RADAR PRECODING FOR SPECTRUM SHARING BETWEEN MATRIX COMPLETION BASED MIMO RADARS AND A MIMO COMMUNICATION SYSTEM

Bo Li and Athina Petropulu

ECE Department, Rutgers, The State University of New Jersey, Piscataway NJ
{paul.bo.li,athinap}@rutgers.edu

ABSTRACT

The paper investigates a new framework for spectrum sharing between a MIMO-MC radar (MIMO radar using matrix completion) and a MIMO communication system, based on radar transmit precoding. The radar transmit precoder is jointly designed with the communication codewords so that the SINR at the radar receiver is maximized while meeting certain rate and power constraints at the communication system. By shaping the transmit beam, the proposed approach results in enhanced SINR at the receive antennas. Unlike prior works, there is no need for sharing the transmit waveforms with the communication system; only the precoding matrix needs to be shared. Therefore, the proposed scheme is less vulnerable to adversaries. Simulation results demonstrate that the proposed method improves the radar SINR and the matrix completion accuracy over previous approaches.

Index Terms— MIMO radar, matrix completion, spectrum sharing, precoding, alternating optimization

1. INTRODUCTION

The operating frequency bands of communication and radar systems often overlap, causing one system to exert interference to the other. For example, the high UHF radar systems overlap with GSM communication systems, and the S-band radar systems partially overlap with Long Term Evolution (LTE) and WiMax systems [1–3]. Spectrum sharing targets at enabling radar and communication systems to share the spectrum efficiently by minimizing interference [2–7].

This paper investigates the problem of spectrum sharing between a MIMO communication system and a matrix completion (MC) based, collocated MIMO radar (MIMO-MC) system [8–10]. MIMO radars transmit different waveforms from their transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center for further processing. Based on the forwarded data, the fusion center populates a matrix, referred to as the "data matrix", which is then used by standard array processing schemes for target estimation. For a relatively small number of targets, the data matrix is low-rank [8], thus allowing one to fully reconstruct it (under certain conditions) based on a small, uniformly sampled set of its entries. This observation is the basis of MIMO-MC radars; the RX antennas forward to the fusion center a small number of pseudo-randomly sampled values of the target returns, along with their sampling scheme, with each RX antenna partially filling a column of the data matrix. Subsequently, the full data matrix is recovered using MC techniques. MIMO-MC radars maintain the high resolution of MIMO radars, and at the same time enable savings in communication power and bandwidth by reducing the amount of data that need to be sent to the fusion center [8–10]. Compared to the compressive sensing (CS) based MIMO radars, MIMO-MC radars achieve data reduction while avoiding the basis mismatch issues inherent in CS-based approaches [11].

In our previous work [12] and [13], spectrum sharing schemes for the coexistence of a MIMO-MC radar and a MIMO communication system were proposed. Those schemes assumed that radar waveforms were shared with the communication system; knowledge of the radar waveforms enables suppression of the interference due to the radar at the communication system via direct subtraction. However, sharing of radar waveforms could compromise the radar system; if during the sharing process the waveforms were intercepted by an adversary, they could be used as the jamming signals to jam the radar. In this paper, we propose a new spectrum sharing scheme that uses radar TX precoding. The radar waveforms are chosen as rows of a random orthonormal matrix $\mathbf{S}$ [10]. Instead of sharing $\mathbf{S}$, the radar shares with the communication system its TX precoding matrix. Since the procoding matrix is on the radar TX side, even if it is leaked, it cannot be used to jam the radar RX antennas. Therefore, the new scheme is more secure. In addition, using radar TX precoding introduces new degrees of freedom for system design, which can improve the overall performance. The radar TX precoder is jointly designed with the communication codewords to maximize the signal to interference plus noise ratio (SINR) at the MIMO-MC radar receiver, while meeting certain rate and power constraints at the communication system. The optimization problem is solved by alternating optimization w.r.t the radar precoding matrix and the communication codeword covariance matrix. Given the communication codeword covariance matrix, the optimization w.r.t. the radar TX precoding matrix is a nonconvex problem. A new slack variable is introduced to solve the problem via applying the alternating optimization again. Simulation results show that the proposed spectrum sharing scheme improves the radar SINR and matrix completion accuracy over our previous approach [13], in which no radar precoding was considered.

The paper is organized as follows. Section 2 introduces the signal model when the MIMO-MC radar and communication systems coexist. The problem of spectrum sharing is studied in Sections 3. Numerical results and conclusions are provided in Sections 4-5.
Notation: $\mathcal{CN}(\mu, \mathbf{\Sigma})$ denotes circularly symmetric complex Gaussian distribution with mean μ and covariance matrix $\mathbf{\Sigma}$. $|\cdot|$ and $\mathrm{Tr}(\cdot)$ denotes the matrix determinant and trace, respectively. The set $\mathbb{N}_L^+$ is defined as $\{1, \ldots, L\}$. Subscripts $\cdot m$ and $n\cdot$ denote the m-th column and the n-th row of a matrix, respectively.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a MIMO communication system which coexists with a MIMO-MC radar system as shown in Fig. 1. The radar system

This work was supported by NSF under Grant ECCS-1408437.

operates as the primary system for target detection/estimation. The communication system shares the same carrier frequency as a secondary system. The radar operates in two phases; in the first phase the TX antennas transmit and the RX antennas receive, while in the second phase, the RX antennas forward their measurements to a fusion center. The communication system interferes with the radar system during both phases. In the following, we will address the interference during the first phase only. The interference during the second phase can be viewed as the interference between two communication systems, and addressing this problem has been covered in the literature [14, 15].

Suppose that the two systems have the same symbol rate and are synchronized in sampling time (see [13] for the mismatched case). The signal received by the radar and communication RX antennas during L symbol durations can be respectively expressed as

$$\mathbf{\Omega} \circ \mathbf{Y}_R = \mathbf{\Omega} \circ (\mathbf{DPS} + \mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2 + \mathbf{W}_R), \tag{1a}$$

$$\mathbf{Y}_C = \mathbf{HX} + \mathbf{G}_1\mathbf{PS}\mathbf{\Lambda}_1 + \mathbf{W}_C, \tag{1b}$$

where

- $\mathbf{P} \in \mathbb{R}^{M_{t,R}\times M_{t,R}}$ is the transmit precoding matrix; $\mathbf{D} \in \mathbb{C}^{M_{r,R}\times M_{t,R}}$ denotes the target response matrix, which depends on the target reflectivity, angle of arrival and target speed; $\mathbf{D}$ is low rank if the number of targets is much smaller than numbers of radar TX and RX antennas (details can be found in [10]); $\mathbf{S} = [\mathbf{s}(1), \cdots, \mathbf{s}(L)]$ is the sampled waveform matrix. In this paper, we consider targets falling in the same range bin.
- $\mathbf{Y}_R \in \mathbb{C}^{M_{r,R}\times L}$ denotes the received signal at the radar RX antennas. $\mathbf{\Omega}$ is an $M_{r,R} \times L$ dimensional random sampling matrix with binary entries and $\circ$ denotes the Hadamard product. The multiplication of $\mathbf{\Omega}$ by $\mathbf{Y}_R$ point by point represents the sampling over randomly selected symbol intervals at each radar RX antenna. The sub-sampling rate, p, equals $\|\mathbf{\Omega}\|_0/LM_{r,R}$. Only the sampled entries of $\mathbf{Y}_R$ are forwarded to the fusion center, where the low-rank matrix $\mathbf{DPS}$ is completed and used for target detection and estimation [8–10, 16].
- $\mathbf{Y}_C \in \mathbb{C}^{M_{r,C}\times L}$ denotes the received signal at the communication RX antennas. The columns of $\mathbf{X} \triangleq [\mathbf{x}(1), \dots, \mathbf{x}(L)]$ are codewords from the code-book of the communication system. $\mathbf{W}_{R/C}$ is the additive noise matrix with i.i.d random entries with distribution $\mathcal{CN}(0, \sigma^2_{R/C})$.
- $\mathbf{H} \in \mathbb{C}^{M_{r,C}\times M_{t,C}}$ denotes the communication channel, where $M_{r,C}$ and $M_{t,C}$ denote respectively the number of RX and TX antennas of the communication system [14]; $\mathbf{G}_1 \in \mathbb{C}^{M_{r,C}\times M_{t,R}}$ denotes the interference channel from the radar TX antennas to the communication system RX antennas [3, 4, 7]; $\mathbf{G}_2 \in \mathbb{C}^{M_{r,R}\times M_{t,C}}$ denotes the interference channel from the communication TX antennas to the radar RX antennas. It is assumed that the channels remain the same over L time-slots.
- The diagonal matrix $\mathbf{\Lambda}_i, i \in \{1, 2\}$ are with entries $e^{j\alpha_{il}}$, which denote the random phase offsets between the MIMO-MC radar and the communication system [13]. The phase offsets result from the random phase jitters of the oscillators at the transmitter and the receiver Phase-Locked Loops.

It is assumed that the MIMO channels $\mathbf{H}$, $\mathbf{G}_1$ and $\mathbf{G}_2$ are perfectly known at the communication transmitter. In practice, the channel state information can be periodically communicated among the communication receiver, transmitter and the radar system through a pilot channel [3, 17].

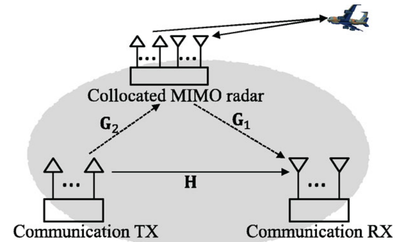

Fig. 1. A MIMO communication system sharing spectrum with a collocated MIMO radar system

In our previous work [12] and [13], we considered the same model as in (1a-1b), except that $\mathbf{P} = \mathbf{I}$, and assumed that $\mathbf{S}$ was shared with the communication system for interference subtraction at the communication receiver. In this paper, we choose $\mathbf{S}$ a random orthonormal matrix [10]. The communication system is aware that $\mathbf{S}$ is orthonormal but has no access to the specific realization of $\mathbf{S}$. We also introduce a radar TX precoder $\mathbf{P}$, which can be designed to focus the power on targets as well as to reduce the interference to the communication system. Instead of sharing $\mathbf{S}$, the radar shares $\mathbf{P}$ with the communication system. Sharing $\mathbf{S}$ would introduce security threat if $\mathbf{S}$ were intercepted by an adversary. On the other hand, $\mathbf{P}$ is on the radar TX side, thus the jammer cannot benefit from $\mathbf{P}$ even the jammer knew $\mathbf{P}$. Therefore, the proposed approach is expected to be more secure than that of [13].

In the next section, the precoding matrix $\mathbf{P}$ and the communication codeword $\mathbf{X}$ are jointly designed to maximize the SINR at the MIMO-MC radar receiver while maintaining certain communication system rate and power constraints.

3. THE PROPOSED SPECTRUM SHARING METHOD

Matrix $\mathbf{S}$ is obtained through performing the Gram-Schmidt orthogonalization on a matrix whose entries are i.i.d Gaussian random variables. Note that the entries of $\mathbf{S}$ are not independent anymore. However, based on [18, Theorem 3], if $M_{t,R} = \mathcal{O}(L/\ln L)$, the entries of $\mathbf{S}$ can be approximated by i.i.d Gaussian random variables with distribution $\mathcal{N}(0, 1/L)$.

For any $l \in \mathbb{N}_L^+$, the interference plus noise at the communication system has zero mean and covariance matrix $\mathbf{R}_w \triangleq \mathbf{R}_{\text{int}} + \sigma_C^2\mathbf{I}$, where $\mathbf{R}_{\text{int}} = \mathbf{G}_1\mathbf{P}\mathbb{E}\{\mathbf{s}(l)\mathbf{s}^H(l)\}\mathbf{P}^H\mathbf{G}_1^H = \frac{1}{L}\mathbf{G}_1\mathbf{P}\mathbf{P}^H\mathbf{G}_1^H$. However, due to the random phase offset, the interference is not Gaussian. The capacity of the model in (1b) is unknown without knowledge of the distribution of the phase offset. In this paper, we are interested in a lower bound of the capacity. It is shown in [19] that Gaussian noise with covariance matrix equal to the actual noise covariance is the worst-case noise for additive noise channels. The lower bound of the capacity is given by

$$\underline{C}(\mathbf{R}_x, \mathbf{P}) \triangleq \log_2\left|\mathbf{I} + \mathbf{R}_w^{-1}\mathbf{H}\mathbf{R}_x\mathbf{H}^H\right|,$$

which is achieved when $\mathbf{x}(l)$ is distributed as $\mathcal{CN}(0, \mathbf{R}_x)$. The total transmit power of the communication TX antennas equals

$$\mathbb{E}\{\text{Tr}(\mathbf{X}\mathbf{X}^H)\} = \mathbb{E}\left\{\text{Tr}\left(\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^H(l)\right)\right\} = L\text{Tr}(\mathbf{R}_x).$$

According to (1a), the total interference power (TIP) exerted at the radar RX nodes equals

$$\text{TIP} \triangleq \mathbb{E}\{\text{Tr}(\mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2\mathbf{\Lambda}_2^H\mathbf{X}^H\mathbf{G}_2^H)\} = L\text{Tr}\left(\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H\right). \tag{2}$$

Since the radar only forwards part of $\mathbf{Y}_R$ to the fusion center, only the term $\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2)$ represents effective interference to the radar system. Based on this observation, we define the *effective interfer-*

ence power (EIP) at the radar RX node as

$$\begin{aligned}
\text{EIP} &\triangleq \mathbb{E}\left\{\text{Tr}\left(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2)\left(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2)\right)^H\right)\right\}\\
=&\mathbb{E}\left\{\text{Tr}\left([\mathbf{G}_{21}\mathbf{x}(1)\ldots\mathbf{G}_{2L}\mathbf{x}(L)][\mathbf{G}_{21}\mathbf{x}(1)\ldots\mathbf{G}_{2L}\mathbf{x}(L)]^H\right)\right\}\\
=&\mathbb{E}\left\{\text{Tr}\left(\sum_{l=1}^{L}\mathbf{G}_{2l}\mathbf{x}(l)\mathbf{x}^H(l)\mathbf{G}_{2l}^H]\right)\right\}=\sum_{l=1}^{L}\text{Tr}\left(\mathbf{G}_{2l}\mathbf{R}_x\mathbf{G}_{2l}^H\right)\\
=&\text{Tr}\left(\left(\sum\nolimits_{l=1}^{L}\boldsymbol{\Delta}_l\right)\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H\right)=\text{Tr}\left(\boldsymbol{\Delta}\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H\right),
\end{aligned}$$

where $\mathbf{G}_{2l}\triangleq\boldsymbol{\Delta}_l\mathbf{G}_2$ with $\boldsymbol{\Delta}_l=\text{diag}(\boldsymbol{\Omega}_{\cdot l})$ and $\boldsymbol{\Delta}\triangleq\sum_{l=1}^{L}\boldsymbol{\Delta}_l$. We note that the EIP is a re-weighted version of the TIP.

Similarly, we could derive the *total signal power* (TSP) at the radar RX node as TSP $\triangleq$ Tr $\left(\mathbf{D}\mathbf{P}\mathbf{P}^H\mathbf{D}^H\right)$ and the *effective signal power* (ESP) of the target echo signal forwarded to the fusion center as ESP $\triangleq$ Tr $\left(\boldsymbol{\Delta}\mathbf{D}\mathbf{P}\mathbf{P}^H\mathbf{D}^H\right)/L$. We assume that the information of targets contained in $\mathbf{D}$ is given as *a priori*. In practice, such information could be obtained in various ways. For example, in tracking applications, the target parameters obtained from previous tracking cycles are provided to focus the transmit power onto directions of interest. In this paper, we jointly design the radar precoding matrix $\mathbf{P}$ and communication codeword covariance matrix $\mathbf{R}_x$ to maximize the *effective* SINR while maintaining certain communication rate and power consumption at both the radar and communication transmit node.

If the MIMO-MC radar shares its random sampling scheme with the communication system, the spectrum sharing problem can be formulated as

$$(\mathbf{P}_1)\quad\max_{\mathbf{R}_x\succeq 0,\boldsymbol{\Phi}\succeq 0}\ \text{ESINR}\triangleq\frac{\text{Tr}\left(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\right)}{\text{Tr}\left(\boldsymbol{\Delta}\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H\right)+pP_{wR}}\tag{3a}$$

$$\text{s.t. }\ \underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})\geq C,\tag{3b}$$

$$L\text{Tr}\left(\mathbf{R}_x\right)\leq P_C, L\text{Tr}\left(\boldsymbol{\Phi}\right)\leq P_R,\tag{3c}$$

where $\boldsymbol{\Phi}\triangleq\mathbf{P}\mathbf{P}^H/L$ is positive semi-definite, and $P_{wR}=LM_{r,R}\sigma_R^2$. The constraint of (3b) restricts the communication rate to be at least C, in order to support reliable communication and avoid service outage. The constraints of (3c) restricts the total communication and radar transmit power to be no larger than P_C and P_R, respectively. Problem $(\mathbf{P}_1)$ is nonconvex w.r.t. both $\mathbf{R}_x$ and $\boldsymbol{\Phi}$. A solution can be obtained via alternating optimization. Let $(\mathbf{R}_x^n,\boldsymbol{\Phi}^n)$ be the variables at the n-th iteration for $n=1,2,\ldots$. We first solve $\mathbf{R}_x^n$ while fixing $\boldsymbol{\Phi}$ to be $\boldsymbol{\Phi}^{n-1}$:

$$\begin{aligned}(\mathbf{P}_{\mathbf{R}})\quad&\min_{\mathbf{R}_x\succeq 0}\text{Tr}\left(\boldsymbol{\Delta}\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H\right)\\ &\text{s.t. }\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi}^{n-1})\geq C, L\text{Tr}\left(\mathbf{R}_x\right)\leq P_C.\end{aligned}\tag{4}$$

The above problem is convex w.r.t. $\mathbf{R}_x$ and can be solved efficiently using the interior point method [20]. In fact, a semi-closed-form solution can be obtained via solving the dual of $(\mathbf{P}_{\mathbf{R}})$, that is

$$(\mathbf{P}_{\mathbf{R}}\text{-D})\ \max_{\lambda_1,\lambda_2\geq 0}\left\{\min_{\mathbf{R}_x\succeq 0}\mathcal{L}_1(\mathbf{R}_x,\lambda_1,\lambda_2)\right\},$$

$$\begin{aligned}\text{where }\mathcal{L}_1(\mathbf{R}_x,\lambda_1,\lambda_2)&=\text{Tr}\left(\left(\mathbf{G}_2^H\boldsymbol{\Delta}\mathbf{G}_2+\lambda_2\mathbf{I}\right)\mathbf{R}_x\right)\\ &-\lambda_1\log_2\left|\mathbf{I}+\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{R}_x\right|+\lambda_1 C-\lambda_2 P_C/L;\end{aligned}$$

and $\lambda_1\geq 0$ and $\lambda_2\geq 0$ are the dual variables associated with the communication capacity and the transmit power constraints, respectively. For the detailed solution, please refer to [13, Lemma 1].

Then, the obtained $\mathbf{R}_x^n$ is used to solve the following problem for $\boldsymbol{\Phi}^n$:

$$\begin{aligned}(\mathbf{P}_{\boldsymbol{\Phi}})\quad&\max_{\boldsymbol{\Phi}\succeq 0}\text{Tr}\left(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\right)\\ &\text{s.t. }\underline{C}(\mathbf{R}_x^n,\boldsymbol{\Phi})\geq C, L\text{Tr}\left(\boldsymbol{\Phi}\right)\leq P_R.\end{aligned}\tag{5}$$

Notice that $\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})$ is actually a convex function of $\boldsymbol{\Phi}$. The constraint $\underline{C}(\mathbf{R}_x^n,\boldsymbol{\Phi})\geq C$ imposes a nonconvex feasible set on $\boldsymbol{\Phi}$. Therefore, the problem in (5) is nonconvex. We could express $\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})$ as follows

$$\begin{aligned}\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})&=\log_2\left|\mathbf{R}_w+\mathbf{H}\mathbf{R}_x\mathbf{H}^H\right|-\log_2\left|\mathbf{R}_w\right|\\ &=\log_2\left|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H+\tilde{\mathbf{R}}_x\right|-\log_2\left|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H+\sigma_C^2\mathbf{I}\right|,\end{aligned}\tag{6}$$

where $\tilde{\mathbf{R}}_x\triangleq\sigma_C^2\mathbf{I}+\mathbf{H}\mathbf{R}_x\mathbf{H}^H$. We can see that $\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})$ is in the form of a concave function plus a convex function. To overcome the nonconvex constraint in (5), the second term on the right-hand-side (RHS) of (6) will be addressed by the following lemma:

Lemma 1 ([20]). *For any positive definite matrix* $\mathbf{Y}\in\mathbb{C}^{N\times N}$, *it holds that*

$$\log_2\left|\mathbf{Y}^{-1}\right|=\max_{\boldsymbol{\Psi}\succeq 0}\left\{-Tr(\mathbf{Y}\boldsymbol{\Psi})+\log_2\left|\boldsymbol{\Psi}\right|+N\right\},$$

where the maximum is attained when $\boldsymbol{\Psi}=\mathbf{Y}^{-1}$.

Applying Lemma 1 to $\underline{C}(\mathbf{R}_x^n,\boldsymbol{\Phi})$ via setting $Y\equiv\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H+\sigma_C^2\mathbf{I}$, the optimization problem in (5) turns to be

$$\begin{aligned}(\mathbf{P}_{\boldsymbol{\Phi}\boldsymbol{\Psi}})\ &\max_{\boldsymbol{\Phi}\succeq 0}\ \text{Tr}\left(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\right),\ \text{s.t. }\ L\text{Tr}\left(\boldsymbol{\Phi}\right)\leq P_R,\\ &\log_2\left|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H+\tilde{\mathbf{R}}_x\right|+\max_{\boldsymbol{\Psi}\succeq 0}\log_2\left|\boldsymbol{\Psi}\right|\\ &-\text{Tr}((\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H+\sigma_C^2\mathbf{I})\boldsymbol{\Psi})+M_{r,C}\geq C.\end{aligned}\tag{7}$$

Problem $(\mathbf{P}_{\boldsymbol{\Phi}\boldsymbol{\Psi}})$ involves two matrix variables, *i.e.*, $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$. Again, alternating optimization is applied as an inner iteration. During the n-th outer alternating iteration, let $(\boldsymbol{\Phi}^{nk},\boldsymbol{\Psi}^{nk})$ be the variables at the k-th inner iteration. $\boldsymbol{\Phi}^{nk}$ is initialized as $\boldsymbol{\Phi}^{n-1}$ for $k=0$. Given $\boldsymbol{\Phi}^{n(k-1)}$, $\boldsymbol{\Psi}^{nk}$ is obtained as follows

$$\boldsymbol{\Psi}^{nk}=\arg\max_{\boldsymbol{\Psi}\succeq 0}\log_2\left|\boldsymbol{\Psi}\right|-\text{Tr}\left(\left(\mathbf{G}_1\boldsymbol{\Phi}^{n(k-1)}\mathbf{G}_1^H+\sigma_C^2\mathbf{I}\right)\boldsymbol{\Psi}\right).$$

From Lemma 1, the closed-form solution of $\boldsymbol{\Psi}^{nk}$ is

$$\boldsymbol{\Psi}^{nk}=\left(\mathbf{G}_1\boldsymbol{\Phi}^{n(k-1)}\mathbf{G}_1^H+\sigma_C^2\mathbf{I}\right)^{-1}.\tag{8}$$

Based on $\boldsymbol{\Psi}^{nk}$, $\boldsymbol{\Phi}^{nk}$ is obtained by solving the following optimization problem

$$\begin{aligned}(\mathbf{P}'_{\boldsymbol{\Phi}})\ &\max_{\boldsymbol{\Phi}\succeq 0}\ \text{Tr}\left(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\right),\ \text{s.t. }\ L\text{Tr}\left(\boldsymbol{\Phi}\right)\leq P_R,\\ &\log_2\left|\mathbf{I}+\mathbf{G}_1^H(\tilde{\mathbf{R}}_x^n)^{-1}\mathbf{G}_1\boldsymbol{\Phi}\right|-\text{Tr}\left(\mathbf{G}_1^H\boldsymbol{\Psi}^{nk}\mathbf{G}_1\boldsymbol{\Phi}\right)\geq C',\end{aligned}$$

where $C'\triangleq C+\sigma_C^2\text{Tr}(\boldsymbol{\Psi}^{nk})-\log_2\left|\tilde{\mathbf{R}}_x^n\boldsymbol{\Psi}^{nk}\right|-M_{r,C}$. $(\mathbf{P}'_{\boldsymbol{\Phi}})$ is convex w.r.t. $\boldsymbol{\Phi}$. Similar to $(\mathbf{P}_{\mathbf{R}})$, a semi-closed form solution can be obtained via solving the dual of $(\mathbf{P}'_{\boldsymbol{\Phi}})$, that is

$$(\mathbf{P}'_{\boldsymbol{\Phi}}\text{-D})\ \max_{\nu_1,\nu_2\geq 0}\left\{\min_{\boldsymbol{\Phi}\succeq 0}\mathcal{L}_2(\boldsymbol{\Phi},\nu_1,\nu_2)\right\},$$

$$\begin{aligned}\text{where }\mathcal{L}_2(\boldsymbol{\Phi},\nu_1,\nu_2)&=\text{Tr}\left(\left(\nu_2\mathbf{I}+\nu_1\mathbf{G}_1^H\boldsymbol{\Psi}^{nk}\mathbf{G}_1-\mathbf{D}^H\boldsymbol{\Delta}\mathbf{D}\right)\boldsymbol{\Phi}\right)\\ &-\nu_1\log_2\left|\mathbf{I}+\mathbf{G}_1^H(\tilde{\mathbf{R}}_x^n)^{-1}\mathbf{G}_1\boldsymbol{\Phi}\right|+\nu_1 C'-\nu_2 P_R/L;\end{aligned}$$

and $\nu_1\geq 0$ and $\nu_2\geq 0$ are the dual variables associated with the communication capacity and the radar transmit power constraints, respectively. We can see that $\mathcal{L}_2(\boldsymbol{\Phi},\nu_1,\nu_2)$ exhibits a similar form as $\mathcal{L}_1(\mathbf{R}_x,\lambda_1,\lambda_2)$. Therefore, we could also apply [13, Lemma 1] to solve $(\mathbf{P}'_{\boldsymbol{\Phi}}\text{-D})$.

It is easy to show that the value of SINR is nondecreasing during the alternating iterations. Also, the SINR has a upper bound. Therefore, the algorithm converges. The complete spectrum share algorithm proposed in this section is summarized in Algorithm 1.

Algorithm 1 The proposed spectrum sharing method.

Input: $\mathbf{D}, \mathbf{H}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{\Omega}, P_R, P_C, C, \sigma_C^2, \delta_1, \delta_2$
Initialization: $\mathbf{\Phi}^0 = P_R/M_{t,R}\mathbf{I}$
repeat
  $\mathbf{R}_x^n \leftarrow$ Solve problem ($\mathbf{P_R}$) using interior point method or ($\mathbf{P_R}$-D) using [13, **Algorithm** 1] with fixed $\mathbf{\Phi}^{n-1}$;
  $\mathbf{\Phi}^{nk} \leftarrow \mathbf{\Phi}^{n-1}$ for $k = 0$;
  repeat
    $\mathbf{\Psi}^{nk} = \left(\mathbf{G}_1\mathbf{\Phi}^{n(k-1)}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}\right)^{-1}$;
    $\mathbf{\Phi}^{nk} \leftarrow$ Solve problem ($\mathbf{P}'_{\mathbf{\Phi}}$) using interior point method or ($\mathbf{P}'_{\mathbf{\Phi}}$-D) with fixed $\mathbf{\Psi}^{nk}$ and $\mathbf{R}_x^n$;
    $k \leftarrow k+1$
  until $|\mathrm{ESP}^k - \mathrm{ESP}^{k-1}| < \delta_1$
  $\mathbf{\Phi}^n \leftarrow \mathbf{\Phi}^{nk}$
  $n \leftarrow n+1$
until $|\mathrm{ESINR}^n - \mathrm{ESINR}^{n-1}| < \delta_2$
Output: $\mathbf{R}_x = \mathbf{R}_x^n, \mathbf{P} = \sqrt{L}(\mathbf{\Phi}^n)^{1/2}$

The above spectrum sharing method requires that the radar subsampling matrix $\mathbf{\Omega}$ is shared with the communication system. Alternatively, we can respectively substitute ESP and EIP by TSP and TIP in the objective of ($\mathbf{P}_1$). If the diagonal entries of $\mathbf{\Delta}$ are identical and equal to pL, it should not matter whether $\mathbf{\Omega}$ is shared or not. The average of $\mathbf{\Delta}$ resulted from the uniformly random sub-sampling matrix $\mathbf{\Omega}$ indeed has identical entries. This means that the proposed algorithm performs almost the same whether $\mathbf{\Omega}$ is shared or not.

Unlike our previous works in [12, 13], the radar waveform $\mathbf{S}$ is not required to share with the communication system. Spectrum sharing is achieved by the joint design of the radar precoding matrix and communication codeword covariance matrix without compromising the confidentiality of the radar waveforms.

4. NUMERICAL RESULTS

For the simulations, we set the number of time-slots $L = 32$, the noise variance $\sigma_C^2 = \sigma_R^2 = 0.01$ and the number of antennas $M_{t,R} = 4, M_{r,R} = M_{t,C} = 8, M_{r,C} = 4$. The MIMO radar system consists of collocated TX and RX antennas forming half-wavelength uniform linear arrays. The radar waveforms are chosen from rows of a random orthonormal matrix [8]. There are two far-field stationary targets at angle $\pm 60°$ w.r.t. to the arrays. For the communication capacity and power constraints, we use $C = 20$ bits/symbol and $P_C = LM_{t,C}$ (the power is normalized by the power of radar waveform). The uniformly random sampling scheme $\mathbf{\Omega}$ is adopted for the MIMO-MC radar. The TFOCUS package [21] is used for low-rank matrix completion at the radar fusion center. The interference channels $\mathbf{G}_1$ and $\mathbf{G}_2$ are generated with independent entries distributed as $\mathcal{CN}(0, 0.01)$. The channel $\mathbf{H}$ has independent entries, distributed as $\mathcal{CN}(0,1)$. The communication covariance matrix and the radar precoding matrix are jointly optimized according to different criteria of Section 3. The obtained $\mathbf{R}_x$ is used to generate $\mathbf{x}(l) = \mathbf{R}_x^{1/2}\mathrm{randn}(M_{t,C}, 1)$. We use the ESINR and MC relative recovery error as the performance metrics, with the later defined as $\|\mathbf{DPS} - \hat{\mathbf{DPS}}\|_F / \|\mathbf{DPS}\|_F$. For comparison, we implement a "no radar precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$, and selfish communication" scenario, where the communication system minimizes the transmit power to achieve certain average capacity without any concern about the interferences it exerts to the radar system. We also implement a scenario with no radar precoding but with $\mathbf{R}_x$ being designed to minimize the interferences it exerts to the radar system while achieving certain communication capacity. Our previous approach in [13] is also implemented, *i.e.*, $\mathbf{S}$ is shared with the communication receiver, and there is no $\mathbf{P}$.

Fig. 2 shows the ESINR and the MC relative recovery errors results for different radar sub-sampling rate p. The radar transmit power budget is $P_R = 10LM_{t,R}$. Fig. 3 shows the ESINR and the MC relative recovery errors results for different radar transmit power budget P_R's. The radar power budget per antenna ranges from 5 to 20 times of the communication power budget per antenna. The radar sub-sampling rate is $p = 0.8$. We can see that the joint design of $\mathbf{P}$ and $\mathbf{R}_x$ significantly outperforms the uniform radar power allocation approach in terms of both ESINR and MC relative recovery error under all levels of p and P_R. Compared with our previous approach with $\mathbf{S}$ shared, the proposed method in this paper could achieve even higher ESINR and lower MC relative recovery error. The reason is that the target prior information in $\mathbf{D}$ facilitates the design of $\mathbf{P}$ to focus the radar power on the targets while nulling the interference to the communication receiver.

The radar precoder plays an important role in the proposed spectrum sharing method. The green and black curves in Fig. 2 and 3 illustrate that the design of $\mathbf{R}_x$ could not benefit the MIMO-MC radar when no radar precoding is considered. We could also assert that the proposed spectrum sharing method performs almost the same whether $\mathbf{\Omega}$ is shared with the communication system or not.

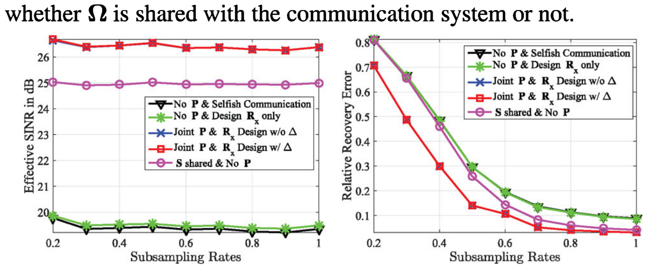

Fig. 2. Spectrum sharing under different sub-sampling rates. $M_{t,R} = 4, M_{r,R} = M_{t,C} = 8, M_{r,C} = 4$.

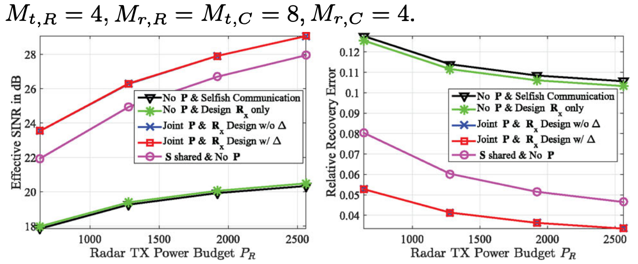

Fig. 3. Spectrum sharing under different radar transmit power budget P_R. $M_{t,R} = 4, M_{r,R} = M_{t,C} = 8, M_{r,C} = 4$.

5. CONCLUSIONS

We have considered a new framework for spectrum sharing between MIMO communication and MIMO-MC radar systems. Spectrum sharing is achieved by joint design of the radar precoding matrix and communication codeword covariance matrix without compromising the confidentiality of the radar waveforms. Simulation results show that the radar precoder plays a key role in improving the radar SINR and matrix completion recovery accuracy over previous approaches.

6. REFERENCES

[1] "Radar spectrum regulatory overview," *[online] 2013*, http://www.darpa.mil/WorkArea/DownloadAsset.aspx?id=2147486331, (Accessed: July 2014).

[2] A. Lackpour, M. Luddy, and J. Winters, "Overview of interference mitigation techniques between wimax networks and ground based radar," in *20th Annual Wireless and Optical Communications Conference*, April 2011, pp. 1–5.

[3] S. Sodagari, A. Khawar, T. C. Clancy, and R. McGwier, "A projection based approach for radar and telecommunication systems coexistence," in *IEEE Global Telecommunication Conference*, Dec 2012, pp. 5010–5014.

[4] A. Babaei, W. H. Tranter, and T. Bose, "A practical precoding approach for radar/communications spectrum sharing," in *8th International Conference on Cognitive Radio Oriented Wireless Networks*, July 2013, pp. 13–18.

[5] H. Deng and B. Himed, "Interference mitigation processing for spectrum-sharing between radar and wireless communications systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 3, pp. 1911–1919, July 2013.

[6] S. Amuru, R. M. Buehrer, R. Tandon, and S. Sodagari, "MIMO radar waveform design to support spectrum sharing," in *IEEE Military Communication Conference*, Nov 2013, pp. 1535–1540.

[7] A. Khawar, A. Abdel-Hadi, and T. C. Clancy, "Spectrum sharing between s-band radar and lte cellular system: A spatial approach," in *IEEE International Symposium on Dynamic Spectrum Access Networks,*, April 2014, pp. 7–14.

[8] S. Sun, A. P. Petropulu, and W. U. Bajwa, "Target estimation in colocated MIMO radar via matrix completion," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, May 2013, pp. 4144–4148.

[9] D. S. Kalogerias and A. P. Petropulu, "Matrix completion in colocated MIMO radar: Recoverability, bounds and theoretical guarantees," *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 309–321, Jan 2014.

[10] S. Sun, W. U. Bajwa, and A. P. Petropulu, "MIMO-MC radar: A MIMO radar approach based on matrix completion," *arXiv preprint arXiv:1409.3954*, 2014.

[11] Y. Yu, A. P. Petropulu, and H. V. Poor, "MIMO radar using compressive sampling," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 146–163, Feb 2010.

[12] B. Li and A. P. Petropulu, "Spectrum sharing between matrix completion based MIMO radars and a MIMO communication system," in *2015 IEEE International Conference on Acoustics, Speech and Signal Processing*, April 2015, pp. 2444–2448.

[13] B. Li, A. P. Petropulu, and W. Trappe, "Optimum design for coexistence between matrix completion based MIMO radars and a MIMO communication system," *arXiv preprint arXiv:1507.01982*, 2015.

[14] R. Zhang and Y. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 88–102, Feb 2008.

[15] R. Zhang, Y. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 102–114, May 2010.

[16] S. Sun and A. P. Petropulu, "On transmit beamforming in MIMO radar with matrix completion," in *2015 IEEE International Conference on Acoustics, Speech and Signal Processing*, April 2015, pp. 2774–2778.

[17] M. Filo, A. Hossain, A. R. Biswas, and R. Piesiewicz, "Cognitive pilot channel: Enabler for radio systems coexistence," in *2nd International Workshop on Cognitive Radio and Advanced Spectrum Management*, May 2009, pp. 17–23.

[18] T. Jiang, "How many entries of a typical orthogonal matrix can be approximated by independent normals?," *The Annals of Probability*, vol. 34, no. 4, pp. 1497–1529, 2006.

[19] S. N. Diggavi and T. M. Cover, "The worst additive noise under a covariance constraint," *IEEE Transactions on Information Theory*, vol. 47, no. 7, pp. 3072–3081, Nov 2001.

[20] S. Boyd and L. Vandenberghe, *Convex optimization*, Cambridge university press, 2004.

[21] S. R. Becker, E. J. Candès, and M. C. Grant, "Templates for convex cone problems with applications to sparse signal recovery," *Mathematical Programming Computation*, vol. 3, no. 3, pp. 165–218, 2011.