Joint Interleaving and Selected Mapping Technique with LCT for PAPR Reduction in OFDM Signals

Ehab F. Badran, Abdulellah K. Aljaroushah, and Darwish A. E. Mohamed
Department of Electronics and Communications Engineering,
Arab Academy for Science, Technology and Maritime Transport,
Alexandria, Egypt.
Email: ebadran@aast.edu

***Abstract*— One of the major problems that faces any wireless communication system that apply orthogonal frequency division multiplexing (OFDM) is the peak-to-average power ratio (PAPR). In this paper, a joint interleaving and selected mapping (SLM) method is proposed to reduce PAPR in OFDM signals. Then, linear companding transform (LCT) is employed for farther PAPR reduction. Joining interleaving and SLM is accomplished by generating randomly interleaved data streams and multiply it by a random phases sequences (SLM). Then the resultant sequence with lowest PAPR is processed by LCT. Simulations results show that the proposed technique simultaneously decrease bit error rate (BER) over AWGN and reduce PAPR by 4.5dB.**

Keywords*— *Interleaving; orthogonal frequency division multiplexing (OFDM); peak to average power ratio (PAPR); linear companding transform (LCT).

I. INTRODUCTION

OFDM systems divide the high-rate data stream into multi lower-rate data streams to be sent over orthogonal subcarriers, the bandwidth of each subcarrier must be smaller than the channel coherence bandwidth, therefore each subcarrier will experience relatively a flat fade [1]. The symbol duration increases in the time due to the lower-rate parallel subcarriers; the relative amount of dispersion in time caused by multipath delay spread is decreased. Intersymbol interference (ISI) caused by multipath distortion, and the Doppler shifts, frequency and phase offset in an OFDM system causes loss the orthogonality among the sub-carriers it is called Intercarrier interference (ICI). ISI and ICI can be overcame by using guard time (Cyclic Prefix) [2]. OFDM is used in many wireless systems such as wireless local area networks IEEE802.11 (WLAN), worldwide interoperability for microwave access IEEE802.16 (WiMAX), power line communication (PLC), asymmetric digital subscriber line (ADSL), digital audio broadcasting systems (DAB) and digital video broadcasting (DVB). Wireless applications based on OFDM system provide us greater immunity to multi-path fading and impulsive noise.

High peak-to-average power ratio (PAPR) is one of the major practical problems that faces the OFDM signal. It causes a distortion when passed through a nonlinear power amplifier (PA) region, there are many techniques ([5]-[10] and the references there in) to solve this problem some of them are clipping, companding, selective mapping, active constellation Extension (ACE), Partial Transmit Sequence (PTS), Turbo Coded OFDM.

In this paper, the signal interleaving is applied to reduce PAPR of OFDM signals by using three different types of interleaving techniques, namely matrix interleaving, helical scan interleaving and random interleaving, and the best of them is to be selected to combine it with SLM which use phase sequences to be multiplied by the interleaved signal then the sequence with the lowest PAPR is passing through LCT. In section 2, three different types of interleaving techniques are studied. In section 3, combined techniques are proposed and tested for PAPR reduction. Section 4 presents the simulation results of the BER performance comparison. A conclusion warps this paper.

II. INTERLEAVING METHODS OF OFDM SIGNAL TO REDUCE PAPR

Consider an OFDM system with N subcarriers and input data sequence $\boldsymbol{X} = \{X_k\}_{k=0}^{N-1}$. Thus, after applying the IFFT, OFDM signal can be written as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}}, \;\; n = 0,1,.,N-1. \quad (1)$$

The PAPR is the relation between the maximum peak power of the discrete OFDM signal to the average power of discrete OFDM signal can be expressed by:

$$PAPR = \frac{\max\{|x(n)|^2\}}{E\{|x(n)|^2\}}, \quad (2)$$

where $E[\cdot]$ is the expectation operator.

An oversampled OFDM signal by oversampling factor of L can be written as:

$$^{L}x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{NL}kn}, \;\; n = 0,1,..,NL-1. \quad (3)$$

Oversampling by $L \geq 4$ is proved in [11] to be sufficient to have the PAPR of discrete OFDM signals almost similar to analog OFDM signals. In this paper L=4 is assumed.

In this paper, the main three different types of interleaving methods are applied to reduce the PAPR. These methods are helical scan interleaving, matrix interleaving, and random interleaving.

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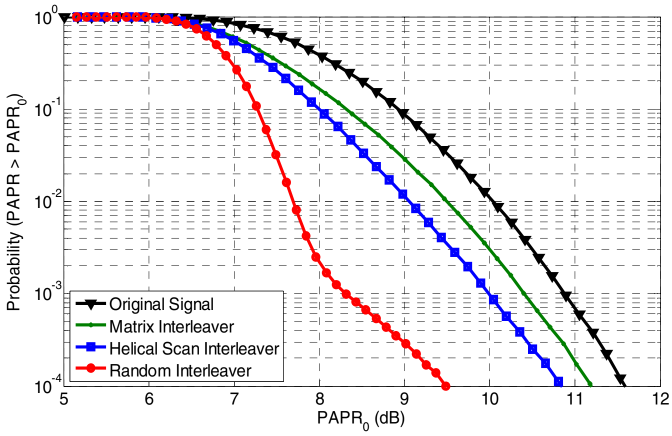

Figure 1 PAPR of Matrix Interleaving, Helical Scan Interleaving and Random Interleaving compared to conventional OFDM with 16QAM and N=512.

- *Helical Scan Interleaving:* Permute input symbols by selecting matrix elements along diagonals.
- *Matrix Interleaving:* Permute input symbols by filling a matrix by rows and emptying it by columns.
- *Random Interleaving:* Reorder the input symbols using a random permutation.

The interleaving permutation can be applied either before or after modulation (mapping). P is the number of permutations for each input sequence of the IFFT and each permutation gives different PAPR. The sequence with the lowest PAPR is selected to be sent through the channel. The main challenge that faces the analog signal is multipath fading channel, however the OFDM signal has resistance to multipath fading by using the Cyclic Prefix (CP). At the OFDM receiver, the index of the selected permutation sequence is required to recover the data sequence that was sent. This index is sent as a side information to the receiver over a secured channel. In this paper $P = 8$ is chosen, thus eight different interleaving sequences are assumed.

Fig. 1 shows the PAPR of the matrix interleaved, helical scan interleaved and random interleaved OFDM signals are compared to PAPR of the conventional OFDM and they give improvement by 0.5dB, 1dB and 2.75dB respectively for the probability 10^{-3} the best of them was random interleaving which can be expressed as:

$$\boldsymbol{I}_p = \boldsymbol{I}\,(\boldsymbol{X}, S_P) \quad p = 1,2,..,P, \tag{4}$$

where $\boldsymbol{I}$ is the interleaving operator and $\boldsymbol{X}$ is the input data sequence, S_p is the p^{th} permutation sequence, $\boldsymbol{I}_p$ is the p^{th} permutated sequence.

III. Combined Techniques for the PAPR Reduction of OFDM Signal

The random interleaving (RI) is the best among interleaving techniques so it is selected to combine it with other technique.

A. Selected Mapping (SLM) Technique

In SLM the input sequence $\boldsymbol{X} = \{X_k\}_{k=0}^{N-1}$ is element by element multiplied by U uncorrelated different phase sequences. Let the N-length phase shift vector $\{\boldsymbol{B}_u\}_{u=0}^{U-1}$

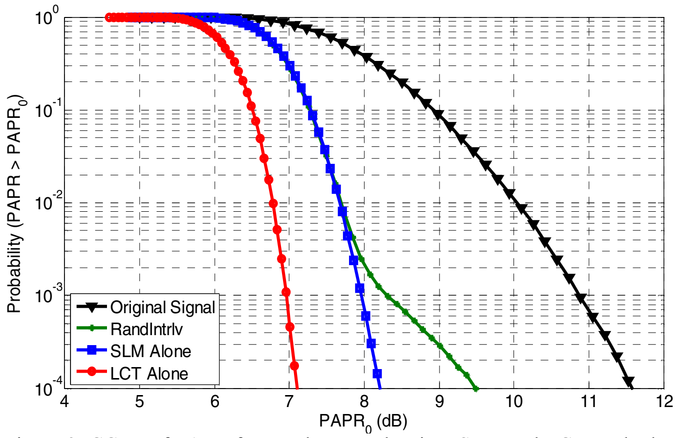

Figure 2. CCDF of PAPR for Random Interleaving, SLM and LCT methods. 16QAM and N=512

where $\boldsymbol{B}_u = \{e^{j\varphi_{u,n}}\}_{n=0}^{N-1}$. This multiplication yields $\{\boldsymbol{X}_u\}_{u=0}^{U-1}$ sequences; the sequence $\boldsymbol{x}^u = IFFT\{\boldsymbol{X}_u\}$ with the lowest PAPR is transmitted. $\boldsymbol{x}^u = [x_{u,0}\ x_{u,1}\ \ldots.x_{u,N-1}]\,, 1 \le u \le U$. Let $\boldsymbol{x}^{\tilde{u}}$ denotes the sequence with the lowest PAPR. The elements of $\boldsymbol{x}^u$ can be expressed as

$$x_{u,n} = \sum_{k=1}^{N-1} X_k e^{j\varphi_{u,k}} e^{j\frac{2\pi nk}{N}}, \quad n = 0,1,..,N-1 \tag{5}$$

B. Linear Companding Transform (LCT)

LCT is an efficient technique to minimize the PAPR. LCT with two inflexion points was proposed in [12]. In that technique, the OFDM signal passed through LCT can be expressed as:

$$y(n) = \begin{cases} a_1 x(n) & |x(n)| \le v_1 \\ a_2 x(n) & v_1 < |x(n)| \le v_2 \\ a_3 x(n) & |x(n)| > v_2 \end{cases} \tag{6}$$

At the receiver side, the received signal can be expressed as $r(n) = y(n) + w(n)$, where $w(n)$ is the additive white Gaussian noise (AWGN). Thus the expanded signal is written as [12]:

$$\tilde{x}(n) = \begin{cases} \dfrac{1}{a_1} r(n) & n \in \varphi_1 \\ \dfrac{1}{a_2} r(n) & n \in \varphi_2 \\ \dfrac{1}{a_3} r(n) & n \in \varphi_3 \end{cases}, \tag{7}$$

where $\varphi_1 = \{n\ \forall |x(n)| \le v_1\}$, $\varphi_2 = \{n\ \forall\ v_1 < |x(n)| \le v_2\}$, and $\varphi_3 = \{n\ \forall |x(n)| > v_2\}$ and the LCT parameters is $a_1 = 2$, $a_2 = 1$,$a_3 = 0.45$, $v_1 = 20\%$, and $v_2 = 40\%$ of the maximum signal magnitude.

In LCT, the OFDM frame peaks are compressed individually to reduce the PAPR, thus LCT can reduced the PAPR more than random interleaving and SLM techniques. Fig. 2 shows CCDF of PAPR comparison between LCT, SLM, RI, and conventional OFDM techniques. The LCT technique gives improvement by 2.75dB, 3dB and 4dB over SLM, RI, and conventional OFDM respectively for a CCDF value of 10^{-3}.

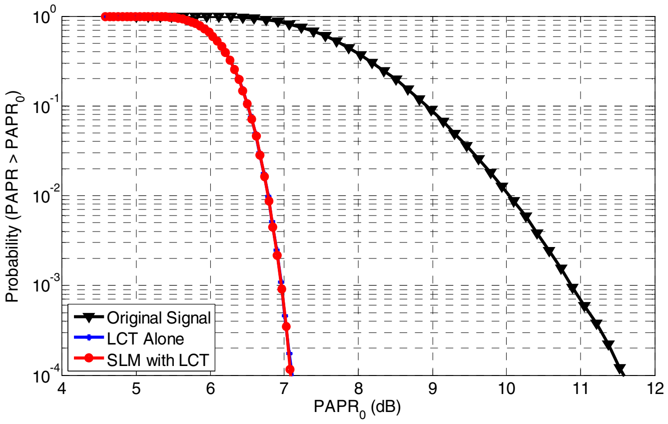

Figure 3. CCDF of PAPR for LCT and SLM with LCT .16QAM, N=512

Fig. 3 shows that there is no difference between the CCDF of PAPR of the LCT technique by itself and a combined SLM and LCT technique.

C. The Proposed Joint PAPR Reduction Technique.

The proposed technique combines random interleaving, SLM and LCT operations successively. The block diagram of the proposed system is shown in Fig. 4. In the proposed technique, four random interleaving permutations and two random phase sequences are considered. By applying random interleaving; four interleaved sequences $\{\boldsymbol{I}_p\}_{p=0}^{3}$ are generated then after multiplying each one of them by two phase shift vectors $\{\boldsymbol{B}_u\}_{u=0}^{1}$ which gives eight different sequences $\{\boldsymbol{X}_{p,u}\}_{p=0,u=0}^{p=3,u=1}$ and after applying IFFT over $\boldsymbol{X}_{p,u}$ eight OFDM frames $\boldsymbol{x}_{p,u} = IFFT\{\boldsymbol{X}_{p,u}\}$ are generated each with different PAPR. The OFDM frame $\boldsymbol{x}_{\tilde{p},\tilde{u}}$ with minimum PAPR is selected. The elements of these eight OFDM frames can be expressed as

$$x_{p,u}(n) = \sum_{k=1}^{N-1} X_{p,u}(n) e^{j\varphi_{u,k}} e^{j\frac{2\pi nk}{N}}, \quad n = 0,1,..,N-1. \quad (8)$$

The selected OFDM frame $\boldsymbol{x}_{\tilde{p},\tilde{u}}$ is then passed through the LCT to apply expansion or compression over the selected frame elements according to the LCT parameters of thresholds values v_1, v_2 and scaling factors u_1, u_2, and u_3 to give more reduction of the PAPR. The LCT transformed OFDM sequence $y(n)$ is achieved by applying equation (6) over the selected OFDM frame $\boldsymbol{x}_{\tilde{p},\tilde{u}}$.

At the receiver, inverse LCT is applied as in equation (7) and a side information of the value of $\tilde{p}$ and $\tilde{u}$ are also received at the receiver side over a secure channel. In order to be able to retrieve the data sequence by multiplying the output of the LCT element by element by the conjugate of the selected phase vector and then apply the deinterleaving process.

D. PAPR Performance of The Suggested Techniques

The CCDF of the PAPR performance versus the value of the PAPR values in dB of the suggested joint PAPR reduction techniques is presented in this subsection.

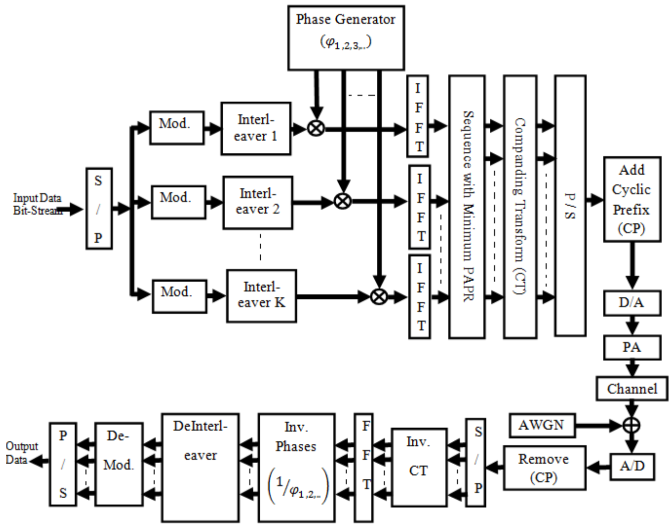

Figure 4. OFDM System with proposed technique to reduce PAPR

Fig. 5 compares the PAPR performance of joint RI and SLM technique, joint SLM and LCT technique, joint RI and LCT technique, and joint RI, SLM, and LCT technique. Improvements over the traditional OFDM by 3dB, 4dB, 4.5dB and 4.6dB respectively for a probability of 10^{-3}. While at CCDF probability of 10^{-4} the proposed technique improves 0.2 dB than joint RI and LCT technique. Fig. 6 (a) and (b) demonstrate the proposed technique and conventional OFDM system with QPSK and 16QAM modulators for N subcarriers 64,128 and 256. At probability of 10^{-3} the PAPR is reduced by 4.5dB over the conventional OFDM signal.

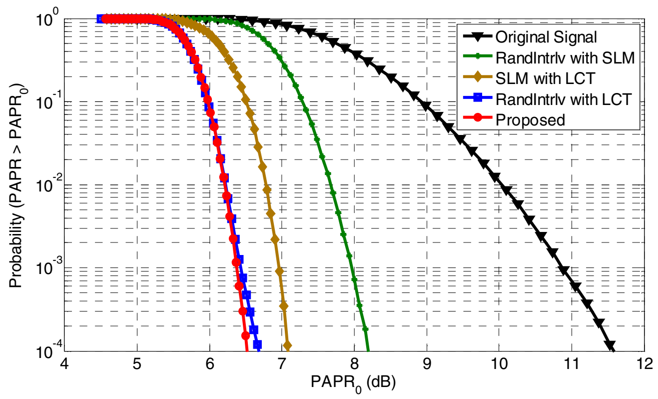

Figure 5. CCDF of PAPR with the proposed technique 16QAM, N=512.

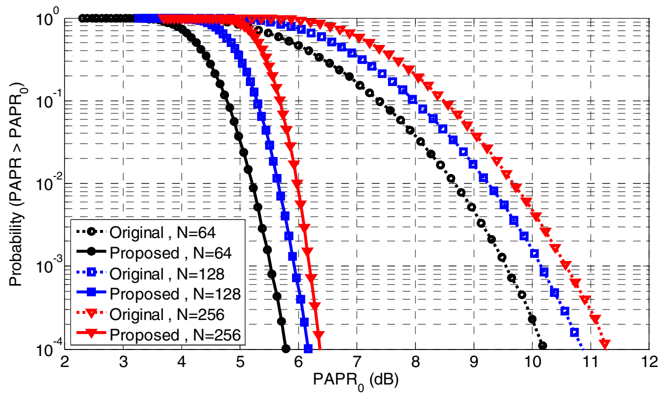

Figure 6.(a) CCDF of PAPR with the proposed technique applied on QPSK

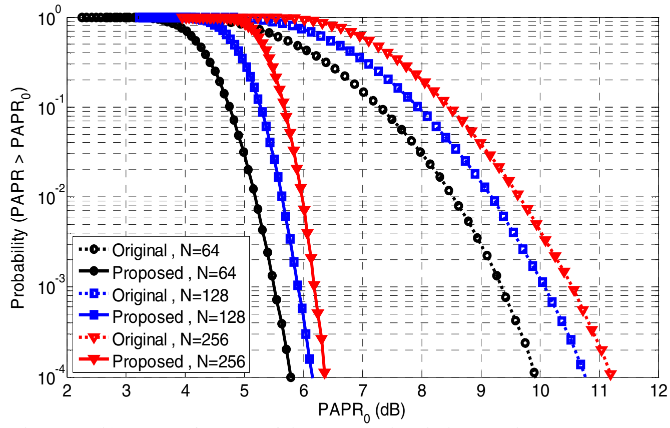

Figure 6. (b)CCDF of PAPR of the proposed technique and conv. OFDM both using 16QAM

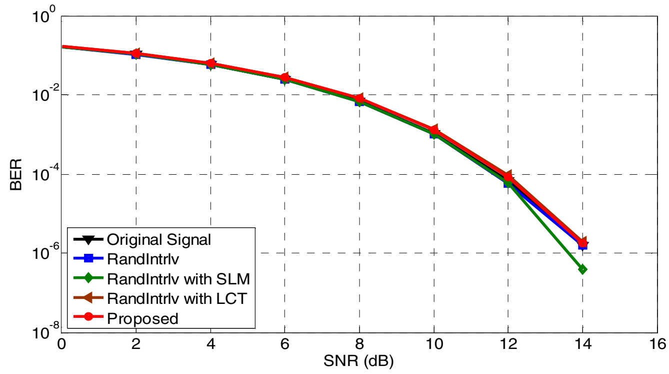

Figure 7(a). BER performance in an AWGN channel by using QPSK,*N*=256

IV. BER PERFORMANCE

The transmitted OFDM signal is applied to a power amplifier (PA) before its propagation over the channel. The parameters of the PA are as follows, a is the amplifier gain, A_{sat} is the saturation level and α is a positive number in range 2 to 3 to control nonlinearity characteristics of the PA. $\alpha = 2$ is assumed as in [12]. The saturation level is given by

$$A_{sat} = \sqrt{P_{in}.IBO}\,, \tag{9}$$

where P_{in} is the power of the PA input signal and IBO is the input power backoff. In this paper IBO =7dB is assumed. The PA output signal can be written as

$$y_A(t) = \frac{a|y(t)|}{\left[1+\left(\frac{|y(t)|}{A_{sat}}\right)^{2\alpha}\right]^{\frac{1}{2\alpha}}} e^{j\emptyset(t)} \tag{10}$$

A. BER Over AWGN Channel

In Fig. 7a and 7b, the BER curves are shown for QPSK modulation and number of subcarriers of N = 256 and N= 512 respectively. It can be seen that the BER performances are almost the same. In Fig. 8a and 8b, the BER curves for 16QAM modulation are shown, with N = 256 and 512 subcarriers respectively the SNR required for the BER$= 10^{-4}$ is improved by 2.5dB for both N = 256 and N = 512, while in BER 10^{-5} the improved by 4.25dB and 4.5dB respectively.

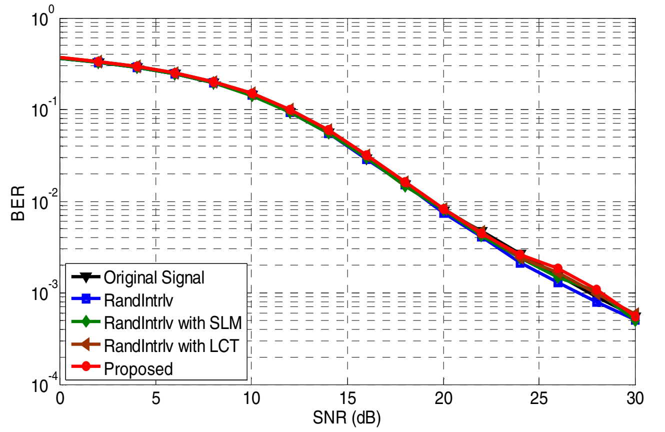

Figure 7(b) BER performance in an AWGN channel by using QPSK,*N*=512

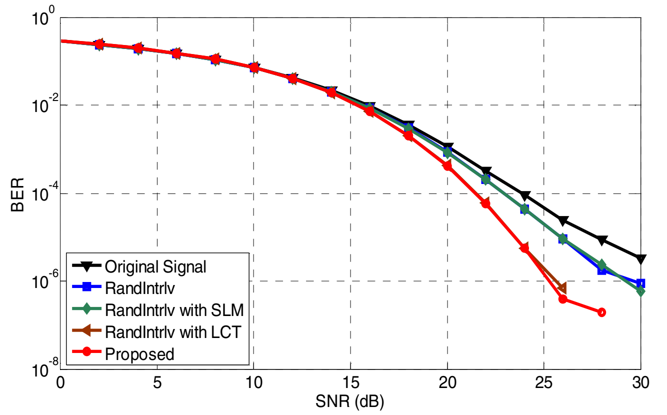

Figure 8(a). BER performance over AWGN channel using 16QAM, *N*=256.

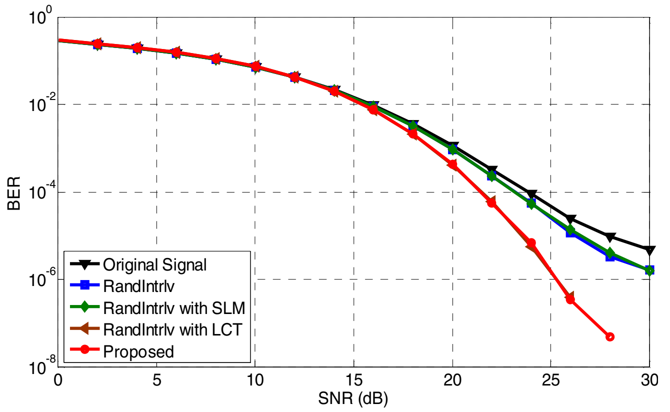

Figure 8(b). BER performance over AWGN channel using 16QAM, *N*=512

B. BER Over Fading Channel

Fig. 9 shows the BER curves over fading channel using QPSK modulation where N = 256 subcarriers the BER performance is almost the same between original signal without PAPR reduction and proposed technique PAPR reduction, which no significant improvement is achieved between original signal and proposed technique for the QPSK modulation.

Fig. 10 demonstrate the BER curves over fading channel using 16QAM modulation with *N*=256 subcarriers the result show that the SNR required for the BER=10^{-2} is improved by 0.5dB.

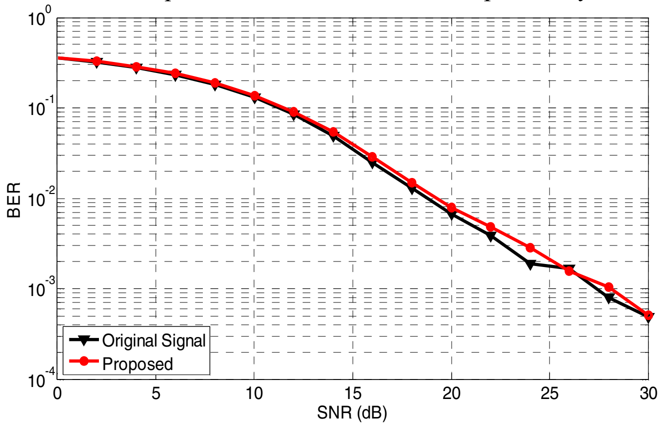

Figure 9. BER performance over fading channel using QPSK modulator and *N*=256.

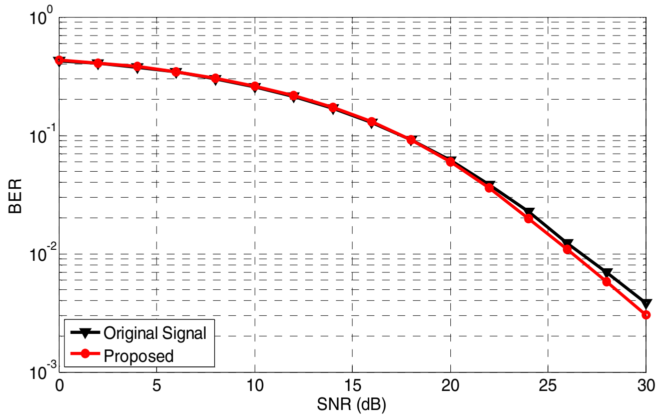

Figure 10. BER performance over fading channel for 16QAM and *N*=256.

V. CONCULSION

In this paper, several combined PAPR techniques were studied, then a joint PAPR technique to reduce the PAPR of the OFDM signal is proposed. This technique combines three basic PAPR reduction methods random interleaving, SLM and LCT. The proposed joint technique reduced the PAPR and decreased the BER in presence nonlinear power amplifier.

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