Results on Spectrum Sharing between a Radar and a Communications System

M. R. Bell* N. Devroye† D. Erricolo† T. Koduri† S. Rao† D. Tuninetti†

Abstract — **The increasing demand for wireless ubiquitous services necessitates new spectrum allotments or a re-design of current systems to effectively manage coexistence between different types of technologies. In particular, the S-band (2-4 GHz) is in high demand for both surveillance and weather radar, communication satellites and commercial wireless networks. In the US there is strong political pressure for portions of radar spectrum to be shared with commercial wireless services. In order to effectively design coexistence strategiesit is crucial to understand how current radar and communications systems would affect each other if they were to coexist. As a starting point, this paper investigates a simple setup where one radar coexists with one communications system and considers the effect each has on the other as their relative distance is varied. Despite its simplicity, this model qualitatively validates known experimental results on the effect of communications systems on radars, and thus could be a useful tool to design coexistence schemes for future networks.**

1 Introduction

The high demand for wireless services and shortage of new spectral resources has recently motivated the design of efficient spectral sharing mechanisms between systems that are traditionally characterized by very different objectives and therefore design criteria, such as for example sensing and data transmission. Deciding what / where services may share spectrum has been a subject of recent debate: in the US, the White House's interest in *unleashing the broadband revolution* [1] has led DARPA (Defense Advanced Research Projects Agency) to launch the SSPARC (Shared Spectrum Access for Radar and Communications) program and NSF (National Science Foundation) the EARS (Enhancing Access to the Radio Spectrum) program to encourage research in this direction. One of the "beachfront" portions of spectrum is the S-band (2-4 GHz), in which at present several radar systems (i.e., air surveillance and weather) and wireless communication systems (i.e., Wi-Fi and WLAN) operate, and is hence a good candidate for spectrum sharing.

The effects of interference between radar and communications systems were already considered in the 50s [2], but have only recently really come to the forefront because of the many-fold increase of wireless data traffic due to smartphones, tablets and real-time video streaming. The effects of amplitude and frequency modulated audio signals on radars was studied in [3], concluding that FM interference behaves like AWGN but AM/DSB interference behaves quite differently. More recently, effectiveness of mitigation techniques such as notch filtering and 16, 32, and 64 pulse coherent integration were studied in [4]. Experimental analysis carried out by the National Telecommunications and Information Administration (NTIA) of interference of digital signals on radars concluded that radar performance significantly deteriorates when the interference to noise level is as low as -6 to -9 dB [5, 6].

Contrary to reports such as [5, 6], which have experimentally demonstrated the effect of communication interference on various radar bands, there are few experimental studies, to the best of our knowledge, that demonstrate the effect of unaltered radar interference on a communications system. In [7], the authors simulate the effect of a rotating radar on a WiMax receiver, demonstrating the effect on bit and packet error rates, and [8] qualitatively proposes a model for how radar receivers may saturate the receivers of other services. In [9], coexistence issues for WiMax and radar systems in the S-band are again discussed qualitatively. In [10], and their subsequent related works, projection based schemes are proposed for cognitive MIMO radars to null out interference caused to communications receivers. We are not aware of an analytical model available to justify and predict the effect of interference on for example missing or false targets for radars, or data rate and outage probability for communication systems.

As a starting point towards a theory of coexistence, we consider the effects of radar and communication interference on each other when the two systems do not alter their processing, which is very different in nature, and present simulation results based on our model.

2 System Model

We consider simplified models of radars and communication systems: a mono-static radar system that coexists on the same spectral band with a coded single-carrier communications system. The geometry of the problem is shown in Fig. 1: the radar is located at equal distance d from a communication transmitter and from the communication receiver; the wireless communication transmitter and receiver are separated by a distance D; the radar target is at a distance R from the radar and lies exactly on the path from the radar to the communi-

*School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, 47907, USA, e-mail: mrb@ecn.purdue.edu.

†Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL, 60607, USA, e-mail: devroye, derric1, tkodur2, srao30, danielat @uic.edu.

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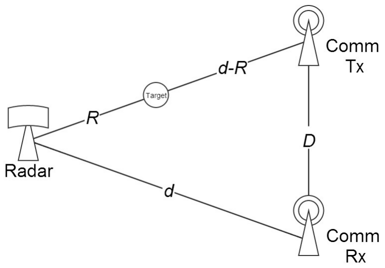

Figure 1: Geometry of the problem.

cation transmitter (meant to capture the worst case interference scenario). For simplicity, we do not consider clutter, multi-path or fading at this point.

Radar System Model: We consider a single pulse up-chirp mono-static radar system, with co-located transmit and receive antennas where a single target remains in the same range-cell for the complete duration and has zero Doppler. The received energy per pulse satisfies $E_{\text{Rrx}} \propto 1/R^4$ The transmitted radar signal can be written as

$$R_{\text{tx}}(t) = \sum_{\ell} \Re\{\sqrt{E_{\text{Rtx}}}\, p_R(t-\ell T_R)\sqrt{2}e^{j2\pi f_R t}\}, \quad (1)$$

and the radar return can be modeled as

$$R_{\text{rx}}(t) = \sum_{\ell} \Re\{\sqrt{E_{\text{Rrx}}}\, p_R(t-\ell T_R-\tau_0)\sqrt{2}e^{j2\pi f_R t}\} + n_R(t), \quad (2)$$

where $p_R(t)$ is the unit-energy radar waveform of pulse of duration τ_R and bandwidth W_R (roughly equal to $1/\tau_R$), T_R is the pulse repetition interval, f_R is the carrier frequency, $\tau_0 = 2R/c$ is the delay in time for the radar return due to the target, $c = 3 \times 10^8$ m/s is the speed of light, $n_R(t)$ is an additive white Gaussian noise with power spectral density β_R^2. The radar receiver forms the ambiguity function / matched filtering at zero Doppler as [11]

$$A(\tau;0) = E_{\text{Rrx}}\, \phi_R(\tau_0-\tau) + \tilde{n}_R(\tau), \quad (3)$$

where $\phi_R(\cdot)$ is the correlation function of $p_R(t)$, and $\tilde{n}_R(\tau)$ is a proper-complex zero-mean Gaussian noise with correlation function $\mathbb{E}[\tilde{n}_R(\tau)\tilde{n}_R(\tau-x)] = \beta_R^2\, \phi_R(x)$, and tries to find its maximum, which without noise occurs at $\tau = \tau_0$. The maximum is then compared to a threshold to determine the presence or absence of a target at the corresponding delay or range.

Metrics for radar performance include the tradeoff between the probabilities of detection P_{d} and false alarm P_{fa}. Assume that in addition to the Gaussian noise of variance β_R^2 in (3) another additive disturbance is present. As a starting point, assume that the interference is Gaussian with power P_{GR} and independent of the Gaussian noise $\tilde{n}_R$. Let $\text{SNR}_R := E_{\text{Rrx}}/\beta_R^2$ be the (useful) Signal to Noise Ratio and $\text{INR}_R := P_{\text{GR}}/\beta_R^2$ be the Interference to Noise Ratio at the radar receiver. With this, P_{fa} and P_{d}, for a threshold-based coherent detector based on (3), with threshold T, in presence of a Gaussian interference becomes [11]

$$P_{\text{fa}} = \frac{1}{2}\text{erfc}\left(\frac{T/\beta_R^2}{\sqrt{\text{SNR}_R(1+\text{INR}_R)}}\right), \quad (4)$$

$$P_{\text{d}} = \frac{1}{2}\text{erfc}\left(\text{erfc}^{-1}(2P_{\text{fa}}) - \sqrt{\frac{\text{SNR}_R}{1+\text{INR}_R}}\right). \quad (5)$$

Modeling the interference from a co-existing digital data system as a Gaussian interference might not be appropriate in the model depicted in Fig. 1, as the central limit theorem might not hold (i.e., too few communication transmitters). Therefore we proceed next to model a digital data interferer.

Communication System Model: The pass-band transmitted signal of a digital (linearly) amplitude modulated signal can be written as [12]

$$C_{\text{tx}}(t) = \sum_{m} \Re\{x_m \sqrt{E_{\text{Ctx}}}\, p_C(t-mT_C)\sqrt{2}e^{j2\pi f_C t}\}, \quad (6)$$

where E_{Ctx} is the transmit energy per pulse, $p_C(t)$ is the unit-energy pulse-shaping waveform (for example a root-raised-cosine) of bandwidth W_C (roughly equal to $1/T_C$), T_C is the symbol interval, f_C is the carrier frequency of the communication system, and x_m is the (possibly coded) digital data sequence from a finite constellation (for example a QAM). Assuming that the channel between the communication transmitter and receiver is frequency non-selective and only corrupted by Gaussian noise, the pass-band received signal is

$$C_{\text{rx}}(t) = h\, C_{\text{tx}}(t) + n_C(t) : |h|^2 \propto \frac{1}{D^2}, \quad (7)$$

where h is the channel attenuation, and $n_C(t)$ is an additive white Gaussian noise with power spectral density β_C^2. Assuming that the pulse shape $p_C(t)$ satisfies the 2nd Nyquist criterion, perfect synchronization and coherent matched-filter detection, then the communication decoder will decide on the transmitted sequence $\{x_m\}$ based on the sufficient statistics

$$y_m = \sqrt{\frac{E_{\text{Ctx}}}{D^2}}x_m + z_m, \quad (8)$$

where $\{z_m\}$ is a sequence of i.i.d. zero-mean Gaussian random variables with variance β_C^2.

The metrics for communication system performance will be the probabilities of bit and symbol error. For example, for uncoded square M-QAM transmission the average probability of symbol error is [12]

$$P_{\text{QAM}} = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\,\text{SNR}_C}{M-1}}\right)\right)^2,$$

where $Q(x) = 1/2\,\text{erfc}(x/\sqrt{2})$ and $\text{SNR}_\text{C} \propto \frac{E_\text{Ctx}}{D^2 \beta_C^2}$. For coded transmission, upper bounds on the probability of error as a function of SNR_C are known as a function of the code parameters [13].

If, in addition to the Gaussian noise of variance β_C^2, another additive disturbance is present, then we must modify the noise term in (8) appropriately. If the overall disturbance could be modeled as Gaussian with power $\beta_C^2 + P_\text{GC} := \beta_C^2(1 + \text{INR}_C)$, then whatever analytical performance measure we had as a function of SNR_C would still be valid if SNR_C is replaced by $\text{SNR}_C/(1 + \text{INR}_C)$. Unfortunately, if the additional interference is a radar signal like (1), the Gaussian assumption is not valid. A better model is discussed next.

Mutual Interference Model: With the communication signal in (6) as extra (duly attenuated) additive interference to the radar received signal in (2), the noise term in the ambiguity function in (3) must be modified as follows: the extra additive "noise" term $\sqrt{E_\text{Rtx}}\,\text{COMM}(\tau)$ must be added, where

$$\text{COMM}(\tau) := h_C \sqrt{E_\text{Ctx}} \sum_m x_m\, p_R(t-\tau) \qquad (9)$$
$$\cdot \int p_C(t - U_C - mT_C)\, e^{j2\pi(f_C - f_R)(t-\tau)}\, dt,$$

where U_C, uniformly distributed on $[0, T_C]$, models the asynchronism among the two systems, and h_C is an attenuation such that $|h_C|^2 \propto 1/d^2$. If $\text{COMM}(\tau)$ in (9) is absent, the tradeoff between P_fa and P_d is given by (4)-(5) with $\text{INR}_R = 0$.

By a similar reasoning, with the radar signal in (1) as extra (duly attenuated) additive interference to the communication received signal in (7), the noise term in (8) must be modified as follows: the extra additive "noise" term RAD_m must be added where

$$\text{RAD}_m := h_R \sqrt{E_\text{Rtx}} \sum_\ell \int p_C(\tau - mT_C) \qquad (10)$$
$$\cdot\, p_R(\tau - U_R - \ell T_R)\, e^{j2\pi(f_R - f_C)\tau}\, d\tau,$$

where U_R, uniformly distributed on $[0, T_R]$, models the asynchronism among the two systems, and h_R is an attenuation such that $|h_R|^2 \propto 1/d^2$. If RAD_m in (10) is absent, y_m in (8) is a sufficient statistic for x_m and the minimum average symbol-by-symbol error rate is obtained by minimum-distance decoding [12].

Table 1: Parameters for the simulation

Symbol	Parameter	Value
f_R	Radar carrier frequency	2700 MHz
W_R	Radar bandwidth	20 MHz
T_R	Radar pulse repetition interval	3.1 ms
τ_R	Radar pulse width	0.88 μ s
SNR_R	Radar SNR	10 dB
W_C	Communication bandwidth	20 MHz
T_C	Communication symbol time	0.88 μs
SNR_C	Communication SNR	4 dB

Note that in deriving (9) and (10) we assumed that neither the radar nor the communication system change their mode of operation. The point of evaluating the performance with *unchanged* systems is to understand if mutual interference is a problem and if so to gain insight into which of the model parameters mostly affect the performance degradation. With $\text{COMM}(\tau)$ in (9) and RAD_m in (10), it appears difficult to analytically evaluate performance metrics for the two systems, but this is the ultimate goal of this framework.

3 Numerical Results

In order to assess the performance degradation due to mutual interference, we performed numerical simulations. We report here only the effect of the communications on the radar for sake of space. The simulations reported here used the parameters listed in Table 1. The radar target was always at a fixed distance R resulting in $\text{SNR}_R = 10$ dB. For the communication system we fixed D resulting in $\text{SNR}_C = 4$ dB, and performance was evaluated by varying the relative distance between the two systems d (see Fig. 1), which in turns varies INR_R and INR_C. For the radar system we used an up-chip waveform, a pencil radar antenna with scan interval 71.4 sec, beam width 0.9 deg and gain 45 dBi. For the communication system we used a rate 1/2 convolutional code mapped onto a QPSK modulation and pulse shaped with root-raised cosine with roll-off 0.22.

The Receiver Operating Curves (ROC) are shown in Fig. 2 for $\text{INR}_R = 0$ dB in four possible cases: (i) stand-alone radar ($\text{INR}_R = 0$ in eqs.(4)-(5)), (ii) radar in the presence of communication interference (frequency separation of the carriers of the two systems is 0MHz, received interference still received at level of 0 dB) as in (9), (iii) radar in the presence of communication interference (frequency separation of the carriers of the two systems is 15MHz, received interference still received at level of 0 dB) as in (9), and (iv) radar in the presence of additional Gaussian interference (eqs.(4)-(5), also at $\text{INR}_R = 0$ dB). Notice that, interestingly, Gaussian interference appears to be less detrimental than the communications interference to an unaltered radar system in terms of ROC curve. The reason for the

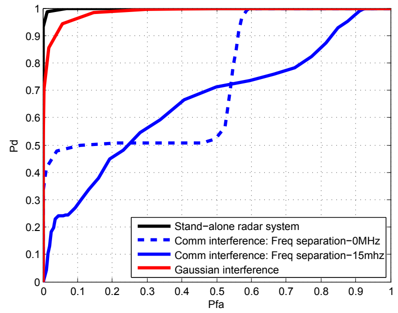

Figure 2: ROC curves for co-existing radar and comm.

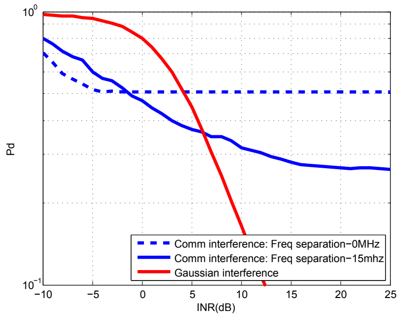

Figure 3: P_{d} vs. INR_R at fixed $\mathrm{SNR}_R = 10$ dB. Without comm. interference, $P_{\mathrm{fa}} = 0.01$ and $P_{\mathrm{d}} = 0.98$.

interesting ROC shape in Fig. 2 is under investigation.

The degradation of P_{d} with increasing INR (received from either the Gaussian or communication system) is shown in Fig. 3. We see that for Gaussian interference, the degradation starts at -5 dB which is similar to that observed through real experiments in [5]. Real communications interference (rather than Gaussian) appears to affect the degradation at even smaller received INR levels, but are less detrimental than Gaussian interference at higher INRs.

Overall, it is clear that communications and Gaussian interference affect the performance of a radar quite differently and hence the former should not be modeled as Gaussian.

4 Conclusions

We analyzed the effect of interference of communication signals on the performance of radars and vice-versa based on a simple models. For radars, our simulation results are consistent with experimental results and can be summarized as follows: low power communication interference can degrade the radar performance significantly, and communication interference should not be modeled as Gaussian interference of the same power.

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