Research of Constructing Method of Complete Complementary Sequence in Integrated Radar and Communication

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Abstract—**Integration of radar and communication on the electronic war platform is an effective method to reduce their volume, electromagnetic interference. Most attention are drawn for the integrated radar and communication system based on signal-sharing. Following the principle of spread spectrum techniques, the paper proposes a new expansion method of complete complementary code family which not only expands the length of code ， and also facilitates the separation of multiple uses for integrated system. Subsequently, a dual-use architecture is proposed for both radar and communication. Finally, computer simulation shows that the expansion method has the characteristic of anti-echo eclipse.**

Keywords- integration of radar and communication; complete complementary code; correlation function; DFT; serial matrix expansion method

I. INTRODUCTION

The integration of multiple functions such as radar tasks and communication applications has attracted substantial interest in recent years and sparked a number of research initiatives. As radar and communication system both are RF systems, a combination of both may be possible. An important research area essential for the development of such integrated systems include the design of signals and sequence sets that can cope with the often stringent demands posed on delay resolution, doppler tolerance, and multi-access interference suppression. The design of intelligent waveforms[1][2], that are suitable for simultaneously performing both data transmission and radar sensing, has proposed at some literature. A typical waveform is the spread spectrum signal which a well-known candidate for integrated radar and communication, the work reported in [3][4]investigated the integrated system based on pseudo random(PN) sequence. One of the main drawbacks of m-sequences don't have ideal correlation to simultaneously satify radar and communication, moreover large sets of m-sequences as needed with mulitiple-access techniques have typically rather poor crosscorrelation properties. The polyphase sequences[5][6] are optimized only with respect to the doppler tolerence and muliple-access techniques. Unfortunately, Welth[7] found that there is no such ideal access code set that the cross-correlation of an access code set is zero for any relative time shift and its auto-correlation is zero for any relative time shift except for the origin. However, complete complementary code[8][9] is a collection of sequence sets, called a sequence family in this paper, with the property that the autocorrelation sum in each sequence set is zero except for zero shift and the cross-correlation sum between each pair of distinct sequence sets is zero for all shifts.

The rest of this paper is as follows. In section Ⅱ, after the definition complementary codes and complete complementary codes, and we shows that an (N,N,N)- complete complementary codes is obtained by DFT matrix. In section Ⅲ, the paper proposes the expansion method by serial matrix technique. In section Ⅳ, we will demonstrate the system construction and signal design of integrated radar and communication. Finally, section Ⅴ and Ⅵ simulates and concludes the paper.

II. COMPLETE COMPLEMENTARY CODES

A. Complementary cods

Let us consider a pair of equally long sequences $X_n = \{x(1), x(2), \ldots, x(n)\}$ and $Y_n = \{y(1), y(2), \ldots, y(n)\}$, where n is the length of two sequences. If the sum of their auto-correlation satisfies the below relationship:

$$R_{x,x}(\tau)+R_{y,y}(\tau)=\sum_{i=0}^{n-1}(x_i x_{i+\tau}^{*}+y_i y_{i+\tau}^{*})=\begin{cases}0 & ,\tau>0\\ 2n, for\,\tau=0\end{cases} \quad (1)$$

Here, τ denotes the number of time shifts and * denotes a complex conjugate. $R_{x,x}(\tau)$ and $R_{y,y}(\tau)$ respectively denotes the auto-correlation of sequence X_n and Y_n. Then the two sequences are considered to be complementary codes. In the words, complementary codes are codes in which the sum of the auto-correlation function between the n codes is zero for all time shifts other than the $\tau=0$. Complete complementary codes means that the cross-correlation function between two pair of complementary codes is completely zero. Let $\{X_n, Y_n\}$ and $\{X_n^{'}, Y_n^{'}\}$ respectively denote two pairs of complementary codes of length n. If the sum of the cross-correlation functions between the codes is zeros for all time shifts, the two pairs of complementary codes $\{X_n, Y_n\}$ and $\{X_n^{'}, Y_n^{'}\}$ are considered to be complete complementary codes.

$$R_{XX'} + R_{YY'} = \sum_{i=0}^{n-1} (x_i x'^{*}_{i+\tau} + y_i y'^{*}_{i+\tau}) = 0 \quad for\,\tau = 0, \ldots, n-1 \tag{2}$$

For extending the definition of complete complementary codes, this paper we consider X be a family of M sequence sets each of which includes N indexed sequences of length L as following

$$X = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{M-1} \end{bmatrix} = \begin{bmatrix} x_{0,0} & x_{0,1} & \cdots & x_{0,N-1} \\ x_{1,0} & x_{1,1} & \cdots & x_{1,N-1} \\ \vdots & \vdots & \cdots & \vdots \\ x_{M-1,0} & x_{M-1,1} & \cdots & x_{M-1,N-1} \end{bmatrix} \tag{3}$$

Furthermore $x_{m,n} = [x_{m,n}(0), x_{m,n}(1), \cdots, x_{m,n}(L-1)]$. X_m and X are respectively said to be an (N,L) sequence set and (M,N, L) sequence family. We define $X_i (i = 0,1,\cdots,N-1)$ as complementary code, where M rows represent M auto-complementary sequences sets. Any two of which are complete complementary codes. The sum of the correlation functions satisfy the property such that, for any j, k=0,1,$\cdots n$-1,

$$\sum_{j=0}^{n-1} R_{x_{j,k} y_{j,k}}(\tau) = \begin{cases} \mathrm{A} & if\ x = y \quad and \quad \tau = 0 \\ 0 & otherwise \end{cases} \tag{4}$$

Where $R_{x_j y_j}(\tau)$ is the correlation function for the finite length sequence X_j and Y_j. A denotes a constant independent of the index i and k.

B. The construction method of complete complementary codes

Let us consider collection of equally long sequences which are constructed by DFT(discrete fourier transform) matrix method. For an arbitrary integer $N \ge 2$, the Nth DFT matrix is defined by

$$F_N^0 = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{N-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(N-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & w^{N-1} & w^{2(N-1)} & \cdots & w^{(N-1)^2} \end{bmatrix} \tag{5}$$

Where $w = \exp(j\frac{2\pi}{N})$ and j means the imaginary unit. For each $k (0 \le k \le N-1)$ denote by F_N^k, the matrix obtained by repeating the cyclic shift with respect to the rows for k times. Then $F_N = \{F_N^k : k = 0,1,\cdots,N-1\}$ is an (N,N,N)-complete complementary codes. Considering the example of N=4, by definition $F_4^0, F_4^1, F_4^2, F_4^3$ can be written as follows:

$$F_4^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w^9 \end{bmatrix}, \quad F_4^1 = \begin{bmatrix} 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w^9 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$F_4^2 = \begin{bmatrix} 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w^9 \\ 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \end{bmatrix} \quad F_4^3 = \begin{bmatrix} 1 & w^3 & w^6 & w^9 \\ 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w^6 \end{bmatrix} \tag{6}$$

Where $w = \exp(j\frac{2\pi}{4})$. The individual auto-correlation function of complementary code F_4^0 , F_4^1 , F_4^2 , F_4^3 set are equal to zero except for the zero shift. The sum of the cross-correlation function of any two different sets in the (4,4,4) sequence family F4 are equal to zero at every shift.

III. THE EXPANSION METHOD

In practice, there are demands for longer code lengths in complementary code by DFT matrix method. However, If the matrix dimension N is increased in the preceding section 2.2, the complication of DFT method will be increased. The paper propose a serial matrix expansion method based on complete complementary code. The idiographic expansion method can be easily showed by:

$$\begin{bmatrix} A_0 & A_1 & A_2 & \cdots & A_{n-1} \\ B_0 & B_1 & B_2 & \cdots & B_{n-1} \end{bmatrix}$$
$$\Rightarrow \begin{cases} \begin{bmatrix} A_0B_0 & A_1B_1 & A_2B_2 & \cdots & A_{n-1}B_{n-1} \\ A_0\overline{B_0} & A_1\overline{B_1} & A_2\overline{B_2} & \cdots & A_{n-1}\overline{B_{n-1}} \end{bmatrix} \\ \begin{bmatrix} B_0A_0 & B_1A_1 & B_2A_2 & \cdots & B_{n-1}A_{n-1} \\ B_0\overline{A_0} & B_1\overline{A_1} & B_2\overline{A_2} & \cdots & B_{n-1}\overline{A_{n-1}} \end{bmatrix} \end{cases}$$
$$\Rightarrow \begin{cases} \begin{cases} \begin{bmatrix} A_0B_0A_0\overline{B_0} & A_1B_1A_1\overline{B_1} & A_2B_2A_2\overline{B_2} & \cdots & A_{n-1}B_{n-1}A_{n-1}\overline{B_{n-1}} \\ A_0B_0\overline{A_0}B_0 & A_1B_1\overline{A_1}B_1 & A_2B_2\overline{A_2}B_2 & \cdots & A_{n-1}B_{n-1}\overline{A_{n-1}}B_{n-1} \end{bmatrix} \\ \begin{bmatrix} A_0\overline{B_0}A_0B_0 & A_1\overline{B_1}A_1B_1 & A_2\overline{B_2}A_2B_2 & \cdots & A_{n-1}\overline{B_{n-1}}A_{n-1}B_{n-1} \\ A_0\overline{B_0}\,\overline{A_0}\,\overline{B_0} & A_1\overline{B_1}\,\overline{A_1}\,\overline{B_1} & A_2\overline{B_2}\,\overline{A_2}\,\overline{B_2} & \cdots & A_{n-1}\overline{B_{n-1}}\,\overline{A_{n-1}}\,\overline{B_{n-1}} \end{bmatrix} \end{cases} \\ \begin{cases} \begin{bmatrix} B_0A_0B_0\overline{A_0} & B_1A_1B_1\overline{A_1} & B_2A_2B_2\overline{A_2} & \cdots & B_{n-1}A_{n-1}B_{n-1}\overline{A_{n-1}} \\ B_0A_0\overline{B_0}A_0 & B_1A_1\overline{B_1}A_1 & B_2A_2\overline{B_2}A_2 & \cdots & B_{n-1}A_{n-1}\overline{B_{n-1}}A_{n-1} \end{bmatrix} \\ \begin{bmatrix} B_0\overline{A_0}B_0A_0 & B_1\overline{A_1}B_1A_1 & B_2\overline{A_2}B_2A_2 & \cdots & B_{n-1}\overline{A_{n-1}}B_{n-1}A_{n-1} \\ B_0\overline{A_0}\,\overline{B_0}\,\overline{A_0} & B_1\overline{A_1}\,\overline{B_1}\,\overline{A_1} & B_2\overline{A_2}\,\overline{B_2}\,\overline{A_2} & \cdots & B_{n-1}\overline{A_{n-1}}\,\overline{B_{n-1}}\,\overline{A_{n-1}} \end{bmatrix} \end{cases} \end{cases} \tag{7}$$

Furthermore, this method can be applied repeatedly by supposing that $A_0B_0A_0\overline{B_0}$, $A_1B_1A_1\overline{B_1}$, $A_2B_2A_2\overline{B_2}$, $A_{n-1}B_{n-1}A_{n-1}\overline{B_{n-1}}$, $A_0B_0\overline{A_0}B_0$, $A_1B_1\overline{A_1}B_1$, $A_2B_2\overline{A_2}B_2$, $A_{n-1}B_{n-1}\overline{A_{n-1}}B_{n-1}$ in topmost matrix of second brace respectively represent A_0 , A_1 , A_2 , A_{n-1} , B_0 , B_1 , B_2 , B_{n-1} in matrix of formula (7). The code length will be about $N \cdot 2^{r+1}$ (r is integer, N is the length of individual sequence in set.) if the expansion method is applied r times. According to the formula(6) and (7), sequence set F_4^i denote $\left[F_4^i(0), F_4^i(1), F_4^i(2), F_4^i(3)\right]$ (i=0,1,2,3), two pairs of complete complementary code $\{F_4^0, F_4^1\}$ and $\{F_4^2, F_4^3\}$ can construct respectively longer expansion codes. The concrete method shows as follows:

$$\begin{Bmatrix} \begin{bmatrix} F_4^0(0) & F_4^0(1) & F_4^0(2) & F_4^0(3) \\ F_4^1(0) & F_4^1(1) & F_4^1(2) & F_4^1(3) \end{bmatrix} \\ \begin{bmatrix} F_4^2(0) & F_4^2(1) & F_4^2(2) & F_4^2(3) \\ F_4^3(0) & F_4^3(1) & F_4^3(2) & F_4^3(3) \end{bmatrix} \end{Bmatrix} \tag{8}$$

$$\Rightarrow \begin{cases} F1 = \begin{bmatrix} F_4^0(0)F_4^1(0) & F_4^0(1)F_4^1(1) & F_4^0(2)F_4^1(2) & F_4^0(3)F_4^1(3) \\ F_4^0(0)\overline{F_4^1(0)} & F_4^0(1)\overline{F_4^1(1)} & F_4^0(2)\overline{F_4^1(2)} & F_4^0(3)\overline{F_4^1(3)} \end{bmatrix} \\ F2 = \begin{bmatrix} F_4^2(0)F_4^3(0) & F_4^2(1)F_4^3(1) & F_4^2(2)F_4^3(2) & F_4^2(3)F_4^3(3) \\ F_4^2(0)\overline{F_4^3(0)} & F_4^2(1)\overline{F_4^3(1)} & F_4^2(2)\overline{F_4^3(2)} & F_4^2(3)\overline{F_4^3(3)} \end{bmatrix} \end{cases}$$

Here, $\overline{d}$ denotes the negative of d ($\overline{d}$ represent the complex conjugate of d when d is complex). The following shows an example of applying this serial matrix expansion method once. Specifically, two pairs of $\{F_4^0, F_4^1\}$ and $\{F_4^2, F_4^3\}$ for a code length of 4 are expanded to a code length of 8 and show as follows:

$$\begin{aligned} &F_4^0(0)F_4^0(1)F_4^0(2)F_4^0(3) = [(1,1,1,1), \quad (1,i,-1,-i), \quad (1,-1,1,-1), \quad (1,-i,-1,i)]; \\ &F_4^1(0)F_4^1(1)F_4^1(2)F_4^1(3) = [(1,i,-1,-i), \quad (1,-1,1,-1), \quad (1,-i,-1,i), \quad (1,1,1,1)]; \\ &F_4^2(0)F_4^2(1)F_4^2(2)F_4^2(3) = [(1,-1,1,-1), \quad (1,-i,-1,i), \quad (1,1,1,1), \quad (1,i,-1,-i)]; \\ &F_4^3(0)F_4^3(1)F_4^3(2)F_4^3(3) = [(1,-i,-1,i), \quad (1,1,1,1), \quad (1,i,-1,-i), \quad (1,-1,1,-1)] \end{aligned} \tag{9}$$

Here, we will confirm the property of complete complementary code in formula(8) which obtain longer versions in the following section by applying the expansion method. An expanded code family of length 8 is showed as follows:

$$F1 = \begin{bmatrix} (1,1,1,1,1,i,-1,-i),(1,i,-1,-i,1,-1,1,-1), & (1,-1,1,-1,1,-i,-1,i), & (1,-i,-1,i,1,1,1,1) \\ (1,1,1,1,-1,-i,1,i),(1,i,-1,-i,-1,1,-1,1), & (1,-1,1,-1,-1,i,1,-i),(1,-i,-1,i,-1,-1,-1,-1) \end{bmatrix} \tag{10}$$

$$F2 = \begin{bmatrix} (1,-1,1,-1,1,-i,-1,i),(1,-i,-1,i,1,1,1,1), & (1,1,1,1,1,i,-1,-i),(1,i,-1,-i,1,-1,1,-1) \\ (1,-1,1,-1,-1,i,1,-i),(1,-i,-1,i,-1,-1,-1,-1),(1,1,1,1,-1,-i,1,i), & (1,i,-1,-i,-1,1,-1,1) \end{bmatrix} \tag{11}$$

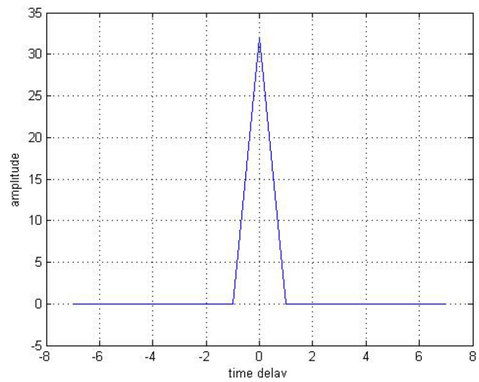

Fig1. auto-correlation of F1

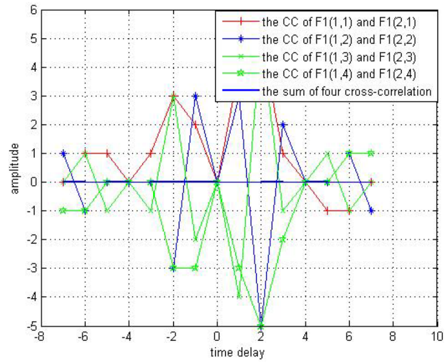

Fig2. cross-correlation of F1

Fig1 show auto-correlation of F1. These have a sidelobe level of zero. The figure is ideal Kronecker delta function. In the figure 2, the cross-correlation functions in matrix (10) obtained as follows:

$$C(\tau) = [\underbrace{0,0,0,\cdots\cdots,0}_{15}] \tag{12}$$

According to Fig1 and Fig2, constructed new sequences family F_1 and F_2 are complete complementary codes.

IV. System Construction

The construction of Kth user based on spread spectrum techniques is shown in fig3. $[A_0, A_1, A_2, A_3]$ and $[B_0, B_1, B_2, B_3]$ denote two pairs of complete complementary code which are expanded by serial matrix expansion method. The transmitting signal consists of four carrier phase-coded pulses with the different frequencies F_0+f_1, F_0+f_2, F_0+f_3, F_0+f_4. The Kth's user data are input and are respectively multiplied with two pair of complete complementary code $[A_0, A_1, A_2, A_3]$ and $[B_0, B_1, B_2, B_3]$. The frequency different between two adjacent carrier is inverse number of the chip width. The demodulated

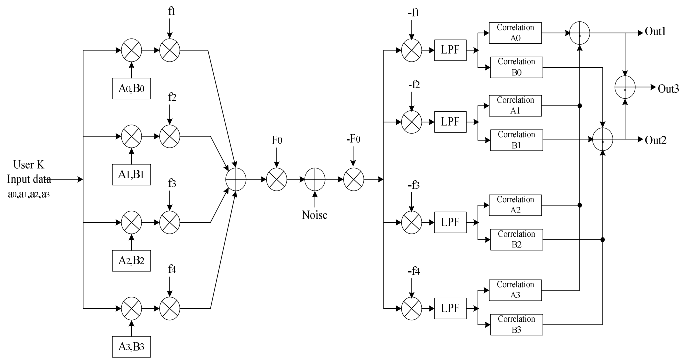

Fig3: The user Kth's system diagram of transmitter and receiver

codes $[A_0, A_1, A_2, A_3]$ or $[B_0, B_1, B_2, B_3]$ are followed by the correlation processing. In formula (7), the bottommost four matrixes in brace can be applied to four users as spread codes. Therefore, if the expansion method is applied n times, the number of user will be about 2n in system. The correlating results of codes A_0, A_1, A_2, A_3 are output at Out1. The correlating results of codes B_0, B_1, B_2, B_3 are output at Out2.

Out3 is the sum of Out1 and Out2. The expression of Out1 and Out2 is $[0,\ldots,0,2N,0,\ldots,0]$ or $[0,\ldots,0,-2N,0,\ldots,0]$. So the delta function and inverse delta function indicate 1, -1 for communication. The output signal of Out3 means received pulses for the radar.

V. SIMULATION

In order to improving sufficiently high spreding factor and the ratio of peak to sidelobe, the best performance is obtained with long sequence. Therefore, echo-elipse becomes a serious problem for short-range targe. In the case of echo-eclipse, conventional complementary signal has very high sidelobes for the reason of complementary feature is destructed. But when the paper adopt serial metrix expansion method to construct complete complementary code, the coded signal will get whose incomplete codes also have perfect correlation characteristic. We first construct an (2,4,32)-complete complementary codes by using expansion method in section Ⅲ. Fig3 show the auto-correlation when received signal is a lack of four code elements. Fig4 show the rate of peak to sidelobe in different number of absent codes. In the figure, when the length of received signal have the half of transmitted signal, the amplitude of mainlobe have max value and the level of sidelobe is equal to zeros.

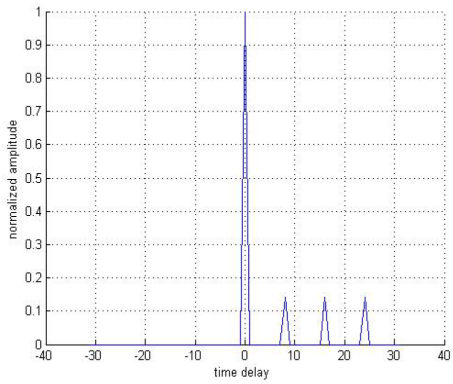

Fig3 the auto-correlation of lack of 4 code elements in received signal

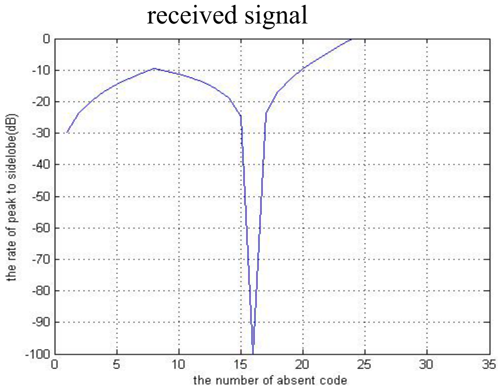

Fig4. the rate of peak to sidelobe in different number of absent codes

VI. CONCLUDE

Tremendous efforts have been devoted to develop the spread frequency technology for integrated system over the past years. In the paper, we have considered several properties of complete complementary code, and present a serial matrix expansion method to construct a new sequence. The property of the correlation can not only avoid mutually jam between different users, but alse increase the capacity of users. Finally, simulation show that the sequence which is constructed by expansion method in section Ⅲ have a excellent characteristic of anti-echo eclipse.

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