A Novel Approach for Embedding Communication Symbols into Physical Radar Waveforms

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***Abstract*—Due to constantly increasing demand from commercial communications, defense applications are losing spectrum while still striving to maintain legacy capabilities, not to mention the need for enhanced performance. Consequently, ongoing research is focused on developing multi-function methods to share spectrum between radar and military communication. One approach is to incorporate information-bearing communication symbols into the emitted radar waveforms. However, varying the radar waveform during a coherent processing interval (CPI) causes range sidelobe modulation (RSM) that results in increased residual clutter in the range-Doppler response, thus leading to reduced target visibility.**

Here a novel approach is proposed to embed information into radar emissions while preserving constant envelope waveforms with good spectral containment. Information sequences are implemented using continuous phase modulation (CPM) and phase-attached to a polyphase-coded frequency-modulated (PCFM) radar waveform, the implementation of which is also derived from CPM. The resulting communication-embedded radar waveforms therefore maintain high power and spectral efficiency. More importantly, the adjustable parameterization of the proposed approach enables direct control of the degree of RSM by trading off bit error rate (BER) and/or data throughput.

I. Introduction

The radio spectrum is a fixed resource with an exponentially increasing demand from commercial communication applications [1]. The resulting erosion of radar spectrum to meet this communication demand is creating additional strain on defense applications that must already operate in congested and contested environments. As such, ongoing research is focused on improving spectral efficiency (e.g. dynamic spectrum access [2]) or developing methods to share spectrum between multiple functions (e.g. radar and communication sharing spectrum [3]).

Generally speaking, spectrum sharing can take two forms: cohabitation or co-design. Where the former tends primarily to address the interference that separately operated systems could cause to one another, the latter involves cooperative control within the same system. Here we investigate the co-design problem, in particular the realization of a single dual-function system with both radar and communication capabilities. At first glance, communications and radar appear to be similar. However, successful communication requires maximizing the entropy embedded in the transmitted waveform [4] while radar waveforms require coherent, restrictive forms to maximize detection performance [5]. Thus a dual-function system that performs radar and communication simultaneously involves a performance trade-off between these functions.

This research was performed while Cenk Sahin held an NRC Research Associateship award at the Air Force Research Laboratory, Wright-Patterson AFB, OH. This work was supported in part by the Naval Research Laboratory - Radar Division, the Air Force Research Laboratory - Sensors Directorate, and by a subcontract with Matrix Research, Inc. for research sponsored by Air Force Research Laboratory under Prime Contract #FA8650-14-D-1722.

Aside from the more obvious approaches of time-sharing or frequency sub-banding, the notion of radar/communication spectrum sharing necessitates the use of some manner of waveform diversity [6]–[9]. As a general principle, waveform diversity can involve the exploitation of the available time, frequency, coding, spatial, and polarization degrees-of-freedom. For example, other work has examined the embedding of low probability of intercept (LPI) communications into radar clutter [10]–[12], using a small set of different radar waveforms where each represents a different communication symbol [13], [14], modulating a communication signal onto the spatial sidelobes of a radar beam [15], using 4G communication signals to also serve as short-range radar emissions for automotive applications [16], dual radar/communication emissions from a common transmit aperture [17], [18], tandem hopping of communications within spectral gaps of the radar emission [19], [20], and phase-modulating a linear FM (LFM) waveform [21], [22]. The latter formulation is particularly relevant for the proposed approach.

As is the case with most radar applications, some communication systems require spectrally contained symbols with high power efficiency (e.g. aeronautical telemetry [23]). To meet this need, a family of constant envelope signalling schemes was developed, collectively denoted as continuous phase modulation (CPM) [24]. The continuous phase feature of CPM signals leads to high spectral efficiency while the constant envelope feature translates to robustness against the distortion introduced by non-linear components in the transmitter (e.g. the power amplifier). As a result, the transmitter power amplifier can be operated in saturation such that the available power is efficiently converted into radiated power. Due to its favorable features, CPM is used in the Bluetooth wireless standard [25] and two variants of shaped-offset quadrature phase-shift keying (SOQPSK) modulation, a type of CPM, are standardized for military applications (SOQPSK-MIL) [26] and aeronautical telemetry (SOQPSK-TG) [23].

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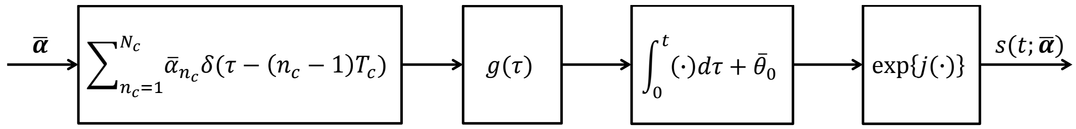

Fig. 1. Block diagram of CPM-based PCFM implementation for radar [27].

Maintaining both power efficiency and spectral efficiency is of great interest for radar systems to maximize "energy on target" and to limit the spectral roll-off for sufficient spectral containment. Accordingly, a CPM-based framework was recently used in [27] to implement arbitrary polyphase radar codes as physically realizable *continuous* frequency-modulated (FM) waveforms. The results in [27] show that the polyphase-coded FM (PCFM) implementation results in significantly superior spectral containment compared to the derivative phase-shift keying (DPSK) and the minimum-shift keying (MSK) implementations [28]. It was subsequently demonstrated in [29] how optimization of the resulting FM waveform can be achieved via determination of an underlying optimal code.

The notion of pulse agility (or waveform agility), in which the radar waveform is allowed to change on a pulse-to-pulse basis, was examined in [13] as a means to incorporate a communication function into the radar emission, where the set of possible waveforms serves as a communication symbol alphabet. The primary issue with varying the radar waveform during a coherent processing interval (CPI) is the clutter *range sidelobe modulation* (RSM) [13], [30] that arises because the pulse compression of different waveforms leads to different sidelobe structures. When Doppler processing is carried out across the CPI of pulsed echoes the presence of RSM induces a partial loss of coherency, the consequence of which is increased residual clutter after cancellation, and thus degraded target visibility. In [13], filter design to mitigate RSM for a given set of waveforms was addressed via the development of the iterative joint least squares (JLS) algorithm. However, the JLS is only suitable for transmitting $1-2$ bits per pulse because the performance diminishes as the number of waveforms increases. In [14] a closed form solution is derived for the JLS approach for moving target indication (MTI) radar, though the new form is likewise only applicable to low data rates.

Here a new approach to embed communications into radar is formulated in which information-bearing sequences are modulated with CPM and *phase-attached* to a PCFM-implemented *fixed* radar waveform. In other words, the phase of the resulting radar/communication emission is the phase addition of the *base radar waveform* and the communication waveform. The result is a constant envelope, continuous phase, communication-embedded radar waveform for each pulse, thus ensuring high power and spectral efficiency. More importantly, adjustable implementation parameters provide control of the degree of RSM by trading off bit error rate (BER) and/or data throughput. Phase-modulating an LFM waveform as a means of embedding communications into radar was proposed in [21], [22]. Specifically, in [21] information sequences modulated with MSK are multiplied by an LFM pulse while in [22] information sequences modulated with phase-shift keying (PSK) [31] having an adjustable phase parameter are multiplied by a higher rate pseudorandom binary sequence (i.e. spread spectrum) and then a discretized LFM pulse. The following represents a generalization of such approaches that is applicable to arbitrary FM radar waveforms, with the subsequent radar+communication waveform retaining the FM structure, which is attractive from a hardware perspective. The impact on RSM is likewise examined.

II. CPM-Based Radar-Embedded Communications

Consider the polyphase radar code given by $\overline{\boldsymbol{\theta}} = [\overline{\theta}_0, \cdots, \overline{\theta}_{N_c}]$, where N_c denotes the number of chips per pulse and $|\overline{\theta}_{n_c}| \leq \pi$ for all $n_c = 0, \cdots, N_c$ [27], [29]. The CPM-based PCFM implementation—converting the discrete polyphase radar code into a continuous-phase FM waveform—is shown in Fig. 1. The sequence $\overline{\boldsymbol{\alpha}} = [\overline{\alpha}_1, \cdots, \overline{\alpha}_{N_c}]$, $|\overline{\alpha}_{n_c}| \leq \pi$ for all $n_c = 1, \cdots, N_c$, denotes the phase changes between successive chips of $\overline{\boldsymbol{\theta}}$ (see (1) in [27] for the derivation of $\overline{\boldsymbol{\alpha}}$ from $\overline{\boldsymbol{\theta}}$) and is referred to as the PCFM code. The overline notation $\overline{(\cdot)}$ is used with $\overline{\boldsymbol{\theta}}$ and $\overline{\boldsymbol{\alpha}}$ to emphasize that both sequences remain the same from pulse to pulse. The length-N_c train of impulses, with the n_c-th impulse scaled by $\overline{\alpha}_{n_c}$, is convolved with the *shaping filter* $g(t)$ and later integrated to produce the continuous phase of the radar waveform, denoted by

$$\psi(t;\overline{\boldsymbol{\alpha}}) = \int_0^t g(\tau) * \left[\sum_{n_c=1}^{N_c} \overline{\alpha}_{n_c}\delta(\tau-(n_c-1)T_c)\right] d\tau + \overline{\theta}_0, \quad (1)$$

where T_c is the time duration of the chip interval, $*$ denotes convolution, and $\overline{\theta}_0$ is the initial phase value. The shaping filter $g(t)$ has time support $[0, T_c]$, and the area under $g(t)$ is unity. The resulting PCFM radar waveform of time duration $T = N_c T_c$ is

$$s(t;\overline{\boldsymbol{\alpha}}) = \sqrt{P_t}e^{j\psi(t;\overline{\boldsymbol{\alpha}})}, \quad (2)$$

where P_t is the transmit power. We refer to $s(t;\overline{\boldsymbol{\alpha}})$ as the *base radar waveform*.

Now consider the communication symbol sequence $\boldsymbol{\beta} = [\beta_1, \cdots, \beta_{N_s}]$ drawn from the M-ary symbol alphabet $\{\pm1, \pm3, \cdots, \pm(M-1)\}$ where N_s is the number of symbols per pulse and $M = 2^m$, with m the number of bits/symbol. The symbol sequence $\boldsymbol{\beta}$ is modulated as a CPM waveform with symbol interval T_s and the communication shaping filter

$g_c(t)$ resulting in the signal phase $\phi(t;\boldsymbol{\beta})$ given by [24]

$$\phi(t;\boldsymbol{\beta}) = h\pi \int_0^t g_c(\tau) * \left[\sum_{n_s=1}^{N_s} \beta_{n_s}\delta(\tau-(n_s-1)T_s)\right] d\tau, \tag{3}$$

where $h = \frac{k}{p}$ is the *modulation index* with k and p mutually prime integers. It is important to note that the modulation index h controls the magnitude of the total phase change due to a communication symbol transmission while the magnitude of the maximum phase change is $h\pi(M-1)$ according to the M-ary symbol alphabet. The communication shaping filter $g_c(t)$ has time duration LT_s with L a positive integer, and the area under $g_c(t)$ is unity. When $L = 1$ the CPM signal is said to be *full-response*; otherwise (i.e. $L > 1$), it is *partial-response*. Partial-response CPM generally results in superior spectral containment at the expense of increased communication receiver complexity [24]. For the sake of illustration we focus on the full-response CPM. The signal phase $\phi(t;\boldsymbol{\beta})$ has the same duration (pulsewidth $T = N_cT_c$) as the base radar waveform. It follows that the chip duration and the communication symbol duration are related by

$$\frac{T_s}{T_c} = \frac{N_c}{N_s}. \tag{4}$$

For a given base radar waveform, i.e. fixed N_c and T_c, the communication symbol duration can be increased/decreased for a decreased/increased symbol rate.

To transmit the communication sequence $\boldsymbol{\beta}$ within the radar emission, $\phi(t;\boldsymbol{\beta})$ is phase-attached to the base radar waveform $s(t;\overline{\boldsymbol{\alpha}})$ which results in the information-bearing continuous-phase radar/communication waveform

$$\tilde{s}(t;\overline{\boldsymbol{\alpha}},\boldsymbol{\beta}) = \sqrt{P_t}e^{j(\psi(t;\overline{\boldsymbol{\alpha}})+\phi(t;\boldsymbol{\beta}))}. \tag{5}$$

The inspiration for combining distinct CPM-based waveforms in phase stems from the multi-rate CPM scheme introduced in [32]. We emphasize that $\boldsymbol{\beta}$ is a random communication sequence, while $\overline{\boldsymbol{\alpha}}$ is a fixed PCFM radar code. The base radar waveform $s(t;\overline{\boldsymbol{\alpha}})$ maintains a degree of similarity among the set of changing waveforms in the CPI that is uniquely specified by the modulation index h, the communication symbol alphabet size M, and the symbol duration T_s.

As h decreases the radar/communication waveforms become more similar, and for the limiting case of $h = 0$ they become identical radar waveforms (i.e. no communication takes place). Given that greater similarity among the waveforms reduces the severity of the RSM, a smaller modulation index h is desired from a radar performance perspective. As the number of bits/symbol m increases, the phase changes due to the communication sequence become greater in magnitude, and hence the waveforms become less similar. Consequently, a smaller m is desired from the radar performance perspective to reduce the severity of the RSM. Finally, as the communication symbol interval T_s increases, the degree of phase change due to the communication component decreases. Put another way, the total (possible) phase change due to the communication sequence decreases because the total phase change is limited

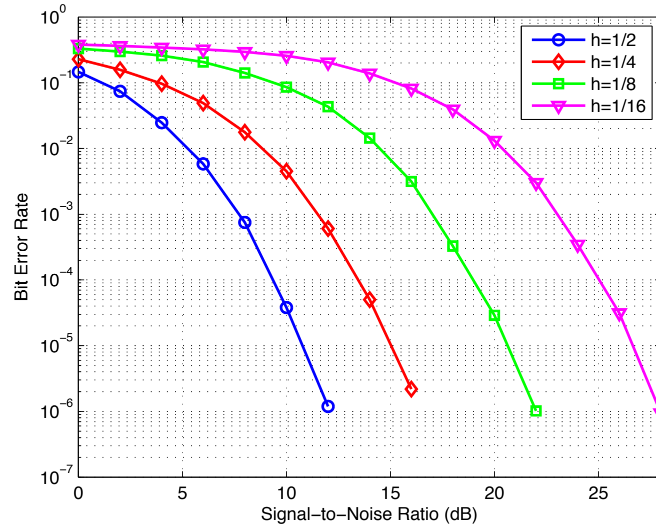

Fig. 2. The bit error rate (BER) of binary CPM with rectangular shaping filter ($T_s = T_c$) phase-attached to a PCFM-implemented LFM radar waveform for various values of the modulation index h.

by the number of symbols/pulse N_s, which is inversely proportional to the symbol duration T_s. This arrangement would likewise increase the similarity of the radar/communication waveforms and hence reduce the severity of the RSM. In addition, the total communication component duration N_sT_s can be selected smaller than the radar pulsewidth $T = N_cT_c$. Since the radar/communication waveforms are non-identical only for the duration of the communication component, this strategy would make the communication-embedded radar waveforms more similar.

Assuming knowledge of the base radar waveform and ideal synchronization at the communication receiver, demodulation via multiplication by $e^{-j\psi(t;\overline{\boldsymbol{\alpha}})}$ realizes

$$r(t;\boldsymbol{\beta}) = \sqrt{P_r}e^{j\phi(t;\boldsymbol{\beta})} + n(t), \tag{6}$$

where $n(t)$ is the resulting noise process and P_r is the received power. The optimal detection of $\boldsymbol{\beta}$ is achieved by the Viterbi algorithm [33] which operates on the $2p$-state trellis modeling the CPM modulator. From a communication performance perspective it is undesirable to reduce h, as doing so would increase the BER for a given signal-to-noise ratio (SNR) at the receiver [31] or require a higher SNR to maintain a certain BER. If channel coding is employed, the coding rate must be lowered as h is decreased to satisfy a given probability of decoding error constraint [34], [35]. Reducing the number of bits/symbol m decreases the data throughput since the number of bits per pulse is mN_s. For communications, therefore, it is desirable to increase m. Similarly, it is undesirable to increase the symbol interval T_s, as this strategy reduces the number of symbols/pulse N_s, and hence reduces the data throughput mN_s bits/pulse. To summarize, the proposed communication-embedded radar system allows for a trade-off between radar performance and communication performance according to the adjustable parameters h, m, and T_s.

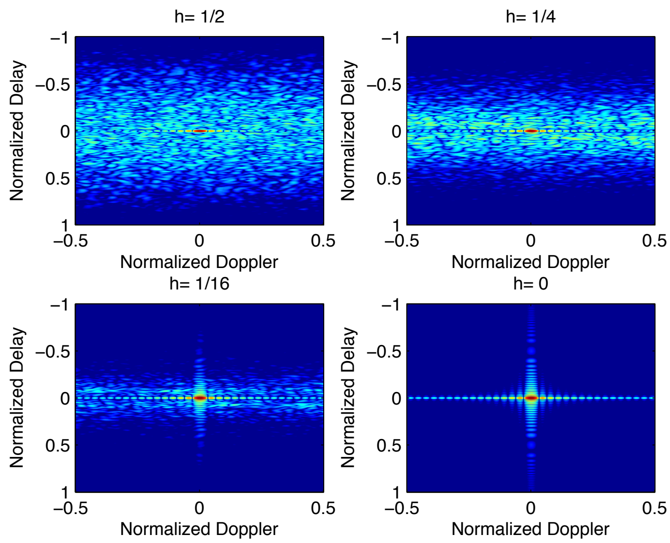

Fig. 3. The range-Doppler response from a point target as a function of the CPM modulation index h produced after pulse compression matched filtering and Doppler processing without windowing.

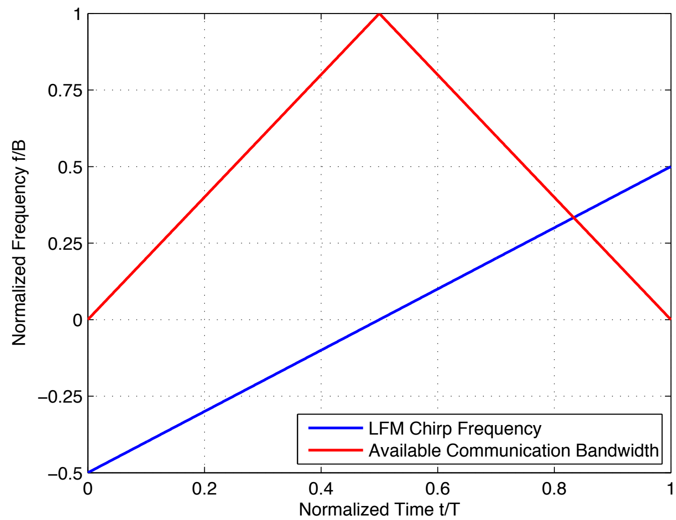

Fig. 4. LFM frequency and bandwidth available for communication normalized by available bandwidth B as a function of time normalized by pulse duration $T = N_c T_c$.

As an example, we examine the BER resulting from the use of binary CPM with a rectangular shaping filter ($T_s = T_c$) that is phase-attached to a PCFM-implemented LFM radar waveform [29] as a function of communication receiver SNR for modulation indices $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, and $\frac{1}{16}$. The BER curves are shown in Fig. 2. The required SNR to achieve a given BER increases with decreasing modulation index h. Figure 3 illustrates the radar range-Doppler response to a point target for modulation indices $h = 0, \frac{1}{2}, \frac{1}{4}$, and $\frac{1}{16}$. The case with $h = 0$ is included as a baseline where no RSM is present. The radar PCFM code length is $N_c = 64$, and the number of pulses in the CPI is $K = 32$. A random binary information sequence is generated for each pulse. The radar receive processing consists of pulse compression matched filtering and Doppler processing without windowing. As previously conjectured, reducing the modulation index reduces the RSM, which is observed where the radar-Doppler response for $h = \frac{1}{16}$ is very similar to the $h = 0$ response while the RSM for $h = \frac{1}{2}$ is severe. The degree of RSM can also be reduced through the use of appropriately designed mismatched filters [30], [36]. Here, however, we only focus on the system description and the various trade-offs between radar and communication performance.

III. Spectral Content

Given the inherent relationship between bandwidth and symbol rate, it is instructive to consider the spectral broadening of this communication-embedded radar mode relative to the spectral content of the base radar waveform. This effect can be observed by expressing $\tilde{s}(t; \overline{\boldsymbol{\alpha}}, \boldsymbol{\beta})$ from (5) as the product

$$\tilde{s}(t; \overline{\boldsymbol{\alpha}}, \boldsymbol{\beta}) = \left(\sqrt{P_t} e^{j\psi(t;\overline{\boldsymbol{\alpha}})}\right) e^{j\phi(t;\boldsymbol{\beta})}, \tag{7}$$

and noting that the Fourier transform of $\tilde{s}(t; \overline{\boldsymbol{\alpha}}, \boldsymbol{\beta})$ is the convolution of the Fourier transforms of the two multiplicands in (7). The spectrum of $e^{j\phi(t;\boldsymbol{\beta})}$, and thus $\tilde{s}(t; \overline{\boldsymbol{\alpha}}, \boldsymbol{\beta})$, broadens with increasing h and m and with decreasing T_s [31]. However, for LFM or other chirp-like radar waveforms (e.g. non-linear FM) the spectral broadening can be eliminated by the use of *guard symbols* equal to 0 at the beginning and end of the pulse. For chirp-like radar waveforms, these guard symbols translate into guard bands at the edges of the base radar waveform spectrum.

First note that the quantity $\frac{d\psi(t;\overline{\boldsymbol{\alpha}})}{dt}$ can be viewed as the (baseband) *carrier frequency* of the communication waveform $e^{j\phi(t;\boldsymbol{\beta})}$. In the case of an LFM base radar waveform with bandwidth B, the carrier frequency $f_c(t) = \frac{d\psi(t;\overline{\boldsymbol{\alpha}})}{dt}$ is time-varying and given by

$$f_c(t) = -\frac{B}{2} + \frac{tB}{T}, \text{ where } 0 \leq t \leq T, \tag{8}$$

and $T = N_c T_c$ is the pulse duration. Assuming that the communication-embedded radar waveform is constrained to occupy the same band as the base radar waveform, the bandwidth $W(t)$ available for communication is a function of time given by

$$W(t) = 2 \min\left\{ \left| f_c(t) + \frac{B}{2} \right|, \left| f_c(t) - \frac{B}{2} \right| \right\}, \tag{9}$$

where $0 \leq t \leq T$. The normalized LFM frequency and the normalized bandwidth available for communication are shown in Fig. 4 as a function of normalized time $\frac{t}{T}$ during the pulse.

Now assume that the power of CPM waveforms with parameters $h, M, g_c(t)$ and T_s is sufficiently contained within the band $[-B_c/2, B_c/2]$ (i.e. the out-of-band power is negligible) for $B_c < B$. Then, by transmitting communication symbols only during the time interval $[t_1, t_2]$, where $B_c \leq W(t)$ for $t_1 \leq t \leq t_2$, the spectrum occupied by the communication-embedded radar waveforms can be limited to $[-B/2, B/2]$. This arrangement is equivalent to transmitting

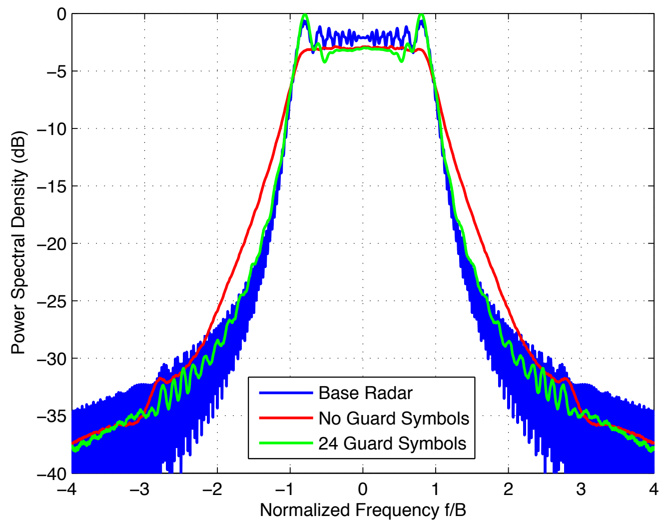

Fig. 5. PSD of the base radar waveform (dB) and communication-embedded radar PSDs (averaged over different communication sequences) for cases without guard symbols, and with $2N_g = 24$ guard symbols.

null communication symbols at the beginning and end of the pulse, i.e. $\beta_j = 0$ for $1 \leq j \leq N_g$ and $N_s - N_g + 1 \leq j \leq N_s$, for some N_g determined by the parameters $h, M, g_c(t)$ and T_s and the radar bandwidth B.

Given that a null symbol is equivalent to setting $h = 0$ or $m = 0$, this arrangement can alternatively be formulated as a special case of a time-varying modulation index or number of bits/symbol, i.e. $h(t) = h$ or $m(t) = m$ for $t_1 \leq t \leq t_2$ and $h(t) = 0$ or $m(t) = 0$ otherwise. When either quantity is defined as a function of time the communication bandwidth B_c becomes a function of time as well denoted by $B_c(t)$. A more spectrally efficient implementation from a communication performance perspective would aim to achieve $B_c(t) \approx W(t)$ by varying $h(t)$ and/or $m(t)$ throughout the pulse. Such an implementation would allow $h(t)$ and/or $m(t)$ to increase/decrease toward/away from the middle of the pulse (an up-down staircase pattern), which results in reduced BER and/or increased data throughput, respectively.

An an example, Fig. 5 compares the power spectral density (PSD) of the base radar waveform with the PSDs realized by communication-embedded radar waveforms (averaged over multiple independent information sequences) with and without guard symbols. The number of PCFM chips is $N_c = 64$ while the CPM parameters are $h = \frac{1}{4}$ and $m = 1$ bit/symbol. The symbol duration is equal to the chip duration ($N_s = N_c = 64$ symbols/pulse). There are $N_s - 2N_g$ and N_s communication symbols transmitted, with and without guard symbols, respectively. It is observed that the use of guard symbols eliminates the spectral broadening that otherwise occurs when no guard symbols are employed.

IV. Conclusions

A novel approach to embed communication symbols into radar emissions has been introduced that allows for a trade-off between radar performance and communication performance via adjustable parameters while still maintaining high power and spectral efficiency. For chirp-like radar waveforms, the spectral broadening arising from this form of communication-embedded radar can be addressed with the use of guard symbols.

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