

Hadamard based SLM using Genetic Algorithm for PAPR Reduction in OFDM Systems

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Abstract— In the recent years there is a drastic improvement in the communication systems. The discovery of Orthogonal Frequency Division Multiplexing technique (OFDM) plays a major role in the high data rate communication system. However, the main drawback of OFDM systems is high Peak to Average Power Ratio (PAPR). Many of the existing PAPR reduction methods have drawbacks such as high complexity, increased Bit Error Rate (BER) and excess redundant information. Recently, precoding techniques has been investigated for PAPR reduction. Due to its low complexity and reduced cost of implementation this technique is an attractive emerging method that yields a substantial reduction of PAPR. In this paper, a novel precoding based on Hadamard Selected Mapping (HSLM) method and genetic algorithm (GA) has been proposed for PAPR reduction in OFDM systems.

Keywords— Bit Error Rate; precoding techniques; Peak to Average Power ratio; Genetic Algorithm; Selected Mapping.

I. INTRODUCTION

The ever increasing thirst for more information in the recent past has bolstered demand for high speed data networks. Orthogonal frequency division multiplexing (OFDM) was a major breakthrough in realizing high speed data transmission and efficient spectrum utilization. High peak to average power ratio PAPR remains a major challenge in implementation of OFDM systems which outweighs all the advantages it has to offer. The phase of the input signal is modified by the Inverse Fast Fourier Transform (IFFT) block at OFDM transmitter causing a high PAPR. When all the signals are added together with the same phase after the modulation operation, at the transmitter, it leads to high PAPR, which may cause nonlinear distortion in the high-power amplifiers (HPA) and increase the power consumption at the transmitter.

Some of the currently existing methods in literature [1] for PAPR reduction are clipping and filtering[2], companding [3], Selective Mapping [4], tone reservation and tone injection [6], partial transmission sequence [5], coding schemes [7], active constellation mapping etc. While all these techniques have been successful at reducing PAPR they cease to be attractive due to various limitations such as distortion effects which lead to increased BER, excess side information, increased computational complexity, a high coding overhead and the necessity of a handshake between transmitter/receiver. Recently, however precoding methods have become an attractive option for PAPR reduction.

The advantages of precoding techniques are multi fold. As data is transformed or coded before transmission in pre-coding techniques, it saves time and ensures high speed transmission. Further, it has less complexity considerably and results in no distortion. Necessity for handshake between the transmitter side and the receiver side is eliminated. Each of the entries of the precoding matrix should be carefully designed in such a way to reduce PAPR.

Selected Mapping (SLM) is one of the most commonly proposed methods for PAPR reduction. However, it has several disadvantages. The large number of IFFT transforms needed result in high computational complexity and side information needs to be transmitted which reduces the data rate. If the side information is misinterpreted at the receiver the entire OFDM symbol is recovered with errors. Thus, the side information cannot be sent without stringent protection and consequently more loss of data transmission is incurred. Thus, this paper aims to propose a hybrid precoding technique in order to improve the PAPR in OFDM systems.

The organization of the paper is as follows. Section II focuses on the background information about OFDM and PAPR. Section III focuses on the existing SLM based method for PAPR reduction. The proposed method is presented in Section IV and the results are discussed in section V. Finally, Section VI concludes major findings of the paper.

II. BASIC OFDM SYSTEM

A. System Model

The OFDM signals are generated by means of multiple modulated carriers that are transmitted in parallel until they get shaped as OFDM symbols. The implementation of this model is relatively simple and spectrally efficient. OFDM systems make use of the IFFT that translate the signals from frequency to time domain and the vice versa is carried out at the receiver by Fast Fourier Transform (FFT). The system model for the generation of OFDM symbols is shown in Fig.1. Initially, all the data bits are mapped on the constellation map by using modulation techniques such as Binary Phase Shift Keying (BPS) or Quadrature Phase Shift Keying (QPSK) etc. It is followed by serial to parallel converter that converts the data bits from serial to parallel.

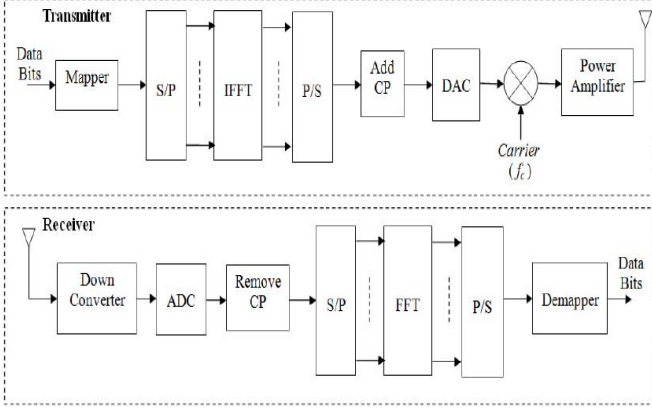


Fig 1, Block diagram of OFDM system

If there are N subcarriers assumed in the system, then N symbols are transformed from serial transmission to parallel communication. These N symbols then undergo IFFT. The discrete time domain representation of the symbol is represented as follows:

$$x(l) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\left(\frac{2\pi}{N}\right)kl}, l = 0, 1, 2, \dots, N-1 \quad (1)$$

where $X(k)$ refer to the discrete time Fourier transformed symbol at the k^{th} subcarrier after carrying out constellation mapping.

The continuous time domain representation of the generated OFDM symbol is given by the Eq. (2):

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi f_i t}, 0 < t < NT \quad (2)$$

where $f_i = i\Delta f = (i/NT)$ refer to the fundamental frequency and T signifies the symbol period of the OFDM system. The next step involves adding a cyclic prefix. The portion of the OFDM symbol is copied attached as a prefix at the start of the symbol. This is carried out to avoid Inter Symbol Interference (ISI). As a rule of thumb, the duration of the cyclic prefix is typically 10-25% of the OFDM symbol.

At the receiver side the exact opposite process takes place. Firstly the cyclic prefix is removed then the data sequence is converted from series to parallel. Then the FFT of this block is computed then it is converted back to serial from parallel. The data bits are demodulated and the information is recovered.

B. Peak to average power ratio (PAPR)

The PAPR is the peak to average power ratio. It is caused due to the presence of the IFFT block where all the OFDM signals are added together in phase leading to high peak power. A high PAPR is undesirable for an OFDM system as it causes the input signal to be outside the range of the high power amplifier at the transmitter as well as increase the power requirement of the system.

If $x(t)$ refers to the continuous time signal and if its discrete time version is given as $x[n]$, then the PAPR of the system is defined as the ratio of maximum instantaneous power of the transmitted OFDM symbols to the average power. It is expressed as follows:

$$PAPR(x[n]) = \frac{\max |x[n]|^2}{E[|x[n]|^2]} \quad (3)$$

where $E[\cdot]$ is the expectation operator. It has to be noted that the PAPR is estimated per OFDM symbol.

C. Precoding methods

The precoding methods as shown in Fig.2, prior to OFDM modulation, the modulated data of each OFDM block is multiplied by a precoding matrix. If all OFDM blocks have the same precoding matrix all the processing required in block-based optimization techniques become unnecessary. The research in [8] proposes and analyzes a design procedure for good precoding schemes.

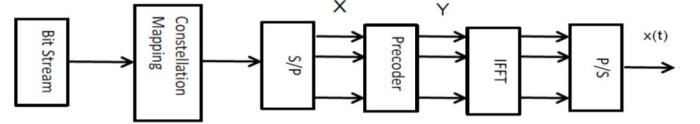


Fig 2: Block diagram of OFDM system with precoding

Precoding methods use multiple transforms or matrices in an attempt to lower the peak of OFDM system. Here the modulated symbol is multiplied with the precoding matrix before it is passed through the IFFT block. Walsh Hadamard Transformation is a simple linear transformation. The complexity of the system is not increased due to WHT. The equations (4) show the mathematical representation of WHT:

$H_1 = [1]$ and $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and it can be generalized as follows:

$$H_N = \frac{1}{\sqrt{2N}} \begin{bmatrix} H_N & H_N \\ H_N & H_N^{-1} \end{bmatrix} \quad (4)$$

H_N^{-1} is the binary complement of H_N .

Walsh functions are square waves which assume a value of +1 or -1. The main reason for high PAPR is the high auto correlation property of the IFFT block. By using WHT to lower the autocorrelation of the input sequence the occurrence of high peaks can be reduced vis-a-vis to the original OFDM system as shown by the work in [9]. This technique does reduce PAPR without the transmission of any side information. Further analysis of this work is done in [10] where the WHT is compared to the Normalized complex Hadamard Transform (NCHT) and Unified complex Hadamard transform (UCHT). The results display that the NHCT has a better PAPR compared to the other two methods.

III. EXISTING METHODS

A. Conventional SLM method

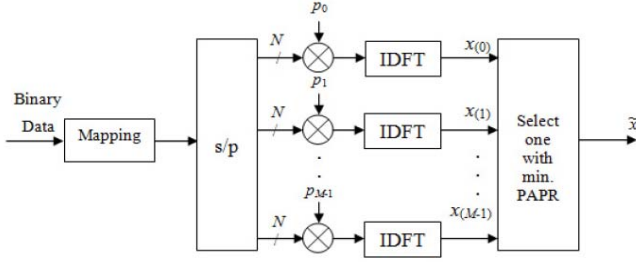


Fig. 3 Block diagram of conventional SLM method

The block diagram of conventional SLM method is shown in Fig. 3. Firstly, all the data bits are mapped on the constellation map by using a modulation technique such as Binary Phase Shift Modulation, Quadrature amplitude modulation etc. In the next step a serial to parallel converter converts the data bits from serial to parallel. Let us suppose that there are N subcarriers. Then, N of the symbols are converted from serial to parallel. Now a set of M vectors are generated. Each of these vectors are sets of N phasors. The OFDM block of N symbols is multiplied separately by each of these phasors to generate M samples. The PAPR of each of these M samples are then calculated. Next the IFFT of all these samples is computed. The sample with the least PAPR with N symbols is selected for transmission.

B. WHT-SLM method

The results of the regular SLM method can be improved when the SLM technique is combined with a WHT precoding matrix [10].

The block diagram for the WHT-SLM method is shown in Fig. 4.

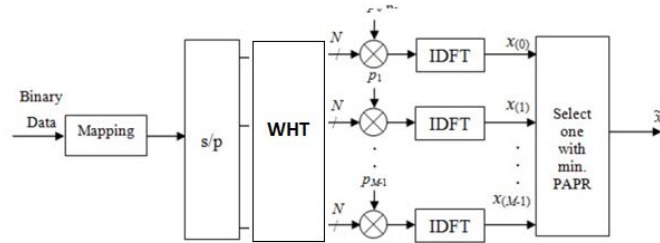


Fig 4 block diagram of WHT-SLM method

At the transmitter, the data bits are mapped to the form sequence symbols. Later, these symbol sequences are separated into blocks of length N . After that, each block $x = [x_0, x_1, \dots, x_{N-1}]$ is transformed to vector $v = [v_0, v_1, \dots, v_{N-1}]$ by multiplying x with the Hadamard matrix. The equation (5) for the same is given below

$$v = H_N \cdot x \quad (5)$$

Now, let us assume that there are D candidate phase sequences. The vector v is multiplied by each of the D phase sequences given by $\{\phi^{(d)}\}_{d=0}^{D-1}$. The IFFT of each of the candidate sequences is computed and the transmitter selects that sample which has the minimum PAPR using equation (6).

$$\tilde{d} = \operatorname{argmin} \operatorname{PAPR}(v \cdot e^{j\phi^{(d)}}), d = 0, 1, \dots, D-1 \quad (6)$$

Considering the multipath transmission in the channel, the OFDM symbol construction is followed by the insertion of the Cyclic Prefix (CP) to lessen the effect of ISI. Alternatively, these CP are removed before carrying out FFT at the receiver. The received signal after signal transmission over the channel is expressed as follows:

$$\begin{aligned} r_n &= h_n v_n^{(\tilde{d})} + w_n \\ &= h_n v_n e^{j\phi_n^{(\tilde{d})}} + w_n, n = 0, 1, \dots, N-1 \end{aligned} \quad (7)$$

where w refer to the zero mean of the AWGN channel and $N_0/2$ is the variance of the real and imaginary components. Here, it is assumed that the response of the channel, h_n , is perfectly known and hence the r_n are considered independent.

At the receiver side, the main aim is to correctly determine the phase sequence \tilde{d} that was transmitted. In this system, it is assumed that the transceiver consider the same phase sequences set as represented by $\{\phi^{(d)}\}_{d=0}^{D-1}$ and share the same Hadamard matrix H_N . The estimation of the transmitted SLM phase sequence, \tilde{d} , depends on the following properties:

- 1) For a complex-Gaussian signal s , the Kurtosis k_2 is defined as the ratio of the fourth moment $M_{4,2}(s)$ and the second moment $M_{2,1}(s)$ of the signals as shown in equation (8):

$$k_2 = \frac{M_{4,2}(s)}{M_{2,1}(s)} = \frac{E(\|s\|^4)}{E(\|s\|^2)^2} \quad (8)$$

- 2) The Kurtosis k_2 of a white Gaussian noise is equal to 2

Since the kurtosis of a signal is a measure of how outlier prone the data is the receiver should select the one with the minimum PAPR.

While these methods improve PAPR, one of the shortcomings is that for the sake of simplicity the only phasors that were selected belonged to $\{\pm 1, \pm j\}$. Furthermore depending on the value of M we would have to multiply the sample with all the M phasors and compute PAPR of each of them before we reach the optimum result.

In order to consider a large number of possibilities and chose and optimum phase factor an optimization technique is needed. This paper uses the Genetic algorithm for the same. The next section explains the proposed method using the genetic algorithm.

IV. PROPOSED METHOD

A. Genetic Algorithm

The Genetic algorithm is a metaheuristic algorithm that is based on the natural process of evolution. The benefit in the use of genetic algorithm is it repeatedly modifies the population of individual solutions by selecting individuals at random from the present population and uses them to produce solution for the next generation. Over successive generations, the population "evolves" toward an optimal solution. The stopping criteria could be a wide variety of things such as maximum number of generations, time span, or an optimal fitness function value.

B. Methodology

A random sample set is taken initially and the PAPR of all the initial samples are calculated. Then the ones with the lowest PAPR are used as parents of the next generation. Based on a cross over or a mutation a new sample set is created for the next generation and the same process is repeated for a certain number of generations. Eventually we will get the optimum phase factor for best PAPR.

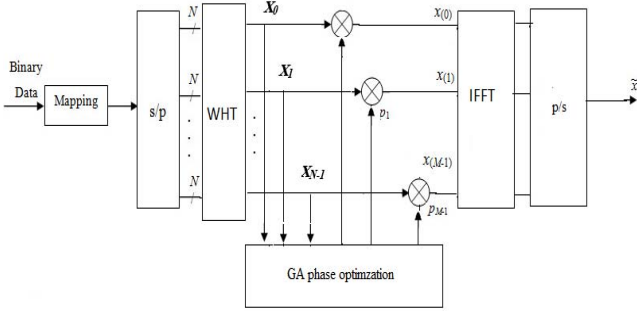


Fig. 5 Block diagram of WHT-SLM with GA phase optimization

The Figure 5 shows the block diagram of the genetic algorithm based WHT-SLM method. The methodology of the proposed method is similar to that of the already existing WHT-SLM method except instead of having a predefined set of phase sequences, each of which is tested to find minimum PAPR the method proposes the use of the genetic algorithm to find the best phase sequence p . The algorithm for implementing GA optimization to find best phase sequence is given below.

1. Randomly select the size of the first population and initial population. During the selection, each gene refers to the vector of phase factor candidate.
2. Calculate the PAPR value for each gene by multiplying the signal X with the assumed set of phase spin factors, which is defined by means of fitness function.
3. The step is followed by the selection of genes with minimum PAPR value.
4. After the selection, crossover and mutation is carried out for all the genes to produce new genes.
5. Step 2 is repeated using new generated population and the process is repeated until termination or reach of the maximum number of generations. The phase rotation

factors with lowest PAPR is used as the transmitted data and it is communicated to the receiver.

After the phase sequence is determined through the genetic algorithm, the phase sequence p is multiplied with vector v from eqn 3.2. The IFFT of this block is performed and the signal is transmitted to the receiver.

V. RESULTS AND DISCUSSION

The proposed method was simulated using MATLAB 2013a software. This section contains all the details of the simulation and results that were obtained.

The specifications for the OFDM systems that were simulated are given in table 1.

TABLE 1: Simulation Parameters of the WHT-SLM-OFDM system with Genetic Algorithm

Specification	Value
Initial population	30
No of iterations	50
Crossover probability	0.8
Mutation probability	0.2

Figure 6 is PAPR vs CCDF plot. Computer simulations for the conventional OFDM method, conventional SLM method, WHT method and the HSLM methods were performed. Two cases were considered for $N=128$ and $N=256$.

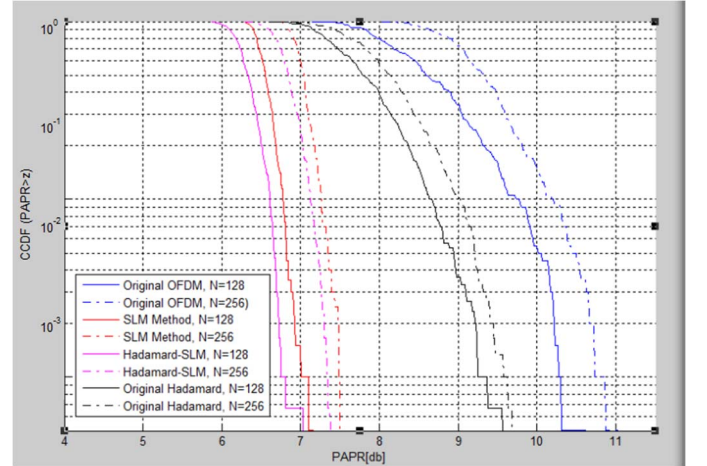


Fig 6 CCDF plot for different OFDM based systems $N=128, 256$ and $D=8$

The results show that the method proposed i.e. HSLM method which combines a precoding method with a normal SLM system has the best PAPR. It can be seen that as the number of subcarriers increases the PAPR also increases because of the large number of signals that are getting added in phase at the IFFT block. There is a significant improvement in PAPR as compared to the using the conventional Hadamard precoding method and the conventional SLM method. This shows that the hybrid method has a better performance as compared to the individual performance of both the methods individually.

Figure 7 shows the simulation for the WHT-SLM OFDM system with Genetic algorithm to select the optimum phase factor at the transmitter. The computer simulations have been

done for the values $N=16, 64, 128, 256$. The modulation technique used in this system was 4 QAM (quadrature amplitude modulation).

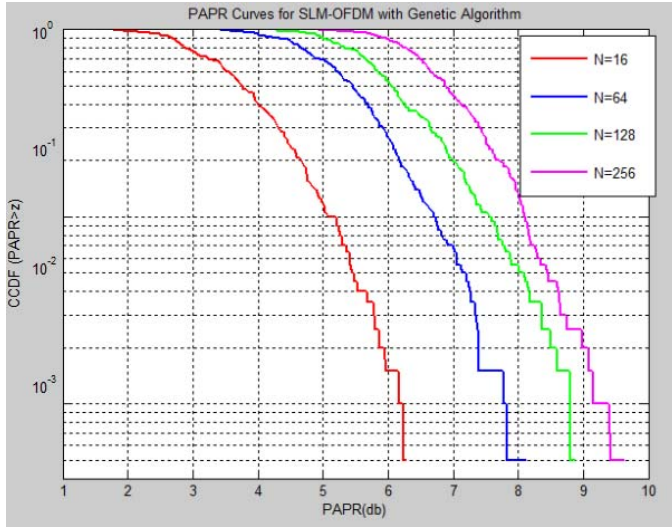


Fig 7 CCDF plots WHT-SLM-OFDM system with GA for $N=16, 64, 128, 256$

The results of the simulation shows that WHT SLM with GA optimization shows similar trends as all SLM based methods. The PAPR of the systems increases with an increase in the number of subcarriers.

In figure 8, simulations for the conventional SLM method, WHT method and the HSLM method, and the WHT-SLM method with GA optimization were performed. Two cases were considered for $N=128$ and $N=256$.

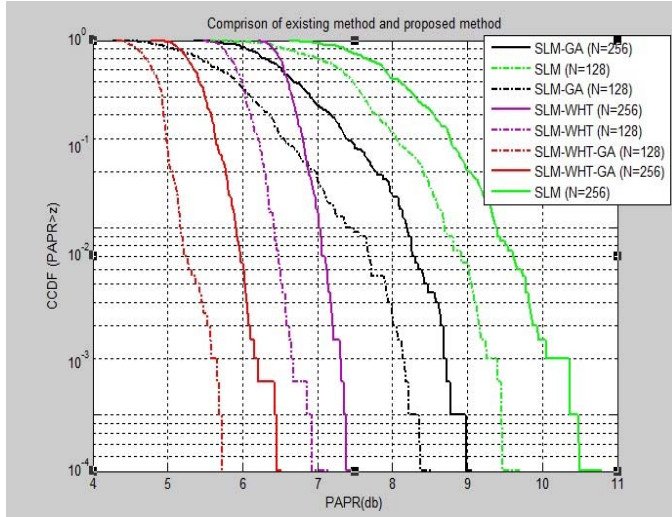


Fig 8 CCDF plot for different SLM based OFDM systems $N=128, 256$ and $D=8$

The graph shows that the PAPR of the system is improved after using the genetic algorithm. Also, increase in the number of subcarriers increases the PAPR also. There is a significant improvement in PAPR as compared to the using the HSLM method and the conventional SLM method.

The PAPR performance of all the methods has been tabulated in table 2. As seen from the table, the proposed method shows significant improvement in the PAPR as compared the existing methods.

TABLE 2: Comparison of PAPR of all SLM based methods for $N=128, 256$

Method	PAPR (dB)
SLM ($N=128$)	9.5
SLM($N=256$)	10.5
SLM-GA ($N=128$)	8.2
SLM-GA ($N=256$)	8.8
SLM-WHT($N=128$)	6.9
SLM-WHT($N=256$)	7.4
SLM WHT-GA ($N=128$)	5.7
SLM WHT-GA($N=256$)	6.5

TABLE 3: Comparison of performance parameters of all SLM based methods.

Method	BER Increase	Bit Rate Loss	Implementation Complexity	Power Increase
Conventional OFDM	Yes	No	low	no
SLM-OFDM	No	Yes	medium	no
WHT-OFDM	No	No	low	no
WHT-SLM-OFDM	No	No	high	no
WHT-SLM-OFDM with GA	no	yes	high	no

The Table 3 below shows the relevant comparisons for all the OFDM based systems that were simulated and analyzed in the paper. Many solutions to the PAPR problem have been explored in the past as seen in Table 3. All of them have seen varied levels of success and involve various tradeoffs among crucial factors such as bit rate, complexity, error rate, costs and power requirements. It is crucial to identify which parameters are more important and realize the best possible trade off among these parameters. It can be concluded from the table above that the proposed method achieves a PAPR improvement of approximately 4 dB over the regular SLM

method. However, the better performance is at the cost of increase in computational complexity of the system.

VI. CONCLUSION

This paper is focused on precoding methods to fix the PAPR issue. Precoding techniques are lucrative as they are easy to implement and produce good results. After an exhaustive literature survey a blind SLM technique was adopted. It used the Walsh hadamard transform in liaison with the conventional SLM method in order to simplify the decoding process at the receiver. The implementation of this method was successful and the results obtained were as expected. The implementation of this method was also very successful and the results obtained confirmed the hypothesis that the use of an optimization algorithm would enhance PAPR performance.

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