OFDM WAVEFORM DIVERSITY DESIGN FOR MIMO SAR IMAGING

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ABSTRACT

Multiple-input and multiple-output (MIMO) synthetic aperture radar (SAR) should use the waveforms that have a big product of time-width and frequency-width and a constant modulus. In this case, the Barker code-based waveforms are not suitable. In this paper, a kind of orthogonal frequency diversion multiplexing (OFDM) chirp diverse waveform is presented for MIMO SAR imaging. The waveform performance is evaluated by the radar ambiguity function. Numerical results show that the waveform has the superiorities of high range resolution, constant modulus, large time-banwidth product, implementation simplicity, and low ambiguity function sidelobes.

***Index Terms*—** Multiple-input and multiple-output (MIMO) radar, synthetic aperture radar (SAR), MIMO SAR, waveform diversity, orthogonal frequency diversion multiplexing (OFDM).

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar has received much attention in recent years; however, little work about MIMO synthetic aperture radar (SAR) has been investigated [1, 2, 3]. Note that MIMO SAR is different from the general MIMO radar in that aperture synthesis is employed in the MIMO SAR [4]. MIMO SAR provides potential solutions to resolving the disadvantages of conventional single-antenna SAR in high-resolution wide-swath remote sensing, ground moving target indication, three-dimensional imaging, etc. Given that MIMO SAR is in its infancy, there is no clear definition of what it is. As shown in Fig. 1, it is generally assumed that independent signals are transmitted through different antennas, and these signals, after propagating through the environment, are received by multiple antennas.

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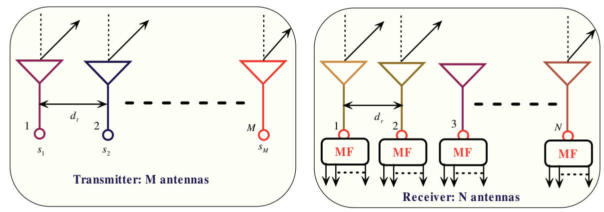

Fig. 1. Illustration of a MIMO SAR system.

Different from phased-array SAR, in MIMO SAR each antenna should transmit a unique waveform, orthogonal to the waveforms transmitted by other antennas. To obtain a high range resolution, the autocorrelation of the transmitted waveforms should be close to an impulse function. Moreover, since SAR is usually placed inside an airplane or satellite, a high average transmit power is required; hence, the waveforms should have a large product of time-width and frequency-width. Certainly the waveforms should also have good ambiguity characteristics such as range resolution, Doppler tolerance, adjacent-band interference, and matched filtering sidelobe performance. A typical orthogonal waveform is the Baker codes [5, 6]; however, the Barker code-based waveforms often have only a single carrier frequency. Consequently they are not suitable for high-resolution SAR imaging.

In this paper, we describe an orthogonal frequency division multiplexing (OFDM) chirp diverse waveform [7] for MIMO SAR imaging. The proposed OFDM chirp diverse waveform has a large time-bandwidth product and almost constant modulus. The remaining sections are organized as follows. The OFDM chirp waveform is designed in Section 2. Ambiguity function analysis and simulation results are provided in Section 3. Discussions are provided in Section 4. This paper is concluded in Section 5.

2. OFDM CHIRP DIVERSE WAVEFORM

As chirp (called also as linearly frequency modulation (LFM)) waveform has been widely used in different radars due to its

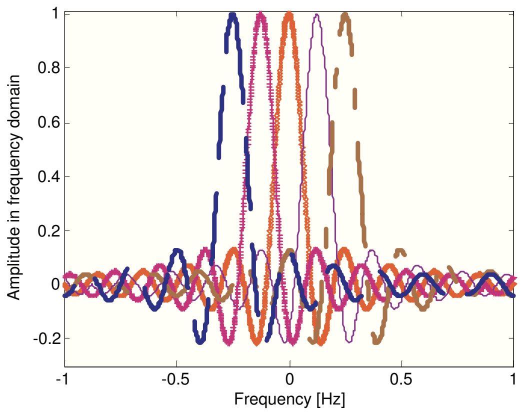

Fig. 2. An illustration of an OFDM signal spectra.

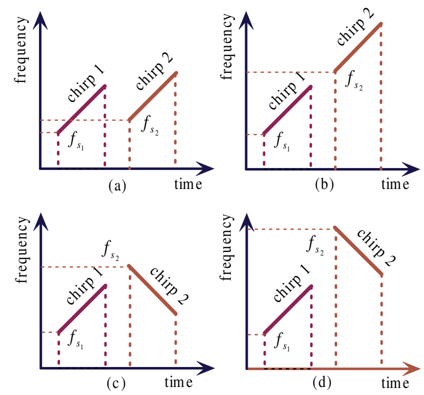

Fig. 3. Different combinations between two chirp waveforms.

good properties such as high range resolution, constant modulus, Doppler tolerance and implementation simplicity , from a practical point of view we think that a good waveform should be designed based on chirp signals. We describe the OFDM chirp waveform from a general chirp waveform which can be represented by the starting frequency f_s, chirp rate k_r, and chirp duration T_p. Neglecting amplitude and carrier frequency terms, a chirp waveform can be expressed by

$$s(t) = \mathrm{rect}\left[\frac{t}{T_p}\right]\exp\left[j2\pi\left(f_s t + \frac{1}{2}k_r t^2\right)\right] \tag{1}$$

OFDM concept was originally developed as a wideband communication modulation technique. The primary disadvantage of using OFDM in communications lies in that time and frequency synchronization is crucial to ensure subcarrier orthogonality; however, sensitivity to time and frequency synchronization is beneficial for radar systems because radar receiver usually uses a stored version of the transmitted signal to matched filtering the received signals. Since the subcarriers are orthogonal, the OFDM signal spectrum can be considered as a sum of the frequency-shifted sinc functions in frequency domain as shown in Fig. 2, where the overlapped neighboring sinc function are spaced by Δf.

Different from conventional OFDM waveforms, we use chirp signals as the OFDM symbols. From also a practical point of view, we suppose the chirp signals have equal frequency bandwidth with inverse or equal chirp rate. The baseband OFDM chirp waveform with P subcarriers and M temporal chips each with duration T_c can be expressed as

$$\begin{aligned} s(t) = &\sum_{p=0}^{P-1}\sum_{m=0}^{M-1} a_{p,m}u(t-mT_c)\exp\left(j2\pi\Delta f_{p,m}(t-mT_c)\right) \\ &\times \exp\left(j\pi k_{p,m}(t-mT_c)^2\right) \end{aligned} \tag{2}$$

where $\Delta f_{p,m}$ and $k_{p,m}$ denote respectively the corresponding baseband frequency and chirp rate for the subcarriers, and $u(t)$, $0 \le t \le T_c$. The OFDM pulse duration T_p should be satisfactory with $MT_c \ll T_p$. Each temporal chip is modulated by a complex weight

$$a_{p,m} = A_{p,m}\exp\left(j\phi_{p,m}\right) \tag{3}$$

The complex weight's amplitude is limited to $A_{p,m} \in [0, A_{\max}]$ where $A_{\max}$ is the largest allowable amplitude and the weight's phase is limited to $\phi_{p,m} \in [0, 2\pi)$.

Consider the different combinations between two chirp waveforms shown in Fig. 3, to obtain a good orthogonality between the OFDM chirp diverse waveforms, the subcarrier frequency separation should be optimally chosen. It is obvious that the performance of cross-correlation suppression will improve with the increase of the separation between two starting frequencies. However, for a given requirement of frequency bandwidth, this means a wider transmit/receive bandwidth for the radio frequency hardware. We concluded previously that using the subchirp signals with an equal chirp rate and non-overlapped frequency coverage or the subchirp signals with an inverse chirp rate and overlapped frequency coverage offer a satisfactory suppression of the cross-correlation components [8]. As a compromise, the subchirp signals which occupy adjacent starting frequency or overlapping frequency coverage with inverse chirp rate are used for the OFDM chirp diverse waveform. Figure 4 gives an example OFDM chirp diverse waveform, where the number of subcarriers and the number of chips are used for illustration purpose only.

3. AMBIGUITY FUNCTION AND NUMERICAL SIMULATION RESULTS

The performance of the OFDM chirp diverse waveform can be evaluated by the ambiguity function, which is an effective tool used to predict the matched filtering output performance when there are time-delay mismatch and/or Doppler

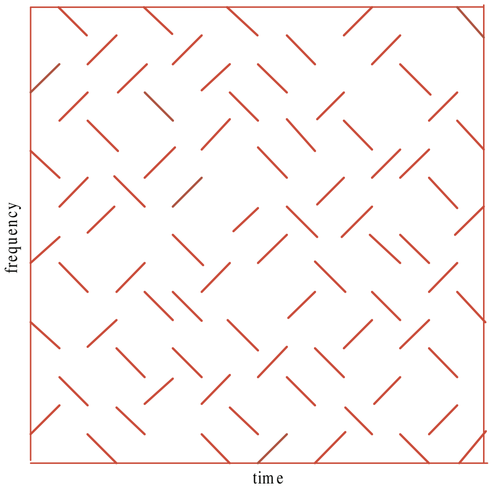

Fig. 4. An example pulse of the OFDM chirp waveform.

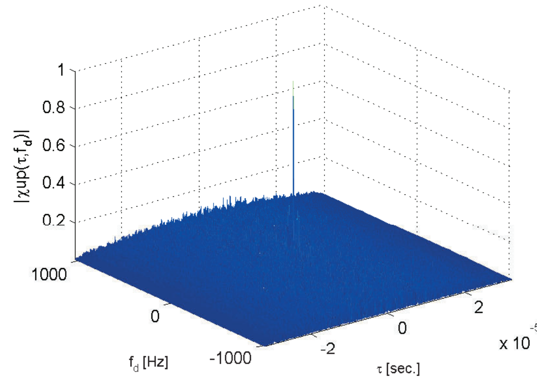

Fig. 5. Ambiguity function of the OFDM chirp waveform.

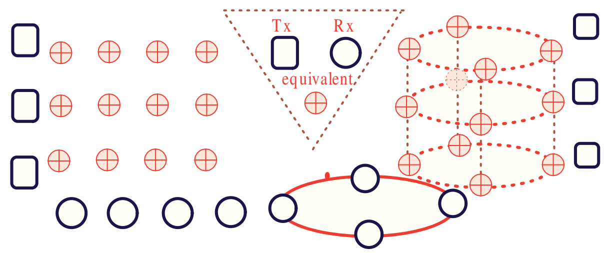

Fig. 6. Equivalent two-dimensional and three-dimensional phase centres.

mismatch. The general radar ambiguity function of signal $s(t)$ is defined as

$$|\chi(\tau, f_d)| \triangleq \left| \int_{-\infty}^{\infty} s(t)s^*(t+\tau)e^{j2\pi f_d t}\mathrm{d}t \right| \tag{4}$$

where $(\cdot)^*$ is a conjugate operator. This ambiguity function indicates the matched filtering output in the receiver when a delay mismatch τ and a Doppler mismatch f_d occur.

As modulation and scalar multiplication will not change the shape of the ambiguity function, we define the ambiguity function of the OFDM chirp waveform as

$$\begin{aligned}\chi(\tau, \nu) &= \int_{\infty}^{\infty} s(t)\hat{s}^*(t+\tau)e^{j\nu t}\mathrm{d}t \\ &= \int_{-\infty}^{\infty} \sum_{p=0}^{P-1}\sum_{m=0}^{M-1} a_{p,m}a^*(p,m)u(t-mT_c)u(t+\tau-mT_c) \\ &\quad \times \exp\left[j2\pi\Delta f_{p,m}(t-mT_c)\right] \\ &\quad \times \exp\left[-j2\pi\Delta f_{p,m}(t+\tau-mT_c)\right] \\ &\quad \times \exp\left[j\pi k_{p,m}(t-mT_c)^2\right] \\ &\quad \times \exp\left[-j\pi k_{p,m}(t+\tau-mT_c)^2\right]e^{j\nu t}\mathrm{d}t\end{aligned} \tag{5}$$

where $\hat{s}^*(t)$ is the filter matched to the expected signal $s(t)$, τ is a temporal mismatch between the filter and the signal, and ν is the Doppler frequency in radians per second. This equation can be evaluated though numerical computation.

Suppose the OFDM chirp diverse waveform shown in Figure 4 has the following parameters: The number of subcarriers is 16, the number of chips is 16, the subcarrier bandwidth is 10 MHz, and the chip duration is $3\mu s$, Figure 5 gives the corresponding ambiguity function. As the value of $|\chi(0,0)|$ represents the matched filtering output without any mismatch, the shaper the function $|\chi(\tau,\nu)|$, the better range resolution and Doppler resolution can be obtained for the radars. It can be noticed that the OFDM chirp diverse pulse has a satisfactory ambiguity performance in both range resolution and Doppler frequency resolution.

4. DISCUSSIONS

While compared to conventional SAR imaging, MIMO SAR provides increased degrees-of-freedom due to the virtual phase centres [9]. Two-dimensional and three-dimensional equivalent phase centres can be synthesized midway between each transmitter-receiver pair. Figure 6 shows an example two-dimensional array and an three-dimensional cylindrical array. Note that many other forms of two-dimensional three-dimensional equivalent phase centres may be possible for the MIMO SAR using OFDM waveforms [10].

One application potential of the MIMO SAR using OFDM chirp waveform is ground moving targets indication (GMTI). MIMO SAR based GMTI is interesting because the increased degrees-of-freedom enables that a large effective aperture can be obtained [11]. Figure 7 shows the geometry of a MIMO SAR GMTI system, where the SAR platform

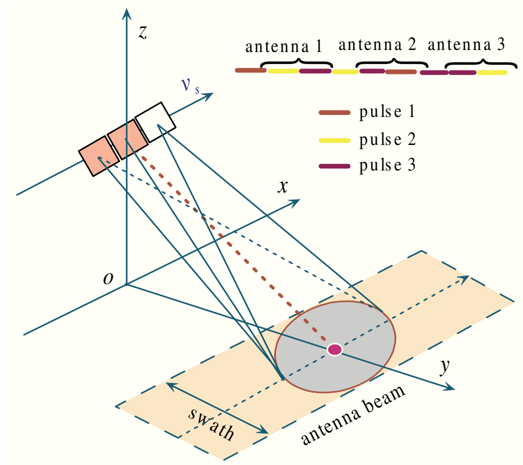

Fig. 7. Illustration of MIMO SAR GMTI with multiple azimuth antennas.

is moving along x-axis at a constant speed. Note that it is different from the conventional SAR with three antennas in azimuth, because in MIMO SAR the waveforms transmitted by the three antennas are independent but in conventional SAR the same waveform is transmitted by the three antennas. As the multiple antennas are displaced in azimuth direction, we can perform clutter suppression like the DPCA-based GMTI technique [12]. Next, GMTI can be obtained by the interferometry processing algorithm [11].

5. CONCLUSION

MIMO SAR is a recently proposed remote sensing concept. It has been shown that MIMO SAR can be used to improve remote sensing system performance by utilizing the increased degrees-of-freedom. This paper presents a kind of OFDM chirp waveforms for MIMO SAR imaging. The ambiguity function performances are analyzed. Numerical results are also provided. It is shown that the OFDM chirp waveforms have the superiorities of high range resolution, constant modulus, big product of time-width and frequency-width, Doppler tolerance, implementation simplicity, and low ambiguity function sidelobes. Our subsequent work is planned to develop high-precision MIMO SAR image formation algorithms.

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