

A Low-Complexity Modified SLM with New Phase Sequences for PAPR Reduction in OFDM System

V.Sudha, Bhukya Anilkumar, M.S.Samatha, D. Sriram Kumar

Department of Electronics and Communication Engineering

National Institute of Technology, Tiruchirappalli
Tamil Nadu, India

Abstract— The major issue in orthogonal frequency division multiplexing (OFDM) system is a high peak-to-average power ratio (PAPR) of transmitting signal. Selective mapping (SLM) is the promising technique to reduce PAPR in OFDM system. However, it suffers from high computational complexity. In our proposed method, to achieve low computational complexity, the input data sequences are partitioned into two sub blocks in such a way that the candidate signals are generated by performing circular convolution for the first sub block and by conventional SLM in the second sub block. The candidate signals of modified SLM are obtained by combining two sub block candidate signals. The proposed method reduces PAPR marginally along with reduction in computational complexity. To further reduce PAPR, different phase sequences namely Riemann, New Riemann and chaotic sequences are used in C-SLM. The Matlab simulation shows that, the modified SLM with new Riemann sequence results in high PAPR reduction than other phase sequences. In addition, it provides significant reduction computational complexity compared to C-SLM.

Keywords— *chaotic sequence; complementary cumulative distribution function (CCDF); orthogonal frequency division multiplexing (OFDM); peak to average power ratio (PAPR); Riemann sequence ; selective mapping (SLM).*

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a potential candidate for next generation wireless communication due to its high spectral efficiency, high speed transmission, interference rejection capability in multipath environments. The major limitation in OFDM system is it exhibits a high peak to average power ratio (PAPR). Therefore, the transmitter requires high power amplifiers with large dynamic range which leads to degradation in power efficiency and increase in system cost. In the literature, several PAPR reduction methods have been proposed [1] like clipping [2], clipping and filtering [3], Iterative clipping and filtering [4], companding [5] to reduce PAPR by distorting the OFDM signal which increases the bit error rate (BER). Selective mapping (SLM) [6] and partial transmit sequence (PTS) [7] algorithms are promising to reduce PAPR without affecting BER performance of the system. Among both, SLM is simple to implement, but it suffers from high computational complexity. There have been some modifications done in SLM to achieve better PAPR reduction such as precoded SLM [8] but there is no reduction in computational complexity. Hence, to reduce the

computational complexity, many low complexity SLM [9-11] algorithms have been proposed.

However, they do not improve PAPR reduction performance. The low complexity SLM structure proposed in [12] reduces both complexity and PAPR. To further enhance PAPR reduction, different phase sequences such as Riemann sequence, modified Riemann sequence and chaotic sequences are applied. It has been shown that the proposed algorithm with new Riemann sequence provides PAPR reduction with less computational complexity than C-SLM.

This paper is organized as follows, definition of PAPR is given in section II. C-SLM algorithm and different phase sequences are discussed in section III. The Proposed algorithm is explained in section IV. Section V presents simulation results and discussions. Section VI presents the computational complexity analysis. Finally, the conclusion is given in section VII.

II. DEFINITION OF PAPR

In an OFDM system, N parallel sub carriers are used to carry the information symbols simultaneously. Let X_k ($0 \leq k \leq N-1$) denote k^{th} modulated symbol of an information symbol. The output of N -point IFFT of X_k being the OFDM symbol over a symbol period T_s , it can be expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{i2\pi \frac{n}{N} k}, \quad 0 \leq n \leq N-1 \quad (1)$$

PAPR of (1) can be defined as the ratio of instantaneous peak power to average power i.e.

$$PAPR = 10 \log_{10} \frac{\max \{|x_n|^2\}}{E \{|x_n|^2\}} \text{ dB} \quad (2)$$

Where $E[\cdot]$ denotes expectation operation.

Complementary cumulative distribution functions (CCDF) are used to measure the amount of PAPR reduction and can be expressed as

$$CCDF = \Pr(PAPR > PAPR_o) = (1 - (1 - e^{-PAPR_o})^N) \quad (3)$$

III. C-SLM ALGORITHM AND PHASE SEQUENCES

A. C-SLM [6]

C-SLM is an attractive PAPR reduction technique because of the simplicity in its structure, ease of implementation and compatibility with any type of modulation. However, the system suffers from high computational complexity. To generate B candidate signals SLM requires 'B' LN-point IFFT blocks. The block diagram of C-SLM scheme is depicted in Fig.1. The transmitter generates a different set of alternate candidate signals representing same information and selects one candidate with minimum PAPR for transmission.

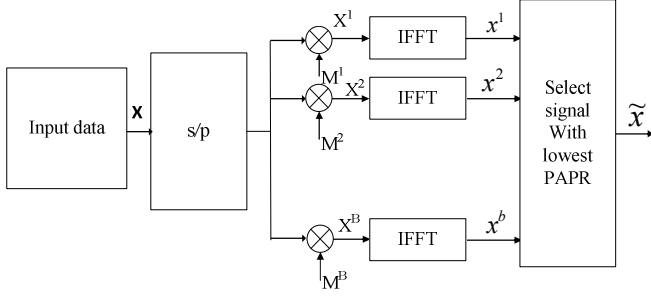


Fig. 1. Block diagram of SLM

The randomly generated input data sequence of length N is converted to parallel form using serial to parallel converter, i.e.,

$$X = [X_0, X_1, X_2, \dots, X_{N-1}]^T \quad (4)$$

Then it is multiplied with randomly generated phase rotation sequence M of the length N .

$$M^{(b)} = [m_0^{(b)}, m_1^{(b)}, \dots, m_{N-1}^{(b)}]^T, b=1, 2, \dots, B, \quad (5)$$

the n^{th} element of the random phase sequence $m_n^{(b)} = e^{j\theta_n^{(b)}}$, where $\theta_n^{(b)} \in (0, 2\pi)$. The candidate signals are generated as follows

$$X^b = X * M^b \quad (6)$$

$$= [X_0 m_0^{(b)}, X_1 m_1^{(b)}, \dots, X_{N-1} m_{N-1}^{(b)}]^T \quad (7)$$

$$x^b = \text{IFFT}(X^b), \quad 1 \leq b \leq B \quad (8)$$

Each candidate signal has different PAPR value. Finally, the candidate with low PAPR (\tilde{x}) will be selected and transmitted as shown in Fig.1. In general, the elements of the phase vectors are chosen from the set $\{\pm 1, \pm j\}$ to simplify the array multiplication.

The lowest PAPR signal \tilde{x} is selected such that

$$\tilde{x} = \underset{1 \leq i \leq B}{\text{argmin}} (\max_{0 \leq n \leq N-1} |x^{(i)}|) \quad (9)$$

The SLM suffers from high computational complexity. So, it requires complex hardware to function at a faster rate and this in turn increases the cost of system implementation and power consumption at the transmitter side.

To overcome these limitations, we proposed low complexity modified SLM. However, the PAPR reduction capability of SLM is determined by the each element in the phase rotation sequence and total number of candidates. Hence, to obtain PAPR reduction the phase sequence with low auto correlation property, namely Riemann, modified Riemann and chaotic sequences are employed in our proposed method.

B. Riemann sequence [14]

Riemann sequences are obtained by removing the first row and column of the matrix A from (10). The rows of the Riemann matrix serve as phase sequence for the C-SLM.

$$A(i, j) = \begin{cases} i - 1, & \text{if } i \text{ divides } j \\ -1, & \text{otherwise} \end{cases} \quad (10)$$

The elements in the u^{th} row of Riemann matrix are 'u' (or) -1. $1 \leq u \leq N$. From (10) the Riemann matrix of order 4 can be written as

$$r_4 = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 2 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \quad (11)$$

The elements with the value -1 are phase factor, they introduce the phase change, whereas the elements with the value 'u' produce the amplitude change, therefore the power level of some data sub carriers gets increased by the factor u^2 .

C. New Riemann sequence

The new Riemann sequence (c_{new}) is obtained by multiplying the Riemann matrix (with another matrix called centering matrix (c_n)). The centering matrix can be expressed as follows

$$c_n = I_n - \frac{1}{n} a_n \quad (12)$$

Where I_n = Identity matrix, a_n = matrix with all the elements are 1's.

The resultant matrix rows are used as phase sequence in the modified SLM method.

$$C_{\text{new}} = c_n * r_n \quad (13)$$

D. Chaotic sequence

The Chaotic sequences are non-periodic sequence in addition, it exhibits random behavior. Therefore, it can be used as phase sequence for the C-SLM. The M ary chaotic sequence $c_n \in \{0, 1, 2, \dots, M-1\}$, $1 \leq n \leq N$, of length N , is obtained by [14]

$$C_n = \frac{My_{n+1}}{2} + \frac{M}{2}, \quad (14)$$

$$y_{n+1} = f(y_n) = 1 - \alpha y_n^2, \alpha \in [1.4105, 1.99],$$

$$y_n \in (-1, 1) \quad (15)$$

IV. PROPOSED SYSTEM MODEL

High PAPR in an OFDM is due to the amount of correlation of the input symbols to IFFT. The idea behind C-SLM is to generate highly uncorrelated data symbols from the actual input data symbols by multiplying it with uncorrelated random sequences and the selection of appropriate random sequence is difficult.

The structure of the proposed system model is shown in Fig.2. To reduce complexity, a perfect sequence [11] is proposed to replace a bank of IFFT blocks in the C-SLM. As these perfect sequences satisfy periodic autocorrelation property and the elements have same magnitude they are used as phase rotation vectors.

In this paper, the input data sequences are partitioned into two sub blocks like partial transmit sequence (PTS). The first sub-block uses perfect sequences as a phase vector to generate v number of sub block candidate signals whereas C-SLM algorithm is applied to the second sub block. However, while applying C-SLM instead of using random phase sequence Riemann, Modified Riemann, and chaotic sequences are applied to reduce PAPR. The first and second sub block candidates are combined to get ' $2V$ ' number of candidate signals. Finally, the candidate with minimum PAPR is selected and transmitted.

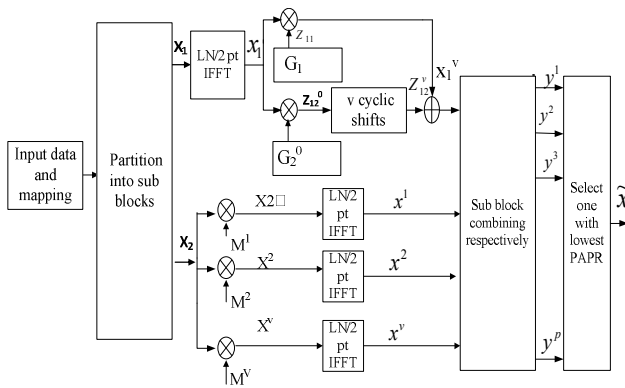


Fig. 2. The proposed system model

Steps for the proposed algorithm are as follows:

1. QPSK modulated data sequence X is partitioned into two disjoint sub blocks, namely X_1, X_2 .

2. Two sub blocks are over sampled by the factor L i.e. $(L-1)N/2$ zeros are inserted in X_1, X_2 as follows

$$X_1 = \left[X_0, X_1, \dots, X_{\frac{N-1}{4}}, \underbrace{0, 0, \dots, 0}_{(L-1)\frac{N}{2}}, X_{\frac{N}{4}}, \dots, X_{\frac{N-1}{2}} \right]^T \quad (16)$$

$$X_2 = \left[X_{\frac{N}{2}}, X_{\frac{N}{2}+1}, \dots, X_{\frac{3N-1}{4}}, \underbrace{0, 0, \dots, 0}_{(L-1)\frac{N}{2}}, X_{\frac{3N}{4}}, \dots, X_{N-1} \right]^T \quad (17)$$

3. Perform $LN/2$ point IFFT operation for the first sub block (16), and then apply circular convolution to time domain signal x_1 using G_1, G_2^0 base vectors. The resultant signal is Z_{11}, Z_{12}^0 .

4. To generate x_1^v candidate signals, Add Z_{11}, Z_{12}^v , Where Z_{12}^v is a v^{th} right circular shift of Z_{12}^0 . The n^{th} element of Z_{12}^v can be written as

$$Z_{12}^v = Z_{12}^0[(n-v)_{LN/2}] \quad (18)$$

The output of first sub block candidate signal is

$$x_1^v = Z_{11} + Z_{12}^v \quad (19)$$

5. The second sub block (17) is multiplied with rows of the Riemann matrix (11), to generate v sub block candidates, X_2 is multiplied with the first ' v ' rows of the Riemann matrix.

6. Perform $V * LN/2$ point IFFT operation.

7. The candidate signals of modified SLM (P) are obtained by combining two sub block candidate signals as follows

$$X^p = X_1^v + w(X_2^v) \quad (20)$$

Where $w = +1$ and -1 , correspondingly for 0 degree and 180 degree phase shift of second sub block candidate signals. The maximum number of candidate signals that can be generated are two times the sub block candidate signals ($2v$).

8. Finally, the candidate, which has lowest PAPR is selected and transmitted.

$$\tilde{x} = \arg \min_{1 \leq i \leq p} \left(\max_{0 \leq n \leq N-1} |y^{(p)}| \right) \quad (21)$$

The same procedure is repeated for New Riemann, the chaotic phase sequence when applied to second sub block.

V. SIMULATION RESULTS AND DISCUSSION

MATLAB simulation has been carried out to assess the PAPR characteristic of the proposed modified SLM scheme. The parameters used for simulation are shown in Table 1. Figs.3 , 4 compare the PAPR performance of the proposed system model with different phase sequences such as Riemann, New Riemann and chaotic sequence for $P=64, 128$ and the number of sub carriers $N=256, 512$ respectively. For the below simulation 10^5 random OFDM symbols are generated.

TABLE I. MATLAB SIMULATION PARAMETERS

Parameters	Description
Number of Subcarriers (N)	256,512
Over sampling factor (L)	4
Number of candidate signals (P)	64,128
Modulation scheme	QPSK
Types of phase sequences	Riemann, New Riemann, Chaotic

From the Figs 3, 4, proposed modified SLM with New (or) modified Riemann sequence achieves better PAPR reduction than other sequences. Table II compares the PAPR of proposed modified SLM with various phase sequences deduced from Fig.3. If $B=64$, PAPR reduction performance of proposed system using Riemann and New Riemann sequence is similar and as B increases New Riemann sequence achieves better PAPR reduction than Riemann. However, PAPR reduction performance of chaotic sequence is same as C-SLM.

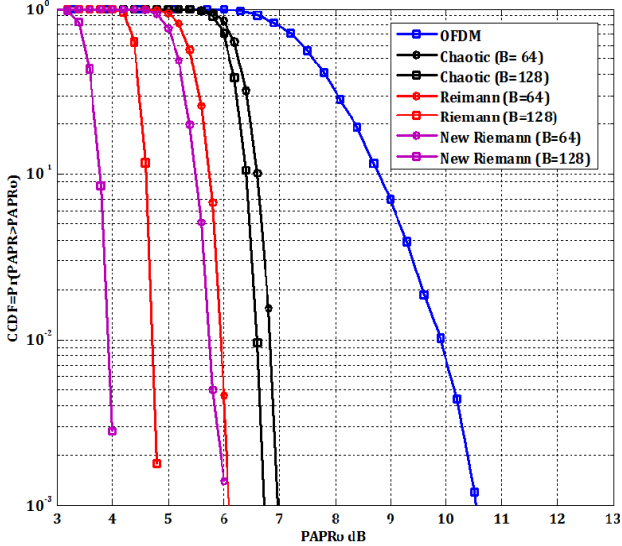


Fig. 3. Comparison of PAPR of proposed model under different phase sequences with $N=256$

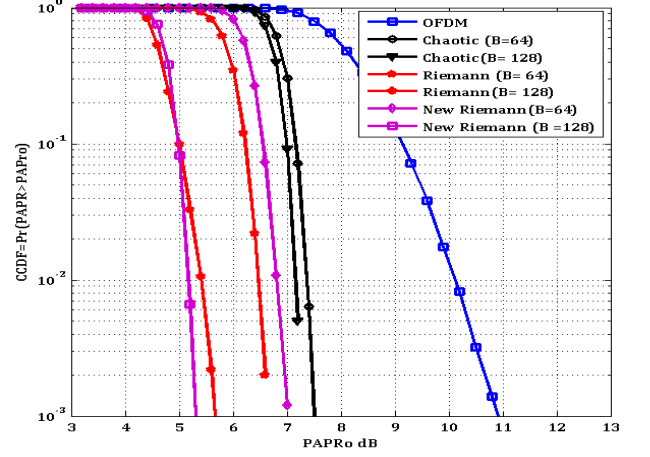


Fig. 4. Comparison of PAPR of proposed model under different phase sequences with $N=512$

Fig.4 compares the PAPR reduction performance of the proposed system with $N=512$. At the clip rate of $CCDF = 10^{-2}$ dB and the number of candidates $B=128$, the proposed system with a New Riemann sequence has 5dB reduction, Riemann sequence has 4.6 dB reduction and chaotic sequence has 2.8dB reduction than OFDM system.

TABLE II. COMPARISON OF PAPR FOR PROPOSED MODIFIED SLM

Number of sub carriers	Type of phase sequence	No .of candidates (P)	PAPR(dB)
256	New Riemann	64,128	6.1,6
	Riemann		7,4
	Chaotic		4.8,6.7
512	New Riemann	64,128	7,6.8
	Riemann		7.5,5.2
	Chaotic		5.6,7.2

VI. COMPUTATIONAL COMPLEXITY ANALYSIS

C-SLM requires M -IFFT blocks to generate M candidate signals. However, one IFFT requires $N/2 \log_2 N$ complex multiplications and $N \log_2 N$ complex additions [9]. The number of complex multiplications n_{mul} and additions n_{add} required for the proposed modified SLM is as follows

$$n_{mul} = v * \frac{LN}{4} \log_2 \frac{LN}{2} + \frac{LN}{4} \log_2 \frac{LN}{2} + \frac{LN}{2} * v \quad (22)$$

$$n_{add} = v * \frac{LN}{2} \log_2 \frac{LN}{2} + \frac{LN}{2} \log_2 \frac{LN}{2} + 2 * \frac{3LN}{2} + \frac{vLN}{2} + \frac{iLN}{2} \quad (23)$$

Where i = number of candidate signals, v = number of circular shifts and L = Oversampling factor.

Thus, the Computational Complexity Reduction Ratio (CCRR) of the proposed SLM method over the conventional SLM methods is expressed as [8]

$$CCRR = \left(1 - \frac{\text{complexity of proposed algorithm}}{\text{complexity of C-SLM}} \right) * 100\% \quad (24)$$

From Table III, we can understand that the number of complex multiplications and additions required for our proposed algorithm is reduced to 76.95% and 67.78% respectively than C-SLM.

TABLE III. COMPUTATIONAL COMPLEXITY REDUCTION RATIO OF PROPOSED SYSTEM OVER C-SLM

Computational blocks	SLM	Proposed modified SLM	CCRR%
No.of.Multiplications	81,920	18,880	76.95
No.of.Additions	1,31,027	42,112	67.78
IFFT blocks	64	33	-

VII. CONCLUSION

This paper presents a low-complexity modified SLM with different phase sequences for PAPR reduction in OFDM system. The simulation results show that the PAPR reduction performance of proposed low complexity modified SLM with New Riemann sequence is better than C-SLM. Moreover, the time domain signal property of OFDM has been exploited for the first half of the data block to get low computational complexity. Hence, the proposed low complexity modified SLM can reduce both PAPR and computational complexity better than C-SLM.

REFERENCES

[1] Han S. H. & Lee I. H. (2005). An overview of peak-to-average power ratio reduction techniques for multicarrier transmission. *Wireless Communications, IEEE*, 12(2), 56-65.

[2] Ochiai, Hideki, and Hideki Imai. "Performance analysis of deliberately clipped OFDM signals." *Communications, IEEE Transactions on* 50.1 (2002): 89-101.

[3] Li, Xiaodong, and Leonard J. Cimini Jr. "Effects of clipping and filtering on the performance of OFDM." *Vehicular Technology Conference, 1997, IEEE 47th*. Vol. 3. IEEE, 1997.

[4] Zhu, Xiaodong, et al. "Simplified Approach to Optimized Iterative Clipping and Filtering for PAPR Reduction of OFDM Signals." (2013): 1-11.

[5] Huang, Xiao, et al. "Commanding transform for reduction in peak-to-average power ratio of OFDM signals." *Wireless Communications, IEEE Transactions on* 3.6 (2004): 2030-2039.

[6] Cimini Jr, Leonard J., and Nelson R. Sollenberger. "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences." *Communications Letters, IEEE* 4.3 (2000): 86-88.

[7] Bauml, R.W.; Fischer, R.F.H.; Huber, J.B., Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping. *Electronics Letters*, Vol. 32 no. 22, pp. 2056,2057,24Oct1996.

[8] Hasan, M. M. (2013). VLM precoded SLM technique for PAPR reduction in OFDM systems. *Wireless personal communications*, 73 (3), 791-801.

[9] Jeon, H. B., No, J. S., & Shin, D. J. (2011). A low-complexity SLM scheme using additive mapping sequences for PAPR reduction of OFDM signals. *Broadcasting, IEEE Transactions on*, 57(4), 866-875.

[10] Wang, S. H., Sie, J. C., Li, C. P., & Chen, Y. F. (2011). A low-complexity PAPR reduction scheme for OFDMA unicast systems. *Wireless Communications, IEEE Transactions on*, 10(4), 1242-1251.

[11] Li C. P., Wang S. H. & Wang C. I. (2010). Novel low-complexity SLM schemes for PAPR reduction in OFDM systems. *Signal Processing, IEEE Transactions on*, 58(5), 2916-2921.

[12] Sudha, V., Bhukya Anilkumar, and D. Sriramkumar. "Low-complexity modified SLM method for PAPR reduction in OFDM systems." In *Electronics and Communication Systems (ICECS), 2015 2nd International Conference on*, pp. 1324-1328. IEEE, 2015.

[13] Wen-Xiang Lin, Jia-Chin Lin† and Yu-Ting Sun‡, "Modified Selective Mapping Technique for PAPR Reduction in OFDM Systems", 12th International Conference on ITS telecommunications, 2012.

[14] N.V. Irulapati, V.K. Chakka and A. Jain, "SLM based PAPR reduction of OFDM signal using new phase sequence", *Electronics letters* 19th november 2009 vol. 45 no. 24.