PAPR Reduction in SLM-OFDM using Lehmer Random Number Generator

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***Abstract*—One of the popular methods of modulation in high speed wireless communication is Orthogonal frequency division multiplexing (OFDM).However, the time domain version of signals suffer from large envelope variations, which are often characterized by the Peak-to-Average Power Ratio (PAPR).One of the promising PAPR reduction techniques for OFDM is selective-mapping (SLM). In this paper we introduce an alternate way to generate phase sequence using Lehmer random number generator (or) multiplicative congruential generator (MCG).Simulations compare PAPR reduction performance using other sequences such as Hadamard and Riemann. Observations drawn from results depict that the proposed sequence provides PAPR reduction equivalent to random sequence with less side information (SI) and better than Hadamard. Hence, MCG can be used as a good alternative for Hadamard and random sequence.**

***Keywords*—complementary cumulative distribution function(CCDF);Lehmer random number generator (RNG); multiplicative congruential generator(MCG);OFDM;PAPR;SLM.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is one of the popular multi carrier modulation technique used for high speed wireless communication. It offers various advantages like considerable high spectral efficiency, multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency[1].Hence, OFDM has been adopted in various wireless communication standards, including European digital audio broadcasting, terrestrial digital video broadcasting, satellite-terrestrial interactive multiservice infrastructure in China and also has been considered (or) approved by many IEEE standard working groups, such as IEEE 802.11a/g/n, IEEE 802.15.3a and IEEE802.16d/e[2].

However, OFDM system design still has some unresolved issues one of them being large envelope variations in time domain often referred to as peak-to-average power ratio (PAPR) [3].There have been various PAPR reduction techniques developed like clipping and filtering, coding schemes, PTS(Partial Transmit Sequences), SLM, nonlinear companding, Tone Reservation(TR), Tone Injection(TI), constellation shaping, phase optimization[1].

Among the above mentioned methods SLM is widely used due to its ease of implementation and the ability to obtain relatively better PAPR reduction by modifying the signal without distortion. Some of the aspects that need to be considered to improve the effect of SLM technique are 1. Selection of effective phase sequence set 2.Complexity reduction 3.Side information reduction.

In this paper, we have proposed a new technique to generate phase sequence using Lehmer Random Number Generator (RNG) for SLM which provides similar PAPR performance as random sequence. However, it requires less side information when compared to random sequence.

The paper is organized in the following manner. A brief, background of SLM technique and its PAPR calculation is presented in section II. The steps involved in the generation of phase sequence using Lehmer Random Number generator are discussed in section III. The simulation results are shown in section IV. Finally, conclusion is given in section V.

II. SLM Technique

A. Basic Principle

SLM uses the fact that slight variation in phase of the frequency domain data will impact PAPR of an OFDM signal. In SLM PAPR reduction is achieved by multiplying original data input $X = [X_0, X_1, ..., X_{N-1}]^T$ with C statistically independent phase vectors $R^c = [r_0^c, r_1^c, ..., r_{N-1}^c]^T$ (c=0,1,2,…C-1) where N is number of data subcarriers and C is number of candidate signals required and determining the PAPR of individual data combination. The combination with lowest PAPR is used for transmission.

The above mentioned process is carried out as follows:

Let X be the original OFDM signal and R^c be the SLM phase vector

1. Each data block X of length N is element wise multiplied by $R^c = [r_0^c, r_1^c, ..., r_{N-1}^c]^T$ (c=0,1,2…C-1) to produce C individual data combinations in frequency domain.
The candidate signals generated can be expressed as

$$X^c = [X_0 r_0^c, X_1 r_1^c, ..., X_{N-1} r_{N-1}^c]^T, (c = 0,1,2..., C-1)$$

Where X^c are the candidates formed from original data sequence

2. Each candidate X^c is then transformed in time domain using IFFT as shown

$$x^c = IFFT\{X^c\} = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k r_k^c e^{\frac{j2\pi nk}{N}} \quad (1)$$

Where $0 \le k \le N-1$, $0 \le n \le N-1$ and IFFT is the inverse fast Fourier transform function.

3. PAPR is calculated for each time domain signal x^c using the expression in equation (2)

$$PAPR\{x^c\} = \frac{\max\{|x^c|^2\}}{E\{|x^c|\}^2} \quad (2)$$

Where E{.} represents the expectation function which is used to evaluate the average power of time domain signal.

4. Candidate signal with lowest PAPR is chosen based on the values calculated in step 3 and is used for transmission.

$$PAPR_{\min} = \min_{0\le c\le C-1}(PAPR\{x^c\}) \quad (3)$$

Let the signal corresponding to the lowest PAPR chosen be

$$x^{c_{\min}} = IFFT\{X.R^{c_{\min}}\} \quad (4)$$

Along with the data phase sequence which corresponds to the minimum PAPR version i.e. $R^{c_{\min}}$ (where $c_{\min}$ is the index corresponding to the minimum PAPR signal. The value $c_{\min}$ should lie in the range $0 \le c_{\min} \le C-1$) must be transmitted as side information (SI). [4]

A block diagram of conventional SLM-PAPR model for OFDM is shown in Fig.1

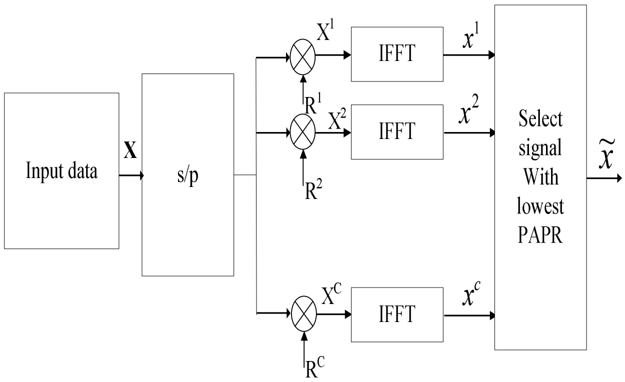

Fig. 1. The structure of C- SLM

In order to measure the PAPR reduction performance of SLM, Complementary Cumulative Distribution Function (CCDF) is used and it can be expressed as

$$\Pr(PAPR_{\min} > PAPR_0) = (1-(1-e^{-PAPR_0})^N)^C \quad (5)$$

III. LEHMER RANDOM NUMBER GENERATOR

This section gives a brief introduction on how random sequence is generated using Lehmer random number generator.

The Lehmer random number generator is based on a general recursive relation as follows

$$X_{n+1} = (aX_n + c) \bmod m \quad (6)$$

Where X_0, *a*, *c*, *m* are integers.

If *c*=0, the generator is often known as multiplicative congruential generator (MCG) (or) Lehmer RNG. In this case, modulus m is a prime number (or) a power of prime number, the multiplier *a* is a primitive root modulo *m* and seed X_0 is co-prime to *m*. [5]

To obtain a sequence of a particular period we follow the method described:

Choose '*a*' and '*m*' values such that *m* is a proper prime [6] in base '*a*'. So that the period of sequence generated is *m-1*.

For example:

If a=2, m=11(which is a full repetend prime in base 2) and X_0=1 then the sequence generated is

1,2,4,8,5,10,9,7,3,6,1,2,4…

The period of above generated sequence is 10 i.e. *m-1*.

The list of full repetend prime or proper prime less than thousand for a particular base *b* is provided by [7].

To obtain phase sequence from the above generated sequence we perform the following steps:

1. Check whether the number generated is odd or even.

2. Replace the even numbers with '-1' and the odd numbers with '1'.

Using the above mentioned procedure the phase sequence generated for the example shown will be:

[1,-1,-1,-1,1,-1,1,1,1,-1,…]

The sequence will repeat after 10 as it is periodic with period *m-1*, if period is *m*.

To generate an N-point phase sequence we should choose *m* such that *m-1*≥N-1.

A visualization of random number generation is shown in Fig.2.

In the proposed method candidates are generated from a single sequence by using cyclic shift operation on Lehmer sequence (Ls).The maximum number of cyclic shifts possible depends on the period of the sequence i.e. if period of sequence is *m-1*, we can perform utmost *m-1* cyclic shifts.

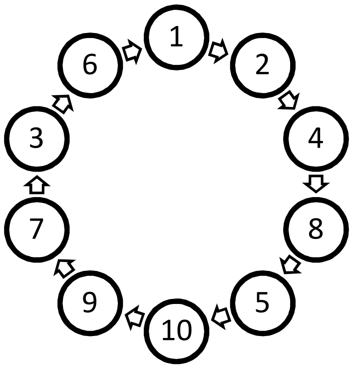

Fig. 2. Number cycle for Random number generated in the example

During implementation the number of cyclic shifts is decided based on number of candidates desired.

The basic block diagram of the proposed scheme is as shown in Fig.3

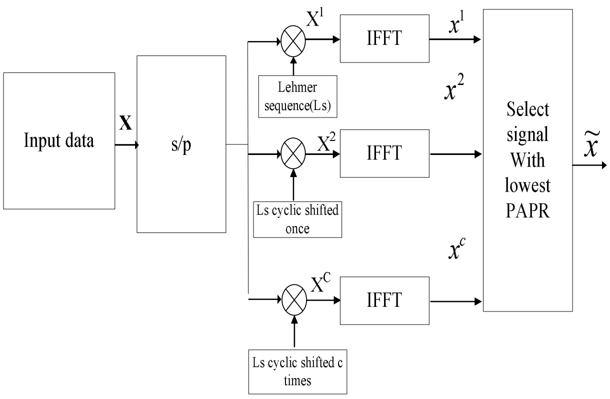

Fig. 3. Basic block diagram for SLM using Lehmer RNG phase sequence

IV. Simulation Results and Discussion

Simulation evaluates and compares the auto correlation function and the PAPR reduction performance of an SLM with different sequences, i.e. Hadamard, Riemann, Random and proposed sequence generated through Lehmer Random number generator.

The auto-correlation function is a standard method to measure the degree of correlation within the sequence or periodic nature.Fig.4 compares the maximum autocorrelation magnitudes of various sequences with proposed sequence and it depicts that sequence generated by Lehmer RNG has low correlation magnitude compared to Hadamard and Riemann sequences except at zero lag and slightly greater than random sequences.

In OFDM, the high peak occurs at the output of IFFT when the input data sequence applied at the input of IFFTs is strongly correlated. Therefore, to reduce the auto correlation at the input of IFFT, the input data sequences are multiplied with another sequence with low autocorrelation magnitude. The random sequences used in conventional SLM provide good PAPR reduction due to highly uncorrelated nature of elements in the phase sequence. Since, the proposed sequence also has correlation properties similar to a random sequence as shown in Fig.4.The simulation results confirm that the proposed sequence can achieve same PAPR reduction performance as a random sequence used in C-SLM.

Figs.5, 6 depicts the PAPR reduction performance of an SLM-OFDM system for N=128,256 respectively. In this, we compared the PAPR reduction performance of C-SLM with different number of candidates (C) i.e. C=8, 16 using different sequences and different number of subcarriers (N). The parameter used for simulations are listed in Table I. The simulation result shows the proposed Lehmer sequence achieves PAPR reduction same as that of C-SLM also it does not affect the bandwidth efficiency unlike C-SLM.

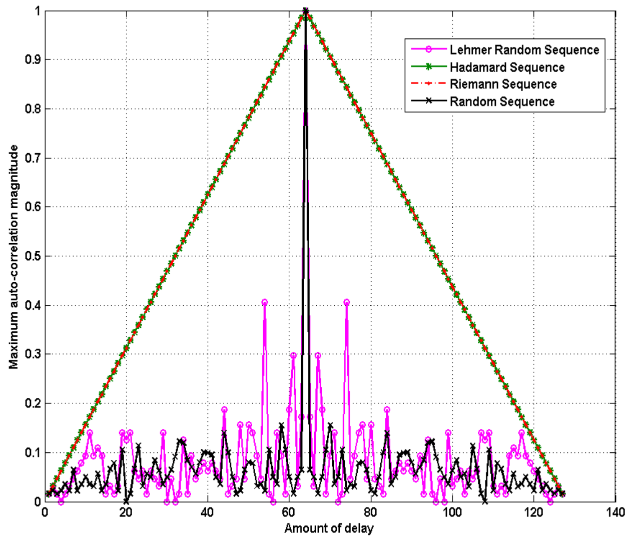

Fig. 4. Autocorrelation function of various sequences

TABLE I: Simulation Parameters used for C-SLM

Simulation Parameters	***Value***
Number of subcarriers (N)	128,256
Oversampling factor (L)	4
Modulation	QPSK
Number of candidates (C)	8,16
Phase sequences	Hadamard, Riemann, Random sequence, Lehmer sequence(a=2,m=1187,X_0=1)
Number of IFFT blocks	C

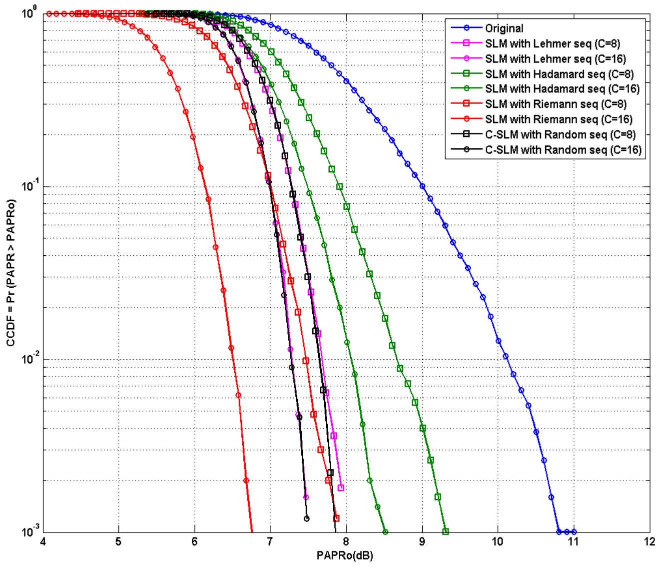

Fig. 5. The PAPR reduction performance of different candidates with N=128

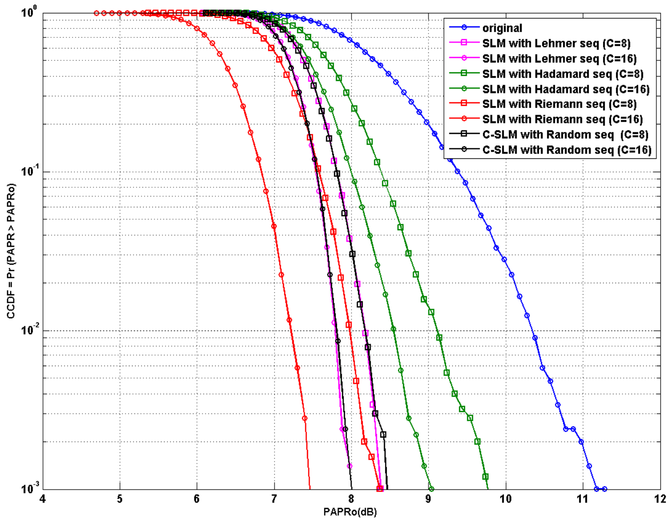

Fig. 6. The PAPR reduction performance of different candidates with N=256

For all the above simulations we have used a=2, m=1187 and X_0=1.However, as most of the applications use N=128, 256, 1024 therefore the above combination can be used for all the values of N$\leq m-1$.

V. CONCLUSION

The proposed sequence exhibits similar auto correlation properties as random number sequence and also provides approximately same PAPR reduction as evident from Fig.5, 6.

Hence, the proposed sequence can be used as an alternative for random number sequence due to its capability of reducing side information during transmission.

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