COMMUNICATION ANALYSIS OF INTEGRATED WAVEFORM BASED ON LFM AND MSK

Liu Zhipeng, Chen Xingbo, Wang Xiaomo, Shanfeng Xu, Feng Yuan

[1]*China Academy of Electronics and Information Technology, Beijing 100041, China*
zhipengliu@live.cn

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Abstract

The integrated waveform has some impacts on radar and communication. Two aspects on communication's impact are analyzed: ①the limit of the modulated data and ②the bit error rate. The time-frequency analysis method is adopted to analyze the limit of the modulated data. After complex deduction, 推理 the bandwidth of the integrated waveform can extend when the codes are not be designed while the bandwidth will not extend with the reasonable designed codes. The bit error rate of integrated waveform is deduced based on the matched filtering in detail. The bit error rate of integrated waveform is the same as that of MSK, and this is also proved by the simulation result.

1 Introduction

Radar and Communication are regarded as different fields due to their different functions in a long time. But from the principle of realizing own functions, they are the processes of electromagnetic wave's emission and receipt. Moreover, as the technology's development, the radio frequency front ends will be increasingly replaced by the digital field; and the structure of radar and communication will be more and more similar to each other. Therefore, the integration of radar and communication has the theoretical and realizing basis[1~5].

From another aspect, due to transmitting power's limit, the communication radio station placed on the plane has three drawbacks: ①the limited transmitting distance, ②the slow transmitting speed and ③the bad anti-interference. Thus, it's difficult to meet requirements when a large data need to be sent. But the communication data can be remotely transmitted with the help of the radar's strong transmitting power. Meanwhile, the directional and high-gain radar antenna will highly improve the anti-interference and anti-investigation of the communication.

The integration of radar and communication is a new subject and only America states openly that the communication research of the active phased array radar is being developed. In 2005, Northrop Grumman, L-3 and Lockheed Martin jointly developed an experimental research on the integration of radar and communication by the broadband active phased array radar. In this experiment, multiple beams performing different tasks were produced by antenna aperture of the active phased array radar. The data
孔径
were transmitted by a modified version of the common data link (CDL) waveform [6].

The waveform design is very important in the integration of radar and communication. The paper [7] designed an integrated waveform based on the plus-minus frequency modulation rate. Obviously, the orthogonality is the key in the design. The papers [8] and [9] proposed the carrier of MSK signal could be designed as the LFM signal, and then the integrated waveform was produced. Based on the paper [8], this paper develops the detailed communication analysis. In the first section, this paper introduces the mathematic principle of the integrated waveform. And then, the short time Fourier transform (STFT) of integrated waveform is deduced 推理 in detail. The limit of modulated data's amount is analyzed by the mathematical equations and simulation results. Next, the bit error rate is deduced and the result also is given. Finally, a conclusion is given.

2 Integrated Waveform

相位连续频移键调制

MSK signal is a special continuous phase frequency shift key (CPFSK). The characteristics include: ①the continuous phase, ②the constant envelop, ③the high use-ratio of frequency spectrum, ④the low bit error rate and so on. The k-th code's signal can be expressed as:

$$s_k(t)=\cos\left(2\cdot\pi\cdot f_c\cdot t+\boxed{\frac{a_k\cdot\pi\cdot t}{2\cdot T}}+\varphi_k\right) \qquad (1)$$

In (1), $a_k=\pm1$ ($a_k=+1$ when the code is "1"; $a_k=-1$ when the code is "0"), f_c is the carrier, T is the duration time of one code, φ_k is the initial phase of the k-th code.

Similarly, the integrated waveform is to change the carrier f_c in (1) as $f_c+\mu\cdot t/2$ (μ is frequency modulation). Thus, the k-th code signal of the integrated waveform can be expressed as[9]:

LFM

$$s_k(t)=\cos\left(\boxed{2\cdot\pi\cdot f_c\cdot t+\pi\cdot\mu\cdot t^2}+\frac{a_k\cdot\pi\cdot t}{2\cdot T}+\varphi_k\right) \qquad (2)$$

In (2), $(k-1)\cdot T\le t\le k\cdot T$.

After complex operations, (2) can be expressed as orthogonal form:

$$s_k(t)=p_k\cdot\cos\left(\frac{\pi\cdot t}{2\cdot T}\right)\cos\left(2\cdot\pi\cdot f_c\cdot t+\pi\cdot\mu\cdot t^2\right)$$
$$-q_k\cdot\sin\left(\frac{\pi\cdot t}{2\cdot T}\right)\sin\left(2\cdot\pi\cdot f_c\cdot t+\pi\cdot\mu\cdot t^2\right) \qquad (3)$$

$\cos\left(\frac{a_k\pi t}{2T}+\varphi_k\right)=\cos\varphi_k\cdot\cos\frac{a_k\pi t}{2T}-\sin\varphi_k\sin\frac{a_k\pi t}{2T}$

$=p_k\cos\frac{a_k\pi t}{2T}$

$\sin\left(\frac{a_k\pi t}{2T}+\varphi_k\right)=\cos\varphi_k\sin\left(\frac{a_k\pi t}{2T}\right)-\sin\varphi_k\cos\left(\frac{a_k\pi t}{2T}\right)$

$=a_k\cos\varphi_k\sin\left(\frac{a_k\pi t}{2T}\right)$

$=q_k$ ~~cos~~

$\sin\varphi_k = 0$

$\Uparrow$

$\Rightarrow \varphi_k = \pm\pi$

In (3), $p_k = \cos(\varphi_k) = \pm 1$, $q_k = a_k \cdot \cos(\varphi_k) = \pm 1$. According to (3), the integrated waveform based on LFM and MSK can be produced by the diagram shown in Figure 1. The simulation results on the integrated waveform are given in Figure 2.

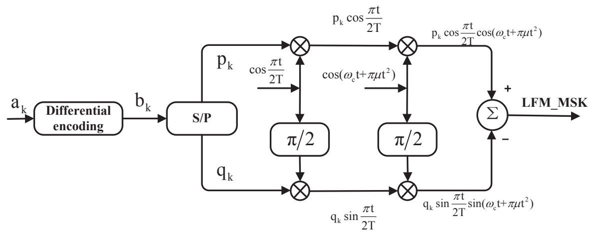

Figure 1. Principle diagram of LFM_MSK waveform

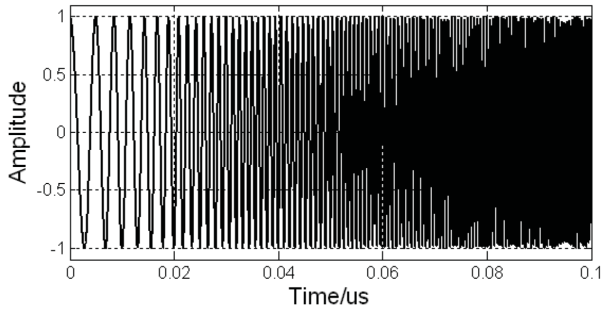

Figure 2. Simulation result of the integrated waveform

3 Limit of Modulated Data's Amount

The integrated waveform discussed in this paper is based on the pulse structure. Thus, it's impossible to send data continually like continuous wave. Moreover, due to the limitation of the radar's bandwidth, the modulated data's number has a limitation. This is different from the MSK signal. In analysis, the time-frequency analysis method-the short time Fourier transform (STFT) is applied.

For a given signal $x(t)$, the STFT is defined as

$$STFT_x(t,f) = \int x(\tau)\cdot \boxed{g^*(\tau - t)} \cdot \exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau \tag{4}$$

In (4), $g(t)$ is the window function, and generally is a 对称实函数 symmetrical real function, namely $g^*(t) = g(t)$.

The amplitude's square operation is performed for both sides of equal sign in (4).

$$\left|STFT_x(t,f)\right|^2 = \left|\int x(\tau)\cdot g^*(\tau - t)\cdot\exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau\right|^2 \tag{5}$$

Define

短时傅里叶变换频谱图，matlab中的短时傅里叶变换函数

$$S_x(t,f) = \left|STFT_x(t,f)\right|^2 \tag{6}$$

$S_x(t,f)$ is named as the spectrogram of the signal $x(t)$.

According to (3), the integrated waveform based on LFM and MSK can be expressed as

$$x(t) = \sum_{k=1}^{N} rect\left(\frac{t-(k-1)T}{T}\right)\cdot$$

$$\exp\left(j\cdot\pi\left(2\cdot f_c\cdot t + \mu\cdot t^2 + p_k\cdot q_k\frac{t}{2\cdot T} + \frac{1-p_k}{2}\right)\right) \tag{7}$$

According to the definition, STFT of the signal $x(t)$,

$$STFT_x(t,f) = \int\sum_{k=1}^{N} rect\left(\frac{\tau-(k-1)\cdot T}{T}\right)$$

$$\exp\left(j\cdot\pi\left(2\cdot f_c\cdot\tau + \mu\cdot\tau^2 + p_k\cdot q_k\frac{\tau}{2\cdot T} + \frac{1-p_k}{2}\right)\right)$$

$$\cdot g^*(\tau - t)\cdot\exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau \tag{8}$$

STFT meets the additive property, namely

$$x_1(t) + x_2(t) \Leftrightarrow STFT_1(t,f) + STFT_2(t,f) \tag{9}$$

And for the signal $x(t)$, when $t\in[t_1,t_2]$, variable t in $STFT_x(t,f)$ which is the STFT of $x(t)$ also meets $t\in[t_1,t_2]$. Thus, (8) can be rewritten as

$$STFT_x(t,f) = \exp(-j\cdot\pi\cdot\mu\cdot t^2)\cdot\sum_{k=1}^{N} rect\left(\frac{t-(k-1)\cdot T}{T}\right)\cdot$$

$$\int\exp\left(j\cdot 2\cdot\pi\cdot\left(f_c + \frac{p_k\cdot q_k}{4\cdot T} + \mu\cdot t\right)\cdot\tau + j\cdot\pi\frac{1-p_k}{2}\right)$$

$$\cdot\exp\left(j\cdot\pi\cdot\mu\cdot(\tau - t)^2\right)\cdot g(\tau - t)\cdot\exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau \tag{10}$$

Let

$$\Gamma(f) = \int\exp(j\cdot\pi\cdot\mu\cdot\tau^2)\cdot g(\tau)\cdot\exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau \tag{11}$$

It's not difficult to find that $\Gamma(f)$ is the Fourier transform of the windowed LFM signal $\exp(j\cdot\pi\cdot\mu\cdot\tau^2)\cdot g(\tau)$. Thus, (10) is the time shift and frequency shift of the signal $\exp(j\cdot\pi\cdot\mu\cdot\tau^2)\cdot g(\tau)$'s Fourier transform. According to the time shift and frequency shift of Fourier transform:

$$\begin{cases} s(t)\Leftrightarrow S(f) \\ s(t-t_0)\Leftrightarrow S(f)\cdot\exp(-j\cdot 2\cdot\pi\cdot f\cdot t_0) \\ s(t)\cdot\exp(j\cdot 2\cdot\pi\cdot f_0\cdot t)\Leftrightarrow S(f-f_0) \end{cases} \tag{12}$$

The STFT of the integrated waveform is

$$STFT_x(t,f) = \exp(-j\cdot\pi\cdot\mu\cdot t^2)\exp\left(-j\cdot 2\cdot\pi\cdot\left(f-(f_0+\mu\cdot t)\right)\cdot t\right)$$

$$\sum_{k=1}^{N} rect\left(\frac{t-(k-1)T}{T}\right)\exp\left(j\cdot\pi\frac{1-p_k}{2} + j\cdot\pi\frac{p_k\cdot q_k}{2\cdot T}t\right)$$

$$\Gamma\left(f - \left(f_0 + \frac{p_k\cdot q_k}{4\cdot T} + \mu\cdot t\right)\right) \tag{13}$$

When the window function is Gaussian function, namely $g(t) = \exp\left(-t^2/(2\cdot\sigma^2)\right)$, (11) can be rewritten as

$$\Gamma(f) = \int\exp(j\cdot\pi\cdot\mu\cdot\tau^2)\cdot\exp\left(-\frac{\tau^2}{2\cdot\sigma^2}\right)\cdot$$

$$\exp(-j\cdot 2\cdot\pi\cdot f\cdot\tau)\,d\tau \tag{14}$$

where σ is the constant. The integral phase of (14) is

$$\Phi = j\cdot\pi\cdot\mu\cdot\tau^2 - \frac{\tau^2}{2\cdot\sigma^2} - j\cdot 2\cdot\pi\cdot f\cdot\tau \tag{15}$$

To gain the stationary phase, derive(15), and let the derivation equal 0.

$$\tau' = \frac{\mathrm{j}\cdot 2\cdot\pi\cdot f}{\mathrm{j}\cdot 2\cdot\pi\cdot\mu - \frac{1}{\sigma^2}} \tag{16}$$

Substitute (16) into (14)

$$\Gamma(f) = \frac{2\cdot\sqrt{\pi}}{\sqrt{(2\cdot\pi\cdot\mu)^2+\sigma^{-4}}}\exp\left(-\frac{1}{2}\cdot\frac{(\mathrm{j}\cdot 2\cdot\pi\cdot f)^2}{\mathrm{j}\cdot 2\cdot\pi\cdot\mu-\sigma^{-2}}\right)$$

$$= \frac{2\cdot\sigma^2\cdot\sqrt{\pi}}{\sqrt{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}}\cdot\exp\left(-\mathrm{j}\cdot\frac{\pi\cdot\mu\cdot(2\cdot\pi\cdot f)^2}{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}+\frac{2\cdot(\sigma\cdot\pi\cdot f)^2}{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}\right) \tag{17}$$

Then, substitute (17) into (13)

$$STFT_x(t,f) = \frac{2\cdot\sigma^2\cdot\sqrt{\pi}}{\sqrt{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}}\exp\left(-\mathrm{j}\cdot\pi\cdot\mu\cdot t^2\right)\cdot$$
$$\exp\left(-\mathrm{j}\cdot 2\cdot\pi\cdot\left(f-(f_c+\mu\cdot t)\right)\cdot t\right)\cdot$$
$$\sum_{k=1}^{\mathrm{N}} rect\left(\frac{t-(k-1)\cdot\mathrm{T}}{\mathrm{T}}\right)\exp\left(\mathrm{j}\cdot\pi\frac{1-p_k}{2}+\mathrm{j}\cdot\pi\frac{p_k\cdot q_k}{2\cdot\mathrm{T}}\cdot t\right)$$
$$\cdot\exp\left(-\mathrm{j}\cdot\frac{\pi\cdot\mu\cdot\left(2\cdot\pi\cdot f-\left(f_c+\frac{p_k\cdot q_k}{4\cdot\mathrm{T}}+\mu\cdot t\right)\right)^2}{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}\right.$$
$$\left.+\frac{2\cdot\left(\sigma\cdot\pi\cdot f-\left(f_c+\frac{p_k\cdot q_k}{4\cdot\mathrm{T}}+\mu\cdot t\right)\right)^2}{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}\right) \tag{18}$$

According to (6), the spectrogram of the integrated waveform is

$$S_x(t,f) = A^2\cdot$$
$$\exp\left(-\frac{\left(2\cdot\pi\cdot\sigma\cdot\left(f-\left(f_c+\mu\cdot t+\sum_{k=1}^{\mathrm{N}} rect\left(\frac{t-(k-1)\cdot\mathrm{T}}{\mathrm{T}}\right)\frac{p_k\cdot q_k}{4\cdot\mathrm{T}}\right)\right)\right)^2}{(2\cdot\pi\cdot\sigma^2\cdot\mu)^2+1}\right) \tag{19}$$

It's obvious that the spectrum of the integrated waveform arranges along the line $f = f_c+\mu\cdot t+\sum_{k=1}^{N} rect\left(\frac{t-(k-1)\cdot\mathrm{T}}{\mathrm{T}}\right)\frac{p_k\cdot q_k}{4\cdot\mathrm{T}}$. On the basis of (19), the extension of the spectrum can be calculated, and then the number's limit of modulated data can be gained.

We can assume that the bandwidth is $B=\mu\cdot T_p=\mu\cdot N\cdot T$ (N is the number of modulated data). If the spectrum has no extension, f should be confined at $[f_c \quad f_c+B]$.

a) In the extreme case, if the modulated data are all -1 namely $p_k\cdot q_k=-1$, the spectrum can break the down limit f_c. Assume the extended amount of modulated data is n_1, n_1 can meet the following equation:

$$f_c = f_c+\mu\cdot n_1\cdot\mathrm{T}-\frac{1}{4\cdot\mathrm{T}} \tag{20}$$

After calculation, $n_1=\frac{1}{4\cdot\mu\cdot\mathrm{T}^2}$.

b) In the other extreme case, if the modulated data are all +1 namely $p_k\cdot q_k=-1$, the spectrum can break the up limit. Assume the extended amount of modulated data is n_2, n_2 can meet the following equation:

$$f_c+B = f_c+\mu\cdot(\mathrm{N}-n_2)\cdot\mathrm{T}+\frac{1}{4\cdot\mathrm{T}} \tag{21}$$

After calculation, $n_2=\frac{1}{4\cdot\mu\cdot\mathrm{T}^2}$.

According to the two above extreme case, the maximum number of extended codes is $\left\lfloor\frac{1}{2\cdot\mu\cdot\mathrm{T}^2}\right\rfloor$ where $\lfloor\ \rfloor$ means to round numbers.

In order to restrict the extension of the spectrum, one method is to put the modulated data in the middle of the LFM signal, and the edge has no data. But this leads to discontinuous phase, and then a more large extension will happen. Another method is to make the n_1 initial codes "+1", and make the n_2 tail codes "-1". This can maintain the phase's continuance and has no extension of the spectrum. The ratio between the amount of codes n_1+n_2 for preventing spectrum's extension and the total codes amount is

$$\rho = \frac{1}{2\cdot\mu\cdot\mathrm{T}^2}\Big/N = \frac{N}{2\cdot B\cdot T_p} \tag{22}$$

It's obvious that the ratio coefficient is a line increase along with the total codes amount when the radar's time-bandwidth product is a fixed value.

But, to reduce the affect on the radar, the the n_1^{th} code should be set as "-1" and the n_2^{th} tailing code should be set as "+1". So

$$\rho = \left\lfloor\frac{1}{2\cdot\mu\cdot\mathrm{T}^2}+2\right\rfloor\Big/N \tag{23}$$

Tab.1 lists some computation results on the ratio coefficient, the total codes amount, the time-bandwidth product and the bit rate. The bit rate is the result after wiping off the extensive codes. Meanwhile, the duty ration of the radar is 10%.

Table 1. Results of extensive codes amount, ratio coefficient and bit rate under different time-bandwidth product and total codes amount

Time-bandwidth Product	Total Codes Amount	Extensive Codes Amount	Ratio Coefficient	Bit Rate
1E4	500	16	3.2%	484 kbps
	1000	52	5.2%	948 kbps
	3000	452	15.07%	2.55 Mbps
3E4	500	8	1.6%	164

				kbps
	1000	20	2.0%	326.67 kbps
	3000	152	5.07%	949.33 kbps

Figure 3 and Figure 4 simulate the spectrum of the integrated waveform with uncorrected codes and with corrected codes. The corrected codes mean that the n_1 initial codes are "+1" and the n_2 tail codes are "-1"; moreover, the n_1^{th} code is "-1" and the n_2^{th} tailing code is "+1" (the values of n_1 and n_2 depend on (20) and (21)). For a more obvious comparison, the spectrum of LFM signal is also placed in Figure 3 and Figure 4. The design bandwidth is restricted at dotted line block. Obviously, after correcting codes, the spectrum of the integrated waveform has no extension.

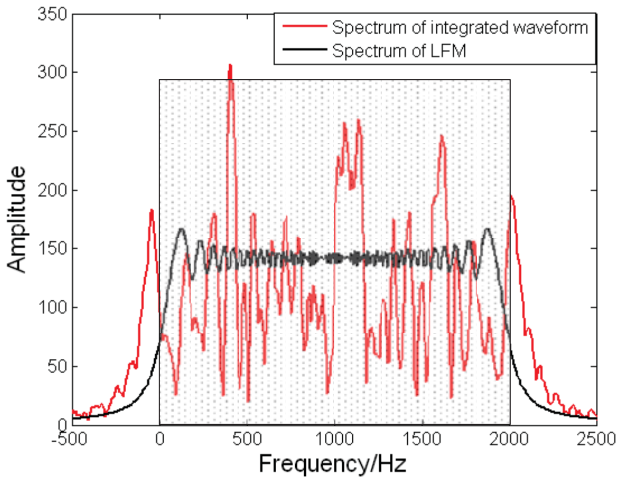

Figure 3. Spectrum of integrated waveform with uncorrected codes and LFM

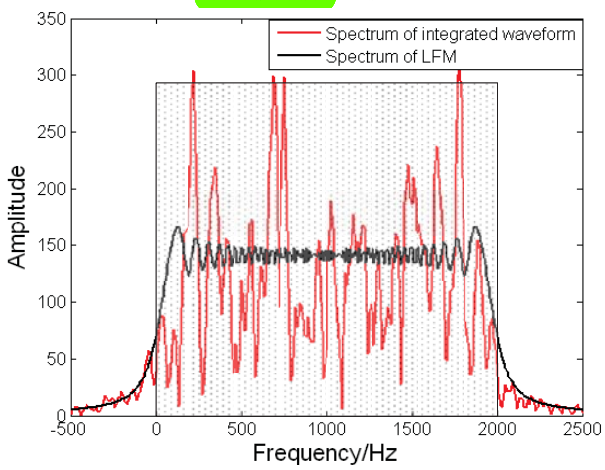

Figure 4.Spectrum of integrated waveform with corrected codes and LFM

4 Bit Error Rate BER

Under the white Gaussian noise channel, the bit error rate of the receiver based on the matched filtering is

$$P_e = Q\left(\left\|s_{T_0}\right\| / \sqrt{N_0/2}\right) \tag{24}$$

Where the concrete expression of $Q(\cdot)$ is

$$Q(t) = \frac{1}{\sqrt{2\cdot\pi}} \int_t^{\infty} \exp\left(-\frac{\tau^2}{2}\right) d\tau \tag{25}$$

$N_0/2$ is the spectrum density of the white Gaussian noise; T_0 is the best sampling time. The expression of $\left\|s_{T_0}\right\|$ is

$$\begin{aligned} \left\|s_{T_0}\right\|^2 &= \frac{1}{4}\int_{-\infty}^{+\infty} \left[s_0(T_0 - t) - s_1(T_0 - t)\right]^2 dt \\ &= \frac{1}{4}\int_{-\infty}^{+\infty} \left[s_0(u) - s_1(u)\right]^2 du \end{aligned} \tag{26}$$

The Euclidean distance d between signals $s_0(u)$ and $s_1(u)$ is defined as

$$d^2 = \left\|s_0(u) - s_1(u)\right\|^2 = \int_{-\infty}^{+\infty} \left[s_0(u) - s_1(u)\right]^2 du \tag{27}$$

The bit error rate depends on the minimum Euclidean distance $d_{\min}$. According to (3) when the p_k and q_k has different values, the signal $s(u)$ has four types. The minimum Euclidean distance of the four types is $2\cdot \mathrm{T}$ by computation. Hence

$$P_e = Q\left(\sqrt{2\cdot \mathrm{T}/N_0}\right) \tag{28}$$

It's obvious that the bit error rate is same as that of MSK signal. Figure 5 simulates simulation values and theoretical values of the bit error rate under different signal-noise-rates (SNR), and the results show that simulation values is very close to the theoretical value.

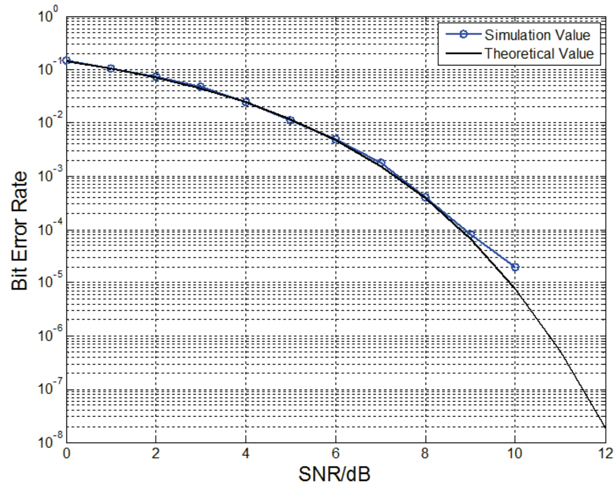

Figure 5. Bit error rate of integrated waveform

5 Conclusion 时频分析

By the time-frequency analysis of the integrated waveform based on LFM signal and MSK signal, the spectrum of the integrated waveform can extend. Hence, it's necessary to design the codes. After designing the codes reasonably, the spectrum's extension gains the obvious control. By the bit error rate's analysis of the integrated waveform based on LFM signal and MSK signal, it's found that the bit error rate is corresponding to that of MSK signal, and also proved by the simulation results.

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