Sequence Optimization for Integrated Radar and Communication Systems Using Meta-heuristic Multiobjective Methods

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***Abstract*—In real-world engineering problems, several conflicting objective functions have often to be optimized simultaneously. Typically, the objective functions of these problems are too complex to solve using derivative-based optimization methods. Integration of navigation and radar functionality with communication applications is such a problem. Designing sequences for these systems is a difficult task. This task is further complicated by the following factors: (i) conflicting requirements on autocorrelation and crosscorrelation characteristics; (ii) the associated cost functions might be irregular and may have several local minima. Traditional or gradient based optimization methods may face challenges or are unsuitable to solve such a complex problem. In this paper, we pose simultaneous optimization of autocorrelation and crosscorrelation characteristics of Oppermann sequences as a multiobjective problem. We compare the performance of prominent state-of-the-art multiobjective evolutionary meta-heuristic algorithms to design Oppermann sequences for integrated radar and communication systems.**

I. Introduction

Real-world problems such as sequence set designs for integrated radar and communication systems [1] have multiple conflicting design objectives subject to various design constraints [2], [3]. In particular, radar applications typically require only a single sequence with excellent autocorrelation (AC) characteristics such that high range resolution and accurate target detection can be achieved. On the other hand, communication applications require sets of sequences to allow multiple users simultaneous access to a shared transmission medium. In this case, sequence sets are needed that offer excellent crosscorrelation (CC) characteristics that assure minimum interference among the sequences of the set. However, good AC comes at the expense of bad CC and vice versa which in turn requires to establish a trade-off between these conflicting objectives. Given that a solution to this problem can be found, the lack of single hardware platforms for integrated radar and communication systems can be addressed which reduces costs and probability of intercept [1]. In this paper, we use a multiobjective optimization approach that facilitates sequence set design for integrated radar and communication systems.

Due to the aforementioned conflicting objectives, true or best single optimal solutions do not exist. Therefore, finding a set of solutions that represents a trade-off among different objectives is desirable. This set of solutions is known as the Pareto-optimal solution set. Compared to single objective optimization problems, multiobjective optimization problems (MOOPs) are difficult and complex to solve using classical optimization methods [4]–[6]. Multiobjective evolutionary meta-heuristic algorithms have the ability to find the Pareto-optimal solution set in a single run [2], [4], [7]–[9]. As a result, they present an alternative tool to solve MOOP.

Traditional approaches to solve MOOP combine individual objective functions into a single objective function [2], [9]. Methods like utility theory and weighted sum method are applied to determine a single objective function [9]. Such an approach requires a proper selection of the utility functions or weights even for simple problems. Another drawback of using this approach is to find a scaling factor amongst different objectives. Yet, another drawback of the weighted sum approach is that a small change in the weights results in different solutions. Thus, only a single solution is available rather than a set of solutions which prevents to find a trade-off between different objectives. However, given the multiple objectives in MOOP, obtaining a set of good solutions representing a trade-off among multiple objectives is often desirable.

Alternatively, an entire Pareto-optimal solution set or a representative subset that are nondominated with respect to each other can be found using multiobjective evolutionary meta-heuristic algorithms. A set of Pareto-optimal solutions representing a trade-off between different objectives is often preferred to single solutions. A trade-off between different objective refers to a certain amount of gain in the value of one objective in return to the loss of value of the other objective.

The rest of the paper is organized as follows. Section II describes the MOOP formulation. In Section III, a sequence optimization problem for integrated radar and communication systems is posed. Numerical results are given in Section IV. In Section V, a sequence design example is presented. Finally, Section VI concludes the paper.

II. Multiobjective Optimization – Preliminaries

Multiobjective optimization algorithms are often employed to optimize a vector of functions with or without constraints [10]. Without loss of generality, we consider a minimization MOOP in continuous search space. Mathematically, a MOOP

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is formulated as [2]

$$MOOP: \begin{cases} \min\ (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_K(\mathbf{x})) \\ \text{s.t. } \mathbf{x} \in \Omega \subseteq \mathbb{R}^K \end{cases} \tag{1}$$

where design variables in D-dimensional design space are represented by $\mathbf{x} = (x_1, x_2, \ldots x_D)$, and objective space consisting of K objective functions is represented by $\mathbb{R}^K$. The symbol Ω denotes the feasible region or the feasible set of decision vectors in the search space, where

$$\Omega = \{\mathbf{x}_{i,\text{Lower}} \leq \mathbf{x} \leq \mathbf{x}_{i,\text{Upper}}\},\ i = 1, 2, \ldots, K \tag{2}$$

In other words, the goal of MOOP is to find a set of design vectors $\mathbf{x}$ that simultaneously optimize the K objectives while satisfying the given constraints, if any [10]. As MOOP involves multiple objectives, dominance and Pareto-optimality are used to extract an entire set of solutions along the Pareto-front (PF) from a solution space [2].

III. Joint Optimization of Average Mean-Square Correlation Metrics

In order to account for the sequence design challenges associated with integrated systems, Oppermann sequences may serve as an alternative to conventional sequence designs [11], [12]. This is because Oppermann sequences can be designed for a wide range of correlation properties [13]. For any given sequence length N, Oppermann sequences are defined by three parameters (m, n, p). The phase $\phi_k(i)$ of the ith element $u_k(i)$ of the kth Oppermann sequence $\mathbf{u}_k = [u_k(0), u_k(1), \ldots, u_k(N-1)]$ of length N taken from a set $\mathcal{U}$ of sequences is given as

$$\phi_k(i) = \frac{\pi}{N}\left[k^m(i+1)^p + (i+1)^n + k(i+1)N\right] \tag{3}$$

where $1 \leq k \leq N-1$, $0 \leq i \leq N-1$, and integer k is relatively prime to length N. The maximum size U of set $\mathcal{U}$ of Oppermann sequences is obtained as $N-1$ for prime sequence length N. A particular family of Oppermann sequences is defined by the real-valued parameters m, n, and p. All sequences in a family have the same magnitude of the AC function for a fixed combination of m, n, and p. In [13], it has been shown that the magnitude of the AC function depends only on the parameter n, if the parameter $p = 1$. For this case, the AC magnitude of each sequence $\mathbf{u}_k$ follows the expression [13]

$$|C_k(l)| = \left|\frac{1}{N}\sum_{i=0}^{N-1-l} \exp\left\{\frac{j\pi}{N}\left[(i+1)^n - (i+l+1)^n\right]\right\}\right| \tag{4}$$

A. Problem Formulation

The trade-off in sequence optimization for integrated radar and communication systems involves simultaneous optimization of multiple objectives. The problem of simultaneously finding the optimal values of parameters m and n in (3) may be posed as MOOP as follows:

$$P: \begin{cases} \min\limits_{m,n}((f_1(\mathbf{x}), f_2(\mathbf{x})) \\ \text{s.t. } 0 \leq m, n \leq 4 \end{cases} \tag{5}$$

where the two objective functions are given in terms of average mean-square autocorrelation R_{ac} and crosscorrelation R_{cc} as

$$f_1(\mathbf{x}) \triangleq R_{ac} = \frac{1}{U}\sum_{x_1=1}^{U}\sum_{\substack{l=1-N \\ l\neq 0}}^{N-1} |C_{x_1,x_1}(l)|^2 \tag{6}$$

$$f_2(\mathbf{x}) \triangleq R_{cc} = \frac{1}{U(U-1)}\sum_{x_1=1}^{U}\sum_{\substack{x_2=1 \\ x_2\neq x_1}}^{U}\sum_{l=1-N}^{N-1} |C_{x_1,x_2}(l)|^2 \tag{7}$$

and $C_{x_1,x_1}(\cdot)$ and $C_{x_1,x_2}(\cdot)$ denote the aperiodic AC and aperiodic CC function, respectively.

In an ideal case, finding the optimal values of parameters m and n in (3) that simultaneously minimize both R_{ac} and R_{cc} is desirable. However, a trade-off between R_{ac} and R_{cc} has to be found, as good R_{ac} comes at the expense of bad R_{cc} and vice versa. A theoretical bound describing the relationship between R_{ac} and R_{cc}, that can be utilized to find such a trade-off, is given in [14] as

$$R_{cc}(M-1) + R_{ac} > U - 1 \tag{8}$$

According to [14], a theoretical PF in the objective function space for sequence design optimization can be obtained by replacing the greater operator "$>$" in (8) by equality "$=$". In view of integrated radar and communication systems, the objective is therefore to simultaneously find the optimal values of parameters m and n in (3). The optimal values of m and n will result in Oppermann sequences such that: (i) R_{ac} and R_{cc} are as close as possible to the theoretical PF, and (ii) the trade-off between R_{ac} and R_{cc} has a good spread in the feasible region Ω of the search space.

IV. Numerical Results

In this section, the application of meta-heuristic multi-objective algorithms for sequence optimization for integrated radar and communication systems is illustrated with the help of a numerical example. In particular, Oppermann sequences defined by their phases in (3) of length $N = 31$ are considered. Furthermore, the considered sequence family offers parameters m and n for optimization given parameter $p = 1$.

In order to solve Problem P posed in (5), we use state-of-the-art multiobjective algorithms, specifically, NSGA-2 [10], SPEA-2 [15], PESA-2 [16], SMS-EMOA [17], MOEA/D [18], and MOWCA [19].

A. Methodology

In order to have a fair comparison, the settings listed in Table I are used for all the algorithms. The algorithms were executed on a personal computer with Intel Pentium i5 processor running at 2.80 GHz and 6 Gigabytes of RAM. The algorithms were executed 30 times with different random seeds for each run.

TABLE I. Parameter Settings

Parameter	Value
Population size (N_p)	20
Number of generations (G)	500
Number of runs (R)	30
Number of function evaluations (*NFE*)	10000

Initially, individual members of the population in the algorithms are randomly distributed in D-dimensional search space. In terms of Oppermann sequences, the individual population members are represented by the values of m and n in D-dimensional search space and are used to generate the phases of Oppermann sequences in (3). The problem search space refers to an interval in which the search for the optimal values of m and n is carried out. Until the objective defined in (5) is met, finding the optimal values of m and n continues for the pre-defined number of function evaluations (NFE).

B. Comparison of the Algorithms

In this section, we study the performance of multiobjective algorithms using the parameter settings given in Tables I and II. The parameter settings in Table II are default recommended settings in the original references. We did not make a special effort to fine-tune them.

Box plots are depicted in Fig. 1(a)-(b) giving a performance comparison of the algorithms in terms of the inverted generational distance metric (IGD) [2] and the hypervolume (HV) metric [2]. The median and mean values of the metrics are depicted by the straight line in the box and marker '$\circ$', respectively. The whiskers above and below the box give additional information about the spread of the metrics. The '$+$' markers in these plots represent the outliers that are beyond the lower and upper whisker, i.e., lowest or worst value for each metric obtained during 30 independent runs.

From Fig. 1, it can be seen that MOWCA obtains the smallest mean value compared to the other algorithms with respect to the IGD metric. However, MOWCA is shown to have only a slightly better mean value compared to SMS-EMOA for the IGD metric. For the HV metric, MOEA/D-WS is shown to have achieved the best mean value compared to the other algorithms. The SMS-EMOA is shown to have consistently produced lower outliers and lowest spread for IGD and HV (see Fig. 1 (a)-(b)).

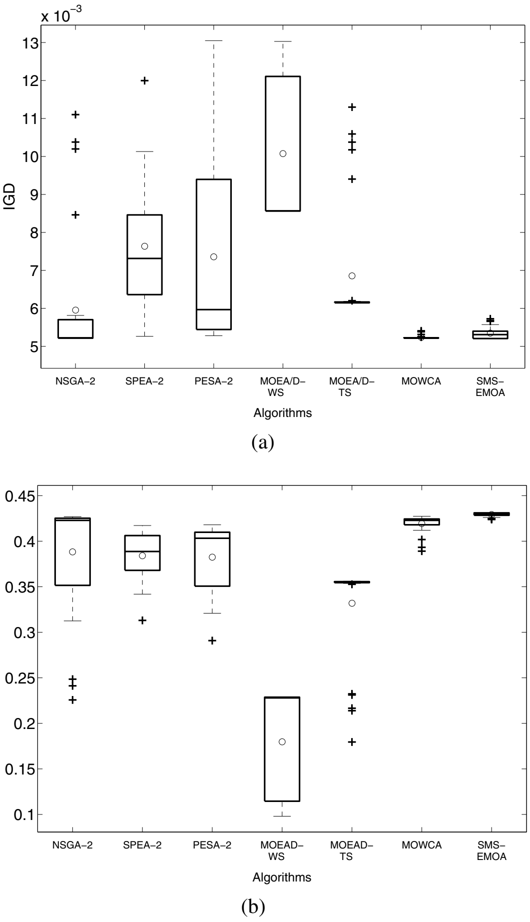


Fig. 1. Performance of different algorithms: (a) IGD, (b) HV.

V. Design Example

Finally, we study the effectiveness of the algorithms in solving problem P. A graphical comparison is used to analyze the PF obtained by each algorithm to identify [20]: (i) if the solutions found by each algorithm lie on the true PF, (ii) how uniform is the distribution of solutions along the true PF.

The coverage and distribution of the solutions of the considered algorithms for problem P for Oppermann sequences of length $N = 31$ is illustrated in Fig. 2 (a)-(g). These figures show the theoretical PF (dotted line) according to (8) and the final PF obtained by the different algorithms for the two opposing design objectives.

It can be seen from Fig. 2 (g) that SMS-EMOA shows the best coverage and distribution of the final solutions among all the algorithms, followed by NSGA-2 (Fig. 2 (a)). The other algorithms such as SPEA-2 (Fig. 2 (b)), PESA-2 (Fig. 2 (c)), MOWCA (Fig. 2 (d)), MOEA/D-WS (Fig. 2 (e)), and MOEA/D-TS (Fig. 2 (f)) show inferior coverage and distribution compared to SMS-EMOA and NSGA-2. The solutions obtained by SPEA-2, PESA-2, MOWCA, MOEA/D-WS, and MOEA/D-TS show concentration of the final solution in certain regions along the PF (see Fig. 2 (b), (c), (e), (f)). This may be due to the use of mutation and crossover operators (Table I). Specifically, solutions with low crosscorrelation are subjected to mutation while crossover operations result in new solutions with poor crosscorrelation. This results in poor performance of these algorithms in terms of coverage and distribution along the true PF. The MOEA/D-WS shows the worst performance in terms of coverage and distribution of the final solutions (Fig. 2 (e)). Some of the final solutions obtained by MOWCA show deviation from the true PF (Fig. 2 (d)).

The poor performance of MOEA/D-WS may be credited to the cost function in (5) defined in terms of average mean-square autocorrelation and crosscorrelation being extremely irregular and may have several local minima. As a result, MOEA/D-WS and MOEA/D-WS may get stuck in local minima such that they cannot achieve a good approximation of the solutions along the PF. According to [18], MOEA/D lacks effective diversity maintenance strategy. This may have contributed to the poor performance of MOEA/D-WS and MOEA/D-TS compared to the other algorithms.

Now, we turn to the results presented in Table III. For radar applications, it is desirable to have small R_{ac} values, while communication applications requiring small R_{cc} values. Integrated radar and communication systems require respective trade-offs between R_{ac} and R_{cc} depending on the emphasis of the particular system at hand.

In view of (4), the small R_{ac} value obtained for radar applications given in Table III (a) is independent of parameter m. It shall be noted that the desired small value of R_{ac} for radar applications implies large spectral spreading [21]. On the

TABLE II. PARAMETER SETTINGS FOR NSGA-2, SPEA-2, PESA-2, SMS-EMOA, MOWCA

Parameters	NSGA-2	SPEA-2	PESA-2	SMS-EMOA	MOEA/D	MOWCA
Crossover probability (p_c)	0.9	0.7	0.5	0.9	0.5	–
Mutation probability (p_m)	0.5	0.3	0.5	0.5	0.5	–
Crossover distribution index (η_c)	20	20	20	20	–	–
Mutation distribution index (η_m)	20	8	8	8	20	–
Scaling factor (F)	–	–	–	–	0.5	–
Archive size	–	20	20	–	–	20
Tournament round	–	1	–	–	–	–
Number of bisection	–	–	5	–	–	–
Closest neighbour number (T)	–	–	–	–	10	–
Number of weight vectors (W)	–	–	–	–	20	–
Number of streams and rivers (N_{sr})	–	–	–	–	–	4
Maximum allowable distance between river and sea (d_{max})	–	–	–	–	–	10^{-5}

TABLE III. SMS-EMOA DESIGNS FOR OPPERMANN SEQUENCES OF LENGTH $N = 31$

(a) Radar applications: Small R_{ac}

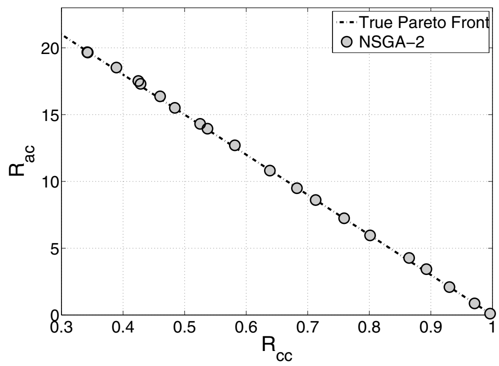
m	n	R_{ac}	R_{cc}
1.0022	2.0055	0.1112	0.9966

(b) Communication applications: Small R_{cc}

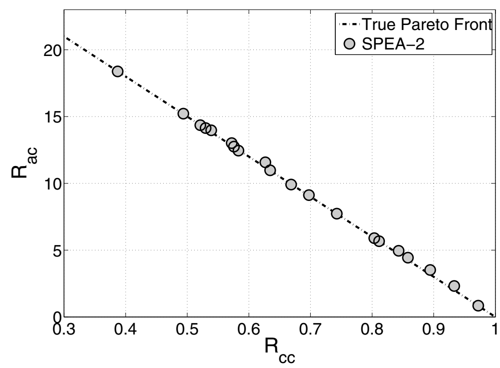
m	n	R_{ac}	R_{cc}
1.0037	1.0000	19.6774	0.3418

(c) Integrated Radar and communication applications: Trade-off between R_{ac} and R_{cc}

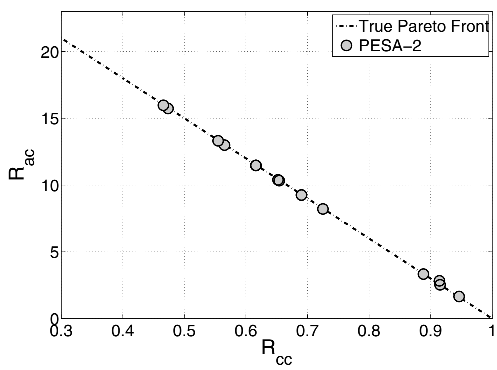
m	n	R_{ac}	R_{cc}
1.0038	1.2411	18.7445	0.3730
1.0038	1.2951	17.7839	0.4051
1.0039	1.3306	16.8175	0.4374
1.0038	1.3578	15.8553	0.4695
1.0039	1.3806	14.8896	0.5018
1.0038	1.4011	13.8971	0.5349
1.0038	1.4197	12.9024	0.5681
1.0038	1.4376	11.8975	0.6017
1.0038	1.4548	10.8839	0.6356
1.0038	1.4719	9.8618	0.6698
1.0037	1.4895	8.8352	0.7041
1.0038	1.5085	7.7973	0.7388
1.0038	1.5296	6.7721	0.7731
1.0037	1.5574	5.6963	0.8092
1.0036	1.5996	4.5956	0.8461
1.0037	1.6514	3.4906	0.8832
1.0034	1.7093	2.3848	0.9202
1.0032	1.8047	1.2546	0.9581



(a)



(b)



(c)

other hand, the small R_{cc} value reported in Table III (b) for communication applications implies low interference between different sequences. Table III (c) shows the trade-off between R_{ac} and R_{cc}, obtained by SMS-EMOA for integrated radar and communication systems. Note that SMS-EMOA has been chosen here, as it outperforms the other considered algorithms in terms of the distribution of solutions along the PF (see Fig. 2 (g)). Results for other algorithms have been omitted for the sake of brevity. Apparently, R_{ac} decreases along the PF with an increase of R_{cc}.

A. Comparison of Computational Time

Here, the computational time taken by each algorithm to solve the MOOP posed in (5) is compared. Fig. 3 shows the average computation time taken by each algorithm to execute a fixed NFE. The results are averaged over 30 independent runs. It can be seen that PESA-2 has taken the least amount of time to solve problem P followed by NSGA-2. The algorithms MOEA/D-WS, MOEA/D-TS and MOWCA are shown to consume more or less the same amount of time to solve P. Although, SMS-EMOA is shown to have the best coverage and distribution of final solutions along the PF, it is found to be expensive in terms of computational time compared to all the other algorithms. This is attributed to the complexity involved in the calculation of HV which has a complexity of order $O({N_p}^3, K^2)$ [17].

In summary, it can be concluded that sequence design for integrated radar and communication systems is a computationally expensive MOOP even for short sequences. Given the practical limitations in real-world engineering problems, an optimization task must also be completed in a reasonable computation time. This can be achieved by striking a good balance between the search operators (exploitation and exploration) [2].

VI. Conclusions

In this paper, we have formulated a sequence design optimization problem for integrated radar and communication system as a multiobjective optimization problem. We have studied the capabilities of some recently developed and well known meta-heuristic multiobjective optimization algorithms to solve the sequence design optimization problem. In particular, the problem of simultaneously minimizing average mean-square autocorrelation and average mean-square crosscorrelation by finding the optimal values of parameters m and n of Oppermann sequences has been studied.

Most of these algorithms are found to perform well on various mathematical test functions. However, our results indicate that not all of the considered algorithms are able to solve this optimization problem. The poor performance of some of the algorithms is thought to be due to the cost function being defined in terms of average mean-square correlations which may be extremely irregular and may have several local minima. As a result, some of these algorithms may get stuck in local minima, and thus cannot achieved a good approximation of the nondominated solutions along the PF.

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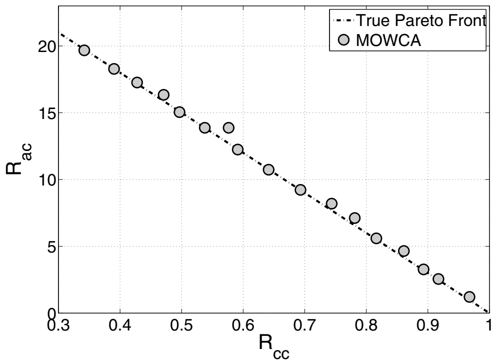
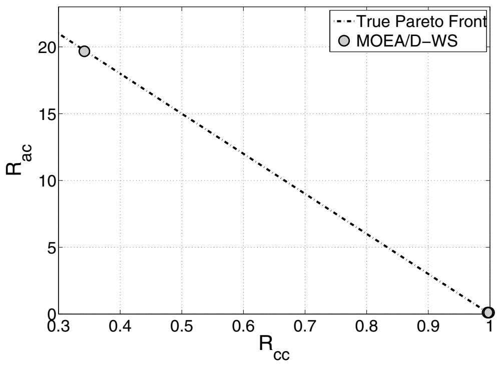


(d)



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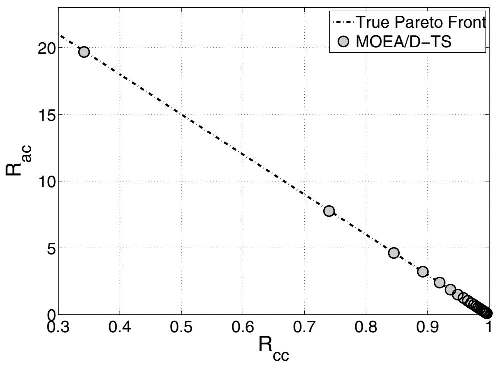
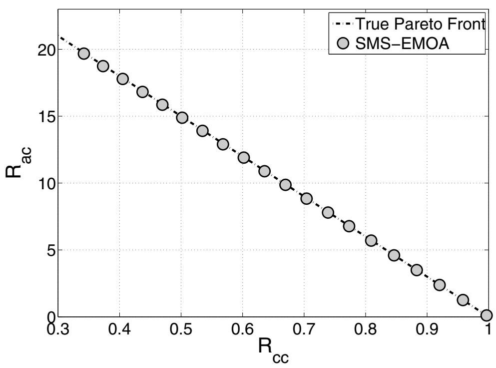


(f)



(g)

Fig. 2. PF of Oppermann sequences ($N = 31$): (a) NSGA-2, (b) SEPA-2, (c) PEAS-2, (d) MOWCA, (e) MOEA/D-WS, (f) MOEA/D-TS, (g) SMS-EMOA.

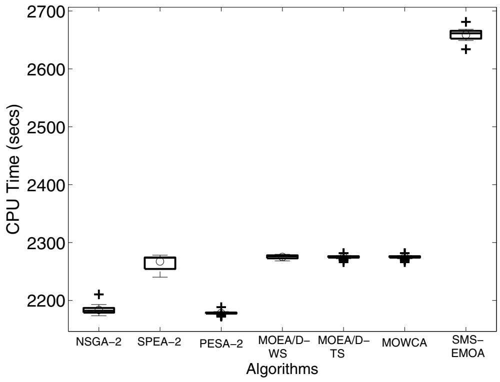


Fig. 3. Comparison of the computational time of the considered algorithms.

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