Novel Method for The Low Autocorrelation OFDM Chirp Waveform

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***Abstract*—Waveforms with low autocorrelation(AC) are desired in many communication or signal processing applications. Because of high PAPR and high autocorrelation analysis in the traditional OFDM chirp signal, we present a convex optimization method for design and synthesize the OFDM chirp waveforms. Furthermore, based on the Gerchberg-Saxton(GS) algorithm, we optimize our method and minimized the peak sidelobe level (PSL) and weighted sum of sidelobe levels(WSSL).Using the optimal solution for the relaxed convex problem, we present an efficient algorithm to find sequences with low PAPR and low autocorrelation**

***Index Terms*—ODFM Chirp, auto-correlation, weighted sum of sidelobe levels**

I. INTRODUCTION

This paper was inspired by the advanced OFDM chirp waveform design[1] which introduced a novel waveform design method. Obviously, sequence with low autocorrelation (AC) are crucial components in communications and signal processing applications. The sequences with low AC can be used as probing signals to estimate the channle impulse response[2]-[4], or as training sequences for correlation-based synchronization[5]-[7] and frequency offset estimation[8]. However, the low AC property reduces or eliminates self-interference caused by multi-paths like waveform[1]. In order to realize the low AC performance, Lung-Sheng Tsai[8] proves a new algorithm based on Gerchberg-Saxton(GS) [14]algorithm.

The pricple of the algorithm is using the spectral constraints on transmitted signals in communication field in order to avoiding causing interference. For example, in radar applications, signals must avoid occupying the bands reserved for navigation or military operations[9].

Our main goal in this work is to apply and advance the above algorithm in reality and apply it in the OFDM chirp waveform design. Part II is the introduction of the advanced OFDM chirp waveform design.Part III is the definition and basic principle of the GS algorithm.Part IV is the advanced algorithm and application in the OFDM chirp signal schem.The last section of the paper conduct the simulation and analysis of the new method for low AC and low PAPR.

II. BASIC METHOD FOR OFDM CHIRP SIGNAL

Linear frequency modulation (LFM) is one of the most popular and widely applied pulse compression signal which broaden the bandwidth size by linear modulating on frequency of signals.

The definition of LFM according to[5]is:

$$x(t) = \cos(\pi \frac{\beta}{\tau} t^2) \qquad (0 \le t \le \tau) \tag{1}$$

In this equation, τ means pulse width and β refers to maximum bandwidth range. There are two kinds of LFM signals, the so-called up-chirp and down-chirp signal. We mainly make example of the up-chirp signal and the complex form of the signal is:

$$x(t) = e^{j\pi\beta\frac{t^2}{\tau}} = e^{j\theta(t)} \qquad (0 \le t \le \tau) \tag{2}$$

We describe basic OFDM. OFDM is a spread-spectrum transmission technique where the signal is comprised of multiple carriers. The technique is based on the carriers constituting the signal being mathematically orthogonal, as a result of the uniform frequency spacing in between the carriers. The OFDM-modulated signal can be represented by:

$$S_n(t) = \sum_{k=0}^{N-1} S_{n,k} e^{j2\pi k\Delta f t}, \qquad 0 \le t \le T_s \tag{3}$$

Where T_s, Δf and N are the symbol duration, the sub-channel space, and the number of sub-channels of OFDM signals, respectively.

A cyclic prefix(CP) or guard interval is critical for OFDM to avoid inter block interference(IBI) caused by the delay spread of wireless channels. They are usually inserted between adjacent OFDM blocks.

III. PINCIPLE OF GS ALGORITHM

We aim to design an OFDM sequence x under spectral constraints so that the AC property of x is optimized. To formulate this design problem, first the goodness of the AC property needs to be quantified. There are two commonly practiced metrics for this quantification. The first metric is the peak sidelobe level(PSL) od the AC function at specified time-lags. We denote the specified time-lags of interest by a set τ . The second metric is the weighted sum of the sidelobe levels(WSSL) of the AC function. We denote the weighting coefficients by an two metrics are formally formulated as:

$$PSL(\tau) = \max_{n\in\tau} |\theta_x(n)| \tag{4}$$

$$WSSL(x) = \sum_{n=0}^{L-1} x_n |\theta_x(n)|^2 \tag{5}$$

The above metrics are used when the periodic AC function is considered.

We describe the spectral constrains by a vector $m = [m_0, \cdots, m_{L-1}]^T$,where m_k denote the maximum allowable power on the k-th subcarrier. This vector m is referred to as a spectrum mask. Our main goal is to obtain sequences satisfying the spectrum-mask constraint and with minimized PSL or WSSL.

A. Sufficient Condition to minimize PSL of WSSL

To formulate the sequence design problem as a convex optimization problem, we start with reformulation the expressions of the constraints and the objective function. We define the IDFT matrix as $G = [g_{0,}g_1, \cdots g_{L-1}]^T$, we have $\theta_x(n) = g_n^T m$, and the two metrics, PSL and WSSL, can be written as

$$PSL(\tau) = \max_{n\in\tau} |g_n^T m| \tag{6}$$

$$\text{WSSL}(w) = \sum_{n=0}^{L-1} w_n |g_n^T m| \tag{7}$$

As describe above, the sufficient condition indicates that the optimality can be achieved by properly arranging the power spectrum of a sequence, independent of the phase at each subcarrier. This property also implies that the phase at each subcarrier. This property also implies that the phase of the elements of the AC-optimized sequence in the transform domain are free variables that ca be used to further the design sequence exhibiting other desired characteristics,e.g., low PAPR, While the optimality of the AC property is maintained. In the remainder of this section, we will present methods to design spectrally constrained sequences with both low correlation .

B. Gerchberg-Saxton Algorithm

Gercgberg-Saxton(GS) algorithm deals with a nearest vector problem in the following form:

The GS algorithm is an iterative algorithm that updates the variable alternatively in each iteration, and it eventually acheives a local or global minimum solution. In this algorithm, the initial values for are randomly assigned. In the k-th iteration, this algorithm performs two main steps.

IV. ADVANCED METHOD AND APPLICATION

A. Advanced OFDM chirp signals

We suppose that the length of the input data is N, and N subcarrier are required in the receiver. The three OFDM waveform signals are superposed and therefore return to a receiver. Hence, the received signal contains total 3N subcarrier components from both waveforms, so that DFT with the length N fails to demodulate the signal. Thus, the modulator(IDFT) and the demodulator(DFT) should both have 3N to satisfied the requirement..

The following is the equation of the signal:

$$\mathbf{S}_1[p] \begin{cases} \exp(-j\pi \dfrac{(p.\Delta f)^2}{K_r}) & \dfrac{p}{3} \quad is \quad \text{int}\,eger \\ 0 & else \end{cases} \tag{8}$$

Use the same method, we derive the second signal:

$$\mathbf{S}_2[p] \begin{cases} \exp(-j\pi \dfrac{((p-1).\Delta f)^2}{K_r}) & \dfrac{p-1}{3} \quad is \quad \text{int}\,eger \\ 0 & else \end{cases} \tag{9}$$

And the third signal:

$$\mathbf{S}_3[p] \begin{cases} \exp(-j\pi \dfrac{((p-2).\Delta f)^2}{K_r}) & \dfrac{p-2}{3} \quad is \quad \text{int}\,eger \\ 0 & else \end{cases} \tag{10}$$

As we all known, the frequency domain form is the Fourier Transform of the time domain. So, we can get the signals in different form

$$S[\overline{p}] = \mathscr{F}\{\text{s[n]}\} \tag{11}$$

According to the form ,s[n] denotes the discrete time samples of a complex chirp signal with length of N. Using the equation 2.4, we generate three input date sequence by the zero interleaving and shift as follows:

$$\mathbf{S}_1[p] = \{S[0], 0, 0, S[1], 0, 0, S[2]\cdots S[N-1], 0, 0\} \tag{12}$$

$$\mathbf{S}_2[p] = \{0, S[0], 0, 0, S[1], 0, 0\cdots 0, S[N-1], \quad 0\} \tag{13}$$

And

$$\mathbf{S}_3[p] = \{0, 0, \quad S[0], 0, 0, S[1], 0, \cdots 0, 0 \quad S[N-1]\tag{14}$$

Where p=0,1,2...3N-1.Both data sequences contain total 3N components, respectively.

Nest, we list the discrete form of the three waveforms in time domain :

$$s_1[n] = s[n]rect[\frac{n}{N}] + s[n-N]rect[\frac{n-N}{N}] + s[n-2N]rect[\frac{n-2N}{N}] \tag{15}$$

So the second signal should be phase shift:

$$s_2[n]=(s[n]rect[\frac{n}{N}]+s[n-N]rect[\frac{n-N}{N}]+s[n-2N]rect[\frac{n-2N}{N}])\exp(jn\frac{2\pi}{3N})$$

Simplify this equation as:

$$s_2[n]=s_2[n]\exp(jn\frac{2\pi}{3N}) \qquad (16)$$

And in the same way, we can easily get the third signal:

$$s_3[n]=s_1[n]\exp(jn\frac{4\pi}{3N}) \qquad (17)$$

From the foremore equation, we can easily get the advanced OFDM chirp signal waveform design and conduct the simulation and analysis in the next section in order to get the low AC result. And we observed that the AC properties of the quantized sequences are slightly degraded in these examples.

B. *Analysis and Simulation of the waveform*

Based on the algorithm before, we conduct the simulation of the three waveform and then get the three waveform respectively. In order better to compare the waveforms, we put the real part and imaginary part of the three waveforms together respectively.

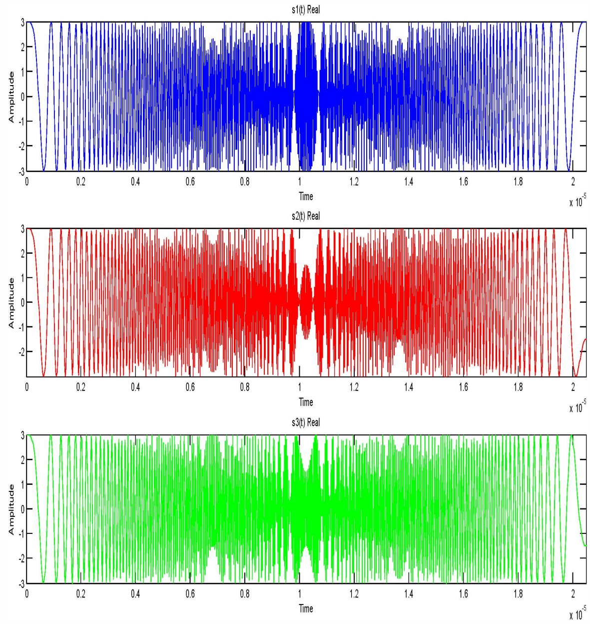

Fig.1 Real part of the three waveforms

From the equation, we know the phase shift of the three waveforms. The Fig.1 satisfied the requirement and make clear Simulation of the three waveforms.

Also, we put the imaginary part above:

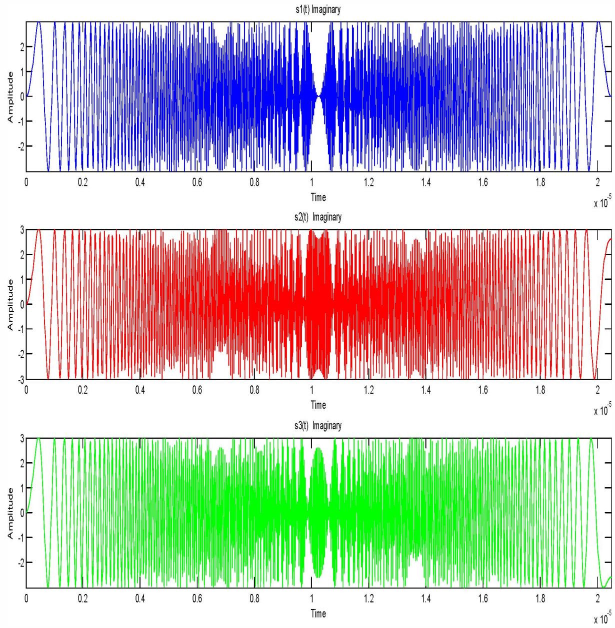

Fig.2 Imaginary part of the waveforms

V. SIMULATION AND ANALYSIS

Based on the GS algorithm, we set 64 subscarriers and the spectrum mask m is setted as 64. Then, we illustrate the OFDM sequences which achieve the minimum WSSL. The weighting vector is set as:

$$w_n=\begin{cases}1, & n\in t\\ 0, & n\in[0,L-1]\notin t\end{cases} \qquad (18)$$

We denote the specified time-lags of interest by a set t. Fig.3 illustrates the sequence which achieves the minimum WSSL. The corresponding PAPR is only slightly greater than 1.

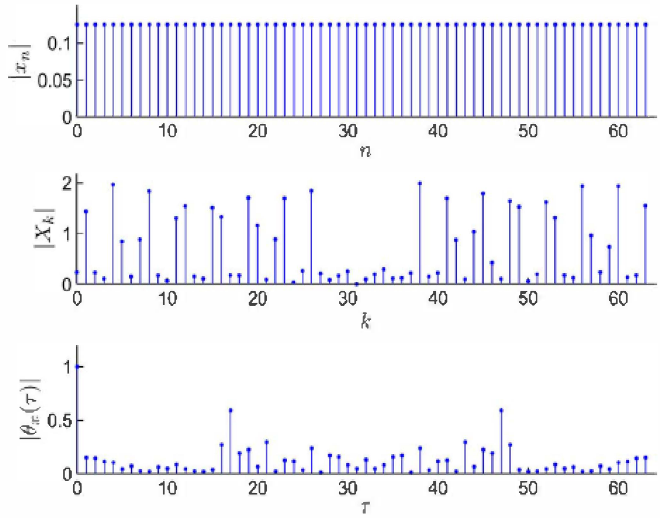

Fig.3 A sequence with minimum WSSL and AC function

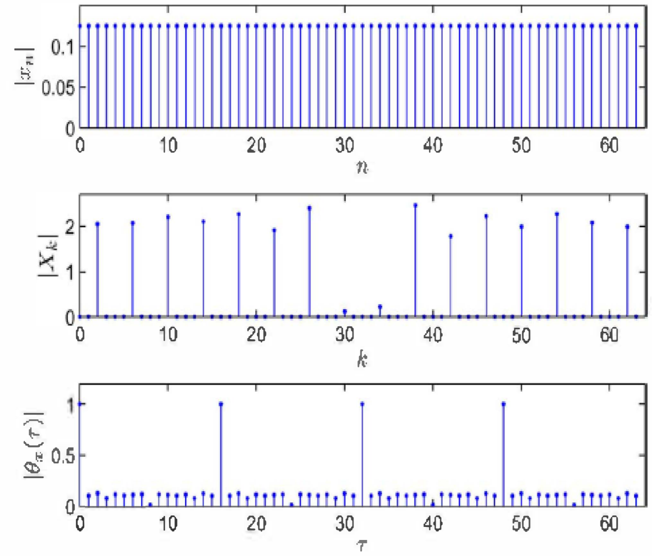

Fig.4 improved method with minimum PSL and AC function

Fig.3 and Fig.4 illustrate the performance of the same waveform respectively.TIg.3 realize the minimum WSSL 0.0914 and PSL 0.154 while the Fig.4 realize the WSSL 0.0851 and PSL 0.125. Because the WSSL and PSL decide the AC function , the improved method will demonstrate better performance.

From the two figure above, we clearly see the advance method for the novle OFDM Chirp waveform optimization. Using the GS algorithm, we succeedly decrease the PSL and WSSL and realize the better performance in the design and analysis ogf the new waveform.

VI. CONCLUSION

This paper presents approaches to the synthesis of OFDM sequences with low AC under spectral constraints through convex optimization techniques. Based on the fact that the power spectrum and the AC function of a sequence constitute a Fourier transformation pair, we showed that, given a sequence, the PSL and WSSL of its periodic AC function can be represented a s convex functions of its power spectrum.

Due to the contribution of the designing of the advanced OFDM chirp signals, we conduct the signal waveform optimize and verification. Besides, we get the low AC property perfect waveform. It's the ideal waveform and which contribute a lot to our constructed work.

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