LFM Signal Sampling and Reconstruction Based on Compressed Sensing and Swept-frequency Theory

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Abstract—**Linear frequency modulated (LFM) signal is widely used in radar, sonar and communication system. In some application scenarios, the LFM signal should have ultra-wide band (UWB), with the result that the general hardware sampling system cannot satisfy the requirement of Nyquist sampling rate. Theory of compressed sensing (CS) enables the successful reconstruction of sparse signal by sampling at a sub-Nyquist sampling rate. However, LFM are not sparse enough in the traditional Fourier transformation domain. In this paper, we propose a method based on the theory of Factional Fourier Transformation (FRFT) domain to improve the performance of LFM signal reconstruction. First, we present an orthogonal discrete FRFT matrix as the sparse dictionary which will reduce the sparsity level. Second, a swept-frequency modulator based on FRFT domain, has been implemented to estimate parameters of LFM signal. Besides, it also improves the signal reconstruction result and reduces sampling rate. Finally, simulations are taken on testing the proposed framework on LFM signals, and evaluation results demonstrate high feasibility and efficiency of this method.**

Keywords-linear frequency modulated; compressed sensing; fractional fourier transformation; analog-to-information; swept-frequency

I. INTRODUCTION

Linear frequency modulated (LFM) signals are a kind of frequency linear change in a continuous time which are often used in radar system. LFM radar system has a frequency beyond GHz. A significant challenge of processing these UWB signals is the requirement for sampling at Shannon-Nyquist rate. The high sampling rate caused a lot of pressure to A/D converter technology today. Therefore how to reduce the sampling rate and reconstruct the signal accurately is a problem worthy of studying.

According to compressed sensing (CS) [1] theory, sparse signal can be recovered with a sub-Nyquist rate sampling rate directly in the sparse domain or recovered after undergoing an invertible linear transformation. Compressed Sensing requires the signal is compressible, in other words, the signal is sparse in some domain (such as Fourier basis, wavelet basis, etc.). There are three key elements need to be explicit while using CS theory: Firstly, compressive measurement method should be found to compress the signal. Secondly, a sparse dictionary should be established in which the signal have sparse representation. Finally, an algorithm should be proposed to reconstruct signal efficiently.

CS suggests a new framework for analog-to-information convertor (AIC) as an alternate to common ADC devices. Kirolos [2] developed an AIC framework, which can be used to directly sample analog signal at a sampling rate much lower than Shannon-Nyquist rate. The AIC architecture is based on a wideband pseudorandom demodulator and a low rate sampler that can efficiently acquire a large class of compressible signals. They have proved that AIC is an effective sampling system for implementing the random measurement procedure in the CS theory.

Since LFM is not sparse in traditional Fourier Transformation domain, Fourier sparse basis cannot represent the signal well. There are some tools to analyze LFM signal, such as Short-Time Fourier Transformation (STFT), Gabor transformation and wavelet transformation. These methods can process LFM signal and gain a good performance. However, there are still some shortcomings. Fractional Fourier Transformation (FRFT) [5] is a promotion of Fourier transformation. It is especially suitable to process LFM signal. LFM signal has aggregation behavior in the correct degree FRFT domain, and it will be displayed as a pulse function. In other words, LFM signal is sparse enough at special angle in FRFT domain. Therefore, an orthogonal discrete FRFT dictionary matrix can be used as sparse basis applying for the signal reconstruction. In practical, the parameters of signal are not visible to us, we are always in a blind condition. Hence, finding a method to get the key element of signal is necessary. Swept-frequency is a proper solution to that problem. It is used to analyze and extract the parameters of signal in frequency domain. Similarly if we implement the idea of swept-frequency into FRFT, we will obtain the optimal angle of LFM signal. Further, the parameters gained by swept-frequency module can also be used in frequency modulator which can reduce the sparsity level of LFM signal. According to CS theory, a sparser signal will lead to a better reconstruction performance.

In this paper, we propose a dictionary based on FRFT which could represent LFM signal sparsely without evident disadvantage. Beyond that, we implement swept-frequency modulator to module and obtain the key element of the signal. Our proposed scheme will remarkably enhance the reconstruction performance and reduce the compression ratio.

The rest of this paper is organized as follow. A brief related work will be presented in Section II. The basic theory of technology will be discussed in section III. The proposed scheme of sampling and reconstruction, including the swept-frequency modulator and sparse dictionary, will be descripted in detail in Section IV. Then, the simulation results are presented in Section V. Finally conclusions are given in Section VI.

II. RELATED WORK

Detecting and processing LFM signal has been an important research topic. Since the time-frequency analysis method keeps on breaking through, researchers study the signal from different transformation domain, and many time-frequency analyzing tools has been found.

As [11] proposed, Short-Time Fourier Transformation (STFT) takes advantage of window technology to process the LFM signal. But there are some drawbacks that cannot be avoided. Firstly, it is difficult to choose a proper window function. Secondly, this processing method based on short time stability exists a low time-frequency resolution. These disadvantages will influence the performance in a large extent. And [10] discusses that Gabor frame is suitable to process short pulse such as LFM. But there are still some disadvantages of Gabor. Firstly, choosing a proper window is still a question as same as STFT. Next, Gabor dictionary is a redundant dictionary. The window becomes smaller and the dictionary multiplies more rapidly. However, precise reconstruction need a small enough window. It is difficult to balance the performance and efficiency. [8][9][14] use the matching dictionary as sparse basis to reconstruct LFM echo. But this is an ideal situation, because we should know all the information about the target LFM signal, otherwise it will not reconstruct the signal successfully. Besides, the matching dictionary has a poor performance to endure Gaussian noise. In practical, we might reconstruct the signal in a blind condition which means we will not know the precise parameters of signal. [12][13] has proposed to process the LFM signal in Wavelet domain. Wavelet transformation function is not unique in wavelet analysis. Different wavelet function has different performance, however it is not sparse enough for LFM signal in wavelet transformation domain. Hence, the reconstruction performance is limited. [15] has proposed that the DFRFT is a well candidate to be sparse dictionary, however the sensing matrix it used is Gaussian matrix which is difficult to implement in practical, beyond that there is room for performance improvement.

III. BACKGROUND

A. CS and AIC

1) Compressed sensing (CS)

The theory of CS was developed by Candès [4] and Donoho [1] in 2006. Traditional sampling is limited by Shannon-Nyquist rate. However, CS theory suggested that, if a signal has a sparse representation in a certain dictionary, one can sample the signal at a sub-Nyquist rate, and reconstruct it accurately with a high probability.

Consider a signal $\mathrm{X} \in R^n$, which can be sparsely represented over a fixed dictionary $\mathbf{\Psi} \in \boldsymbol{R^{n\times k}}$ that is assumed to be redundant, meaning that $\mathrm{k} > \mathrm{n}$. $\mathbf{\Psi}$ is called sparse basis or sparse dictionary. Thus, the signal can be described as

$$\mathbf{X} = \mathbf{\Psi\theta} \tag{1}$$

Consider a general linear products between $\mathbf{X}$ and a collection of vectors $\{\varphi_i\}_{i=1}^m$ as $y_i = \langle X, \varphi_i \rangle$. Arrange the measurements y_i in an $\mathrm{m} \times 1$ vector $\mathbf{Y}$ and the sampling vectors ${\varphi_i}^T$ as rows in an $\mathrm{m} \times \mathrm{n}$ matrix $\mathbf{\Phi}$, then $\mathbf{Y}$ can be written as

$$\mathbf{Y} = \mathbf{\Phi\Psi\theta} \tag{2}$$

The original $\mathbf{X}$ can be reconstructed from $\mathbf{Y}$ by exploring its sparse expression. We seek the sparsest one among all possible $\hat{\theta}$. This reconstruction requires the solution of

$$\min\|\hat{\theta}\|_0, \mathrm{s.t}\ \mathbf{Y} = \mathbf{\Phi\Psi}\hat{\boldsymbol{\theta}} = \boldsymbol{D}\hat{\boldsymbol{\theta}}$$

where $\mathbf{D} = \mathbf{\Phi\Psi}$ is define as the equivalent dictionary.

To solve the problem, different sub-optimal strategies are used in practice such as Orthogonal Matching Pursuit (OMP) and Basis Pursuit (BP). In this paper, we adopt OMP as the optimization method.

2) Random demodulation structure of AIC

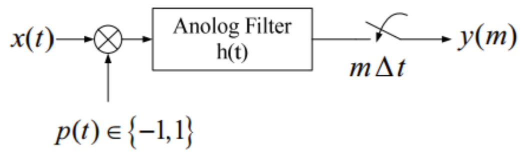

Figure 1 The structure of random demodulation

The signal acquisition system consists of three main components: demodulation, filtering, and uniform sampling. As seen in Figure 1, the signal is modulated by a pseudo-random maximal-length PN sequence of ± 1, besides its alternate between values at of faster than the Nyquist frequency of the input signal. Finally, the signal is sampled at rate $1/\Delta\mathrm{t}$ using a traditional ADC.

Supposing the signal has finite information rate, then it is reasonable to assume that it can be represented using a finite number of parameters per unit time in some continuous basis. More concretely, let the analog signal $\mathrm{x}(t)$ be composed of a discrete, finite numbers of weighted continuous basis or dictionary components, as is shown in equation (1).

Although this system process analog signal in continuous-time signal domain, the sampled value can abstract into mathematic model. The analog input signal is composed of a finite and discrete number of components of $\mathbf{\Psi}$. Then we will separate out an expression for each element $v_{m,n} \in \mathrm{V}$

$$v_{m,n} = \int_{-\infty}^{\infty} \psi_n(\tau) p_c(\tau) h(m\Delta t - \tau) d\tau \tag{3}$$

We could reconstruct the input signal with CS theory through this model.

B. FRFT and Discrete-FRFT

As generalization of Fourier transform, fractional Fourier transform (FRFT) is especially suitable for processing LFM signal. The FRFT uses a transform kernel which allows the signal in time-frequency domain to be projected onto a line of arbitrary angle α. The definition is given as

$$X_p\,(u) = \int_{-\infty}^{+\infty} x(t)\ \widetilde{K}_p(u,t) \mathrm{d}t \tag{4}$$

The kernel $\widetilde{K}_p(u,t)$ is defined as

$$\widetilde{K}_p = \begin{cases} C_\alpha \cdot e^{j\pi(u^2 cot\alpha - 2utcsc\alpha + t^2 cot\alpha)}, \alpha \neq n\pi \\ \delta(t-u), \alpha = 2n\pi \\ \delta(t+u), \alpha = 2(n\pm 1)\pi \end{cases} \quad (5)$$

C_α is a parameter with respect to α,

$$C_\alpha = \sqrt{1 - jcot\alpha}\ , \alpha = \frac{p\pi}{2}$$

There are many algorithms to discrete FRFT into matrix. One of most classic algorithm has been proposed by Pei [6]. It has some good features, such as fast speed and low complex degree. Besides the matrix is suitable for sparse dictionary due to its orthogonality and reversibility. The basic idea of discretizing signal is sampled at a proper interval in both time and FRFT domain. The final equation can be written as:

$$K_p(m,n) = \sqrt{\frac{sin\alpha - jcos\alpha}{2M+1}} \cdot e^{\frac{j}{2}cot\alpha m^2 \Delta u^2 - \frac{2\pi nm}{2M+1} + \frac{j}{2}cot\alpha n^2 \Delta t^2} \quad (6)$$

When $M = N$, we get the sparse dictionary as follow:

$$K_{-p} = K_p^* = [K_p(m,n)]^* \quad (7)$$

C. Swept-frequency implemented in FRFT domain

FRFT is an efficient tool for analyzing the time-frequency signal. Swept-frequency based on FRFT will have effecting on non-stationary signal processing. Traditional method to sweep signal is limited in time or frequency domain. However, LFM signal has energy aggregation behavior in FRFT. Taking advantage of this character, we will efficiently detect the signal.

LFM signal model can be described as:

$$x(t) = a_0 e^{j\varphi_0 + j2\pi f_0 t + \pi\mu_0 t^2} \quad (8)$$

f_o represents the center frequency, K is the slope of the frequency modulation. Now simplify the LFM model as $x(t) = e^{j\pi K t^2}$, and transform LFM signal into fractional Fourier domain.

The α angle ($\alpha = \frac{\pi}{2} \cdot p$) fractional Fourier transforms LFM can be described as:

$$X_p(u) = \begin{cases} \sqrt{\frac{1+jtan\alpha}{1+ktan\alpha}} exp\left(j\pi \frac{u^2}{2} \frac{k - tan\alpha}{1+ktan\alpha}\right) \alpha - arctank - \frac{\pi}{2} \neq n\pi \\ \delta(u), \alpha - arctank - \frac{\pi}{2} = n\pi \end{cases} \quad (9)$$

It is obvious that LFM signal will turn into a pulse function, while α is equal to $arctank + \frac{\pi}{2} + n\pi$ in FRFT domain. This is the theoretical basis of estimating LFM parameters. According to equation (9), a different α results in a different $X_p(u)$ value, therefore we can draw a two-dimensional plane about (α, u). The basic idea to swept-frequency is revolving the angle α and scanning at (α, u) plane. According to threshold peak point on this plane of the two-dimensional searching, we can detect LFM signal and estimate its parameters. The procedure of swept-frequency can be described as:

$$\begin{gathered} \{\hat{\alpha}_0, \hat{u}_0\} = \arg\max_{\alpha,u} |X_\alpha(u)|^2 \\ \hat{\mu}_0 = -cot\hat{\alpha}_0 \\ \hat{f}_0 = \hat{u}_0 csc\hat{\alpha}_0 \\ \hat{\varphi}_0 = \arg\left[\frac{X_{\hat{\alpha}_0}(\hat{u}_0)}{A_{\hat{\alpha}_0} e^{j\pi\hat{u}_0^2 cot\hat{\alpha}_0}}\right] \\ \hat{a}_0 = \frac{|X_{\hat{\alpha}_0}(\hat{u}_0)|}{\Delta t |A_{\hat{\alpha}_0}|} \end{gathered} \quad (10)$$

Among them, we don't know the exact values of α_0, φ_0, f_0 and μ_0 of LFM. As a linear transformation, FRFT reserves signal's phase information, therefore we will effectively estimate the slope of frequency, central frequency, amplitude and phase of a LFM signal.

IV. Proposed Scheme

A. Sampling Framework

The design of our framework is shown in Figure 3. Random demodulation is implemented in AIC framework to sample the signal at a sub-Nyquist speed directly. The DFRFT matrix is used as sparse dictionary instead of Inverse Discrete Fourier Transformation (IDFT). The Orthogonal Matching Pursuit (OMP) algorithm can recover the signal from a NP-hard problem. Besides, the whole framework consists of three important components. The detail can be described as follows:

- Swept-frequency modulator implemented in our framework, this module has two components which are parameter estimation module and frequency modulator. Parameter Estimation Module estimates the parameters of LFM, especially slope of frequency modulation and optimal angle in FRFT domain. Frequency modulator is used to eliminate the frequency time-varying of LFM. The purpose is to reduce the LFM sparsity level in FRFT domain, whose consequence is to make the signal become more easy-handling for AIC.
- Analog-to-information convertor (AIC), this module implemented by Random Demodulation is used to directly sample signal at a sub-Nyquist rate. Besides, OMP algorithm will recover the signal precisely.
- Discrete FRFT Sparse Dictionary, relying on the estimated optimal angle in FRFT domain, can be used as sparse dictionary.

B. Swept-frequency Modulator

Swept-frequency modulator is composed by two components: the first part is parameters estimated module, and the second part is frequency modulator.

1) Parameter Estimation Module

In practical, we do not know the exact parameters about the received LFM signal. However, in CS framework, prior knowledge of the signal is necessary for generating sparse basis. Therefore, this module is used to estimate necessary parameters for reconstruction. As an important component of the proposed scheme, precise and fast frequency sweeping and parameters estimation influences the reconstruction accuracy in a certain degree.

The implementations of parameter estimation depends on the idea of swept-frequency which has been mentioned in section II. However, it still meets problem in practical application. In the processing, we scan the signal by rotating variable α. If we wish a precise estimation, it is better to reduce the scope, which means the computational complexity will be multiplied.

To solve the complexity of computation, we use an optimizing search method. This method can be divided in two parts. Firstly, rotating angle α at (α, u) plane with a big step and getting a pre-best angle. After a rough but fast operation, an inaccuracy angle has been obtained. Secondly, on the basis of the rough angle, we could take advantage of Quasi-Newton method to conduct the iterative search. Then we will gain the precise estimated value. The iterative procedure can be written as follows:

$$\begin{bmatrix} \hat{\alpha}_{n+1} \\ \hat{u}_{n+1} \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_{n} \\ \hat{u}_{n} \end{bmatrix} - \lambda_n H_n \begin{bmatrix} \dfrac{\partial |X_\alpha(u)|^2}{\partial p} \\ \dfrac{\partial |X_\alpha(u)|^2}{\partial u} \end{bmatrix}_{\substack{\alpha=\hat{\alpha}_n \\ u=\hat{u}_n}} \tag{11}$$

$\hat{\alpha}_n$ and $\hat{u}_n$ are result which is gained by the nth time search. λ_n is the step coefficient. H_n is an measurement matrix which is created by $|X_\alpha(u)|^2$ at point $(\hat{\alpha}_n, \hat{u}_n)$. By Quasi-Newton method, every iterative computation is a one-dimension search and first-order partial derivative of function. Through simulation, optimizing search method illustrated by equation (11) will gain a precise value no more than 4 times iteration, and the results reach or approach Crammer-Rao lower limit. This algorithm is characterized by low computational complexity and accuracy processing.

2) Frequency Modulator

Frequency Modulator consists of two multipliers whose function is instantaneous frequency linear changing. One formula of down frequency modulator can be written as:

$$s(t) = e^{-j\pi\hat{K}t^2}$$

$\hat{K}$ stands for the estimated slope of frequency module. The main idea of down frequency modulator is to eliminate the frequency shifty of LFM before being processed by AIC framework. According to equation (8), after LFM is processed by frequency modulator, its frequency time-varying will be reduced. The post-process signal is more easily handling to AIC. After LFM is processed by AIC, the module's duty is recovering the signal. And an up frequency modulator is similar to down, and its function is about $-\hat{K}$. Its duty is to recover the lost frequency information and make results perfect.

By the idea of swept-frequency, we will estimate the time-frequency varying component of LFM signal. Then, we will construct modulation signal $s(t)$ by the preliminary estimated parameters of signal, and the modulation will be used to module the origin signal. The frequency time-varying component will be slashed by this method. After modulating, the signal will be sparser in FRFT domain. According to CS theory, a signal with lower sparsity level will get more probability to reconstruct successfully and gain better reconstruction performance.

C. Discrete Fractional Fourier Transformation (DFRFT)

According to [3], an empirical study of the minimum measurement rate required to accurately reconstruct signals with the random demodulator has been proposed. The results are phrased in terms of the Nyquist rate W, the sampling rate R, and the sparsity level K. The compression rate R/W measures the improvement in the sampling rate over the Nyquist rate.

The results lead us to an empirical rule for the sampling rate necessary to recover random sparse signals using the demodulator system. The formula can be written as:

$$R \approx 1.7K \log(W/K + 1) \tag{12}$$

As equation (12) is shown that, with the sparsity level turns to a lower value, the necessary sampling rate R will be smaller. In other words, if we could reduce sparsity of signal in some dictionary, it is reasonable to recovery the signal at a lower sampling rate with a high probability successful reconstruction.

DFRFT sparse dictionary will represent the LFM more sparsely than IDFT and other sparse dictionary. According to CS theory, a low sparse level means that the signal will be reconstructed successfully with less samples and lower sampling speed in a high probability. As shown in Figure 2 we compare the sparse level between DFRFT and IDFT sparse basis, and the input signal is LFM.

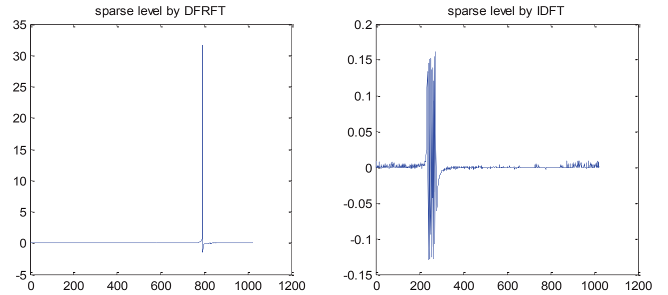

Figure 2 Sparse level comparing

Left side figure stands for LFM signal represented by DFRFT dictionary, and the sparse level is nearly 2. And right side figure stands for IDFT dictionary whose sparse level is about 30. As a conclusion, DFRFT dictionary gets a better performance to represent LFM signal than IDFT does. Therefore, according to equation (12), DFRFT dictionary will obtain a better performance than IDFT dictionary.

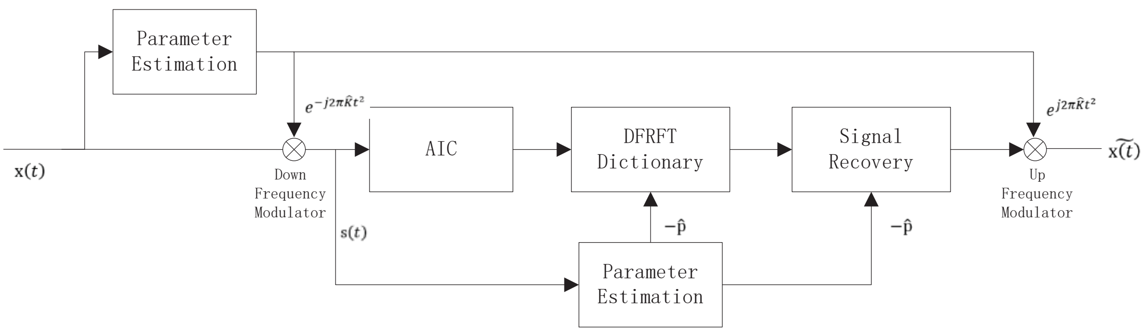

Figure 3 The proposed framework

V. Simulation and Analysis

In order to measure the performance of our design, some simulations are carried on as follow. The experimental signal is LFM with UWB. To simplify the model, we just reconstruct a single LFM signal. The value of amplify A_i is 1, carrier frequency f_0 is assumed to be 20GHz, the bandwidth is 200MHz. The Nyquist sampling rate $f_s = 40.4GHz$, and we have collected 1024 samples to analyze performance in different condition. SNR is used as a metric to compare the quality of reconstructed analog waveform, as defined as

$$\mathrm{SNR} = 20\log_{10}\left(\frac{\|x\|}{\|x - x^{\#}\|}\right)$$

where $\|\cdot\|$ is the Euclidean norm, x is signal as measured by sampling framework, and $x^{\#}$ is constructed signal vector. The unit of SNR is dB.

First of all, we consider that whether the proposed method is utility in free noise condition. Secondly, the impacts of Gaussian distribution noise based on the quality of reconstructed signal are evaluated. Then, we discuss about compression ratio performance improvement in this method. Finally, how much an inaccuracy estimated of parameters influences the performance is analyzed.

A. *Noise Free Condition*

We choose Inverse Discrete Fourier Transformation (IDFT) sparse dictionary as reference. The compression ratio is fixed on 2. And 500 trials are repeated in the same condition. As shown in Figure 4, DFRFT dictionary with frequency modulator labeled by red line stands for our proposed framework. DFRFT dictionary without frequency modulator and traditional IDFT dictionary serve as references. If we only use DFRFT dictionary to reconstruct signal, there might be 2dB better than IDFT dictionary. But if we add a frequency modulator, there will be nearly 15dB improvement than IDFT dictionary. We can see that our framework has gained a better performance in a noise-free condition. As section IV discussed that, after modulated the frequency time-varying component will be slashed, and the signal will be sparser which will lead to a better reconstruction performance.

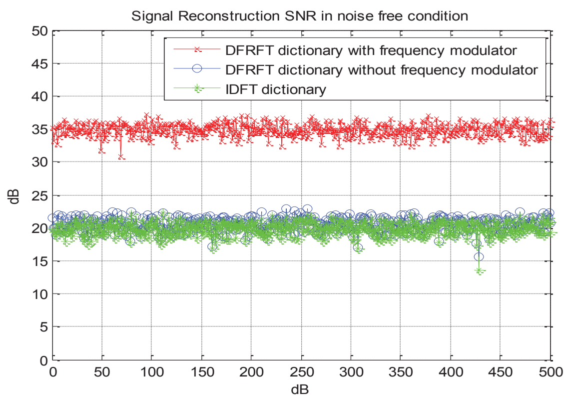

Figure 4 Reconstruction SNR in noise free condition

B. *Noise Condition*

In order to investigate the ability of noise endurance we still use the three methods to observe the reconstruction performance with a fixed compression ratio. We assume that the input SNR ranging from -15 dB to 25dB in increment of 5dB are tested, which is acceptable to real conditions.

We compare the results in different noise level for 1000 repeated trials at each specific noise level. From Figure 5, when the input SNR is below 5dB, it means the channel condition is not good, the signal will merge in noise. Although the slope of frequency will still be estimated precisely, and the accurate ratio is above 90% after experiments, it does not bring enough gains, which is because CS theory is sensitive to noise. Our scheme gains nearly 0.8dB better than IDFT dictionary when input SNR is below 5dB. We can see that the reconstruction performance is getting better as the input SNR increasing. When the input SNR is above 5dB, our framework shows an obviously better performance than DFRFT without frequency modulator or traditional IDFR dictionary.

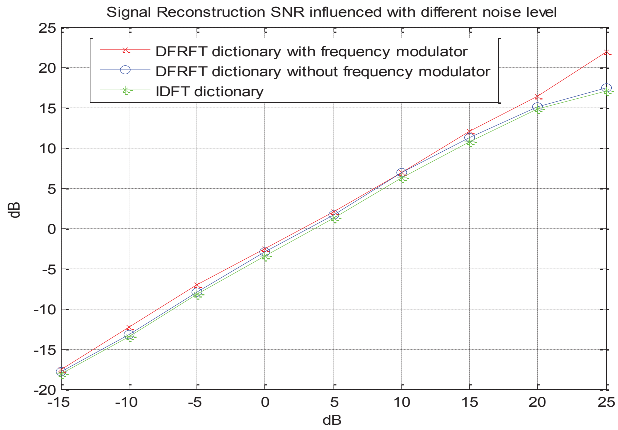

Figure 5 Reconstruction SNR influenced with different noise level

C. *Compression Ratio Analysis*

Compression ratio (CR) is the value of dividing Nyquist sampling speed by the actual sampling speed. CR is an important indicator to measure the efficiency of an AIC framework. The ultimate aim is reducing compression ratio, meanwhile gaining a wonderful performance of reconstruction.

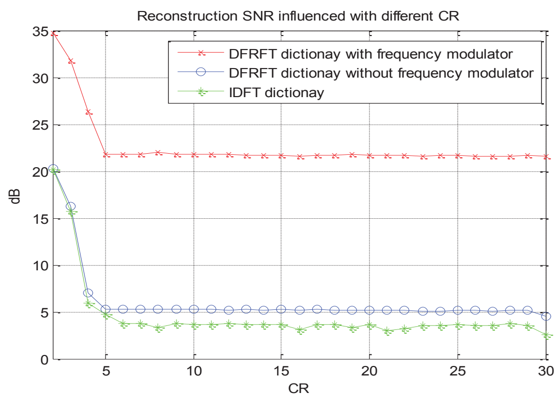

Figure 6 Reconstruction SNR influenced with different CR

As shown in Figure 6, when the CR is less than 5, reconstruction performances of the three methods are acceptable. But with the CR increasing, the results become poorer. In addition when the CR is greater than 5, the reconstruction results become undesirable. Even if the CR is greater than 30, we can still reconstruct the signal successfully and gain a better performance than others. As mentioned in equation (12) that, a sparser signal will need fewer samples and gain a better reconstruction performance with fewer samples. As section IV has proved that, our scheme will reduce the sparsity level of signal, therefore we will gain a satisfied result with a larger compression ratio.

D. Estimating Accuracy Ratio Analysis

Swept-frequency is used to estimate the optimal angle in FRFT domain. How much will this module impact on reconstruction results. Figure 7 has given the answer. Here we just talk about influence brought by frequency modulator, because sparse dictionary is not well affected. We control the estimated value to compare with a no frequency modulator framework in a noise free condition.

Estimated accuracy ratio is calculated by function below:

$$\mathrm{AR} = \left(1 - \frac{|K - \widehat{K}|}{K}\right) \times 100\%$$

AR means accuracy ratio, $\widehat{K}$ represents the estimated value of slope of frequency modulation and K represents the true value, besides K is assumed to be a positive number.

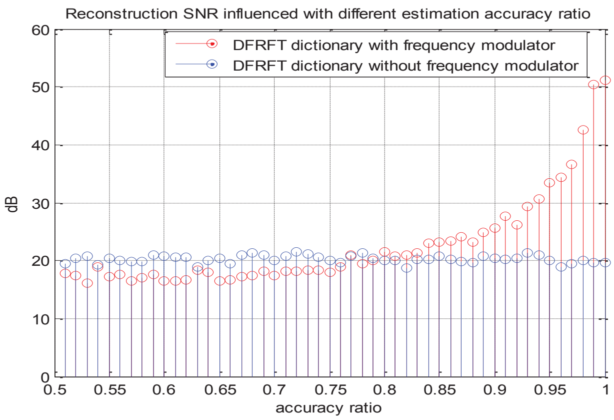

Figure 7 Reconstruction SNR influenced with different estimation accuracy ratio

When accuracy ratio is more than 80%, the reconstruct performance is better than no frequency modulator framework. In practical, according to the theory in section Ⅱ, the accuracy ratio is always more than 80%, even though the slope of frequency modulation is big enough or the channel condition is bad. So we can draw a conclusion that, our proposed scheme will enhance the reconstruction performance with a certain degree, even the channel condition is poor.

VI. Conclusion

In this paper, we proposed a structure of signal reconstruction method which is used to sample and reconstruct LFM waveforms at a sub-Nyquist sampling rate. FRFT is proposed to process the LFM signal because of energy concentration property. The DFRFT matrix is used as sparse dictionary which adjusts the signal sparser than IDFT sparse dictionary. Beyond that, we implement swept-frequency modulator to estimate the parameters and slash the frequency time-varying character which will make signal sparser and more easy-handling by AIC framework. Our proposed framework will reconstruct the signal successfully without any prior knowledge. Besides, we enhance the reconstruction performance with lower sampling rate and higher probability. Simulation results and the performances of processing the LFM with UWB have proved the feasibility of our scheme.

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