

A Novel Low Complexity High Efficiency Hybrid PAPR Reduction for OFDM Systems

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Abstract—In this paper, we propose a novel hybrid multiplicative-additive technique to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. This technique consists of two inverse fast Fourier transform (IFFT) blocks. The input symbols of the first IFFT are the mapped symbols, whereas the input symbols of the second IFFT are the summations of the absolute value of the real part of the outer signal constellation points and zeros symbols. First, the output of the two IFFT blocks is partitioned into four subblocks, which are subsequently used to rearrange the subblocks with padding zeros in a specific manner. Then, a new optimization scheme is introduced, in which only a single two-phase sequence and four iterations needs to be applied. Numerical analysis shows that the hybrid proposed technique achieves better bit error rate (BER) and PAPR reduction performance than partial transmit sequences (PTS) multiplicative technique and tone reservation (TR) additive PAPR technique. The other salient feature of this scheme is that no side information (SI) is needed which increases transmission efficiency.

Index Terms—OFDM, PAPR, PTS, TR.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is becoming increasingly popular in various broadband communication systems, such as worldwide interoperability in microwave access (WiMAX), digital video broadcasting (DVB) and long term evolution (LTE). OFDM can be used either as a multicarrier modulation technique or as a multiplexing technique [1]. Moreover, OFDM possesses some essential features for wireless communications. These feature include bandwidth efficiency, immunity against the intersymbol interference (ISI) caused by multi-path delay spread, robustness against impulse noise, and multi-path delay spread tolerance [1]. In OFDM systems, the envelope of the time domain signal will change because several subcarrier components are added after the inverse fast fourier transform (IFFT) operation [2], [3]. Accordingly, OFDM systems have a high the peak-to-average power ratio (PAPR) due to the superposition of many subcarriers, in which the high dynamic ranges is required for a nonlinear device such as a power amplifier (PA) to avoid the amplitude clipping of the signal [1], [4]. However, the higher the PAPR, the more fluctuation in the signal amplitude, causes nonlinear distortions and spectral spreading [3].

Numerous PAPR reduction techniques have been proposed to reduce the PAPR. These techniques can be classified into signal scrambling (multiplicative) techniques such as selective

level mapping (SLM) and partial transmit sequences (PTS) [4], [6], [12] and additive techniques such as tone reservation (TR), tone injection (TI), peak cancellation, and clipping and filtering [3], [5], [7].

PTS technique is the most attractive technique in multiplicative technique, because it is a distortionless technique and offers good PAPR reduction performance. In the PTS technique based on the phase optimization approach, the given phases are manipulated to reduce the peak of the OFDM signals to an acceptable level. Therefore, PTS is a distortionless technique for PAPR reduction [4], [7], [8]. The main problems in the PTS (multiplicative) technique is that the complexity and it requires sending of side information to the receiver [10], [11].

TR and TI are an efficient additive technique to reduce PAPR in multicarrier systems [5], [7]. TR technique is based on adding data block-dependent signals to the original multicarrier signals to reduce their peaks [5], [7]. The added signals are computed at the transmitter and stripped off at the receiver. The added signals are dummy signals holding no information. Therefore, the TR technique is also known as the dummy sequence insertion technique. The main problems in the TR and TI (additive) technique are that the complexity and it needs side information required by the receiver.

II. SYSTEM MODEL

In OFDM systems, the data stream with rate R bps is mapped to phase shift keying (PSK) or quadrature amplitude modulation (QAM). A set of N mapped signal is converted to N parallel stream by using a serial-to-parallel converter. These sets are known as the OFDM symbol. Afterwards, the IFFT with length N is used to produce orthogonal data subcarriers. Then, all orthogonal subcarriers are transmitted simultaneously over the symbol interval T . The complex baseband OFDM signal $x(t)$ with N orthogonal subcarriers can be written as [11], [12]].

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi k \Delta f t} \quad (1)$$

where $\Delta f = \frac{1}{T}$ is the subcarrier spacing, X_k is the k_{th} frequency domain signals in OFDM.

The PAPR of the signal is defined as [3]:

$$PAPR(x(t)) = \frac{\max_{0 \leq t \leq T} [|x(t)|^2]}{E[|x(t)|^2]} \quad (2)$$

where $E[\cdot]$ is expected value operator.

The high peaks appear when N different mapped symbols phases in (1) are accumulated constructively [2]. Generally, the complementary cumulative distribution function (CCDF) is used to capture the statistical PAPR properties of the PAPR in OFDM system, and is defined as:

$$CCDF(PAPR_0) = Pr\{PAPR > PAPR_0\} \quad (3)$$

where $PAPR_0$ is a constant.

A. Tone reservation technique

In the TR technique, the transmitter and the receiver know the set of data carrying subcarriers [5]. Different methods have been proposed by researchers to construct the reduction signals. However, the generation methods of these signals are all iterative and may require sometimes tens of loops [1], [7].

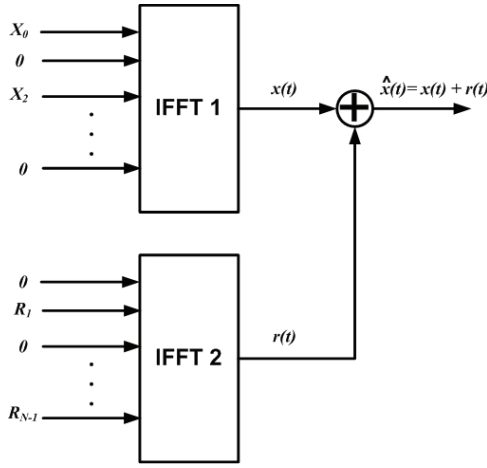


Fig. 1. Block diagram of TR technique.

The TR technique can reduce the PAPR value by utilizing the reserved subcarriers, which are not used for data transmission. Based on the TR technique, the baseband signal in (1), can be expressed as:

$$\hat{x}(t) = x(t) + r(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N-1} (X_k + R_k) e^{j2\pi k \Delta f t} \quad (4)$$

where $r(t)$ ($\mathbf{r} = [r_0, r_1, \dots, r_{N-1}]^T$) is the peak-canceling signal in time domain, and $\mathbf{R} = [R_0, R_1, \dots, R_{N-1}]^T$ is the peak-canceling signal vector in frequency domain, which is generated by using Q peak reduction tones (PRTs). The Q PRTs do not carry any data information, and they are only used for reducing the PAPR.

The modulated data signal X_k and the peak reduction signal R_k are restricted to stand in disjoint subcarriers tones as shown in Fig. 1, that is:

$$X_k + R_k = \begin{cases} X_k, & \text{if } k \in K^c \\ R_k, & \text{if } k \in K \end{cases}$$

where $K = \{k_0, k_1, \dots, k_{Q-1}\}$ is the dummy data signal (PRTs) on the k th subcarrier, and K^c is the set of the

remaining subcarriers in N used for the modulated data signal. However, $R_k \neq 0$ only if $k \in K$. \mathbf{r} is designed to minimize the maximum value of $\hat{x}(t)$. A minimax CF optimization problem is defined as:

$$\mathbf{r}^{opt} = \underbrace{\arg \min_r}_{r} \|\hat{x}\|_{\infty} \quad (5)$$

where $\|\cdot\|_{\infty}$ is the ∞ -norm of a vector.

Equation (10) can be reconstructed as a quadratically constrained quadratic program (QCQP) problem, which causes high complexity. However, there are several proposed techniques based on TR to reduce complexity, for example, the simple suboptimal techniques to define the value of r is the iterative clipping and filtering (ICF). In additional, The main problems in the TR technique are that the complexity and it needs side information (SI) required by the receiver.

B. Partial transmit sequence technique

In the C-PTS technique, the incoming serial random data vectors at the transmitter are mapped into QAM symbols and then converted from serial to parallel streams:

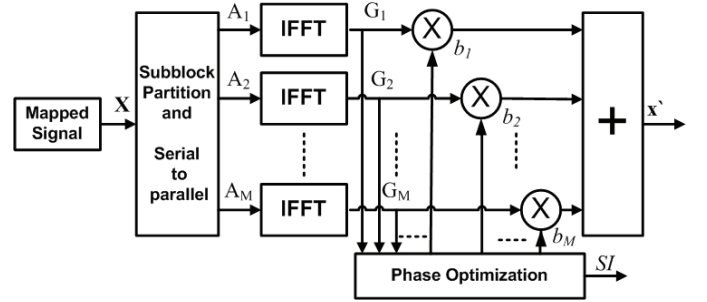


Fig. 2. Block diagram of PTS technique.

$$\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T \quad (6)$$

Then, \mathbf{X} is partitioned into M disjoint subblocks as shown in Fig. 2, which are represented by the vectors \mathbf{X}_m ($1 \leq m < M$) of length V , where $N = MV$ for integers M and V . For $m = 1, \dots, M$, let the matrix \mathbf{A} be a zero-padded of \mathbf{X}_m , which can be written as:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1,M} \\ A_{21} & A_{22} & \cdots & A_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{LN,1} & A_{LN,2} & \cdots & A_{LN,M} \end{bmatrix} \quad (7)$$

where L is the oversampling factor. Then, let the matrix \mathbf{G} be the zero-padded IFFT of \mathbf{A} , which can be written as:

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1,M} \\ G_{21} & G_{22} & \cdots & G_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ G_{LN,1} & G_{LN,2} & \cdots & G_{LN,M} \end{bmatrix} \quad (8)$$

Next, the time domain sequences can be combined to minimize the PAPR, this is done by applying the complex phase rotation factors $\mathbf{b} = [b_1, b_2, \dots, b_M]^T$. The resulting time domain signal after combination can be written as:

$$\mathbf{x}' = \mathbf{G}\mathbf{b} \quad (9)$$

where $\mathbf{x}' = [x'_1, x'_2, \dots, x'_{LN}]$ is the block of optimized signal samples. Hence, the objective of PTS technique is to come out with an optimal phase factor for the subblock set that minimizes the PAPR. The objective of the optimization problem is to identify optimum phases $\hat{\mathbf{b}}$ that satisfy:

$$\{\hat{b}_1, \hat{b}_2, \dots, \hat{b}_M\} = \underbrace{\arg \min}_{\{b_1, b_2, \dots, b_M\}} \left(\max_{1 \leq k < LN} \left| \sum_{m=1}^M b_m G_{k,m} \right| \right) \quad (10)$$

where $b_m \in \{\pm 1, \pm j\}$ and ($W = 4$), where W is the number of phase weight factors. b_1 can be set equal to 1 without loss of performance [1, 3]. Therefore, in the PTS technique, it is necessary to test W^{M-1} sets of distinct possible candidate vectors \mathbf{b} to satisfy (10). Accordingly, the computational complexity of the PTS technique increases exponentially with M .

At the receiver, after the N -point fast Fourier transform (FFT), the frequency domain sequence can be written as:

$$\mathbf{A}' = FFT(\mathbf{x}') \quad (11)$$

Then, the vector \mathbf{A}' is partitioned into M disjoint subblocks, which are represented by the vector \mathbf{A}'_m ($1 \leq m < M$) of length V , where $N = MV$ for certain integers M and V . For $m = 1, \dots, M$, let the matrix \mathbf{A} be the zero-padded of \mathbf{A}'_m , which can be written as:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1,M} \\ A_{21} & A_{22} & \cdots & A_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N,1} & A_{N,2} & \cdots & A_{N,M} \end{bmatrix} \quad (12)$$

So, using the inverse phase rotation vector $\hat{\mathbf{b}}$, we can recover the signal as follows:

$$\hat{\mathbf{x}} = \mathbf{A}\hat{\mathbf{b}} \quad (13)$$

As noted, in the PTS technique, only the phase information is changed. Accordingly, no out-of-band radiation occurs.

III. PROPOSED TECHNIQUE ANALYSIS

In the proposed technique, the input symbols of the first IFFT are the mapped symbols, whereas the input symbols of the second IFFT are the summations of the absolute value of the real part of the outer signal constellation points and zeros symbols. First, the output of the two IFFT blocks is partitioned into four subblocks, which are subsequently used to rearrange the subblocks with padding zeros in a specific manner. Then, a new optimization scheme is introduced, in which only a single two-phase sequence $\{0, 1\}$ needs to be applied. After summation, the original constellation points are shifted to the right or left based on the weighting phase. If the weighting

phase is 1, then the original constellation is shifted to the right with changes to the sign of the phase of constellation points. If the weighting phase is 0, then the original constellation is shifted to the left without changes to the sign of the phase of constellation points. and also the phase of the output of the first IFFT is rotated when the weighting phase is 1. The proposed technique does not require extra subcarriers for PRTs.

At the transmitter of the proposed technique, as shown in Fig. 3, the incoming serial random data vector is initially mapped into QAM symbols and then converted from a serial stream to a parallel stream, such that:

$$\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T \quad (14)$$

where N is the number of points of the IFFT. When X passes through the first IFFT block, the N -point IFFT output can be expressed as follows:

$$\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T \quad (15)$$

Then, we calculate C as follows:

$$C = \hat{C} \times N \quad (16)$$

where \hat{C} denote the absolute value of the real part of the outer signal constellation points.

In case of 4-QAM, we can calculate C as follows:

$$C = \sum_{n=0}^{N-1} |\Re\{X_n\}| \quad (17)$$

Then, inserting C to the first point of the second N -point IFFT, and the rest of points are all zeros, the output of second N -point IFFT can be derived as follows:

$$\mathbf{c} = [c_0, c_1, \dots, c_{N-1}]^T \quad (18)$$

Then, the output of the first and second N -point IFFT will be partitioned into four subblocks. Then, we rearrange the subblocks with padding zeros, as shown in Fig. 3. The output can be expressed as a matrix with size $4 \times N$. Each row in the following matrix is referred to as a subblock:

$$\mathbf{d}_{4 \times N} = \begin{pmatrix} x_{0,\dots,k-1} & 0_{0,\dots,k-1} & c_{0,\dots,k-1} & c_{k,\dots,2k-1} \\ 0_{0,\dots,k-1} & x_{k,\dots,2k-1} & 0_{0,\dots,k-1} & 0_{0,\dots,k-1} \\ 0_{0,\dots,k-1} & 0_{0,\dots,k-1} & x_{2k,\dots,3k-1} & 0_{0,\dots,k-1} \\ c_{2k,\dots,3k-1} & c_{3k,\dots,4k-1} & 0_{0,\dots,k-1} & x_{3k,\dots,4k-1} \end{pmatrix} \quad (19)$$

where $k = (N/4)$, $x_{0,\dots,n-1}$ are k th-1 samples of the first IFFT, $c_{0,\dots,n-1}$ are k th-1 samples of the second IFFT and $0_{0,\dots,n-1}$ are k th-1 padding zeros.

Then, a new phase optimization scheme is applied. Only two phase sequences $\{0, 1\}$ are required. First, all phase sequence possibilities are generated using a 16×4 encoder (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111\}$), as shown in Table I. Then, the phase of each subblock is

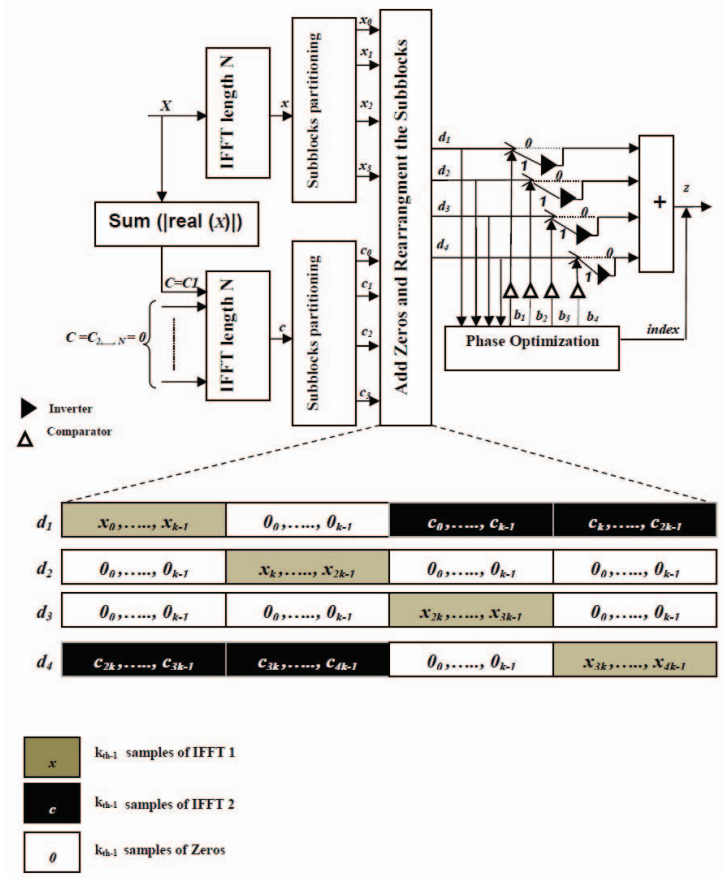


Fig. 3. Block diagram of the proposed technique.

TABLE I
CANDIDATE PHASE SEQUENCES USING AN 16×4 ENCODER

Index	1	2	3	4
$\{b_1 \ b_2 \ b_3 \ b_4\}$	$\{0 \ 0 \ 0 \ 0\}$	$\{0 \ 0 \ 0 \ 1\}$	$\{0 \ 0 \ 1 \ 0\}$	$\{0 \ 0 \ 1 \ 1\}$
Index	5	6	7	8
$\{b_1 \ b_2 \ b_3 \ b_4\}$	$\{0 \ 1 \ 0 \ 0\}$	$\{0 \ 1 \ 0 \ 1\}$	$\{0 \ 1 \ 1 \ 0\}$	$\{0 \ 1 \ 1 \ 1\}$
Index	9	10	11	12
$\{b_1 \ b_2 \ b_3 \ b_4\}$	$\{1 \ 0 \ 0 \ 0\}$	$\{1 \ 0 \ 0 \ 1\}$	$\{1 \ 0 \ 1 \ 0\}$	$\{1 \ 0 \ 1 \ 1\}$
Index	13	14	15	16
$\{b_1 \ b_2 \ b_3 \ b_4\}$	$\{1 \ 1 \ 0 \ 0\}$	$\{1 \ 1 \ 0 \ 1\}$	$\{1 \ 1 \ 1 \ 0\}$	$\{1 \ 1 \ 1 \ 1\}$

converted based on the proposed weight of phase rotation, as follows:

$$\mathbf{z} = \sum_{m=1}^4 (-1)^{b_m} \mathbf{d}_m \quad (20)$$

where $b_m \in \{0, 1\}$, m is the row number of the matrix $\hat{\mathbf{x}}_{4 \times N}$ in (19).

As shown in Fig. 3, the comparator will detect whether the phase factor is 0 or 1. If the weight of the phase factor is 0, the phase of elements of subblock does not change and passes directly to the summation unit. If the weight of the phase factor is 1, the phase is rotated when passing

through the inverter and then passed to the summation unit. As the first step, the PAPR of the combined signal can be calculated. For example, we check if the PAPR at $b_1 = 1$ is the lowest. Then, all phase sequences with $b_1 = 0$ will be omitted (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111\}$). Thus, half of these sequences of Table I is omitted. Afterward, we check b_2 of the rest of the phase sequences (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111\}$). If the PAPR at $b_2 = 1$ is the lowest, then all phase sequences with $b_2 = 0$ will be omitted (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{1000, 1001, 1010, 1011\}$). Thus, half of these sequences are omitted. Afterward, we check b_3 of

the rest of the phase sequences (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{1100, 1101, 1110, 1111\}$). If the PAPR at $b_3 = 1$ is the lowest, then all phase sequences with $b_3 = 0$ will be omitted (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{1100, 1101\}$). Finally, one of the last two sequences (i.e. $\{b_1 \ b_2 \ b_3 \ b_4\} = \{1110, 1111\}$) will be the best sequence with minimum PAPR. Then, the best sequence will convert to an index, as shown in Table I. For example, if the best sequence is $\{1110\}$, it will be converted to its index, which is 15 as shown in Table I. Then, we minimize the sample number 15 in OFDM symbol with $1/M$ as the minimum power between the first of 16 samples of OFDM symbol.

At the receiver side, an encoder similar to that in the transmitter is used. The first of 16 samples of the OFDM symbols are checked to determine the minimum sample power, and its index is inserted into the encoder to generate the phase sequence. For example, if the index of sample with minimum power is 9, then the input of encoder number 9 will be ON and the output will be $\{1001\}$. As such, the proposed technique does not require sending of side information. Complexity due to the number of complex multiplications and number of complex additions relies on the number of iterations. The number of iterations in proposed technique is only 4. Clearly, the proposed technique achieves the lowest computational complexity among other low-complexity multiplicative PAPR reduction techniques.

IV. NUMERICAL RESULTS

Simulations were conducted using MATLAB to evaluate and compare the performance of the proposed technique with the C-PTS technique, and the original OFDM. We utilized 16-QAM signaling with various IFFT length of $N = 128, 256, 512$, $W = 2$, and oversampling factor of $L=4$. We generated 10^5 random OFDM symbols to obtain the complementary cumulative distribution function (CCDF). To obtain the complementary cumulative distribution function (CCDF), 10^5 random OFDM symbols were generated. The CCDFs of the proposed technique, C-PTS and the original OFDM for various numbers of subcarriers $N = \{128, 256, 512\}$ are presented in Fig. 4. The range of PAPR reduction achieved by adopting proposed technique is 2.7dB up to 3.2dB compared to original OFDM. From the simulation results shown, shorter IFFT length will achieve higher PAPR reduction compared to longer IFFT length where in this case, for $N = 128$ the reduction is 3.2dB while for $N = 512$ IFFT length the reduction is only 2.7 dB. Apart from that, a comparison between PAPR reduction performance which conducted by using C-PTS and proposed technique is carried out to determine the ability of proposed technique to decrease PAPR of an OFDM signal. Fig. 4 shows that proposed technique out performs C-PTS technique as the PAPR reduction is higher compared to C-PTS technique. The range of PAPR reduction achievement is from 1.14dB up to 1.18dB. Among various IFFT lengths adopted, 128-IFFT lengths achieved the highest improvement which is as much as 1.18dB. It is evident that the proposed technique yields 2.7dB to 3.2dB reduction in the PAPR with respect to

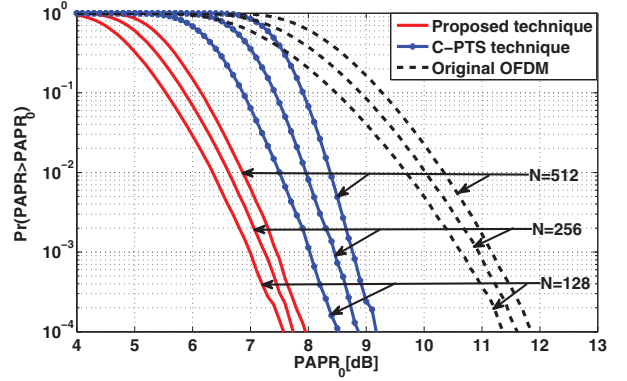


Fig. 4. CCDF of PAPR of the proposed technique for different N subcarriers compared with the C-PTS and original OFDM for $M=4, L=4$.

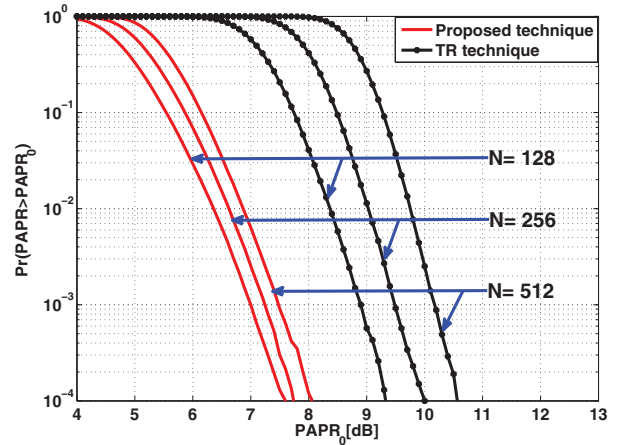


Fig. 5. CCDF of PAPR of the proposed technique for different N subcarriers compared with the TR.

the original OFDM transmission with only 4 iterations at a CCDF of 10^{-4} .

Fig. 5 shows comparison in PAPR reduction performance between the proposed technique and TR technique with 10 iterations. From this figure, it is evident that the PAPR reduction of the proposed technique is slightly more superior compared to that of TR technique. For example, as to the TR technique, the proposed technique can achieve 2.63dB, 2.32dB, 1.21dB, 1.96dB PAPR reduction when $N = \{512, 256, 128\}$ at a CCDF of 10^{-4} , respectively.

In Fig. 6, the analytical BER expressions for M-ary QAM signaling in additive white gaussian noise (AWGN) and multipath Rayleigh fading channel [13] are respectively given as:

$$P_e = \frac{2(U-1)}{M \log_2 U} Q \left(\sqrt{\frac{6E_b}{N_o} \cdot \frac{\log_2 U}{U^2 - 1}} \right) \quad (21)$$

$$P_e = \frac{U-1}{U \log_2 U} \left(1 - \sqrt{\frac{3\gamma \log_2 U / (U^2 - 1)}{3\gamma \log_2 U / (U^2 - 1) + 1}} \right) \quad (22)$$

where γ and U denote E_b/N_o and the modulation order,

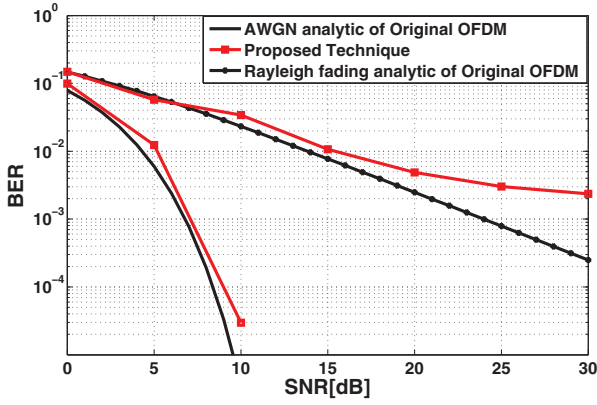


Fig. 6. BER performance for OFDM system and proposed technique with 4-QAM and $N=256$.

respectively, while $Q(\cdot)$ is the standard Q-function defined as

$$Q(\cdot) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \quad (23)$$

We utilized 4-QAM signaling with IFFT length of 256 in order to obtain the bit error rate (BER) performance in additive white gaussian noise (AWGN) and multipath Rayleigh fading channel (with the maximum delay of 15 samples). The results are presented in Fig. 6. In Fig. 6, the performance bounds are obtained by ignoring the effect of the high power amplifier (HPA) and directly transmitting the OFDM signals through AWGN and Rayleigh fading channels. Notably, BER performance in AWGN channel and the Rayleigh fading channel is consistent with the analytical result.

V. CONCLUSION

We proposed a novel hybrid PAPR reduction technique for OFDM systems with no side information and low computational requirement that results in low complexity. A new partitioning and optimization schemes are established. In optimization scheme, only a two-phase sequence is required with only four iterations. Thus, compared to the other PAPR reduction techniques is considered unique while giving superior performance.

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