FFT to c_{clip} to obtain C_{clip}.
4) Enforce all ACE constraints on $\mathrm{C_{clip}}$ by projecting exterior points onto the region of increased margin while setting all the remaining directions to zeros. The frequency-domain resulting signal is termed $\mathrm{C_{ext}}$. Modulate C_{ext} to get c.
5) Compute the optimization factor μ using the LSA. The expression of μ is as follows:

$$\mu=\frac{\sum_{n\epsilon p}|c(n)||c_{clip}(n)|}{\sum_{n\epsilon p}|c(n)|^2} \tag{15}$$

6) Compute $x[n]=x[n]+\mu*c[n]$ (16)

IV. SIMULATION RESULTS AND DISCUSSION

In the following simulations, we set up a critically sampled OFDM system with $\mathrm{N}=256$ subcarriers employing QPSK modulation per complex symbol. The clipping ratio of 3dB is chosen. The proposed LSA- POCS-ACE method is evaluated and compared to the conventional POCS-ACE algorithm. The proposed scheme is evaluated through the CCDF, BER, and the Power Spectral Density (PSD).

A. Comparison of CCDF plots

In the first two simulations, we compare the CCDF of the new scheme LSA- POCS-ACE with the conventional POCS-ACE for several iterations where QPSK constellation is employed. The maximum number of POCS iterations is set to 30 and the clipping threshold is set to$A=3\mathrm{dB}$.

The POCS-ACE approach is simulated with the abovementioned parameters and the PAPR results are shown in Fig. 3. At a clipping level of 10^{-4} the gain is about 4.5dB after thirty POCS iterations. The fact that the number of POCS-ACE iterations is high enables the assertion that the the POCS-ACE method converges slowly and exhibits high complexity.

Fig.4 displays the PAPR reduction performances of the LSA-POCS-ACE method applied to the OFDM system with 256 sub-channels employing QPSK. For this simulation, the number of iterations is set only to one and the result is very encouraging. By way of illustration, at the clipping level of 10^{-2}, our proposed scheme had a gain of more than 4dB over the original signal without any PAPR reduction scheme. Additionally, when compared with the conventional POCS-ACE scheme, the LSA-POCS-ACE using only one iteration outperforms the thirty iterations of the conventional POCS-ACE scheme.

We can conclude from these CCDF results that the proposed LSA-POCS-ACE scheme performs better than the conventional one. Additionally, a decrease in complexity is evident since it reduces the number of iterations to just one, leading to faster convergence.

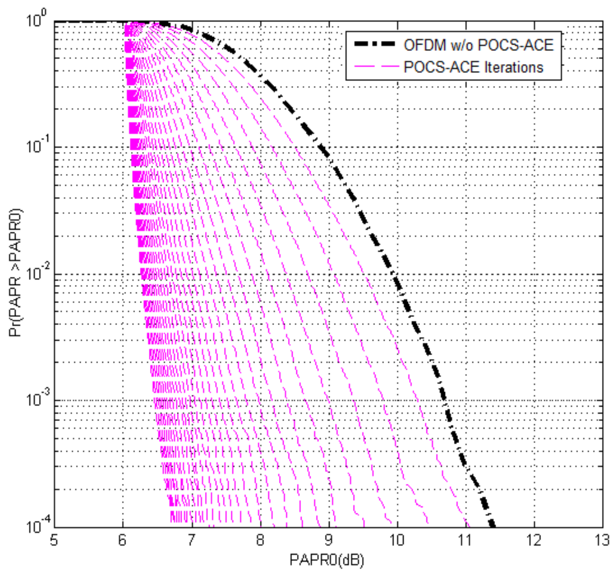

Fig. 3. PAPR results of the POCS-ACE method applied to OFDM system with 256 sub-channels employing QPSK

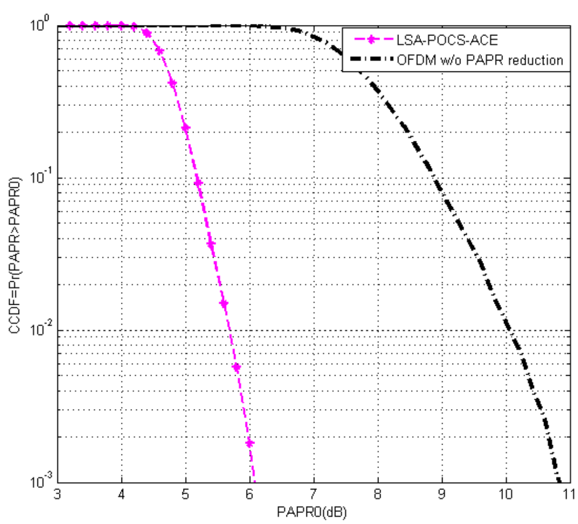

Fig. 4. PAPR reduction performances of the LSA-POCS-ACE method applied to OFDM system with 256 sub-channels employing QPSK

B. Comparison of BER performances

To evaluate the performance of any PAPR reduction scheme and compare it with other schemes and amongst many criterions of evaluation, BER qualifies as an additional parameter to CCDF which is widely used to assess the performance of PAPR reduction techniques.

The following simulation will examine the BER performance of the anticipated LSA-POCS-ACE scheme when compared with that of the conventional POCS-ACE scheme through an Additive White Gaussian Noise (AWGN) channel.

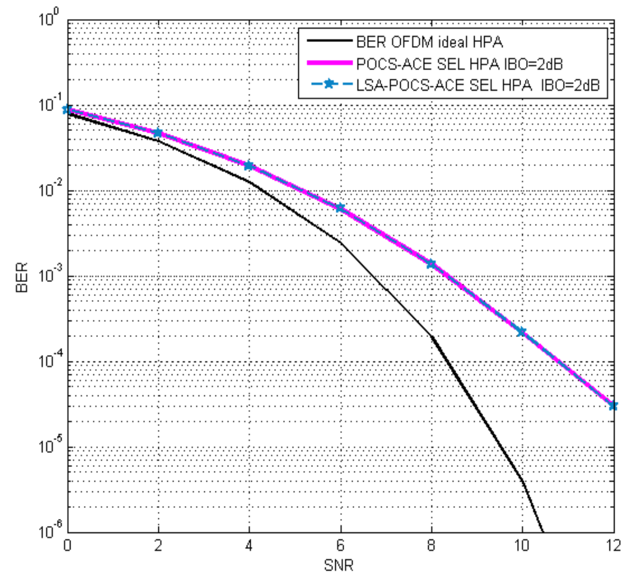

Fig. 5. BER performances of OFDM system over AWGN channel when the LSA-POCS-ACE and conventional POCS-ACE schemes are employed with a SEL amplifier

Fig. 5 shows the BER performances of the OFDM system through AWGN channel when the LSA-POCS-ACE and conventional POCS-ACE schemes are used. The Soft Envelope Limiter (SEL) amplifier is used. The value of the input backoff (IBO) is set to 2dB. For this simulation, the number of POCS iterations is set to 15 while it is just fixed to one for the LSA-POCS-ACE PAPR reduction technique.

The black curve marked with "BER OFDM ideal HPA" is obtained without PAPR reduction scheme and with ignoring the effect of amplifier, which means directly transmitting out the original OFDM signal.

As shown in Fig.5, the one-iteration LSA-POCS-ACE scheme can offer the same BER performance when compared with the conventional POCS-ACE scheme with several iterations.

C. Comparison of Power Spectral Density

To compare the out-of-band radiation of the signals before and after PAPR reduction approach, Fig. 6 draws the spectrum of the OFDM signals through the same SEL amplifier as mentioned above. The number of subcarriers is set to $N = 512$. The curve with the proposed LSA-POCS-ACE scheme that with the conventional POCS-ACE method and the one without PAPR reduction are presented. The higher the shoulder of spectrum is, the more severe the spectral spreading becomes. This results in higher interference between the sub-bands of the OFDM system. With the POCS-ACE methods employed to reduce the PAPR, the out-of-band radiation is reduced as with the one calculated for LSA-POCS-ACE scheme.

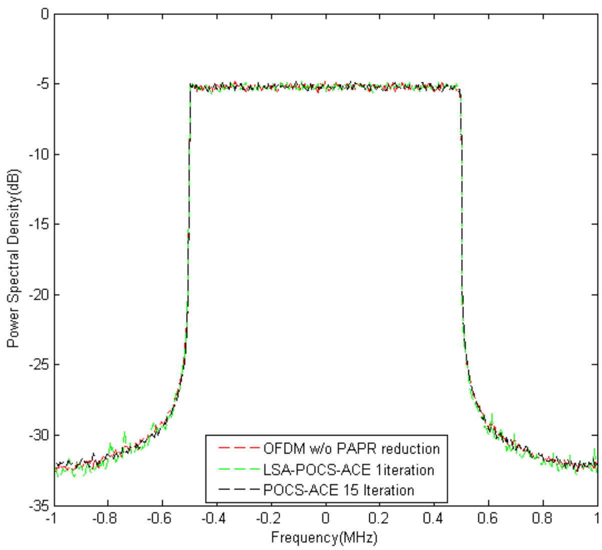

Fig.6. Comparison of the power spectral density effects of the proposed LSA-POCS-ACE with that of the conventional POCS-ACE

The spectrum with the proposed LSA-POCS-ACE technique and the one with the conventional POCS-ACE are generated only in the usable frequency band. The results confirm that the PAPR reduction techniques studied here can reduce the components out of the system frequency band.

V. Conclusion

In the present study and in order to reduce the PAPR of OFDM signals, we suggested a novel PAPR reduction scheme based on the conventional POCS-ACE scheme. The novel scheme is termed LSA-POCS-ACE. The proposed scheme is characterized by fast convergence when compared with the conventional POCS-ACE. This is due to the use of the least squares approximation algorithm. The proposed LSA-POCS-ACE scheme entirely exploits the filtered clipping noise to generate the peak-canceling signal by multiplying it with an optimization factor. More precisely, the proposed LSA-POCS-ACE scheme makes the amplitude of the generated new peak-canceling signal approximate to that of the original clipping noise. Therefore, the proposed LSA-POCS-ACE scheme can obtain a good PAPR reduction with a fast convergence rate (i.e., two iterations at most). The conducted simulation results also showed that the proposed LSA-POCS-ACE scheme offers an excellent PAPR reduction with a much lower computational complexity, compared with that of the traditional POCS-ACE method.

As an extension to this work, we can reapply the same algorithm to a Filter Bank Multicarrier system (FBMC) and extend it to the Multiple Input Multiple Output (MIMO) context.

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