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PAPR reduction in OFDM using Goppa codes

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Abstract— **For many years, different methods were tried to reduce the PAPR of OFDM and are broadly classified as distortion or distortionless schemes, each of which has some merits and demerits. But the inherent frequency diversity benefits of a transmitted OFDM signal on multipath fading channels can only be exploited by use of channel coding. Hence in this paper, PAPR reduction was tried with coding method to exploit the advantage of OFDM systems. Since cyclic codes have been proved earlier as the suitable coding method to reduce PAPR, binary cyclic Goppa codes using BPSK carriers have been considered.**

Simulations show that the proposed scheme shows good PAPR reduction compared to uncoded OFDM and with the possibility to be extended with error detection and correction capability it can be used as a PAPR reduction method and can be experimented over several modulation techniques.

The process is illustrated by means of an example, and it is shown that there is an improvement possible for a (16,8,5) and (12,4,5) Goppa coded symbols compared to the uncoded ones.

Keywords—OFDM, Coded OFDM, PAPR , Goppa codes

I. INTRODUCTION

An OFDM signal divides a high rate information bit sequence into several parallel low rate sequences and use these to modulate a number of orthogonal subcarriers by Fourier transform techniques and has many advantages in a number of communication systems. But one of the major drawback of the OFDM transmitted signal is a high peak-to-average power ratio (PAPR), which degrades the efficiency of the linear power amplifier.

In an OFDM system, a group of N input bit sequence is encoded into a block of BPSK bits *s(l)* , *(l=1,...N)* where the duration of each bit is T seconds. These N symbols are serial to parallel converted and modulated using N orthogonal sub-carriers, $\{e^{j2\pi f_k t},..., e^{j2\pi f_{N-1+k} t}\}$ with the *l*-th subcarrier frequency $f_l = \frac{1}{NT}$ (Hz), and *k= 0 or 1*. Thus the OFDM modulated signal *s(t)* for a block duration NT is given by

$$s(t) = \frac{1}{N}\sum_{l=k}^{N-1+k} s(l)e^{j2\pi f_l t} \tag{1}$$

$$= \frac{1}{N}\sum_{l=k}^{N-1+k} s(l)e^{\frac{j2\pi lt}{\mathrm{NT}}} \tag{2}$$

Discretizing *s(t)* at *t= nT*, where *(n = 1,... N), s(n)* $\triangleq$ *s(nT)*

$$= \frac{1}{N}\sum_{l=1}^{N} s(l)e^{\frac{j2\pi \ln}{\mathrm{N}}}$$

which is equivalent to *N*-pt. IDFT of the *N* modulated symbol *s(l)*. The peak to mean average power (PMEPR) is $10\log_{10}(16/4) = 6.02$ dB for a 4 bit data word represented by s(t) and envelope power is calculated as *s(t)s*(t)* where power of each carrier is normalized to 1 watt. After applying parity codes [1], PMEPR reduces to $10\log_{10}(7.07/4)= 2.48$ dB, and without a change in data rate. Therefore reduced envelope power results in increase in efficiency of power amplifier. The energy per transmitted bit decreases, it is offset by the increased spectral efficiency of the multicarrier schemes.

II. CYCLIC CODING

A multicarrier modulated signal can be further written as

$$s(t) = \sum_{l=1}^{N} s(l)\cos[(\omega + l\Delta\omega t) + \varphi_l] \tag{3}$$

considering k as 1 and $\{\varphi_l\}_{l=1}^{N} = 0$ or $\pi, 0 \le t \le T$ and $\Delta\omega = \frac{2\pi}{T}$. Separating *s(t)* into odd and even terms, it can be shown that

$$|s(t)| \le 2\sum_{\substack{i=1 \\ odd}}^{N}\left|\cos(\frac{\Delta\omega t}{2} + \alpha_i)\right| \tag{4}$$

where $\alpha_i = \frac{\varphi_{i+1}-\varphi_i}{2}$.

Assuming *N=4L*, where *L* is an integer and taking minimum of the maximum value of eq (4), $|s(t)| \le 2\times\frac{2}{\sqrt{2}}\times\frac{N}{4}$, which shows a decrease in PAPR by 3 dB compared to uncoded multicarrier modulation. For *i=1,3,...N-1*, it follows that $|\alpha_{i+1} - \alpha_i| = \frac{\pi}{2}$. Therefore the phase of every fourth carrier at $i = 1, 4, ...N$ is given as

$$\varphi_{i+3} - \varphi_{i+2} = \varphi_{i+1} - \varphi_i + \pi \tag{5}$$

which is an algorithm for cyclic codes [2]. It is known that when *N* signals are added with the same phase, a high peak power equal to *N* times the average power is produced in the *N*-point IFFT to produce an OFDM symbol. Therefore scrambling of phases results in lesser PAPR.

If the data word s(*l*) can be written as $s = [d_1, d_2, \ldots d_N]^T$, then $X_l = [G]_M \times [S]_{N\times 1} = [X_1 \ldots X_{M-1}]^T$ where G is the generator matrix and X_l is the coded data.

After IFFT, the OFDM signal is given by

$$x(t)=\sum_{l=1}^{M} X_l e^{\frac{j2\pi lt}{T}}, 0\le t\le T \quad (6)$$

$$=\sum_{l=1}^{M}\sum_{n=1}^{M} g_{mn} d_n e^{\frac{j2\pi lt}{T}}$$

Let $p_{mn}=g_{mn}d_n$ and

$$PAPR_{\max}=\max PAPR(t) \;=\frac{1}{M}\max_{0\le t\le T}\Big(\sum_{n=1}^{M}\Big|\sum_{l=1}^{M} p_{mn} e^{\frac{j2\pi lt}{T}}\Big|^2\Big)$$

$$PAPR_{\max}=\frac{1}{M}\max_{0\le t\le T}\Big(\sum_{n=1}^{M}\Big|\sum_{l=1}^{M} p_n(t)\Big|^2\Big) \quad (7)$$

where $p_n(t)=p_{mn}e^{\frac{j2\pi lt}{T}}$ is the average energy per transmitted symbol. Therefore PAPR depends on number of subcarriers M and the entries of the coding matrix G and can be reduced by proper selection of the matrix so that the peak amplitudes of the M functions do not occur at the same time within the interval 0 to T.

III. GOPPA CODES

Let C be the collection of all possible transmitted codewords in the OFDM system then a good OFDM code $C(n,k,d)$ is the one with large $n(C)$, where n is the length of the code word, low $PAPR(C)$ and high $d(C)$ where d is the minimum distance and $PAPR(C)=\max_{0\le t\le T}(p_n(t))^2$ and its value is approximated by its discrete n samples $p_n\left(\frac{lT}{n}\right)$.

A block coding scheme was used to reduce PAPR where a four bit data symbol was given as an input to a multicarrier transmission system[1]. The PAPR is high for certain symbols 0000, 0101, 1010 and 1111 as seen in Fig. 1 and can force the high power amplifier in the final stage of the system to operate with large power back-off in order to avoid non-linear distortions.

Since the most power efficient operating point of a power amplifier is at or near its saturation region, PAPR reduction is essential otherwise the cost of the device and its power consumption will increase.

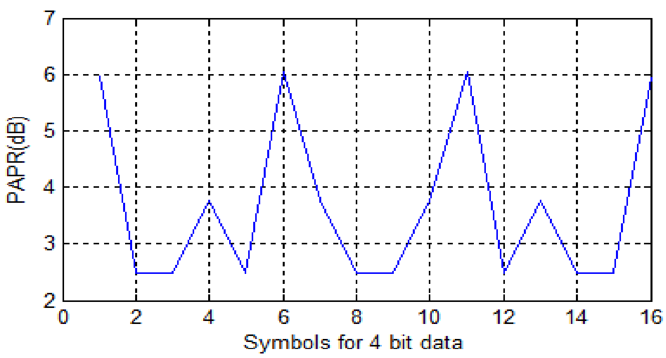

Fig. 1. PAPR for uncoded 4 bit data

Employing similar concept of PAPR reduction using cyclic codes, a search has led to Goppa codes, a binary code with good error correction capability. It has been shown that the double error-correcting Goppa codes can become cyclic by extending an overall parity check to it with a defined Goppa polynomial and location sets [4][8].

Goppa codes, commonly referred as $\Gamma(L, g(z))$ codes, have some of the largest minimum distances possible for linear codes[3][5]. These codes have found extensive use in cryptography and have good distance properties. It is defined by the location set $L=\{\alpha_1,...,\alpha_n\}\subseteq GF(q^m)$ and a Goppa polynomial $g(z)=g_0+g_1z+...+g_tz^t=\sum_{i=0}^{t} g_iz^i$, of degree t and coefficients from $GF(q^m)$, such that $g(\alpha_i)\ne 0$ for all $\alpha_i\in L$. If the code vector using Goppa codes is denoted by $c=(c_1,...,c_n)$ over $GF(q)$ then it is associated with a function $R_c(z)=\sum_{i=1}^{n}\frac{c_i}{z-\alpha_i}$ (8)

where $\frac{1}{z-\alpha_i}$ is the unique polynomial with $(z-\alpha_i).\frac{1}{z-\alpha_i}\equiv 1(\bmod g(z))$. Therefore the Goppa $\Gamma(L, g(z))$ code consists of all vectors such that $R_c(z)=0(\bmod g(z))$. (9)

It is a linear code with the dimensions *(n,k,d)* where $k\ge n-mt$ and $d\ge t+1$, where d is the minimum distance, k is the length of the information bits and n is the length of the codeword. For a [12,4,≥5] Goppa code, with an extension field $GF(2^4)$, the irreducible polynomial $k(X)=X^4+X+1$, and has degree 4.

Here $g(z)=(z+\alpha)(z+\alpha^{14})$ and $L=\{\alpha^i \,|\, 2\le i\le 13\}$ and the dimensions of the code are *q=2, m=4, n=12* and *t=2*. The peak power contribution for such a code is shown as follows in fig. (2) and table (1), where the all-zero sequence shows very high power compared to other symbols. Therefore certain symbols including the all zero symbol with high PAPR can be avoided[1]. Considering the increase of carriers from 4 to 12, the power requirement would be definitely high after coding but if the threshold of permissible PAPR is kept at 5 dB, then there are only four symbols which fall under non-permissible codes as seen in Fig, 3. Furthur research can be done to make these codes permissible by applying hybrid techniques of PAPR reduction. The extra 8 bits in each of the 4 bit information symbols can be utilised for error correction and detection using Berlekamp-Massey algorithm.

A Goppa[16,8,≥5] code was also tried with 8 information bits and 16 carriers with dimensions *q=2, m=4, n=16* and *t=2*, and an increased code rate from ⅓ to ½ . Here the irreducible polynomial is $k(X)=X^4+X^3+1$ and the binary separable polynomial is $g(z)=z^2+z+\alpha$.

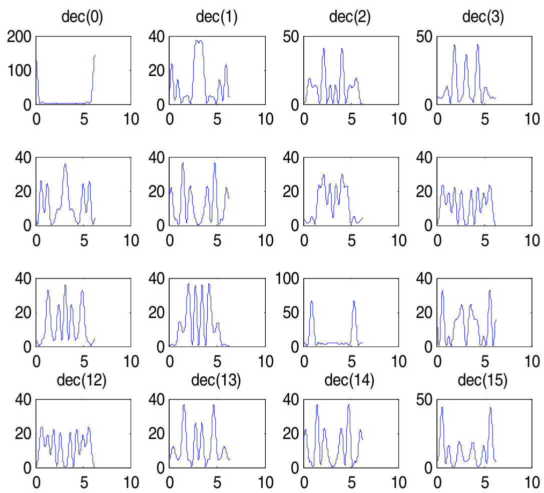

Fig. 2. Peak power of Goppa (12,4,5) coded symbols

In Fig. 4, it is observed that the PAPR of 8 bit uncoded data symbols are very high for 3 symbols and is greater than 8 dB, whereas the coded symbols in Fig. 5 shows PAPR greater than 8 dB. In addition, the overall PAPR for all other symbols in (16,8,5) Goppa codes are restricted to 8 dB, which considering the increased data rate from 4 bits in (12,4,5) to 8 bits in (16,8,5) Goppa codes and with the possibility of the redundant bits to be used for error detection and correction [6] shows an improved performance.

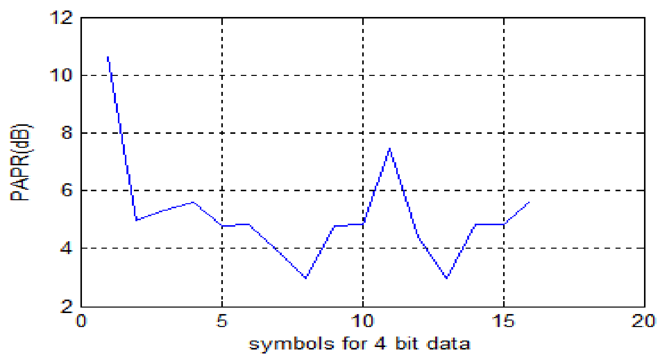

Fig. 3. PAPR for coded Goppa (12,4,5) symbols

If again a choice of only permissible codes among all coded symbols are made, then it further reduces to 4.8 dB and about 7 dB for (12,4,5) and (16,8,5) codes respectively.

TABLE I. COMPARISON OF 4 BIT UNCODED WITH 12 BIT GOPPA (12,4,5) CODED DATA

4 bit data	12 bit coded data	Uncoded PAPR(dB)	Coded PAPR(dB)
0000	000000000000	5.97	10.61
0001	111101010100	2.48	4.98
0010	010011110010	2.48	5.30
0011	101110100110	3.77	5.60
0100	001010111000	2.48	4.78
0101	110111101100	6.04	4.82
0110	011001001010	3.76	3.92
0111	100100011110	2.48	2.99
1000	010100110001	2.48	4.77
1001	101001100101	3.76	4.81
1010	000111000011	6.04	7.45
1011	111010010111	2.48	4.36
1100	011110001001	3.77	2.99
1101	100011010010	2.48	4.82
1110	001101111011	2.48	4.81
1111	110000101111	5.97	5.65

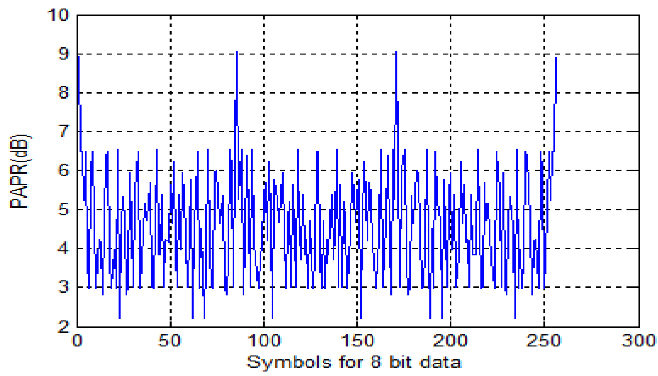

Fig. 4. PAPR for uncoded 8 bit data symbol

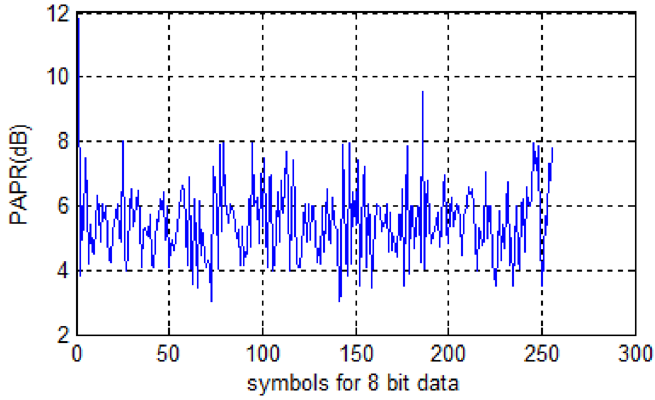

Fig. 5. PAPR for (16,8,5) Goppa coded symbols

Considering that Goppa polynomial have good error correcting performance and have efficient decoding algorithms, both the codes can be tried for efficient utilization of redundant bits. Goppa codes combined with probabilistic schemes like partial transmit sequences can also be exploited in the future to selectively reduce the PAPR of non-permissible codes.

IV. CONCLUSION

In continuation to the above discussions there can be permissible code words, that is number of possible code words that can be transmitted in an OFDM system with low PAPR the search of which is exhaustive specially for long codes. It can be selected and reduced by probabilistic methods of PAPR reduction like PTS (partial transmit sequences) and SLM (selected mapping) both of which have searching complexity. In addition to traditional SLM scheme, in which only frequency-domain phase rotation is used to generate the candidate signals, frequency-domain cyclic shifting, complex conjugate, and subcarrier reversal operations can also be tried and reduce implementation complexity to increase the diversity of the transmit signals.[10]. A T-OFDM system is also proposed in which the computational complexity and PAPR of conventional precoded OFDM systems were reduced by using a Walsh-Hadamard matrix [11].

Since Goppa codes are a very large class of codes, near to random codes and are easy to generate and possesses an interesting algebraic structure other example of these codes can also be exploited.

A cyclic code is a good candidate for a scrambling subcode as shown in the simulation result. The OFDM-BPSK method presented in this paper can be extended to any M-ary coherent modulation technique to improve the PAPR statistics and its error correction capability can also be studied to efficiently use the redundant bits in the code.

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