OFDM RADAR WAVEFORM DESIGN WITH SPARSE MODELING AND CORRELATION OPTIMIZATION

Sheng-Juan Cheng, Wen-Qin Wang⋆ *, Huaizong Shao*

School of Communication and Information Engineering,
University of Electronic Science and Technology of China, Chengdu, 611731, China.
Email: shengjuancheng@gmail.com; ⋆wqwang@uestc.edu.cn; hzshao@uestc.edu.cn.

ABSTRACT

Large time-bandwidth product waveform diversity design is a challenging topic in multiple-input multiple-output radar high-resolution imaging because existing methods usually can generate only two large time-bandwidth product waveforms. This paper proposes a new low peak-to-average ratio (PAR) orthogonal frequency division multiplexing chirp waveform diversity design through randomly subchirp modulation. This method can easily yield over two orthogonal large time-bandwidth product waveforms. More waveforms means that more degrees-of-freedom can be obtained for the system. The waveform performance is evaluated by the ambiguity function. It is shown that the designed waveform has the superiorities of a large time-bandwidth product which means high range resolution and low transmit power are allowed for the system, almost constant time-domain and frequency-domain modulus, low PAR and no range-Doppler coupling response in tracking moving targets.

Index Terms— Waveform diversity design, MIMO radar, OFDM waveform, chirp diverse waveform, ambiguity function.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular choice for radar waveform because it offers many advantages such as robustness against multipath fading and simple synchronization [1]. OFDM waveforms have been shown to be suitable for radar applications [2, 3]. A sparse modeling-based frequency-hopping code design is proposed in [4], but monochromatic frequency signals are used in each frequency interval and consequently results in poor peak-to-average ratio (PAR) performance. The up- and down-chirp waveforms are used in [5], but there are only two pseudo-orthogonal waveforms. In [1], a novel OFDM chirp waveform design scheme is proposed, which is further improved in [6]. This scheme allows the designed two waveforms to occupy common spectral support. It can be easily extended to generate over two pseudo-orthogonal waveforms, but they will generate large grating lobes [7]. For each transmitted signal, a separate receiving beam is needed to suppress the grating lobes [8]. A space-time coding Alamouti waveform scheme is presented in [9]. We have proposed an OFDM chirp waveform design method [10], which uses chirp signals instead of monochromatic frequency signals.

In this letter, we use sparse modeling to design OFDM chirp waveforms and optimally design the waveform parameters to achieve improved performance. First, we calculate the block coherence measure of the sensing matrix and then select hopping frequencies of the waveforms. Thereafter, we adaptively adjust the waveform amplitudes during each hopping interval to yield improved correlation and ambiguity performance. Relative to [10], this letter optimally designs the waveforms with mathematical optimization, whereas the waveform is heuristically designed in [10].

2. SIGNAL MODEL

Consider an MIMO radar with M_T transmit antennas and M_R receive antennas arranged in linear arrays. The transmit and receive inter-elements are separated by d_T and d_R, respectively. Furthermore, we assume that these arrays form an angle θ with respect to a far-field target. Suppose the waveform emitted from the mth antenna consists of Q subchirp signals:

$$u_m(t) = \sum_{q=0}^{Q-1} b_{m,q} e^{j2\pi c_{m,q}\Delta f t + j\pi k_r t^2} \phi(t - q\Delta t) \tag{1}$$

where k_r is the subchirp rate, Δf is the subchirp bandwidth, $c_{m,q}$ is the hopping frequency index, $b_{m,q}$ is the subchirp amplitude, and the baseband signal is $\phi(t) = 1, 0 < t < \Delta t$ with Δt being the subchirp duration.

The cross-correlation between any two subchirps with hopping frequency indexes $c_{m,q}$ and $c_{m',q'}$ during one fre-

This work described in this paper was supported by the Program for New Century Excellent Talents in Universities under grant NCET-12-0095 and Sichuan Province Science Fund for Distinguished Young Scholars under grant 2013JQ0003.

quency hopping interval can be expressed as

$$\begin{aligned}&\frac{1}{\Delta t}\int_{0}^{\Delta t} e^{j2\pi c_{m,q}\Delta f t + j\pi k_r t^2}\left(e^{j2\pi c_{m',q'}\Delta f t + j\pi k_r t^2}\right)^{*}\mathrm{d}t \\ &= \operatorname{sinc}\left[\pi\left(c_{m,q} - c_{m',q'}\right)\Delta f \Delta t\right]\end{aligned} \tag{2}$$

where * is the conjugate operator and $\operatorname{sinc} = \sin(x)/x$. To make the two subchirps orthogonal, we should choose $\Delta f = \frac{i}{\Delta t}$, with i being a positive integer.

According to (1), the OFDM chirp waveform diversity design amounts to optimally choose $c_{m,q}$ and $b_{m,q}$ for all the subchirps. We assume that each $c_{m,q}$ takes a value from the set $\{1, 2, \cdots, G\}$, where G is a positive integer which represents the number of frequency intervals. To ensure waveform orthogonality, in each hopping interval q we should make $c_{m,q} \neq c_{m',q}, \forall m \neq m'$. Thus, we arange $c_{m,q}$ and $b_{m,q}$ into a code matrix $\mathbf{C} \in \mathbb{C}^{M_T \times Q}$ and an amplitude matrix $\mathbf{B} \in \mathbb{C}^{M_T \times Q}$, respectively. The $\mathbf{C}$ describes all the waveform frequencies, whereas $\mathbf{B}$ stores all the waveform amplitudes.

The received signal in discrete time at the kth receiver is

$$\begin{aligned} y_k(n) = &\sum_{m=1}^{M_T}\sum_{q=0}^{Q-1} \alpha_{m,q} b_{m,q} e^{j2\pi c_{m,q}\Delta f (nT_s - \tau) + j\pi k_r (nT_s - \tau)^2} \\ &\times \phi(nT_s - q\Delta t - \tau) e^{j2\pi \nu n T_s} e^{j2\pi d_r \sin\theta(\gamma m + k)/\lambda} + e_k(n) \end{aligned} \tag{3}$$

where $\alpha_{m,q}$ is the target reflection coefficient, τ and ν represent the time delay and Doppler shift, respectively, and $e_k(n)$ is the corresponding additive noise, λ is the carrier wavelength, $\forall n = 1, \ldots, N$ with N being the total number of samples at each receiver during one processing interval, T_s is sampling interval, and $\gamma = d_T/d_R$.

3. OPTIMAL OFDM CHIRP WAVEFORM DESIGN

3.1. Sparse Modeling

We use sparse modeling to represent the MIMO radar data model in (3). For each of the R targets, the unknown parameters include the attenuation, delay, and Doppler. We discretize the delay-Doppler space into V uniformly spaced grid points. Only R of these grid points correspond to the true target parameters. Let τ_v and ν_v represent the delay and Doppler corresponding the vth grid point, respectively.

Considering only the phase terms in (3) and defining $v \in \{1, \ldots, V\}$, we have [4]

$$\begin{aligned} \psi_{m,k,q}(n, v) = &e^{j2\pi c_{m,q}\Delta f (nT_s - \tau_v) + j\pi k_r (nT_s - \tau_v)^2} \\ &\times \phi(nT_s - q\Delta t - \tau_v) e^{j2\pi \nu_v n T_s} e^{j2\pi d_R \sin\theta(\gamma m + k)/\lambda}. \end{aligned} \tag{4}$$

We stack $\left\{\psi_{m,k,q}(n, v)\right\}_{n=1}^{N}$ into an N-element column vector

$$\psi_{m,k,q}(v) = \left[\psi_{m,k,q}(1, v), \cdots, \psi_{m,k,q}(N, v)\right]^T \tag{5}$$

where ^{T} is the transpose operator. Suppose M_R receiving antennas, the $\left\{\psi_{m,k,q}(v)\right\}_{k=1}^{M_R}$ is further stacked into an NM_R-element column vector $\psi_{m,q}(v)$. Each vector corresponds to a different transmitter and hopping interval, and we stack the columns corresponding to the same hopping interval together. Now, for each grid point v, we stack the column vectors into an $NM_{\mathrm{R}} \times M_T Q$ matrix $\mathbf{\Psi}(v)$ and arrange $\{\mathbf{\Psi}(v)\}_{v=1}^{V}$ into an $NM_{\mathrm{R}} \times VM_T Q$ matrix $\mathbf{\Psi}$, which is just the dictionary matrix that defines the basis elements of our sparse representation.

Next, we use the sparse model to optimally design $u_m(t)$ (see (1)) and choose $b_{m,q}$ and $c_{m,q}$. Let $\mathbf{\Psi}(v)$ and $\mathbf{\Psi}(v')$ denote the vth and v'th blocks of Ψ, respectively. Each block contains $M_T Q$ columns correspond to distinct transmitter and hopping interval. To make $c_{m,q} \neq c_{m',q}, \forall m \neq m'$, we remove the columns of $\mathbf{\Psi}(v)$ that are the same to the corresponding columns in $\mathbf{\Psi}(v')$. Let

$$D_{v,v'} = M_T Q - d_{v,v'} \tag{6}$$

where $d_{v,v'}$ denotes the number of same columns in $\mathbf{\Psi}(v)$ and $\mathbf{\Psi}(v')$. Define the correlation matrix $\mathbf{M}_{v,v'}$ for each pair of $\mathbf{\Psi}$ blocks as

$$\mathbf{M}_{v,v'} = \mathbf{\Psi}^H(v)\mathbf{\Psi}(v'). \tag{7}$$

where ^{H} is the conjugate transpose operator. Note that each entry of $\mathbf{M}_{v,v'}$ contains the auto-correlations between distinct columns of the selected blocks. Using these notations, we can define the block coherence measure as

$$\mu_B = \max_{v \neq v'} \frac{1}{D_{v,v'}} \lambda_{\max}^{1/2}\left(\mathbf{M}_{v,v'}^H \mathbf{M}_{v,v'}\right) \tag{8}$$

where $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue.

Minimizing the block coherence measure implies a theoretical guarantee for sparse support recovery of signals. Since $\lambda_{\max}^{1/2}\left(\mathbf{M}_{v,v'}^H \mathbf{M}_{v,v'}\right)$ is independent of $\mathbf{C}$, it can be ignored in the code matrix design. Analogous to [4], we define

$$\beta(\mathbf{C}) = \max_{v,v' \neq v} \frac{1}{D_{v,v'}} = \max_{v,v' \neq v} d_{v,v'} = \max_{v,v \neq v'} \sum_{m,q,q' = q - \frac{\tau_v - \tau_{v'}}{\Delta t}} \xi_{mqmq'} \tag{9}$$

where $\xi_{mqmq'}$ denotes the number of elements with $c_{m,q'} = c_{m',q'}$. Then, the optimal code matrix selection can be formulated as the following optimization problem:

$$\mathbf{C}_{\mathrm{opt}} = \arg\min_{\mathbf{C}} \{\beta(\mathbf{C})\}. \tag{10}$$

3.2. Code Matrix Optimization

The impacts of undesired mutual interferences between distinct waveforms can be equivalently evaluated by the cross-correlation function:

$$R_c(u_m, u'_m, \tau) = \int_{-\infty}^{+\infty} u_m(t) u^*_{m'}(t-\tau)\mathrm{d}t$$
$$= \int_{-\infty}^{+\infty} \left(\sum_{q=0}^{Q-1} b_{m,q} e^{j2\pi c_{m,q}\Delta f t + j\pi k_r t^2} \phi(t - q\Delta t) \right)$$
$$\times \left(\sum_{q=0}^{Q-1} b_{m',q} e^{j2\pi c_{m',q}\Delta f (t-\tau) + j\pi k_r (t-\tau)^2} \phi(t - q\Delta t - \tau) \right)^* \mathrm{d}t. \tag{11}$$

If $b_{m,q} = b_{m',q}$ and $c_{m,q} = c_{m',q}$, namely $u_m(t) = u_{m'}(t)$, it simplifies to the auto-correlation function, $R_a(u_m, \tau)$. Equation (11) implies that, to achieve low mutual interferences, $c_{m,q}$ in different waveforms should be as distinct as possible, namely,

$$\mathbf{C}_{\mathrm{opt}} = \arg\min_{\mathbf{C}} \{\mathrm{Rep}(\mathbf{C})\} \tag{12}$$

where $\mathrm{Rep}(\mathbf{C})$ is the maximal number of identical $c_{m,q}$. Therefore, we formulate the following joint optimization problem:

$$\begin{aligned} \mathbf{C}_{\mathrm{opt}} &= \arg \min_{\mathbf{C}} \{\beta(\mathbf{C})\} \\ \text{s.t. } &\arg \min_{\mathbf{C}} \{\mathrm{Rep}(\mathbf{C})\}. \end{aligned} \tag{13}$$

Since (13) cannot be resolved easily and there may have multiple solutions, we use an iterative algorithm to design the code matrix. Supposing $d = 1$ and $d_{\mathrm{all}} = 1$ at the beginning, we search for the matrix that satisfies the following constraint:

$$\max_{v, v' \neq v} d_{v,v'} \le d, \quad \forall v, v'. \tag{14}$$

If no such a matrix exists, we use $d = d + 1$ for the next iteration and search the matrix again. Otherwise, we set flag = 1 and check whether the matrix satisfies

$$\mathrm{Rep}(\mathbf{C}) \le d_{\mathrm{all}}. \tag{15}$$

If so, we set Endflag = 1 and terminate the algorithm. If not, let $\mathrm{Count}_{\mathrm{all}} = \mathrm{Count}_{\mathrm{all}} + 1$ and if $\mathrm{Count}_{\mathrm{all}} \le \mathrm{Count}_{\mathrm{max}}$, then the possible matrix is searched again. Otherwise, d_{all} is updated by $d_{\mathrm{all}} + 1$ for next iteration. The algorithm is finalized when the desired code matrix is obtained.

3.3. Waveform Amplitude Optimization and Mutual Interferences Suppression

It is necessary to further optimize the OFDM chirp waveform amplitudes. Moreover, the auto-correlation sidelobes and cross-correlation interferences should be effectively suppressed. Mathematically, these optimization problems can be formulated as

$$\mathbf{B}_{\mathrm{opt}} = \min_{\mathbf{B}} \left\{ \eta_1 \sum_{m=0}^{M_T} \max_{|\tau| \ge 1/\Delta f} |R_a(u_m, \tau)| + \eta_2 \sum_{m=1}^{M_T - 1} \sum_{m'=m+1}^{M_T} \max_{\tau} |R_c(u_m, u_{m'}, \tau)| \right\} \tag{16}$$

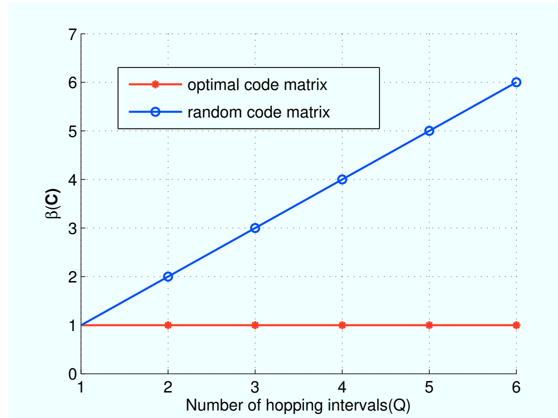


Fig. 1: $\beta(C)$ as a function of the number of hopping intervals.

where η_1 and η_2 are the weighting parameters. It can be resolved by In GA algorithm [11].

4. DESIGN EXAMPLES AND PERFORMANCE ANALYSIS

First, we provide the code matrix $\mathbf{C}$ and amplitude matrix $\mathbf{B}$ design examples. Let $M_T = 4$, $Q = 4$, and $G = 8$. Using the iterative code selection algorithm presented in Section III, we get

$$\mathbf{C}_{\mathrm{opt}} = \begin{bmatrix} 4 & 3 & 8 & 5 \\ 5 & 2 & 4 & 6 \\ 8 & 6 & 1 & 7 \\ 1 & 7 & 3 & 2 \end{bmatrix}. \tag{17}$$

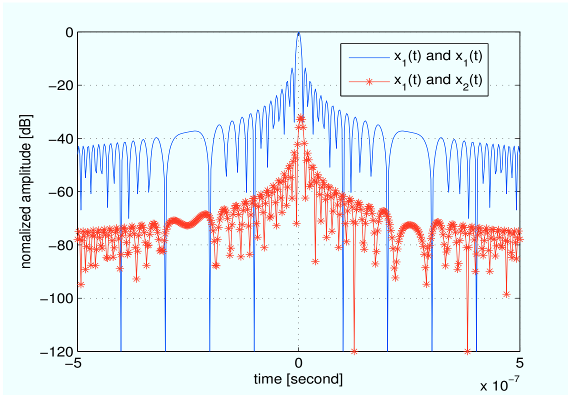
Figures 1 compares the $\beta(\mathbf{C})$ of the optimal code matrix $\mathbf{C}_{\mathrm{opt}}$ and a random code matrix whose columns are chosen uniformly from the set of possible columns. Note that all the results are obtained with 1000 independent Monte Carlo simulations. We intend to make $\beta(\mathbf{C})$ as low as possible. It is noticed that our proposal achieves much lower $\beta(\mathbf{C})$ than that of the random code matrix.

Using the proposed algorithm in Section III to solve the amplitude selection problem, we obtain

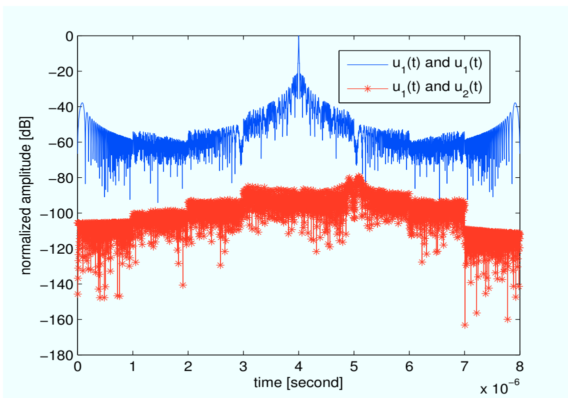
$$\mathbf{B} = \begin{bmatrix} 0.0289 & 0.9448 & 0.9987 & 0.9060 \\ 0.5410 & 0.0515 & 0.1724 & 0.3344 \\ 0.0249 & 0.0405 & 0.1298 & 0.0168 \\ 0.3771 & 0.1066 & 0.1150 & 0.0202 \end{bmatrix}. \tag{18}$$

The maximum energy for each waveform can be arranged to different frequency hopping steps. For instance, the four waveforms represented by (18) emit their maximum energy at the second, first, third and fourth frequency hopping steps, respectively.

Furthermore, we compare the proposed method with the method proposed in [1], which generates two waveforms denoted by $x_1(t)$ and $x_2(t)$. Suppose the two design schemes



(a) for $x_1(t)$ and $x_2(t)$ designed by [1].



(b) $u_1(t)$ and $u_2(t)$ designed by our method.

Fig. 2: Correlation comparisons between our method and [1].

have the same waveform parameters, namely, $B = 50$ MHz, $T_p = 1\ \mu s$, $M_T = 2$, $Q = 2$, $G = 8$, $\eta_1 = 1$, $\eta_2 = 1$, $\Delta t = 1\ \mu s$ and $\Delta f = 50$ MHz. Figure 2 compares their auto- and cross-correlation results. It is seen that our proposal yields better cross-correlation suppression performance.

5. CONCLUSION

We have proposed an OFDM chirp waveform design scheme by exploiting combinational sparse modeling and joint optimization of the waveform code matrix and amplitude matrix. By adopting sparse modeling, the design of the waveforms amounts to choosing the optimal code matrix and amplitude matrix for all the waveforms in the hopping intervals. Numerical results show that the advantages of high range resolution, large time-bandwidth product and low peak-to-average ratio. More importantly, when compared to existing methods, it can easily generate more pseudo-orthogonal waveforms with satisfactory ambiguity function performance.

6. REFERENCES

[1] J. H. Kim, M. Younis, and A. Moreira, “A novel OFDM chirp waveform scheme for use of multiple transmitter in SAR,” *IEEE Geoscience and Remote Sensing Letters*, vol. 10, no. 3, pp. 568–572, May 2013.

[2] S. Sen, “PAPR-constrained pareto-optimal waveform design for OFDM-STAP radar,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 52, no. 6, pp. 3658–3669, June 2014.

[3] D. Garmatyuk and M. Brenneman, “Adaptive multicarrier OFDM SAR signal processing,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 49, no. 10, pp. 3780–3790, October 2011.

[4] S. Gogineni and A. Nehorai, “Frequency-hopping code design for MIMO radar estimation using sparse modeling,”*IEEE Transactions on Signal rocessing*, vol. 60, no. 6 , pp. 3022–3035, June 2012.

[5] J. Mittermayer and J. M. Martínez, “Analysis of range ambiguity suppression in SAR by up and down chirp modulation for point and distributed targets,” in *Proceedings of the IEEE Geoscience and Remote Sensing Symposium*, Toulouse, France, June 2003, pp. 4077–4079.

[6] J. Wang, X. Liang, C. Ding, L. Chen, L. J. Zhou, and Y. W. Dong, “An improved OFDM chirp waveform used for MIMO SAR system,” *Science China: Information Sciences*, vol. 57, no. 6, pp. 1–9, June 2014.

[7] W.-Q. Wang, “MIMO SAR chirp modulation diversity waveform design,” *IEEE Geoscience and Remote Sensing Letters*, vol. 11, no. 9, pp. 1644–1648, September 2014.

[8] C.-Z. Meng, J. Xu, X.-G. Xia, T. Long, E.-K. Mao, J. Yang, and Y.-N. Peng, “MIMO-SAR waveform separation based on inter-pulse phase modulation and range-Doppler decouple filtering,” *Electronics Letters*, vol. 49, no. 6, pp. 420–422, March 2014.

[9] F. He, Z. Dong, and D. Liang, “A novel space-time coding Alamouti waveform scheme for MIMO-SAR implementation,” *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 2, pp. 229–233, February 2015.

[10] W.-Q. Wang.,“Mitigating range ambiguities in high PRF SAR with OFDM waveform diversity,” *IEEE Geoscience and Remote Sensing Letters*, vol. 10, no. 1, pp. 101–105, January 2013.

[11] R. A. Horn and C. R. Johnson. Matrix Analysis. Cambrige, Cambrige University Press, 1985.