OFDM PAPR Reduction by Convex Optimization: A Power Amplifier Point-Of-View

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Abstract **— In this paper we evaluate the application of convex optimization for PAPR reduction on OFDM 802.11a signal type. A radio frequency power amplifier is measured and characterized while excited by both original and optimized OFDM signals. A state-of-art test setup was used for the purpose. Figure of merits such as power added efficiency, in-band errors, and out-of-band spectral emissions are investigated for their relevance and a study of the power distribution in the excitation signal is evaluated.**

Index Terms **— Communication system performance, convex optimization, dirty radio, power amplifiers, power added efficiency, spectrum mask.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a widely used modulation scheme in today's wireless communication systems [1-3]. The power amplifier (PA) is a key component in a wireless communication chain as it holds the highest power level in the system. Its nonlinearity is of great importance regarding in-band errors and out-of-band interference [4]. Its power added efficiency (PAE) is of high relevance as it reflects the amount of wasted energy in a communication system [4].

OFDM signals are characterized by their high bandwidth efficiency and robustness against frequency fading due to multipath propagation. However, their major drawback is in the signal high peak to average ratio (PAR) which limits the performance of the PA [4]. Due to that, a large number of dBs have to be backed-off to keep a linear operation at the PA level, which will result in drastically losses in its power efficiency [5].

Many methods have been proposed to reduce the PAPR of OFDM signals by clipping, envelope tracking, or by redistributing the nonlinear energy of the free subcarriers [6-9]. In [10], PAR minimization was presented as a convex optimization problem. A new power spectrum distribution was developed by optimizing the time domain peak power subject to error vector magnitude (EVM) constraints. It is processed by a power redistribution of the signal over its sub-carriers, including the power-free ones. By exploiting the fast Fourier transform (FFT) and its inverse (IFFT) structure of OFDM signal, a customized interior point method (IPM) can be adapted which find the near-to-global optimum PAR with fast and low complex algorithm [11]. This optimization approach has been taken further in [12-13] by adding spectral mask constraint and minimizing the EVM while keeping the PAR below a minimum threshold.

To our knowledge, no previous literature presents in-depth experimental validation on the impact of the PAR reduction of the signal on the power performance of the amplifier. Most of previous investigations were based on simulations.

In this paper, we experimentally evaluate the utility of the optimization method introduced by [10] applied to a WLAN 802.11a OFDM type signal, with respect to how the PAR optimization influences the power amplifier PAE and out-of-band emissions. A WLAN 802.11 a signal is generated and optimized using convex optimization IPM algorithm as described in Sec. II. The signal is used to excite a radio frequency (RF) power amplifier and performance characterization is performed using a state-of-art measurement setup described in Sec. III. In Sec. IV, an evaluation of the PA power performance is presented and compared to the non-optimized case. Finally conclusions are drawn in Sec. V.

II. OFDM PAR In A Convex form

This section gives a brief overview on the convex formulation of the OFDM PAR minimization problem and on the used IPM algorithm [10].

A WLAN 802.11a using OFDM signal with 48 data, 4 pilot, 12 free subcarriers and 128 symbols, was used in the validation process. The 52 subcarriers (data/pilot) were modulated using 16-quadrature amplitude modulation (QAM). The baseband OFDM signal was generated by dividing the information data into multiple data streams, each of was passed to a subcarrier for modulation. The modulated data streams were sent in parallel on the orthogonal subcarriers. The frequency constellation was then time domain transformed through IFFT. Cyclic prefix insertion as well as windowing was then applied for time-spreading handling and intersymbol interference elimination. The time domain symbols were packed in a series form and sent for in-phase and quadrature (IQ) modulation [1].

Consider $\mathbf{c_0}=(c_{0,1},\ldots,c_{0,n})^T$ an ideal OFDM frequency constellation, which is a complex-valued vector of length n. $\mathbf{c_0}$ will be modified by a factor $\Delta \in \mathbb{C}^n$ during the optimization process; $\mathbf{c} = \mathbf{c_0} + \Delta$. Its average error vector magnitude (EVM) is defined as [1]

$$\mathrm{EVM} = \sqrt{\frac{\frac{1}{d}\sum_{i=i_1}^{i_d}\left|c_i - c_{0,i}\right|^2}{P_0}} \qquad (1)$$

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where d is the number of modulated subcarriers, **c** and $\mathbf{c_0}$ are scaled to the same average power for evaluation, and P_0 is the average power of the modulation scheme used.

The time domain signal, $\mathbf{x}=\text{IFFT}_l[\mathbf{c}]$ and $\mathbf{x} \in \mathbb{C}^{nl}$, is found by applying an l-times oversampled IFFT to the constellation **c**. The oversampling or zero-padding is mainly used to accurately estimate the time domain peak levels. Let $p \in \mathbb{R}$ be the time domain peak level for which the aim of minimization. The PAR is defined as the ratio of the highest signal peak power to its RMS level.

$$\text{PAR} = \frac{p(x)}{\text{RMS}(x)}. \quad (2)$$

Minimizing the OFDM PAR based on a constrained EVM and a lower bounded average data power, as given in [10], is

$$\begin{aligned} \text{minimize} \quad & p = \max_i \left(|x_i|^2 \right) \\ \text{subject to} \quad & \| x_i \| \le p, \quad i=1,\dots,nl \\ & \| \mathbf{S\Delta} \| \le \varepsilon \\ & \Re\left[\mathbf{c_0}^H \mathbf{S} \left(\mathbf{c} - \mathbf{c_0} \right) \right] \ge -\frac{\varepsilon^2}{2}. \end{aligned} \quad (3)$$

where ε is a real-valued positive parameter proportional to the allowed EVM, given by

$$\varepsilon = \text{EVM}_{\max} \sqrt{dP_0}\,, \quad (4)$$

and $\text{EVM}_{\max}$ is the maximum allowed EVM for a given bit error rate. $\mathbf{S} \in \mathbb{R}^{n\text{x}n}$ is a diagonal matrix defined as

$$S_{i,i} = \begin{cases} 1 & \text{if carrier i contains data,} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

The last constraint in (3) put a lower bound on the average data power, formulated from

$$\|\mathbf{Sc}\|^2 \ge \|\mathbf{Sc_0}\|^2, \quad (6)$$

Solving convex minimization problems needs efficient algorithm to reduce the objective function in a fast and convergent way. Using a logarithm-barrier-IPM algorithm with computation of the direction and step size of the variables reduction in each iteration, is efficient to reduce the barrier function value [10], and find the global optimum solution (p^*, $\mathbf{c}^*$, $\mathbf{x}^*$) that solve the PAR minimization problem. The reader is referred to [10]-[11] for details on the solver.

The algorithm starts with a strictly feasible point (**c**, p). The next step would be to find a search direction ($\mathbf{v}$, v_p) and a step size α that update the feasible point (**c**, p) with a factor $\alpha\mathbf{v}$ and αv_p respectively. The updating procedure is set to respect the feasibility condition of the point and reduce the barrier function value [11]. The procedure is iterated until a global optimum point ($\mathbf{c}^*$, p^*) is reached [10].

III. Measurement Setup

Major characteristics of testing power amplifiers are accuracy and ability to track fast variations in the signal envelope. As the aim was to validate the power performance improvement in a PA, a state-of-art measurement system is needed, see Fig. 1.

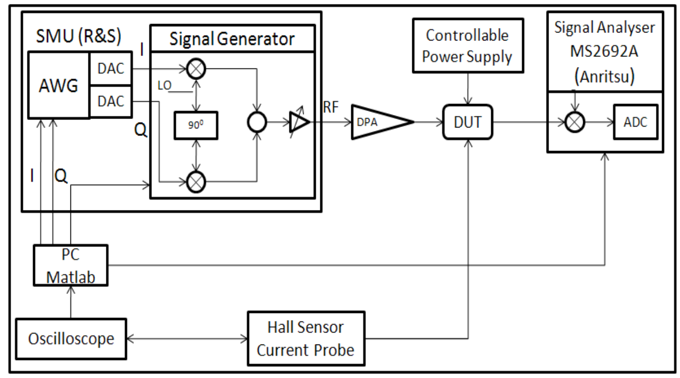

Fig. 1. The measurement setup

The test-bed was mainly based on an R&S SMU200A vector signal generator, an Anritsu MS2692A signal analyzer, an Agilent 54610B oscilloscope, an Agilent N2783A high bandwidth hall sensor current probe, an Ericsson LDMOS 3G highly linear driver amplifier, an Agilent E3631A controllable power supply, and a personal computer (PC).

To accurately monitor the drain current vector and get accurate PAE readings, a high bandwidth hall sensor current probe is used. Measuring the current envelope through an oscilloscope allow an envelope-tracking dynamic power consumption up to 100MHz.

To study the performance of the test-setup, an evaluation of the EVM in function of the input power (P_{in}) is carried out, where the PA is replaced by a connector. EVM was evaluated between the baseband signal transmitted and the received one. An average system error of -45dB was found which shows good performance regarding in-band system error.

For the measurement showed in the current paper, a class AB LDMOS high power (48dBm) amplifier was used. The amplifier has the capability of handling PARs as high as 14dB. WLAN 802.11a use a bandwidth of 20MHz in real application; however to eliminate any error caused by the 3G amplifier due to flatness variation, the amplifier was operated at 2GHz with a gain variation of 0.4dB over an 80MHz bandwidth.

IV. Results And Evaluation

An OFDM signal with 20MHz bandwidth was generated based on 802.11a standards. It has 64 subcarriers with 128 OFDM symbols, a cyclic prefix of ¼, an oversampling rate of 4 and 13.7dB PAR after being hamming windowed. The optimized signal after 3 Newton iterations has a PAR of 9.9dB.

The main goals in reducing PAR are extending the input power level (P_{in}) corresponding to the saturation region, reducing the back-of margin, and allowing an efficient use of the available power. Fig. 2 shows the PAE of the amplifier as function of P_{in} for both reference signal and optimized one.

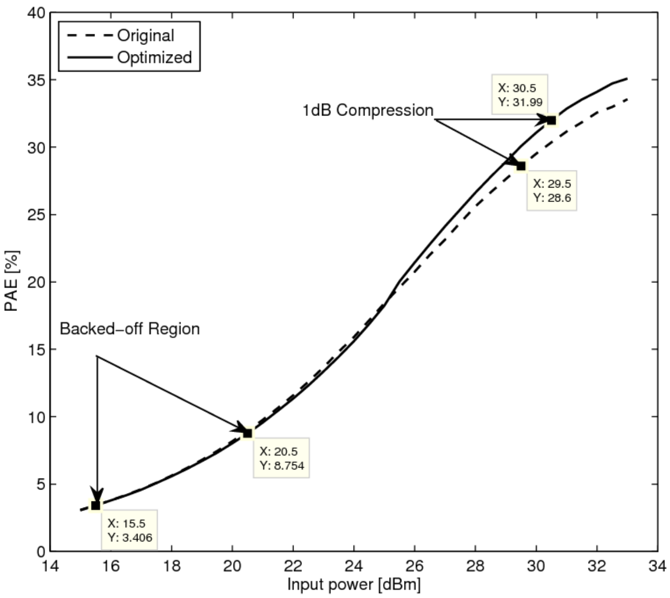

Fig. 2. Power added efficiency of the device under test vs input power for the reference signal (dashed line) and PAR-optimized signal (solid line).

As shown in Fig. 2, the input power level relative to the saturation region of the PA was extended by 1dB which improved the PAE by 3.4% near compression point. While considering the backed-off region by each relative PAR, an improvement of 5.4% was found.

A main constraint in the optimization process is to keep the in-band error, represented by EVM, below a specified standard limit [1]. Fig.3 presents the EVM for both reference and optimized signal at the output stage of the amplifier. Despite the difference between EVM levels at back-of region, the EVM of the optimized signal is still accepted by the standards. Such behavior is expected as the EVM constraint in the optimization algorithm was set to the maximum allowed value by the standards for a 16-QAM, -19dB.

Adding extra power to the sideband free sub-carriers will increase the bandwidth of the signal which might lead to interference with neighboring channels. Fig. 4 presents the adjacent channel power ratio (ACPR) as function of the input power. As shown, the optimized signal has a higher ACPR than its original counterpart at the low side of the channel. Such behavior is mainly due to RF impairments in the system which might cause a channel leakage from the main channel that bias the value of ACPR for that side. Those RF impairments can be characterized by I-Q imbalance, frequency offset, non ideal filters and analog to digital converters impairments.

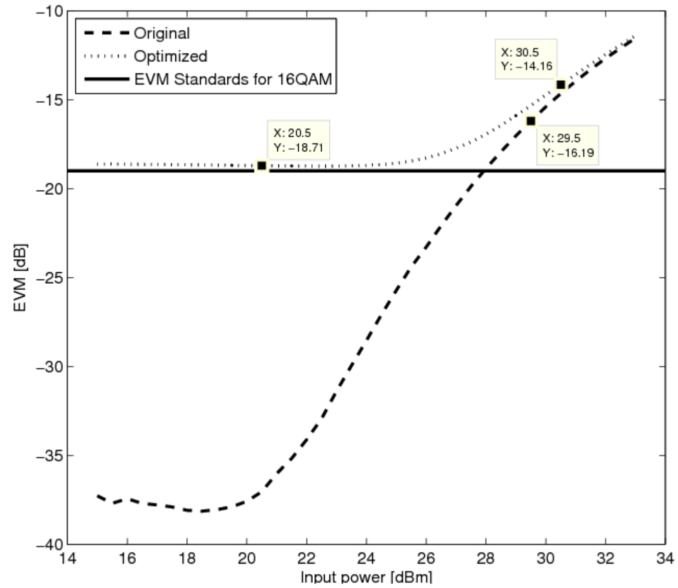

Fig. 3. Error vector magnitude of the device under test versus input power P_{in} for the reference signal (dashed line) and PAR-optimized signal (dotted line).

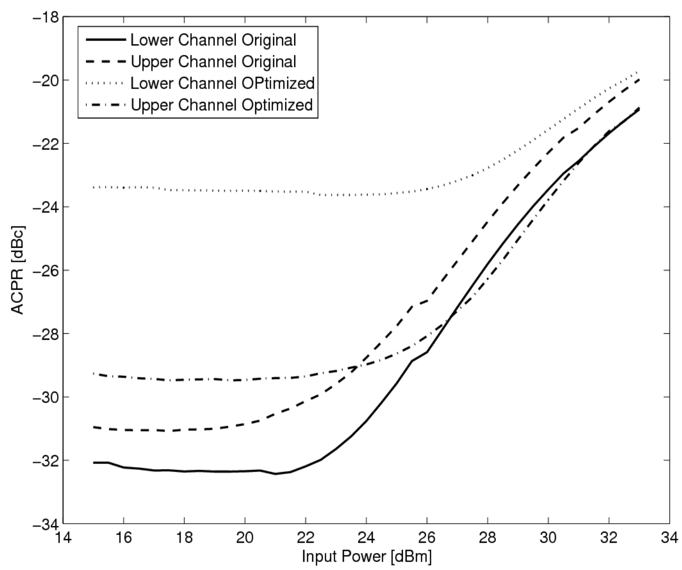

Fig. 4. Comparison of adjacent channel power ratio for both reference and PAR- optimized signal, at lower and upper side f the main channel.

Reducing the PAR should in general extend the saturation point of the PA. However, opposite to what was expected, just 1dB improvement was found near compression, which raises remarks behind such behavior. A study of the complementary cumulative density function (CCDF) of the peak distribution in the measured signal would justify such result. Fig. 5 presents the CCDF of the PAR in both original and optimized measured signal, when normalized to their relative highest peak.

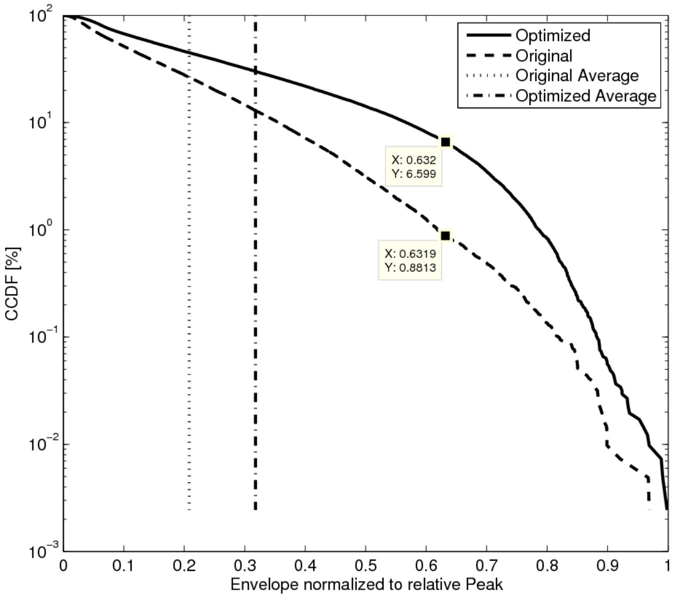

Fig. 5. Complementary cumulative density function of the signal peaks to average ratio for the reference signal (dashed line) and the PAR-optimized signal (solid line).

By analyzing Fig. 5, one can realize that the reduction of the signal peaks was done on the price of increasing the lower peaks density distribution, which turns to limit the improvement effect near saturation. In other words, despite the reduction of the high peaks in the signal, the optimization algorithm allowed the smaller peaks to increase which led to an approximate same nonlinear energy in the signal as compared to the non-optimized case.

III. CONCLUSION

Optimizing OFDM signals based on convex optimization algorithm for PAR reduction was evaluated on a RF PA. It showed a limited improvement in power performance near compression; an extra 1 dB in output power and 3.4% in efficiency. While in the backed-of region it had 5.4% PAE improvement and a gain of 5dB in output power. A reason for such behavior is the increase in lower peak density distribution in the signal.

Spectral emission was considered to be a draw back in the method which needs further investigations in the algorithm in order to meet the standard requirements for emission. The main cause to the spectral leakage is the absence of the power free-subcarriers. It is advised to adjust the minimization algorithm to consider part of the sub-carriers as guard intervals.

Even though PAR reduction by convex optimization is a more complicated method in terms of digital signal processing, it is worth to investigate and apply as it is free in terms of hardware adjustments. This result is believed to be of significance in a world where every dB is worth a Billion.

ACKNOWLEDGEMENT

This work was supported by Ericsson AB, Freescale Semiconductor Nordic AB, Infineon Technologies Nordic AB, Knowledge Foundation, NOTE AB, Rohde&Schwarz AB and Syntronic AB.

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