OFDM PAPR Reduction Based on Nonlinear Functions without BER Degradation and Out-Of-Band Emission

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***Abstract*—Peak-to-Average Power Ratio (PAPR) reduction techniques based on nonlinear functions (clipping, companding techniques, etc) are the most straightforward PAPR reduction methods. These techniques reduce the PAPR at the cost of generating distortions. In this paper, we prove basing on Bussgang theorem [1] that in context of PAPR reduction techniques using nonlinear functions, the uncorrelated component resulting in nonlinear process of the input signal amplitude is the component for PAPR reduction. Based on this result, we propose to reduce PAPR of multicarrier systems by exploiting the unused subcarriers embedded in their spectrums to carry the component for PAPR reduction whitout BER degradation and without Out-Of-Band (OOB) emission.**

Later in the paper, the classical clipping function is used to reduce the PAPR of the Wireless Local Area Networks (WLAN) system based on IEEE 802.11a/g standard whitout BER degradation and OOB emission.

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Nonlinear functions.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a popular modulation technique used in many new and emerging broadband technologies either wired like ADSL (asymmetric digital subscriber line) or wireless as in DAB (digital audio broadcasting), DVB-T (digital video broadcasting-terrestrial), WiLAN (Wireless LAN), and so forth [2]. The main advantage of OFDM is its robustness to multi-path fading, its great simplification of channel equalization and its low computational complexity implementation based on using Fast Fourier Transform (FFT) techniques [3].

Despite many advantages, a major drawback of OFDM is its high Peak-to-Average Power Ratio (PAPR) problem, which makes system performance very sensitive to nonlinear distortions [4, 5]. Indeed, when the OFDM signal with high PAPR passes through a nonlinear device, the signal may suffer significant nonlinear distortions and severe power penalty which is unaffordable for battery powered portable wireless terminals.

To reduce the PAPR of OFDM signals, several PAPR reduction techniques have been proposed [6, 7]. In this paper we focus on PAPR reduction techniques based on nonlinear functions. Two well known examples are clipping techniques which use a clipping function for PAPR reduction and companding techniques [(a)] which use a compression function at the transmitter side for PAPR reduction.

This paper propose to reduce the OFDM PAPR by using a nonlinear function called "function for PAPR reduction" without degrading the BER and without any OOB power. The remainder of this paper is organized as follows: section II introduces the OFDM system. Section III describes PAPR reduction techniques based on nonlinear functions in adding signal context and analyzes the adding signal for PAPR reduction , while section IV establishes the principle of PAPR reduction without BER degradation and OOB emission. In section V, the classical clipping function is used to reduce the WLAN PAPR under the constraint of no BER degradation and no OOB emission, while in section VI, a conclusion is drawn.

II. OFDM System Model

In this section, we review the basics of the OFDM transmitter and define the OFDM PAPR.

In OFDM systems, N data symbols X_k, $k = 0, \cdots, N-1$, are modulated on a set of N orthogonal subcarriers. The baseband time-domain signal $s(t)$ is

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, \quad 0 \le t \le T_s \ , \tag{1}$$

where N data symbols X_k form an OFDM symbol $\mathbf{X} = [X_0, \cdots, X_{N-1}]$, $f_k = \frac{k}{T_s}$, and T_s is the time duration of the OFDM symbol.

By sampling $s(t)$ defined in Eq.(1) at frequency $f_s = \frac{NL}{T_s}$, where L is the oversampling factor, the discrete-time OFDM symbol can be written as

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{NL}nk}, \quad 0 \le n \le NL-1 \ . \tag{2}$$

When $L = 1$, the above equation reduces to the Nyquist rate sampling case. Eq.(2) can be implemented by using a length-

[(a)]In a companding technique, a compression is used in transmitter side for PAPR reduction while an expanding is used in receiver side for better performance in BER.

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(NL) IFFT operation with the input vector

$$\mathbf{X}_{\text{ext}} = \left[\begin{array}{ccc} X_0, \cdots, X_{\frac{N}{2}-1}, & \underbrace{0, \cdots, 0}_{(L-1)N \text{ zeros}} & X_{\frac{N}{2}}, \cdots, X_N \end{array} \right].$$

Thus, $\mathbf{X}_{\text{ext}}$ is extended from $\mathbf{X}$ by using the so-called zero-padding scheme, i.e., by inserting $(L-1)N$ zeros in the middle of $\mathbf{X}$.

The PAPR of the signal $s(t)$ may be defined as

$$\text{PAPR}[s] \triangleq \frac{\max\limits_{t \in [0, T_s]} |s(t)|^2}{\mathcal{P}_s}, \tag{3}$$

where $\mathcal{P}_s = E\left\{|s(t)|^2\right\}$ is the signal power and $E\{.\}$ is the statistical expectation operator. Note that, in order to avoid aliasing the out-of-band distortion into the data bearing subcarriers and in order to accurately describe the PAPR, an oversampling factor $L \geq 4$ is required.

III. Characterization of PAPR reduction Scheme based on Nonlinear Function

In this section, we first describe PAPR reduction techniques based on nonlinear functions in adding signal context and then we analyze the adding signal for PAPR reduction called "peak-reducing signal". Let us start by a definition and a proposition.

A. Definition and Remark

1) Definition: Let $f(.)$, be a function and let $s(t) = |s(t)| e^{j\varphi(t)}$ be a single or multicarrier signal, where $\varphi(t)$ is the phase of $s(t)$. The function $f(.)$ is a function of $s(t)$ PAPR reduction if the PAPR of $s(t)$ is strictly inferior to the PAPR of $\tilde{s}(t) = f[|s(t)|] e^{j\varphi(t)}$, i.e., it exists $\Delta\text{PAPR} > 0$ such us

$$\text{PAPR}[\tilde{s}] + \Delta\text{PAPR} = \text{PAPR}[s] \ \text{ (in dB)},$$

where ΔPAPR is the gain in PAPR reduction.

2) Remark: If $f(.)$ is a function of PAPR reduction then $f(.)$ is a nonlinear function. Indeed, a signal $s(t)$ and its scaled version $\alpha s(t)$ have the same PAPR, i.e., PAPR$[s]$ = PAPR$[\alpha s]$, where α is a scalar. To be changed and in particular to be reduced, the transformation must be nonlinear, i.e., $f(.)$ must be nonlinear.

B. Proposition

In context of PAPR reduction based on a nonlinear function, the uncorrelated component resulting in nonlinear process of the input signal amplitude is the component for PAPR reduction.

Proof: Let $f(.)$ be a function of $s(t)$ PAPR reduction, $s(t)$ can be a single or multicarrier signal.

(i) According to the above definition (III-A1), it exists $\Delta\text{PAPR} > 0$ such us

$$\text{PAPR}[\tilde{s}] + \Delta\text{PAPR} = \text{PAPR}[s] \ \text{ (in dB)},$$

(ii) As $f(.)$ is a nonlinear function (refer to the remark), using the Bussgang decomposition [1], the signal $\tilde{s}(t) = f[|s(t)|] e^{j\varphi(t)}$ can be written as

$$\begin{aligned} \tilde{s}(t) &= f[|s(t)|] e^{j\varphi(t)} \\ &= \alpha s(t) + d(t), \end{aligned} \tag{4}$$

where $\alpha = \frac{\mathcal{R}_{\tilde{s}s}(0)}{\mathcal{R}_{ss}(0)}$. The component $d(t)$ is uncorrelated with the input signal $s(t)$, i.e., $\mathcal{R}_{sd}(\tau) = E[d(t+\tau) s^*(t)] = 0$.

(iii) As, $s(t)$ and its scaled version $\alpha s(t)$ have the same PAPR, i.e., PAPR$[s]$ = PAPR$[\alpha s]$, and as

$$\begin{aligned} \text{PAPR}[\alpha s + d] + \Delta\text{PAPR} &= \text{PAPR}[\tilde{s}] + \Delta\text{PAPR} \\ &= \text{PAPR}[s] \\ &= \text{PAPR}[\alpha s]; \end{aligned}$$

(iv) Therefore, PAPR$[\alpha s + d] <$ PAPR$[\alpha s]$; so $d(t)$ is the component for $s(t)$ PAPR reduction.

C. PAPR reduction techniques based on Nonlinear Functions in adding signal context

In adding signal context, the PAPR is reduced by adding a signal called sometimes "peak reducing signal" or "peak canceling signal". Many well known PAPR reduction techniques of the literature such us Tone Reservation (TR) [8], Tone Injection (TI) [8, 9] or Active Constellation Extension (ACE) [10] are known as adding signal techniques. In [11], it is shown that any form of clipping can be formulated as an adding signal technique.

PAPR reduction techniques based on Nonlinear Functions as particularly clipping techniques can be modeled as adding signal techniques which can be illustrated by the structure in Fig.1.

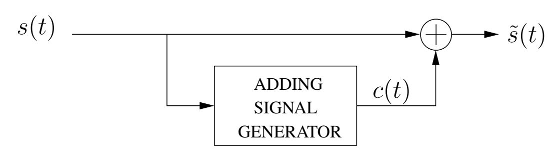

Fig. 1: Adding signal scheme for PAPR reduction, where $c(t)$ is the peak-reducing signal.

Now, let us suppose $f(.)$ the function of $s(t)$ PAPR reduction, the signal $\tilde{s}(t)$ at the output of PAPR reduction scheme is expressed as

$$\tilde{s}(t) = s(t) + c(t). \tag{5}$$

From Eq.(4) and Eq.(5), the peak-reducing signal $c(t)$ is expressed as

$$c(t) = (\alpha - 1) s(t) + d(t). \tag{6}$$

We see from Eq.(6) that, the peak-reducing signal depends on the uncorrelated component resulting in the nonlinear process of the input signal amplitude.

D. Properties of the peak-reducing signal

The statistical properties of signals that pass through nonlinear devices have been widely investigated in the past [12, 13]. In this subsection, using the statistical properties of uncorrelated component resulting in the Bussgang decomposition investigated in the past [12, 13], we derive the statistical properties of the peak-reducing signal. Indeed, in [13], it is shown that the correlation function for the uncorrelated component $d(t)$ is

$$\mathcal{R}_{dd}(\tau) = \sum_{n=1}^{\infty} b_n \left[\frac{\mathcal{R}_{ss}(\tau)}{\mathcal{P}_s}\right]^{2n+1}, \tag{7}$$

where $\mathcal{R}_{ss}(\tau)$ is the correlation function for $s(t)$ and b_n is expressed by

$$b_n = \frac{1}{\mathcal{P}_s}\frac{1}{n+1}\left\|\int_{\mathcal{D}(r)} r f(r)\, p(r)\, .L_n^{(I)}\left(\frac{r^2}{\mathcal{P}_s}\right) dr\right\|^2, \tag{8}$$

where $\mathcal{D}(r) = \{r : 0 \leq r \leq \infty\}$ is the domain of integration, $p(r)$ is the probability density function of the input signal amplitude and $L_n^{(I)}(x)$ is the Laguerre function expressed by

$$L_n^{(I)}(x) = \frac{x^{-k}e^{x}}{n!}\left(\frac{d}{dx}\right)^n \left(x^{n+k}.e^{-x}\right). \tag{9}$$

Using Eq.(9) and as $d(t)$ is uncorrelated with $s(t)$, the correlation function for $c(t)$ is

$$\mathcal{R}_{cc}(\tau) = |\alpha-1|^2 \mathcal{R}_{ss}(\tau) + \mathcal{R}_{dd}(\tau). \tag{10}$$

Substituting Eq.(7) into Eq.(10) we find

$$\mathcal{R}_{cc}(\tau) = |\alpha-1|^2 \mathcal{R}_{ss}(\tau) + \sum_{n=1}^{\infty} b_n \left[\frac{\mathcal{R}_{ss}(\tau)}{\mathcal{P}_s}\right]^{2n+1}. \tag{11}$$

The peak-reducing signal power spectrum density (PSD) $\mathcal{S}_{cc}(\upsilon)$ is expressed by

$$\begin{aligned}\mathcal{S}_{cc}(\upsilon) &= \mathrm{FT}\{\mathcal{R}_{cc}(\tau)\}\\ &= \left\{\begin{array}{c} |\alpha-1|^2 \mathcal{S}_{ss}(\upsilon)\\ + \sum\limits_{n=1}^{\infty} \frac{b_n}{\mathcal{P}_s^{2n+1}}.\left[\mathcal{S}_{ss}(\upsilon) \otimes_1 \cdots \otimes_{2n+1} \mathcal{S}_{ss}(\upsilon)\right]\end{array}\right.,\end{aligned} \tag{12}$$

where FT$\{.\}$ is the Fourier Transform operator and υ the frequency variable. It should know that, Eq.(12) is the power spectrum density of Eq.(6).

From Eq.(12), we can see that $c(t)$ is distributed over a wider bandwidth than $s(t)$.

In [12, 13], it is shown that the uncorrelated component resulting in the Bussgang decomposition is responsible to in-band (IB) and out-of-band (OOB) distortions. As $c(t) = (\alpha-1)s(t) + d(t)$, the peak-reducing signal is implicitly responsible to in-band (IB) and out-of-band (OOB) distortions. Thus, a trade-off must be done between PAPR reduction and distortions generated.

IV. Principle of PAPR reduction without BER degradation and OOB emission

In this section, we propose to process the peak-reducing signal $c(t)$ in order not to degrade the BER of system and not to generate OOB distortion. For that, a cleverness is to use the unused subcarriers embedded in the spectrum of multicarrier standards to carry the peak-reducing signal. Indeed, without unused subcarriers of multicarrier standards, PAPR can be reduced without BER degradation but with OOB emission or without OOB emission but with BER degradation.

In this paper, we use a FFT/IFFT pair-based digital filtering to filter the peak-reducing signal in order to carry it only on the unused subcarriers of multicarrier standards. Let $\mathcal{I}$ and $\mathcal{O}$ be the set of the in-band indices and out-of-band indices respectively. Let $\mathcal{R} = \{i_0, \cdots, i_{\mathrm{N}_r-1}\}$ be the locations of the unused subcarriers [(b)] and $\mathcal{R}^c$ the complement of $\mathcal{R}$ in $\mathcal{I}$.

The FFT/IFFT pair-based filter consists of a FFT followed by an IFFT operation. The forward FFT transforms c_n back to the frequency-domain. The discrete frequency components of c_n on the unused subcarriers $\mathcal{R}$ are passed unchanged while the data subcarriers $\mathcal{R}^c$ and the OOB components $\mathcal{O}$ are setted to zero, i.e,

$$\tilde{C}_k = \left\{\begin{array}{ll} C_k, & k \in \mathcal{R}\\ 0, & k \in (\mathcal{R}^c \cup \mathcal{O})\end{array}\right. . \tag{13}$$

The IFFT operation transforms $\tilde{C}_k$, $k = 0, \ldots,$ NL - 1 back to the time domain. This results in the filtered peak-reducing signal $\tilde{c}_n$ at the output of the filter-based FFT/IFFT. Because of $\mathcal{R} \cap \mathcal{R}^c = \emptyset$, the BER of the system is not degraded. Because of $\tilde{C}_k = 0$ for $k \in \mathcal{O}$, there is not OOB emission.

It is obvious that the filtering eliminates a part of the peak-reducing signal which causes a peaks regrowth. In order to reduce as much as possible the PAPR, the process of adding signal computation followed by filtering must be repeated several times. The new PAPR reduction scheme including the filtering and the iterations is illustrated in Fig.2 in discrete time-domain.

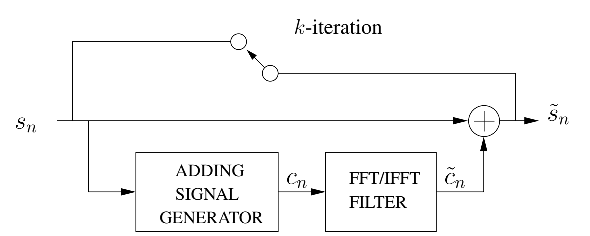

Fig. 2: Adding signal scheme for PAPR reduction including FFT/IFFT-based filter and the iteration process.

V. Illustration by the classical clipping function in a WLAN System context

In this section, we consider the classical clipping function which is a function for PAPR reduction. This function is used to reduce the PAPR of the local-area-networks (WLAN)

[(b)]It is assumed that, the unused subcarriers are located in the OFDM spectrum, i.e., $\mathcal{R} \subset \mathcal{I}$.

system based on IEEE 802.11a/g standards under the condition of none BER degradation and none OOB power emission.

A. *Clipping function*

Clipping function is used in clipping techniques to clip the high peaks of signals. In this paper we consider the classical clipping technique [14] whose clipping function is expressed as

$$f_{clip}(r) = \begin{cases} r, & r \leq \mathrm{A} \\ \mathrm{A}, & r \leq \mathrm{A} \end{cases},$$

where, r is the amplitude of the signal and A is the clipping level. In [11] , it is shown that any form of clipping (in particular, the classical clipping) can be formulated as an adding signal technique as illustrated in Fig.1.

B. *The IEEE 802.11a/g standards based WLAN system*

In WLAN IEEE 802.11a/g standard IFFT size (N) is 64. Out of these 64 subcarriers, 48 subcarriers are used for data, while 4 subcarriers are used for pilots. The rest 12 subcarriers are unused (null) subcarriers located at the positions $\mathcal{R} = \{0, 27, \cdots, 37\}$ of the IFFT input. Only these unused subcarriers shall be utilized for WLAN PAPR reduction.

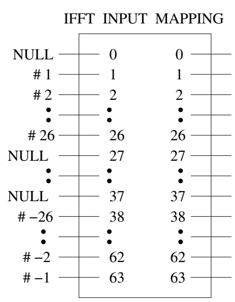

Fig. 3: Inputs and outputs of IFFT

Fig.3 shows the IEEE 802.11a/g Standard specifications: Subcarriers (data, pilots, unused) positions [15].

C. *Simulation Results*

In this subsection, we evaluate the performance of the classical clipping function-based nonlinear function for WLAN PAPR reduction.

System Parameter	Parameter Value
Modulation Scheme	16-QAM
Number subcarriers	N = 64
Number of data subcarriers	48
Number of pilot subcarriers	4
Oversampling Factor	L = 4
Channel Model	AWGN

Table. I: Simulation Environment

The distribution based on a Complementary Cumulative Distribution Function (CCDF) is used to evaluate the performance in terms of PAPR reduction of the system, the BER metric is used to evaluate the transmission performance of the system over an AWGN channel and the Power Spectral Density (PSD) of signals will be evaluated.

Fig.4 shows the peak power reduction results for the PAPR reduction technique based on the classical clipping function $f_{clip}(.)$ at different iterations. The gain in PAPR reduction ΔPAPR increases as the number of iterations increases. For example, at 10^{-3} of the CCDF, ΔPAPR is about .75 dB, 1.75 dB and 2.75 dB for one, three and five iterations respectively.

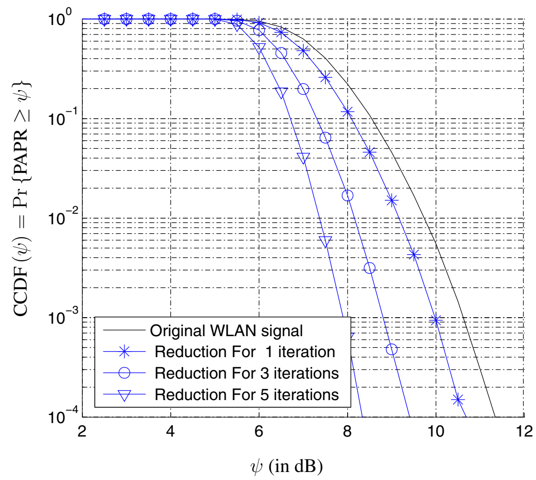

Fig. 4: PAPR reduction performance for $\frac{\mathrm{A}}{\sqrt{\mathcal{P}_s}} = 5$ dB.

Fig.5 shows the transmission performance of the system over an AWGN channel. It is obvious that, whatever the number of iteration, the BER of the system after PAPR reduction is not degraded. Indeed, the unused subcarriers embedded in the spectrum of the WLAN standard are used to carry the peak-reducing signal, so the BER of data subcarriers with the technique is the same as that of the original WLAN system.

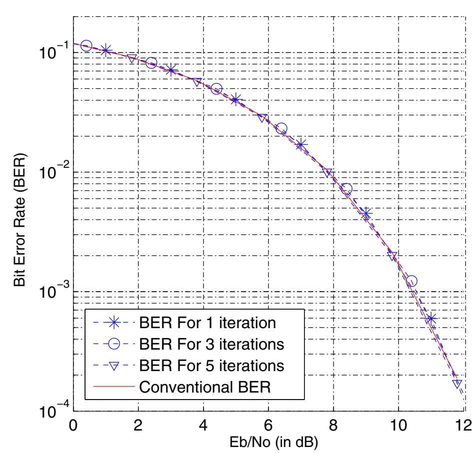

Fig. 5: BER performance for $\frac{\mathrm{A}}{\sqrt{\mathcal{P}_s}} = 5$ dB.

Fig.6 and Fig.7 show the PSD of signals before and after PAPR reduction at different iterations. Fig.7 is a zoom-in of Fig.6. They show a growth of the spectrum under the unused subcarriers after PAPR reduction. Additional, the level of spectrum under the unused subcarriers increases as the number of iterations increases.

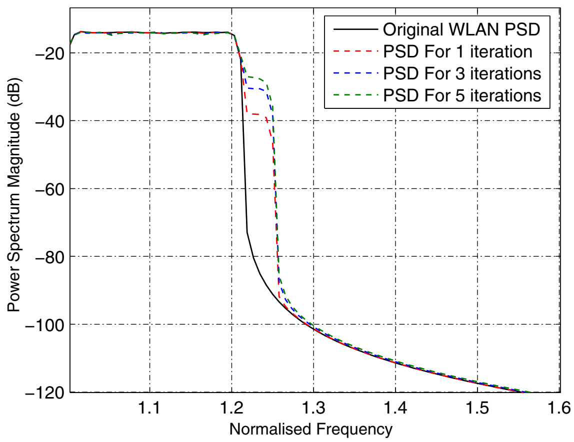

Fig. 6: Power Spectral Density (PSD) of signals for $\frac{A}{\sqrt{\mathcal{P}_s}} = 5$ dB.

Because of the peak reducing signal is carried only by the unused subcarriers of the WLAN standard, so it is obvious that the spectrum of the standard under the unused subcarriers after PAPR reduction grows. However, the iteration process increases the level of signal power under the unused subcarriers that is why the gain in PAPR reduction increases as the number of iterations increases as shown in Fig.4.

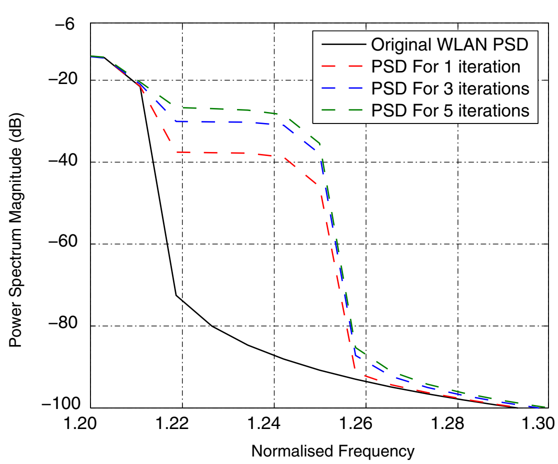

Fig. 7: Zoom on the PSD of signals for $\frac{A}{\sqrt{\mathcal{P}_s}} = 5$ dB.

VI. Conclusion

This paper is focused on the PAPR reduction techniques which use nonlinear functions to reduce the high fluctuations of the signals amplitude. These techniques reduce the PAPR at the cost of generating distortions. Based on Bussgang theorem, we have proven for these techniques that, the uncorrelated component resulting in nonlinear process of the input signal amplitude is the component for PAPR reduction. Using this result, we have proposed to reduce PAPR of multicarrier systems by exploiting the unused subcarriers embedded in their spectrums to carry the component for PAPR reduction. As the component for PAPR reduction is carried by the unused subcarriers of standards, so there is no BER degradation nor OOB emission. Then, the classical clipping function is used to illustrate these assertions in a WLAN system based on IEEE 802.11a/g standard.

The work carried out in this paper has generated some areas of possible future study.

- According to simulation results, the gain in PAPR reduction increases as the number of iterations increases. It must bear in mind that, the system complexity grows linearly with the number of iterations. So, a cleverness to get more PAPR reduction with less iterations is to exploit, in additional of the unused subcarriers of standards, the OOB subcarriers (created by oversampling) to carry the peak reducing signal.
- Another possible future study is to find an optimal nonlinear function which give more PAPR reduction with less iterations by formulating the problem as an optimization problem.

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