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A Hybrid Companding Transform Technique for PAPR Reduction of OFDM Signals

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Abstract— **Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technique for wireless communication applications. However, an OFDM signal has a large Peak-to-Average Power Ratio (PAPR), which can result in significant distortion when passed through a nonlinear device, such as a transmitter power amplifier. Number of techniques has been proposed for reducing the PAPR in OFDM systems. Among them Companding is a well-known technique for the PAPR reduction of OFDM signals. Recently, a piecewise linear companding technique is proposed to extenuate companding distortion. In this paper, a joint piecewise linear companding technique and Discrete Cosine transform (DCT) method is proposed to reduce peak-to-average of OFDM signal. Simulation results show that the proposed technique may obtain significant PAPR reduction while maintaining good performance in the Bit Error Rate (BER) and Power Spectral Density (PSD) compared to piecewise linear companding technique.**

Keywords- OFDM; PAPR; Discrete cosine transform; Companding Distortion;

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been recently seen rising popularity in wireless applications. Keeping in view of the less complexity in implementing the OFDM using Fast Fourier Transform (FFT), high spectral efficiency and vigour to frequency selective fading channel OFDM has gained reputation in a number of applications such as Digital Video and Audio Broadcasting (DVB/DAB), Wireless Local Area Networks (WLAN) and Wireless Metropolitan Area Networks (WMAN). Despite many advantages, OFDM affect from high fluctuation of the transmitted envelope signal. To characterize the envelope fluctuations of OFDM signals Peak to Average Power Ratio (PAPR) mostly used by relating peak value and mean value power. When non linear power amplifier (PA) is used, then high PAPR causes serious degradation in performance.

Several techniques are proposed to reduce PAPR in OFDM signals [1, 2]. Selective level mapping (SLM) partial transmit sequence (PTS), Hadamard transform, and companding techniques are proposed in [3-9, 11]. In these techniques, companding techniques gain more attention due to their flexibility and simplicity. The concept of companding technique was first introduced in [5], which uses the μ-law companding technique for the reduction PAPR by increasing the average power of the signal while keeping the peak power remains unchanged. Later on, exponential companding (EC) was developed in [6], which can improve PAPR reduction by altering the distribution of OFDM signals while keeping average power constant. In recent times, a nonlinear companding technique is proposed [7] with a linear function which changes the Gaussian distributed signal into distribution form. The nonlinear companding technique reduces the PAPR with the expense of high computational complexity. The two piecewise companding (TPWC) technique proposed in [8] compress large signal amplitudes and expand small ones by using two different piecewise functions. Thus companding techniques decrease PAPR by producing companding distortion. Recently, a piecewise linear companding technique was investigated in [9] to decrease the companding distortion alters the signals linearly with amplitudes close to the given companded peak amplitude and chopping the signals with amplitudes over a given companded amplitude. Park *et al.* [11] proposes Hadamard transform technique may reduce the occurrence of the high peaks when compared to the original OFDM system. Discrete cosine transform may reduce PAPR of OFDM signal while the error probability of system is not increased.

In this paper, the authors propose a hybrid companding transform technique for reducing PAPR by combining a piecewise linear companding technique and Discrete Cosine Transform (DCT). The DCT operation can be applied to reduce the PAPR of OFDM signals before OFDM signal is modulated using an IFFT (Inverse Fast Fourier Transform). This further reduces the PAPR of the OFDM signal the piecewise linear companding technique is applied after the IFFT operation. This technique will be compared with the original system and piecewise linear companding technique for reduction of PAPR.

This paper starts with the introduction of the topic in section I. Section II presents a typical OFDM system with a formulation of PAPR problem. Piecewise linear Companding technique and Discrete Cosine Transform are introduced in section III and section IV. Section V provides signal processing steps to implement a PAPR

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reduction technique by combing piecewise linear companding transform and DCT. Simulation results are presented in section VI and conclusion is given in VII.

II. FORMULATION OF PAPR PROBLEM

Generally, N independent data symbols are modulated by using baseband modulation techniques like phase-shift keying (PSK) and quadrature amplitude modulation (QAM). OFDM signal is nothing but sum of those N independent modulated data symbols. In discrete-time domain, since the Nyquist rate samples might not represent the peaks of the continuous-time signal, so for getting better approximation of PAPR the researchers do oversampling of the OFDM signal. The oversampled time-domain OFDM symbols $X = [x_0, x_1, ..., x_{LN-1}]^T$ can be calculated as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . e^{j2\pi \frac{kn}{NL}}, \quad 0 \le n \le NL-1, (1)$$

Where $n = 0,1,..., LN-1$ time is index and L is the oversampling factor. Usually, $L \geq 4$ is used to accurately describe the PAPR of the continuous-time signal. For getting oversampling of OFDM signal by inserting $(L-1)N$ zeros in the middle of N length vector, i.e.

$$\hat{X}_e = \left[X_0, X_1, ..., X_{\frac{N}{2}-1}, \underbrace{0,....,0}_{N(L-1)}, X_{\frac{N}{2}}, ..., X_{N-1} \right]^T. \quad (2)$$

It is clear that $x = IFFT_{LN}\{X_e\}$. For a large N (e.g. $N \geq 64$), the real and imaginary parts are approximated as independent and identically distributed Gaussian random variables with zero mean and a variance σ_x^2. Based on this approximation, the signal amplitude $|x_n|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x) = \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}} \qquad x \geq 0. \qquad (3)$$

The cumulative density function (CDF) of $|x_n|$ is therefore

$$F_{|x_n|}(x) = \Pr ob\{|x_n| \le x\} = \int_0^x \frac{2y}{\sigma_x^2} e^{-\frac{y^2}{\sigma_x^2}} dy = 1 - e^{-\frac{x^2}{\sigma_x^2}}, \; x \geq 0 \qquad (4)$$

The PAPR of OFDM signal in a given frame is defined as

$$PAPR_X = \frac{\max_{n\in[0, LN-1]}\{|x_n|^2\}}{E\{|x_n|^2\}}. \qquad (5)$$

It is more helpful to consider the PAPR as a random variable and utilize a statistical description given by the complementary cumulative density function (CCDF), defined as the probability that the PAPR of exceeds an assigned level $\gamma_0 > 0$, i.e.

$$CCDF_X(\gamma_0) = \Pr ob\{PAPR_X > \gamma_0\} = 1 - (1 - e^{-\gamma_0})^N \qquad (6)$$

The companding transform can be applied to the original signal x_n before it is converted into analog waveform and amplified by the HPA. The companded signal is denoted as $s_n = C(x_n)$, where $s_n = C(\cdot)$ is the companding function that only changes the amplitude of x_n. In the case of additive Gaussian white noise (AWGN) channel, the received signal $r_n = s_n + \upsilon_n$ can be recovered by the inverse companding function $C^{-1}(\cdot)$, namely, $x'_n = C^{-1}(s_n + \upsilon_n) = x_n + C^{-1}(\upsilon_n)$, where υ_n is channel noise.

III. PIECEWISE LINEAR COMPANDING TECHNIQUE

When the original signal x_n is companded with a given peak amplitude A_c, the piecewise linear companding technique shown in Fig. 1 clips the signals with amplitudes over A_c and linearly changing the signals with amplitudes close to A_c. Then, the companding function of the piecewise linear companding technique is

$$s(n) = C\{x(n)\} = \begin{cases} x(n) & |x(n)| \le A_i \\ mx(n) + (1-m)A_c & A_i < |x(n)| \le A_c, \\ sgn(x(n))A_c & |x(n)| > A_c \end{cases} \quad (7)$$

where $sgn(x)$ is the sign function.

Next, the inverse companding function at the receiver is

$$y(n) = C^{-1}(r(n)) = \begin{cases} r(n) & |r(n)| \le A_i \\ (r(n) - (1-m)A_c)/m & (1-m)A_c < |r(n)| \le A_c, \\ sgn(r(n))A_c & |r(n)| > A_c \end{cases} \quad (8)$$

where A_c, A_i and m are the parameters specified by the piecewise linear companding technique. The PAPR value of the piecewise linear companding technique is getting theoretically by the determined value of A_c. As the average signal power is maintained constant, then according to the definition of PAPR in (5), with a theoretical PAPR preset value, A_c can be determined $A_c = \sigma_x 10^{PAPR_{preset}/20}$. By using the A_c value remaining parameters A_i and m can be obtained by solving

$$\int_{A_i}^{A_c} \left(mx + (1-m)A_c\right)^2 f_{|x_n|}(x)\,dx + \int_{A_c}^{\infty} A_c^2 f(x)dx = \int_{A_i}^{\infty} x^2 f_{|x_n|}(x)\,dx. \qquad (9)$$

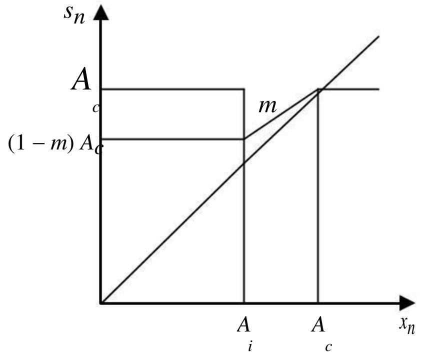

Fig. 1. Piecewise linear companding transform

IV. DISCRETE COSINE TRANSFORM

The DCT decorrelates the data sequence such as the Hadamard transform. DCT is applied to decrease the autocorrelation of the input sequence before the IFFT operation is applied. In brief the authors review the DCT in this section. The one-dimensional DCT of length N can be formulated as following:

$$X_c(k) = \alpha(k)\sum_{n=0}^{N-1} x(n)\cos\left[\frac{\pi(2n+1)k}{2N}\right], \quad (10)$$

for $k = 0,\ldots, N-1$

Similarly, the inverse transformation can be formulated as

$$x(n) = \sum_{k=0}^{N-1}\alpha(k)X_c(k)\cos\left[\frac{\pi(2n+1)k}{2N}\right], \quad (11)$$

for $n = 0,\ldots, N-1$

For both Equations (10) and (11) $\alpha(k)$ is defined as

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{N}}, & for \quad k = 0 \\ \sqrt{\frac{2}{N}}, & for \quad k \neq 0 \end{cases} \quad (12)$$

Equation (10) can be expressed in matrix form as

$$X_c = C_N x \quad (13)$$

where X_c and $\boldsymbol{x}$ are both vectors of dimension $N \times 1$, and C_N is a DCT matrix of dimension $N \times N$. The rows (or column) of the DCT matrix, C_N are orthogonal matrix vectors. The authors can reduce the peak power of OFDM signals by using orthogonal property of the DCT matrix.

According to [10], there is a relation between the autocorrelation function (ACF) of an aperiodic input vector and the PAPR of an OFDM signal. Assume $\rho(i)$ is the ACF of a signal vector, X, then:

$$\rho(i) = \sum_{k=0}^{N-1-i} X_{k+i}X_k^* \qquad for \quad i = 0,1,\ldots, N-1 \quad (14)$$

where the superscript * denotes the complex conjugate. Then, the PAPR of the transformed OFDM signal is bounded by [11]:

$$PAPR \leq 1 + \frac{2}{N}\sum_{i=1}^{N-1}|\rho(i)| \quad (15)$$

Let $\lambda = \sum_{i=1}^{N-1}|\rho(i)|$, a signal with a lower PAPR in OFDM systems can produce a lower λ. For the reduction PAPR the authors reduce the $\rho(i)$ by applying DCT transform before IFFT.

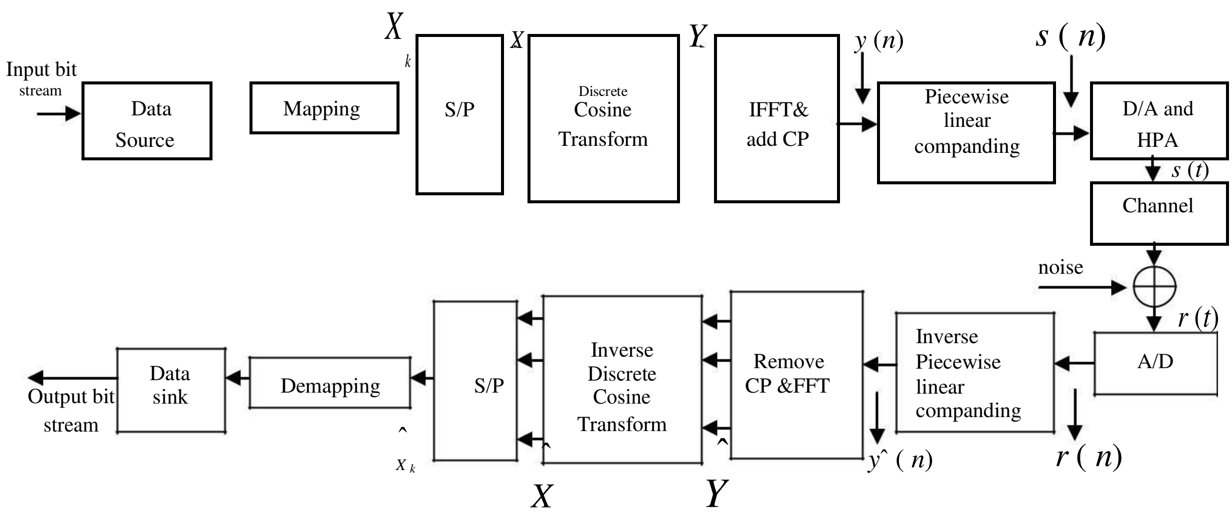

Fig. 2. Block diagram of a OFDM system with DCT-Piecewise Companding.

V. PROPOSED TECHNIQUE

In this section, the authors propose a hybrid companding transform technique to reduce the PAPR of OFDM signal by combining piecewise linear companding technique and DCT transform. The coming input data stream is firstly transformed by discrete cosine transform, and then the transformed data stream is given as input to IFFT signal processing unit. The OFDM system with proposed technique is shown at Fig.2.

The hybrid companding transform technique processing steps are given below:

Step1: firstly DCT transform is applied to the sequence X, i.e. $Y = HX$

Step2: apply IFFT to DCT transformed signal, **y**=IFFT (**Y**), where $y = [\, y(1)\; y(2) \; .... \; y(N)]^T$

Step3: apply piecewise linear companding transform to y, i.e. $s(n) = C\{y(n)\}$

Step4: apply piecewise linear decompanding transform to received signal $r(n)$, i.e. $\hat{y}(n) = C^{-1}\{r(n)\}$.

Step5: apply FFT transform to the signal $\hat{y}(n)$, i.e. $\hat{Y} = FFT(\hat{y})$, where $\hat{y} = [\hat{y}(1)\; \hat{y}(2) \; .... \; \hat{y}(N)]^T$

Setp6: apply inverse DCT transform to the signal $\hat{Y}$, i.e. $\hat{X} = H^T \hat{Y}$. Then the signal $\hat{X}$ is demapped to bit stream.

VI. SIMULATION RESULTS

Computer simulation results are presented in this section to evaluate the performance of the proposed technique i.e. hybrid companding transform with respect to the PAPR reduction, BER and PSD performance. The number of subcarrier N = 256 and oversampling factor L is 4 considered as per IEEE 802.16 standards. The baseband modulation techniques 4-QAM and 16-QAM are considered in the simulations. Solid State Power Amplifier (SSPA) is considered as a model for input-output characteristics of the nonlinear region by passing companded signals through HPA. This model is formulated by

$$|z(t)| = \frac{|y(t)|}{\left(1 + \left(\frac{|y(t)|}{A_{sat}}\right)^{2p}\right)^{\frac{1}{2p}}} \qquad (14)$$

where A_{sat} is the saturation level, and p = 2 is selected for this paper.

A. Performance in PAPR Reduction

Fig. 3 plots the simulated Complementary Cumulative Distribution Function (CCDF) of PAPR of the proposed technique and previous companding techniques. The authors observed from Fig. 3, the proposed technique can draw good PAPR reduction. Given that CCDF= 10^{-3}, the proposed technique with $PAPR_{preset}$ =4dB is 0.2dB, $PAPR_{preset}$ =4.5dB is 0.3dB and $PAPR_{preset}$ =5dB is 0.5dB superior over piecewise linear companding technique with their respective $PAPR_{preset}$ value.

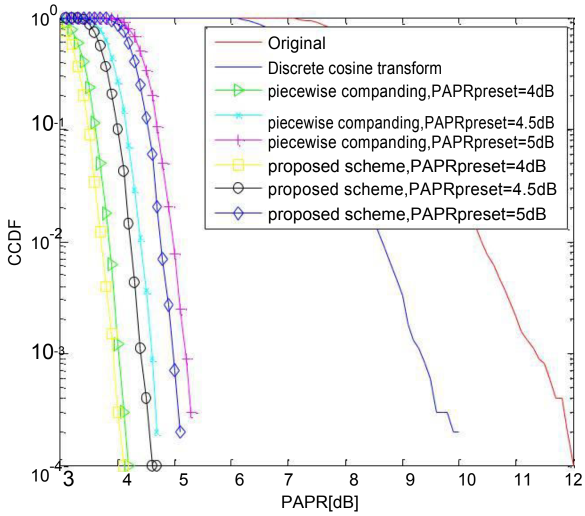

Fig.3. CCDFs of original OFDM signal and different PAPR reduction techniques.

B. BER Performance

The impact of the proposed hybrid companding transform technique on the BER performance is discussed in this section. BER versus Signal to Noise Ratio (SNR) curves with different companding transforms under an AWGN channel using 4-QAM and 16-QAM are shown in Fig. 4 and 5 respectively. It is observed that BER performance is improved in 4-QAM modulation, with the proposed technique. At BER level of 10^{-3}, the proposed technique with =4dB exceeds linear companding technique by 0.35dB.

It is also observed that in 16-QAM, the BER performance of proposed technique has performance floor at high SNR because of the output of the proposed companding function is not continuous. At a BER level of 10^{-2}, the proposed technique with $PAPR_{preset}$ =4dB is superior over the piecewise linear technique by 0.3dB of their respective $PAPR_{preset}$ value.

Fig. 6 depicts the BER performance using 4-QAM modulation with SSPA passing through an AWGN channel. It can be seen that the BER performance of the proposed technique with SSPA model is also sufficient.

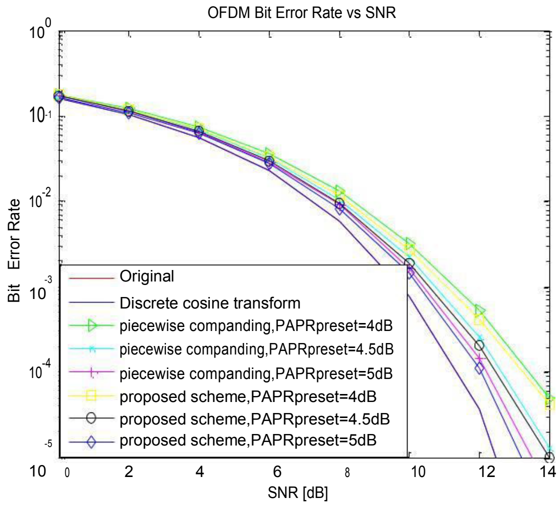

Fig. 4. BER Performance of original OFDM signal and different techniques over AWGN channel with 4-QAM modulation.

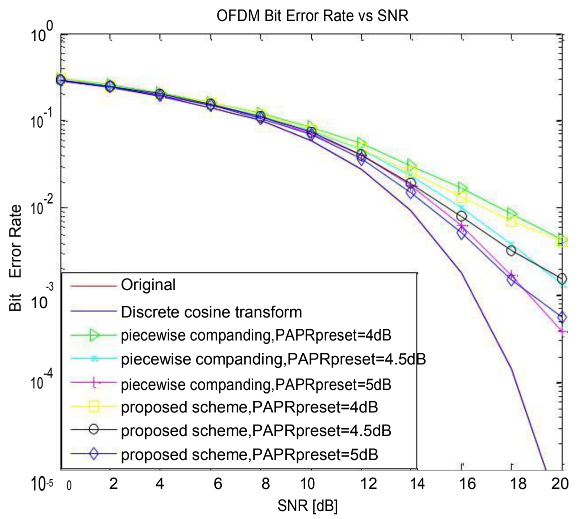

Fig. 5. BER Performance of original OFDM signal and different techniques over AWGN channel with 16-QAM modulation.

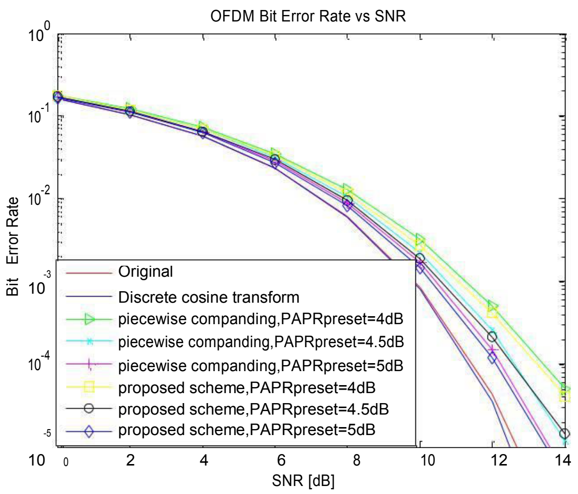

Fig. 6. BER Performance of original OFDM signal and different techniques with SSPA over AWGN channel with 4-QAM modulation.

C. PSD Performance

The PSD performances for the original OFDM signal and the other companded OFDM signals are given in Fig.7. The PSDs are computed by means of nonparametric method to get PSD comparison among different companding techniques.

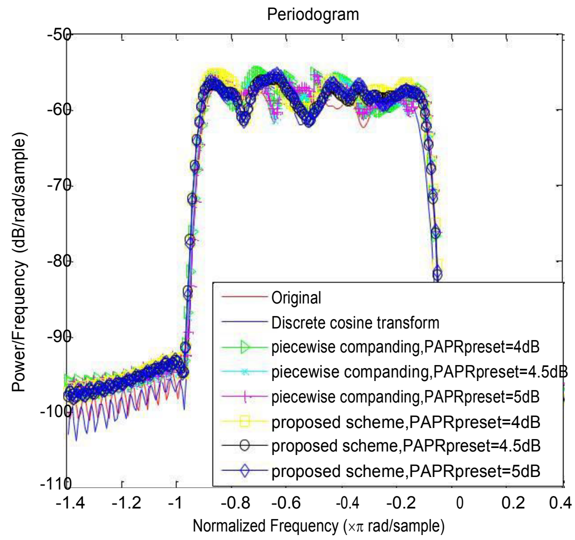

Fig. 7. PSDs of the different companding techniques.

VII. CONCLUSION

In this paper, a hybrid companding transform technique is proposed to reduce PAPR in OFDM signals and the results are compared with the existing linear companding transform technique. The PAPR is reduced by 0.8dB considering the preset value as 5dB. The BER performance under an AWGN channel using 4-QAM is improved by 0.35dB and maintaining the same performance in 16QAM.The PAPR reduction and the improvement in BER performance are achieved without scarifying the PSD performance.

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