

# *PAPR Reduction Of FrFT-Based MB-OFDM Ultra Wide Band Signals*

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**Abstract**—Multi-band orthogonal frequency division multiplexing (MB-OFDM) is being considered as a promising candidate for UWB systems due to its higher spectral efficiencies and better system performance. The classical Fourier transform scheme may fail when the channel is doubly selective. Chirp-like basis (fractional Fourier transform based) can be used instead of exponential functions to combat this problem. However, just like in the Fourier transform based MB-OFDM, fractional Fourier transform based MB-OFDM (FrFT based MB-OFDM) UWB also suffers from the high Peak to Average Power Ratio (PAPR) problem. In this paper, we have analyzed the MB-OFDM UWB signals in the fractional domain, and evaluated the effects of the well-known OFDM PAPR reduction methods such as selected mapping (SLM) and partial transmit sequences (PTS) to FrFT based MB-OFDM UWB through computer simulations.

**Keywords**— MB-OFDM UWB; FrFT; PAPR; SLM; PTS

## I. INTRODUCTION

UWB technology is a wireless protocol for high-speed data transmission and has recently received a lot of interest from the wireless manufacturing and user community [1]. In February 2002, the FCC opened up 7,500 MHz of spectrum (from 3.1 GHz to 10.6 GHz) for UWB use. The signals whose fractional bandwidth is larger than 0.20-0.25 can be named as UWB signals. This technology promises to deliver data rates that can scale from 110 Mb/s at a distance of 10 meters up to 480 Mb/s at a distance of 2 meters in realistic multi-path environments all while consuming very little power and silicon area [2]. During the past years, UWB systems can be classified in two different types named single and multi-band systems respectively.

In multi-band orthogonal frequency division multiplexing (MB-OFDM), the 7.5GHz of spectrum is divided into 14 bands each of which is above 500 MHz. Based on this, five band groups are defined. Band group 1 is used for mandatory mode and the remaining band groups are reserved for future use. The relationship between center frequency and band number is given:  $2904 + 528 \times n_b, n_b = 1 \dots 14$  (MHz). The transmitted MB-OFDM symbols are time-interleaved across the sub-bands and hop between 14 band center frequencies according to the specified time-frequency code [3].

OFDM is a special technology of multicarrier modulation which ensures mutual orthogonality in the frequency domain because of employing the sinusoidal basis function. Since the addition of cyclic prefix to each symbol, the systems based OFDM can easily handle intersymbol interference (ISI)

resulting from time dispersion. When the channel is, however, frequency-dispersive, interchannel interference (ICI) may degrade an OFDM system performance to intolerable levels. To combat this problem, a new MB-OFDM system has been proposed in [4]. The novel core is to employ orthogonal signal basis of the chirp type to match time-varying characteristics of the channel. The significant method related to the fractional Fourier transform (FrFT) can be implemented at no extra cost with respect to fast Fourier transform.

In general, all the systems based on OFDM have the same disgusting drawbacks, including the potentially large peak to average ratio (PAPR), which causes high signal peak powers when a great number of independently modulated signals are added. Its high PAPR saturates the power amplifier, limits the system capacity and thus causes intermodulation distortion. Therefore, reducing the PAPR is of practical interest. Two classical and effective methods to alleviate the problem have proposed in [5], [6] and are referred to as selected mapping (SLM) and partial transmit sequences (PTS). In this paper, we extend the PAPR study and introduce these two approaches into the MB-OFDM UWB system based on FrFT.

The reminder of this paper is organized as follows. In section 2, a MB-OFDM system model based on FrFT and the PAPR expressions are analyzed. We describe the SLM and PTS schemes for the PAPR reduction in section 3. In section 4, some analytical evaluations and computer simulation results are given and we show the comparison between two methods in the two corresponding systems respectively. In the last section we conclude the paper.

## II. SYSTEM AND PAPR DESCRIPTION

The system model, in continuous-time version, can be presented as follows. The transmitted signals can be described using a complex baseband signal notation. At the transmitter, a set of the binary bit streams are changed into data blocks by the serial-to-parallel buffer and then mapped through QPSK modulation. The 128 complex-value tone coefficients are sent to the input of inverse fast transform function, and the output of transform is given below:

$$s(t) = \sum_{k=-\infty}^{+\infty} \sum_{n=0}^{N-1} d_{k,n} g_n(t - kT) \quad (1)$$

Where  $d_{k,n}$  is the  $n^{\text{th}}$  modulation vector of the  $k^{\text{th}}$  symbol,  $N$  subcarriers, and  $T=312.5$  nanosecond is the MB-OFDM symbol period. Each symbol period has 60.6

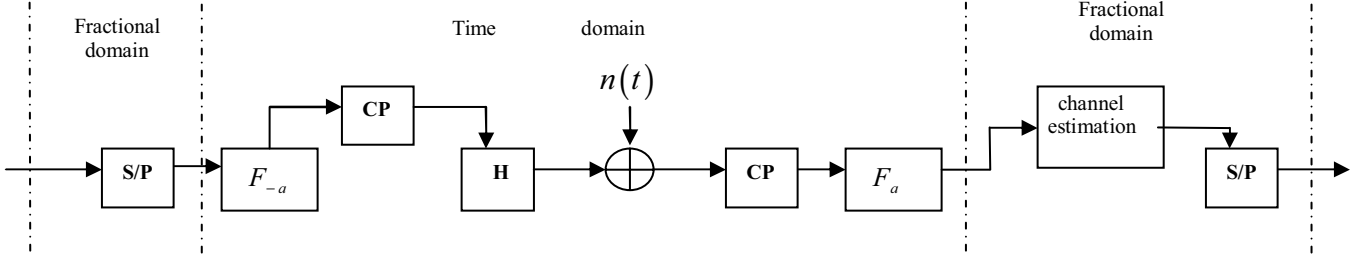


Figure 1. FrFT-based MB-OFDM system block diagram

nanosecond cyclic prefix (CP) for robustness against multi-path and 9.5 nanosecond guard interval (GI) to provide sufficient time for the transmitter and receiver switching between different sub-channels, so the total symbol length of 165-sample contains 32-sample prefix and 5-sample guard suffix in the process of the simulations [7]. Let's assume  $g_n(t-kT) = f_{a,n}(t-kT)$ , the novel MB-OFDM system is based on the chirp type that corresponds to the FrFT signal basis in the time-frequency plane when  $f_{a,n}(t)$  is defined:

$$f_{a,n}(t) = \sqrt{1-j \cot a} \exp j\pi \left( t^2 \cot a + (n \sin a/T)^2 \cot a - 2nt/T \right), \quad (2)$$

where  $a$  is related to the FrFT transform order  $p$  by  $a = \pi/2 \times p$ . In particular, if  $a = \pi/2$ , that is  $p=1$ , (1) converts to:

$$s(t) = \sum_{k=-\infty}^{+\infty} \sum_{n=0}^{N-1} d_{k,n} e^{j2\pi n(t-kT)/T}, \quad t \in [0, T], \quad (3)$$

which is the general expression for a baseband FFT-MB-OFDM transmitted signal. The selection of the function  $f_{a,n}(t)$  is motivated by the fact that exactly as the exponential (sinusoidal) basis of the FT has an impulse output in the Fourier domain, the basis  $f_{a,n}(t)$  corresponds to an impulse in the fractional Fourier domain. Fig.1 above depicts a MB-OFDM UWB system based on FrFT model. The method of cyclic prefix based on chirp periodic signal has proposed in [8].

Unfortunately, because of the summation of independently multi-carriers in the transmitter, the only drawback of all OFDM systems, which also contain the MB-OFDM system, is that its peak-to-average ratio maybe slightly higher than that of single-carrier UWB systems and pulsed-based MB approaches. Let's assume  $s(t)$  is the transformed time domain signal, the definition of PAPR is expressed as:

$$PAPR = \frac{P_{peak}}{P_{average}} = \frac{\max \{ |s(t)|^2 \}}{E \{ |s(t)|^2 \}}. \quad (4)$$

According to central limit theorem, when  $N$  is large, both the real and imaginary part of  $s(t)$  are Gaussian distributed, while the  $s(t)$  amplitude is Rayleigh distributed. So the power of MB-OFDM signal is  $\chi^2$  distributed with two degrees of

freedom. We usually describe the PAPR performance in terms of the complementary cumulative distribution function (CCDF), which is defined as the probability that the PAPR of the transmitted signal exceeds a given threshold  $\lambda$ . That is,

$$\Pr\{PAPR > \lambda\} = 1 - \Pr\{PAPR \leq \lambda\} = 1 - (1 - e^{-\lambda})^N. \quad (5)$$

If  $D$  statistically independent MB-OFDM frames represent the same information, the probability of PAPR is given by:

$$\Pr\{PAPR > \lambda\} = 1 - (\Pr\{PAPR \leq \lambda\})^D = 1 - (1 - e^{-\lambda})^N)^D. \quad (6)$$

Hence, to avoid nonlinear distortion and spectral spreading of the transmitted signals, many methods to reduce the PAPR are proposed. Considering complexity and redundancy, in this paper we only put the SLM and PTS schemes into the MB-OFDM based on FrFT and make the comparison between the two schemes.

### III. PAPR REDUCTION ANALYSIS

Unlike clipping PAPR reduction algorithms, both SLM and PTS schemes, which belong to multiple signal representation (MSR) techniques of probabilistic approaches, are distortionless PAPR reduction methods.

Since the FrFT-MB-OFDM method is a generalization of FFT-MB-OFDM method, there exists a potential to extend the application of the techniques developed to reduce the PAPR for FFT-MB-OFDM systems to FrFT-MB-OFDM systems [9].

#### A. The SLM technique

The core idea of SLM is selecting one particular signal with better properties from a set of different signal sequences representing the same information. Let's assume  $M$  statistically independent vectors  $P^{(u)} = (P_0^{(u)}, P_1^{(u)}, \dots, P_{N-1}^{(u)})$ ,  $u = 1, 2, \dots, M$ . Where  $P_i^{(u)} = \exp(j\varphi_i^{(u)})$  is the rotation factor,  $\varphi_i^{(u)}$  is homogeneous distributed in  $[0, 2\pi)$ . In our system based on FrFT, after mapping of QPSK, the fractional domain signals are multiplied with  $M$  vectors  $P^{(u)}$ , resulting in  $M$  different sequences  $X^{(u)}$  of the length  $N$ , that is:

$$X^{(u)} = X \times P^{(u)} = (X_0 P_0^{(u)}, X_1 P_1^{(u)}, \dots, X_{N-1} P_{N-1}^{(u)}). \quad (7)$$

As seen in Fig.2. Then, all  $M$  sequences are modulated into the time-domain using IFrFT and the lowest PAPR one is chosen for transmission.

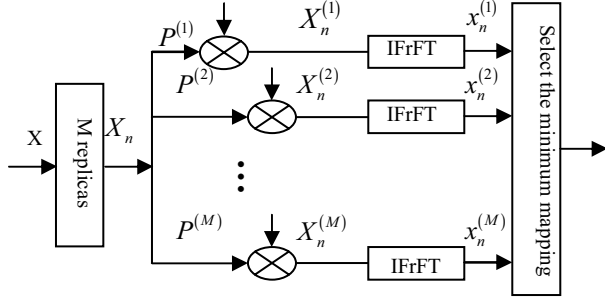


Figure 2. SLM block diagram

Considering the number of selected sequences through IFFT modulation, SLM algorithm needs  $M-1$  IFFT extra operations for each MB-OFDM symbol, leading to the system complicated. In addition, the receivers need to know the chosen independent vectors to ensure the correct recovery of the transmitted signals.

#### B. The PTS technique

The PTS scheme, which is the same as SLM, is that forming some replicas from one information sequence and choosing the lowest PAPR one for transmission, but it is a special case by optimally combining signal subblocks. In the PTS approach, the input data block is partitioned into non-overlapping subblocks or clusters which are combined to minimize the PAPR [6]. Generally, three methods of partition have been provided: adjacent partition, pseudo-random partition and interleaved partition. First, we define data symbol  $X = (X_0, X_1, \dots, X_{N-1})$  with  $N$  subcarriers, then the vector  $X$  is partitioned into  $V$  disjoint subblocks  $\{X_v, v=1, 2, \dots, V\}$ ,

$X = \sum_{v=1}^V X_v$ , each of which contain the same number subcarriers. And then  $V$  subblocks are combined as follows:

$$X' = \sum_{v=1}^V b_v X_v. \quad (8)$$

Where  $b_v = \exp(j\phi_v)$  is the phase-rotation factor and  $\phi_v \in [0, 2\pi)$ , which are called side information. The  $V$  vectors  $X_v$  convert to time-domain symbols using IFFT, that is,

$$x' = \sum_{v=1}^V b_v \text{IFFT}\{X_v\} = \sum_{v=1}^V b_v x_v. \quad (9)$$

$x_v$  are so-called partial transmit sequence. The phase-rotation factors are so carefully chosen by the optimization block that follows a specific algorithm that the optimized combination of the transmitted symbols, which produce the lowest PAPR. Then the optimum signal combination will be transmitted. Fig.3 is a block diagram of the PTS technique.

As we described in SLM, a general question is that the complexity consideration of the system design. In PTS algorithm, the large number of IFFT operations is also an oppressive burden for MB-OFDM symbols. Therefore, we decrease the calculated complexity through not only the

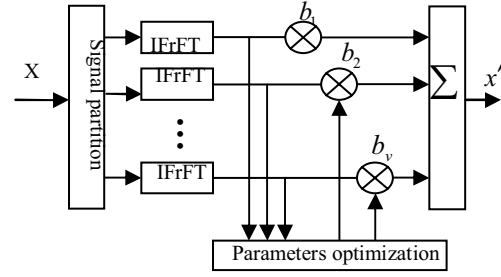


Figure 3. PTS block diagram

limitation of  $b_v$  value range, but also using the proper partitioned methods.

### IV. SIMULATION RESULTS

Computer simulations will run to verify and extend the analytical evaluation of the previous sections.

#### A. Simulation of SLM

In our simulation experiments, MB-OFDM UWB signals are first through QPSK and 128 subcarriers modulations. For simplicity, we choose  $M=4$  for the PAPR reduction of SLM technique. Literatures showed that the larger the value of  $M$  is, the better the result is [9]. To make the comparison between FrFT and FFT schemes in system, the fractional order  $p=1$  and  $p=0.001$  with  $10^4$  blocks of data generated are performed. The selection of the optimum order  $p$  is given in [4]. The results of simulations are shown in Fig.4.

It is observed that the FFT-system PAPR is effectively decreased about 2.5 dB corresponding to that of the original transmitted signal when the CCDF is  $10^{-4}$ , while the FrFT-system performance is optimized about 2 dB more. This clearly implies that the MB-OFDM signals based on FrFT perform better characteristics than the ones based on FFT in the fixed fractional domain.

#### B. Simulation of PTS

In the sequel, we also choose  $N=128$ , QPSK,  $V=4$  for the PAPR reduction of PTS technique in simulation experiments. Interleaved partition is performed. Using the method of order selection in [4], we optimize the fractional domain order ( $p=1$  and  $p=0.01$ ) and make the comparison of the FrFT-MB-OFDM and FFT-MB-OFDM system. However, the  $p$  order here is different with the one in SLM. Fig.5 below plots the overall simulation results of our analyzed system.

Noted that, the FFT-system PAPR is decreased about 2 dB corresponding to that of the original signal at the CCDF of  $10^{-4}$ , while the performance of the FrFT based system is much better than the FFT-MB-OFDM system about 4 dB. We easily noticed that if the  $p$  is optimum, the PAPR of system is distinctly decreased though PTS in certain fractional domain.

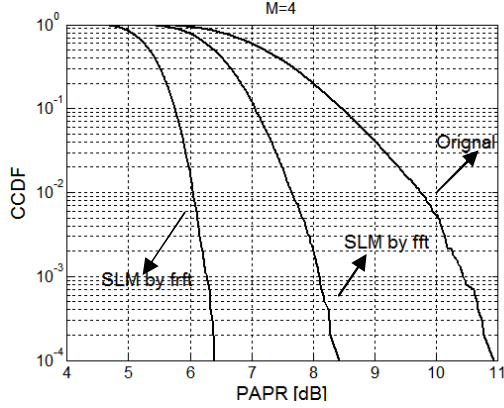


Figure 4. PAPR reduction performance of the SLM method

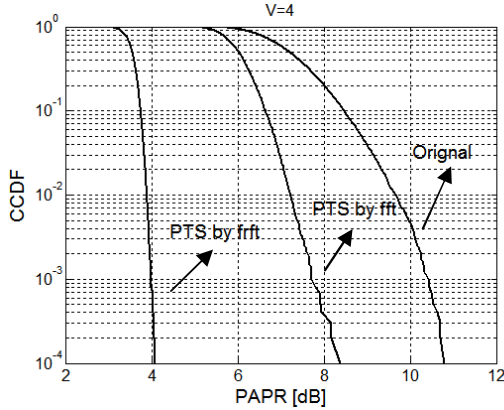


Figure 5. PAPR reduction performance of the PTS method

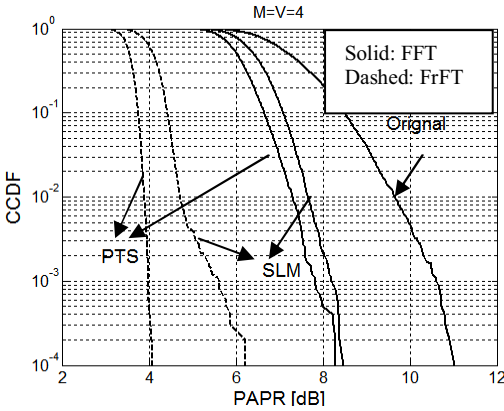


Figure 6. PAPR reduction performance of SLM and PTS methods

### C. Comparison of SLM and PTS

PTS can be viewed as a generalization of the SLM algorithm. In the past few years, the two methods were both evaluated and compared only in the frequency domain by many people. Now, we carry on this work and introduce the comparison in the fractional domain.

In the computer experiment below, to compare the SLM and PTS in the fractional domain, we choose the parameters as follows:  $M=V=4$ ,  $(1, -1, j, -j)$  for the phase rotation factor of SLM and the side information of PTS,  $p=1$  (FFT) and  $p=0.01$  (FrFT) respectively, noted that it is not the best

FrFT order for SLM,  $N=128$  and QPSK modulation. From Fig.6, it is evidently noted that the PTS greatly outperforms the SLM not only in the frequency domain but also in the certain (not all) fractional domain. The fact is that two techniques contain different optimum fractional orders.

## V. CONCLUSION

In this paper, we introduced the conventional methods of PAPR reduction into the fractional domain. Theoretical as well as simulation results suggested that FrFT-MB-OFDM systems outperform FFT-MB-OFDM systems in some certain condition. However, when the angle  $\alpha$  is not optimum, such conclusions may be altered correspondingly. Furthermore, we have shown that it is possible to extend methods and processes of MB-OFDM system based on FFT to MB-OFDM system based on FrFT.

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