Waveform Design for Radar-Embedded Communications

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***Abstract*—This paper considers the embedding of a covert communication signal amongst radar backscatter by means of a tag/transponder that lies within the illuminated area of the radar. Past approaches have operated on an *inter-pulse* basis whereby a communication symbol/identifier is relayed to an intended receiver by imparting a Doppler-like phase-shift to each of a successive series of incident radar pulses. In contrast, the approach proposed in this paper operates on an *intra-pulse* basis whereby the incident radar waveform at the tag/transponder is "re-modulated" into one of a set of different waveforms each representing a different communication symbol. The particular design issues for these re-modulated waveforms are discussed and three general design methods are proposed. The effectiveness of the different methods is assessed in terms of the probability of communication error as a function of the respective powers of the embedded communication signal, the masking radar backscatter, and noise. The relative "covertness" of the resulting waveforms is also discussed.**

I. Introduction

Generally speaking, the notion of radar-embedded communications can be summarized in the following manner. A given radar, which may or may not be cooperative, illuminates a given area in order to extract desired information (*e.g.* moving targets, images) from the resulting backscatter. The task of embedded communication is undertaken by a tag/transponder within the radar-illuminated area which operates upon the incident radar illumination and subsequently reflects/retransmits the altered backscatter towards some desired receiver, with the normal radar backscatter masking the presence of the embedded signal. Given that the desired receiver (which may or may not be the radar) possesses prior knowledge of the set of possible embedded signals, each of which represents a communication symbol or uniquely identifies a particular tag/transponder, the receiver can extract the communication information by coherently estimating the most likely embedded signal.

Previous approaches to radar-embedded communications [1–3] have focused upon radar illumination consisting of numerous (perhaps 100's) of pulses such as is encountered in synthetic aperture radar (SAR) applications. The method of embedding the communication signal used by these approaches is to phase modulate the incident radar illumination on an *inter-pulse* basis in the same manner that radial target motion induces a Doppler phase-shift over the sequence of pulses. A particular inter-pulse phase-shift sequence is applied over a known coherent processing interval (CPI) and either can be used as a unique identifier for a specific tag/transponder or can represent one of a set of communication symbols used by a specific tag/transponder to relay information. The general method of inter-pulse modulation for radar-embedded communications is covert due to the presence of the ambient radar backscatter yet inherently provides a low data-rate on the order of *bits-per-CPI*.

In contrast, the approach proposed here operates upon a single pulse individually (with generalization to multiple pulses being straightforward if desired). Thus the new data rate is in terms of *bits-per-pulse* instead of *bits-per-CPI*. The new approach relies on the waveform-level diversity that results from phase re-modulating the incident radar waveform into one of K different communication waveforms, each of which acts as a communication symbol representing some pre-determined bit sequence (or as a unique identifier for one of several different tags/transponders). Given knowledge of the possible embedded communication waveforms, an intended receiver recovers the embedded information by determining which of the possible communication waveforms is most likely to be present within a given radar pulse-repetition-interval (PRI). As such the intended receiver selects the highest likelihood among a set of known possibilities while an eavesdropper (which presumably has no knowledge of the particular set of possible communication waveforms) is forced to ascertain if an embedded signal even exists among the masking interference of the radar backscatter, thereby maintaining the covert nature of the embedded signal.

Embedding a communication signal into the radar backscatter in this manner raises some pertinent technical issues that must be addressed. These issues are 1) the design of communication waveforms given the incident radar waveform, 2) the appropriate method for extraction of the embedded signal at the receiver, and 3) the trade-off between covertness and the rate of communication errors at the receiver. In the remainder of the paper these issues are discussed and are utilized to develop appropriate strategies for communication waveform design based upon the particular incident radar waveform.

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II. INTRA-PULSE MODULATION

The proposed radar-embedded communication method operates by re-modulating an incident radar waveform into one of K different communication waveforms. In order to recover a given embedded communication waveform at an intended receiver, the set of communication waveforms must be both sufficiently separable from one another in order to minimize symbol estimation errors and sufficiently separable from the incident radar waveform in order to minimize the effects of interference. Of course, if the communication waveforms are too separable from the radar illumination (*e.g.* frequency shifted out of band) then the natural masking supplied by the radar backscatter cannot be exploited. As such, a logical choice is to generate communication waveforms that reside in (or very near to) the passband of the incident radar illumination yet are "temporally coded" so as to possess a relatively low cross-correlation with the radar waveform.

In general, radar waveforms fully occupy their passband thus leaving no spectral region within the passband in which to embed a communication waveform. Stated another way, this implies that insufficient design degrees-of-freedom exist with which to generate a communication waveform solely occupying the radar passband. However, it is well known that radar emissions are not strictly confined to their passband and exhibit a "bleeding" effect into the surrounding spectrum [4] as depicted in Fig. 1. This effect can be exploited to provide a covert region very near to the radar passband in which the embedded communication signal can reside. Furthermore, by expanding the bandwidth of the communication waveforms to encompass some of the surrounding spectrum of the radar illumination, sufficient design degrees-of-freedom become available with which to obtain suitable communication waveforms.

While one could inject any given signal to act as a communication waveform, a more deliberate design strategy may be employed in order to ensure that the embedded communication signal is sufficiently separable from the radar backscatter at an intended receiver while also maintaining a desired level of covertness (*i.e.* not too different from the radar illumination). Thus the design procedure must inherently be dependent upon the illuminating radar waveform. We shall consider three particular approaches to communication waveform design, all of which are based upon an eigen-decomposition of the collection of delay shifts of the incident radar waveform. Note that this approach is applicable to both "continuous" and "discrete" radar waveforms as long as the entire waveform bandwidth is taken into account (*e.g.* the extraneous bandwidth due to transitions for discrete waveforms).

Consider a radar waveform $s(t)$ having a bandwidth B that is illuminating a given area. A tag/transponder within the illuminated area can obtain a nominal discrete representation of the radar waveform by sampling the incident illumination at the Nyquist rate of B complex samples/sec. The length of the resulting nominally-sampled waveform is denoted as N. Of course, at the Nyquist sampling rate the radar waveform completely occupies the discrete spectrum. In order to accommodate the design of appropriate communication waveforms, the incident illumination is alternatively sampled at a rate of MB complex samples/sec where M is an over-sampling factor and dictates how much additional spectrum is to be utilized to embed a covert communication waveform. Thus the length of the over-sampled waveform representation is NM.

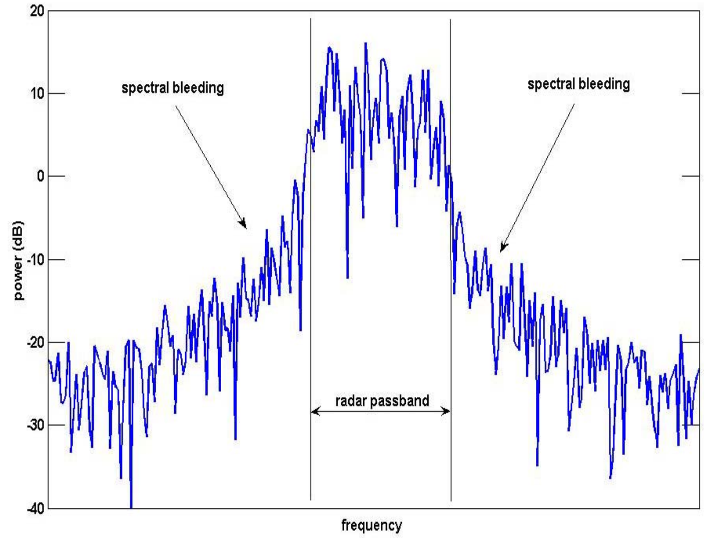

Fig. 1. Radar spectral "bleeding" effect

Let us denote the vector $\mathbf{s} = [s_0 \; s_1 \; \cdots \; s_{NM-1}]^T$ as the factor-of-M over-sampled discrete representation of the radar waveform. Based on the radar waveform $\mathbf{s}$, the design of appropriate communication waveforms is performed in the context of the ambient radar backscatter. The radar backscatter can be mathematically modeled as a convolution operation between the radar waveform and a swath of range profile illuminated by the radar. This operation can alternatively be expressed as a matrix multiplication of the form

$$\mathbf{Sx} = \begin{bmatrix} s_{NM-1} & s_{NM-2} & \cdots & s_0 & 0 & & 0 \\ 0 & s_{NM-1} & & s_1 & s_0 & & 0 \\ & & \ddots & \vdots & \vdots & \ddots & \\ 0 & 0 & & s_{NM-1} & s_{NM-2} & \cdots & s_0 \end{bmatrix} \mathbf{x} \qquad (1)$$

where $\mathbf{x}$ is a length $2NM-1$ vector comprised of range samples of the surrounding radar backscatter. Knowledge of the particular values in $\mathbf{x}$ is not required in order to design appropriate communication waveforms, though the average power of the surrounding backscatter may be utilized to control the power level of the embedded signal and thus control the probability of intercept.

The eigen-decomposition of the correlation of $\mathbf{S}$ is

$$\mathbf{SS}^H = \mathbf{V\Lambda V}^H \qquad (2)$$

where $\mathbf{V} = [\mathbf{v}_0 \ \mathbf{v}_1 \ \cdots \ \mathbf{v}_{NM-1}]$ contains the NM eigenvectors, $\mathbf{\Lambda}$ is a diagonal matrix comprised of the associated eigenvalues (assumed to be in order of increasing magnitude), and $(\bullet)^H$ is the Hermitian operator. For example, over-sampling the mathematical function for the P3 code (which is a sampled version of a continuous LFM waveform) by a factor of $M = 2$ yields the eigenvalues (dB scale) shown in Fig. 2. The nominally-sampled radar waveform length is $N = 100$. Note that the dominant space is approximately rank N and is relatively flat while the eigenvalues associated with the non-dominant space have a considerably greater spread and thereby provide a space within which to design appropriate communication waveforms. It is the eigenvectors in $\mathbf{V}$ associated with the dominant and non-dominant spaces that shall be utilized to obtain suitable covert communication waveforms.

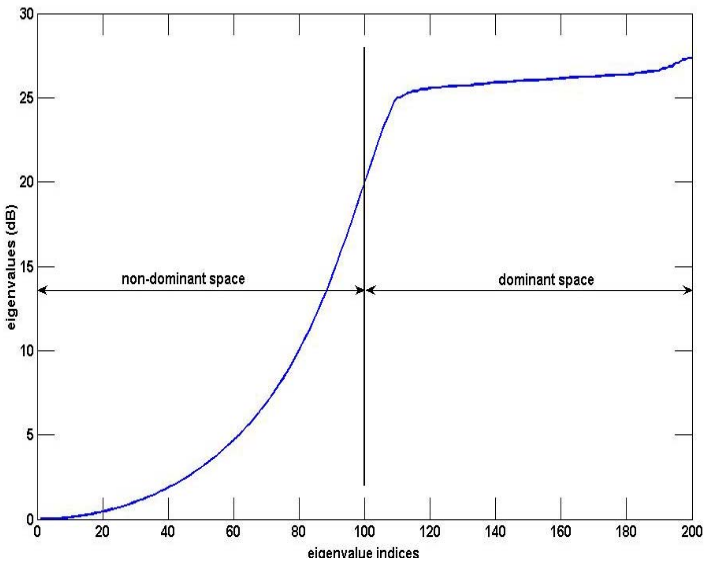

Fig. 2. Eigenvalues of correlation matrix for $M = 2$ over-sampled radar waveform

Embedding a communication waveform in the spectral region immediately surrounding the radar passband provides a natural means with which to maintain a low probability of intercept due to the masking "interference" provided by the radar. However, this interference introduces some obstacles at the intended receiver that must be addressed in order to ensure satisfactory communication performance as measured by the symbol-error-rate (SER) or bit-error-rate (BER). Over the time period within which the embedded communication signal arrives at the intended receiver, the total received discretized signal of length NM (synchronization assumed) has the form

$$\mathbf{r} = \mathbf{c}_k + \mathbf{S}\tilde{\mathbf{x}} + \mathbf{v} \tag{3}$$

where $\mathbf{c}_k$ for $k = 1, 2, \cdots, K$ is the k^{th} communication waveform, $\tilde{\mathbf{x}}$ is some set of $2NM-1$ samples of radar backscatter which are not necessarily the same as those in $\mathbf{x}$ from (1), and $\mathbf{v}$ is a vector of NM additive noise samples. To ensure a low probability of intercept, the received power of the embedded signal $\mathbf{c}_k$ must remain sufficiently below that of the masking radar backscatter resulting from the $\mathbf{S}\tilde{\mathbf{x}}$ term. As such, because it cannot account for the presence of this significant interference term, the standard coherent filter matched to $\mathbf{c}_k$ may not provide adequate performance. However, the received signal model of (3) has parallels to CDMA multi-user detection [5], thus numerous estimators already exist that may be exploited to extract the embedded communication signal.

Due to its relative simplicity and because it does not require knowledge of relative power levels, we shall consider the decorrelator receiver [6]. Given the set of communication waveforms and the matrix comprising the shifts of the radar waveform, the $NM \times (2NM + K - 1)$ matrix $\mathbf{C}$ can be formed by appending the K communication waveforms to $\mathbf{S}$ as

$$\mathbf{C} = [\mathbf{S} \ \mathbf{c}_1 \ \cdots \ \mathbf{c}_K] \tag{4}$$

which models all the possible signal components (radar and communication) that may be present in the received signal $\mathbf{r}$. The k^{th} decorrelator-like filter can thus be obtained as

$$\mathbf{w}_k = \left(\mathbf{C}\mathbf{C}^H\right)^{-1} \mathbf{c}_k \quad \text{for} \quad k = 1, 2, \cdots, K \,. \tag{5}$$

Extraction of the embedded communication signal is then achieved by selecting the communication symbol that satisfies

$$\hat{k} = \arg\left\{ \max_k \left\{ \left| \mathbf{w}_k^H \mathbf{r} \right| \right\} \right\}. \tag{6}$$

III. Communication Waveform Design

It is assumed that the tag/transponder and the intended receiver each obtain an identical over-sampled discrete representation of the illuminating radar waveform and thus an identical set of eigenvectors $\mathbf{V}$. This assumption is justified by the fact that radar illumination is generally very high power and has undergone only one-way path loss. Based on the set of eigenvectors, three design approaches are considered.

A. *Eigenvectors-as-waveforms* (*EAW*)

The straightforward approach to communication waveform design/selection is to simply utilize a subset of the non-dominant eigenvectors directly as communication waveforms such as

$$\mathbf{c}_k = \mathbf{v}_k \quad \text{for} \quad k = 1, 2, \cdots, K \,. \tag{7}$$

In so doing, to minimize the effects of the radar interference on communication symbol estimation at the receiver, the eigenvectors to be selected should be associated with eigenvalues that are relatively small (in comparison to those in the dominant space) and of commensurate magnitude. Having the associated eigenvalues possess similar magnitude acts to minimize symbol estimation errors in (6) which may be induced by disparate levels of correlation with the ambient radar backscatter.

If one selects the eigenvectors corresponding to the K smallest eigenvalues, then of the three design approaches to be discussed, this approach tends to result in the lowest communication symbol-error-rate. However, this approach tends to be problematic in terms of maintaining a low probability of intercept because an eavesdropper presumably would possess the same access to the transmitted radar waveform and could therefore obtain these communication waveforms through eigen-decomposition as well. Thus covertness is only maintained through the eavesdropper's lack of knowledge regarding the exact dimensionality of the communication waveform.

B. *Weighted-Combining* (*WC*)

As a trade-off to generate communication waveforms that are more covert at the cost of a slightly higher symbol-error-rate, we consider the weighted combining of multiple non-dominant eigenvectors. Denoting a set of L non-dominant eigenvectors as $\tilde{\mathbf{V}}_{\mathrm{ND}} = [\mathbf{v}_0 \ \mathbf{v}_1 \ \cdots \ \mathbf{v}_{L-1}]$, a set of K communication waveforms can be formed by combining the L non-dominant eigenvectors as

$$\mathbf{c}_k = \tilde{\mathbf{V}}_{\mathrm{ND}}\, \mathbf{b}_k \quad \text{for} \quad k = 1, 2, \cdots, K \tag{8}$$

where each $\mathbf{b}_k$ is a different $L \times 1$ weight vector known only to the tag/transponder and the intended receiver.

Because the set of L non-dominant eigenvectors can correspond to a larger spread of eigenvalue magnitudes, the resulting communication waveforms will generally be more correlated to the radar waveform than in the previous approach and thus somewhat higher communication errors will ensue. However, by selecting the weight vectors $\mathbf{b}_k$ to contain relatively constant modulus terms, then on average the set of K communication waveforms will be equally correlated to the ambient radar backscatter. Thus the symbol-error-rate is manageable at the tag/transponder by controlling the power of the embedded signal relative to the radar interference. Also, the increased correlation to the radar waveform provides an implicit benefit regarding low probability of intercept as the embedded communication waveform is now more difficult to discern from the ambient radar backscatter.

C. *Dominant-Projection* (*DP*)

Finally, instead of concentrating upon the non-dominant space in order to generate the communication waveforms, we shall consider projecting away from the dominant space. This approach becomes particularly useful in situations where relative sampling offsets of the incident radar waveform may result in a slightly different representation of $\mathbf{s}$ for the tag/transponder and the intended receiver thus producing some variations in the set of eigenvectors. The projection approach, however, considers the dominant space as a whole and thus is less susceptible to differences in individual eigenvectors.

The projection approach to communication waveform design utilizes a set of K "random" $NM \times 1$ vectors denoted as $\mathbf{d}_k$ that are known to both the tag/transponder and the intended receiver. For the first L eigenvectors in $\mathbf{V}$ corresponding to the non-dominant space, the first communication waveform is found as

$$\mathbf{c}_1 = \left(\mathbf{I} - \tilde{\mathbf{V}}_{\mathrm{D},0}\, \tilde{\mathbf{V}}_{\mathrm{D},0}^H\right) \mathbf{d}_1 \tag{9}$$

where $\tilde{\mathbf{V}}_{\mathrm{D},0} = [\mathbf{v}_L \ \mathbf{v}_{L+1} \ \cdots \ \mathbf{v}_{NM-1}]$ is the $NM - L$ eigenvectors comprising the dominant space of $\mathbf{V}$. To obtain the second communication waveform, the matrix

$$\mathbf{S}_{\mathrm{P},1} = [\mathbf{S} \ \mathbf{c}_1] \tag{10}$$

is formed and used to obtain a new eigen-decomposition as

$$\mathbf{S}_{\mathrm{P},1}\, \mathbf{S}_{\mathrm{P},1}^H = \mathbf{V}_{\mathrm{P},1}\, \mathbf{\Lambda}_{\mathrm{P},1}\, \mathbf{V}_{\mathrm{P},1}^H \ . \tag{11}$$

The second communication waveform is then found as

$$\mathbf{c}_2 = \left(\mathbf{I} - \tilde{\mathbf{V}}_{\mathrm{D},1}\, \tilde{\mathbf{V}}_{\mathrm{D},1}^H\right) \mathbf{d}_2 \tag{12}$$

where $\tilde{\mathbf{V}}_{\mathrm{D},1} = [\mathbf{v}_{L-1} \ \mathbf{v}_L \ \cdots \ \mathbf{v}_{NM-1}]$ is the $NM - L + 1$ eigenvectors comprising the dominant space of $\mathbf{V}_{\mathrm{P},1}$. In general, the k^{th} communication waveform is obtained by determining the eigen-decomposition of the correlation matrix of

$$\mathbf{S}_{\mathrm{P},k} = [\mathbf{S} \ \mathbf{c}_1 \ \cdots \ \mathbf{c}_k] \tag{13}$$

and then projected out the dominant $NM - L + k - 1$ eigenvectors from the vector $\mathbf{d}_k$.

IV. Simulation Results

To ascertain the relative performance between the three communication waveform design approaches we consider a length $N = 100$ nominally-sampled linear frequency modulated (LFM) radar waveform that is over-sampled by a factor $M = 2$. The LFM waveform is modeled using the mathematical function for the P3 code [7] (Nyquist-sampled version of LFM). The number of communication symbols is set to $K = 4$ so that the embedded data rate is 2 bits-per-pulse. The ambient radar backscatter and noise from (3) are modeled as white Gaussian. The embedded communication signal, average radar backscatter (the interference), and noise are each scaled to achieve the levels of signal-to-interference ratio (SIR) and SNR for which the symbol-error-rate (SER) is to be determined. The eigenvectors-as-waveforms (EAW) approach uses the 1st four eigenvectors as these correspond to the smallest eigenvalues. The other two approaches set $L = 90$ as the non-dominant space for the purpose of waveform design. The random vectors for weighted-combining (WC) and dominant-projection (DP) are constant-modulus random-phase and complex Gaussian, respectively.

Figures 3, 4, and 5 illustrate the SER performance for the EAW, WC, and DP approaches, respectively as SNR varies for SIR values of –30 dB, –35 dB, and –40 dB. For the EAW approach, the decorrelator and matched filter performance is nearly identical because the waveforms correspond to the lowest eigenvalues and thus possess very little correlation with the radar interference. In contrast, the other two approaches utilize a considerably larger space for waveform design and as a result the benefit of the decorrelator over the matched filter is evident.

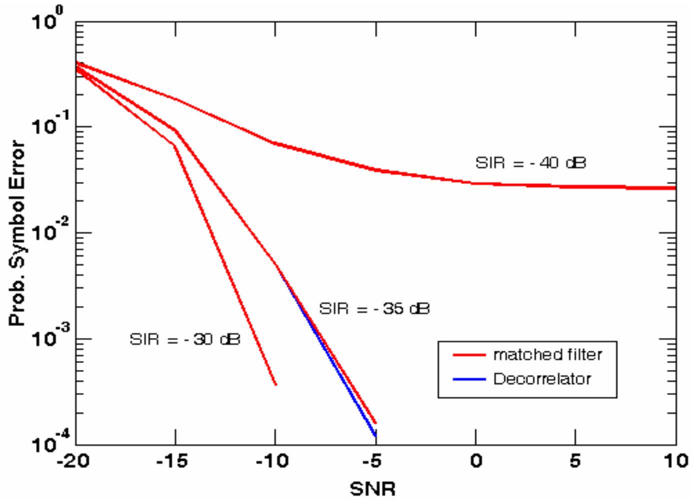

Fig. 3. Symbol-error-rate for eigenvectors-as-waveforms

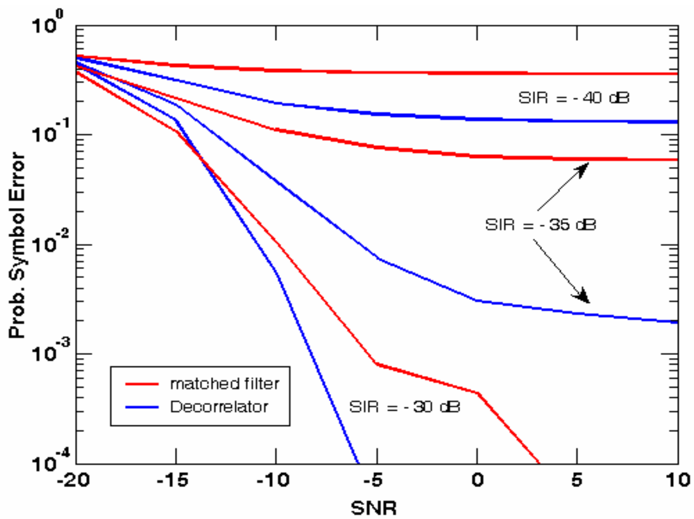

Fig. 4. Symbol-error-rate for weighted-combining

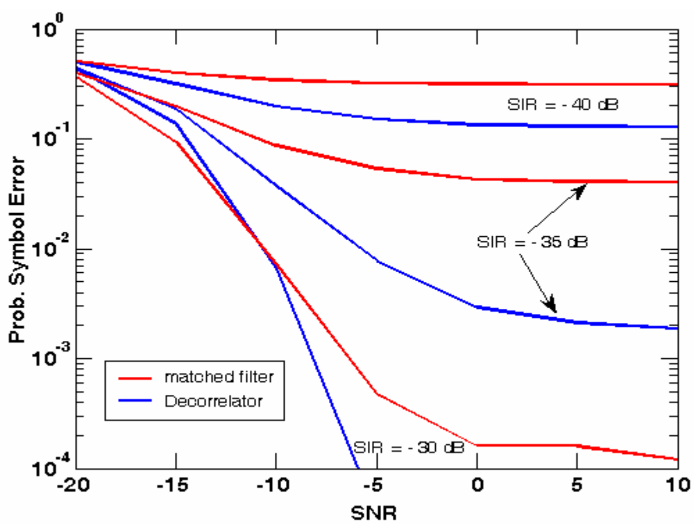

Fig. 5. Symbol-error-rate for dominant-space projection

For both receivers, it is observed that the performance of the WC and DP approaches are quite similar, which is to be expected as they both utilize roughly the same space for communication waveform design. The EAW approach is found to yield a marked SER performance gain over the other two waveform design approaches when the matched filter is employed, though the use of the decorrelator greater reduces this performance gap as long as the SNR is not too low (the decorrelator may suffer from noise enhancement effects at too low an SNR [5,6]). Note that it has been observed (not shown here) that given knowledge of the over-sampled dimension *MN*, the presence of an embedded communication waveform is significantly more difficult to discern when the WC and DP approaches are employed relative to EAW and thus yield waveforms with considerably lower probability of intercept.

CONCLUSIONS

Design strategies and issues for *intra-pulse* radar-embedded communications have been considered which result in a substantial data-rate increase over previous *inter-pulse* schemes. Three approaches for communication waveform design have been proposed which provide different degrees of communication error performance and probability of intercept. Future work in this regard will address practical implementation issues as well as means to expand the waveform design space in order to achieve even higher data-rates.

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