Solving Doppler Ambiguity by Doppler Sensitive Pulse Compression Using Multi-Carrier Waveform

R. F. Tigrek[#1], W. J. A. de Heij[#2], P. van Genderen[*]

[#]*Thales Surface Radar, The Netherlands*

[1]*Delftechpark 24, 2628 XH Delft, The Netherlands*

[2]*Haaksbergerstraat 49, 7554 PA Hengelo, The Netherlands*

[1]recep.tigrek@nl.thalesgroup.com

[2]wim.deheij@nl.thalesgroup.com

[*]*IRCTR, Delft University of Technology*
Mekelweg 4, 2628 CD Delft, The Netherlands

P.vanGenderen@IRCTR.TUDelft.nl

Abstract— **A processing technique, which exploits the structure of the multi-carrier signal to obtain range and radial velocity information from the echo signal, is proposed. The novel processing technique executes the Doppler matched filter bank digitally, and solves the ambiguity in the radial velocity measurements in one train of multi-carrier pulses as a result of Doppler sidelobes of the ambiguity function being lower. The processing technique, being independent of the phase coding on the multiple carriers, enables integration of communication and radar waveforms.**

I. INTRODUCTION

Use of radar signals composed of multiple carriers is proposed in the literature with the aim of improving the radar performance. OFDM (Orthogonal Frequency Division Multiplexing) waveform, along with a coding scheme, is proposed in [1], [2] to obtain low range sidelobe levels while limiting the spectral extent of the waveform; the Doppler response is not inspected in detail. More generic multi-carrier waveform that consists of transmission of the same multi-carrier waveform twice is proposed in [3], with no coding on its carriers, causing range ambiguity and high PAPR (Peak to Average Power Ratio). Another scheme using generic multi-carrier structure is presented in [4], with bank of FIR filters estimating the radial velocity. Use of OFDM waveform for SAR applications is proposed in [5], where the radial velocity measurement is not considered, with emphasis on the double use of the waveform for both radar and communication purposes.

In this paper an OFDM waveform transmit and receive scheme and the corresponding processing technique are proposed for the measurement of the target range and radial velocity. The novelty of the processing technique is the mitigation of the ambiguity in the radial velocity by exploiting the multi-carrier structure of the waveform and unambiguous range measurement at the same time. The processing does not have strong requirements on coding; thus, the communications between multiple radar units can be embedded in the radar signal.

In Section II the multi-carrier radar waveform structure is presented. The processing scheme is explained in Section III, and the radial velocity resolution and ambiguity are given in Section IV. Conclusions are given in Section V.

II. THE MULTI-CARRIER RADAR WAVEFORM STRUCTURE

The OFDM waveform is proposed as the multi-carrier radar waveform. The OFDM waveform $p(t)$ is the sum of carriers $p_k(t)$, given as

$$p(t)=\sum_{m=0}^{N-1} x_m \exp\{j\phi_m\}\exp\{j2\pi m\Delta f t\}, \qquad (1)$$

where $x_m \exp\{j\phi_m\}$ is the amplitude and phase of the complex symbol modulating the carrier m, and N is the number of carriers. $T=1/\Delta f$ is the duration of one OFDM waveform unit, called a chip, with Δf the carrier spacing.

The proposed radar signal consists of consecutive OFDM chips, where each chip is preceded by guard interval. The guard interval is generated by replicating the end portion of the OFDM chip that is to be transmitted afterwards, which is called a cyclic prefix (CP). The guard interval duration is chosen such that

$$T\alpha > \frac{2R_{\max}}{c}, \qquad (2)$$

where α is the ratio of the guard interval to the chip length, as seen in Fig. 1; thus, the echo received from the maximum range of interest comprises a complete OFDM chip. The consecutive OFDM chips, while they constitute a continuous waveform (CW), can be regarded as the pulses in a pulse train.

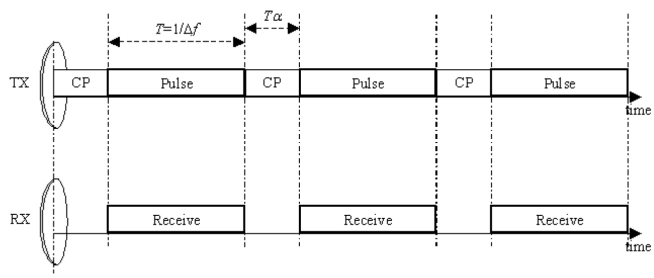

Fig. 1 The transmitting and receiving timing scheme

III. The Processing Scheme

The processing scheme is based on the decoding of the received OFDM waveform by an FFT. To accomplish this task, the receiving of the echoes start as soon as the transmission of the cyclic prefix is ended and the transmission of the actual OFDM chip is started. The receiving continues as long as the OFDM chip duration, after which the receiving ends and the cyclic prefix of the next OFDM chip is transmitted. Transmit and receive progression is given in Fig. 1. The received and downconverted echo of one OFDM chip for this scheme is

$$s(t) = \sum_{m=0}^{N-1} \exp\left\{ \begin{array}{l} j2\pi m \Delta f \left(1 - \frac{2v}{c}\right)\left(t - \frac{2R}{c}\right) \\ - j2\pi f_c \frac{2R}{c} - j2\pi f_c \frac{2v}{c}\left(t - \frac{2R}{c}\right) \end{array} \right\} \exp\{j\phi_m\} \tag{3}$$

where R is the range and v is the radial velocity of the point target, and f_c is the RF carrier frequency. In (3) the time t starts with the transmission of the actual OFDM chip.

For the rest of the paper, the signal and the processing steps are expressed in terms of matrix operations on discrete samples. The received echo is expressed as

$$\mathbf{s} = \psi \mathbf{\Gamma} \boldsymbol{\beta} \mathbf{A} \boldsymbol{\varphi}, \tag{4}$$

where $\mathbf{s}$ is the column vector representing the time samples of the received echo, and for critical sampling,

$$\psi = \exp\left\{ - j2\pi f_c \left(1 - \frac{2v}{c}\right)\frac{2R}{c} \right\}, \tag{5}$$

$$\mathbf{\Gamma} = diag\left\{1, \gamma, \gamma^2, \ldots, \gamma^{N-1}\right\}, \tag{6}$$

$$\gamma = \exp\left\{ - j2\pi f_c \frac{2v}{c} \frac{1}{N\Delta f} \right\}, \tag{7}$$

$$\boldsymbol{\beta} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \beta & \beta^2 & & \beta^{N-1} \\ 1 & \beta^2 & \beta^4 & & \beta^{2(N-1)} \\ \vdots & & & \ddots & \vdots \\ 1 & \beta^{N-1} & \beta^{2(N-1)} & \cdots & \beta^{(N-1)^2} \end{bmatrix}, \tag{8}$$

$$\beta = \exp\left\{ j2\pi \left(1 - \frac{2v}{c}\right)\frac{1}{N} \right\}, \tag{9}$$

$$\mathbf{A} = diag\left\{1, \alpha, \alpha^2, \ldots, \alpha^{N-1}\right\}, \tag{10}$$

$$\alpha = \exp\left\{ - j2\pi \Delta f \left(1 - \frac{2v}{c}\right)\frac{2R}{c} \right\}, \tag{11}$$

$$\boldsymbol{\varphi}^T = \left[\exp\{j\phi_0\} \quad \exp\{j\phi_1\} \quad \ldots \quad \exp\{j\phi_{N-1}\}\right]. \tag{12}$$

The $\boldsymbol{\beta}$ matrix is the same as an Inverse Discrete Fourier Transform (IDFT) matrix of the same size when $2v/c<<1$, which will be the assumption in this paper.

The pulse compression is achieved by compensating for the starting phases of the carriers, which is essentially matched filtering. Instead of correlating the received echoes with the transmitted waveform, the received echoes are de-modulated through FFT algorithm and the recovered phases are compensated by multiplying with the complex conjugates of the transmitted phases.

The loss in the pulse compression gain due to Doppler effect is inspected in [6], with a measure of the filter spacing in terms of Doppler frequency to limit the compression loss to 1 dB. The loss in pulse compression gain due to the Doppler effect is because of the transmitted signal's being mismatched to the received echo with the Doppler effect. Doppler filter banks, which consist of filters matched to Doppler shifted replicas of the transmitted signal, are employed to mitigate the loss in the pulse compression gain.

OFDM waveform is composed of a number of orthogonal carriers, and the Doppler effect on the OFDM waveform can be considered as the shift of the carriers by an amount determined by the radial velocity of the reflector. The structure of the OFDM enables the Doppler compensation in a straightforward manner by implementing a cyclic shift of the FFT output in the receiver. In this manner no separate hardware is needed to implement the Doppler compensated matched filtering banks or to generate frequency shifted replicas of the reference signal.

Thus, the received echo samples are processed as

$$\mathbf{y} = \mathbf{F}^{-1}\mathbf{P}\mathbf{C}^{-1}\mathbf{F}\mathbf{s}, \tag{13}$$

where $\mathbf{P}$ is the compensation operation for the starting phases, $\mathbf{C}^{-1}$ is the compensation operation for the Doppler shift, which is the cyclic shift, $\mathbf{F}$ is the Discrete Fourier Transform (DFT) matrix, and $\mathbf{F}^{-1}$ is the IDFT matrix. The processing scheme is given as block diagram in Fig. 2. The Doppler effect on the received echo $\mathbf{s}$ is confined to the $\mathbf{\Gamma}$ matrix, due to the assumption that $2v/c<<1$.

When $\mathbf{s}$ in (13) is replaced by (4), the consequent multiplication of the DFT matrix $\mathbf{F}$ in (13) by the diagonal $\mathbf{\Gamma}$ matrix in (4) is equivalent to multiplying each column of the DFT matrix with the corresponding diagonal element of the $\mathbf{\Gamma}$ matrix. As the elements of the $\mathbf{\Gamma}$ matrix are powers of the same complex exponential, this multiplication is equivalent to the cyclic shift of the rows of DFT matrix when

$$\gamma = \exp\left\{ - j2\pi f_c \frac{2v}{c} \frac{1}{N\Delta f} \right\} = \exp\left\{ - j2\pi\varepsilon \frac{1}{N} \right\}, \tag{14}$$

where ε is a positive integer. The frequency shift resulting from the Doppler effect is equal to an integer multiple of the carrier spacing Δf in this case. For example, for $\varepsilon=2$, the DFT matrix is shifted upwards in a cyclic manner by 2 rows.

Such shifting operation can be realized also through multiplication from left by an identity matrix $\mathbf{I}$ cyclically

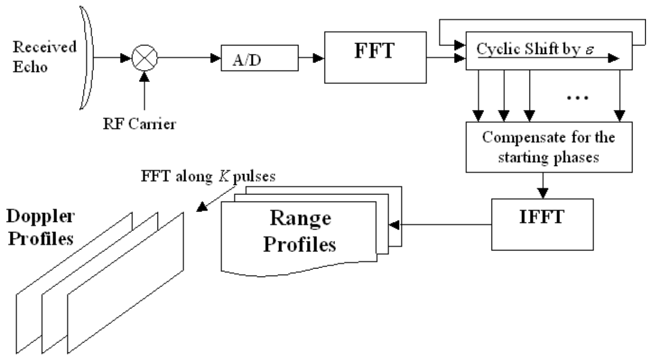

Fig. 2 The processing scheme block diagram

shifted in the same manner as the DFT matrix would be, inverse of which is the $\mathbf{C}^{-1}$ in (13). The range of the cyclic shift values ε that is to be swept during the processing is determined by

$$\frac{-v_{\max} 2f_c}{c\Delta f} \leq \varepsilon \leq \frac{v_{\max} 2f_c}{c\Delta f}, \qquad (15)$$

where $v_{\max}$ is the maximum velocity of interest. This method is valid when the received waveform is padded with zeros before the FFT operation to obtain over-sampling.

Radial velocity is measured through the phase variation from pulse to pulse for a range bin, the same as the other pulse compression techniques.

IV. The Doppler Resolution and Ambiguity

As the consecutive OFDM chips are processed as a pulse train, the resolution and ambiguity calculations are the same as that of a regular pulse train. The unambiguous radial velocity defined for conventional pulse burst processing is given as

$$v_{unam} = \frac{cf_p}{2f_c}, \qquad (16)$$

where f_p is the pulse repetition frequency and f_c is the RF carrier. For the continuous waveform consisting of OFDM chips and CP, the unambiguous radial velocity is modified to

$$v_{unam} = \frac{c\Delta f}{2f_c(1+\alpha)}, \qquad (17)$$

where Δf is the carrier spacing and α is the ratio of the length of the cyclic prefix to the actual chip length.

The radial velocity resolution is related to the time on target through the equation

$$v_{res} = \frac{c}{2T_{dwell} f_c}, \qquad (18)$$

where T_{dwell} is the time on target. For single pulse the radial velocity resolution is

$$v_{res,1} = \frac{c\Delta f}{2f_c}, \qquad (19)$$

and for the pulse train the radial velocity resolution is

$$v_{res,K} = \frac{c\Delta f}{2K(1+\alpha)f_c}, \qquad (20)$$

where K is the number of OFDM chips forming the pulse train.

The processing technique lowers the Doppler sidelobes by combining the ambiguity function of a non-modulated pulse train by the ambiguity function of the single OFDM chip. The periodic ambiguity function (PAF) for coherent train of K pulses is given in [7] as

$$|\chi(\tau, f_d)| = |\chi_T(\tau, f_d)| \left| \frac{\sin(K\pi f_d T(1+\alpha))}{K\sin(\pi f_d T(1+\alpha))} \right|, \qquad (21)$$

where $|\chi_T(\tau, f_d)|$ is the single-period PAF, or the ambiguity function of the single OFDM chip, and for our waveform

$$\tau = \frac{2R}{c}, f_d = f_c \frac{2v}{c}. \qquad (22)$$

While in [7] the expression in (21) is considered invalid for continuous wave signals and pulse trains with α<1, the assumption that the maximum delay in the echoes is less than the guard interval, as stated in (2), allows its use.

We may write the ambiguity function for single OFDM chip as

$$|\chi_T(\tau, f_d)| = \left| \int_{-\infty}^{\infty} p(t)p^*(t+\tau)\exp\{j2\pi f_d t\} dt \right|, \qquad (23)$$

and changing the orders of the integral and summation gives

$$|\chi(\tau, f_d)| = \left| \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} \begin{pmatrix} \exp\{j\phi_m\} \\ \exp\{-j\phi_k\} \end{pmatrix} \int_0^T \left(\exp\begin{Bmatrix} j2\pi\Delta f((m-k)t - k\tau) \\ + j2\pi f_d t \end{Bmatrix} \right) dt \right| \qquad (24)$$

The limits of the integral are modified as 0 and T according to the timing scheme given in Fig. 1.

The expected value of the single-period PAF for the OFDM waveform can be used to assess the expected improvement of the measurement ambiguity. Following a similar approach as in [6], only the terms with k=m are non-zero, and the expected value of the single-period PAF for the OFDM waveform is

$$|E[\chi(\tau, f_d)]| = \left| \sum_{k=0}^{N-1} \exp\{-j2\pi k\Delta f\tau\} \int_0^T (\exp\{j2\pi f_d t\}) dt \right| \cdot \qquad (25)$$

Solving the integral yields

$$|E[\chi(\tau, f_d)]| = T\left| \operatorname{sinc}(\pi f_d T) \sum_{k=0}^{N-1} \exp\{-j2\pi k\Delta f\tau\} \right| \cdot \qquad (26)$$

For the zero delay Doppler cut, (26) is simplified further as

$$|E[\chi(0, f_d)]| = NT|\operatorname{sinc}(\pi f_d T)| \qquad (27)$$

The expected zero delay Doppler cut of the ambiguity function given in (27), superimposed that of the single OFDM chip and unmodulated pulse train, is given in Fig. 3 for α=0,25.

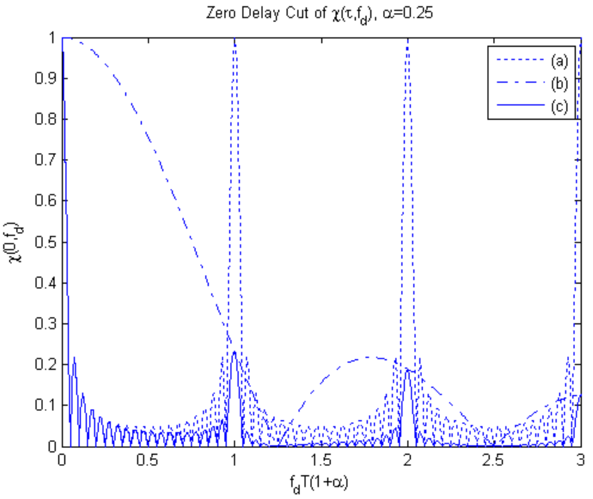

Fig. 3 The expected zero delay Doppler cut of the ambiguity functions of (a) the unmodulated pulse train, (b) the single OFDM pulse modulated with uniformly distributed random phases, (c) the train of the OFDM pulses

The normalized level of the ambiguity sidelobes of the zero delay Doppler cut, as seen in Fig. 4, is

$$A_{s,OFDM} = \left| \operatorname{sinc}\left(\frac{s\pi}{1+\alpha} \right) \right|, \tag{28}$$

where s is a positive integer and $A_{s,OFDM}$ is the amplitude of the ambiguity at

$$f_d = (s+1)\frac{\Delta f}{(1+\alpha)}. \tag{29}$$

Fig. 4 shows that the maximum range of interest determines the lower bound of α while the levels of the ambiguities determine the upper bound.

With the ambiguity in the radial velocity measurement due to the use of the coherent pulse train solved, the next ambiguity in the radial velocity measurement occurs when

$$f_d = N\Delta f \Rightarrow v_{unam} = \frac{f_d c}{2 f_c} = \frac{cN\Delta f}{2 f_c}. \tag{30}$$

The ambiguity is the result of the Doppler frequency shift being equal to the signal bandwidth. In this case, the cyclic shift ε is equal to the number of the carriers, N, which is ambiguous to the zero radial velocity.

While the expression in (30) holds for the narrow band cases, additional deterioration in wide band case may limit the measurable maximum radial velocity before the ambiguous velocity is approached.

V. Conclusions

The multi-carrier waveform structure of OFDM offers an opportunity to solve the ambiguity in the radial velocity measurements without the need to transmit multiple trains of pulses, reducing the required time on target remarkably. Furthermore, the Doppler compensation of multi-carrier waveform is accomplished in a more efficient manner than that of single-carrier phase-coded waveforms.

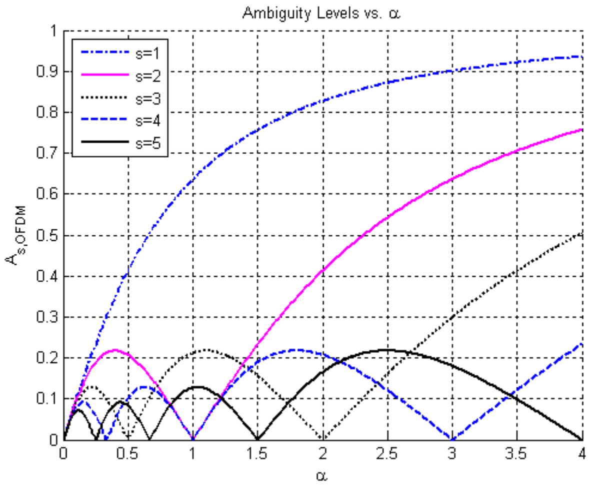

Fig. 4 Relative levels of the first 5 radial velocity ambiguities for different guard interval relative durations

The waveform structure and the processing scheme are considered as good candidates for use in radar networks responsible for surveillance of areas for slow moving targets and persons. The communication function can be embedded into the radar waveform, as provided by the processing scheme functioning with random phase coding on the carriers. Such dual use of the waveform and the hardware provides the radar network with robust communications infrastructure.

The investigation in this paper focuses on demonstrating the capability of the multi-carrier structure to solve the Doppler ambiguity. The performance of the processing scheme considering noise and other effects, along with the reduction of the Doppler sidelobes by coding and weighting will be the topic of further analysis.

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