A Joint Design of Transmit Waveforms for Radar and Communications Systems in Coexistence

Anish Turlapaty and Yuanwei Jin
Department of Engineering and Aviation Sciences
University of Maryland Eastern Shore, Princess Anne, MD 21853
Email: {aturlapaty, yjin}@umes.edu

Abstract— **In this paper, we propose a dynamic spectrum allocation approach for the coexistence of a radar system with a communications system whose operating frequency ranges overlap. We develop a combined mutual information criterion for the joint waveform and power spectrum design to optimize the performance of the radar and communications systems. The performance is evaluated in terms of the minimum separation distance between the coexisting systems under the constraint that a predefined operational SINR must be achieved for both systems. Significant performance gains close to 3.5 dB for the communication system and 1 dB for the radar system are observed in terms of the minimum separation distance compared to the scenario when the waveform with equal power allocation across the frequency bandwidth is used.**

I. Introduction

Spectrum congestion becomes a growing problem as the advance of communication technologies has resulted in scarcity of the frequency spectrum, thus limiting the operational capabilities of these competing systems due to mutual interference. For example, the broadband wireless access system such as WiMax and 4G long term evolution (LTE) mobile communications system are expected to be allocated to the frequency band which has been traditionally occupied by S-band radar systems. The call for spectrum sharing is to allow co-existence between various wireless systems that operate in the same or close frequency band while achieving normal operational performance levels. For example, the cognitive radio technology attempts to sense the under utilized spectrum and dynamically allocates resources to the secondary users between communications systems [1]. The spectrum sharing problem is different when the primary user is a radar system. Traditionally, fixed spectrum allocation has been used to prevent interference for radar and communications systems that operate in close ranges. A common approach to prevent interference from communication systems to critical radar operations has been through regulation. For example, OfCom, the British spectrum regulator, would require existing S-band radar license holders to upgrade their receiver filters to reduce interference caused by the new 4G LTE mobile communications systems that operate in adjacent frequency bands [2]. Waveform design and power control are other major approaches to address the radar-communications systems coexistence problem. In [3], a mutual information (MI) based waveform design is considered under the constraints of total transmission power, radar clutter in the environment and the interference to a coexisting communications system. When antenna arrays are used, spatial degrees of freedom can be exploited to address the coexistence problem. In [4], a null space projection method is proposed to exploit the spatial diversity provided by MIMO radars to direct the main beam into the null subspace of the communications system's channel matrix. In [5], [6], a combined system design based on orthogonal frequency division multiplexing (OFDM) waveform modulation is proposed. The key idea is to utilize OFDM waveforms as radar signals encoded with the communication signals under a single RF platform.

In this paper, we focus on the coexistence problem for a wide-band radar and a multi-carrier communications system. Key requirements for the radar and communications systems to coexist are their abilities to sense the spectrum usage by the coexisting systems and to subsequently adjust their transmission power spectrum so that both systems are operational. We call this strategy frequency selectivity of the transmitted waveforms. In a wide band radar, this can naturally be achieved by adaptive design of transmit waveforms while for a communications system, this can be accomplished by using multi-carrier modulation such as OFDM [6], [7]. In this paper, we proposed a joint design of the radar transmission waveform's power spectrum and the power spectral density of the multi-carrier communications system. From the co-existence perspective, the goal of radar waveform design is to accomplish radar functions such as estimation and detection while mitigating interference to the co-existing communications system. The objective of communications power spectrum design is to maintain a sufficient throughput while reducing interference to the radar. In this paper, we use minimal separation distance as a performance indicator. For simplicity, no antenna array is utilized for both systems.

II. Signal Models

We consider a wide-band monostatic radar system that coexists with a multi-carrier communications system. We let d_1 and d_2 denote the separation distance between the radar

This work is supported in part by the Army Research Laboratory, the Office of Naval Research, and the Army Research Office under grant no. W911NF-11-1-0160 and the National Science Foundation under grant no. CMMI-1126008.

978-1-4799-2035-8/14/$31.00@2014 IEEE

and the communications transmitter, and between the radar and the communications receiver, respectively. We begin with the time domain signal model and then transform it into the frequency domain.

A. Radar Signal Model

The received signal model consists of models for the target response, interference and noise. The transmitted signal $s(t)$ strikes the target and a fraction of the EM energy is reflected to the radar receiver. Assuming an extended target, let $g(t)$ denote the target response which is modeled as a zero-mean complex Gaussian random process [8], where $|g(t)|$ follows a Rayleigh distribution [9]. The received radar signal can be written as

$$y(t) = g(t) * s(t) + y_i(t) + w(t) \tag{1}$$

where $w(t)$ is the receiver noise. We omit the impact of clutter response for the moment for simplicity purposes. $y_i(t)$ is the interference defined by

$$y_i(t) = \beta(t) * u(t - \tau_1) \tag{2}$$

where $\beta(t)$ represents the impulse response of the interference channel between the communication system's transmitter to the radar receiver. $u(t)$ is the transmitted communications signal and the delay τ_1 is related to the separation distance d_1. Here, $y_i(t)$ is assumed to be a zero-mean complex Gaussian random process. Applying the Fourier transform to (1) we obtain

$$Y(f) = G(f)S(f) + Y_i(f) + W(f) \tag{3}$$

where $G(f)$, $S(f)$ and $W(f)$ denote the Fourier spectra of $g(t)$, $s(t)$ and $w(t)$, respectively. $Y_i(f) = \beta(f)U(f)$, where the interference channel frequency response function (FRF) $\beta(f) = e^{-j2\pi f\tau_1}\mathcal{F}\{\beta(t)\}$ and the signal spectrum $U(f)$ is defined in (18) of Section II-B. $e^{-j2\pi f\tau_1}$ is the random phase component. Let

$$\mathcal{F}_r = \{f_k; k = 1, \cdots, Q_r\} \tag{4}$$

denote the set of equal spaced discrete frequencies representing the operational range of the radar (see, also [10]). Thus, from (3), the equivalent discrete signal spectrum at f_k can be written as

$$Y(k) = G(k)S(k) + Y_i(k) + W(k) \tag{5}$$

Since the Fourier transform is a linear operation, the response spectrum $G(k)$ is also a zero mean complex Gaussian random process [11], defined by

$$G(k) \sim \mathcal{CN}\left(0, \sigma_G^2(k)\right) \tag{6}$$

where $\sigma_G^2(k)$ is the spectral variance of the target channel response. We assume $\sigma_G^2(k)$ is known [8]. Next, let $\Psi(k) = |S(k)|^2$, the transmit power is constrained by

$$\frac{1}{Q_r}\sum_{k=1}^{Q_r}\Psi(k) \le E_r \tag{7}$$

where E_r is the radar total transmission power. The interference spectrum $Y_i(k) = \beta(k)U(k)$ is distributed as

$$Y_i(k) \sim \mathcal{CN}\left(0, L_\beta(k)\boldsymbol{\chi}(k)\right) \tag{8}$$

where $L_\beta(k)$ is the path loss from the communications transmitter to the radar receiver, modeled by

$$L_\beta(k) = \frac{G_T A_\gamma \lambda_k^2}{4\pi d_1^2} \tag{9}$$

where G_T is the antenna gain of the communications transmitter, A_γ is the radar antenna's effective aperture in the direction of interference source, and λ_k is the wavelength at k-th frequency. The noise spectrum at k-th frequency is assumed to be a white Gaussian process defined by

$$W(k) \sim \mathcal{CN}\left(0, \sigma_w^2\right) \tag{10}$$

Hence the probabilistic model of the radar signal $Y(k)$ is

$$Y(k) \sim \mathcal{CN}\left(0, \sigma_Y^2(k)\right) \tag{11}$$

where the variance is given by

$$\sigma_Y^2(k) = \Psi(k)\sigma_G^2(k) + L_\beta(k)\boldsymbol{\chi}(k) + \sigma_w^2(k) \tag{12}$$

B. Signal Model for the Communications System

We consider a multi-carrier communications system such as a multi-carrier OFDM. The main advantage of using OFDM scheme is separation of the operational bandwidth into frequency non-selective sub-bands. Specifically this division allows for sub-carrier specific power allocation such as water filling based on observed radio channel response function [12]. The signal arriving at the communications receiver is

$$x(t) = h(t) * u(t) + x_i(t) + v(t) \tag{13}$$

where $u(t)$ is the communications signal which can be modeled as complex Gaussian process for large number of subcarriers [13] and $h(t)$ is the communication channel impulse response function. The interfering signal due to radar illumination is given by

$$x_i(t) = \alpha(t) * s(t - \tau_2) \tag{14}$$

where the signal strength depends on the gain of the radar antenna in the direction of communications receiver and the separation distance d_2. $\alpha(t)$ is the interference channel impulse response functions. Applying the Fourier transform to (13), we obtain the equivalent frequency domain signal model:

$$X(f) = H(f)U(f) + X_i(f) + V(f) \tag{15}$$

where $X_i(f) = \alpha(f)S(f)$ and $\alpha(f) = e^{-j2\pi f\tau_2}\mathcal{F}\{\alpha(t)\}$. $e^{-j2\pi f\tau_2}$ is the random phase term due to the separation distance d_2. Similar to (4), we let

$$\mathcal{F}_c = \{f_l; l = 1, \cdots, Q_c\} \tag{16}$$

denote the set of discrete frequencies for the communications system. Next consider the signal spectra at $f_l \in \mathcal{F}_c$, we obtain

$$X(l) = H(l)U(l) + \alpha(l)S(l) + V(l) \tag{17}$$

where the noise $V(l) \sim \mathcal{CN}\left(0, \sigma_v^2\right)$ and the signal spectrum is ZMCGRP

$$U(l) \sim \mathcal{CN}\left(0, \boldsymbol{\chi}(l)\right) \tag{18}$$

and constrained by the total power E_c

$$\frac{1}{Q_c} \sum_{l=1}^{Q_c} \chi(l) \leq E_c \tag{19}$$

Next we assume the signal component $X_s(l) \triangleq H(l)U(l)$ and the interference component $X_i(l) = \alpha(l)S(l)$ follow complex Gaussian distributions [14], i.e.,

$$X_s(l) \sim \mathcal{CN}\left(0, L_h(l)\boldsymbol{\chi}(l)\right) \tag{20}$$

$$X_i(l) \sim \mathcal{CN}\left(0, L_\alpha(l)\Psi(l)\right) \tag{21}$$

where $L_h(l)$ and $L_\alpha(l)$ are the path loss from communication transmitter to the receiver (with separation distance d_c) and from the radar to the communication receiver (with separation distance d_2), respectively

$$L_h(l) = \frac{G_T A_R \lambda_l^2}{4\pi d_c^2}; \ \ L_\alpha(l) = \frac{\gamma A_R \lambda_l^2}{4\pi d_2^2} \tag{22}$$

A_R is the effective aperture area of the communications receiver antenna and γ is the gain of the radar antenna in the direction of the communications receiver.

III. Waveform Design

We assume that the frequency separations Δf within the respective bandwidths are same for both systems. Now we define the union set F_u and the intersection (overlapping bands) set $\mathcal{F}_o$ of frequency bands for the two systems as

$$\mathcal{F}_u = \mathcal{F}_r \cup \mathcal{F}_c; \ \mathcal{F}_o = \mathcal{F}_r \cap \mathcal{F}_c \tag{23}$$

The radar only set $\mathcal{F}_1$ and the communications system only set $\mathcal{F}_2$ are given by

$$\mathcal{F}_1 = \mathcal{F}_r - \mathcal{F}_o; \ \mathcal{F}_2 = \mathcal{F}_c - \mathcal{F}_o \tag{24}$$

The total number of frequencies in $\mathcal{F}_u$ is Q_u. For $f_q \in \mathcal{F}_1$, i.e., for radar only frequencies, an optimal parameter estimation for an extended target can be achieved by maximizing mutual information between its frequency response $g(q)$ and the measurements $Y(q)$ [8]. Since the signal $Y(q)$ in (11) is a complex Gaussian process, the mutual information at f_q is given by

$$I_r(q) = \log\left(1 + \frac{\Psi(q)\sigma_G^2(q)}{L_\beta(q)\boldsymbol{\chi}(q) + \sigma_w^2}\right) \tag{25}$$

For $f_q \in \mathcal{F}_2$, i.e., for a communications system only frequencies, the channel capacity can be reached by maximizing the information rate which is related to mutual information between the measurements $X(q)$ and the transmitted signal $U(q)$ [15]. Since the signal $X(q)$ in (11) is Gaussian distributed, the information rate is $I_c = \sum_q I_c(q)$, where each $I_c(q)$ is defined by

$$I_c(q) = \log\left(1 + \frac{L_h(q)\boldsymbol{\chi}(q)}{L_\alpha(q)\Psi(q) + \sigma_v^2}\right) \tag{26}$$

For the frequencies in the overlapping band $\mathcal{F}_o$, the received signals of the radar and communications system can be seen as a combined signal vector $Z(q)$ with a combined parameter vector $J(q)$, defined by

$$Z(q) = [Y(q) \ X(q)]^T; \ \ J(q) = [G(q) \ U(q)]^T \tag{27}$$

For non-adaptive waveform, the uncertainty regarding the parameter vector is minimized with sufficient number of measurements. This convergence can be accelerated if the mutual information with the observations is maximized through adaptive waveform transmission [16]. Hence, the goal of the waveform design is to maximize the combined mutual information

$$I(Z(q), J(q)) = H(Z(q)) - H(Z(q)|J(q)) \tag{28}$$

We note that from the received signal models (1) and (13), the target impulse response $g(t)$, the communications signal $u(t)$, interference channel impulse responses $\alpha(t)$ and $\beta(t)$, and the receiver noise $w(t)$ and $v(t)$ are all independent of each other. The mutual independence enables a decomposition of the combined mutual information as

$$I(Z(q), J(q)) = I_r(q) + I_c(q) \tag{29}$$

Note, however, that even though the mutual information terms are separable, the definitions of $I_r(q)$ and $I_c(q)$ have common terms $\Psi(q)$ and $\boldsymbol{\chi}(q)$. Hence, the joint mutual information of the radar-com system is defined as

$$I_{\text{tot}} = \sum_{q=1}^{Q_u} I_{\text{tot}}(q) \tag{30}$$

where

$$I_{\text{tot}}(q) = \begin{cases} I_r(q) & \text{if } f_q \in \mathcal{F}_1 \\ I_c(q) & \text{if } f_q \in \mathcal{F}_2 \\ I_r(q) + I_c(q) & \text{if } f_q \in \mathcal{F}_o \end{cases} \tag{31}$$

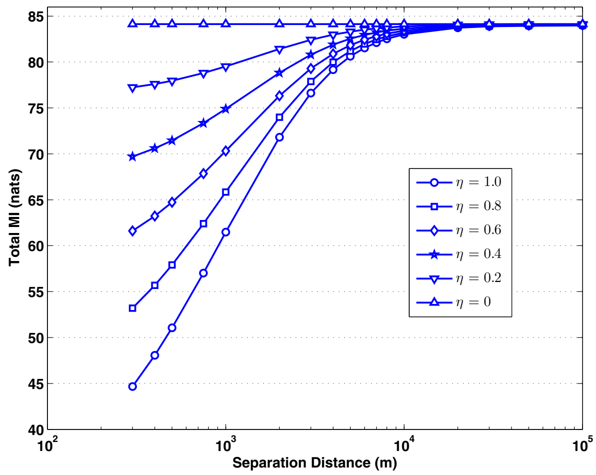
Under the given models and assumptions, our goal is to design both the radar transmit waveform spectral density $\Psi(q)$ and the communication power level $\chi(q)$ at each of the q-th carrier. where E_c is the average transmission power of the communications system. We propose that the desired signal power spectra can be jointly designed by maximizing total mutual information I_{tot}. Hence the constrained optimization problem can be written as

$$\text{find} \ \ \Psi(k), \chi(l) \in \arg \max_{\Psi(k), \chi(l)} I_{\text{tot}} \tag{32}$$

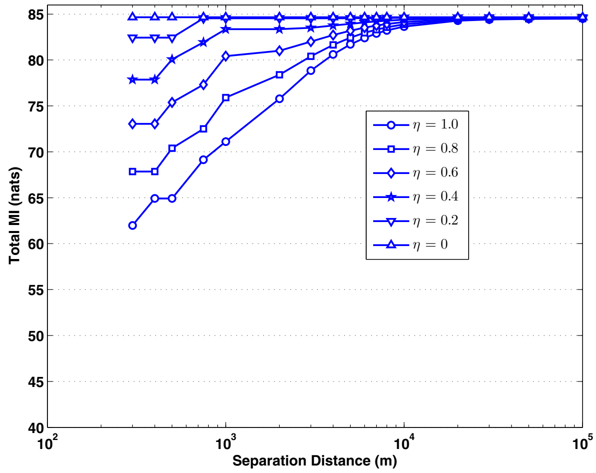
subject to the power constraints (7) and (19).

IV. Numerical Results

For a joint design of waveforms for two co-existing radar and communications systems, the goal is to achieve mutually assured satisfactory performance in terms of respective operational SINR. The respective SINR for two systems are defined

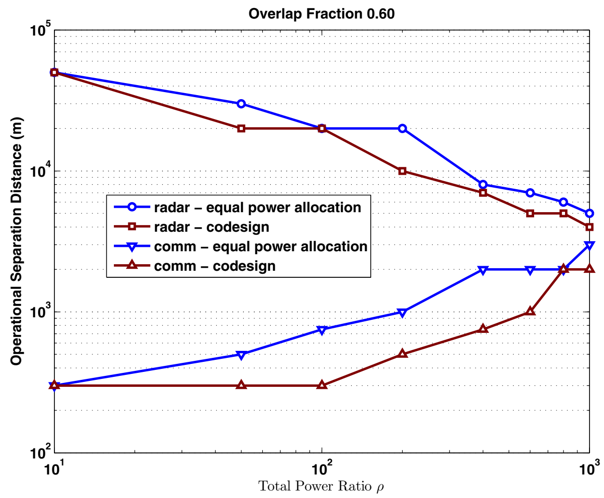


(a)

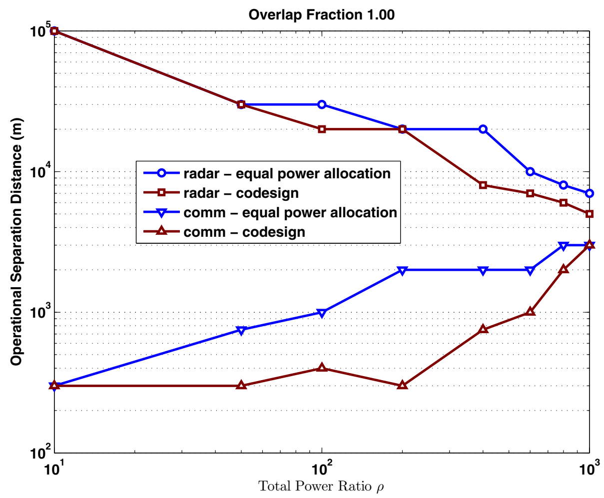


(b)

Fig. 1. Total Mutual Information vs. Separation Distance d. (a) Equal power allocation waveform. (b) CDMI waveform.



(a)



(b)

Fig. 2. Minimum Operational Distance vs. Total Power Ratio. (a) Overlap fraction $\eta = 0.6$. (b) Overlap fraction $\eta = 1.0$

as

$$\text{SINR}_r = \sum_{k=1}^{Q_r} \frac{\Psi(k)\sigma_G^2(k)}{L_\beta(k)\chi(k) + \sigma_w^2} \quad (33)$$

$$\text{SINR}_c = \sum_{l=1}^{Q_c} \frac{L_h(l)\chi(l)}{L_\alpha(l)\Psi(l) + \sigma_v^2} \quad (34)$$

For simplicity, we assume that the communications transmitter and receiver are at equal distance from the radar, i.e., $d = d_1 = d_2$. Hence, consider the transmitter/receiver located on a circle of radius d with the radar being at the center, the separation distance d is defined as the minimum distance at which at least a desired SINR level is achieved (8 dB is chosen for simulations). In the simulations, we adopt the radar spectral variance model for an extended target [8]

$$\sigma_G^2(k) = a_0 e^{-a_1(f_k - f_c)^2} L_G(k) \quad (35)$$

where f_c is the center frequency of the bandwidth defined in $\mathcal{F}_r$. The radar cross section of the extended target at its peak frequency is $a_0 = 1$. $L_G(k)$ is the two-way path loss from the radar to the target defined by

$$L_G(k) = \frac{G_R^2 \lambda_k^2}{(4\pi)^3 R^4} \quad (36)$$

The radar antenna gain G_R in the bore sight is assumed to be 50 dB. The range of the target is assumed to be $R = 50$ km. The transmit power of the communications system E_c is normalized to 1. E_r, the transmit power of the radar system is at $\rho = E_r/E_c$ times that of E_c. γ the sidelobe level of the radar antenna relative to the main beam (γ assumed to -30 dB). The transmit and receive antennae are assumed to be omni-directional and thus the corresponding gains G_T, G_R are equal to 1. At a very large separation distance (i.e., no mutual interference is present), the maximum SNR is set at 12 dB. Note that the mutual interference will not vanish for the two systems simultaneously. The RFI from the communications system to the radar is usually more severe (in terms operation distance) because of the high gain of radar antenna and two way path loss of radar signals. The overlap ratio η is

$$\eta = \frac{Q_o}{\min(Q_r, Q_u)} \tag{37}$$

where Q_o, Q_r and Q_u are the size of the set $\mathcal{F}_o$, $\mathcal{F}_r$ and $\mathcal{F}_u$, respectively. Radar bandwidth is fixed as $1.0-3.0$ GHz. The bandwidth of communication system $\mathcal{F}_c$ is varied from $2.0-3.0$ GHz at $\eta = 1$ to $3.0-4.0$ GHz at $\eta = 0$. Next, under the assumed models and parameters, we solve the optimization problem in (32) numerically using convex optimization by the active-set constrained nonlinear method [17].

We call the resulting spectra as mutual information based co-designed waveform (CDMI). Fig. 1(a) depicts the total mutual information I_{tot} when equal power is allocated for each channel at f_q. It is evident that I_{tot} monotonically increases with an increasing separation distance d at a fixed η. While as the overlap fraction η decreases, I_{tot} improves in that the situation of spectrum congestion is lessened. Fig. 1(b) shows the trend of MI I_{tot} when CDMI waveforms are used. Significant improvement in I_{tot} is observed in the range of $0 < \eta < 1$, which demonstrates the effectiveness of the proposed method to achieve spectrum sharing. Fig. 2(a) presents the minimum operational separation distance d as a function of the total power ratio ρ.

Fig. 2(a) and Fig. 2(b) correspond to the overlap ratios of $\eta = 0.6$ and $\eta = 1.0$, respectively. These plots clearly show that proposed CDMI waveform uniformly outperforms the equal allocation waveforms. As expected, the separation distance of the radar system uniformly decreases and the distance for the co-existing communication system degrades with increase of the power ratio ρ. The improvement in the communications system's gain in operation distance varies between 3 and 4 dB when the power ratio ρ is in the orders of 100. As the ratio ρ approaches 1000 the gains are closer to or less than 1 dB. For the radar system, the operational gains vary between 0.5 and 3 dB. The key reason for the gains in operational distance is because of adaptation of the transmit power spectrum of the both systems in response to the spectral usage by the other co-existing system. The radar waveform not only depends on the target spectral response but also on the statistics of the radio communication and interference channels. Similarly, the power allocation in communications system which traditionally adapts only to the radio channel characteristics also has to adjust to the radar signal's spectrum occupancy which ultimately depends on target spectral statistics.

V. Conclusion and Discussions

A novel approach is proposed for dynamic spectrum allocation problem of the coexistence between radar and communications systems operating in the S-band. A unified mutual information criterion is used for the joint design of radar waveform and communications power spectrum in order to optimize the overall system performance. The operational separation distance between the coexisting systems is evaluated at different overlapping and total transmit power ratios. When radar transmit power is of the order of several 100 times greater than that of the communications system, a performance gain of 3 dB for the communications system and approximately 1 dB for the radar is observed at multiple overlapping ratios.

However, the proposed approach only addresses how to design the magnitude of the waveform spectrum, not on the phase terms. Hence, further analysis is needed to discuss how the optimization algorithm will impact radar waveform properties that could lead to degradation of other critical radar performance metrics including range resolution and range sidelobe level. Future research would focus on the impact of proposed waveform design approach on radar ambiguity function which allows examination of both the Doppler and range resolutions as well as the sidelobe levels.

References

[1] L. Lu, X. Zhou, U. Onunkwo, and G. Li, "Ten years of research in spectrum sensing and sharing in cognitive radio," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 1, pp. 28, 2012.

[2] Ofcom, "Coexistence of S Band radar systems and adjacent future services," Tech. Rep. December, Ofcom, 2009.

[3] S. Amuru, R. M. Buehrer, R. Tandon, and S. Sodagari, "On the co-existence of mimo radar with communication systems," available online.

[4] S. Sodagari, A. Khawar, T. C. Clancy, and R. McGwier, "A projection based approach for radar and telecommunication systems coexistence," in *IEEE Global Communications Conference (GLOBECOM)*, 2012, pp. 5010–5014.

[5] D. Garmatyuk, J. Schuerger, Y. T. Morton, K. Binns, M. Durbin, and J. Kimani, "Feasibility study of a multi-carrier dual-use imaging radar and communication system," in *Radar Conference (EuRAD)*, 2007, pp. 194–197.

[6] A. Turlapaty, Y. Jin, and Y. Xu, "Range and velocity estimation of radar targets using weighted OFDM signals," in *IEEE Radar Conference*, Cincinnati, OH, May 2014.

[7] E. F. Casas and C. Leung, "OFDM for data communication over mobile radio FM channels. I. analysis and experimental results," *IEEE Transactions on Communications*, vol. 39, no. 5, pp. 783–793, 1991.

[8] M. R. Bell, "Information theory and radar waveform design," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1578–1597, 1993.

[9] R. V. Ostrovityanov and F. A. Basalov, *Statistical Theory of Extended Radar Targets*, Artech House, Dedham, MA, 1985.

[10] A. Turlapaty and Y. Jin, "A unified approach for parameter estimation and waveform design by free-energy principle," (submitted for publication).

[11] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing: Concepts and Techniques*, Prentice-Hall, Englewood Cliffs, NJ, 1993.

[12] S. T. Chung and J. M. Cioffi, "Rate and power control in a two-user multicarrier channel with no coordination: the optimal scheme versus a suboptimal method," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1768–1772, 2003.

[13] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Transactions on Communications*, vol. 49, no. 2, pp. 282–289, 2001.

[14] T. Cui and C. Tellambura, "Joint data detection and channel estimation for OFDM systems," *IEEE Transactions on Communications*, vol. 54, no. 4, pp. 670–679, 2006.

[15] R. Gallager, *Principles of Digital Communications*, Cambridge University Press, Cambridge, UK, 2008.

[16] A. Turlapaty and Y. Jin, "Sequential Bayesian parameter estimation by cognitive multi-antenna arrays," (submitted for publication).

[17] S.-P. Han, "A globally convergent method for nonlinear programming," *Journal of Optimization Theory and Applications*, vol. 22, no. 3, pp. 297309, July 1977.