Adaptive OFDM Integrated Radar and Communications Waveform Design Based on Information Theory

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***Abstract*— To improve the effectiveness of limited spectral resources, an adaptive orthogonal frequency division multiplexing integrated radar and communications waveform design method is proposed. First, the conditional mutual information (MI) between the random target impulse response and the received signal, and the data information rate (DIR) of frequency selective fading channel are formulated. Then, with the constraint on the total power, the optimization problem, which simultaneously considers the conditional MI for radar and DIR for communications, is devised, and the analytic solution is derived. With low transmit power, the designed integrated waveform outperforms the fixed waveform (i.e., equal power allocation). Finally, several simulated experiments are provided to verify the effectiveness of the designed waveform.**

***Index Terms*— Transmit power adaptation, integrated radar and communications, orthogonal frequency division multiplexing, conditional mutual information, data information rate.**

I. Introduction

AT PRESENT, the integrated system has attracted much attention, since it has advantages in reducing the system size, weight, and power consumption, and mitigating electromagnetic interference, and a multitude of scenarios of application. In [1], radar, communications, and electronic warfare are integrated in a single platform with the array antennas, signal processing and display hardware shared. It is studied in [2] that the intelligent transportation system (ITS) employs communications device to convey traffic information and utilizes the radar device to sense the traffic circumstance, which motives the integration of radar and communications.

The integration of radar and communications in hardware is easy to be attained with present technology [3]. It is more crucial to exploit an integrated waveform simultaneously performing the radar and communications functions to improve the availability of limited spectral resources. To this end, there have been many literatures regarding orthogonal frequency division multiplexing (OFDM) concentrating on the realization of radar and communications functions and the corresponding signal processing [3], [4]. Moreover, in [5] the total power is split between the data symbols and training symbols which are employed to perform the radar function. The inner bounds on performance of the joint radar and communications system in terms of data information rate (DIR) and estimation information rate for the receiver are investigated in [6]. The conditional mutual information (MI) and channel capacity of the integrated radar and communications system (IRCS) based on multiple input multiple output (MIMO) is addressed in [7] without optimizing the transmitted waveform. Actually, it is possible to improve the performance of both radar and communications by using flexible waveforms. To the best our knowledge, it is also an open problem to design the optimal integrated waveform with the constraint on the total power.

For radar waveform design, an important tool is to utilize the information theory [8]–[10]. The pioneering research in [9] maximizes the conditional MI between a random extended target and the reflected signal to optimize the transmitted waveform. Afterwards, the investigation in [10] states that minimizing the minimum mean square error (MMSE) in estimating the target impulse response is equivalent to maximizing the conditional MI. For communications waveform design, a crucial design criterion is to maximize the throughput (rate) by adaptively assigning the transmit power according to the channel state information (CSI) [11]–[13].

For the integrated radar and communications waveform design, we consider the extended target and frequency selective fading channel. And we propose an adaptive integrated waveform design method to improve the conditional MI and DIR of the OFDM IRCS.

The remainder of this letter is organized as follows. In Section II, the integrated signal model is established and the conditional MI and DIR are formulated. The adaptive waveform design method is proposed in Section III. In Section IV, several numerical simulations are presented. Finally, conclusions and future work are given in Section V.

II. Problem Description and Modeling

In this section, the integrated signal model and the radar and communications performance criteria of the IRCS are created.

A. Integrated Signal Model

For the IRCS, the transmitted N_s consecutive integrated OFDM symbols can be described as

$$s(t) = e^{j2\pi f_c t} \sum_{n=0}^{N_s-1} \sum_{m=0}^{N_c-1} a_m c_{m,n} e^{j2\pi m \Delta f (t-nT_s)} \cdot \mathrm{rect}\left[(t-nT_s)/T_s\right] \quad (1)$$

where f_c is the center frequency, N_c is the number of subcarriers, the complex weight transmitted over the m-th subcarrier is a_m which will be designed in the following, $c_{m,n}$ is the phase

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code of the m-th subcarrier, n-th OFDM symbol and conveys communications information, $\Delta f = 1/T$ is the subcarrier interval with the duration of elementary OFDM symbol T, T_{s} is the duration of each completed OFDM symbol, and satisfies $T_{\mathrm{s}} = T + T_{\mathrm{g}}$ with the cyclic prefix (CP) T_{g}, and rect $[t/T_{\mathrm{s}}]$ is rectangle function, which is equal to one for $0 \leq t \leq T_{\mathrm{s}}$, and zero, otherwise.

B. Conditional Mutual Information

For radar target identification and classification, it is important to achieve much more conditional MI to improve the accuracy of estimation of target impulse response. In this subsection, the conditional MI that the IRCS can obtain with the integrated OFDM waveform is addressed.

Suppose that the impulse response $g(t)$ of an extended target is a Gaussian random process. Hence, the received signal is

$$y(t) = \int_{-\infty}^{\infty} g(\tau)\, s(t-\tau)\, d\tau + n(t) \tag{2}$$

where $n(t)$ is complex additive white Gaussian noise (AWGN) with zero mean and power spectral density $N(f)$.

Following the guidelines in [9], the conditional MI between the target impulse response and the received signal is

$$I\left(y(t); g(t) | s(t)\right) = \frac{\Delta f T_{\mathrm{p}}}{2} \sum_{m=0}^{N_{\mathrm{c}}-1} \log_2 \left(1 + \frac{|S(f_m)|^2 |G(f_m)|^2}{N(f_m)\, T_{\mathrm{p}}} \right) \tag{3}$$

with the assumption that in the frequency interval $\Theta_m = \left[f_m, f_{m+1}\right]$, $S(f) \approx S(f_m)$, $G(f) \approx G(f_m)$, and $N(f) \approx N(f_m)$, where $f_m = f_{\mathrm{c}} + m\Delta f$ is the m-th subcarrier frequency, $T_{\mathrm{p}} = N_{\mathrm{s}} T_{\mathrm{s}}$ is the signal duration, and $S(f)$ and $G(f)$ are the Fourier transform of $s(t)$ and $g(t)$, respectively.For simplicity, the conditional MI is referred to as MI in this letter.

Define $Q(f) = |S(f)|^2$, the following can be achieved

$$\begin{aligned} Q(f) = T_{\mathrm{s}}^2 &\sum_{n=0}^{N_{\mathrm{s}}-1} \sum_{m=0}^{N_{\mathrm{c}}-1} \sum_{n'=0}^{N_{\mathrm{s}}-1} \sum_{m'=0}^{N_{\mathrm{c}}-1} a_m a_{m'}^* c_{m,n} c_{m',n'}^* \\ &\cdot e^{-j2\pi(f-f_{\mathrm{c}})(n-n')T_{\mathrm{s}}} e^{-j\pi(m-m')\Delta f T_{\mathrm{s}}} \\ &\cdot s_{\mathrm{a}}\left(\pi(f-f_m)T_{\mathrm{s}}\right) s_{\mathrm{a}}\left(\pi(f-f_{m'})T_{\mathrm{s}}\right) \end{aligned} \tag{4}$$

where $(\cdot)^*$ is complex conjugation, and $s_{\mathrm{a}}(t) = \sin t/t$.

In practice, the communications code $c_{m,n}$ is determined by the carried information. If the phase of the communications code $c_{m,n}$ is random with uniform distribution (Phase shift keying modulation is widely used in communications), which can be attained by precoding [14], the following can be gotten

$$\mathrm{E}\left[c_{m,n} c_{m',n'}^*\right] = \begin{cases} 1, & m = m', n = n' \\ 0, & \text{else} \end{cases} \tag{5}$$

where $\mathrm{E}[\cdot]$ is the expectation operator. Thus, $\mathrm{E}[Q(f)]$ can be simplified to

$$\mathrm{E}[Q(f)] = T_{\mathrm{s}}^2 N_{\mathrm{s}} \sum_{m=0}^{N_{\mathrm{c}}-1} |a_m|^2 [s_{\mathrm{a}}(\pi(f-f_m)T_{\mathrm{s}})]^2 \tag{6}$$

If N_{c} or N_{s} is sufficiently large, $Q(f)$ will approach to its mean value $\mathrm{E}[Q(f)]$. In practice, there are hundreds of subcarriers. Hence, the following approximation is reasonable

$$Q(f) \approx T_{\mathrm{s}}^2 N_{\mathrm{s}} \sum_{m=0}^{N_{\mathrm{c}}-1} |a_m|^2 [s_{\mathrm{a}}(\pi(f-f_m)T_{\mathrm{s}})]^2 \tag{7}$$

In practice, since $s_{\mathrm{a}}(\pi(f_m - f_{m'})T_{\mathrm{s}})$ is far less than one for $m \neq m'$, we can make the approximation $T_{\mathrm{s}}^2 N_{\mathrm{s}} \sum_{m'=0, m'\neq m}^{N_{\mathrm{c}}-1} |a_{m'}|^2 [s_{\mathrm{a}}(\pi(f_m - f_{m'})T_{\mathrm{s}})]^2 \approx 0$, for $m = 0, 1, \cdots, N_{\mathrm{c}}-1$. Therefore, at the frequency $f = f_m$, $Q(f)$ can be approximately

$$Q(f_m) = T_{\mathrm{s}}^2 N_{\mathrm{s}} |a_m|^2 \tag{8}$$

Substituting (8) into (3) yields

$$I\left(y(t); g(t) | s(t)\right) = \frac{1}{2} \Delta f T_{\mathrm{p}} \sum_{m=0}^{N_{\mathrm{c}}-1} \log_2 (1 + p_m \upsilon_m) \tag{9}$$

where $p_m = |a_m|^2$, and $\upsilon_m = N_{\mathrm{s}} T_{\mathrm{s}}^2 |G(f_m)|^2 / \left(N(f_m)\, T_{\mathrm{p}}\right)$ can be regarded as the signal to noise ratio (SNR) in the m-th subchannel.

C. Data Information Rate

For communications, the DIR is a paramount performance metric. Particularly, in the frequency selective fading channel, the DIR can be improved by suitably assigning the transmit power in each subchannel.

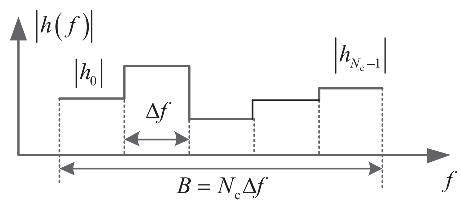

Fig. 1. The frequency response of communications channel.

Without loss of generality, we assume that the communications channel is slowly time variant and frequency dependent. The frequency fading response $h(f)$ is illustrated in Fig. 1. Hence, the total DIR of AWGN channel is formulated as [14]

$$\begin{aligned} C_{\mathrm{t}} &= \sum_{m=0}^{N_{\mathrm{c}}-1} \Delta f \log_2 \left(1 + |a_m|^2 |h_m|^2 / \sigma_{\mathrm{c}}^2\right) \\ &= \sum_{m=0}^{N_{\mathrm{c}}-1} \Delta f \log_2 (1 + p_m \varpi_m) \end{aligned} \tag{10}$$

where p_m represents the transmit power of the m-th subchannel, h_m indicates the channel frequency response of the m-th subchannel, σ_{c}^2 is the noise power in the communications channel, and $\varpi_m = |h_m|^2/\sigma_{\mathrm{c}}^2$ can be interpreted as the SNR in the m-th communication subchannel.

III. Adaptive Integrated Waveform Design

In this section, we devote to the OFDM integrated radar and communications waveform design to simultaneously maximize the MI for radar and DIR for communications.

With the constraint on the entire power, the optimization problem can be formulated as

$$\hat{\mathbf{p}} = \arg\max_{\mathbf{p}\in\Re^{N_c}} \left\{ \frac{w_r}{2F_r} \Delta f T_p \sum_{m=0}^{N_c-1} \log_2(1+p_m \upsilon_m) + \frac{w_c}{F_c} \sum_{m=0}^{N_c-1} \Delta f \log_2(1+p_m \varpi_m) \right\} \tag{11a}$$

$$\text{subject to } \mathbf{1}_{N_c}^{\mathrm{T}} \mathbf{p} \le 1, \quad p_m \ge 0, \; m = 0, 1, \cdots, N_c - 1 \tag{11b}$$

where $\mathbf{p} = \left[p_0 \; p_1 \; \cdots \; p_{N_c-1} \right]^{\mathrm{T}}$ is an $N_c \times 1$ vector containing the transmit power, $\mathbf{1}_{N_c}$ represents an $N_c \times 1$ vector of ones, the positive w_r and w_c are weighting factors specified by the user, and satisfy that $w_r + w_c = 1$. F_r and F_c are the maximum MI in (9) and DIR in (10) under the constraint in (11b), respectively.

The objective function in (11a) is concave, since it is the affine combination of two concave functions. Moreover, the inequality constraint in (11b) is convex. Therefore, the optimization problem in (11) is convex [15]. And the optimization problem is solvable under Karush-Kuhn-Tucker (KKT) conditions [15]:

$$\mu - \mu_m = w_r \upsilon_m \Delta f T_p / [2 \ln 2 F_r (1 + p_m \upsilon_m)] + w_c \varpi_m \Delta f / [\ln 2 F_c (1 + p_m \varpi_m)] \tag{12a}$$

$$\mu \left(\sum_{m=0}^{N_c-1} p_m - 1 \right) = 0 \tag{12b}$$

$$\mu_m p_m = 0 \tag{12c}$$

$$\mu \ge 0, \quad \mu_m \ge 0, \; m = 0, 1, \cdots, N_c - 1 \tag{12d}$$

where μ, and μ_m, for $m = 0, 1, \cdots, N_c - 1$ are Lagrange multipliers.

The following optimal solution can be obtained:

$$\hat{p}_m = \frac{1}{2} \left[\mu' (\alpha' + \beta') - (\upsilon'_m + \varpi'_m) + \sqrt{[(\varpi'_m - \upsilon'_m) + \mu'(\alpha' - \beta')]^2 + 4\mu'^2 \alpha' \beta'} \right]^+ \tag{13}$$

where $[x]^+ = \max\{x, 0\}$, $\alpha' = w_r \Delta f T_p / (2 \ln 2 F_r)$, $\beta' = w_c \Delta f / (\ln 2 F_c)$, $\upsilon'_m = 1/\upsilon_m$, $\varpi'_m = 1/\varpi_m$, and $\mu' = 1/\mu$ satisfies that

$$\left(\sum_{m=0}^{N_c-1} \hat{p}_m - 1 \right) = 0 \tag{14}$$

The positive μ' can be achieved by a simple bisection search over the interval $0 < \mu' \le 1/\min_m \left\{ \alpha' / (\upsilon'_m + 1) + \beta' / (\varpi'_m + 1) \right\}$, for $m = 0, 1, \cdots, N_c - 1$, where $\min_m \{x_m\}$, for $m = 0, 1, \cdots, N_c - 1$, represents the minimum value in the set $\{x_0, x_1, \cdots, x_{N_c-1}\}$. Once μ' is obtained, the optimal power allocation can be gotten using (13).

In practice, the target impulse response $g(t)$ is unknown. The OFDM IRCS transmits the integrated OFDM waveform with equal power allocation. Employing the received signal, the MMSE estimation of $g(t)$ can be achieved. Then, the maximum MI F_r and the maximum DIR F_c are calculated. Finally, the optimal integrated waveform is created. The devised waveform will be transmitted in the following period.

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
Duration of CP	1 us	Number of subcarriers	128
Subcarrier spacing	0.25 MHz	Number of OFDM symbols	10

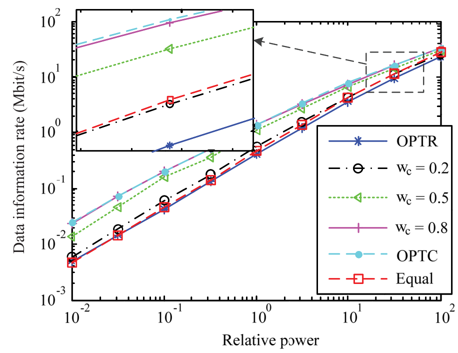

Fig. 2. The variation of data information rate with relative power.

IV. SIMULATION

In this section, several numerical examples are presented to verify the effectiveness of the proposed method. In all the simulations, the noise is complex AWGN, and both the frequency response of the communications channel and the target frequency response are subject to standard normal distribution. Other simulation parameters are shown in Table I.

For comparison, the performance of equal power allocation scheme in [7] which is modified to meet the simulation scenario, i.e., the power is equally allocated in each sub-channel, is plotted, denoted as "Equal". Furthermore, the performance of optimal radar F_r and performance of optimal communications F_c, are also illustrated, denoted as "OPTR", and "OPTC", respectively. To verify the performance of the proposed method, the standard Monte Carlo technique with 1000 independent trails is employed and the average results are presented.

A. Communications Performance

Under different power allocation approaches, the dependence of DIR on the relative power is shown in Fig. 2. The relative power is the total transmit power normalized by noise power. As expected, the DIR is enhanced with the increase of weighting factor for communications. For optimal radar, the DIR is poor, since its target is to maximize the radar performance. The performance of equal power allocation is inferior when the relative power is low. But it is improved with the increase of relative power, because with the relative power

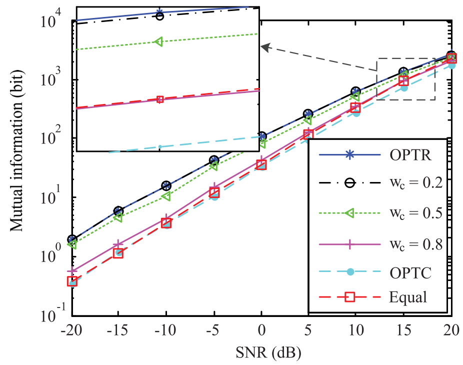

Fig. 3. The variation of mutual information with SNR.

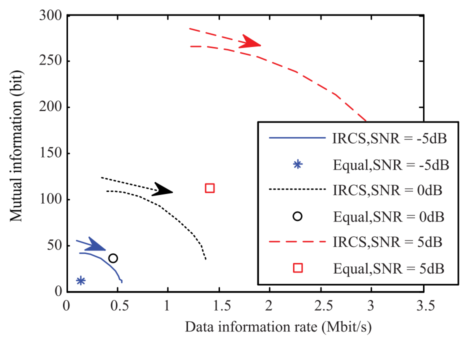

Fig. 4. The optimal tradeoff curve.

increasing, the difference of power allocation for each subchannel is gradually decreasing, i.e., the equal power allocation is close to the optimal power allocation for communications.

B. Radar Performance

In Fig. 3, the radar performance is evaluated under different power allocation schemes. It is obvious that the radar performance is improved with the decrease of weighting factor for communications. When the SNR is low, the equal power allocation is unfavorable. However, with the increase of SNR, it becomes better due to the decrease of difference between equal power allocation and optimal power allocation for radar. For optimal communications, the radar performance is inferior since the DIR for communication is maximized without considering the MI.

C. Optimal Tradeoff Curve

In Fig. 4, the optimal tradeoff curve is shown. The weighting factor for communications is increasing from 0 to 1 along the direction of the arrow. As expected, the DIR is enhanced with the weighting factor increasing, and the MI and DIR is improved with the increment of SNR. Moreover, in most cases, the designed waveform outperforms the fixed waveform. In practice, by using the optimal tradeoff curve, the weighting factor is selected to meet the demands for MI and DIR.

The previous simulation results enable us to conclude that compared with the equal power allocation the designed integrated waveform can achieve more excellent radar and communications performance with low total transmit power. And in most cases, neither the radar nor the communications performance is optimal for the OFDM IRCS. It is inescapable to make a tradeoff between the radar and communications performance. That is the cost to be paid for the OFDM IRCS to improve the availability of frequency spectrum.

V. CONCLUSION

In this letter, the adaptive OFDM integrated radar and communications waveform design is investigated. With the constraint on the entire power, an optimal integrated waveform design problem is devised, and the explicit solution in close-form is derived. In most cases, the designed integrated waveform makes a compromise between the radar and communications performance, neither of which is optimal. However, with low transmit power the designed integrated waveform is more pleasurable than the fixed waveform. Moreover, the radar and communications performance can be adjusted by the weighting factors. In future work, we will investigate the robust waveform design when the estimated values of target impulse response and channel frequency response are not exactly correct.

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