PEAK TO AVERAGE POWER RATIO ANALYSIS OF A CONSTANT ENVELOPE CHIRPED OFDM

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Abstract —Orthogonal Frequency Division Multiplexing (OFDM) using Fractional Fourier Transform (FrFT) normally known as Chirped OFDM is studied. The Constant Envelope of the Chirped OFDM is then obtained by phase modulation of the chirped OFDM and then its Peak to Average Power Ratio (PAPR) is analyzed.

The study shows the Constant Envelope Chirped OFDM (CE-COFDM) has low PAPR than convectional OFDM. Different modulation Index of the phase modulator produces different PAPR values of CE-COFDM. Finally the data rate was increased and the study shows it produces minor increase in PAPR.

Keywords— Orthogonal Frequency Division Multiplexing, Fractional Fourier Transform, Peak to Average Power Ratio.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) provides high data rate transmissions over frequency selective fading channels and it is easily implemented by the use of accurate Fast Fourier Transform (FFT). The ability of OFDM to combat multipath effects of wireless channel motivated the adoption of OFDM as a standard in Digital Audio Broadcast (DAB), Terrestrial Video Broadcasting, IEEE 802.11a/g, WiMAX and LTE

Regardless of its widely applicability, OFDM still suffers from high Peak to Average Power Ratio (PAPR) and sensitivity to time and frequency synchronization problems. Reducing PAPR of the system will increase energy utilization efficiency of the system which is a major problem for a hand held (smart) devices as they have more computational capability, So by reducing PAPR battery life for a hand held devices is increased.

Different schemes are adopted in reducing PAPR of an OFDM categorized into transmitter enhancement, receiver enhancement and signal transformation techniques. While transmitter enhancement techniques includes tone reservation, trellis-shaping, multiple signal representation, selected mapping, partial transmit sequences, signal clipping, peak cancellation and peak windowing, Receiver enhancement techniques includes signal reconstruction, maximum likelihood decoding and interference cancellation Signal transformation technique is implemented by producing Constant Envelope of the OFDM by phase modulating the OFDM subcarriers.

Both [1] and [2] shows that Constant Envelope OFDM (CE-OFDM) performs better than the first two techniques in reducing PAPR of an OFDM with a reduced complexity.

The realization of Fractional Fourier transform (FrFT) by Mendlovic and Ozaktas in 1993 for optical signal processing and its realization in optical instruments made FrFT to be a study focus of a lot of researchers [3]. Interpretation of the FRFT as a “rotation” operator in the time–frequency plane and as a generalization of the Fourier transform has generated significant interest of FrFT in the signal processing community [4] especially in OFDM. The FrFT can provides different modulated subcarrier by using different FrFT order hence combating different types of noise and interference in different types of channel.

As shown by [5], [6] and [7] chirped OFDM has better performance than the convectional OFDM. Since Chirped OFDM performs better than convectional OFDM more studies of OFDM are now based on it and this paper intends to study PAPR reduction in a Constant Envelope Chirped OFDM (CE-COFDM) system.

The rest of this paper is organized as follows. In section II, the system model based on FRFT is described. The PAPR system performance is discussed in Section III. Section IV discusses the findings of the research and Section V gives the conclusion of this research.

II. SYSTEM MODEL

For an OFDM system shown the Figure 1 the high data rate signal must first be serially to parallel converted to produce M low rate parallel symbols x_n of N maximum size.

$$x = x_n(t) = x_0, x_1, \ldots x_{N-1} \qquad (1)$$

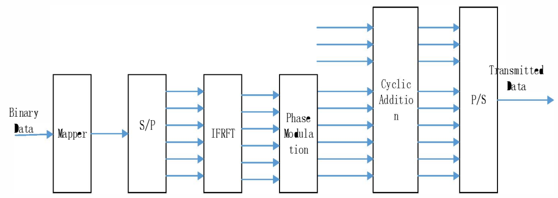

Figure 1. Constant Envelope FRFT OFDM

Each of the subcarrier is then modulated by Inverse Fractional Fourier Transform (IFrFT) to produce frequency transform of the original signal at a specified angle α depending on the frequency selectivity of the channel in use.

$$x_\alpha(u) = [x_n]_\alpha(u) = \{F^{-p}x(t)\} = \int_{-\infty}^{\infty} x(t)K_\alpha(t,u)dt \qquad (2)$$

where

$\alpha = p * \pi/2$,

p is real number caller the order of FrFT, and

$$K_\alpha(t,u) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}} e^{j((u^2+t^2)/2)\cot\alpha - jut\operatorname{cosec}(\alpha)} & \text{if } \alpha \text{ is not a multiple of } \pi \\ \delta(t-u) & \text{if } \alpha+\pi \text{ is a multiple of } 2\pi \\ \delta(t+u) & \text{if } \alpha \text{ is a multiple of } 2\pi \end{cases}$$

where

$j = \sqrt{-1}$, u is frequency, t is time, $\cot(\alpha)$ and $\operatorname{cos}ec(\alpha)$ are cotangent and cosecant of the FrFT angle α respectively.

The phase modulator will produce s_n, $0 \le n \le N-1$ constant envelope phase modulated OFDM signal as given by equation 3.

$$s_n = A_n e^{j\theta_n} \qquad (3)$$

where

A_n is a constant Amplitude of the nth subcarrier through a phase modulator,

θ_n is the phase of the nth subcarrier, and

$$\theta_n(i) = 2\pi h x_\alpha(i) + \theta_0(i) \quad , 0 \le i \le N-1$$

h is constant named modulation index, and

$$\theta_0(i+1) = \sum_{i=0}^{N-1} \theta_0(i) + \frac{2\pi}{N} \quad , 0 \le i \le N-1$$

is an angle used to track changes of Phase Modulated CE-COFDM.

Cyclic prefix addition is performed to produce s_{n+n_g}, $0 \le n \le N + N_g - 1$ for Inter Symbol Interference (ISI) cancelation during transmission and lastly parallel to serial is performed to obtain data ready for transmission.

VI: PAPR A ANALYSIS

Peak to average power ratio (PAPR) is the ratio between the maximum power to the average power of the signals.

$$PAPR = \frac{P_{peak}}{P_{avg}} \qquad (5)$$

where

$P_{peak} = \max|s_n|^2$, and

$$P_{avg} = \frac{1}{N}\sum_{n=0}^{N-1}|s_n|^2$$

PAPR is the measure of amount of power the system utilizes in its operation. Because the system needs to amplify the power every time before its transmission, variation of power in different signals components provides complexity in amplifying the signals hence maximum power utilization. So as PAPR increases from 1 then that's how the power utilization of the system increases.

In studying PAPR normally Cumulative Distribution Function (CDF) is used. The CDF of $\max(s_n)$ given by Equation 6 shows the probability that the PAPR is below some threshold level

$$CDF = 1 - e^{\frac{-s^2}{2\sigma^2}} \qquad (6)$$

$$P(PAPR \le x) = \left(1 - e^{\frac{-s^2}{2\sigma^2}}\right)^N \qquad (7)$$

Considering Complementary Cumulative Distribution Function (CCDF) as given by Equation 8.

CCDF will be used in this paper to study power utilization of the CE-COFDM.

$$CCDF = 1 - \left(1 - e^{\frac{-s^2}{2\sigma^2}}\right)^N \qquad (8)$$

IV: SIMULATION RESULTS

In analyzing Constant Envelope FRFT OFM, Figure 1 gives the spectrum of the two systems using 64 bits and 10,000 symbols.

Figure 3 and Figure 4 uses FrFT Size of 64 bits, Cyclic Prefix length of 8 bits, FrFT order of 0.001 and Amplitude of the Phase Modulation being set to constant of 1.

Figure 3 compares CCDF of an OFDM and a CE-COFDM having different modulation index. Figure 4 shows the variation in increasing data rate with the PAPR changes produced.

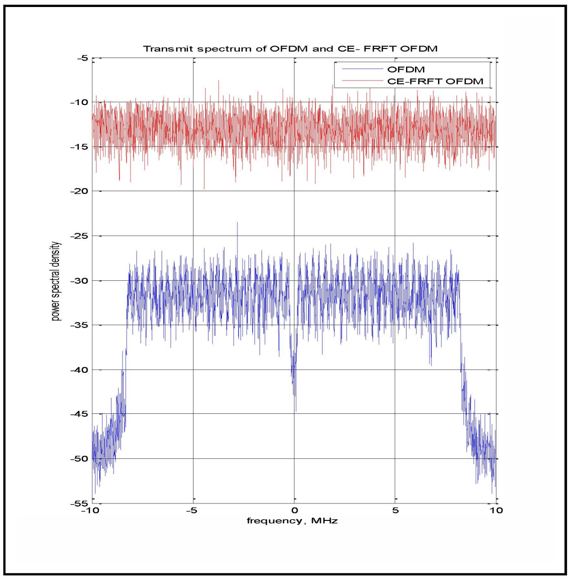

Figure 2. Constant Envelope Chirped OFDM and Conventional OFDM

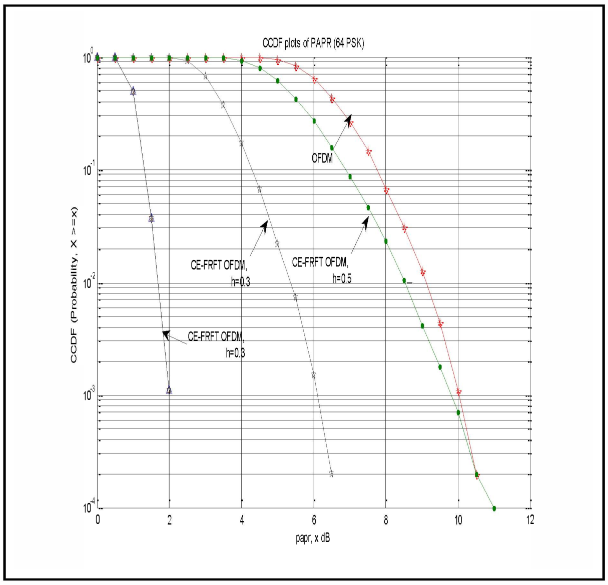

Figure 3. Constant Envelope FRFT-OFDM

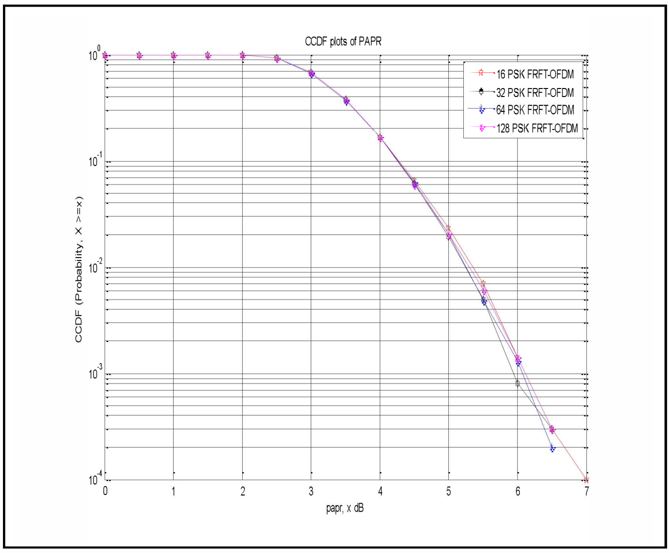

Figure 4. CE-COFDM data rate variation

V: CONCLUSION

Figure 3 shows the Constant Envelope Chirped OFDM has low PAPR compared to a normal OFDM. So by changing modulation index of the CE-Chirped OFDM we can lower the PAPR of the system. Figure 4 shows data rate increase in CE-Chirped OFDM produce considerable low PAPR increase in the system. Meaning the system using CE-Chirped OFDM will use low transmit power with small increase as data rate increases. With further study of FrFT the maximum potential of CE-FrFT OFDM will be explored.

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