Performance of Chirp Spread Spectrum in Wireless Communication Systems

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***Abstract*—This paper analyzes the structure and modem scheme of Chirp-BOK system. In Additive White Gaussian Noise channel, an approach of studying the performance of Chirp-BOK is presented and the BER-SNR result is evidently improved compared with traditional BOK. Furthermore, the BER of multi-user Chirp-BOK system is discussed.**

Keywords-chirp spread spectrum; BOK; bit error rate; multi-user

I. INTRODUCTION

Chirp spread spectrum (CSS) is receiving more and more attention as the IEEE 802.15.4a standard includes it as one of the two optional PHYs. CSS, used only in radar system in the past, becomes a rational choice for low-rate wireless personal networks (LR-WPAN) defined by IEEE 802.15.4a. Based on a linear frequency modulated and wide-banded pulse called chirp, CSS can provide robust performance as well as low power consumption, low time delay and multi-path resistance [1,2].

CSS can combine with digital modulation schemes to achieve better performance and practicability, such as ① binary orthogonal keying (BOK), ② quadrature phase-shift keying (QPSK) and ③ differential quadrature phase-shift keying (DQPSK). Chirp modulation was first introduced by Winkler [3] in 1962. She suggested a binary system using a pair of chirps with opposite chirp rates. In 1973, A J. Berni and W D. Gregg [4] compared the BER of binary chirp modulation with FSK and PSK. [5] and [6] presented a digital modulation scheme of combining CSS and DQPSK and measured its bit error rate (BER) through simulation. In [7], Hengstler et al. proposed a novel approach in multi-user chirp modulation spectrum system (CMSS) and derived the analytical expression of the BER in additive white Gaussian noise channel (AWGN). [8] presented a Chirp-BOK system and its simulated performance estimation.

This paper studies on the structure and modem scheme of Chirp-BOK system, derives the BER expression of single-user Chirp-BOK and confirms the theoretical results through simulations. On the basis of single-user Chirp-BOK, we further study multi-user Chirp-BOK system and then derive and depict its BER expression with matrices.

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II. CHIRP THEORY

Chirp signals are stimulated by surface acoustic wave device of dispersive delay line (SAW DLL). Its wave form can be written as:

LFM:
$$s(t) = a(t)\cos(2\pi f_o t + \pi \mu t^2) \tag{1}$$

where $a(t)$ is the envelope of chirp signal and $a(t)=0$, when $|t| \geq T/2$; f_o is the starting frequency; μ is the rate of frequency change. $\mu > 0$ makes an up-chirp while $\mu < 0$ makes a down-chirp.

One important feature of chirp signal is its autocorrelation function, which is given by [9]:

$$\varphi_{ss}(t) = \sqrt{BT}\,\frac{\sin[\pi BT(1-\frac{|t|}{T})]}{\pi BT}\cos(2\pi f_0 t) \tag{2}$$

$$-T < t < T$$

The envelope has a pulse peak at $t = 0$ and small main-lobe width of $2/B$ as its first zeroes are at $\pm 1/B$ [10].

For up-chirp signal $c_1(t)$ and down-chirp signal $c_2(t)$, $\int_0^T c_1(t)c_2(t)dt \approx 0$ and $\int_0^T c_i(t)c_i(t)dt \approx 1$, $i = 1,2$ when normalized envelope parameter $\sqrt{2/T}$ is added. So up-chirp and down-chirp signals can approximately keep to orthogonal criterion and consequently can be used to form BOK system.

III. CHIRP-BOK ANALYSIS

The block diagram of Chirp-BOK system is shown in Fig. 1.

At the transmitter end, a switch circuit is controlled by the bit stream under transmission. If one bit is '1', an up-chirp signal is sent. If one bit is '0', a down-chirp signal is sent.

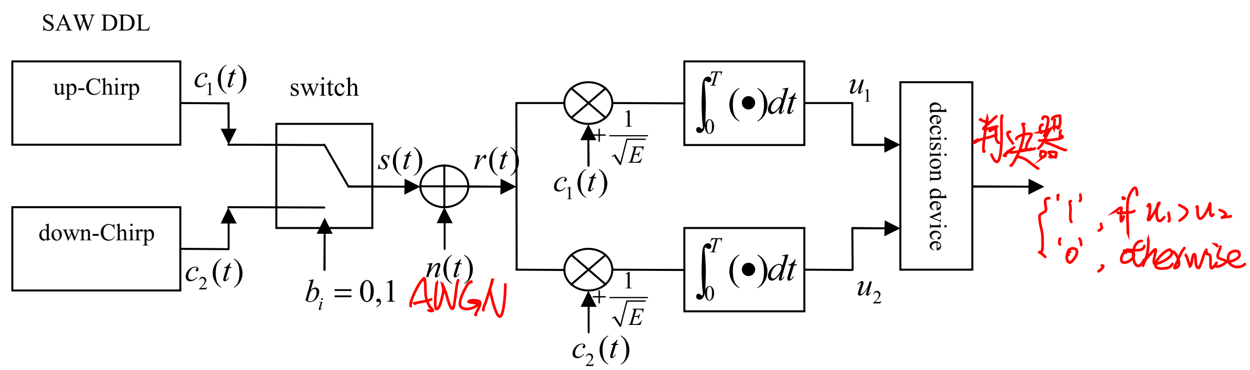

Figure 1. Block diagram of Chirp-BOK system.

An AWGN channel is considered. It adds a white Gaussian noise with zero mean and variance $N_0/2$ to the transmitted signal.

At the receiver end, the received signal is fed into two branches and multiplied with up-chirp $c_1(t)$ and down-chirp $c_2(t)$ respectively. Two decision variables are produced:

$$u_i = \frac{1}{\sqrt{E}}\int_0^T c_i(t)r(t)dt,\ i=1,2 \tag{3}$$

The data symbol is estimated as '1' if u_1 is greater than u_2 and '0' otherwise.

(?)

Within a period of $0 \le t < T$,

$$\begin{cases} c_1(t) = a(t)\cos(2\pi f_0 t + \pi\mu t^2) \\ c_2(t) = a(t)\cos(2\pi f_0 t - \pi\mu t^2) \end{cases} \tag{4}$$

The transmitted signal is

$$s(t) = \begin{cases} c_1(t), '1'\ sent \\ c_2(t), '0'\ sent \end{cases},$$

which can be represented as

发送：
$$s(t) = bc_1(t) + (1-b)c_2(t) \tag{5}$$

where b is the bit ready to be sent. The received signal is composed of transmitted signal and AWGN

接收：
$$r(t) = s(t) + n(t) = bc_1(t) + (1-b)c_2(t) + n(t) \tag{6}$$

The mean of the noise is 0 and the variance is $N_0/2$ as mentioned before.

In the definition of decision variable (3), E is bit energy

$$E = \int_0^T c_i^2(t)dt,\quad i=1,2 \tag{7}$$

If ρ is defined as the cross-correlation coefficient between up-chirp and down-chirp.

$$\rho = \frac{1}{E}\int_0^T c_1(t)c_2(t)dt \tag{8}$$

Then u_1 and u_2 can be expressed as

$$\begin{aligned} u_1 &= \sqrt{E}(b + (1-b)\rho) + \frac{1}{\sqrt{E}}\int_0^T c_1(t)n(t)dt \\ u_2 &= \sqrt{E}((1-b) + b\rho) + \frac{1}{\sqrt{E}}\int_0^T c_2(t)n(t)dt \end{aligned} \tag{9}$$

So

$$\begin{aligned} \Delta = u_1 - u_2 &= \sqrt{E}(2b-1)(1-\rho) \\ &+ \frac{1}{\sqrt{E}}\int_0^T (c_1(t) - c_2(t))n(t)dt \end{aligned} \tag{10}$$

As $n(t)$ is a white Gaussian process, Δ conforms to normal distribution. Its mean is given by $\mu_\Delta = \sqrt{E}(2b-1)(1-\rho)$ and its variance is $\sigma_\Delta^2 = E((\Delta - \mu_\Delta)^2) = N_0$.

Two cases will lead to a bit error: the bit symbol is '0' but $\Delta > 0$ or the bit symbol is '1' but $\Delta < 0$. Therefore, the possibility of a bit error at the receiver end is

$$\begin{aligned} P_e &= P\{\Delta > 0 \mid '0'sent\}P\{'0'sent\} + P\{\Delta < 0 \mid '1'sent\}P\{'1'sent\} \\ &= \frac{1}{2}(P\{\Delta > 0 \mid '0'sent\} + P\{\Delta < 0 \mid '1'sent\}) \end{aligned} \tag{11}$$

Due to the symmetry of normal distribution, P_e is simplified as

对称性

$$\begin{aligned} P_e &= P\{\Delta > 0 \mid '0'sent\} = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_\Delta^2}}\exp[-\frac{(x-\mu_\Delta)^2}{2\sigma_\Delta^2}]dx \\ &= Q(-\frac{\mu_\Delta}{\sigma_\Delta}) = Q(-\sqrt{\frac{E}{N_0}}(2b-1)(1-\rho)) \\ &= \frac{1}{2}erfc(\sqrt{\frac{E}{2N_0}}(1-\rho)) \end{aligned} \tag{12}$$

$\rho = 0$

Under ideal condition, for coherent detected 2FSK theoretical BER is $\frac{1}{2}erfc(\sqrt{E/2N_0})$ [11] and for traditional BOK $\frac{1}{2}erfc(\sqrt{E/4N_0})$ [1]. To validate the theoretically derived BER, a simulation result is carried out by Monte Carlo simulation in MATLAB software and compared with (12). In Fig. 2, above four BERs are depicted. The system parameters of Chirp-BOK are listed in table 1.

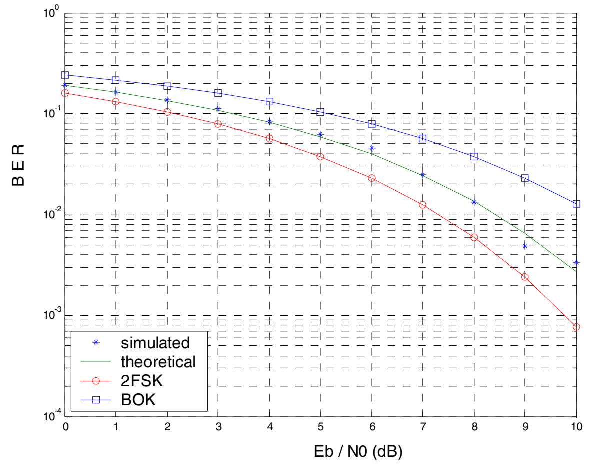

Figure 2. Theoretical and simulated BERs of Chirp-BOK compared with theoretical BERs of 2FSK and BOK.

TABLE I. SYSTEM PARAMETERS OF CHIRP-BOK

T	***f₀***	***B***	***E***	***ρ***
1μs	2.5GHz	64MHz	0.5	2.7149×e-9

As is shown in Fig. 2, the theoretical and simulated BER curves coincide well with each other, both of which are superior to that of BOK and inferior to that of 2FSK. The reason for this can be explained as follows. Chirp signal has an ultra-wide bandwidth and a large time-bandwidth product BT. During transmitting in the channel, both the signal and noise spread on the wide band. But at the receiver, the signal produces a pulse peak due to its autocorrelation while the noise is further spread and has rather small amplitude. However, up-chirp and down-chirp are not strictly orthogonal, which limits the performance of Chirp-BOK.

IV. MULTI_USER CHIRP-BOK ANALYSIS

For a multi-user Chirp-BOK system, N users share the same bandwidth. Every user has the same modem scheme as single-user system but different up-chirps and down-chirps:

$$\begin{aligned} c_{1i}(t) &= a(t)\cos(2\pi f_i t + \pi\mu_i t^2) \\ c_{2i}(t) &= a(t)\cos(2\pi f_i t - \pi\mu_i t^2) \end{aligned} \tag{13}$$

The transmitted signal can be written as

$$s_i(t) = b_i c_{1i}(t) + (1-b_i)c_{2i}(t) \tag{14}$$

where b_i is the bit of the ith user. Received signal can be written as

$$r(t) = \sum_{i=1}^{N} s_i(t) + n(t) \tag{15}$$

For the receiver of jth user, let $B = (b_1\, b_2 \cdots b_N)$, a $1\times N$ matrix consists of all the bits of N users. $I = (11\cdots 1)$ is a $1\times N$ unit matrix. Define two $N\times 1$ matrices C_1 and C_2:

$$C_1 = \begin{pmatrix} c_{11}(t) \\ c_{12}(t) \\ \vdots \\ c_{1N}(t) \end{pmatrix}, \quad C_2 = \begin{pmatrix} c_{21}(t) \\ c_{22}(t) \\ \vdots \\ c_{2N}(t) \end{pmatrix} \tag{16}$$

Then the received signal can be expressed by matrices

$$r(t) = BC_1 + (I-B)C_2 + n(t) \tag{17}$$

The decision variables are given by

$$u_{1j} = \frac{1}{\sqrt{E}}\int_0^T c_{1j}(t)[BC_1 + (I-B)C_2]dt + \frac{1}{\sqrt{E}}\int_0^T c_{1j}(t)n(t)dt \tag{18}$$

$$u_{2j} = \frac{1}{\sqrt{E}}\int_0^T c_{2j}(t)[BC_1 + (I-B)C_2]dt + \frac{1}{\sqrt{E}}\int_0^T c_{2j}(t)n(t)dt \tag{19}$$

As a result ,

$$\begin{aligned} \Delta_j = u_{1j} - u_{2j} &= \frac{1}{\sqrt{E}}[B\int_0^T (c_{1j}(t) - c_{2j}(t))C_1 dt \\ &+ (I-B)\int_0^T (c_{1j}(t) - c_{2j}(t))C_2 dt] \\ &+ \frac{1}{\sqrt{E}}\int_0^T (c_{1j}(t) - c_{2j}(t))n(t)dt \end{aligned} \tag{20}$$

Define two kinds of coefficients

$$\begin{aligned} \xi_i &= \frac{1}{E}\int_0^T [c_{1j}(t) - c_{2j}(t)]c_{1i}(t)dt \\ \zeta_i &= \frac{1}{E}\int_0^T [c_{1j}(t) - c_{2j}(t)]c_{2i}(t)dt \end{aligned} \tag{21}$$

and two matrices.

$$P = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{pmatrix}, \quad Q = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_N \end{pmatrix} \tag{22}$$

Δ_j can be written in matrix form.

$$\Delta_j = \sqrt{E}[BP + (I-B)Q] + \frac{1}{\sqrt{E}}\int_0^T [c_{1j}(t) - c_{2j}(t)]n(t)dt \tag{23}$$

As is discussed before, Δ_j is a variable of normal distribution and $\mu_{\Delta j} = \sqrt{E}[BP + (I-B)Q]$, $\sigma_{\Delta j}^2 = N_0$.

Hence, the BER of the jth user is found to be

$$\begin{aligned} P_e &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_\Delta^2}}\exp[-\frac{(x-\mu_\Delta)^2}{2\sigma_\Delta^2}]dx \\ &= \frac{1}{2} erfc(-\sqrt{\frac{E}{2N_0}}[BP + (I-B)Q]) \end{aligned} \tag{24}$$

Averaging (23) over all N receivers, we get the BER of the multi-user Chirp-BOK system as a whole.

$$BER = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{2} erfc(-\sqrt{\frac{E}{2N_0}}[BP + (I-B)Q]) \tag{25}$$

In a limiting case, substituting $N=1$ into (25) turns B, I, C_1, C_2, P and Q into 1×1 matrices and $P=(1-\rho)$, $Q=(\rho-1)$. Thus, (24) degenerates to the single-user BER result (12). As is illustrated in (24), the elements of P, and Q, namely ξ_i and ζ_i, are determined by $c_{1j}(t)-c_{2j}(t)$. In other words, every user has a different BER. On condition that $c_{1i}(t)$ and $c_{2i}(t)$ are designed properly, the BERs may have no relation with j and each user will have equal performance.

V. Conclusion

This paper derived the BER of Chirp-BOK system through theoretical analysis and validated it by simulation. In AWGN channel, Chirp-BOK performs better than traditional BOK in noise resistance owing to spread spectrum. Furthermore a multi-user Chirp-BOK system is presented and its BER is derived and simplified with matrices.

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