Performance Analysis of Precoded PTS and SLM Scheme for PAPR Reduction in OFDM System

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Abstract—**Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier system which provides high data rate services in wireless communication systems. The major drawback of OFDM is high Peak-to- Average Power Ratio (PAPR). An OFDM with high PAPR requires High Power Amplifiers (HPA) which are very difficult to design and manufacturing becomes complex and costlier. Many PAPR reduction techniques have been proposed out of which the existing technique is Partial Transmit Sequences (PTS) and Selected Mapping (SLM) with optimum PAPR reduction and less complexity. In this paper the analysis of OFDM system, PTS OFDM system and SLM OFDM systemwith various precoding methods like Discrete Fourier Transform (DFT) Precoder, Discrete Hartley Transform (DHT)Precoder and Walsh- Hadamard Transform (WHT) Precoder techniques are analyzed for PTS, SLM for M-QAM is analyzed for better PAPR reduction.**

Keywords- OFDM, PAPR, PTS,SLM, Precoder, DFT, DHT, WHT

I. INTRODUCTION

The data rate of the signal is increasing as the technology is getting expanded in wireless and mobile communications. In Orthogonal Frequency Division Multiplexing (OFDM), the enormous data is cleaved into low rate and these low data rate signal is modulated with N orthogonal subcarriers [1][2][3]. OFDM has wide applications in Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), Wireless LANs and Digital Subscriber Lines (DSL) as it has high spectral and power efficiency. As OFDM is a multicarrier system, it suffers with high PAPR which decreases the efficiency of high power amplifiers (HPA). Many PAPR reduction techniques have been propositioned like distortion based techniques: clipping [4] [5] which may lead to in-band and out-band distortion and Companding [6] reducing the out-band distortion and BER which are not power efficient and probabilistic techniques like clipping and filtering, coding, Tone reservation(TR), selected mapping(SLM), partial transmit sequence(PTS) were implemented [7] [8]. These methods try to make use of subcarrier symbols of OFDM by generating correlation between them. In PTS [9] [10] the computational complexity can be reduced by using less number of IFFTs.
To get significant PAPR of OFDM signal the subcarrier waveforms are manipulated with different shapes [11] [12] [13]. The precoding techniques [14] [15] [16] [17] are distortion-less, maximizes the diversity gain and reduces PAPR. In the proposed method the Precoding(P) techniques like Discrete Fourier Transform (DFT), Discrete Hartley Transform (DHT), and Walsh Hadamard Transform (WHT) is spread over the PTS OFDM system.

II. OFDM SYSTEM

The OFDM signal can be generated by using number of parallel subcarriers. The group of bits are mapped into some constellation points by using different keying techniques like M-PSK, M-QAM. Let us consider N parallel subcarriers which are used to transfer the signal.

$$X = \{X_k \ , k = 0,1, \dots \dots \dots \dots . N\}$$

The N subcarriers are orthogonal and are stored within the time interval T. Each symbol is used to modulate with one of the subcarriers and at last all modulated signals are transferred simultaneously over the time interval T. Therefore each symbol is made on IFFT operation.

The OFDM signal:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{n-1} X_k e^{\frac{j2\pi f_k t}{N}}$$

$$\text{Where} f_k = k\Delta f \ \text{and} \Delta f = \frac{1}{T}$$

In OFDM system, assuming the signal consists of real and imaginary parts which are in frequency domain are converted into time domain by IFFT/IDFT.
Calculating PAPR for continuous time signal is difficult therefore estimating PAPR for discrete time is same as continuous time by oversampling the signal L times.

The OFDM signal:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{n-1} X_k e^{\frac{j2\pi f_k t}{L}}$$

$$for\ k = 0,1, \dots \dots NL - 1$$

III. Peak To Average Power Ratio

According to central limit theorem, the OFDM signal will be close to Gaussian distributed if N is large, therefore the real and imaginary parts are Gaussian distributed and envelope is Rayleigh distributed and power is exponential distributed.
The PAPR for discrete – time version x[n] is:

$$PAPR(x[n]) = \max_{0 \le n \le N-1} \frac{|x[n]|^2}{E[|x[n]|^2]}$$

$$PAPR\{x(t)\} = \frac{P_{peak}}{P_{avg}}$$

The probability of PAPR is analyzed by cumulative distributed function (CDF) within threshold level is $F_z(z) = 1 - e^{-z}$ and if probability of PAPR exceeding threshold level is expressed in complementary cumulative distributed function (CCDF) is

$$\bar{F}_z(z) = 1 - (1 - e^{-z})^n.$$

IV. Proposed Method

Precoded PTS OFDM system:

These input blocks of length N are partitioned into V disjoint subblocks. $X = X_v$.Each partitioned block of length N is precoded by an $L\ X\ N$ matrix (P) because the OFDM signal is oversampled with $L = N + N_p$ is total number of subcarriers and N_p is the overhead subcarriers, where as $p_{i,j}$ are the complex or real numbers of precoding matrix. [14]

$$P = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,N-1} \\ p_{1,0} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ p_{L-1,0} & \cdots & \cdots & p_{(L-1)(N-1)} \end{bmatrix} \quad (1)$$

The precoding matrix transform input X to a new vector of length L is

$$Y_v = PX_v = [Y_{v,0}, Y_{v,1}, \dots \dots \dots Y_{v,L-1}]^T \quad (2)$$

$$\text{Where} y_v = \sum_{m=0}^{V-1} p_{i,m}\, x_m \quad for\ i = 0,1, \dots \dots \dots V-1$$

The precodedsignals are transferred over distinct subcarriers which are called OFDM modulation i.e., the IFFT (Y_v) is:

$$x(t) = \sum_{i=0}^{V-1} Y_i\, e^{\frac{j2\pi i t}{T}} \quad (3)$$

The OFDM modulated signals are computed with complex phase factor

$$b^v = e^{j\varphi v} \quad for \quad v = 0,1,2, \dots .V-1. \quad (4)$$

The Precoded PTS- OFDM signal with N=2048, L=4, M=64 and V=2 expressions in which all V subblocks are optimally

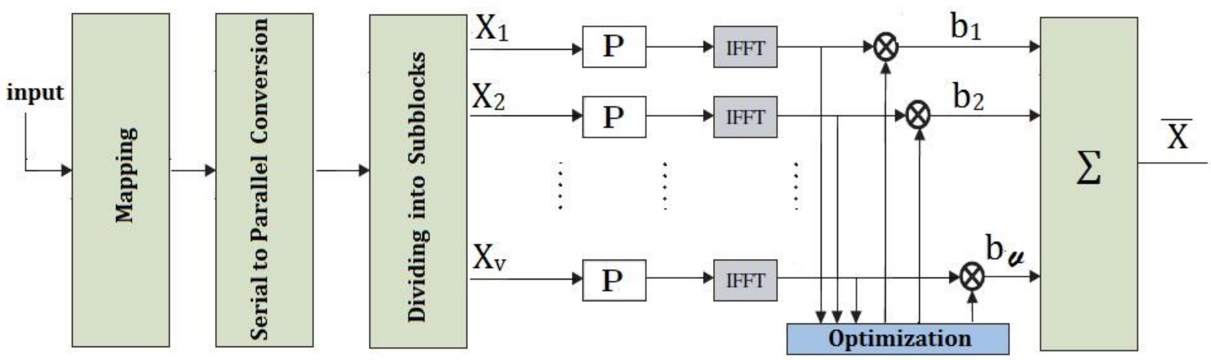

Figure 1. Precoded PTS OFDM System

In this propositioned method an OFDM system with M-QAM and N symbols are considered. As shown in Figure 1. the modulator followed by a serial to parallel converter and partition block then Precoder (P) and PTS method i.e., precoder followed by OFDM modulation computed with weighting factors. The input is mapped and is sorted into blocks which consist of N symbols each. Let the input of OFDM is defined in blocks consisting of N symbols.

$$X = [X_1, X_2, \dots \dots \dots X_N]^T$$

$$\overline{x(b)} = \sum_{i=0}^{V-1} Y_i\, e^{\frac{j2\pi i t}{T}}\, b_v \quad (5)$$

$$= \sum_{i=0}^{V-1} e^{\frac{j2\pi i t}{T}}\, b_v \sum_{m=0}^{V-1} p_{i,m}\, x_m$$

$$\overline{x(b)} = b_v \sum_{m=0}^{V-1} x_m \left(\sum_{i=0}^{V-1} p_{i,m}\, e^{\frac{j2\pi i t}{T}} \right)$$

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The PAPR of the precoded PTS OFDM system is defined as follows:

$$PAPR\ (t) \leq \frac{1}{N}\left(\sum_{m=0}^{V-1}\left|\sum_{i=0}^{V-1} p_{i,m}\, e^{\frac{j2\pi it}{T}}\, b_v\right|\right)^2 \qquad (6)$$

$$PAPR\ (t) \leq \frac{1}{N}\max_{0\leq t\leq T}\left(\sum_{m=0}^{V-1}\left|\sum_{i=0}^{V-1} p_{i,m}\, e^{\frac{j2\pi it}{T}}\, b_v\right|\right)^2$$

Precoded SLM OFDM system:

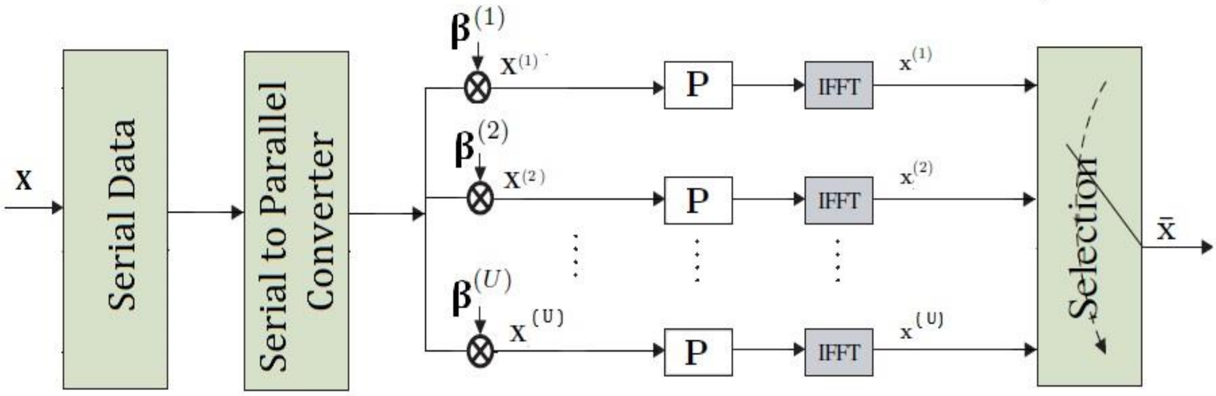

Figure 2. Precoded SLM OFDM System

In Figure 2, the OFDM parallel bits are multiplied with U alternative phase factors so that U alternative X^U signal blocks are generated where all blocks have same information. These alternative OFDM signals are followed by precoder and OFDM modulation. The sequence of lowest PAPR is selected.
The input sequence multiplied with U phase factors is

$$X^u = [X_0\beta_0, X_1\beta_1, \dots\dots\dots\dots X_{N-1}\beta_{N-1}]^T \qquad (7)$$

Where u=1, 2…………U-1 i.e.,$X_k^u = X_k\beta_k^u$

The precoded outputs of the above sequences are:

$$Y_m^u = \sum_{k=0}^{N-1} P_{m,k} X_k^u \qquad (8)$$

The IFFT of above signal is:

$$\bar{x_k^u}(\bar{t}) = \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1} Y_m^u\, e^{j2\pi m\Delta f t}$$

Precoding Techniques

A. Discrete Fourier Transform (DFT) Precoding:

In this the series of N complex numbers $x_0, x_1, x_2, \dots\dots\dots\dots x_{N-1}$ are converted into N complex numbers.

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{\frac{-j2\pi k}{N}} \text{ for } k = 0,1,\dots\dots\dots\dots N-1$$

Because of periodicity always the DFT is implemented via Fast Fourier Transform (FFT). DFT has many properties like linearity, orthogonality, periodicity, convolution, circular convolution and so on.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi k}{N}}$$

B. Discrete Hartley Transform (DHT) Precoding:

The DHT transform was introduced in early 1980s like other transforms such DCT, DFT, This can be applied efficiently through a factorization of transform matrix. DHT is defined as

$$X_H(k) = \sum_{n=0}^{N-1} x(n)\ \mathrm{cas}\,\frac{2\pi nk}{N}$$

Where $cas\emptyset = cos\emptyset + sin\emptyset$. The signals$x(t)$ which are real values are transformed into real- valued.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(n)\ \mathrm{cas}\,\frac{2\pi nk}{N}$$

C. Walsh Hadamard Transform (WHT) Precoding:

The Walsh Hadamard Transform (WHT) is a non-sinusoidal orthogonal transform that decomposes a signal into set of basic functions which are Walsh functions i.e., square waves of values +1 and -1. This transform has no multiplications and is real.

$$y_n = \frac{1}{N}\sum_{i=0}^{N-1} x_i\, WAL(n,i) \qquad for\ i = 0,1,\dots\dots\dots\dots N-1$$

$$x_i = \sum_{i=0}^{N-1} y_n\ WAL(n,i)$$

V. Reduction Efficiencies Of Precoded PTS, Precoded SLM OFDM System

In OFDM systems the number of symbols are 2048, N=64 subcarriers, 64QAM modulations are considered. The simulation results are shown in figures 4 and 5. PAPR reduction efficiency factor was developed based on CCDF information for proposed methods. The PAPR reduction is calculated at the standard probabilities $10^{-2}, 10^{-1}$. Comparing PAPR of OFDM with PTS, Precoded PTS OFDM, comparing PAPR of OFDM with SLM, Precoded SLM OFDM PAPR reduction values are calculated at standard probabilities.

For Therefore,

$$OFDM\ PAPR\ Reduction = Original\ PAPR - Modified\ PAPR \quad (9)$$

$$PAPR\ Reduction\ Efficiency\ (\eta) = \frac{OFDM\ PAPR\ Reduction}{Original\ OFDM\ PAPR}$$

For Precoding PTS OFDM System:

$$OFDM\ PAPR\ REDUCTION = OFDM\ PTS\ PAPR - PRECODED\ PAPR \quad (10)$$

$$PAPR\ Reduction\ Efficiency\ (\eta) = \frac{OFDM\ PAPR\ Reduction}{OFDM\ PTS\ PAPR}$$

For Precoding SLM OFDM System:

$$OFDM\ PAPR\ REDUCTION = OFDM\ SLM\ PAPR - PRECODED\ PAPR \quad (11)$$

$$PAPR\ Reduction\ Efficiency\ (\eta) = \frac{OFDM\ PAPR\ Reduction}{OFDMSLM\ PAPR}$$

VI. Simulation Results

The simulation is done in MATLAB for evaluating the performance analysis of PAPR reduction in OFDM system and modified OFDM system. For analysis the data is randomly generated and then modulated with 64-QAM, 64 subcarriers, L=4. In Table 1: the PAPR reduction efficiencies are calculated from equation 10 for OFDM, hybrid OFDM, PTS OFDM and SLM OFDM. The PAPR reduction efficiency improves for SLM OFDM and deteriorates in PTS OFDM for the probabilities increased from 10^{-2} to 10^{-1}. To reduce computational complexity and hardware complexity the hybrid method is applied which combines SLM and PTS. For U=16 and V=2, the PAPR reduction efficiency is improved

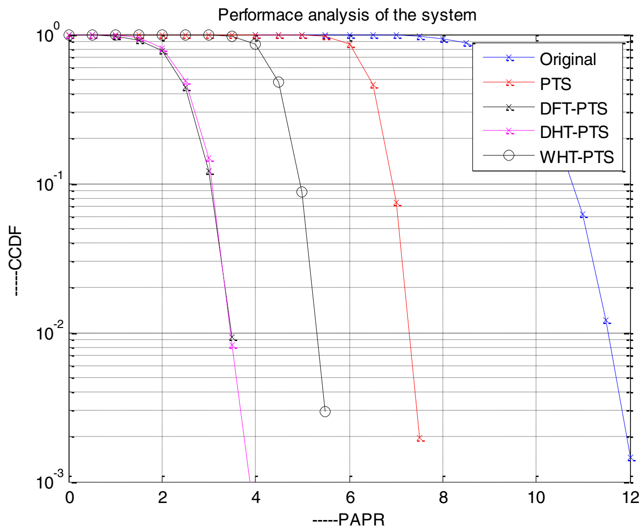

Figure 3. Comparison of PAPR of Precoding with PTS OFDM, PTS OFDM and OFDM

Figure 3. shows CCDF comparison of DFT – Precoder PTS OFDM system, WHT –Precoder PTS OFDM system, DHT– Precoder PTS OFDM system V = 2. In Table 2: the PAPR reduction efficiency is enhanced from equation 10 for all precoding PTS OFDM technique for the probabilities increased from 10^{-2} to 10^{-1}. Compared to the three precoding techniques the performance of DFT- PTS OFDM and DHT-PTS OFDM system enriches than WHT-PTS OFDM system.

Table 1: PAPR values OF OFDM, PTS OFDM, SLM OFDM and their efficiencies at standard probabilities

PAPR > $PAPR_O$	Original OFDM PAPR	OFDM PTS PAPR	OFDM PTS PAPR (η)	OFDM SLM PAPR	OFDM SLM PAPR (η)
10^{-2}	11.5	7.5	34.7	4	65.2
10^{-1}	10.5	7	33.3	3.5	66.6

Table 2: PAPR values of OFDM, PTS OFDM, DFT PTS OFDM, DHT PTS OFDM, WHTPTS OFDM and their efficiencies at standard probabilities

PAPR > PAPRo	OFDM PTS PAPR	DFT-PTS OFDM PAPR	DFT–PTS OFDM (η)	DHT-PTS OFDM PAPR	DHT-PTS OFDM (η)	WHT-PTS OFDM PAPR	WHT-PTS OFDM (η)
10^{-2}	7.5	3.5	53.3	3.5	53.33	5.5	26.66
10^{-1}	7	3	57.1	3	57.14	5	28.57

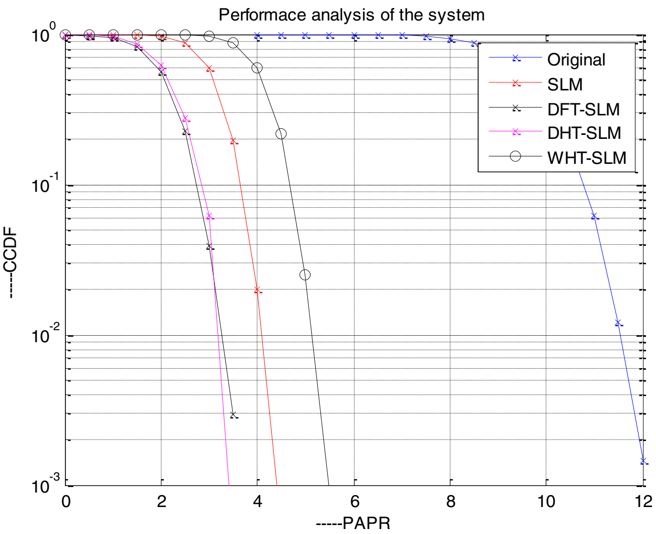

Figure 4. Comparison of PAPR of Precoding with SLM OFDM, SLM OFDM and OFDM

Table 3: PAPR values of OFDM, SLM OFDM, DFT SLM OFDM, DHT SLM OFDM, WHT SLM OFDM and their efficiencies at standard probabilities

PAPR > $PAPR_0$	OFDM SLM PAPR	DFT-SLM OFDM PAPR	DFT-SLM OFDM (η)	DHT-SLM OFDM PAPR	DHT-SLM OFDM (η)	WHT-SLM OFDM PAPR	WHT-SLM OFDM (η)
10^{-2}	4	3.5	12.5	3	25.0	5	-
10^{-1}	3.5	2.5	28.5	3	14.28	5.2	-

Figure 4. Shows CCDF comparison of DFT –PrecoderSLM OFDM system, WHT –PrecoderSLM OFDM system, DHT– PrecoderSLM OFDM system U=16. In Table 3: the PAPR reduction efficiency is enhanced from equation 11 for DFT precoding SLM OFDM technique and impaired for DHT precoding SLM OFDM technique for the probabilities increased from 10^{-2} *to* 10^{-1}. Compared to the three precoding techniques the performance of DFT- SLM OFDM system better than DHT-SLM OFDM system and WHT-SLM OFDM system is inferior to SLM OFDM system

VII. CONCLUSION

In this paper, we have fostered a precoded PTS scheme and precoded SLM scheme for PAPR reduction of OFDM signals, where a principle was created for calculating peak to average power ratio in terms of PAPR reduction efficiency. In PTS OFDM system it is stated that as M increases the PAPR is decreasing with a minute difference in all aspects but as the number of subcarriers are increasing the PAPR is getting demoted but computational complexity increases. In SLM OFDM system though the PAPR is less than PTS technique the hardware complexity increases for finding optimum PAPR. In this method, the input data signal which is N-sequence (real or complex) is transformed into N-sequence in matrix form by using various precoding techniques like DFT, DHT, and WHT and then modulated into time domain to find optimum PAPR. Simulation results show the PAPR in terms of CCDF. The DHT-precoding technique which transforms input signal real values to real values shows good performance and is desirable for using in wireless communications applications.

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