SPECTRUM SHARING BETWEEN MATRIX COMPLETION BASED MIMO RADARS AND A MIMO COMMUNICATION SYSTEM

Bo Li and Athina Petropulu

ECE Department, Rutgers, The State University of New Jersey, Piscataway NJ
{paul.bo.li,athinap}@rutgers.edu

ABSTRACT

Recently proposed multiple input multiple output radars based on matrix completion (MIMO-MC) employ sparse sampling to reduce the amount of data forwarded to the radar fusion center, and as such enable savings in communication power and bandwidth. This paper proposes designs that optimize the sharing of spectrum between MIMO-MC radars and MIMO communication systems, so that the latter interferes minimally with the former. First, the communication system transmit covariance matrix is designed to minimize the effective interference power (EIP) at the radar receiver, while maintaining certain average capacity and transmit power for the communication system. Two approaches are proposed, namely a noncooperative and a cooperative approach, with the latter being applicable when the radar sampling scheme is known at the communication system. Second, a joint design of the communication transmit covariance matrix and the MIMO-MC radar sampling scheme is proposed, which achieves even further EIP reduction.

Index Terms— Collocated MIMO radar, matrix completion, spectrum sharing

1. INTRODUCTION

The operating frequency bands of communication and radar systems often overlap, causing one system to exert interference to the other. For example, the high UHF radar systems overlap with GSM communication systems, and the S-band radar systems partially overlap with Long Term Evolution (LTE) and WiMax systems [1–3]. Spectrum sharing is a new line of work that targets at enabling radar and communication systems to share the spectrum efficiently by minimizing interference effects [2–7].

This paper investigates the problem of spectrum sharing between a MIMO communication system and a matrix completion (MC) based, collocated MIMO radar (MIMO-MC) system [8–10]. MIMO radars transmit different waveforms from their transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center for further processing. Based on the forwarded data, the fusion center populates a matrix, referred to as the "data matrix", which is then used by standard array processing schemes for target estimation. For a relatively small number of targets, the data matrix is low-rank [8], thus allowing one to fully reconstruct it (under certain conditions) based on a small, uniformly sampled set of its entries. This observation is the basis of MIMO-MC radars; the RX antennas forward to the fusion center a small number of pseudo-randomly sampled values of the target returns, along with their sampling scheme, each RX antenna partially filling a column of the data matrix. Subsequently, the full data matrix is recovered using MC techniques. MIMO-MC radars maintain the high resolution of MIMO radars, while at the same time require significantly fewer data to be communicated to the fusion center, thus enabling savings in communication power and bandwidth [8–10]. Compared to the compressive sensing (CS) based MIMO radars, MIMO-MC radars achieve data reduction while avoiding the basis mismatch issues inherent in CS-based approaches [11].

In this paper, the MIMO-MC radar system is considered as the primary user of the channel, while the MIMO communication system is the secondary user. *First*, for a fixed uniformly random radar subsampling scheme, the communication system optimally designs its transmit covariance matrix so that its effective interference power (EIP) exerted to the radar RX node is minimized, while its own average communication capacity and transmit power are kept at a prescribed level. In doing so, two approaches are proposed, namely, a cooperative and a noncooperative approach, depending on whether the communication system has knowledge of the MIMO-MC radar sampling instances. It is shown that when the MIMO-MC radar sampling scheme is known to the communication system, the EIP can be greatly reduced, especially at low subsampling rates. *Second*, a joint-design of the radar sampling scheme and the communication system transmit covariance matrix is proposed, targeting at minimizing the EIP at the radar RX node. Alternating optimization is employed to solve the optimization problem. The candidate sampling scheme needs to ensure that the resulting data matrix can be completed. Recent work [12] indicates that for matrix completion the sampling locations should correspond to a binary matrix with large spectral gap. To reduce the complexity of the search, we propose to search for the optimum sampling matrix among matrices which are row and column permutations of an initial sampling matrix with large spectrum gap. Even before any design is implemented, the MIMO-MC radar is expected to be less susceptible to interference than plain MIMO radars; this is because the interference affects only some entries of the data matrix. As it is shown in the paper, by appropriately designing the communication TX waveforms and/or the radar sampling scheme, the interference can be further reduced.

The paper is organized as follows. Section 2 introduces the signal model when the MIMO-MC radar and communication systems are coexisted. The problem of MIMO communication sharing spectrum with MIMO-MC radar is studied in Sections 3 and 4. Numerical results, discussions and conclusions are provided in Sections 5-7.

Notation: $\mathcal{CN}(\mu, \mathbf{\Sigma})$ denotes the circularly symmetric complex Gaussian distribution with mean μ and covariance matrix $\mathbf{\Sigma}$. $|\cdot|$ and $\mathrm{Tr}(\cdot)$ denotes the matrix determinant and trace, respectively. The set $\mathbb{N}_L^+$ is defined as $\{1, \dots, L\}$. $\mathcal{N}(\mathbf{A})$ and $\mathcal{R}(\mathbf{A})$ denote the null and row spaces of matrix $\mathbf{A}$, respectively. Subscripts $\cdot m$ and $n\cdot$ denote the m-th column and the n-th row of a matrix, respectively.

This work was supported by NSF under Grant ECCS-1408437.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a MIMO communication system which coexists with a MIMO-MC radar system as shown in Fig. 1. The radar system operates as the primary system for target detection/estimation. The communication system shares the same carrier frequency as a secondary system. The MIMO-MC radar operates in two phases; in the first phase the TX antennas transmit and the RX antennas receive, while in the second phase, the RX antennas forward their measurements to a fusion center. The communication system interferes with the radar system during both phases. In the following, we will address the interference during the first phase only. The interference during the second phase can be viewed as the interference between two communication systems, and addressing this problem has been covered in the literature [13, 14].

Suppose that the two systems have the same symbol rate and are synchronized in sampling time (see Section 5 for the mismatched case). Let l denote the sampling time instance according to Nyquist rate sampling. We do not assume perfect carrier phase synchronization between the two systems. The signal received by the radar and communication RX antennas can be respectively expressed as

$$\mathbf{y}_R(l) = \mathbf{\Omega}_l \circ [\mathbf{D}\mathbf{s}(l) + e^{j\alpha_2}\mathbf{G}_2\mathbf{x}(l) + \mathbf{w}_R(l)], \tag{1a}$$

$$\mathbf{y}_C(l) = \mathbf{H}\mathbf{x}(l) + e^{j\alpha_1}\mathbf{G}_1\mathbf{s}(l) + \mathbf{w}_C(l),\ \forall l \in \mathbb{N}_L^+, \tag{1b}$$

where

- $\mathbf{\Omega}_l$ is a random sampling vector with binary entries and $\circ$ denotes the Hadamard product.
- $\mathbf{y}_{R/C}(l)$ and $\mathbf{w}_{R/C}(l)$ respectively denote the received signal and the additive noise at the radar/communication RX antennas. It is assumed that $\mathbf{w}_{R/C} \sim \mathcal{CN}(0, \sigma_{R/C}^2\mathbf{I})$.
- $\mathbf{D} \in \mathbb{C}^{M_{r,R} \times M_{t,R}}$ denotes the target response matrix, where $M_{r,R}$ and $M_{t,R}$ denote respectively the number of RX and TX antennas at the MIMO radar. $\mathbf{D}$ is low-rank and depends on the target reflectivity, angle of arrival and target speed (details can be found in [8, 9]).
- $\mathbf{H} \in \mathbb{C}^{M_{r,C} \times M_{t,C}}$ denotes the communication channel, where $M_{r,C}$ and $M_{t,C}$ denote respectively the number of RX and TX antennas of the communication system [13]; $\mathbf{G}_1 \in \mathbb{C}^{M_{r,C} \times M_{t,R}}$ denotes the interference channel from the radar TX antennas to the communication system RX antennas [3, 4, 7]; $\mathbf{G}_2 \in \mathbb{C}^{M_{r,R} \times M_{t,C}}$ denotes the interference channel from the communication TX antennas to the radar RX antennas. It is assumed that the channels remain the same over L time-slots.
- $\mathbf{s}(l)$ and $\mathbf{x}(l)$ respectively denote the transmit vector at the radar and the communication TX antennas;
- $e^{j\alpha_1}$ and $e^{j\alpha_2}$ denote the unknown phase offsets between the carriers of the radar and the communication systems.

Grouping L samples together, (1) becomes

$$\mathbf{Y}_R = \mathbf{\Omega} \circ (\mathbf{D}\mathbf{S} + e^{j\alpha_2}\mathbf{G}_2\mathbf{X} + \mathbf{W}_R), \tag{2a}$$

$$\mathbf{Y}_C = \mathbf{H}\mathbf{X} + e^{j\alpha_1}\mathbf{G}_1\mathbf{S} + \mathbf{W}_C, \tag{2b}$$

where $\mathbf{Y}_R \triangleq [\mathbf{y}_R(1), \ldots, \mathbf{y}_R(L)]$; the matrices $\mathbf{S}, \mathbf{X}, \mathbf{Y}_C$, $\mathbf{W}_R$, and $\mathbf{W}_C$ are similarly defined; $\mathbf{S}$ contains the radar waveforms, which are typically orthogonal, e.g., Hadamard and Gaussian orthogonal [10]; the columns of $\mathbf{X}$ are codewords from the code-book of the communication system. The capacity achieving codewords are Gaussian, i.e., $\mathbf{x}(l) \sim \mathcal{CN}(0, \mathbf{R}_{xl})$; $\mathbf{\Omega}$ is an $M_{r,R} \times L$ dimensional random sampling matrix with binary entries, whose l-th column is $\mathbf{\Omega}_l$. Only the nonzero entries of matrix $\mathbf{Y}_R$ are forwarded to the fusion center. The subsampling rate p is defined as $\|\mathbf{\Omega}\|_0/LM_{r,R}$.

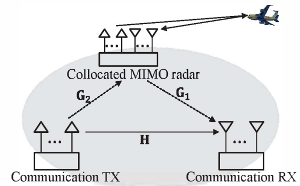

Fig. 1. A MIMO communication system sharing spectrum with a collocated MIMO radar system

It is assumed that the MIMO channels $\mathbf{H}$, $\mathbf{G}_1$ and $\mathbf{G}_2$ are perfectly known at the communication TX. In practice, the channel state information can be periodically communicated among the communication receiver, transmitter and the radar system through a pilot channel [3, 15]. The communication system aims at minimizing its interference to the MIMO-MC radar while maintaining its average capacity over L time-slots by adapting its transmit resources in both time and spatial domain.

3. DESIGN OF COMMUNICATION SYSTEM WAVEFORMS

In this section, we design the communication transmit waveforms, and in particular their covariance matrix, to minimize the interference power at the radar RX node while satisfying the communication rate and power constraints of the communication system. The total transmit power of the communication TX antennas equals

$$\mathbb{E}\{\text{Tr}(\mathbf{X}\mathbf{X}^H)\} = \mathbb{E}\left\{\text{Tr}\left(\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^H(l)\right)\right\} = \sum_{l=1}^{L}\text{Tr}(\mathbf{R}_{xl}),$$

where $\mathbf{R}_{xl} \triangleq \mathbb{E}\{\mathbf{x}(l)\mathbf{x}^H(l)\}$.

According to (2a), the total interference power (TIP) exerted at the radar RX node equals

$$\text{TIP} \triangleq \mathbb{E}\{\text{Tr}(\mathbf{G}_2\mathbf{X}\mathbf{X}^H\mathbf{G}_2^H)\} = \sum\nolimits_{l=1}^{L}\text{Tr}\left(\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H\right). \tag{3}$$

Since the radar only forwards part of $\mathbf{Y}_R$ to the fusion center, only the term $\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X})$ represents effective interference to the radar system. Based on this observation, we define the *effective interference power* (EIP) at the radar RX node as

$$\begin{aligned}
\text{EIP} &\triangleq \mathbb{E}\left\{\text{Tr}\left(\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X})\,(\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X}))^H\right)\right\}\\
&= \mathbb{E}\left\{\text{Tr}\left([\mathbf{G}_{21}\mathbf{x}(1)\ldots\mathbf{G}_{2L}\mathbf{x}(L)][\mathbf{G}_{21}\mathbf{x}(1)\ldots\mathbf{G}_{2L}\mathbf{x}(L)]^H\right)\right\}\\
&= \mathbb{E}\left\{\text{Tr}\left(\sum_{l=1}^{L}\mathbf{G}_{2l}\mathbf{x}(l)\mathbf{x}^H(l)\mathbf{G}_{2l}^H]\right)\right\} = \sum_{l=1}^{L}\text{Tr}\left(\mathbf{G}_{2l}\mathbf{R}_{xl}\mathbf{G}_{2l}^H\right)
\end{aligned} \tag{4}$$

where $\mathbf{G}_{2l} \triangleq \mathbf{\Delta}_l\mathbf{G}_2$ with $\mathbf{\Delta}_l = \text{diag}(\mathbf{\Omega}_l)$. We note that the EIP in time-slot l contains the interference to radar RX antennas corresponding to "1"s in $\mathbf{\Omega}_l$ only.

In the model of (2), both the effective interference channel $\mathbf{G}_{2l}$ and interference power at the communication receiver $\mathbf{R}_{\text{int}l} \triangleq \mathbf{G}_1\mathbf{s}(l)\mathbf{s}^H(l)\mathbf{G}_1^H$ vary slot by slot. The communication system need to use different covariance matrices $\mathbf{R}_{xl}$'s to match the variation of $\mathbf{G}_{2l}$ to minimize the effective interference to the radar system. The channel can be equivalently viewed as a fast fading channel with perfect channel state information at both the transmitter and receiver [16, 17]. Similar to the definition of ergodic capacity [16], the achieved capacity is the average over L time-slots:

$$\text{AC}(\{\mathbf{R}_{xl}\}) \triangleq \frac{1}{L}\sum\nolimits_{l=1}^{L}\log_2\left|\mathbf{I} + \mathbf{R}_{wl}^{-1}\mathbf{H}\mathbf{R}_{xl}\mathbf{H}^H\right|, \tag{5}$$

where $\{\mathbf{R}_{xl}\}$ denotes the set of all $\mathbf{R}_{xl}$'s and and $\mathbf{R}_{wl} \triangleq \mathbf{R}_{\text{int}l} + \sigma_C^2\mathbf{I}$

for all $l \in \mathbb{N}_L^+$.

In the following we will consider two spectrum sharing approaches between the communication and radar systems, namely, a noncooperative and a cooperative, depending on whether the communication system knows the radar sampling scheme. The performance improvement is expected to be higher under higher level of cooperation at the cost of reduced security.

In the *noncooperative approach*, the communication system has no knowledge of $\mathbf{\Omega}$. Therefore, it cannot obtain the expression of EIP as in (4). In this case, the communication system will design its covariance matrix to minimize the TIP in (3) as follows

$$(\mathbf{P}_0) \quad \min_{\{\mathbf{R}_{xl}\}\succeq 0} \text{TIP}(\{\mathbf{R}_{xl}\}) \text{ s.t. } \sum\nolimits_{l=1}^{L} \text{Tr}(\mathbf{R}_{xl}) \leq P_t \quad (6a)$$

$$\text{AC}(\{\mathbf{R}_{xl}\}) \geq C, \quad (6b)$$

where the constraint of (6a) restricts the total transmit power at the communication TX to be no larger than P_t. The constraint of (6b) restricts the communication average capacity during L time-slots to be at least C, in order to provide reliable communication and avoid service outage. $\{\mathbf{R}_{xl}\} \succeq 0$ imposes the positive semi-definiteness on the solution. Let us denote by $\mathbb{X}_0$ the feasible set determined by the above three constraints. Problem $(\mathbf{P}_0)$ is convex and can be solved using the interior point method.

The power constraints of (6a) and (6b) are jointly applied for all L time-slots. The extension to constraints individually applied for each time-slot is straightforward because the convexity of the problem is preserved. Problem $(\mathbf{P}_0)$ is a variant of the Problem $(\mathbf{P}_6)$ in [13] for multichannel spectrum sharing in cognitive radio network.

In the *cooperative approach*, the MIMO-MC radar shares its random sampling scheme with the communication system. Now, the spectrum sharing problem can be formulated as

$$(\mathbf{P}_1) \quad \min \text{ EIP}(\{\mathbf{R}_{xl}\}) \text{ s.t. } \{\mathbf{R}_{xl}\} \in \mathbb{X}_0. \quad (7)$$

Problem $(\mathbf{P}_1)$ has exactly the same constraints as $(\mathbf{P}_0)$. The following theorem compares the minimum EIP achieved by two approaches under the same communication constraints.

Theorem 1. *For any P_t and C, the EIP achieved by the cooperative approaches in $(\mathbf{P}_1)$ is less or equal than that of the noncooperative approach via $(\mathbf{P}_0)$.*

Proof. Let $\{\mathbf{R}_{xl}^{*0}\}$ and $\{\mathbf{R}_{xl}^{*1}\}$ denote the solution of $(\mathbf{P}_0)$ and $(\mathbf{P}_1)$, respectively. We know that $\{\mathbf{R}_{xl}^{*0}\}$ satisfies the constraints in $(\mathbf{P}_1)$, which means that $\{\mathbf{R}_{xl}^{*0}\}$ is a feasible point of $(\mathbf{P}_1)$. The optimal $\{\mathbf{R}_{xl}^{*1}\}$ achieves an objective value no larger than any feasible point, including $\{\mathbf{R}_{xl}^{*0}\}$, does. It holds that $\text{EIP}(\{\mathbf{R}_{xl}^{*1}\}) \leq \text{EIP}(\{\mathbf{R}_{xl}^{*0}\})$, which proves the claim. □

There are certain scenarios in which the cooperative approach outperforms significantly the noncooperative one in terms of EIP. Let us denote by ϕ_1 the intersection of $\mathcal{N}(\mathbf{G}_{2l})$ and $\mathcal{R}(\mathbf{R}_{wl}^{1/2}\mathbf{H})$, and by ϕ_2 the intersection of $\mathcal{N}(\mathbf{G}_2)$ and $\mathcal{R}(\mathbf{R}_{wl}^{1/2}\mathbf{H})$. We know that $\phi_2 \subseteq \phi_1$. Consider the case where ϕ_1 is nonempty while ϕ_2 is empty. This happens with high probability when $M_{r,R} \geq M_{t,C}$ but $pM_{r,R}$ is much smaller than $M_{t,C}$. Problem $(\mathbf{P}_1)$ will guide the communication system to focus its transmission power along the directions in ϕ_1 to satisfy both communication system constraints while introducing zero EIP to the radar system. On the other hand, since ϕ_2 is empty, Problem $(\mathbf{P}_0)$ will guide the communication system transmit power along directions that introduce nonzero EIP.

4. JOINT COMMUNICATION AND RADAR SYSTEM DESIGN FOR SPECTRUM SHARING

In the above described spectrum sharing strategies, the MIMO-MC radar operates with a predetermined pseudo random sampling scheme. In this section, we consider a joint design of the communication system transmit covariance matrices and the MIMO-MC radar random sampling scheme, i.e., $\mathbf{\Omega}$. The candidate sampling scheme needs to ensure that the resulting data matrix can be completed. This means that $\mathbf{\Omega}_I$ is either a uniformly random subsampling matrix [18] or a matrix with a large spectral gap [12].

Recalling that $\mathbf{G}_{2l} = \mathbf{\Delta}_l\mathbf{G}_2$, the EIP is rewritten as:

$$\text{EIP} = \sum_{l=1}^{L} \text{Tr}\left(\mathbf{\Delta}_l\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H\mathbf{\Delta}_l^H\right) = \sum_{l=1}^{L} \text{Tr}\left(\mathbf{\Delta}_l\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H\right).$$

The joint design scheme is formulated as follows

$$(\mathbf{P}_2) \quad \underset{\{\mathbf{R}_{xl}\},\mathbf{\Omega}\text{ binary}}{\arg\min} \text{ EIP} = \sum\nolimits_{l=1}^{L} \text{Tr}\left(\mathbf{\Delta}_l\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H\right)$$
$$\text{s.t. } \{\mathbf{R}_{xl}\} \in \mathbb{X}_0, \mathbf{\Delta}_l = \text{diag}(\mathbf{\Omega}_l), \|\mathbf{\Omega}\|_0 = \lfloor pLM_{r,R}\rfloor.$$

The above problem is nonconvex. A solution can be obtained via alternating optimization. Let $(\{\mathbf{R}_{xl}^n\}, \mathbf{\Omega}^n)$ be the variables at the n-th iteration. We alternatively solve the following two problems:

$$\{\mathbf{R}_{xl}^n\} = \underset{\{\mathbf{R}_{xl}\}\in\mathbb{X}_0}{\arg\min} \sum\nolimits_{l=1}^{L} \text{Tr}\left(\mathbf{\Delta}_l^{n-1}\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H\right), \quad (8a)$$

$$\{\mathbf{\Omega}^n\} = \underset{\mathbf{\Omega}\text{ binary}}{\arg\min} \sum\nolimits_{l=1}^{L} \text{Tr}\left(\mathbf{\Delta}_l\mathbf{G}_2\mathbf{R}_{xl}^n\mathbf{G}_2^H\right), \quad (8b)$$
$$\text{s.t. } \mathbf{\Delta}_l = \text{diag}(\mathbf{\Omega}_l), \|\mathbf{\Omega}\|_0 = \lfloor pLM_{r,R}\rfloor.$$

The problem of (8a) is convex and can be solved efficiently. To avoid the intermediate variable $\{\mathbf{\Delta}_l\}$, we can reformulate (8b) as follows

$$\mathbf{\Omega}^n = \underset{\mathbf{\Omega}\text{ binary}}{\arg\min} \text{ Tr}(\mathbf{\Omega}^T\mathbf{Q}^n) \text{ s.t. } \|\mathbf{\Omega}\|_0 = \lfloor pLM_{r,R}\rfloor, \quad (9)$$

where the l-th column of $\mathbf{Q}^n$ contains the diagonal entries of $\mathbf{G}_2\mathbf{R}_{xl}^n\mathbf{G}_2^H$. Recall that the sampling matrix $\mathbf{\Omega}$ is also required to have large spectral gap needed for good matrix completion performance [12]. However, it is difficult to incorporate such condition in the above optimization problem.

Noticing that row and column permutation of the sampling matrix would not affect its singular values and thus the spectral gap, we propose to optimize the sampling scheme by permuting the rows and columns of an initial sampling matrix $\mathbf{\Omega}^0$:

$$\mathbf{\Omega}^n = \underset{\mathbf{\Omega}}{\arg\min} \text{ Tr}(\mathbf{\Omega}^T\mathbf{Q}^n) \text{ s.t. } \mathbf{\Omega} \in \wp(\mathbf{\Omega}^0), \quad (10)$$

where $\wp(\mathbf{\Omega}^0)$ denotes the set of matrices obtained by arbitrary row and/or column permutations. The $\mathbf{\Omega}^0$ is generated with binary entries and $\lfloor pLM_{r,R}\rfloor$ ones. Meanwhile, $\mathbf{\Omega}^0$ has large spectral gap. One of the matrices that exhibit large spectral gap with high probability is the uniformly random sampling matrix [12]. Brute-force search can be used to find the optimal $\mathbf{\Omega}$. However, the complexity is very high since $|\wp(\mathbf{\Omega}^0)| = \Theta(M_{r,R}!L!)$. By alternately optimizing w.r.t. row permutation and column permutation on $\mathbf{\Omega}^0$, we can solve (10) using a sequence of linear assignment problems [19].

To optimize w.r.t. column permutation, we need to find the best one-to-one match between the columns of $\mathbf{\Omega}^0$ and the columns of $\mathbf{Q}^n$. We construct a cost matrix $\mathbf{C}^c \in \mathbb{R}^{L\times L}$ with $[\mathbf{C}^c]_{ml} \triangleq (\mathbf{\Omega}_{\cdot m}^0)^T\mathbf{Q}_{\cdot l}^n$. The problem turns out to be a linear assignment problem with cost matrix $\mathbf{C}^c$, which can be solved in polynomial time using the Hungarian algorithm [19]. Let $\mathbf{\Omega}^c$ denote the column-permutated sampling matrix after the above step. Then, we permute the rows of $\mathbf{\Omega}^c$ to optimally match the rows of $\mathbf{Q}^n$. Similarly, we construct a cost matrix $\mathbf{C}^r \in \mathbb{R}^{M_{r,R}\times M_{r,R}}$ with $[\mathbf{C}^r]_{ml} \triangleq \mathbf{\Omega}^c_{m\cdot}(\mathbf{Q}^n_{l\cdot})^T$. Again, the Hungarian algorithm can be used to solve the row assignment problem. The above column and row permutation steps are alternately repeated until Tr$(\mathbf{\Omega}^T\mathbf{Q}^n)$ changes relatively little.

It is easy to show that the EIP decreases during the alternating iterations between (8a) and (8b). The proposed joint-design spectrum sharing strategy is expected to further reduce the EIP at the radar RX node compared to the methods in Section 3.

5. ABOUT ASYNCHRONOUS SYSTEMS

In Section 2, the sampling rate of the radar system is assumed to match with the symbol rate of the communication system. Here we consider the mismatch cases. Denote f_s^R and f_s^C the radar sampling rate and the communication symbol rate, respectively. In the case of $f_s^R < f_s^C$, the interference arrived at the radar receiver will be down-sampled. The communication symbols which are not sampled by the radar receiver would introduce zero interference power to the radar RX. Therefore, we only need to design the communication transmit covariance of the communication waveforms which are sampled by the radar receiver. In the case of $f_s^R > f_s^C$, the interference arrived at the radar receiver will be over-sampled. One individual communication symbol will introduce interference to the radar system in $\lfloor f_s^R/f_s^C \rfloor$ consecutive time-slots. Correspondingly, in the expression of the EIP, each individual communication transmit covariance matrix will be weighted by the sum of interference channels for $\lfloor f_s^R/f_s^C \rfloor$ radar time-slots instead of one single interference channel. In the above mismatched cases, the spectrum sharing problem still follows similar form as in Section 3 and 4. Thus, the proposed techniques in this paper can be readily applied.

6. NUMERICAL RESULTS

For the simulations, we set the number of time-slots $L = 32$ and the noise variance $\sigma_C^2 = 0.01$. The MIMO radar system consists of collocated TX and RX antennas forming half-wavelength uniform linear arrays. The normalized Gaussian orthogonal waveforms [8] is used. There is one far-field stationary target at angle $30°$ w.r.t. to the arrays. For the radar system, noise is added at SNR= 25dB. For the communication capacity and power constraints, we use $C = 8$ bits/symbol and $P_t = L$ (the power is normalized by the power of radar waveform). The same uniformly random sampling scheme $\mathbf{\Omega}^0$ is adopted by the radar in the noncooperative and cooperative spectrum sharing (SS) methods. The joint-design SS method uses the same sampling matrix as its initial sampling matrix. The TFOCUS package [20] is used for low-rank matrix completion at the radar fusion center. The interference channels $\mathbf{G}_1$ and $\mathbf{G}_2$ are generated with independent entries distributed as $\mathcal{CN}(0, 0.1)$. The channel $\mathbf{H}$ has independent entries, distributed as $\mathcal{CN}(0, 1)$. The communication covariance is optimized according to different criteria of Sections 3 and 4. The obtained $\mathbf{R}_{xl}$ is used to generate $\mathbf{x}(l) = \mathbf{R}_{xl}^{1/2}$randn$(M_{t,C}, 1)$. We use the EIP and MC relative recovery error as the performance metrics, with the later defined as $\|\mathbf{DS} - \hat{\mathbf{DS}}\|_F/\|\mathbf{DS}\|_F$. For comparison, we also implement a "selfish communication" scenario, where the communication system minimizes the transmit power to achieve certain average capacity without any concern about the interferences it exerts to the radar system.

In the first scenario, we use $M_{t,R} = 4, M_{r,R} = M_{t,C} = 8, M_{r,C} = 4$. We plot the EIP results for 5 different realizations of $\mathbf{\Omega}^0$ in Fig. 2(a). For better visualization, Fig. 2(b) shows the relative recovery errors averaged over all 5 realizations of $\mathbf{\Omega}^0$. The cooperative spectrum sharing (SS) approach (see $(\mathbf{P}_1)$) outperforms its noncooperative counterpart (see $(\mathbf{P}_0)$) in terms of both EIP and MC recovery error. As discussed in Section 3, the EIP is significantly reduced by the cooperative SS when $p < 0.5$, *i.e.*, where $pM_{r,R}$ is much smaller than $M_{t,C}$. For the cooperative SS, the MC recovery performs well even when $p = 0.4$. The cooperative SS performs almost the same as the joint-design method in this scenario.

In the second scenario, we set $M_{t,R} = 16, M_{r,R} = 32, M_{t,C} = 4, M_{r,C} = 4$. In Fig. 3(a), we plot the EIP corresponding to 5 different realizations of $\mathbf{\Omega}^0$. Again, Fig. 3(b) shows the relative recovery error averaged over all realizations of $\mathbf{\Omega}^0$. The cooperative SS approach outperforms the noncooperative SS only marginally. This is due to the fact that $\mathbf{G}_{2l}$ is also full rank. The the joint-design method for SS of Section 4 optimizes $\mathbf{\Omega}$ starting from the sampling matrix used by the other three methods. However, it achieves much smaller EIP and relative recovery error than the other three approaches. This validates the effectiveness of the proposed joint-design method.

We should note that when p decreases, the null space of $\mathbf{G}_{2l}$ expands with high probability and the EIP of the cooperative SS approach is reduced. However, if p is too small, the MC recovery at the fusion center fails. In the above scenarios, the optimal range of p is $[0.4, 0.6]$, where the proposed cooperative and joint-design SS methods reduce the EIP by at least 20% over the "selfish communication method".

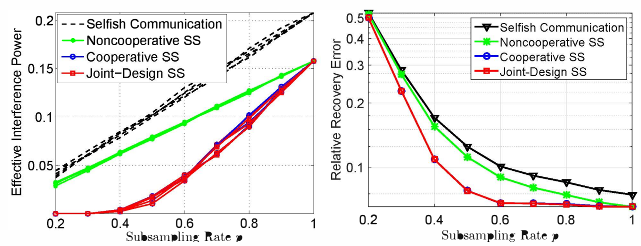

Fig. 2. Spectrum sharing under different sub-sampling rates. $M_{t,R} = 4, M_{r,R} = M_{t,C} = 8, M_{r,C} = 4$.

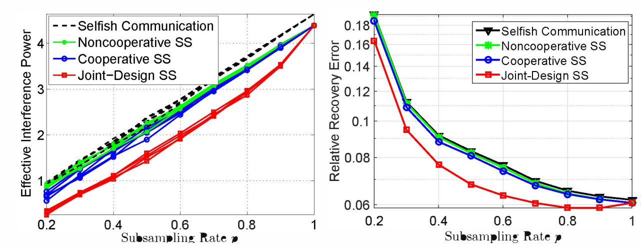

Fig. 3. Spectrum sharing under different sub-sampling rates. $M_{t,R} = 16, M_{r,R} = 32, M_{t,C} = 4, M_{r,C} = 4$.

7. CONCLUSIONS

We have considered spectrum sharing between MIMO communication and MIMO-MC radar systems. We have proposed several strategies to to reduce the interference from the communication TX to the radar. MC-MIMO radars are less susceptible to interference than MIMO radars, due to the fact that the interference affects only some entries of the data matrix. By appropriately designing the communication system waveforms, the interference can be further reduced by at least 20%. Both the theoretical and simulation results have shown that optimal design of the communication waveforms and the sampling scheme at the radar RX antennas can lead to further reduction of the interference.

8. REFERENCES

[1] "Radar spectrum regulatory overview," *[online] 2013*, http://www.darpa.mil/WorkArea/DownloadAsset.aspx?id=2147486331, (Accessed: July 2014).

[2] A Lackpour, M. Luddy, and J. Winters, "Overview of interference mitigation techniques between wimax networks and ground based radar," in *2011 20th Annual Wireless and Optical Communications Conference*, April 2011, pp. 1–5.

[3] S. Sodagari, A Khawar, T.C. Clancy, and R. McGwier, "A projection based approach for radar and telecommunication systems coexistence," in *IEEE Global Telecommun. Conf.*, Dec 2012, pp. 5010–5014.

[4] A Babaei, W.H. Tranter, and T. Bose, "A practical precoding approach for radar/communications spectrum sharing," in *2013 8th International Conference on Cognitive Radio Oriented Wireless Networks*, July 2013, pp. 13–18.

[5] Hai Deng and B. Himed, "Interference mitigation processing for spectrum-sharing between radar and wireless communications systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 3, pp. 1911–1919, July 2013.

[6] S. Amuru, R.M. Buehrer, R. Tandon, and S. Sodagari, "MIMO radar waveform design to support spectrum sharing," in *2013 IEEE Military Commun. Conf.*, Nov 2013, pp. 1535–1540.

[7] A Khawar, A Abdel-Hadi, and T.C. Clancy, "Spectrum sharing between s-band radar and lte cellular system: A spatial approach," in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks,*, April 2014, pp. 7–14.

[8] S. Sun, A. P. Petropulu, and W. U. Bajwa, "Target estimation in colocated MIMO radar via matrix completion," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, May 2013, pp. 4144–4148.

[9] D.S. Kalogerias and AP. Petropulu, "Matrix completion in colocated MIMO radar: Recoverability, bounds and theoretical guarantees," *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 309–321, Jan 2014.

[10] S Sun, W. Bajwa, and A. Petropulu, "MIMO-MC radar: A MIMO radar approach based on matrix completion," *accpted IEEE Transactions on Aerospace and Electronic Systems*, 2014.

[11] Yao Yu, AP. Petropulu, and H.V. Poor, "MIMO Radar using compressive sampling," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 146–163, Feb 2010.

[12] S. Bhojanapalli and P. Jain, "Universal matrix completion," *arXiv preprint arXiv:1402.2324*, 2014.

[13] R. Zhang and Y. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 88–102, Feb 2008.

[14] R. Zhang, Y. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 102–114, May 2010.

[15] M. Filo, A Hossain, AR. Biswas, and R. Piesiewicz, "Cognitive pilot channel: Enabler for radio systems coexistence," in *2009 Second International Workshop on Cognitive Radio and Advanced Spectrum Management*, May 2009, pp. 17–23.

[16] Andrea Goldsmith, Syed Ali Jafar, Nihar Jindal, and Sriram Vishwanath, "Capacity limits of mimo channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, 2003.

[17] David Tse and Pramod Viswanath, *Fundamentals of wireless communication*, Cambridge university press, 2005.

[18] E.J. Candes and Y. Plan, "Matrix completion with noise," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 925–936, June 2010.

[19] Harold W Kuhn, "The hungarian method for the assignment problem," *Naval research logistics quarterly*, vol. 2, no. 1-2, pp. 83–97, 1955.

[20] S. R. Becker, E. J. Candès, and M. C. Grant, "Templates for convex cone problems with applications to sparse signal recovery," *Mathematical Programming Computation*, vol. 3, no. 3, pp. 165–218, 2011.