A New ZCT Precoding Based SLM Technique for PAPR Reduction in OFDM Systems

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***Abstract* — High Peak to Average Power Ratio (PAPR) in the transmitted signal is still a very important challenge in Orthogonal Frequency Division Multiplexing (OFDM) systems. In this paper, we present a new Zadoff-Chu matrix Transform (ZCT) precoded Selected Mapping (SLM) based OFDM (SLM-OFDM) system for PAPR reduction. The proposed system is based on precoding the constellation symbols with ZCT precoder after the multiplication of phase rotation factor and before the Inverse Fast Fourier Transform (IFFT) in SLM-OFDM Systems. At the clipping probability of 10^{-3}, simulation results show that our proposed system can reduce the PAPR up to 5.0 dB with N=64 (System subcarriers) and V=16 (Dissimilar phase sequences) for QPSK modulation. Additionally, ZCT based SLM-OFDM systems also take advantage of frequency variations of the communication channel and can also offer substantial performance gain in fading multipath channels.**

***Keywords* - Peak to Average Power Ratio (PAPR), Zadoff-Chu matrix Transform (ZCT), Orthogonal Frequency Division Multiplexing (OFDM), SLM based OFDM (SLM-OFDM)**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wireline digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference and frequency selective fading [1].

OFDM thwarts Inter Symbol Interference (ISI) by inserting a Guard Interval (GI) using a Cyclic Prefix (CP) and moderates the frequency selectivity of the Multi Path (MP) channel with a simple equalizer [2]. OFDM is widely adopted in various communication standards like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Wireless Local Area Networks (WLAN), Wireless Metropolitan Area Networks (WMAN), Wireless Personal Area Networks (WPAN) and even in the beyond 3G Wide Area Networks (WAN) etc.

However, among others, the Peak to Average Power Ratio (PAPR) is still one of the major drawbacks in the transmitted OFDM signal [3]. For zero distortion of the OFDM signal, the RF High Power Amplifier (HPA) must not only operate in its linear region but also with sufficient back-off. Thus, HPA with a large dynamic range are required for OFDM systems. These amplifiers are very expensive and are major cost components. Thus, if we reduce the PAPR it not only means that we are reducing the cost of OFDM system and reducing the complexity of A/D and D/A converters, but also increasing the transmit power, thus, for same range improving received SNR, or for the same SNR improving range.

The literature is replete with a large number of PAPR reduction techniques. Among them, the schemes like constellation shaping [4], phase optimization [5], nonlinear companding transforms [6], Tone Reservation (TR) and Tone Injection (TI) [7]-[8], clipping and filtering [9], Partial Transmit Sequence (PTS) [10], precoding based techniques [11]-[13], Selected Mapping (SLM) [14]-[16], Precoding based Selected Mapping (PSLM) [17], and phase modulation transform [18]-[20] are popular. The precoding based techniques, however, show great promise as they are simple linear techniques to implement.

In this paper, we present a new ZCT precoded SLM based OFDM (SLM-OFDM) system, for PAPR reduction. In the proposed system we applied ZCT based precoder after the multiplication of phase rotation factor and before the IFFT in the SLM-OFDM systems. This paper is organized as follows. Section II describes the basics of the SLM-OFDM systems, In Section III we present the proposed system model for PAPR reduction, and Section IV presents computer simulation results and section V describe the conclusion.

II. SLM BASED OFDM (SLM-OFDM) SYSTEM

The SLM is a PAPR reduction technique which is based on phase rotations. In SLM-OFDM system, a set of V different data blocks are formed at the transmitter representing the same information and a data block with minimum PAPR is selected for transmission. Fig. 1 shows the block diagram of the SLM-OFDM system.

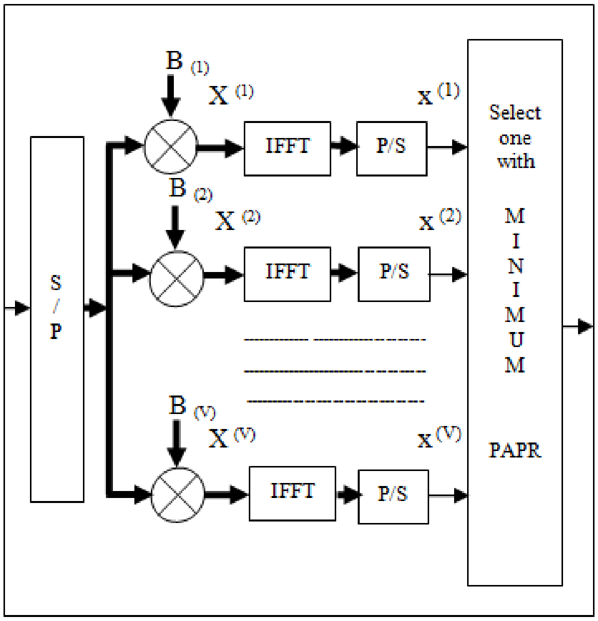

Fig. 1 Block diagram of SLM based OFDM (SLM-OFDM) system.

Suppose data stream after serial to parallel conversion is $X = [X_0, X_1, X_{2...} X_{N-1}]^T$ and each data block is multiplied by V dissimilar phase factors, each length equal to N, $B^{(v)} = [b_{v,0}, b_{v,1}, \dots, b_{v,N-1}]^T (v= 1, 2 \dots V)$, which results in the V altered data blocks. Therefore, the v^{th} phase sequence after multiplied is $X^{(v)} = [X_0 b_{v,0}, X_1 b_{v,1}, \dots, X_{N-1} b_{v,N-1}]^T$ $v=1, 2, 3 \dots V$. Each X_n^v can be defined as:-

$$X_n^v = X_n b_{v,n} \quad , \quad (1 \le v \le V) \tag{1}$$

SLM-OFDM signal with N subcarriers can be written as:-

$$x_n^{(v)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k^v . e^{j2\pi \frac{n}{N} k} \text{ , } n=0,1,2,\dots,N-1 \tag{2}$$

III. Proposed Model

A. Zadoff-Chu (ZC) Sequences and Zadoff-Chu matrix Transform (ZCT)

Zadoff-Chu (ZC) sequences are class of poly phase sequences having optimum correlation properties. ZC sequences have an ideal periodic autocorrelation and constant magnitude. According to [21], the ZC sequences of length N can be defined as:-

$$a_n = \begin{cases} e^{\frac{j2\pi r}{N}\left(\frac{k^2}{2}+qk\right)} & \text{for } N \text{ Even} \\ e^{\frac{j2\pi r}{N}\left(\frac{k(k+1)}{2}+qk\right)} & \text{for } N \text{ Odd} \end{cases} . \tag{3}$$

where k = 0, 1, 2… N-1, q is any integer, r is any integer relatively prime to N. The kernel of the ZCT is defined in (4). For $j = \sqrt{-1}$, the ZCT, A, of size $N = L \times L$ is obtained by reshaping the ZC sequence by $k = m + lL$ as hereunder:-

$$A = \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0(L-1)} \\ a_{10} & a_{11} & \cdots & a_{1(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(L-1)0} & a_{(L-1)1} & \cdots & a_{(L-1)(L-1)} \end{bmatrix} \tag{4}$$

Here m is the row variable and l the column variable. In other words, the $N = L^2$ point long ZC sequence fills the kernel of the matrix transform column-wise.

B. ZCT Precoded SLM based OFDM (SLM-OFDM) System

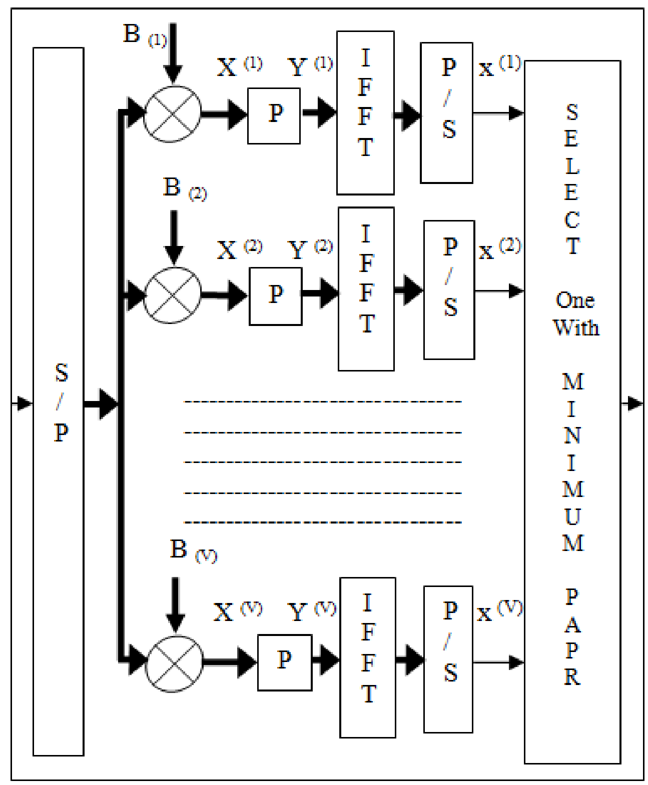

Fig. 2 Block diagram of ZCT Precoded SLM-OFDM System.

Fig. 2 shows the block diagram of the proposed ZCT precoded SLM-OFDM system. Suppose data stream after serial to parallel conversion is $X = [X_0, X_1, X_{2...} X_{L-1}]^T$ and each data block is multiplied by V dissimilar phase factors, each of length equal to L, $B^{(v)} = [b_{v,0}, b_{v,1}, \dots, b_{v,L-1}]^T$ $(v= 1, 2 \dots V)$, which results in the V altered data blocks. Therefore, the v^{th} phase sequence after multiplied is $X^{(v)} = [X_0 b_{v,0}, X_1 b_{v,1}, \dots, X_{L-1} b_{v,L-1}]^T$ $v=1, 2, 3 \dots V$.

Each X_l^v can be defined as:-

$$X_l^v = X_l b_{v,l} \qquad l = 0,1, \ldots L-1, v = 1,2,3, \ldots, V \tag{5}$$

After multiplication of the phase rotation factor, we pass the signal given in equation (5) through the ZCT precoder and the resultant signal can be written as:-

$$Y_m^v = \sum_{l=0}^{L-1} a_{m,l} X_l^v \qquad m = 0,1, \ldots L-1 \tag{6}$$

where $a_{m,l}$ means, m^{th} row and l^{th} column of the precoding ZCT matrix. Expanding equation (6), using column-wise sequence reshaping $k = m + lL$ and putting $r = 1$ and $q = 0$ in equation (3) we get:-

$$Y_m^v = \sum_{l=0}^{L-1} \left(e^{j\frac{\pi r (m+lL)^2}{L^2}}\right) X_l^v$$

$$= e^{j\frac{\pi r m^2}{L^2}} \sum_{l=0}^{L-1} \left(\left(e^{j\pi r l^2} . X_l^v\right) . \; e^{j\frac{2\pi r m l}{L}} \right) \tag{7}$$

where, $m = 0, 1, 2 \ldots L-1$. Equation (7) represents the ZC precoded data/constellations symbols/signal. The complex baseband ZCT-SLM-OFDM signal with L subcarriers can be written as:-

$$x_n^{(v)} = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} Y_m^v . e^{j2\pi \frac{n}{L} m} \text{ , } n=0,\ 1,\ 2\ldots\ L-1 \tag{8}$$

Using equations (7) and (8), we get:-

$$x_n^{(v)} = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} \left\{ e^{j\frac{\pi m^2}{L^2}} \left[\sum_{l=0}^{L-1} \left(\left(e^{j\pi l^2} . X_l^v\right) . \; e^{j\frac{2\pi m l}{L}} \right] \right\} . e^{j2\pi \frac{n}{L} m} \tag{9}$$

where v = *1, 2... V and n = 0, 1, 2... L-1*. Equation (9) represents the ZCT precoded SLM-OFDM signal. The PAPR of signals in equation (2) and (9) can be written as:-

$$PAPR = \frac{\max \left[\left| x_n^{(v)} \right|^2 \right]}{E\left[\left| x_n^{(v)} \right|^2 \right]} \tag{10}$$

where, E [.] denotes expectation.

IV. Simulation Results

Extensive simulations in MATLAB(R) have been carried out to evaluate the performance of our proposed ZCT precoding based SLM-OFDM system. To show the PAPR analysis of proposed system, the data is generated randomly then modulated by QPSK, with 10^5 random OFDM blocks. We evaluate the PAPR statistically by using Complementary Cumulative Distribution Function (CCDF). The CCDF of the PAPR for different precoding based OFDM system's signals are used to express the probability of exceeding a given threshold $PAPR_0$ ($CCDF = Prob\,(PAPR > PAPR_0)$). We also compared our results with conventional OFDM systems, ZCT-OFDM system, SLM-OFDM systems for N=64 and 256 with V=4, 8 and 16.

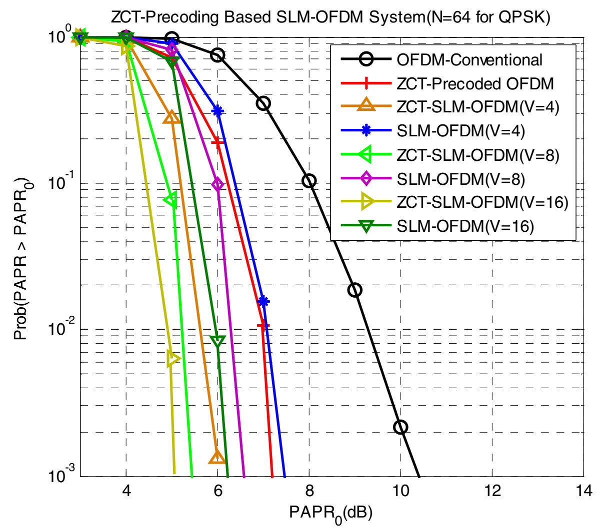

Fig. 3 CCDF of the PAPR of OFDM Original, ZCT-Precoded OFDM, SLM-OFDM and ZCT precoded SLM-OFDM with N=64 & V=4, 8, 16 for QPSK modulation.

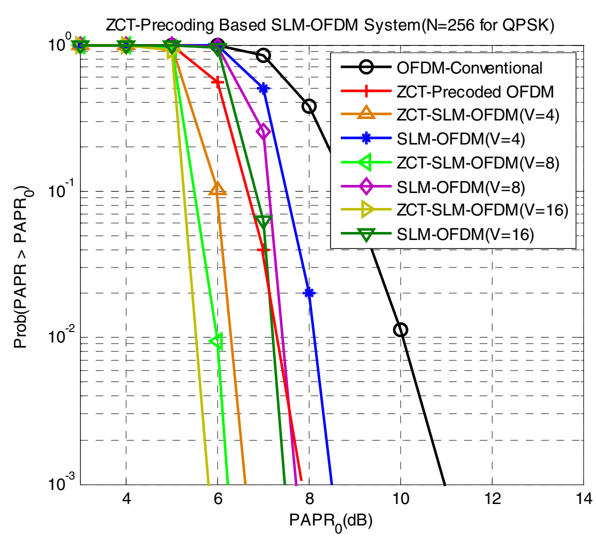

Fig. 4 CCDF of the PAPR of OFDM Original, ZCT-Precoded OFDM, SLM-OFDM and ZCT precoded SLM-OFDM with N=256 & V=4, 8, 16 for QPSK modulation.

Fig. 3 shows the CCDF comparisons of ZCT precoding based SLM-OFDM system with ZCT precoding based OFDM, SLM based OFDM system and OFDM conventional with QPSK modulation for *N*=64. At clip rate of 10^{-3}, the PAPR is 10.3 dB, 7.1 dB, 7.5 dB, 6.5 dB, 6.2 dB, 6.0 dB, 5.4 dB and 5.0 dB, for OFDM Conventional, ZCT precoding based OFDM System, SLM based OFDM (*V*=4), SLM based OFDM (*V*=8), SLM based OFDM (*V*=16), ZCT precoding based SLM-OFDM (*V*=4), ZCT precoding based SLM-OFDM (*V*=8) and ZCT precoding based SLM-OFDM (*V*=16) respectively.

Fig. 4 shows the CCDF comparisons of ZCT precoding based SLM-OFDM system with ZCT precoding based OFDM, SLM based OFDM system and OFDM conventional with QPSK modulation for *N*=256. At clip rate of 10^{-3}, the PAPR is 10.9 dB, 7.8 dB, 8.4 dB, 7.8 dB, 7.5 dB, 6.7 dB, 6.2 dB and 5.8 dB, for OFDM Conventional, ZCT precoding based OFDM System, SLM based OFDM (*V*=4), SLM based OFDM (*V*=8), SLM based OFDM (*V*=16), ZCT precoding based SLM-OFDM (*V*=4), ZCT precoding based SLM-OFDM (*V*=8) and ZCT precoding based SLM-OFDM (*V*=16) respectively.

V. CONCLUSIONS

In this paper, we proposed a new ZCT precoded SLM-OFDM system to reduce the high PAPR, generated by multi carrier modulation in the OFDM systems. From Fig. 3, it is concluded that at clip rate of 10^{-3}, $N = 64$ and $V = 16$, proposed technique can reduce the PAPR up to about 5.0 dB. Our proposed technique can reduce more PAPR if we increase the value of *V,* but with the increase in the value of *V* the computational complexity is also increased. Thus, the value of *V* should be chosen carefully. Additionally, this technique is efficient, signal independent, distortionless, it does not require any complex optimization.

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