Reconfigurable Complex Filtering Methods for PAPR Reduction of OFDM Signals with Low Computational Complexity

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***Abstract*— In this paper, we analyze the reconfigurable complex filtering techniques based on the Tone Reservation (TR) set and data bearing sub-carriers in the time-domain and in the frequency-domain to reduce the Peak-to-Average Power Ratio (PAPR) of OFDM signals. The simulation results of the proposed methods for 16-QAM and 64-QAM modulation are presented. These results show that PAPR of OFDM signals is significantly reduced (about 6.6 dB), and OFDM signals can reach the desired amplitude level after 2-4 iterations.**

Keywords*— *Peak-to-Average Power Ratio (PAPR), Orthogonal Frequency Division Multiplexing (OFDM), Tone Reservation (TR), clipping and filtering, FFT-IFFT-based complex filter, complex FIR filter, reconfigurable complex filter.

I. INTRODUCTION

Due to achievements of high data rate transmission over multipath fading channels and high spectrum efficiency, orthogonal frequency division multiplexing (OFDM) has been widely adopted in various broadcasting standards, such as IEEE 802.11a/g, IEEE 802.16, Digital Video Broadcasting (DVB) systems [1], [2], Digital Audio Broadcasting (DAB) [3], etc. One of the major challenges of OFDM systems is high PAPR due to the superimposing of a large number of subcarrier signals with the same phase. It results in a significant loss of power efficiency due to use a linear PA or causes in-band distortion and out-of-band radiation when it passes through a nonlinear PA. Several PAPR reduction techniques have been proposed [4], [5], including amplitude clipping, clipping and filtering (CAF), coding, partial transmit sequence (PTS), selected mapping (SLM), interleaving, tone reservation, tone injection (TI), and active constellation extension (ACE).

Amplitude clipping is the simplest technique for PAPR reduction. However, it causes in-band distortion and out-of-band radiation. Out-of-band radiation reduces spectral efficiency, but it can be reduced by filtering. Therefore, a filter after envelope clipping is used. This scheme is also called clipping and filtering. Nevertheless, in-band distortion cannot be reduced by this filter and results in an error performance degradation (BER, MER). And the peak will regrow after filtering. To reach a desired amplitude level, a repeated clipping-and-filtering can be used wherein reduced PAPR comes at a cost of increasing the in-band distortion [6]. Alternative methods are based on coding [7] to reduce the occurrence probability of the same phase of the signals. However, coding techniques are limited when the frame size is large and the cost of coding rate loss is very high. In addition, multiple signal representation techniques have been proposed, including PTS [8], SLM [9]. These techniques do not suffer from in-band distortion and out-of-band radiation, but they need a large number of IFFT/FFT calculations to find optimal phase sequences. Therefore, their computational complexity is too high. Another drawback of these methods is that the side information has to be transmitted from the transmitter to the receiver to recover the original data. Incorrectly received side information results in burst errors.

Two PAPR-reduction techniques, ACE and TR are adopted in the DVB-T2 standard and other OFDM communication systems due to absence of the signal distortion and side information. The ACE technique [10] reduces the PAPR by extending some of the outer constellation points in the frequency domain without changing the minimum distance in the signal space. To improve the efficiency of the ACE technique, adaptive ACE scheme is proposed in [11]. ACE provides significant reduction in PAPR for low order constellations. But, it is not used with high order QAM modulation up to 256 points, and it also cannot be used with rotated constellations. The TR technique was proposed by Tellado [12] based on using a small subset of subcarriers to form an impulse-like kernel. In each iteration, the largest peak is reduced by this kernel. In general, the higher number of iterations will give a lower PAPR, but this increases the amount of the processing delay. The number of iterations should be selected taking into account the requirements of the hardware implementations.

In this paper, we propose two filtering techniques to design the peak-canceling signal based on fast Fourier transform (FFT/IFFT) techniques [13] and Finite Impulse Response (FIR) Filter [14], respectively. These techniques can remove multiple peaks in a single iteration. They do not limit the number of peaks in an iteration. The peaks regrowth after

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filtering are reduced in the following iterations. Simulation results show that the proposed algorithms have significant PAPR reduction after the first iteration and could achieve the desired PAPR reduction with a very few iterations.

The remainder of this paper is organized as follow: Section II gives a brief description of the OFDM system basics, PAPR definition and TR-Based PAPR reduction techniques. The disadvantages of the PAPR reduction method adopted in the DVB-T2 standard are analyzed. Section III analyzes and describes two proposed algorithms using FFT/IFFT pair-based filter and the complex FIR filter. The simulation results are proved in section IV. Finally, section V concludes the paper.

II. OFDM Systems and Tone Reservation Technique

In this paper, |.| denotes the magnitude of a signal. $E\{.\}$ is the signal average power. Time and frequency domain vectors are denoted by lower-case and upper-case letters, respectively.

A. OFDM system basics

An OFDM signal is the sum of independent signals $X_0, X_1, ..., X_{N-1}$ modulated by N orthogonal subcarriers, out of which a set of N_r reserved tones called TR set $\Re$, $\Re = \{r_0, r_1, ..., r_{N_r-1}\}$ are reserved for PAPR reduction purposes and N_C subcarriers in the remaining $(N - N_r)$ tones named set $\Re^c$ are used for data transmission. The complex baseband samples of an OFDM signal are computed via an inverse discrete Fourier transform (IDFT or IFFT):

$$x_n + c_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (X_k + C_k) e^{j2\pi kn/N} \tag{1}$$

where X_k and C_k lie in disjoint frequency subcarriers, i.e., $X_k = 0, k \in \Re$ and $C_k = 0, k \notin \Re$.

The PAPR of an OFDM signal before and after peak-reduction are defined by *prePAPR* and *postPAPR*, respectively as follows:

$$\begin{cases} prePAPR = \dfrac{\max\limits_n |x_n|^2}{E\left\{|x_n|^2\right\}} \\ postPAPR = \dfrac{\max\limits_n |x_n + c_n|^2}{E\left\{|x_n + c_n|^2\right\}} \end{cases} \tag{2}$$

To measure the in-band distortion level due to configuration of the filter on the data bearing sub-carriers in the last iteration, we consider the Modulation Error Rate (MER) estimated in frequency domain by:

$$MER\left\{X_k, \hat{X}_k\right\} = \frac{\sum\limits_{k\in\Re^c} \left|X_k - \hat{X}_k\right|^2}{\sum\limits_{k\in\Re^c} |X_k|^2} \tag{3}$$

where $\hat{X}_k$ represents the modulated version of X_k after the PAPR reduction.

B. TR-Based PAPR Reduction Technique

TR-Based PAPR Reduction Technique has been widely utilized in the OFDM systems, since the additional signal causes no distortion on the data bearing subcarriers due to the orthogonality of tones. The idea of TR schemes is to find the value of C_k such that $PAPR(x+c) < PAPR(x)$.

The TR method applied in the OFDM systems, such as in the DVB-T2 standard [1] is implemented by utilizing an impulse-like kernel. The disadvantage of this method consists in detection and removing only one maximum peak in the time-domain per iteration. The desired clipping level and the number of iterations should be selected taking into account the requirements of the hardware modules. Moreover, in fact analog signal peaks after the Digital to Analog Converter (DAC) and the high PA can regrow, since the Nyquist rate sampled OFDM signals cannot correctly represent the peak power of the continuous-time OFDM signals. Hence, the oversampled OFDM signal should be considered to better approximate the peak accuracy. In this case, the number of peaks increases about L times where L represents the oversampling factor. This will lead a large number of iterations needed to meet the target threshold if the traditional TR method is used.

III. Proposed Methods

The proposed methods are based on a hard clipping function of the input OFDM signal and using a complex filter to design a noise (called the correction signal) as close as possible to the clipping noise. The clipped signal can be represented as follows:

$$\underset{0\le n\le N-1}{\bar{f}_n(x_n)} = \begin{cases} x_n, & |x_n| \le A \\ Ae^{j\theta_n}, & |x_n| > A \end{cases} \tag{4}$$

where A is the desired clipping level, and $x_n = |x_n| e^{j\theta_n}$.

The clipping noise is the difference between the samples of the input OFDM signal and its clipped version, and can be obtained as:

$$f_n(x_n) = x_n - \bar{f}_n(x_n) \tag{5}$$

This clipping noise is passed to a filter to obtain the correction signal. The complex filter configured on the TR set is characterized by frequency response as:

$$\underset{0\le k\le N-1}{H_k} = \begin{cases} 1, & k \in \Re \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

The impulse response of this filter is obtained from taking the Inverse Fourier Transform of the frequency response:

$$h_n = IFFT(H) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_k e^{j2\pi kn/N} = \frac{1}{\sqrt{N}} \sum_{k\in\Re} e^{j2\pi kn/N} \quad (7)$$

We could consider a complex filter as an N-order FIR filter which performs a weighted sum (also known as discrete convolution) on a window of N data samples. The output of an N-weight FIR filter is given by:

$$c_n = \sum_{k=0}^{N-1} h_k f_{n-k} = h_n * f_n \quad (8)$$

where (*) is a convolution operation.

The clipping noise f_n is a series of pulses. We assume that there are P peaks exceeding the threshold A, which occur at the locations $n_1, n_2, \ldots, n_P$. S_P is the index set of the peaks of f_n. First, let us consider a special case where only one peak at the location $n_i \in S_P$ is limited. In this case, the clipping noise can be expressed as follows:

$$f_n = f_{n_i} \delta(n - n_i) \quad (9)$$

where $\delta(n)$ is a Dirac delta function.

The z-transform of equation (8) gives:

$$C(z) = H(z)F(z) = f_{n_i} z^{-n_i} H(z) \quad (10)$$

From the equation (10) we see that noise generated by the complex filter is the impulse response h_n which is circularly shifted by n_i to the right and scaled by clipping noise amplitude at that position. After the circular shift, the primary peak ($h_n(0)$) of the impulse response h_n is shifted to the peak location of the clipping noise f_n. Moreover, from the equation (7), the primary peak $h_n(0)$ of the impulse response h_n is the largest and the others ($h_n(1)$ to $h_n(N-1)$) have a value considerably smaller than $h_n(0)$, i.e. the output of this filter is as close as a discrete-time impulse and its peak coincides with the peak of the clipping noise. To cancel the considered peak, the output peak amplitude of the filter has to approximate to sample $f_n(n_i)$ of the clipping noise f_n. Therefore, the impulse response of this filter need to be selected so that $h_n(0) = 1$, i.e.

$$\underset{0\le n\le N-1}{h_n} = \frac{1}{N_r} \sum_{k\in\Re} e^{j2\pi kn/N} = \frac{\sqrt{N}}{N_r} IFFT(H) \quad (11)$$

In general case, the time-domain clipping noise and the z-domain correction signal on the output of complex filter are represented as follows:

$$f_n = \sum_{n_i\in S_P} f_{n_i} \delta(n - n_i) \quad (12)$$

$$C(z) = H(z)F(z) = \sum_{n_i\in S_P} f_{n_i} z^{-n_i} H(z) \quad (13)$$

In the frequency-domain, the clipping noise and the output of the filter can be expressed as:

$$F_k = FFT(f_n) = \sum_{n=0}^{N-1} f_n e^{j2\pi nk/N} = \sum_{n\in S_P} f_n e^{j2\pi nk/N} \quad (14)$$

$$C_k = F_k H_k = \frac{\sqrt{N}}{N_r} \underset{k\in\Re}{F_k} \quad (15)$$

According to the equation (15), only the discrete frequency components F_k on the reserved tones are kept while the others are reset to zero.

The correction obtained on the output of the filter can significantly decrease the peaks on set S_P. However, the correction samples c_n on the other locations are nonzero that causes the peak regrowth after the filter exceeding the clipping level for any of the N samples of the symbol. Hence, generating the peak reduction signal should be repeated several times to suppress secondary peaks.

In [15] iterative estimation and cancellation of clipping noise is proposed. Since, the clipping noise is generated by a known process, it can be recreated and eliminated from the received signal. Hence, in the last iteration, only the out-of-band radiation is removed. Our filter is reconfigured by using data bearing sub-carriers. Similarly, the frequency and impulse responses of the filter in this configuration are given:

$$\underset{0\le k\le N-1}{H_k} = \begin{cases} 1, & k \in \Re^c \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

$$\underset{0\le n\le N-1}{h_n} = \frac{1}{N_c} \sum_{k\in\Re^c} e^{j2\pi kn/N} = \frac{\sqrt{N}}{N_c} IFFT(H) \quad (17)$$

In this filter configuration, a small clipping noise is added to the transmitted signal. Distortion will decrease with increasing number of iterations in using the filter configured on TR set. In practice, modulators such as DVB-T/T2 exciter and etc. require MER about 42 dB, receiver can reconstruct

Algorithm 1: FFT/IFFT-based complex filter

Initialization: Choose the desired clipping threshold A, TR set $\Re$, and a maximum iteration number *MaxIter*, typically, 2-4 iterations.
Runtime: This algorithm is invoked for each OFDM symbol. For the m-th symbol:

1. Initial condition: set $x = x^{(m)}$ and set the initial values of correction to zeros: $c^{(0)} = \left\{c_n^{(0)} = 0 \middle| 0 \le n \le N-1\right\}$.
2. i starts from 1.
3. Calculate $f_n(x + c^{(i-1)})$ by using (5).
4. Calculate F_k and C_k using (14) and (15), respectively; then update the peak reduction signal $c^{(i)}$ as follows: $c^{(i)} = c^{(i-1)} - IFFT(C_k)$
5. Increment the iteration counter, $i = i + 1$. If $i < MaxIter$, return to step 3. Otherwise, go to step 6.
6. Calculate the clipped signal $\overline{f}_n(x + c^{(MaxIter-1)})$ by (4).
7. Convert $\overline{f}_n(x + c^{(MaxIter-1)})$ to the frequency domain. Keep the frequency components on set $\Re^c$ and reset the others to zero. Then, convert these frequency components to the time domain to obtain new signal $\overline{x}$.
8. Transmit $x' = \overline{x} + c^{(MaxIter-1)}$.

Algorithm 2: complex FIR filter

Initialization:
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4. Calculate the correction $c = \left\{c_n \middle| 0 \le n \le N-1\right\}$ by using (8) and (11), and then, update the peak reduction signal $c^{(i)}$ $c^{(i)} = c^{(i-1)} - c$

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7. Put $\overline{f}_n(x + c^{(MaxIter-1)})$ to the input of the filter with impulse response (17) to obtain new signal $\overline{x}$.
8. Transmit $x' = \overline{x} + c^{(MaxIter-1)}$.

the transmitted signals in the bad receiving conditions without clipping noise estimation.

From the above analysis, two proposed techniques can be summarized in Algorithm 1 and Algorithm 2, respectively. The two proposed algorithms differ only by used filtering methods. In the algorithm 1, the complex filter is based on FTT/IFFT pair [13] to design the peak reduction signal while for the algorithm 2, FIR-filters are used to implement that complex filter. In the step 4, these filters are configured on the TR set. This configuration of filter can be iterated. In the step 7, these filters are reconfigured by using the data bearing sub-carriers. The FFT/IFFT-based filter complexity is equivalent to the complexity of utilizing the FFT/IFFT pair while the complexity of algorithm 2 mainly depends on the selected FIR filter architecture. The filter in algorithm 1 calculates the value of $\{C_k\}$ set of correction signal in frequency domain and then converts this set to time domain, whilst that in algorithm 1 works only in time domain.

On the other hand, FFT and IFFT can be efficiently realized on hardware implementation by supporting the IP Core [13] integrated in the design Tools (ISE, Vivado, System Generator for DSP) while a complex domain FIR filter is not available in these design Tools. So, algorithm 1 shall be chosen in our hardware designs. Although, it can be seen that both algorithms give similar results in our tests on Matlab.

IV. Simulation Results

In this section, we evaluate the performance of the proposed algorithms in terms of PAPR reduction capability using the complementary cumulative distribution function (CCDF) system interference MER and the increase in the mean power, where CCDF represents the probability that PAPR exceeds a given threshold, *PAPR*0. We use normalized 16-QAM and 64-QAM modulation symbols as the input of the OFDM system, the number of N = 1024 subcarriers, a randomly selected TR set with 5% of the total subcarriers and a clipping threshold $A \approx 6.6$ dB.

The configuration parameters used for the simulations and simulation results are shown in Tab. I.

TABLE I. The Simulation Parameters and Results

System Parameters	Parameter Value	
Modulation scheme	16-QAM	64-QAM
Number of subcarriers	N = 1024	N = 1024
Number of data subcarriers	999	999
Number of pilot subcarriers	50	50
Number of OFDM symbols	10000	10000
Number of iterations	3	4
Aclip (dB)	6.67 dB	6.62 dB
Simulation Results		
PAPR of original OFDM signal (dB)	13.35	13.24
PAPR after peak reduction (dB)	6.71	6.63
PAPR reduction (dB)	6.64	6.61
MER (dB)	-50.45	-56.76
Average power increase (dB)	0.072	0.082

Fig. 2 and Fig. 3 show the peak power reduction results of the proposed algorithms at different iterations. We observe that the proposed algorithms can obtain the desired amplitude with three and four iterations for 16-QAM and 64-QAM modulation schemes, respectively. After the first iteration, about 4.7 dB PAPR reduction from that of the original OFDM

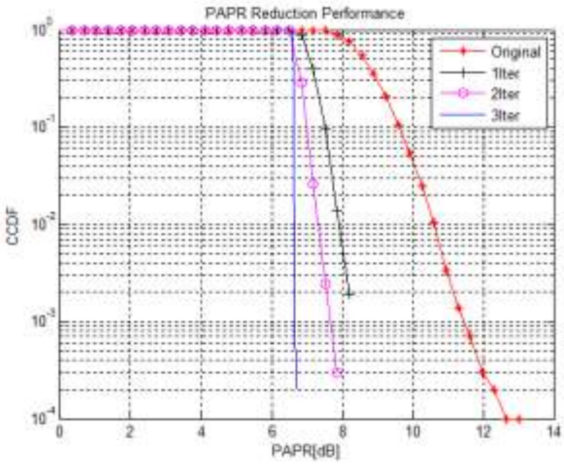

Fig. 1. PAPR CCDF of the proposed algorithms for 16-QAM modulation.

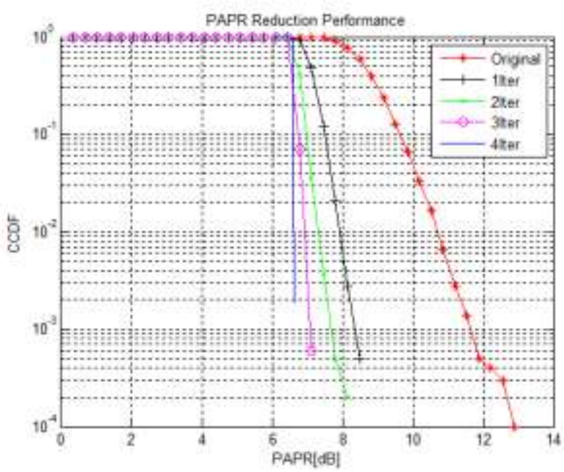

Fig. 2. PAPR CCDF of the proposed algorithms for 64-QAM modulation.

signal can be achieved for both modulation schemes. After three iterations for 16-QAM modulation and four iterations for 64-QAM modulation, PAPR reductions are 6.64 dB and 6.60 dB at -50.45 dB and -56.76 dB MER value, respectively.

The signal average power for our algorithms is nearly unchanged, it is increased about 0.1 dB. This can meet the high power amplifier constraints.

V. CONCLUSIONS

In this paper, we have proposed two reconfigurable complex filters for PAPR reduction of OFDM signal. Configuration of the filter on TR set is iteratively used to significantly suppress peaks of OFDM signal. It does not introduce in-band distortion and out-of-band radiation into the transmitted signal. Reconfiguration of the filter on the data bearing sub-carriers introduces a low clipping noise into the transmitted signals, but peaks quickly down to the desired threshold level, and secondary peaks do not appear. Distortion will decrease with increasing number of iterations in using the filter configured on TR set.

Both algorithms give similar results in terms of PAPR reduction capability, expense of system interference MER and the increase in the mean power. In the future works, we will realize these algorithms in FPGA hardware implementations and combine them with other known schemes to increase the PAPR reduction gain.

REFERENCES

[1] ETSI EN 302 755 V1.4.1, "Digital video broadcasting (DVB); Frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system," July 2015, 188p.

[2] ETSI EN 302 769 V1.3.1, "Digital video broadcasting (DVB); Frame structure channel coding and modulation for a second generation digital transmission system for cable systems," Oct. 2015, 112p.

[3] ETSI EN 300 401 V2.1.1, "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers," Jan. 2017, 124p.

[4] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," IEEE Wireless Communications, vol. 12, no. 2, pp. 56–65, April 2005.

[5] T. Jiang and Y. Wu, "An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals," IEEE Transactions on Broadcasting, vol. 54, no. 2, pp. 257–268, June 2008.

[6] Shang-Kang Deng , Mao-Chao Lin, "Recursive Clipping and Filtering With Bounded Distortion for PAPR Reduction," IEEE Transactions on Communications, vol. 55, no. 1, pp. 227–230, Jan. 2007.

[7] J. A. Davis and J. Jedwab, "Peak-to-mean power control in OFDM, Golay complementary sequences, and Reed-Muller codes," IEEE Transactions on Information Theory, vol. 45, no. 7, pp. 2397–2417, Nov. 1999.

[8] M.V.R. Vittal, K. Rama Naidu, "A novel reduced complexity optimized PTS technique for PAPR reduction in wireless OFDM systems," Egyptian Informatics Journal, Nov. 2016.

[9] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," IEEE Electronics Letters, vol. 32, no. 22, pp. 2056–2057, Sep. 1996.

[10] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," IEEE Trans. Broadcast., vol. 49, no. 3, pp. 258–268, Sep. 2003.

[11] K. Bae, J.G. Andrews, and E.J. Powers, "Adaptive active constellation extension algorithm for peak-to average ratio reduction in OFDM," IEEE Commun. Lett., vol. 14, no. 1, pp. 39–41, Jan. 2010.

[12] J. Tellado, "Peak to average power reduction for multicarrier modulation," Ph.D. dissertation, Stanford Univ., Stanford, CA, 2000.

[13] Xilinx, pg109, "Fast Fourier Transform v9.0," LogiCORE IP Product Guide, 97p, Nov. 2015.

[14] Xilinx, pg149, "FIR Compiler v7.2," LogiCORE IP Product Guide, 131p, Nov. 2015.

[15] H. Chen and M. Haimovish, "Iterative Estimation and Cancellation of Clipping Noise for OFDM Signals," IEEE Commun. Lett., vol. 7, no. 7, pp. 305–307, July 2003.