Towards a Dual-Function MIMO Radar-Communication System

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***Abstract*—Recently, dual-function radar-communication systems in which the radar platform and resources are used for communication signal embedding have emerged as means to alleviate spectrum congestion and ease competition over frequency bandwidth. In this paper, we introduce a new technique for information embedding specific to multiple-input multiple output (MIMO) radar. We exploit the fact that in a MIMO radar system, the receiver needs to know the association of the transmit waveforms to the transmit antennas. However, this association can change over different pulse repetition periods without impacting the radar functionality. We show that by shuffling the waveforms across the transmit antennas over constant pulse repetition periods, a data rate of megabits per second can be achieved for a moderate number of transmit antennas. The probability of error is analyzed and the bounds on the symbol error rate are derived. Simulation examples are provided for performance evaluation and to demonstrate the effectiveness of the proposed information embedding technique.**

I. Introduction

Competition over frequency spectrum between radar and communications could be directly alleviated when both systems are allowed to share the same resources and be deployed from a single platform [1]– [17]. This requires the establishment of dual system functionality where identical signals and a common antenna array are used for both operations. Dual-function radar-communication (DFRC) systems have been recently introduced in a number of papers [18]–[23]. In this paper, we pave the way to establishing a dual-function multiple-input multiple-output (MIMO) radar-communication system by introducing a new DFRC system where the orthogonal radar waveforms, repetitively emitted from different antennas, are exploited for embedding symbols over the pulse repetition intervals. This permits a space-time signal representation lending itself to a MIMO-DFRC system.

Existing waveform-diversity based DFRC techniques either utilize the waveforms as communication symbols [18] or use orthogonal waveforms in tandem with sidelobe modulation [21]. It is important to note that in the aforementioned DFRC systems, the waveforms orthogonality is only necessitated by the communication part of system. Conversely, dual system functionality which builds on MIMO radar platforms demands waveform diversity which should then be used to implement viable communication schemes.

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In this paper, we propose a novel signaling scheme for information embedding in MIMO radar. The basic idea behind this scheme is embedding communication symbols by shuffling the independent waveforms across the transmit antennas. This shuffling is transparent to the radar operation and does not change or compromise the offerings of the MIMO radar configurations. We derive the achievable data rate under the proposed signaling scheme and show that the number of symbols that can be embedded is a factorial of the number of transmit antennas. Toward this end, the large number of degrees of freedom in choosing symbol combinations is exploited for the lowest possible symbol error rate (SER). The latter is investigated as a function of signal-to-noise ratio (SNR) and the direction of the communication receiver.

Since permutations used to shuffle the waveforms are known to the radar, the unshuffling enables restoring the coherent structure of the MIMO radar data, i.e., the primary MIMO radar operation is unaffected by the secondary communication function. In this respect, the only burden incurred by the DFRC system is the additional complexity associated with applying the permutation operation during the transmit mode and reversing the permutation during the radar receive mode.

It is important to note that the proposed signaling scheme, albeit involves both space and time, stops short of implementing space-time coding as it does not aim at creating redundancy in symbol transmission. However, it presents a building block for future extension towards a MIMO DFRC system. The main objective of this paper is to establish relationships between achievable communication data rate and existing MIMO radar resources.

The paper is organized as follows. Section II presents the MIMO radar signal model. Section III describes the proposed information embedding scheme while Section IV gives some performance analysis. Simulation results are given in Section VI and conclusions are drawn in Section VII.

II. MIMO Radar Signal Model

Consider a MIMO radar system comprising M omnidirectional colocated transmit antennas and a receive array of N colocated antennas. Without loss of generality, we assumed that the transmit array is a uniform linear array (ULA) with interelement spacing of d measured in wavelength. It is also assumed that both the transmit and receive arrays are closely spaced such that a target in the far-field would be seen from

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the same direction by both arrays. Let $\phi_m(t)$, $m = 1 \ldots M$, be M orthonormal waveforms, that is,

$$\int_{T_0} \phi_m(t)\phi^*_{m'}(t)dt = \delta(m - m'), \qquad (1)$$

where t is the fast time index, T_0 is the radar pulse width, $(\cdot)^*$ denotes the conjugate, and $\delta(\cdot)$ is the Kronecker delta function.

If Q targets are located in the far-field, the $N \times 1$ complex vector of the received observations can be expressed as [24]–[25]

$$\mathbf{x}\left(t, \tau\right) = \sum_{q=1}^{Q} \alpha_q\left(\tau\right) \left[\mathbf{a}^T\left(\theta_q\right)\mathbf{\Phi}(t)\right] \mathbf{b}\left(\theta_q\right) + \mathbf{n}\left(t, \tau\right) \qquad (2)$$

where τ is the slow time index, i.e., the pulse number, $\alpha_q\left(\tau\right)$ is the reflection coefficient of the q-th target, θ_q is the spatial angle associated with the q-th target, $\mathbf{a}\left(\theta\right)$ and $\mathbf{b}\left(\theta\right)$ are the steering vectors of the transmit and receive arrays, respectively, $(\cdot)^T$ stands for the transpose, $\mathbf{\Phi} \triangleq [\phi_1(t), \ldots, \phi_M(t)]^T$ is the $M \times 1$ vector of orthogonal waveforms, and $\mathbf{n}\left(t, \tau\right)$ is the $N \times 1$ vector of zero-mean white Gaussian noise. In (2), the reflection coefficients $\alpha_q\left(\tau\right),\ q = 1, \ldots, Q$ are assumed to obey the Swerling II target model [26].

At the radar receiver, matched-filtering extracts the received signal components associated with the individual transmitted waveforms. Assuming that the M transmit waveforms satisfy the orthogonality condition at all time-delays and Doppler-shifts within the range and velocity specifications of the radar, matched-filtering the received data to the waveforms yields the $MN \times 1$ extended virtual data vector

$$\begin{aligned}\mathbf{y}\left(\tau\right) &= \mathrm{vec}\left(\int_{T_0} \mathbf{x}\left(t, \tau\right)\mathbf{\Phi}^H\left(t\right)dt\right) \\ &= \sum_{q=1}^{Q} \alpha_q\left(\tau\right)\left[\mathbf{a}\left(\theta_q\right) \otimes \mathbf{b}\left(\theta_q\right)\right] + \tilde{\mathbf{n}}\left(\tau\right) \qquad (3)\end{aligned}$$

where $\mathrm{vec}\left(\cdot\right)$ is the operator that stacks the columns of a matrix into one column vector, $\otimes$ denotes the Kronecker product, $(\cdot)^H$ stands for the Hermitian transpose, and

$$\tilde{\mathbf{n}}(\tau) = \mathrm{vec}\left(\int_{T_0} \mathbf{n}(t, \tau)\mathbf{\Phi}^H(t)dt\right) \qquad (4)$$

is the $MN \times 1$ noise term whose covariance is given by $\sigma_z^2\mathbf{I}_{MN}$ with $\mathbf{I}_{MN}$ denoting the identity matrix of size $MN \times MN$. It is worth noting that, in practice, perfectly orthogonal waveforms with overlapped spectral contents cannot be achieved and, therefore, waveforms with low cross-correlations should be used. The problem of waveform design with low cross-correlations has been extensively studied in the literature (see [27]–[29], and references therein).

III. Proposed Formulations and Information Embedding Scheme

In this section, we show that MIMO radar with transmit waveform shuffling yields the same extended virtual data model at the radar receiver. The section also presents the proposed information embedding scheme based on shuffling the transmit MIMO radar waveforms.

A. MIMO Radar With Waveform Shuffling

Let $\mathbf{P}$ be an $M \times M$ arbitrary permutation matrix. Applying the permutation to the vector of waveforms $\mathbf{\Phi}(t)$ yields the $M \times 1$ vector of shuffled waveforms $\mathbf{\Psi}(t)$, that is

$$\mathbf{\Psi}(t) = \mathbf{P}\mathbf{\Phi}(t). \qquad (5)$$

It is worth noting that the vector of shuffled waveforms is also orthogonal, that is

$$\begin{aligned}\mathbf{\Psi}(t)\mathbf{\Psi}^H(t) &= \mathbf{P}\mathbf{\Phi}(t)\mathbf{\Phi}^H(t)\mathbf{P}^H \\ &= \mathbf{P}\mathbf{P}^T = \mathbf{I}_M, \qquad (6)\end{aligned}$$

where the permutation matrix properties $\mathbf{P}^H = \mathbf{P}^T$ and $\mathbf{P}^{-1} = \mathbf{P}^T$ are applied. Now, transmitting the vector of shuffled orthogonal waveforms, the $N \times 1$ complex vector of the radar received observations can be rewritten as

$$\tilde{\mathbf{x}}(t, \tau) = \sum_{q=1}^{Q} \alpha_q(\tau)\left[\mathbf{a}^T(\theta_q)\mathbf{\Psi}(t)\right]\mathbf{b}\left(\theta_q\right) + \mathbf{n}\left(t, \tau\right). \qquad (7)$$

Matched-filtering the data (7) to the shuffled vector of orthogonal waveform $\mathbf{\Psi}(t)$ yields

$$\begin{aligned}\tilde{\mathbf{y}}(\tau) &= \mathrm{vec}\left(\int_{T_0} \tilde{\mathbf{x}}(t, \tau)\mathbf{\Psi}^H(t)dt\right) \\ &= \sum_{q=1}^{Q} \alpha_q\left(\tau\right)\left[\mathbf{a}\left(\theta_q\right) \otimes \mathbf{b}\left(\theta_q\right)\right] + \breve{\mathbf{n}}(\tau), \qquad (8)\end{aligned}$$

where

$$\breve{\mathbf{n}}(\tau) = \mathrm{vec}\left(\left[\int_{T_0} \mathbf{n}(t, \tau)\mathbf{\Phi}^H(t)dt\right]\mathbf{P}^T\right) \qquad (9)$$

is the $MN \times 1$ additive noise term with covariance $\sigma_n^2\mathbf{I}_{MN}$.

By comparing the virtual signal models (3) and (8), it can be concluded that the MIMO radar with and without waveform shuffling yields the same data model at the radar receiver except for a permutation on the additive noise term. In this respect, the noise term waveform shuffling (9) can be expressed in terms of the noise term (4) using the following relationship

$$\breve{\mathbf{n}}(\tau) = [\mathbf{P} \otimes \mathbf{I}_M]\tilde{\mathbf{n}}(\tau). \qquad (10)$$

Since both sides in (10) have the same statistics, we conclude that waveform shuffling at the input of the transmit array does not change the MIMO radar operation.

B. Proposed Information Embedding

Let $\mathcal{S} = \{\phi_1(t), \ldots, \phi_M(t)\}$ be the set of orthogonal waveforms. The assignment of each of these waveforms to an antenna element is arbitrary and does not affect the radar operation. Such an arrangement is essentially a permutation of the elements of $\mathcal{S}_0$. There are $M!$ possible permutations, $\mathcal{S}_\ell$ for $\ell = 1, \ldots, M!$, with $\mathcal{S}_1 \triangleq \mathcal{S}$ being the reference arrangement. Making use of all possible permutations, the vector of orthogonal transmit waveforms $\mathbf{\Phi}(t)$ can be used to generate the following vectors of shuffled waveforms

$$\mathbf{\Psi}_\ell(t) = \mathbf{P}_\ell\mathbf{\Phi}(t), \quad \ell = 1, \ldots, M!, \qquad (11)$$

where the permutation matrices, $\mathbf{P}_\ell,\ \ell = 1, \ldots, M!$, are of size $M \times M$ each. Suppose that during a certain PRI, the transmitted set of waveforms is $\mathbf{\Psi}_\ell(t)$ and assume that a communications user knows its direction θ_c relative to the stationary MIMO transmit platform. Then, the signal at the output of the communication receiver is modeled as

$$\begin{aligned} r(t) &= \beta_{\mathrm{ch}} \mathbf{a}^T(\theta_c) \boldsymbol{\Psi}_\ell(t) + w(t) \\ &= \beta_{\mathrm{ch}} \mathbf{a}^T(\theta_c) \mathbf{P}_\ell \boldsymbol{\Phi}(t) + w(t), \end{aligned} \tag{12}$$

where β_{ch} is the channel coefficient which summarizes the propagation environment between the MIMO radar transmit array and the communication receiver and $w(t)$ is interference plus noise additive term which is assumed to be Gaussian with zero mean and variance σ_w^2.

The communication receiver has perfect knowledge of the vector of orthogonal waveforms $\boldsymbol{\Phi}(t)$. Matched-filtering the received data (12) to $\boldsymbol{\Phi}(t)$ yields

$$\begin{aligned} \mathbf{r} &= \mathrm{vec}\left(\int_{T_0} r(t) \Phi^H(t) dt \right) \\ &= \beta_{\mathrm{ch}} \mathbf{P}_\ell^T \mathbf{a}(\theta_c) + \mathbf{w} \\ &= \beta_{\mathrm{ch}} \mathbf{s}^{(\ell)}(\theta_c) + \mathbf{w}, \end{aligned} \tag{13}$$

where $\mathbf{s}^{(\ell)} \triangleq \mathbf{P}_\ell^T \mathbf{a}(\theta_c)$ and $\mathbf{w} = \mathrm{vec}\left(\int_{T_0} w(t) \Phi^H(t) dt \right)$. Thus, the communication receiver signal at the output of the matched-filter is a (scaled and noisy) permutation of the steering vector $\mathbf{a}(\theta_c)$, meaning that the permutation matrix $\mathbf{P}_\ell$ can be recovered from the received vector $\mathbf{r}$ by determining the shuffling of the elements of the steering vector $\mathbf{a}(\theta_c)$ provided that they are distinct. *We propose using the permutation matrices $\boldsymbol{P}_\ell$ as the codes and the vectors $\mathbf{s}_\ell$ as the communications symbols that are detected at the receiver*. Thus, a dictionary of $L \leq M!$ unique symbols can be constructed as

$$\mathbb{D} = \left\{ \mathbf{s}^{(1)}(\theta_c), \ldots, \mathbf{s}^{(L)}(\theta_c) \right\}, \tag{14}$$

The cardinality L of the dictionary determines the capacity of the system and will be discussed in section IV.

At this point, two remarks are in order:

Remark 1: The communication receiver knows its direction with respect to the MIMO radar transmit array, and the communication received signal (13) is dependent on θ_c. Therefore, it is expected that the communication receiver's ability to detect the embedded symbol depends on the direction at which the receiver is located. This will be discussed in more details in the paper in Sec. IV.

Remark 2: The MIMO radar receiver has perfect knowledge of the vector of shuffled waveforms $\boldsymbol{\Psi}_\ell(t)$ and, consequently, it is able to apply the correct matched-filters to undo the shuffling, as explained earlier in Sec. III-A. This therefore, does not impact the radar's primary task of estimating the unknown directions of the scatterers (targets) from the received signal.

C. Received Symbol Detection

Let us assume for the purpose of this work that the channel is non-changing and has a unity gain[1]. Since the noise follows a multivariate complex Gaussian distribution with zero mean and covariance matrix $\sigma^2 \mathbf{I}_M$, the pdf of $\mathbf{r}$ is given by

$$\begin{aligned} f(\mathbf{r}|\mathbf{s}^{(\ell)}) &= \frac{1}{(\pi\sigma^2)^M} \exp\left\{ -\frac{\|\mathbf{r} - \mathbf{P}_\ell^T \mathbf{a}(\theta_c)\|^2}{\sigma^2} \right\} \\ &= \frac{1}{(\pi\sigma^2)^M} \exp\left\{ -\frac{\|\mathbf{r} - \mathbf{s}^{(\ell)}\|^2}{\sigma^2} \right\}. \end{aligned} \tag{15}$$

The maximum likelihood detector then becomes

$$\hat{\mathbf{s}}^{(\ell)} = \arg\min_{\mathbf{s}\in\mathbb{D}} \|\mathbf{r} - \mathbf{s}(\theta_c)\|^2. \tag{16}$$

This is equivalent to finding the permutation matrix $\mathbf{P}$ such that

$$\hat{\mathbf{P}}_\ell = \arg\min_{\mathbf{P}} \left\| \mathbf{r} - \mathbf{P}^T \mathbf{a}(\theta_c) \right\|^2. \tag{17}$$

Expressing the minimization in terms of the permutation matrix ensures that the properties of the permutation are expressed in the detector. This minimization, however, is not a simple problem and its implementation in the general case is beyond the scope of this work. For small values of M we can enumerate the symbols $\mathbf{s}^{(\ell)}$ and an exhaustive search can be used to implement the detector. We employ this approach for the purpose of illustration in this paper.

IV. Performance Analysis

Now we turn to the proposed communications scheme. We first establish the maximum achievable bit rate. Then we discuss the dependency of the channel on the spatial angle of the communications receiver with respect to the transmitter and describe a simple strategy for mitigating this dependency. Finally we establish an upper bound on the symbol error rate.

A. Achievable Bit Rate

Given that the radar transmits one symbol per pulse, the symbol rate of the communications system is identical to the pulse repetition frequency (f_{PRF}). The number of bits that can be transmitted per symbol is determined by the total number of unique symbols (or permutations), L, that are available:

$$N_b = \lfloor \log_2(L) \rfloor, \tag{18}$$

where $\lfloor\cdot\rfloor$ denotes the floor function. The resulting bit rate of the system is $N_b \times f_{\mathrm{PRF}}$. In general, L is not a power of 2 and the required number of symbol to transmit the N_b bits is

$$L_s = 2^{\lfloor \log_2(L) \rfloor} \leq L. \tag{19}$$

The fact that only a subset of the available symbols is required offers flexibility in the design of the actual system as well as improved noise immunity. However, a comprehensive study of this task is beyond the scope of this work. In order to illustrate the performance of the proposed strategy, we employ enumeration (for relatively small M) to select the subset of L_s symbols with inter-symbol distances $\|\mathbf{s}^{(k)} - \mathbf{s}^{(\ell)}\|^2$.

[1] In practice, we assume that the channel is estimated accurately. Training sequences can be periodically transmitted to update the channel estimate.

B. Angular Ambiguities

Since the elements of the dictionary $\mathbb{D}$ are permutations on the steering vector $\mathbf{a}(\theta_c)$, L is a function of the number of unique element of $\mathbf{a}(\theta_c)$. For $\theta_c = 0$, that is when the communications receiver is at broadside, all elements of $\mathbf{a}(\theta_c)$ are real and $\mathbb{D} = \{\mathbf{s}^{(1)}(0)\}$ has only one element. We refer to this as the trivial ambiguity, and no information can be embedded via shuffling the transmit waveforms. When $\theta_c > 0$ is small, the phases of the elements of $\mathbf{a}(\theta_c)$ are equally spaced on the unit circle with phase difference $\varphi_c \triangleq 2\pi d \sin(\theta_c)$. As θ_c increases, the angle φ_c increases and we reach an angle θ_M for which we have the largest spread around the unit circle. At this point the minimum phase difference between any two elements of $\mathbf{a}(\theta_M)$ is at its maximum. We call this the maximal spread angle. This occurs when $\varphi_c = 2\pi/M$ and the corresponding spatial angle is given by

$$\theta_M = \sin^{-1}\left(\frac{1}{Md}\right). \tag{20}$$

For $\theta_M < \theta_c$, it is possible that two (or more) elements of $\mathbf{a}(\theta_c)$ have the same value. This happens when their phases are equal modulo 2π. The first non-trivial ambiguity occurs when $(M-1)\varphi_1 = 2\pi$. Solving for θ_1 we get

$$\theta_1 = \sin^{-1}\left(\frac{1}{(M-1)d}\right). \tag{21}$$

In this case, only two elements of $\mathbf{a}(\theta_1)$ overlap, namely $\mathbf{a}_0(\theta_1) = 1$ and $\mathbf{a}_{M-1}(\theta_1) = e^{j2\pi}$ and the cardinality of $\mathbb{D}$ is $L = (M-1)!$. In general ambiguities occur when $(M-n)\varphi = 2\pi$, giving $\varphi_n = \frac{2\pi}{M-n}$ for $n = 1 \dots M-1$. Solving for the spatial angles we have

$$\theta_n = \sin^{-1}\left(\frac{1}{(M-n)d}\right), \ n = 1 \dots M-1. \tag{22}$$

Note that for all angles θ_c other than the points of ambiguity, all elements of $\mathbf{s}^{\ell}$ are distinct and $L = M!$. Fig. 1 shows angular ambiguities and bits per symbol that can be transmitted versus spatial angle for a MIMO system with $M = 6$ and $d = 0.5$. The dotted blue curve gives the minimum angular separation of the elements of the symbol vectors $\mathbf{s}^{(\ell)}$. As predicted by (20), the maximal spread angle is $19.47°$, whereas the first ambiguity occurs for $\theta_c = 23.58°$ in agreement with (21). The number of bits that can be transmitted per symbol is shown by the solid red curve. The system is able to transmit 9 bits/symbols except at the discrete set of angles described in (22), where for each angle the number of bits is reduced depending on the number of elements with unique phases. It is important to note, however, that for angles where the minimum angular separation is small, the performance may be poor if the full number of bits is used. Therefore, it is advantageous to transmit at the maximal spread angle θ_M which gives the best performance. In the following section we describe a scheme to mitigate the ambiguities and steer the performance of the maximal spread angle to any receiver spatial angle.

C. Angular Ambiguity Mitigation

The ambiguities described above can be mitigated by inducing the required phases at the transmitter. Let us suppose we pre-multiply element-wise at the transmitter the vector of orthogonal waveforms, $\mathbf{\Psi}(t)$, by the vector $\mathbf{u} = [1 \; e^{j\varphi} \; \dots \; e^{j(M-1)\varphi}]$. Then the vector of phase-shifted orthogonal waveforms becomes $\tilde{\mathbf{\Psi}}(t) = \mathbf{u} \odot \mathbf{\Psi}(t)$. Clearly, $\tilde{\mathbf{\Psi}}(t)$ still comprises orthogonal waveforms and the radar operations is unchanged. The matched-filtered signal at the communications receiver, however, becomes

$$\begin{aligned}\mathbf{r} &= \beta_{\text{ch}} \mathbf{P}_\ell^T \left(\mathbf{a}(\theta_c) \odot \mathbf{u}\right) + \mathbf{w}. \\ &= \beta_{\text{ch}} \mathbf{s}^{(\ell)} + \mathbf{w}\end{aligned} \tag{23}$$

The received symbol vector now has elements $\mathbf{s}_k^{(\ell)} = e^{jk(\varphi_c + \varphi)}$. Thus, we can induce a specific phase progression, φ_d at the spatial angle θ_c of the communications receiver by setting $\varphi = \varphi_d - \varphi_c$. For example putting $\varphi_d = \varphi_M = \frac{2\pi}{M}$, we can induce the maximum angular spread at any communications receiver θ_c. In this manner, not only are we able to mitigate the ambiguities, but also to deliver the best symbol dictionary to any receiver.

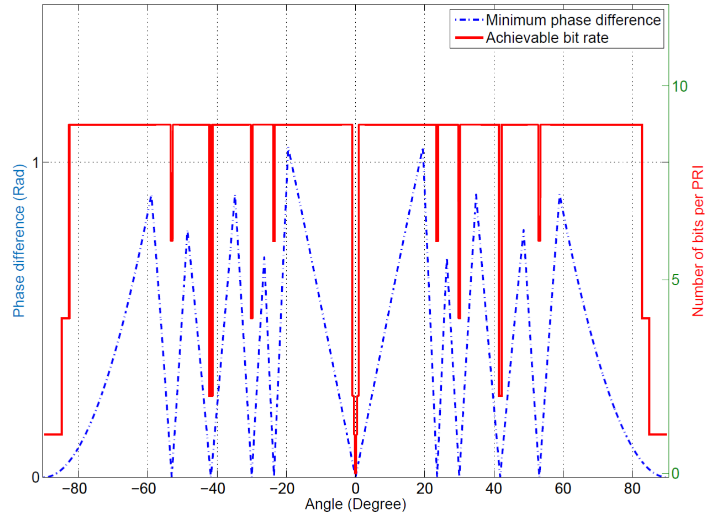

Fig. 1. System capacity as a function of the spatial angle of the communications receiver: the blue curve shows minimum angular separation between the symbol elements, while the red curve gives the number of bits per symbol (or pulse repetition interval) that can be transmitted.

D. Symbol Error Rate

Let us assume without loss of generality that the transmitted symbol be $\mathbf{s}^{(i)}(\theta_c)$. Then a symbol error occurs at the receiver whenever the noise places the received signal closer to another symbol, $\mathbf{s}^{(\ell)}(\theta_c)$, such that $\ell \neq i$. We now seek an upper bound for the probability, p_e, of symbol error. To this end, we proceed to write $p_e = 1 - P(\text{no error})$. Now let us define $d^{(\ell)} \triangleq \|\mathbf{r} - \mathbf{s}^{(\ell)}(\theta_c)\|^2$. Then the probability of a correct symbol detection is given by

$$P(\text{no error}) = P\left(d^{(i)} < d^{(\ell)}, \forall\, \ell = 1 \dots L, l \neq i\right). \tag{24}$$

Now for each symbol $\mathbf{s}^{(\ell)}(\theta_c)$, we have

$$\left\{d^{(i)} < d^{(\ell)}\right\} \supseteq \cap_{k=0}^{M-1} \left\{d_k^{(i)} < d_k^{(\ell)}\right\}. \tag{25}$$

Consequently, we have that

$$P(d^{(i)} < d^{(\ell)}) \geq \prod_{k=0}^{M-1} P\left(d_k^{(i)} < d_k^{(\ell)}\right). \quad (26)$$

The problem on the right hand side is similar to a Phase Shift Keying scenarios where the angular separation between two constellation points is given by $\gamma_k^{(i,\ell)} = \varphi^{(i)} - \varphi^{(\ell)}$. The possible values for γ_k are $= k\varphi_c$ for $m = 1, \ldots, (M-1)$. The probability of error can then be approximated at high SNR by $Q\left(\sqrt{2\rho}\sin\frac{\gamma_k}{2}\right)$, where $Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2)}}\right)$ and erfc(x) is the standard complementary error function. Thus, we get

$$P\left(d_k^{(i)} < d_k^{(\ell)}\right) = 1 - Q\left(\sqrt{2\rho}\sin\frac{\gamma_k}{2}\right). \quad (27)$$

Clearly, an error may only occur if the two entries are not equal. Then we arrive at

$$P(d^{(i)} < d^{(\ell)}) \leq \frac{1}{M}\sum_{k=0}^{M-1} P\left(d_k^{(i)} < d_k^{(\ell)} | \mathbf{s}^{(\ell)}(k)\right). \quad (28)$$

Since there are M elements, the upper bound on the probability of error becomes

$$P(\text{error}) \leq \left[\frac{1}{M}\sum_{k=0}^{M-1}\left(1 - Q\left(\sqrt{2\rho}\sin\frac{\gamma_k}{2}\right)\right)\right]^M. \quad (29)$$

V. Discussion

The MIMO communication system predicates on the channels linking different transmit and receive antennas to be of independent fading. The system implements space-time coding involving repeated symbol transmission across the different antennas, as in the case of space-time block codes. Viewing MIMO radar, with the proposed signaling scheme, as also a MIMO communication system requires: 1) Transmit array configurations with, at least, sufficient subarray spacing which lends itself to independent communication channels seen by the receiver. In this case, each subarray acts as an equivalent transmit antenna in MIMO communications; 2) An assumption that the signals from each subarray, although independently faded by the channel, maintain the same nominal angle of arrivals. The proposed DFRC and signaling scheme using MIMO radar platform inspires a dual-function MIMO radar communication system where both MIMO functionalities are preserved. Details of this system and its performance evaluation are beyond the scope of this paper.

VI. Simulation Results

In order to demonstrate the capacity of the proposed system for establishing communications over the MIMO radar platform, we consider a radar with $M = 6$ antennas. We evaluate the performance of the system by showing the SER first as a function of SNR, and secondly as a function of the communications receiver spatial angle. Figure 2 shows the SER versus SNR for $\theta_c = \theta_M$. We report curves for various numbers of bits/symbol and also include the theoretical upper bound of (29). Firstly, the SER curves show the expected standard behavior of a communications system, with the bit error rate increasing with decreasing SNR and with increasing number of bits/symbol. Secondly we note that the upper bound is valid for SNRs above 12dB and for SERs that are in the region of interest (that is SER less than 10^{-2}. The bound is not useful below SNR = 12dB as the approximations employed in its derivation are not valid.

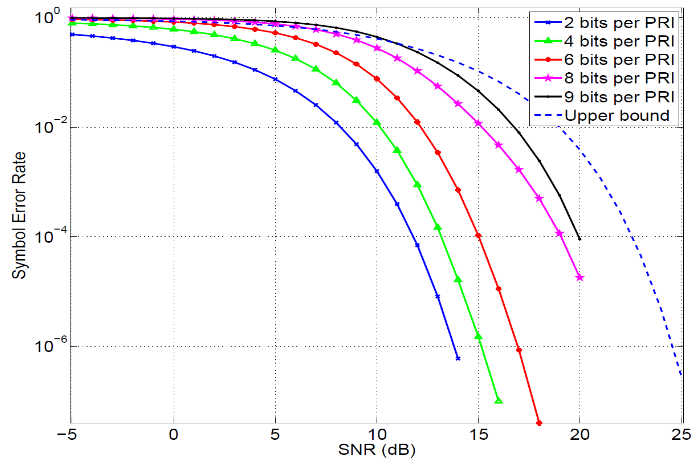

Fig. 2. SER versus SNR for the case where the communications receiver is at direction $\theta_M = 19.47°$.

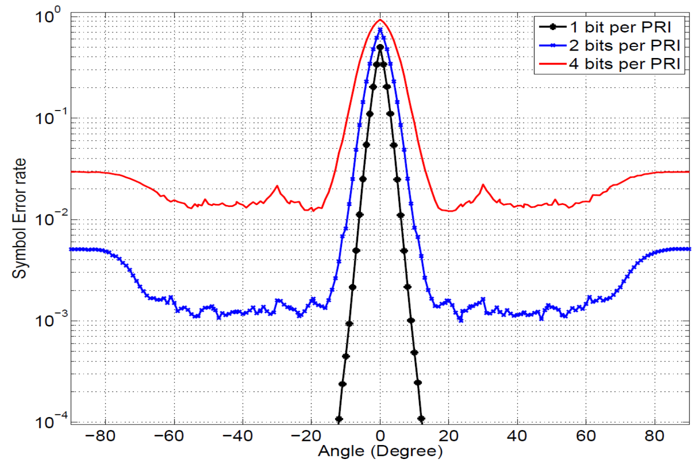

Fig. 3. SER versus spatial angle of the communication receiver with no induced phases at the transmitter. It is clear that the worst performance occurs at broadside ($\theta_c = 0$) where the system has no capacity to transmit information. The SNR employed is 10dB.

The performance as a function of the receiver spatial angle for fixed SNR=10 dB is shown in Figs. 3 and 4. We display performance curves for 1, 2, and 4 bits per symbol. Note that the curves for higher bits per symbol values of 8 and 9 are not shown since their SERs are quite high as can be seen from Fig. 2. The first figure shows the SER performance for the case where no performance steering is employed. As expected, the communications system exhibits its worst performance at broadside where it has no capacity to transmit information. Employing the phase induction method outlined above, we shift the worst performance point to receiver angle $\theta_c = -\theta_M$ and the best achievable performance to $\theta_c = 0°$.

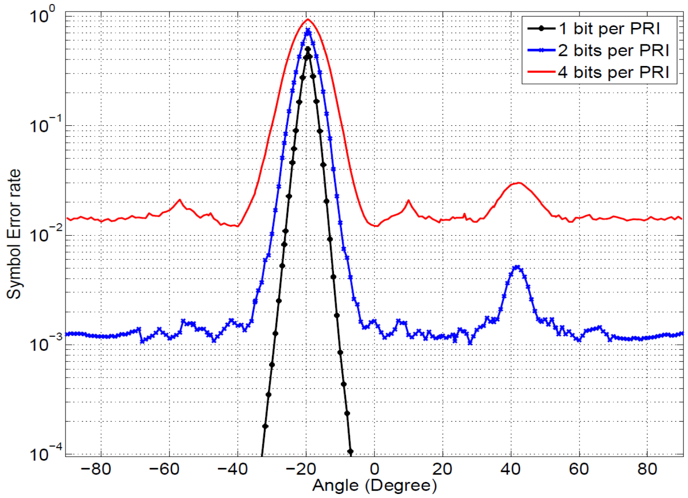

Fig. 4. SER versus spatial angle of the communication receiver with induced phases employed at the transmitter to shift the best performance towards $\theta_c = 0$. The worst performance has now been shifted to $-\theta_M = -19.47°$. The SNR employed is 10dB.

VII. Conclusions

The problem of dual-function cooperative radar-communication system design was considered. It was assumed that the system transmitter is either mounted on a stationary platform or its airborne spatial coordinate is provided to the communication receiver. In either case, the relative angle of the communication receiver to the radar transmitter is assumed known. A new technique for information embedding specific to MIMO radar was introduced. The proposed technique exploits the fact that the MIMO radar receiver requires the knowledge of transmit waveforms-transmit antennas pairing, and does not require pinning a specific waveform to a specific antenna. The flexibility of varying the waveforms across M transmit antennas over different pulse repetition periods can be exploited to embed a large constellation of $M!$ symbols. This allows the transmission of $\log_2(M!)$ bits per PRI. The probability of error was analyzed and the bounds on the symbol error rate were derived. Simulation examples were used to demonstrate the effectiveness of the proposed information embedding technique.

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