Constant Envelope Chirped OFDM for Power-efficient Radar Communication

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***Abstract*—In increasing radar performance, radar waveform and waveform attributes are key elements. Among different waveforms, Linear Frequency Modulation (LFM) or simply chirp waveform attains best performance in terms of Doppler tolerance and time resolution. In analysing LFM waveform a Fractional Fourier Transform provides superior performance in time-frequency selective environment. Although Chirped Orthogonal Frequency Division Multiplexing OFDM is used to provide high data rate transmission it suffers from Peak to average power ratio (PAPR). Constant Envelope chirped OFDM is used to provide 0dB PAPR while providing high data rate transmission, this research studies Constant Envelope chirped OFDM as a radar waveform. Different simulations was conducted to check the performance of CE chirped OFDM in comparison to conventional OFDM and different waveform attributes was studied to check Doppler shift and time delay of the radar signal.**

FRFT

Keywords*—Fractional Fourier Transform, Linear Frequency Modulation, OFDM, Peak to Average Power Ratio (PAPR)

I. Introduction

For any new radar the design or choice of waveforms is a key element to determine its performance. The efficient use of waveforms attributes such as pulse duration, bandwidth, amplitude, and phase or frequency modulation can change radar performance.

Different waveforms have been used to obtain different advantages of the waveform features, Continuous Waves (CW) 连续波 are mostly used to obtain velocity of the target. Pulse compression waveforms can be used differently to attain range or Doppler shift of the target [1].The most used pulse compression technique is linear frequency modulation (LFM) or most referred as chirp waveform. LFM waveform achieves a signal processing gain equal to bandwidth pulse period product compared to a simple pulse at the same peak power [2].

In analysing LFM signals, time-frequency representations tools are used including Wigner Ville distribution (WVD) and Fractional Fourier Transform (FrFT). Although WVD gives fine resolution it exhibits cross terms in multi-component signals which make it difficult to interpret. FrFT is a linear operator and it represents rotation of a signal in time-frequency plane hence will not be influenced by cross terms even if multiple and moving targets exist [3], [4].

Fractional Fourier OFDM or chirped OFDM provides high data rate transmission at fast and low cost implementation in frequency disperse channel. Although chirped OFDM provides more throughput, high spectral efficiency, high bit error ratio (BER) at a reduced inter symbol interference (ISI) and inter carrier interference (ICI) [5], but it still inherit 继承，保留 high peak to average power ratio (PAPR) challenges of convectional 对流的 OFDM which is caused by high amplitude fluctuations of the modulated waveforms [5], [6]. The high PAPR makes chirped OFDM sensitive to nonlinear distortion caused by the transmitter's power amplifier. The fluctuations causes the system to suffer from spectral broadening, intermodulation distortion, and consequently performance degradation [7].Different techniques have been applied to reduce PAPR of OFDM signals, the recently adopted technique called Constant Envelope OFDM provides 0dB PAPR [8], [9]. The constant envelope OFDM signal can be amplified with minimal power back-off hence maximizes power amplifier efficiency and increase system range as more signal power is radiated 发射，散发 into the channel [10].

This paper will study Constant Envelope chirped OFDM waveform and evaluate its performance in radar systems. The rest of the paper is as arranged. 1 Section II gives a detailed information of the Constant envelope chirped OFDM signal and ambiguity function of the waveform and its PAPR analysis. 2 Section III covers the simulation results and analysis of the findings obtained. 3 Section IV gives conclusion of the paper.

II. System Model

A. Constant Envelope chirped OFDM Waveform

In this section the designed radar waveform is explained.

The constant envelope chirped OFDM signal $s(t)$ to be used is given by equation 1. According to [9] the constant envelope OFDM signal is given by

$$s(t) = A \cdot e^{j\varphi(t)} \tag{1}$$

where A is amplitude of the signal, $j = \sqrt{-1}$ and $\varphi(t)$ is real value OFDM signal.

The same expression will be used here except of $\varphi(t)$ being real valued chirped OFDM signal obtained by extending $\varphi(t)$ into real and imaginary part hence doubling bandwidth of the signal hence reducing its probability of detection [index]. The general chirped OFDM signal is provided by Equation 2, the used chirped OFDM is given by Equation 3.

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$$\varphi(t) = 2\pi h_{\mathrm{mod}} \sum_k \sum_i \exp\left(\frac{j2\pi\mu_k t^2}{T}\right) P(t - iT) \tag{2}$$

Figure 1 shows the waveforms, OFDM waveform, chirped OFDM, and Constant Envelope chirped OFDM waveform spectrum.

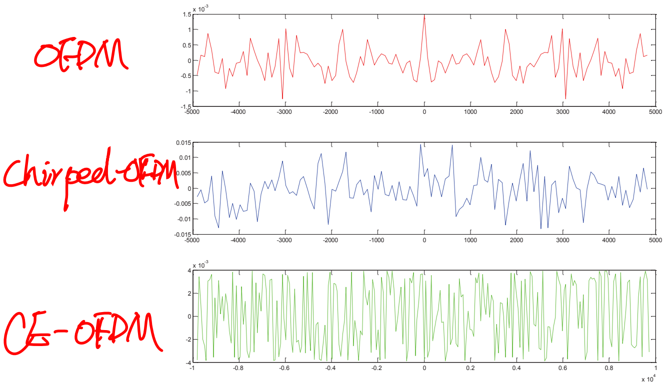

Figure 1. Chirped OFDM and Modified Chirped OFDM spectrum

Substituting Equation 3 to Equation 1, we obtain the radar waveform as represented by Equation 4.

B. Waveform Ambiguity function

The relative radial velocity between the radar, target, and the environmental interference increases complexity in studying radar waveforms. A special set of mathematical functions was generated to allow interpretation of the output of a signal processor due to relative motion of radar and/or target or multiple targets. These functions are called time-frequency autocorrelation functions or ambiguity functions and are based on the transmitted and reflected signal to determine the Doppler shift and time delay of the reflected signal [2]. Direct examination of the ambiguity function surface in the range-velocity ambiguity plane can tell capability of the radar to resolve target and cluster scatters [1], [2]. The ambiguity function of the transmitted signal is given by Equation 5.

$$\left|\chi(\tau, f_d)\right| = \left|\int_{-\infty}^{\infty} s(t) s^*(t+\tau) \exp(j2\pi f_d t) dt\right| \tag{5}$$

Where $\left|\chi(\tau, f_d)\right|$ is Ambiguity function, $s(t)$ is transmitted signal, $s^*(t+\tau)$ is conjugate of the received signal with τ delay and f_d is Doppler frequency.

1 The general Ambiguity function for the designed waveform is given by Equation 6.

2 The Ambiguity function Doppler cut $\left|\chi(0, f_d)\right|$ is used to study the Doppler tolerance of the waveform. The simplified Delay cut is given by equation 7.

3 The ambiguity function delay cut $\left|\chi(\tau, 0)\right|$ is used to study time resolution of the waveform is given by Equation 8.

Further simplification of ambiguity function delay cut $\left|\chi(\tau, 0)\right|$ can be obtained by assumption of *N=1, 2, 3...* and with the help of Bessel functions the time resolution of the waveform can be obtained numerically of which is not coverage of this paper. MATLAB was used in plotting the Ambiguity function Doppler cut.

$$\varphi(t) = 2\pi h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) + 2\pi h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) \tag{3}$$

where, h_{mod} is modulation index, $P(t)$ is a rectangular waveform signal, k is number of carriers, i is size of each carrier, μ_k is the chirp rate, T is period of the signal.

$$s(t) = A\exp\left(2\pi j\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT)\right)\right) \tag{4}$$

模糊函数

$$\left|\chi(\tau, f_d)\right| = A^2 \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\begin{array}{l} \exp\left(2\pi j\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT)\right)\right) \times \\ \exp\left(\left(2\pi(-j)\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k (t+\tau)^2}{T}\right) P((t+\tau) - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k (t+\tau)^2}{T}\right) P((t+\tau) - iT)\right)\right)\right) \exp(j2\pi f_d t) dt \end{array} \right) \right| \tag{6}$$

速度模糊

$$\left|\chi(0, f_d)\right| = A^2 \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\begin{array}{l} \exp\left(2\pi j\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT)\right)\right) \times \\ \exp\left(\left(2\pi(-j)\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k (t)^2}{T}\right) P((t) - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k (t)^2}{T}\right) P((t) - iT)\right)\right)\right) \exp(j2\pi f_d t) dt \end{array} \right) \right| \tag{7}$$

距离模糊

$$\left|\chi(\tau, 0)\right| = A^2 \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\begin{array}{l} \exp\left(2\pi j\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k t^2}{T}\right) P(t - iT)\right)\right) \times \\ \exp\left(\left(2\pi(-j)\left(h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=1}^{M} \cos\left(\frac{2\pi\mu_k (t+\tau)^2}{T}\right) P((t+\tau) - iT) + h_{\mathrm{mod}} \sum_{k=1}^{N} \sum_{i=M+1}^{2M} \sin\left(\frac{2\pi\mu_k (t+\tau)^2}{T}\right) P((t+\tau) - iT)\right)\right)\right) dt \end{array} \right) \right| \tag{8}$$

C. *Peak to Average Power Ratio (PAPR)*

One of the major drawback of OFDM systems is high PAPR. The PAPR is inherintly in chirped OFDM due to independently summation of different signals from each carrier. In reduction of the PAPR the Constant Envelope Chirped OFDM is used and its PAPR for signal $s(t)$ given by Equation 4 is simply given by the ration between the peak power P_{peak} and average power $P_{average}$ of the signal as noted in Equation 9.

$$PAPR = \frac{P_{peak}}{P_{average}} = \frac{\max\left\{\left|s(t)\right|^2\right\}}{E\left\{\left|s(t)\right|^2\right\}} \tag{9}$$

According to central limit theorem, when N is large, both the real and imaginary part of $s(t)$ are Gaussian distributed, and the amplitude of the signal $s(t)$ follows Rayleigh distribution, making its power to follow Chi distribution with two degrees of freedom.

Usually PAPR analysis is performed using the complementary cumulative distribution function (CCDF), which is defined as the probability that the PAPR of the transmitted signal exceeds a given threshold λ as shown by Equation 10.

互补累计分布函数

$$P_r\{PAPR > \lambda\} = 1 - P_r\{PAPR \le \lambda\} = 1 - \left(1 - e^{-\lambda}\right)^N$$

(10)

MATLAB was used to analyse the PAPR of the designed waveform and compared to OFDM and chirped OFDM.

III. SIMULATION RESULTS AND ANALYSIS

In simulation of the constant envelope chirped OFDM waveform, 128 subcarriers was used with BPSK modulation and with bit period of 50 μs. The Ambiguity function of the designed waveform were obtained as illustrated by Figure 2. It shows the relation between the matched filter which will be used by the receiver to obtain the transmitted signal. Presence of small peaks ouside of 0Hz and 0sec shows the cost of designing the matched filter will be small.

Figure 2 shows Ambiguity function with Zero delay cut or doppler tolerance of the designed waveform in comparison to conventional OFDM. Figure 3 shows ambiguity function with zero time delay cut or time resolution of the waveform also compared with the OFDM waveform, and Figure 4 provides Ambiguity function contour representation of doppler tolerance and time resolution for the waveform.

Figure 3 to Figure 5 shows doppler tolerance of the waveform and time resolution is closely to the convectional OFDM

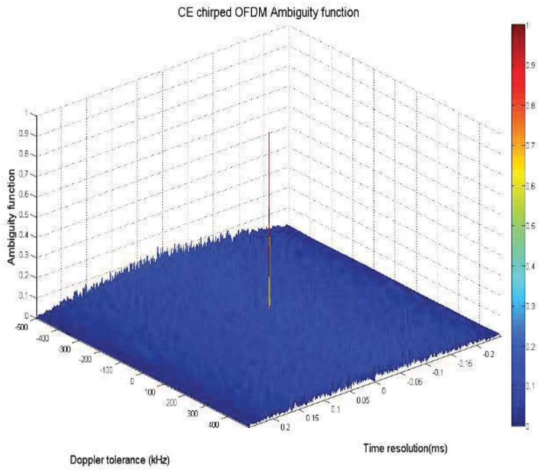

Figure 2. Constant Envelope chirped OFDM waveform Ambiguity function

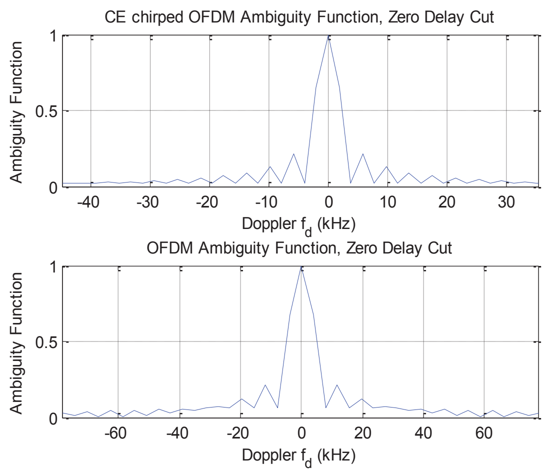

Figure 3. Constant Envelope Chirped OFDM Zero Delay Cut

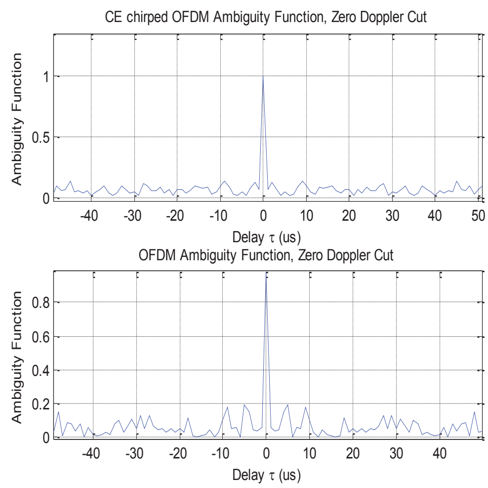

Figure 4. Constant Envelope Chirped OFDM Zero Doppler Cut waveform. Figure 6 gives the PAPR for the designed OFDM with comparison to convectional OFDM. As Figure 6 shows

the designed waveform has 0dB and adding to figure 2 to 5 the performance of the waveform is closely to convectional OFDM

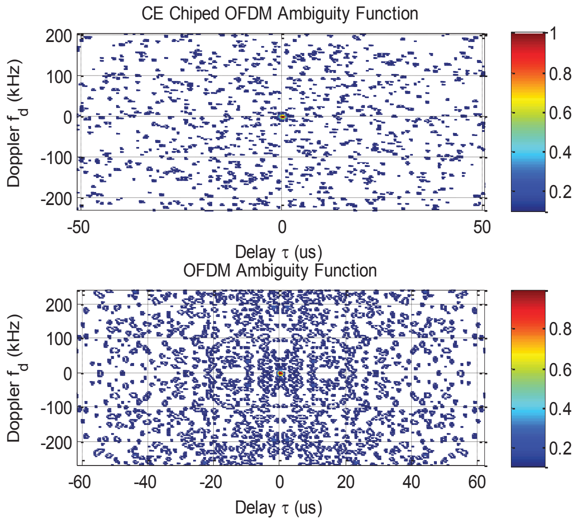

Figure 5: Waveform Ambiguity Function Contour plot.

外形，轮廓，等高线

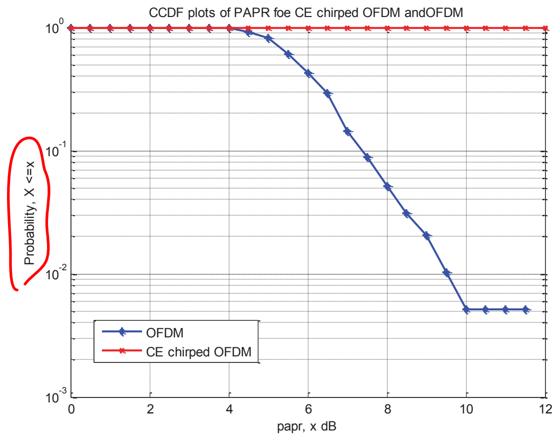

Figure 6. Complementary Cumulative Distribution Function for CE chirped OFDM and OFDM

IV. Conclusion

Different simulations has been conducted for a Constant Envelope chirped OFDM waveform. Figure 1 shows the designed waveform provides evenly distribution of signal spectrum compared to chirped OFDM and convectional OFDM. Figure 2 shows the Ambiguity function variation for the designed waveform and Figure 3, Figure 4 and Figure 5 compares the designed waveform with convectional OFDM and it show the designed waveform behaves the same as convectional OFDM with less time resolution. Figure 6 proves the designed waveform is more power efficient as it provides 0dB PAPR

References

[1] Fred E. Nathanson, J. Patrick Reilly and Marvin N. Cohen, “Radar design principles, signal processing and the environment” second edition, SCITECHpublishing, inc., 1999.

[2] Nadavlevanon and EeliMozeson, “Radar signals” A John Wiley & Sons, inc, publication, 2004

[3] Haldun M. Ozaktas, OrhanAnkan, M. AlperKutay, and GozdeBozdaki, “Digital Computation of the Fractional Fourier Transform” IEEE Transactions on Signal Processing, 1996

[4] Soo-Chang Pei and Man Hung, “Discrete Fractional Fourier Transform” IEEE International Symposium on Circuits and Systems, 1996.

[5] SeungHee Han and Jae Hong Lee, “An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission” IEEE Wireless Communications, 2005

[6] Tao Jiang and Yiyan Wu, “An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals” IEEE Transactions on Broadcasting, 2008.

[7] JingonJoung, Chin KeongHo, and Sumei Sun “Tradeoff of Spectral and Energy Efficiencies: Impact of Power Amplifier on OFDM Systems” Globecom 2012 - Symposium on Selected Areas in Communications

[8] Steve C. Thompson John G. Proakis James R. Zeidler, “Constant Envelope Binary OFDM Phase Modulation” IEEE Military Communications Conference, 2003

[9] Steve C. Thompson, Ahsen U. Ahmed, John G. Proakis, James R. Zeidler and Michael J. Geile, “Constant Envelope OFDM” IEEE Transactions on Communications, 2008

[10] Steve C. Thompson, Ahsen U. Ahmedt", John G. Proakis, James R. Zeidler, 2004 “Constant envelope OFDM phase modulation: spectral containment, signal space properties and performance” IEEE Military Communications Conference