

# New PTS Schemes for PAPR Reduction of OFDM Signals Without Side Information

Hyun-Seung Joo, Kee-Hoon Kim, Jong-Seon No, and Dong-Joon Shin

**Abstract**—In this paper, two partial transmit sequence (PTS) schemes without side information (SI) are proposed for reducing peak-to-average power ratio of orthogonal frequency division multiplexing signals. The proposed schemes do not transmit SI identifying a rotating vector because identifiable phase offset is applied to the elements of each rotating vector. To extract SI from the received signal and recover the data sequence, the maximum likelihood (ML) detector is used. This ML detector exploits the Euclidean distance between the given signal constellation and the signal constellation rotated by the phase offset. By doing pairwise error probability (PEP) analysis, it is investigated how to choose good phase offsets for embedding SI. Also, the performance degradation caused by SI detection failure is analyzed based on PEP. Finally, through numerical analysis, it is shown that the BER performance of the proposed PTS schemes is not degraded compared with the conventional PTS with perfect SI.

**Index Terms**—Orthogonal frequency division multiplexing (OFDM), partial transmit sequence (PTS), peak-to-average power ratio (PAPR), phase offset, side information (SI).

## I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has been adopted as a standard method in various wireless communication systems because it can achieve high-rate data transmission and insure high reliability over the multipath fading environment [1]. However, high peak-to-average power ratio (PAPR) is a critical drawback of OFDM because OFDM signals are generated by summing a large number of independently modulated subcarriers. High PAPR causes significant inter-modulation and out-of-band radiation when OFDM signals pass through nonlinear devices such as high power amplifier (HPA).

To solve the PAPR problem of OFDM signals, many PAPR reduction schemes have been proposed such as clipping [2], [3], coding techniques [4], [5], selected mapping (SLM) [6]–[8], partial transmit sequence (PTS) [9]–[11], tone

reservation (TR) [12], [13], tone injection (TI) [12], and active constellation extension (ACE) [14]. Among them, the PTS scheme is a well-known probabilistic method without causing signal distortion, which selects the OFDM signal with the minimum PAPR among  $U$  alternative signals generated by multiplying rotating factors to  $V$  signal subsequences and summing them.

In order to recover the input symbol sequence in the PTS scheme, the selected signal index, called side information (SI), should be transmitted to the receiver. If the transmitted SI is erroneously detected, the BER performance of the OFDM systems is critically degraded and thus strong error correcting scheme should be applied to SI. Since some subcarriers should be reserved for SI transmission in the PTS scheme, the spectral efficiency of the OFDM systems is degraded.

Recently, there have been several PTS schemes which can recover the original OFDM signal sequence without transmitting SI. In [15], a PTS scheme without SI is proposed, where the SI is considered as a part of channel frequency response. That is, channel estimation is independently performed for each signal subsequence by introducing novel type pilot arrangement. However, for doing this, the conventional equal-spaced pilot tone arrangement has to be modified, which inevitably causes poor channel estimation.

Similar to [15], an additional pilot tone is intentionally inserted at the end of each signal subsequence in [16] and therefore channel estimation on each signal subsequence without SI is separately performed for data symbols. That is, channel estimation can be improved by using an additional pilot tone placed at the end of each signal subsequence. Clearly, it degrades spectral efficiency.

A novel multi-points square mapping scheme is proposed in [17], where an expanded signal constellation is considered. The key idea of this scheme is to use four constellation points to represent one data. However, it is clear that the expansion of the signal constellation leads to a smaller minimum Euclidean distance and thus causes BER performance degradation.

In [18], a PTS scheme without SI is proposed, where the signal constellation is randomly rotated and the original OFDM signal sequence is recovered by using a maximum likelihood (ML) detector. However, this scheme requires high detection complexity to recover the original OFDM signal sequence because the search space by an ML detector is very large due to randomly designed phase offsets.

In [19], the candidates are generated through cyclically shifting each signal subsequence in time domain and

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combining them in a recursive manner. Similar to [18], the ML detector at the receiver can successfully recover the original signal without SI. However, this scheme also needs large amounts of computations because the search space by the ML detector is quite large.

In this paper, two PTS schemes without SI are proposed, which perform low-complexity detection of the selected rotating vector at the receiver. Similar to the method in [20], the proposed PTS schemes embed the SI identifying rotating vectors into the alternative signal sequences by giving identifiable phase offset to the elements of each rotating vector. Specifically, the number of phase offsets has to be properly chosen by considering the number of subsequences of PTS scheme, and the phase rotating factors are phase shifted by these offsets, which is different to the scheme in [20] for a SLM case. To extract SI from the received signal and recover the data sequence, an ML detector is used. This ML detector exploits the Euclidean distance between the given signal constellation and the signal constellation rotated by the phase offset. By doing pairwise error probability (PEP) analysis, it is investigated how to choose good phase offsets for embedding SI. Also, the performance degradation caused by SI detection failure is analyzed based on PEP. Through numerical analysis, it is shown that the BER performance of the proposed PTS schemes is not degraded compared with the conventional PTS with perfect SI.

Contrary to the schemes in [15] and [16], the proposed PTS schemes do not modify the pilot tones, that is, both spectral efficiency and accurate channel estimation are achieved by the proposed PTS schemes. Also, the proposed schemes do not expand constellation points as done in [17]. Instead, the proposed schemes rotate the signal constellation by using a small number of phase offsets. Thus, the proposed schemes show low computational complexity to inspect a smaller search space compared with the schemes in [18] and [19].

The rest of the paper is organized as follows. In Section II, the conventional PTS scheme is explained. In Section III, a new PTS scheme I (P-PTS I) is proposed, and the phase offsets and the subblock phase offset vectors used for embedding the SI into the rotating vectors are described. Also, an ML detector for P-PTS I is explained, and the performance degradation caused by SI detection failure is analyzed. To further lower the detection complexity of P-PTS I, a new PTS scheme II (P-PTS II) is proposed in Section IV. In Section V, the computational complexity of the proposed PTS schemes is analyzed and compared with other PTS schemes. Simulation results are given to show the performance of the proposed PTS schemes in Section VI. Finally, conclusions are given in Section VII.

## II. CONVENTIONAL PTS SCHEME

In an OFDM system, an input symbol sequence  $\mathbf{X} = [X_0 \ X_1 \ \dots \ X_{N-1}]$  is made up of  $N$  complex symbols from a signal constellation  $\mathcal{Q}$  of size  $q$ , where  $N$  is the number of subcarriers. An OFDM signal sequence  $\mathbf{x} = [x_0 \ x_1 \ \dots \ x_{N-1}]$  is generated by applying inverse fast Fourier transform (IFFT)

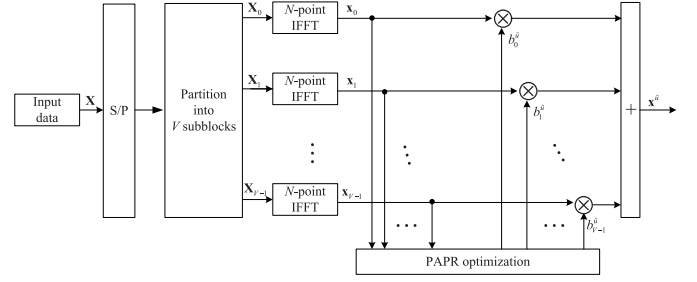


Fig. 1. A block diagram of the conventional PTS scheme.

to  $\mathbf{X}$  as  $\mathbf{x} = \text{IFFT}(\mathbf{X})$ , that is,

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N}k}, \quad 0 \leq n \leq N-1, \quad (1)$$

and the PAPR of the OFDM signal sequence  $\mathbf{x}$  is defined as

$$\text{PAPR}(\mathbf{x}) \triangleq \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{E\{|x_n|^2\}} \quad (2)$$

where  $E\{\cdot\}$  denotes the expectation.

In the PTS scheme, an input symbol sequence  $\mathbf{X}$  is partitioned into  $V$  disjoint symbol subsequences  $\mathbf{X}_v = [X_{v,0} \ X_{v,1} \ \dots \ X_{v,N-1}]$ ,  $0 \leq v \leq V-1$ , which can be expressed as

$$\mathbf{X} = \sum_{v=0}^{V-1} \mathbf{X}_v. \quad (3)$$

For each symbol subsequence  $\mathbf{X}_v$ , all symbols are zero except for  $N/V$  data symbols. The signal subsequence  $\mathbf{x}_v = [x_{v,0} \ x_{v,1} \ \dots \ x_{v,N-1}]$ , called a subblock, is generated by applying IFFT to each symbol subsequence  $\mathbf{X}_v$ . Each subblock  $\mathbf{x}_v$  is multiplied by a rotating factor  $b_v^u$  with a unit magnitude selected from a given alphabet  $\mathbb{B}$  which is practically  $\{\pm 1\}$  or  $\{\pm 1, \pm j\}$ . Also, we denote the cardinality of  $\mathbb{B}$  as  $|\mathbb{B}| = W$ . Summing these  $V$  phase-rotated subblocks, the  $u$ -th alternative signal sequence  $\mathbf{x}^u = [x_0^u \ x_1^u \ \dots \ x_{N-1}^u]$  is obtained as

$$\mathbf{x}^u = \sum_{v=0}^{V-1} b_v^u \mathbf{x}_v \quad (4)$$

where  $0 \leq u \leq U-1$  and  $U = W^{V-1}$ . Let  $\mathbf{b}^u = [b_0^u \ b_1^u \ \dots \ b_{V-1}^u]$  be the  $u$ -th rotating vector, where  $\mathbf{b}^u \in \mathbb{B}^V$  and  $b_0^u = 1$ . In order to choose the alternative signal sequence with the minimum PAPR for transmission, the optimum rotating vector  $\mathbf{b}^{\tilde{u}} = [b_0^{\tilde{u}} \ b_1^{\tilde{u}} \ \dots \ b_{V-1}^{\tilde{u}}]$  is obtained by

$$\mathbf{b}^{\tilde{u}} = \arg \min_{\substack{\mathbf{b}^u \in \mathbb{B}^V \\ \text{and } b_0^u = 1}} \left( \max_{0 \leq n \leq N-1} \left| \sum_{v=0}^{V-1} b_v^u x_{v,n} \right| \right). \quad (5)$$

Fig. 1 shows the OFDM transmitter employing the PTS scheme. It is well known that the PAPR reduction performance of the PTS scheme depends on subblock partitioning scheme. In general, there are three types of subblock partitioning methods such as adjacent, interleaving, and random subblock

partitioning schemes. Among them, random subblock partitioning scheme shows the best PAPR reduction performance, but it shows the largest complexity. Also, in the PTS scheme, a transmitter is required to send the SI on  $\tilde{u}$  to correctly recover the original input symbol sequence at the receiver, which results in some loss of data transmission rate.

### III. PROPOSED PTS SCHEME I

#### A. Embedding Side Information Into Rotating Vectors

To eliminate the transmission of SI, we propose a method to embed the SI into the rotating vectors through phase-shifting them by appropriate offsets. First, a  $V$ -tuple phase offset vector is defined as

$$\mathbf{S}^u = [S_0^u \ S_1^u \ \cdots \ S_{V-1}^u]$$

where  $S_v^u \in \{0, 1, \dots, Z\}$ ,  $0 \leq v \leq V-1$  and  $0 \leq u \leq U-1$ ,  $S_v^u = 0$  implies phase offset 0, and  $Z$  is the number of distinct nonzero phase offsets. To embed the SI on  $u$  into the  $u$ -th rotating vector  $\mathbf{b}^u$ , each rotating factor  $b_v^u$ ,  $0 \leq v \leq V-1$ , is multiplied by  $e^{j\theta_{S_v^u}^u}$ , where  $\theta_{S_v^u}^u$  is the phase offset indicated by  $S_v^u$  as  $0 \leq \theta_{S_v^u}^u < 2\pi$ . Then, the  $u$ -th modified rotating vector  $\bar{\mathbf{b}}^u$  is defined as

$$\begin{aligned} \bar{\mathbf{b}}^u &= [\bar{b}_0^u \ \bar{b}_1^u \ \cdots \ \bar{b}_{V-1}^u] \\ &= [b_0^u e^{j\theta_{S_0^u}^u} \ b_1^u e^{j\theta_{S_1^u}^u} \ \cdots \ b_{V-1}^u e^{j\theta_{S_{V-1}^u}^u}] \end{aligned}$$

and the  $u$ -th alternative signal sequence  $\mathbf{x}^u$  in the proposed PTS scheme is obtained as

$$\mathbf{x}^u = \sum_{v=0}^{V-1} \bar{b}_v^u \mathbf{x}_v. \quad (6)$$

This scheme will be called the proposed PTS scheme I (P-PTS I) and  $\mathbf{x}^{\hat{u}}$  with the minimum PAPR is selected for transmission.

Suppose that the selected OFDM signal  $\mathbf{x}^{\hat{u}}$  is transmitted through noiseless channel. Then, the received OFDM signal is FFTed to obtain the phase-shifted symbol for the  $k$ -th input symbol  $X_k$  belonging to the  $v$ -th symbol subsequence  $\mathbf{X}_v$ , that is,  $R_k = \bar{b}_v^{\hat{u}} X_k$ . Clearly,  $\bar{b}_v^{\hat{u}} X_k$  is a symbol from the signal constellation  $\mathcal{Q}_{\theta_{S_v^{\hat{u}}}}$  which is obtained by rotating the signal constellation  $\mathcal{Q}$  by  $\theta_{S_v^{\hat{u}}}$ , where  $0 \leq \theta_{S_v^{\hat{u}}} < 2\pi$  and  $S_v^{\hat{u}}$  is the  $v$ -th element of the  $\hat{u}$ -th phase offset vector chosen for transmission. The nonzero phase offsets should be selected such that the detected symbol  $\bar{b}_v^{u*} R_k$  does not come back on the signal constellation  $\mathcal{Q}$  when  $S_v^u \neq S_v^{\hat{u}}$ , where  $\bar{b}_v^{u*}$  is the complex conjugate of  $\bar{b}_v^u$ . This phenomenon can help the receiver find the correct SI. Specifically, if  $U$  phase rotating vectors are expressed by  $U$  distinct phase offset vectors  $\mathbf{S}^u$ , the receiver can find the correct SI by observing the deviant Euclidean distance of the detected symbol from the signal constellation  $\mathcal{Q}$ .

Clearly, in order to properly modify  $U$  rotating vectors by using  $V$ -tuple phase offset vectors,  $Z$  must be selected such that

$$(Z+1)^V \geq U = W^{V-1}. \quad (7)$$

Then, the receiver can estimate the SI and recover the input symbol sequence  $\mathbf{X}$  by using ML detector exploiting the

Euclidean distance between the original and rotated signal constellations. In the next subsection, this ML detection will be explained in detail.

#### B. ML Detection for the P-PTS I

From the received signal generated by the modified rotating vector  $\bar{\mathbf{b}}^{\hat{u}}$ , the receiver should find the index  $\hat{u}$  without using SI and recover the input symbol sequence  $\mathbf{X}$ . The received symbol  $R_k$  at the  $k$ -th subcarrier, which belongs to the  $v$ -th symbol subsequence  $\mathbf{X}_v$ , in frequency domain is expressed as

$$R_k = H_k \bar{b}_v^{\hat{u}} X_k + N_k \quad (8)$$

where  $H_k$  is the frequency response and  $N_k$  is an AWGN sample at the  $k$ -th subcarrier with the variance per dimension  $N_0/2$ . It is assumed that channel is quasi-static Rayleigh fading channel and  $H_k$ 's are statistically independent and perfectly known to the receiver, that is, the perfect channel state information (CSI) is assumed.

Now, we will derive the ML detector for the P-PTS I, which exploits the Euclidean distance between the original and rotated signal constellations as follows. Let  $\mathbf{R}_v = [R_{v,0} \ R_{v,1} \ \cdots \ R_{v,N-1}]$ ,  $0 \leq v \leq V-1$ , denote the  $v$ -th received symbol subsequence. The ML detector of the P-PTS I is operated in two steps.

First, the partial metric for each received symbol subsequence  $\mathbf{R}_v$  corresponding to  $\mathbf{X}_v$  is calculated as

$$D_{v,S_v^u} = \sum_{k \in \mathbb{I}_v} \min_{X'_k \in \mathcal{Q}} |R_{v,k} e^{-j\theta_{S_v^u}^u} - \hat{H}_k X'_k|^2 \quad (9)$$

where  $|\cdot|$  denotes the magnitude of a complex number,  $\hat{H}_k$  is the estimated channel response, and  $\mathbb{I}_v = \{I_{v,0}, I_{v,1}, \dots, I_{v,\frac{N}{V}-1}\} \subset \{0, 1, \dots, N-1\}$  is the index set of  $N/V$  data symbols contained in the  $v$ -th symbol subsequence  $\mathbf{X}_v$ . Note that  $\mathbb{I}_v$  is determined by the subblock partitioning scheme. From the process in (9), for each  $v$ -th symbol subsequence, we obtain  $D_{v,S_v^0}, D_{v,S_v^1}, \dots, D_{v,S_v^{U-1}}$ . Also, the corresponding  $X'_k$  for each  $S_v^u$  in (9) are denoted as a vector  $\hat{\mathbf{X}}_v^{S_v^u} = [\hat{X}_{v,0}^{S_v^u} \ \hat{X}_{v,1}^{S_v^u} \ \cdots \ \hat{X}_{v,N-1}^{S_v^u}]$ ,  $0 \leq u \leq U-1$ , where  $\hat{X}_{v,k}^{S_v^u}$  is the constellation point in  $\mathcal{Q}$  for  $X'_k$ , which minimizes the Euclidean distance between  $R_{v,k} e^{-j\theta_{S_v^u}^u}$  and  $\hat{H}_k X'_k$ . Clearly,  $\hat{\mathbf{X}}_v^{S_v^u}$  consists of  $N/V$  detected data symbols with indices in  $\mathbb{I}_v$  and  $N(V-1)/V$  zeros for the removing index positions. This process is repeated for all  $v$ ,  $0 \leq v \leq V-1$ .

Second, by using the partial metrics  $D_{v,S_v^u}$ 's in (9), calculate  $D_u = \sum_{v=0}^{V-1} D_{v,S_v^u}$ ,  $0 \leq u \leq U-1$ , and the index  $\hat{u}$  is obtained by searching  $D_{\hat{u}}$  with the minimum value among  $U$  metric values  $D_u$  as

$$\hat{u} = \arg \min_{0 \leq u \leq U-1} D_u. \quad (10)$$

Then, the detected input symbol sequence at the receiver is finally obtained as

$$\hat{\mathbf{X}}^{\hat{u}} = \sum_{v=0}^{V-1} \bar{b}_v^{\hat{u}*} \hat{\mathbf{X}}_v^{S_v^{\hat{u}}},$$

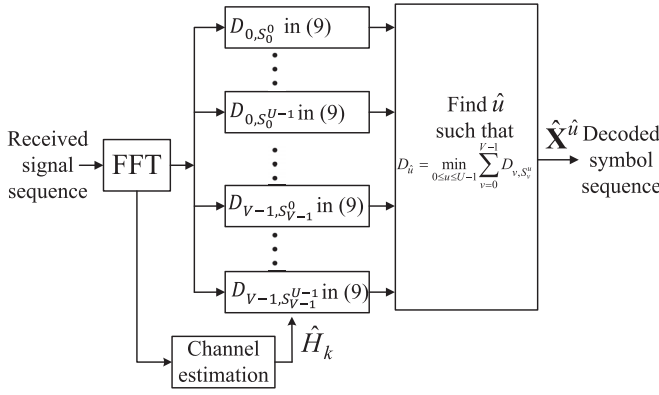


Fig. 2. A block diagram of the ML detector for the P-PTS I.

which is the summation of  $V$  detected symbol subsequences derotated by multiplying  $b_v^{u*}$ ,  $0 \leq v \leq V-1$ . Fig. 2 shows a block diagram of the proposed ML detector for the P-PTS I.

Note that if the minimum  $Z$  to satisfy the condition (7) is chosen,  $Z$  is smaller than  $U$ . Consequently, the overall detection complexity of the P-PTS I to find the index  $\hat{u}$  and the detected symbol sequence  $\hat{\mathbf{X}}^{\hat{u}}$  in (9) and (10) is  $(Z+1)qN|\cdot|^2$  operations if the real additions are ignored because the complexity of  $|\cdot|^2$  operation is much larger than that of real addition.

### C. Design Criteria of $V$ -Tuple Phase Offset Vectors $\mathbf{S}^u$ and Phase Offsets

In this subsection, design criteria of  $V$ -tuple phase offset vectors  $\mathbf{S}^u$  and phase offsets are derived, which minimize the PEP of the ML detector given in (9) and (10). Note that PEP analysis is widely used for analyzing the error correction performance of communication systems with coding schemes such as trellis codes and space-time codes [21].

For convenience of PEP analysis, we assume that the adjacent subblock partitioning is used and the similar analysis can be applied to other subblock partitioning methods. Also, we assume Rayleigh fading channel such that the fading coefficients  $H_k$  in (8) are samples of independent complex Gaussian random variables with zero mean and variance 0.5 per dimension. We consider that each element of the signal constellation is contracted by a scale factor  $\sqrt{E_s}$  so that the average energy of the signal constellation is 1, where  $E_s$  is the average energy of the transmitted symbols. Therefore, the PEP analysis for the proposed ML detector can be performed independently from the average energy of the signal constellation.

Let  $\mathbf{B}^u = [\bar{b}_0^u \cdots \bar{b}_0^u \bar{b}_1^u \cdots \bar{b}_1^u \cdots \bar{b}_{V-1}^u \cdots \bar{b}_{V-1}^u]$ ,  $0 \leq u \leq U-1$ , be an  $N$ -tuple phase vector of modified rotating factors for the adjacent subblock partitioning, where each  $\bar{b}_v^u$ ,  $0 \leq v \leq V-1$ , is repeated  $N/V$  times. Then, we regard the PEP of the proposed ML detector as the probability to determine  $\hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}}$  when  $\mathbf{X} \otimes \mathbf{B}^{\hat{u}}$  is transmitted where  $\otimes$  denotes the component-wise multiplication of two vectors. Assuming the perfect CSI at the receiver, the probability of transmitting  $\mathbf{X} \otimes \mathbf{B}^{\hat{u}}$  and determining  $\hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}}$  by the proposed ML detector can be well

approximated as follows [21]:

$$\Pr(\mathbf{X} \otimes \mathbf{B}^{\hat{u}} \rightarrow \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}} | H_k, k = 0, 1, \dots, N-1) \leq \exp\left(-\frac{d^2(\mathbf{X} \otimes \mathbf{B}^{\hat{u}}, \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}})E_s}{4N_0}\right) \quad (11)$$

where

$$d^2(\mathbf{X} \otimes \mathbf{B}^{\hat{u}}, \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}}) = \sum_{v=0}^{V-1} \sum_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} \left| H_k (X_k b_v^{\hat{u}} e^{j\theta_{S_v^{\hat{u}}}} - \hat{X}_k \hat{b}_v^{\hat{u}} e^{j\theta_{\hat{S}_v^{\hat{u}}}}) \right|^2.$$

Let  $A_k = X_k b_v^{\hat{u}} e^{j\theta_{S_v^{\hat{u}}}} - \hat{X}_k \hat{b}_v^{\hat{u}} e^{j\theta_{\hat{S}_v^{\hat{u}}}}$ . Since  $\mathbb{B}$  is  $\{\pm 1\}$  or  $\{\pm 1, \pm j\}$  and  $A_k = 0$  for  $S_v^{\hat{u}} = \hat{S}_v^{\hat{u}}$ , the PEP in (11) can be rewritten as

$$\Pr(\mathbf{X} \otimes \mathbf{B}^{\hat{u}} \rightarrow \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}} | H_k, k = 0, 1, \dots, N-1) \leq \prod_{v \in \mathbb{V}^{\hat{u}}} \prod_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} \exp\left(-\frac{|H_k A_k|^2 E_s}{4N_0}\right) \quad (12)$$

where  $\mathbb{V}^{\hat{u}}$  is the set of  $v$  such that  $S_v^{\hat{u}} \neq \hat{S}_v^{\hat{u}}$ . Since  $H_k$ 's are independent complex Gaussian random variables with variance 0.5 per dimension and zero mean,  $|H_k|$ 's are independent Rayleigh distributed random variables with the probability density function (PDF)

$$\Pr(|H_k|) = 2|H_k| \exp(-|H_k|^2) \quad \text{for } |H_k| \geq 0.$$

Therefore, the PEP in (12) is averaged with respect to the independent Rayleigh distributions of  $|H_k|$  as follows:

$$\Pr(\mathbf{X} \otimes \mathbf{B}^{\hat{u}} \rightarrow \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}}) \leq \prod_{v \in \mathbb{V}^{\hat{u}}} \left( \frac{1}{\prod_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} (1 + |A_k|^2 E_s / 4N_0)} \right) \cong \prod_{v \in \mathbb{V}^{\hat{u}}} \left( \frac{1}{\prod_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} (|A_k|^2 E_s / 4N_0)} \right). \quad (13)$$

From (13), an upper bound on the PEP for the proposed ML detector in the P-PTS I is obtained as

$$\Pr(\mathbf{X} \otimes \mathbf{B}^{\hat{u}} \rightarrow \hat{\mathbf{X}} \otimes \hat{\mathbf{B}}^{\hat{u}}) \leq \left( \frac{E_s}{4N_0} \right)^{-|\mathbb{V}^{\hat{u}}| \frac{N}{V}} \prod_{v \in \mathbb{V}^{\hat{u}}} \left( \prod_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} |A_k|^2 \right)^{-1}. \quad (14)$$

It is easy to check that in order to minimize the PEP in (14),  $|\mathbb{V}^{\hat{u}}|N/V$  and  $|A_k|^2$  should be maximized, respectively. Thus, the following design criteria for  $V$ -tuple phase offset vectors  $\mathbf{S}^u$  and phase offsets  $\{\theta_0, \theta_1, \dots, \theta_Z\}$  are derived.

- The minimum Hamming distance between  $\mathbf{S}^i$  and  $\mathbf{S}^j$  for  $i \neq j$  should be maximized for a given  $U = W^{V-1}$ .



- In order to obtain large  $|A_k|^2$  in average sense, the average Euclidean distance between signal constellations  $Q_{\theta_{S_i^u}}$  and  $Q_{\theta_{S_j^u}}$  for  $i \neq j$  should be maximized.

Therefore, in the next subsection, it is explained how to design  $V$ -tuple phase offset vectors and phase offsets for the P-PTS I based on this design criteria.

#### D. Design of $V$ -Tuple Phase Offset Vectors and Phase Offsets

If the number of nonzero phase offsets  $Z$  is large, a set of  $V$ -tuple phase offset vectors having large minimum Hamming distance following the first criterion can be easily constructed. However, as  $Z$  increases, the average Euclidean distance between the original and rotated signal constellations is considerably reduced [22], which violates the second criterion, and the detection complexity of the proposed ML detector also increases. Therefore, it is difficult to find the optimal  $V$ -tuple phase offset vectors and the optimal phase offsets to simultaneously satisfy two design criteria. Through the extensive numerical analysis from the viewpoints of BER performance and the implementation, we conclude that  $Z$  should be minimized because the ML detector of the P-PTS I requires  $(Z+1)qN \lceil \cdot \rceil^2$  operations. Thus, the minimum  $Z = \lceil W^{\frac{V-1}{V}} \rceil - 1$  in (7) is used for the P-PTS I, where  $\lceil x \rceil$  takes the least integer greater than or equal to  $x$ .

For simplicity, we consider QAM modulation schemes such as 4-QAM (QPSK), 16-QAM, and 64-QAM and the condition that  $\mathbb{B}$  is  $\{\pm 1\}$  or  $\{\pm 1, \pm j\}$  in this paper. Then, the phase offsets  $\theta_{S_i^u} \in \{\frac{i\pi}{2(Z+1)} | i = 0, 1, \dots, Z\}$  are used for ease of implementation and having a large average Euclidean distance between signal constellations  $Q_{\theta_{S_i^u}}$  and  $Q_{\theta_{S_j^u}}$  for  $i \neq j$ . Finally, each phase offset vector modifies one of  $U$  phase rotating vectors, which is one-to-one mapping because  $(Z+1)^V = U$ .

For  $\mathbb{B} = \{\pm 1\}$ , the minimum  $Z$  satisfying (7) in the P-PTS I is 1. That is, for the binary rotating factors,  $U$  vectors are selected from  $2^V$  binary vectors as  $V$ -tuple phase offset vectors and  $\theta_{S_i^u} \in \{0, \pi/4\}$  which satisfies the second criterion. On the other hands, when  $\mathbb{B} = \{\pm 1, \pm j\}$ , it is not difficult to see that the minimum  $Z$  is 2 for  $V < 5$  and 3 for  $V \geq 5$  to satisfy (7).

For example, when  $V = 3$  and  $\mathbb{B} = \{\pm 1, \pm j\}$ , the P-PTS I takes  $Z = 2$  and uses 16 3-tuple phase offset vectors randomly chosen from all 27 ternary vectors with length 3 and phase offsets  $\theta_{S_i^u} \in \{0, \pi/6, \pi/3\}$  which satisfies the second criterion. Fig. 3 shows 16 rotating vectors  $\mathbf{b}^u$ ,  $V$ -tuple phase offset vectors  $\mathbf{S}^u$ , and modified rotating vectors  $\bar{\mathbf{b}}^u$  for the P-PTS I with  $V = 3$ ,  $\mathbb{B} = \{\pm 1, \pm j\}$ , and  $Z = 2$ .

Note that for the P-PTS I, the total detection complexity to find the index  $\hat{u}$  and detect the symbol sequence  $\hat{\mathbf{X}}^u$  given in (9) and (10) at the receiver is  $\lceil W^{\frac{V-1}{V}} \rceil qN \lceil \cdot \rceil^2$  operations if the real additions are ignored.

#### E. Analysis of Performance Degradation by SI Detection Failure

Clearly, the failure probability of SI mainly depends on SNR. In this subsection, we describe the relationship between them. Similar to the derivation of PEP for the proposed ML

	$b_0^u$	$b_1^u$	$b_2^u$	$S_0^u$	$S_1^u$	$S_2^u$	$\bar{b}_0^u$	$\bar{b}_1^u$	$\bar{b}_2^u$
$u = 0$	1	1	1	0	0	0	1	1	1
$u = 1$	1	1	-1	0	0	1	1	1	$-1 \times e^{j\frac{\pi}{6}}$
$u = 2$	1	1	$j$	0	1	0	1	$1 \times e^{j\frac{\pi}{6}}$	$j$
$u = 3$	1	1	$-j$	1	0	1	$1 \times e^{j\frac{\pi}{6}}$	1	$-j \times e^{j\frac{\pi}{6}}$
$u = 4$	1	-1	1	2	1	0	$1 \times e^{j\frac{\pi}{3}}$	$-1 \times e^{j\frac{\pi}{6}}$	1
$u = 5$	1	-1	-1	1	1	1	$1 \times e^{j\frac{\pi}{6}}$	$-1 \times e^{j\frac{\pi}{6}}$	$-1 \times e^{j\frac{\pi}{6}}$
$u = 6$	1	-1	$j$	0	1	2	1	$-1 \times e^{j\frac{\pi}{6}}$	$j \times e^{j\frac{\pi}{3}}$
$u = 7$	1	-1	$-j$	1	2	2	$1 \times e^{j\frac{\pi}{6}}$	$-1 \times e^{j\frac{\pi}{3}}$	$-j \times e^{j\frac{\pi}{3}}$
$u = 8$	1	$j$	1	0	2	0	1	$j \times e^{j\frac{\pi}{3}}$	1
$u = 9$	1	$j$	-1	2	1	1	$1 \times e^{j\frac{\pi}{3}}$	$j \times e^{j\frac{\pi}{6}}$	$-1 \times e^{j\frac{\pi}{6}}$
$u = 10$	1	$j$	$j$	1	0	0	$1 \times e^{j\frac{\pi}{6}}$	$j$	$j$
$u = 11$	1	$j$	$-j$	1	2	0	$1 \times e^{j\frac{\pi}{6}}$	$j \times e^{j\frac{\pi}{3}}$	$-j$
$u = 12$	1	$-j$	1	0	2	1	1	$-j \times e^{j\frac{\pi}{3}}$	$1 \times e^{j\frac{\pi}{6}}$
$u = 13$	1	$-j$	-1	1	1	0	$1 \times e^{j\frac{\pi}{6}}$	$-j \times e^{j\frac{\pi}{6}}$	-1
$u = 14$	1	$-j$	$j$	2	2	0	$1 \times e^{j\frac{\pi}{3}}$	$-j \times e^{j\frac{\pi}{3}}$	$j$
$u = 15$	1	$-j$	$-j$	2	2	2	$1 \times e^{j\frac{\pi}{3}}$	$-j \times e^{j\frac{\pi}{3}}$	$-j \times e^{j\frac{\pi}{3}}$

Fig. 3. An example of 16 rotating vectors  $\mathbf{b}^u$ , 3-tuple phase offset vectors  $\mathbf{S}^u$ , and modified rotating vectors  $\bar{\mathbf{b}}^u$  for the P-PTS I with  $V = 3$ ,  $\mathbb{B} = \{\pm 1, \pm j\}$ , and  $Z = 2$ .

detector in P-PTS1 in (14), the PEP of an OFDM symbol  $\Pr(X_k \rightarrow \hat{X}_k)$  is lower bounded by

$$\Pr(X_k \rightarrow \hat{X}_k) \leq (|X_k - \hat{X}_k|^2)^{-1} \left( \frac{E_s}{4N_0} \right)^{-1}. \quad (15)$$

Then upper bounds of two inequalities (14) and (15) can be averaged with respect to  $k$  as

$$\begin{aligned} E \left[ \prod_{v \in \mathbb{V}^u} \left( \prod_{k=\frac{Nv}{V}}^{\frac{N(v+1)}{V}-1} |A_k|^2 \right)^{-1} \left( \frac{E_s}{4N_0} \right)^{-|\mathbb{V}^u| \frac{N}{V}} \right] \\ = (E[|A_k|^2])^{-|\mathbb{V}^u| \frac{N}{V}} \left( \frac{E_s}{4N_0} \right)^{-|\mathbb{V}^u| \frac{N}{V}} \end{aligned} \quad (16)$$

and

$$\begin{aligned} E \left[ (|X_k - \hat{X}_k|^2)^{-1} \left( \frac{E_s}{4N_0} \right)^{-1} \right] \\ = (E[|X_k - \hat{X}_k|^2])^{-1} \left( \frac{E_s}{4N_0} \right)^{-1}, \end{aligned} \quad (17)$$

respectively.

The difference becomes

$$\begin{aligned} (16) - (17) &= 4 \left( \frac{E[|X_k - \hat{X}_k|^2]}{(E[|A_k|^2])^{|\mathbb{V}^u| \frac{N}{V}}} \right)^{\frac{1}{|\mathbb{V}^u| \frac{N}{V} - 1}} - \frac{E_s}{N_0} \\ &\approx 4 \frac{(E[|X_k - \hat{X}_k|^2])^{\frac{1}{|\mathbb{V}^u| \frac{N}{V} - 1}}}{E[|A_k|^2]} - \frac{E_s}{N_0}. \end{aligned} \quad (18)$$

Since  $N/V \gg 1$  in the OFDM system,  $(E[|X_k - \hat{X}_k|^2])^{|\mathbb{U}| \frac{N}{V} - 1} \approx 1$ . Therefore, the approximation in (18) is rewritten as

$$(16) - (17) \approx \frac{4}{E_s/N_0} - E[|A_k|^2] \quad (19)$$

where  $E[|A_k|^2]$  depends on the number of phase offsets and used constellations. If (19) is larger than 0, we can say that the failure probability of SI mainly degrades the bit error performance. On the other hand, if (19) is smaller than 0, we can say that the failure probability of SI is not the main factor of bit error compared to the channel noise. That is, in low SNR region the failure probability of SI mainly degrades the bit error performance of the OFDM system. Therefore, the proposed scheme is appropriate to OFDM systems operating in high SNR region, where the desirable SNR region can be calculated from (19).

#### IV. PROPOSED PTS SCHEME II

As already mentioned, in order to minimize the detection complexity, the number of nonzero phase offsets should be minimized. Since it is possible to use  $Z = 1$  for  $\mathbb{B} = \{\pm 1\}$ , the minimum number of phase offsets for the P-PTS I is  $Z = 1$ , which leads to low detection complexity at the receiver. However, in the case of  $\mathbb{B} = \{\pm 1, \pm j\}$ , the minimum number of nonzero phase offsets for the P-PTS I should be larger than one to satisfy (7) except for the rare case when  $V < 3$ , which increases the detection complexity of the P-PTS I. In order to achieve low detection complexity, another PTS scheme is proposed, which can use  $Z = 1$  for the case of  $\mathbb{B} = \{\pm 1, \pm j\}$ . This scheme is called the proposed PTS scheme II (P-PTS II), where we consider the binary phase offsets  $\theta_0 = 0$  and  $\theta_1 = \pi/4$ . It is verified in [22] that if  $Z = 1$ , the Euclidean distance between two signal constellations  $\mathcal{Q}$  and  $\mathcal{Q}_{\frac{\pi}{4}}$  is maximized for QAM modulations such as 4-QAM, 16-QAM, and 64-QAM.

The key idea of the P-PTS II is to further divide the subblocks only for assigning different phase offset by using the linearity of FFT. That is, the  $v$ -th subblock  $\mathbf{x}_v$  is divided into an even subblock  $\mathbf{x}_v^e = [x_{v,0}^e \ x_{v,1}^e \ \cdots \ x_{v,N-1}^e]$  and an odd subblock  $\mathbf{x}_v^o = [x_{v,0}^o \ x_{v,1}^o \ \cdots \ x_{v,N-1}^o]$  which are obtained by applying IFFT to the even-indexed symbols and the odd-indexed symbols in frequency domain having  $N/2$  zeros in an interleaved pattern, respectively. It is easy to derive the following relations

$$\begin{aligned} x_{v,n}^e &= x_{v,n+\frac{N}{2}}^e = \frac{1}{2} \left( x_{v,n} + x_{v,n+\frac{N}{2}} \right) \\ x_{v,n}^o &= -x_{v,n+\frac{N}{2}}^o = \frac{1}{2} \left( x_{v,n} - x_{v,n+\frac{N}{2}} \right) \end{aligned} \quad (20)$$

where  $0 \leq n < N/2$  and  $0 \leq v \leq V-1$ . Then, the  $u$ -th alternative signal sequence of the P-PTS II is generated as

$$\mathbf{x}^u = \sum_{v=0}^{V-1} b_v^u \left( e^{j\frac{\pi}{4} S_{v,e}^u} \mathbf{x}_v^e + e^{j\frac{\pi}{4} S_{v,o}^u} \mathbf{x}_v^o \right). \quad (21)$$

Then, for the binary phase offsets  $S_{v,e}^u, S_{v,o}^u \in \{0, 1\}$ , there are four phase offset pairs  $(S_{v,e}^u, S_{v,o}^u)$  as

$$(S_{v,e}^u, S_{v,o}^u) \in \{(0, 0), (1, 1), (0, 1), (1, 0)\}$$

Therefore, by dividing the subblocks into even-indexed and odd-indexed symbols, the constraint (7) is relaxed to

$$(Z + 3)^V \geq U = W^{V-1}.$$

In the same manner, when  $\mathbb{B} = \{\pm 1, \pm j\}$ , all the  $U$  phase rotating vectors can be fully expressed by using the binary phase offsets  $Z = 1$ .

Since the alternative signal sequences for the P-PTS II are generated differently from the P-PTS I, the metric in (9) should be modified as

$$\begin{aligned} D_{v,S_{v,e}^u} &= \sum_{k \in \mathbb{I}_v^e} \min_{X_k' \in \mathcal{Q}} |R_{v,k} e^{-j\theta_{S_{v,e}^u}} - \hat{H}_k X_k'|^2 \\ D_{v,S_{v,o}^u} &= \sum_{k \in \mathbb{I}_v^o} \min_{X_k' \in \mathcal{Q}} |R_{v,k} e^{-j\theta_{S_{v,o}^u}} - \hat{H}_k X_k'|^2 \end{aligned} \quad (22)$$

where  $\mathbb{I}_v^e$  and  $\mathbb{I}_v^o$  denote the subsets of even indices and odd indices of  $\mathbb{I}_v$ , respectively. By using the partial metrics in (22),  $D_u$  is calculated as

$$D_u = \sum_{v=0}^{V-1} D_{v,S_{v,e}^u} + \sum_{v=0}^{V-1} D_{v,S_{v,o}^u}.$$

Then, similar to the P-PTS I, SI can be estimated by using (10). As a result, the detection complexity of the P-PTS II is only  $2qN |\cdot|^2$  operations which is less than that of the P-PTS I.

#### V. COMPUTATIONAL COMPLEXITY OF THE PROPOSED PTS SCHEMES

In this section, the overall computational complexity of the proposed PTS schemes is analyzed, and compared to those of the conventional PTS scheme and the PTS schemes using ML detection in [18] and [19]. We consider that the number of alternative signal sequences is the same as  $U = W^{V-1}$  for all the PTS schemes. For the computational complexity analysis, the number of complex multiplications  $n_{mul}$  and complex additions  $n_{add}$  will be counted.

The conventional PTS scheme requires  $V$  IFFT operations in order to generate  $V$  subblocks. It is well known that the numbers of complex multiplications and complex additions required by IFFT operation are  $N/2 \log_2 N$  and  $N \log_2 N$ . Thus,  $n_{mul}$  and  $n_{add}$  required for the conventional PTS scheme are  $VN/2 \log_2 N$  and  $VN \log_2 N$ , respectively. To select the alternative signal sequence with the minimum PAPR,  $U$  alternative signal sequences should be generated, for which  $U(V-1)N$  complex additions are required because the rotating factor in  $\{\pm 1\}$  or  $\{\pm 1, \pm j\}$  has a unit magnitude. At the receiver,  $N$ -point FFT and ML detection with perfect SI are performed for the conventional PTS scheme. An  $N$ -point FFT requires  $N/2 \log_2 N$  complex multiplications and  $N \log_2 N$  complex additions similar to the IFFT operation. By regarding  $|\cdot|^2$  as one complex multiplication, the ML detection requires  $(q+2)N$  complex multiplications and  $2qN$  complex additions, where  $q$  is the size of constellation.

Similar to the conventional PTS scheme, the PTS schemes in [18] and [19] require  $V$  IFFT operations which need  $VN/2 \log_2 N$  complex multiplications and  $VN \log_2 N$  complex additions. Also, these PTS schemes require  $U(V-1)N$  complex additions for generating  $U$  alternative signal sequences.

TABLE I  
OVERALL COMPUTATIONAL COMPLEXITY OF PTS SCHEMES

	Overall computational complexity
Conv. PTS	$3(V+1)N \log_2 N + U(V-1)N + (6q+8)N$
PTS [18]	$3(V+1)N \log_2 N + U(V-1)N + (6q+12)WN$
PTS [19]	$3(V+1)N \log_2 N + U(V-1)N + (6q+8)WN$
P-PTS I	$3(V+1)N \log_2 N + U(V-1)N + 4ZN + (6q+8)(Z+1)N$
P-PTS II	$3(V+1)N \log_2 N + U(V-1)N + 8N + 2(6q+8)N$

TABLE II  
CCRRs OF THE PROPOSED PTS SCHEMES OVER THE OTHER PTS  
SCHEMES FOR  $N = 256$ ,  $W = 4$  AND  $V = 4$

	P-PTS I			P-PTS II		
	QPSK	16QAM	64QAM	QPSK	16QAM	64QAM
Conv. PTS	-20.9%	-51.9%	-110.2%	-11.6%	-26.9%	-56.8%
PTS in [18]	8.8%	15.1%	21.1%	15.7%	29.0%	41.8%
PTS in [19]	5.5%	13.2%	20.4%	12.7%	27.4%	41.3%

However, the PTS scheme in [19] does not require additional complexity because it uses cyclic shifting technique, but the PTS scheme in [18] additionally requires  $WN$  complex multiplications because of random rotating factor. For the receiver, one FFT operation is applied to both PTS schemes, which causes  $N/2 \log_2 N$  complex multiplications and  $N \log_2 N$  complex additions. Also, the ML detection for both PTS schemes requires  $(q+2)WN$  complex multiplications and  $2qWN$  complex additions because of the random rotating factors in [18] and the cyclic shifting in [19].

For the proposed PTS schemes,  $VN/2 \log_2 N$  complex multiplications and  $VN \log_2 N$  complex additions are required for  $V$  IFFT operations. In order to generate  $U$  alternative signal sequences, the proposed PTS schemes require  $U(V-1)N$  complex additions. If  $(Z+1)V$  subblocks multiplied by  $Z$  phase offsets are saved in memory, the additional  $ZN$  complex multiplications is required for P-PTS I. For the P-PTS II, the combinations of  $\mathbf{x}_u^e$  and  $\mathbf{x}_u^o$  are pre-computed and saved in memory, which require  $N$  complex multiplications and  $4N$  complex additions by using the IFFT property in (20). At the receiver,  $N/2 \log_2 N$  complex multiplications and  $N \log_2 N$  complex additions by one FFT operation are required to the proposed PTS schemes. The ML detection of the P-PTS I requires  $(q+2)(Z+1)N$  complex multiplications and  $2q(Z+1)N$  complex additions by exploiting  $Z$  phase offsets. For comparison, the P-PTS II with phase offset  $\pi/4$  is considered, where the ML detection requires  $2(q+2)N$  complex multiplications and  $4qN$  complex additions.

In results, the overall computational complexity of the conventional PTS scheme is obtained as

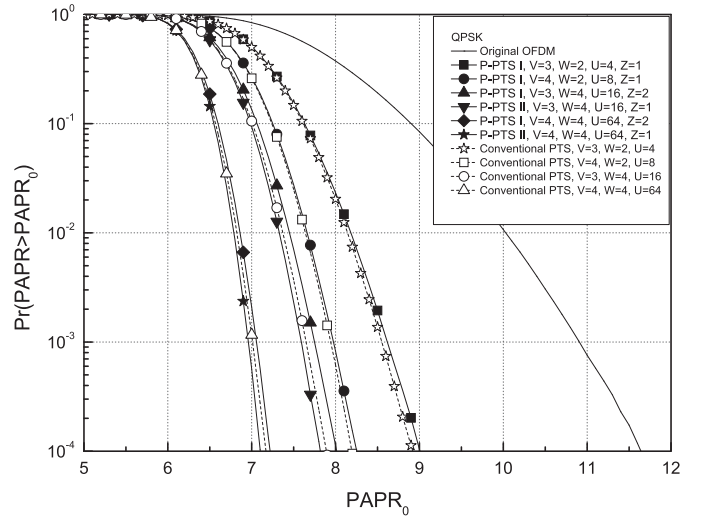
$$\begin{aligned} n_{mul} &= (V+1)N/2 \log_2 N + (q+2)N \\ n_{add} &= (V+1)N \log_2 N + U(V-1)N + 2qN. \end{aligned} \quad (23)$$

And, the overall computational complexity of the PTS scheme in [18] is given as

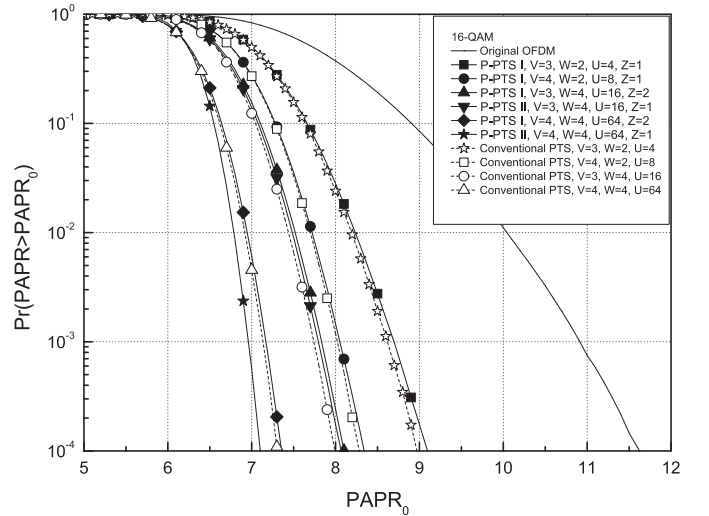
$$\begin{aligned} n_{mul} &= (V+1)N/2 \log_2 N + WN + (q+2)WN \\ n_{add} &= (V+1)N \log_2 N + U(V-1)N + 2qWN. \end{aligned} \quad (24)$$

For the PTS scheme in [18], it becomes

$$\begin{aligned} n_{mul} &= (V+1)N/2 \log_2 N + (q+2)WN \\ n_{add} &= (V+1)N \log_2 N + U(V-1)N + 2qWN. \end{aligned} \quad (25)$$



(a)



(b)

Fig. 4. Comparison of PAPR reduction performance of two proposed PTS schemes and the conventional PTS scheme when  $N = 256$ : (a) QPSK and (b) 16-QAM.

The overall computational complexity for the P-PTS I is expressed as

$$\begin{aligned} n_{mul} &= (V+1)N/2 \log_2 N + ZN + (q+2)(Z+1)N \\ n_{add} &= (V+1)N \log_2 N + U(V-1)N + 2q(Z+1)N, \end{aligned} \quad (26)$$

and for the P-PTS II, it is given as

$$\begin{aligned} n_{mul} &= (V+1)N/2 \log_2 N + N + 2(q+2)N \\ n_{add} &= (V+1)N \log_2 N + U(V-1)N + 4N + 4qN. \end{aligned} \quad (27)$$

If we regard one complex multiplication as four complex additions, overall computation complexity can be summarized in Table I.

To compare the complexity of the proposed PTS schemes with other PTS schemes, the computational complexity reduction ratio (CCRR) of the proposed PTS scheme I or II over the other PTS schemes is defined as

$$\text{CCRR} = \left( 1 - \frac{\text{Complexity of P-PTS I/II}}{\text{Complexity of other PTS}} \right) \times 100[\%]. \quad (28)$$

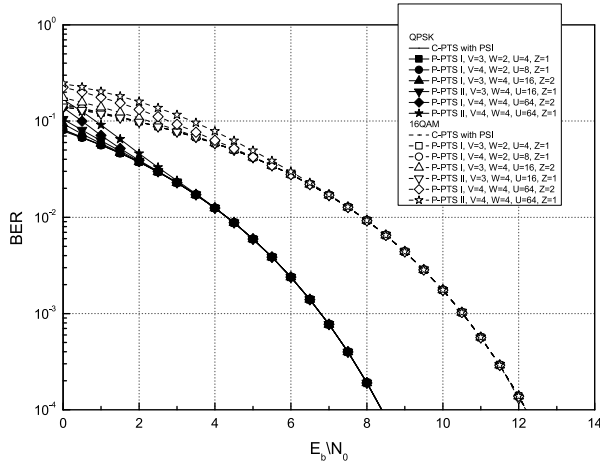


Fig. 5. Comparison of BER of two proposed PTS schemes and the conventional PTS scheme with PSI for  $N = 256$  in the AWGN channel.

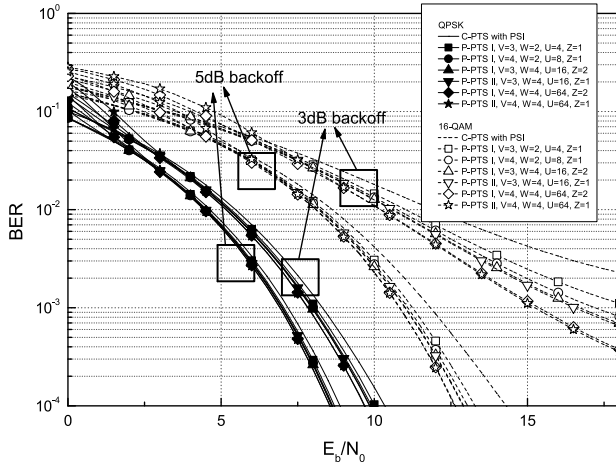


Fig. 6. Comparison of BER of two proposed PTS schemes and the conventional PTS scheme with PSI for  $N = 256$  in the AWGN channel when a nonlinear HPA having backoff values 3 and 5 dB is used.

The CRRs of the proposed PTS schemes over the other PTS schemes for  $N = 256$  with QPSK, 16QAM, and 64QAM are shown in Table II, where  $W = 4$  and  $V = 4$ . From Table II, it is clear that PTS schemes without SI using ML detection cannot avoid the increase of detection complexity. However, the proposed PTS schemes have smaller computational complexity than the PTS schemes without SI in [18] and [19]. Specifically, the P-PTS II is computationally most efficient among PTS schemes without SI using ML detection.

## VI. SIMULATION RESULTS

In this section, simulation results are provided to evaluate the performance of two proposed PTS schemes in terms of PAPR reduction and BER of the ML detector. The simulation has been performed for the OFDM systems modulated by QPSK or 16-QAM when  $N = 256$ .

Fig. 4 compares the PAPR reduction performance of two proposed PTS schemes and the conventional PTS scheme for QPSK and 16-QAM. The PAPR reduction performance of the P-PTS I is slightly worse than that of the P-PTS II. It is

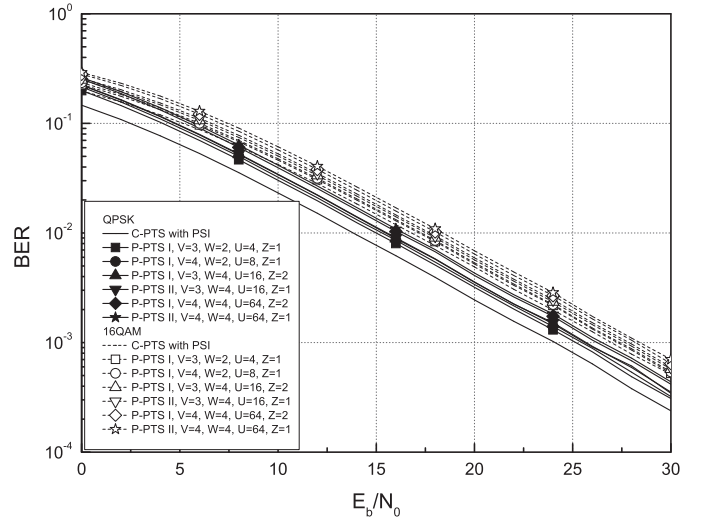


Fig. 7. Comparison of BER of two proposed PTS schemes and the conventional PTS scheme with PSI for  $N = 256$  in the Rayleigh fading channel.

because the average Euclidean distance between alternative signal sequences in the P-PTS I is reduced by using more phase offsets, compared with the P-PTS II. Especially, for QPSK compared with 16-QAM, the gap between the PAPR reduction performance of P-PTS I and P-PTS II is increased because the correlation between alternative signal sequences in P-PTS I increases for QPSK due to less Euclidean distance as the number of phase offsets increases. However, for 16-QAM, the performance degradation between two proposed schemes is negligible, that is, within 0.1 dB.

Fig. 5 compares the BER performance of two proposed PTS schemes when  $N = 256$ , QPSK or 16-QAM, and AWGN channel are assumed. From Fig. 5, the performance degradation of P-PTS I and P-PTS II compared to the conventional PTS with perfect SI (PSI) in low SNR region is induced by detection failure of SI, which is explained in Section III-E. For most of wireless communication systems such as long term evolution (LTE), it requires BER better than  $10^{-2}$ . Therefore, we can see that the degradation of overall throughput by the detection failure of SI is negligible.

Fig. 6 shows the BER performance of two proposed PTS schemes for  $N = 256$ , and QPSK or 16-QAM in the AWGN channel when a nonlinear HPA with backoff values of 3 and 5 dB are used, where HPA is modeled as a soft clipper. The BER performance of two proposed PTS schemes is better than that of the conventional PTS case in the AWGN channel with a nonlinear HPA.

Fig. 7 compares the BER performance of two proposed PTS schemes and the conventional PTS with perfect SI when  $N = 256$ , and QPSK or 16-QAM are used in the Rayleigh fading channel. The conventional PTS scheme with PSI performs slightly better than two proposed PTS schemes in the Rayleigh fading channel but the difference looks negligible.

## VII. CONCLUSION

In this paper, two PTS schemes without SI are proposed for reducing both the PAPR of OFDM signals and increasing



the throughput, which do not transmit SI identifying a rotating vector because identifiable phase offset is applied to the elements of each rotating vector. To find SI of the rotating vector and recover the data sequence at the receiver, the ML detection for the proposed PTS schemes are derived. The simulation results show that for QPSK and 16-QAM, the BER performance and PAPR performance of two proposed PTS schemes are negligibly degraded compared with the conventional PTS with perfect SI.

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