Joint Time-Frequency Alignment for PAPR and OOBE Suppression of OFDM-Based Waveforms

Z. Esat Ankaralı, Alphan Şahin, and Hüseyin Arslan

***Abstract*— Orthogonal frequency division multiplexing (OFDM) waveform has been adopted by many wireless standards due to its numerous advantages. However, OFDM symbols have two critical drawbacks: high out-of-band emission (OOBE) and high peak-to-average power ratio (PAPR). Cyclic prefix (CP) alignment is one of the promising methods that jointly suppresses the OOBE and PAPR of OFDM symbols without introducing extra receiver complexity. However, its suppression performance remains limited in practical scenarios as it only exploits the degrees of freedom (DoFs) provided by the CP part of the OFDM symbols. In this letter, we propose a novel approach which jointly exploits the time domain resources, i.e., CP, and the frequency domain resources that are not effectively used by the receiver, i.e., guard tones and the subcarriers faded by the multipath channel. Thus, the available DoFs are substantially increased and more efficiently utilized for alignment purpose. It is shown that the OOBE and PAPR suppression performance is significantly enhanced with the proposed method as compared with the original approach.**

***Index Terms*— Interference alignment, OFDM, OOBE, PAPR.**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has been the prominent waveform in the literature due to its flexibility in spectrum, robustness against frequency selective channels, and simplicity in equalization. Therefore, it has been adopted in many wireless communication standards such as 5th generation (5G) New Radio, Wi-Fi, and Long Term Evolution. However, high out-of-band emission (OOBE) degrades the spectral compactness of Orthogonal frequency division multiplexing (OFDM) and may cause severe interference on adjacent channels in certain scenarios. Additionally, its high peak-to-average power ratio (PAPR) makes the OFDM signal vulnerable against non-linear distortion due to power amplifiers (PAs), and decreases the coverage range in cellular scenarios. Therefore, maintaining OOBE and PAPR low for OFDM signals is critically important for an efficient deployment of OFDM in future standards including 5G.

Although OOBE and PAPR of OFDM signals are well discussed separately in the literature, only few approaches investigate their joint suppression. In [1], a joint OOBE and PAPR suppression technique is proposed for cognitive radio scenarios. In [2], partial-transmit sequences (PTS) are deployed for the same goal, however, it requires sharing PTS information for each OFDM symbol. Recently, cyclic prefix (CP) alignment (CPA) concept is proposed in [3]. In this method, an additive signal, called alignment signal (AS), is designed for joint OOBE and PAPR suppression and transmitted along with the OFDM symbol. After passing through the wireless channel, the AS aligns with the CP portion, similar to the alignment concept presented in [4], [5], and offers a promising solution to high PAPR and OOBE without introducing any problem at the receiver side. However, the suppression performance of this method heavily depends on the degrees-of-freedom (DoF) provided by the CP size, which may be small in certain scenarios. In addition, due to the constraint of alignment with CP duration, which is a local portion of OFDM symbol in time domain, the AS mostly concentrates around CP and does not effectively reduce the amplitude variation of OFDM symbol. Therefore, CPA is limited especially in PAPR suppression. In [6], channel independent CPA where the alignment is achieved by receive filter is also investigated. However, no solution is provided for the aforementioned problems.

In this letter, we introduce a novel joint time-frequency alignment (JTFA) concept to overcome the shortcomings of original CPA. We allow AS to align with both time and frequency domain resources that are not effectively used at the receiver, i.e. CP, guard tones, and subcarriers that are severely faded by the channel. Thus, not only is the available DoF for AS generation substantially increased as compared to the CPA, but also the power of AS is optimally distributed across time and frequency domains in the sense that it leads to further PAPR and OOBE suppression, jointly.

The remainder of the letter is organized as follows. Section II explains system model and Section III presents the proposed method. Numerical results are provided in Section VI and Section V concludes the letter.

Notation: $\mathbf{I}_N$ represents $N \times N$ identity matrix and $\mathbf{0}_{N\times M}$ is $N \times M$ zero matrix. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $\ker(\cdot)$ denote transpose, conjugate transpose and kernel of a matrix. $\mathcal{CN}(0, \mathbf{C})$ represents a zero mean complex Gaussian distribution with the covariance matrix $\mathbf{C}$. $\mathbb{R}$ and $\mathbb{C}$ denote the real and complex number fields, respectively.

II. SYSTEM MODEL

We consider a single link OFDM-based communication system with N subcarriers. The ith OFDM symbol, $\mathbf{x}_i \in \mathbb{C}^{(N+N_{\mathrm{c}})\times 1}$, is expressed in time domain as

$$\mathbf{x}_i = \mathbf{A}\mathbf{F}^{\mathrm{H}}\mathbf{M}\mathbf{d}_i, \tag{1}$$

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where $\mathbf{d}_i \in \mathbb{C}^{N_s \times 1}$ is the vector of data symbols, $\mathbf{M} \in \mathbb{R}^{N \times N_s}$ is the mapping matrix assigning the data symbols to the selected subcarriers, $\mathbf{F}$ is the N-point DFT matrix, $\mathbf{A} \in \mathbb{R}^{(N+N_c)\times N}$ represents the CP insertion matrix explicitly given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{N_c \times N-N_c} & \mathbf{I}_{N_c} \\ & \mathbf{I}_N \end{bmatrix}, \tag{2}$$

N_c is the number of samples in CP, and N_s is the number of data symbols. In [3], PAPR and OOBE characteristics of OFDM signals are improved by adding an AS, $\mathbf{a}_i \in \mathbb{C}^{(N+N_c)\times 1}$, and the transmitted signal is formed as

$$\mathbf{s}_i = \mathbf{x}_i + \mathbf{a}_i. \tag{3}$$

The AS vector $\mathbf{a}_i$ can be calculated as

$$\mathbf{a}_i = \mathbf{P}\mathbf{u}_i. \tag{4}$$

In (4), $\mathbf{P} \in \mathbb{C}^{(N+N_c)\times \hat{N}_c}$ is an orthonormal precoder matrix[1] that generates the AS vector $\mathbf{a}_i$ from any $\mathbf{u}_i \in \mathbb{C}^{\hat{N}_c \times 1}$ where $\hat{N}_c$ represents the DoF for designing $\mathbf{a}_i$[1]. In CPA, $\hat{N}_c$ corresponds to CP size to avoid interference on the information symbols.

We assume that $\mathbf{s}_i$ passes through an independently and identically distributed multipath channel with L taps whose vector representation is given by $\mathbf{h} = [h_0, h_1, ..., h_L]^{\mathrm{T}} \sim \mathcal{CN}(0, \mathbf{I}_{L+1}/(L+1))$. Power delay profile (PDP) of $\mathbf{h}$ is considered as an exponential decaying channel. We then calculate the ith received signal vector as

$$\mathbf{r}_i = \begin{bmatrix}\mathbf{H}_p \ \mathbf{H}\end{bmatrix}\begin{bmatrix}\mathbf{s}_{i-1} \\ \mathbf{s}_i\end{bmatrix} + \mathbf{n}, \tag{5}$$

where $\mathbf{n} \in \mathbb{C}^{(N+N_c)\times 1} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{N+N_c})$ is an additive white Gaussian noise vector, $\mathbf{H}_p \in \mathbb{C}^{(N+N_c)\times(N+N_c)}$ characterizes the leakage of the previous signal $\mathbf{s}_{i-1}$ on the current signal $\mathbf{s}_i$, and $\mathbf{H} \in \mathbb{C}^{(N+N_c)\times(N+N_c)}$ is the channel convolution matrix.

After discarding the CP, the signal can be expressed as

$$\begin{aligned}\mathbf{y}_i = \mathbf{D}\mathbf{r}_i = \mathbf{DHAF}^{\mathrm{H}}\mathbf{M}\mathbf{d}_i + \mathbf{DH}\mathbf{a}_i \\ + \ \mathbf{DH}_p\mathbf{AF}^{\mathrm{H}}\mathbf{M}\mathbf{d}_{i-1} + \mathbf{DH}_p\mathbf{a}_{i-1} + \mathbf{n},\end{aligned} \tag{6}$$

where $\mathbf{D} \in \mathbb{R}^{N\times(N+N_c)}$ is the CP removal matrix as

$$\mathbf{D} = \begin{bmatrix}\mathbf{0}_{N\times N_c} \ \mathbf{I}_N\end{bmatrix}. \tag{7}$$

Discarding the CP nullifies the third and fourth terms of (6) as the leakage from the $(i-1)$th OFDM symbol falls into ith symbol's CP duration. Then, after the DFT and de-mapping operations, the received signal in frequency domain can be calculated as

$$\tilde{\mathbf{y}}_i = \mathbf{M}^{\mathrm{H}}\mathbf{FDHAF}^{\mathrm{H}}\mathbf{M}\mathbf{d}_i + \mathbf{M}^{\mathrm{H}}\mathbf{FDHP}\mathbf{u}_i + \hat{\mathbf{n}}. \tag{8}$$

The second term in (8) corresponds to the interference because of the AS and $\hat{\mathbf{n}}$ is the noise. Hence, $\mathbf{M}^{\mathrm{H}}\mathbf{FDHP}\mathbf{u}_i$ should be zero for carrying out an interference free transmission. In [3], this is ensured by setting the columns of $\mathbf{P}$ in a way that they span the null space of $\mathbf{DH}$, i.e. $\ker(\mathbf{DH})$. Thus, $\mathbf{P}\mathbf{u}_i$ can be optimized for PAPR and OOBE suppression while $\mathbf{u}_i$ is mapped into the null space of $\mathbf{DH}$, which corresponds to the CP part of the OFDM symbols.

[1]Since $\mathbf{P}$ is an orthonormal matrix, $\mathbf{a}_i^{\mathrm{T}}\mathbf{a}_i = \mathbf{u}_i^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}\mathbf{P}\mathbf{u}_i = \mathbf{u}_i^{\mathrm{T}}\mathbf{u}_i$.

III. Joint Time-Frequency Alignment for Joint OOBE & PAPR Suppression

In the proposed method, to enable joint utilization of time and frequency domain resources, we design the mapping matrix $\mathbf{M}$ such that it discards the guard tones and the subcarriers experiencing a deep channel fading for data transmission. This is done by selecting the active data subcarriers and forming $\mathbf{M}$ with N_s corresponding columns of $\mathbf{I}_N$. The columns of $\mathbf{P}$ span the null space of $\mathbf{\Gamma}$, i.e. $\ker(\mathbf{\Gamma})$, where $\mathbf{\Gamma} = \mathbf{M}^{\mathrm{H}}\mathbf{FDH} \in \mathbb{C}^{N_s\times(N+K)}$. Thus, by generating $\mathbf{a}_i$ as $\mathbf{P}\mathbf{u}_i$, we can allow the AS to align with the CP duration in time domain, the guard tones and the subcarriers faded by the channel in frequency domain. As we also include the frequency domain resources in null space calculation, $\hat{N}_c$ increases to the sum of the CP size, the number of allocated subcarriers experiencing a deep fading and the guard tones. Then, a more effective $\mathbf{u}_i$ can be designed for joint PAPR and OOBE suppression, as compared to CPA.

It is worth noting that we introduce a small loss in capacity due to the faded subcarriers allocated for the alignment purpose. These subcarriers are selected based on a threshold in channel gain, ϕ_{tr}. In other words, the subcarriers experiencing a channel gain below the selected threshold are exploited for alignment purpose and ignored at the receiver side. For instance, the receiver can decide ϕ_{tr} based on a tolerable loss in capacity and feedback that information to the transmitter along with the channel state information (CSI). Then, the transmitter determines the active subcarriers, design AS based on the CSI and ϕ_{tr}, and transmit the OFDM signal combined with AS. Finally, the active subcarriers are selected at the receiver side and data detection is performed. The block diagram of the transceiver with JTFA and illustrations of transmitted and received signals are provided in Fig. 1.

Using singular value decomposition, $\mathbf{\Gamma}$ can be decomposed as $\mathbf{\Gamma} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$, where $\mathbf{U} \in \mathbb{C}^{N_s\times N_s}$ and $\mathbf{V} \in \mathbb{C}^{(N+N_c)\times(N+N_c)}$ are unitary matrices that contain the singular vectors of $\mathbf{\Gamma}$, and $\mathbf{\Sigma} \in \mathbb{R}^{N_s\times(N+N_c)}$ is a diagonal matrix including the singular values of $\mathbf{\Gamma}$, arranged in descending order. As the null space of $\mathbf{\Gamma}$ is spanned by the last $\hat{N}_c$ columns of $\mathbf{V} = [\mathbf{v}_0, \mathbf{v}_1, ..., \mathbf{v}_{N+N_c-1}]$, $\mathbf{P} \in \mathbb{C}^{(N+N_c)\times\hat{N}_c}$ can be created as $\mathbf{P} = [\mathbf{v}_{N+N_c-\hat{N}_c}, \mathbf{v}_{N+N_c-\hat{N}_c+1}, ..., \mathbf{v}_{N+N_c-1}]$.

After guaranteeing the avoidance of AS's interference with the precoding matrix $\mathbf{P}$, $\mathbf{u}_i$ can be optimized for suppressing OOBE and PAPR of the digital signal $\mathbf{x}_i$ as

$$\begin{aligned}\mathbf{u}_i = \underset{\hat{\mathbf{u}}_i}{\arg\min} \ & (1-\lambda)\|\mathcal{F}_O(\mathbf{x}_i + \mathbf{P}\hat{\mathbf{u}}_i)\|_2 + \lambda\|(\mathbf{x}_i + \mathbf{P}\hat{\mathbf{u}}_i)\|_\infty \\ \text{over } & \hat{\mathbf{u}}_i \in \mathbb{C}^{\hat{N}_c\times 1} \\ \text{subject to } & \|\hat{\mathbf{u}}_i\|_2 \le \sqrt{\alpha}\|\mathbf{x}_i\|_2,\end{aligned} \tag{9}$$

where $\mathcal{F}_O$ is the matrix that contains the rows of an oversampled DFT matrix, corresponding to the signal elements in the out-of-band region, α is a power limiting parameter for $\mathbf{a}_i$, and $\lambda \in [0, 1]$ is the weighting factor in the joint OOBE and PAPR optimization. While larger λ yields a lower PAPR, smaller λ leads to a better OOBE suppression. Also, $\|\cdot\|_2$ and $\|\cdot\|_\infty$ represent the 2-norm and infinity norm operators, respectively.

The objective function and the constraint are both convex in (9). Therefore, the problem can be solved by a convex optimization solver. In this study, YALMIP is utilized [7].

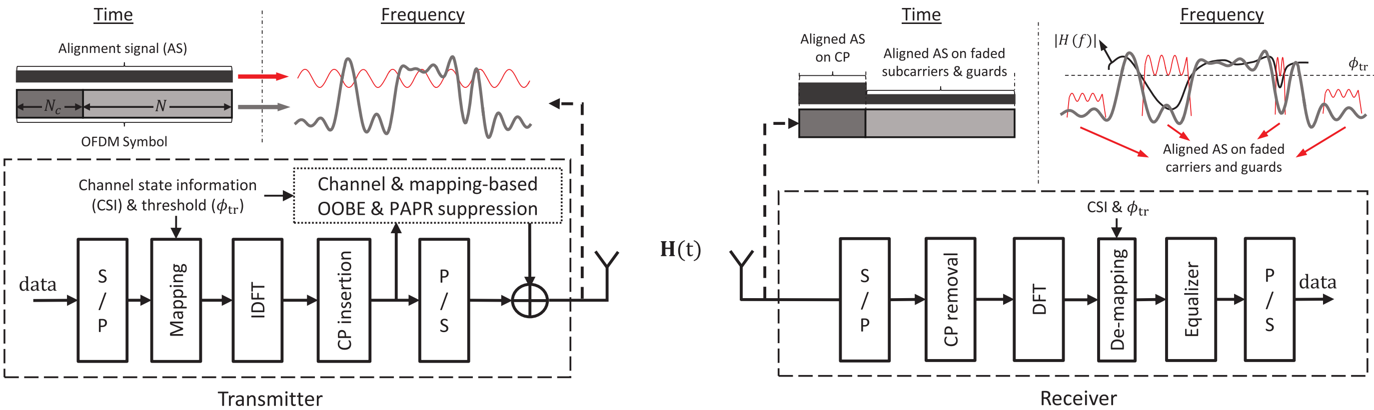

Fig. 1. Block diagram for the proposed system. AS aligns with the CP duration, the guard tones and the faded subcarriers at the receiver side.

IV. Numerical Results

In this section, we demonstrate bit-error rate (BER), OOBE, and PAPR suppression performance of the JTFA through simulations. We consider OFDM symbols with 128 subcarriers and 16 samples for CP, where the 64 subcarriers of each OFDM symbol are utilized for 16-QAM symbols. PDP of the channel is assumed to be a 17-tap exponential decaying function, expressed as $h(\tau) = ae^{-\tau n}$, where a is the normalization factor, n indicates the tap index, τ is the decaying factor, and the amplitude of each tap follows Rayleigh distribution. In the simulations, we set τ to 0.2 unless otherwise stated. The threshold ϕ_{tr} is set to 0.2. We also compare JTFA with CPA [3] and cancellation carrier insertion (CCI) [8]. For the sake of a fair comparison, we use the same set of subcarriers for both CCI and JTFA, and keep the total signal power the same for all the approaches.

In Fig.2, we show the energy distribution of the AS in time domain for CPA and JTFA for $\lambda = 0.99$. When AS is designed based solely on CP duration, as done in CPA method, most of the energy concentrates around the CP duration and dramatically decreases on the middle samples even for large λ values. As shown in Fig.2, the energy of the samples located in the data duration of OFDM symbol is approximately 13 dB weaker than the samples of OFDM symbol for CPA, even when the channel decaying rate, τ, is zero which corresponds to a uniform PDP. As τ increases, which is likely in practical scenarios, AS concentrates further on the edges. Since any sample may have a high amplitude in an OFDM symbol, the AS in CPA is ineffective to cancel the peak sample, which yields a limited PAPR suppression. In JTFA, since time and frequency domain resources are jointly utilized, the energy of the samples of AS are distributed more uniformly across the useful OFDM symbol duration as shown in Fig.2. Hence, JTFA is more effective than CPA to reduce PAPR.

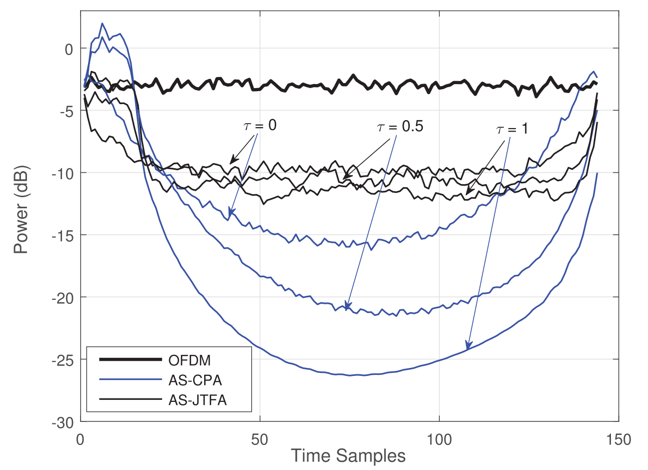

Fig. 2. Power distribution of plain OFDM and AS samples for CPA and JTFA in time for different channel decaying factors ($\alpha = 0.25$, $\phi_{tr} = 0.2$).

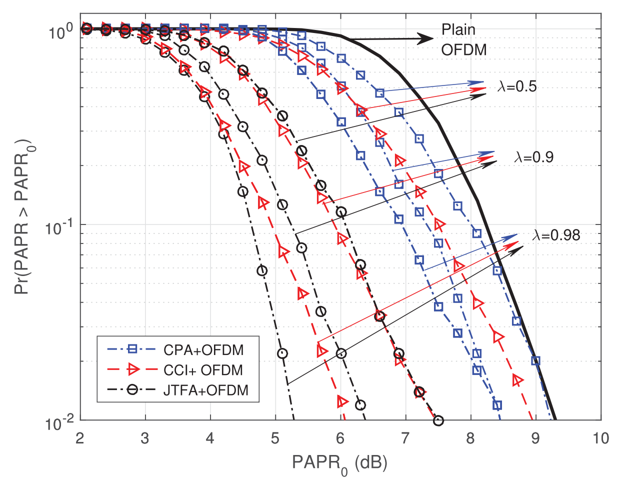

Fig. 3. PAPR performance for CPA, JTFA, and CCI for different λ values ($\alpha = 0.25$, $\tau = 0.2$, $\phi_{tr} = 0.2$).

In Fig. 3, the PAPR performances are provided for JTFA, CPA, and CCI when $\alpha = 0.25$ and $\lambda = \{0.5, 0.9, 0.98\}$. The simulation results show that JTFA method achieves 4 dB suppression while CPA can only suppress less than 1 dB for $\lambda = 0.98$, as compared to plain OFDM symbols. As discussed earlier, this is because of the fact that JTFA is more effective than CPA to cancel the peak sample of OFDM symbols since it exploits the frequency domain resources along with the CP duration. It is also worth noting that CCI remains around 3 dB reduction in PAPR as only frequency domain resources are employed in this scheme. As a result, JTFA is superior to CPA and CCI in terms of PAPR suppression. This is achieved at the expense of approximately 6% capacity loss when $\gamma = 10$ dB, which is calculated based on Shannon's channel capacity.

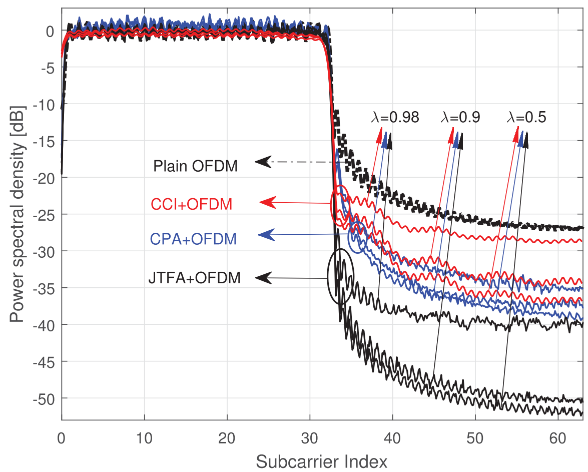

Fig. 4. OOBE performance for CPA, JTFA, and CCI for different λ values ($\alpha = 0.25$, $\tau = 0.2$, $\phi_{\mathrm{tr}} = 0.2$).

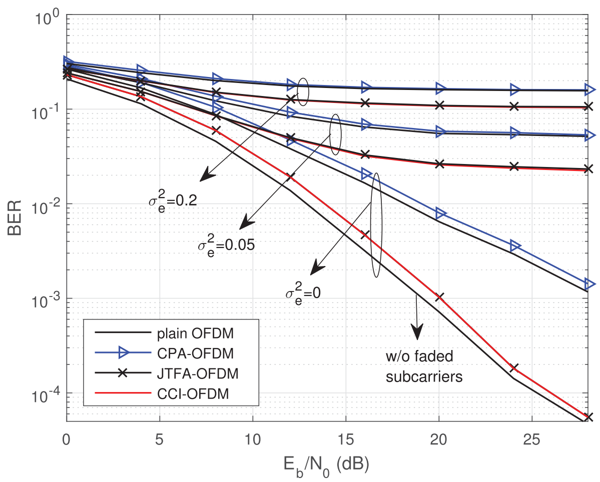

Fig. 5. BER performance for CPA, JTFA, and CCI for different MSEs (σ_e^2) in channel estimation ($\alpha = 0.25$, $\tau = 0.2$, $\phi_{\mathrm{tr}} = 0.2$).

In Fig. 4, we compare OOBE reduction performance of aforementioned schemes. JTFA method reduces the OOBE up to 24 dB when $\lambda = 0.5$, while CPA and CCI provide approximately 13 dB and 10 dB suppressions, respectively. Our simulation results show that JTFA is better than CPA and CCI in both OOBE and PAPR reduction for the investigated λ values.

In Fig. 5, BER results are provided for JTFA, CPA and CCI for different mean squared errors (MSEs) in channel estimation. We quantify MSE as the expected value of the normalized difference between the channel response h and the channel estimated by the receiver $\tilde{h}$, and defined as $\sigma_e^2 = \frac{\mathrm{E}[|\tilde{h}-h|^2]}{\mathrm{E}[|h|^2]}$ where $\mathrm{E}[\cdot]$ denotes the expected value. When there is no channel estimation error, i.e. $\sigma_e^2 = 0$, CPA, JTFA and CCI methods are slightly worse than the plain OFDM generated with the same active subcarriers since a part of the power (20% for $\alpha = 0.25$) is used for either AS or inserted carriers. However, the impact of equalization with an erroneous channel estimation mostly dominates the effect of power discrepancy and BER performances of JTFA and CPA become similar to that of CCI and regular OFDM transmission, respectively, for $\sigma_e^2 > 0$.

V. Conclusion

In this study, we present a joint PAPR and OOBE reduction technique for OFDM systems. We significantly increase the DoF in designing AS by jointly exploiting specific subchannels, i.e. guard tones and subcarriers faded by the channel, and CP in the optimization. Thus, a substantial suppression is obtained in PAPR and OOBE at the cost of a small loss in capacity.

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