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Mixed-modulated linear frequency modulated radar-communications

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Abstract: Increasing congestion of the electromagnetic spectrum has spurred investigation into approaches that provide more efficient spectrum use. While many approaches focus on some type of orthogonality in time, frequency, or coding, another less explored option is mixed modulation of two complementary signals through intentional modulation of pulse. This study discusses intentional modulation of a linear frequency modulated (LFM) chirp radar pulse with a communications signal in order to conduct complementary activities with a combined signal. In order to reduce cross-interference between the signals, the communications message employs spread-spectrum encoding (binary M-sequences) and new modulation scheme with reduced phase change to modulate the LFM waveform. Preferred pair of M-sequences exhibit excellent auto-correlation and low cross-correlation properties that lend themselves to simultaneous transmission of channelised communication signals. The proposed approach is to modulate a radar signal with preferred pair M-sequences using binary reduced phase shift keying modulation that provides a low throughput, communications signal that could be used for administrative or navigation purposes. To measure effect on radar performance, comparison of the radar ambiguity function for the LFM signal is contrasted with combined radar-communication signal.

1 Introduction

The proliferation of commercial wireless devices and increasing bandwidth needs due to high speed data transfer requirements has created increasing congestion of the electromagnetic spectrum. As demand increases, efforts are being made to find more efficient uses of the available spectrum. Among these approaches are research on cognitive radio and dynamic spectrum access to make use of 'white space' of unused spectrum bands for secondary user's transmission [1]. With the evolution of digital signal processing, another promising approach is mixed modulation through intended modulation on pulse (IMOP) to combine radar and communication functions in one waveform [2]. In the past IMOP was challenging due to both the obvious cross interference concerns and the limits of signal processing technology [2]. However, advances in both digital electronics and signal processing have opened the door to re-exploring IMOP as a means to increase spectral efficiency.

In this paper, we propose to develop a novel joint mixed-modulated linear frequency modulated (LFM) waveform to serve radar and communication purposes simultaneously through IMOP. Previously, a LFM waveform has been adopted for communication purposes by employing ① binary phase shift keying (BPSK) and ② pseudo-noise spread spectrum technologies [3–8]. However the focus of this research was in the use of LFM waveform primarily as a communications means and not as a joint radar/communication waveform. Therefore, the deleterious effect on the ambiguity function (AF) and performance of the LFM pulse was not considered at all.

In our proposed approach, we employ a new modulation scheme with reduced phase magnitude to encode the communication messages into the LFM waveform while preserving the radar performance characteristics. This is accomplished by both using longer M-sequences and by selecting preferred pair M-sequences such that two message channels can be encoded into a single pulse. The contribution of this paper is to explore IMOP using spread spectrum encoding of signals with reduced magnitude phase shift keying in order to transmit both radar and communications information in a single radar pulse. The challenge is to find an IMOP modulation scheme that provides adequate signal efficiency and a satisfactory bit error rate while minimising radar performance degradation. In order to reduce the effects of the communications signal on the radar performance and improve message recovery, this paper will explore the use of binary M-sequences and modulation with relaxed phase differences (e.g. $\phi \ll (\pi/2)$). M-sequences exhibit high in-phase and low-out-of-phase auto-correlation properties and have lengths that can satisfy the required duration to ensure adequate bit energy at the receiver. In addition, as will be seen, the auto-correlation and cross-correlation properties of M-sequences offer the opportunity to encode communications in separate channels.

The rest of the paper is organised as follows: ① In Section 2 we review existing LFM chirp based communication schemes. ② Section 3 discusses the novel joint radar/communication LFM waveform via reduced phase shift keying modulation and spread spectrum technologies. ③ In Section 4, we further enhance the communication data rate by employing channelised transmission via preferred pair of pseudo-noise M-sequences. ④ In Section 5, we evaluate the influence of the new design on radar through AF. Conclusion follows.

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2 Existing LFM communication waveform design

2.1 LFM waveform

Since it is one of the more common radar waveforms, we consider a LFM signal as the baseline radar waveform. The basic LFM pulse linearly sweeps a frequency band of width B over a pulse width of T. As a result the instantaneous frequency $f(t)$ varies linearly with time: $f(t) = f_0 + kt$, where f_0 is the starting frequency, k is the rate of frequency increase or chirp rate, and $B = |k|T$. When k is larger than 0, the signal is an up-chirp; when k is less than 0, the signal is a down-chirp. Mathematically, the LFM chirp signal corresponds to:

LFM:

$$x(t) = A_c \cos\left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \phi_I\right), \quad 0 \le t \le T \tag{1}$$

$= A_c \cos[2\pi(f_0 t + \frac{k}{2} t^2) + \phi_1]$

where A_c is the amplitude of the LFM chirp signal, ϕ_I is the initial phase of the chirp signal, and T is the duration of the LFM chirp pulse. At time T, the stopping frequency of the LFM chirp signal is simply $f_1 = f_0 + kT$.

2.2 LFM based communication waveforms

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The inherent capability of interference rejection makes the chirp signal very attractive to communication, where a significant advantage is the low Doppler sensitivity. Multiple LFM chirp based communication waveforms have been developed over the years.

2.2.1 Chirp modulation and BPSK modulated LFM chirp: The first communication scheme employing LFM signals is called chirp modulation. Chirp modulation was patented by Sidney Darlington in 1954 [3] with significant later work performed by Winkler in 1962 [4]. The idea of chirp modulation is very simple: binary data is transmitted by mapping the bits into up-chirps and down-chirps. For instance, over one bit period '1' is assigned a chirp with positive rate k and '0' a chirp with negative rate $-k$.

In chirp modulation, only one bit is transmitted on one LFM chirp signal. To increase data rate and transmit multiple bits on one chirp, it is natural to use shorter symbol duration and modulate multiple information bits sequentially onto the chirp signal [5]. This modulation can be done by simply multiplying the baseband binary pulse amplitude modulation (2PAM) signal with the original LFM chirp signal.

Mathematically, the BPSK modulated LFM chirp signal is:

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$$\begin{aligned} s(t) &= a(t) \cdot x(t) = \sum_{i=0}^{N-1} b_i p(t - iT_b) \cdot x(t) \\ &= \sum_{i=0}^{N-1} b_i p(t - iT_b) \cdot A_c \cos\left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \phi_I\right) \end{aligned} \quad (2)$$

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where $a(t)$ is the baseband 2PAM signal containing N bits, b_i is the ith information bit and $b_i \in \{+1, -1\}$, $p(t)$ is a rectangular pulse with unit height and duration T_b, and since we are transmitting N bits over one LFM chirp waveform $T = NT_b$.

It is important to note that the multiplication of baseband 2PAM signal to the LFM chirp signal is equivalent to introducing a 0 or π phase offset to the unmodulated chirp signal at different data symbols:

b_i

$$s(t) = \sum_{i=0}^{N-1} p(t - iT_b) \cdot A_c \cos\left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \theta_i + \phi_I\right) \quad (3)$$

$b_i \Rightarrow \theta_i$

where $\theta_i = 0$ if $b_i = 1$, and $\theta_i = \pi$ if $b_i = -1$.

At the receiver side, a matched filter receiver can be exploited to obtain excellent BER performance for the BPSK modulated LFM chirp communication signal. At the receiver, received signal $r(t)$ is fed through a matched filter to produce a decision variable r_i, and a hard decision device makes a decision on the estimate of d_i:

$$\hat{d}_i = \begin{cases} 1 & \text{if } r_i > 0 \\ 0 & \text{else} \end{cases} \quad (4)$$

2.2.2 PN-chirp: To enhance the communication performance, pseudo-noise spreading was proposed to be combined with chirp modulated signal. This scheme was first proposed by Baier *et al.* [6] and Kowatsh *et al.* [7, 8] and termed pseudo-noise-chirp (PN-chirp). The idea is also quite straightforward: a pseudo-noise spreading code with M chips is used to represent one data symbol in a chirp modulated signal. This way, a processing gain M of the spread spectrum is exploited and offers significant performance gain to the chirp modulated communication system. In PN-chirp signal, the baseband signal is a direct-sequence spread spectrum signal:

$$y(t) = \sum_{i=0}^{P-1} b_i c(t - iT_b) \quad (5)$$

where $c(t)$ is the pseudo-noise spreading code with M chips with chip duration $T_c = T_b/M$,

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$$c(t) = \sum_{k=0}^{M-1} c_k p_c(t - kT_c) \quad (6)$$

c_k is the kth chip of the spreading code where $c_k \in \{+1, -1\}$, $p_c(t)$ is a rectangular pulse with unit height and duration T_c, and P is the number of bits carried by one LFM pulse.

The transmitted PN-chirp signal corresponds to:

$$\begin{aligned} s(t) &= y(t) \cdot x(t) \\ &= \sum_{i=0}^{P-1} b_i \sum_{k=0}^{M-1} c_k p_c(t - iT_b - kT_c) \cdot A_c \cos \\ &\quad \left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \phi_I\right) \\ &= \sum_{i=0}^{P-1} \sum_{k=0}^{M-1} p_c(t - iT_b - kT_c) \cdot A_c \cos \\ &\quad \left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \theta_{i,k} + \phi_I\right) \end{aligned} \quad (7)$$

where $\theta_{i,k} = 0$ if $b_i c_k = 1$, and $\theta_i = \pi$ if $b_i c_k = -1$.

3 Joint radar/communication LFM waveform design

All existing work on LFM communication waveform uses the LFM waveform primarily as a communications means, not as a joint radar/communication waveform. As a direct result, if such waveforms are used as a radar waveform, the radar performance is significantly degraded. As will be shown later in the numerical simulation results, existing LFM communication waveforms expand the bandwidth of original unmodulated LFM chirp signal and significantly distorts the AF of the radar pulse.

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3.1 Reduced phase shift keying modulated LFM chirp

We now propose to employ a novel modulation to solve this problem. The joint radar/communication signal is transmitted from the radar transmitter and reaches the intended communication receiver and reflects from objects the radar is trying to detect. The radar signal needs to complete a round-trip path to perform its function, and this creates a significant reduction in signal strength. Hence, most radar transmitters transmit signals at very high power compared with normal communication transmitters. On the other hand, the embedded communication signal only needs to reach its target through a one-way trip. Therefore, the received signal enjoys very high signal to noise ratio.

This enables us to adopt a different phase shift keying modulation suitable for our purposes in this scenario. Instead of using phases 0 and π in the signal constellation to represent our binary data like a regular BPSK, we use two constellation points with much smaller phase difference (Fig. 1*a*). Specifically, 星座 we use phase ϕ and phase $-\phi$ to represent the binary data. We call this new modulation scheme binary reduced phase shift keying (binary RPSK, or BRPSK).

The transmitted LFM chirp signal with embedded BRPSK modulation corresponds to:

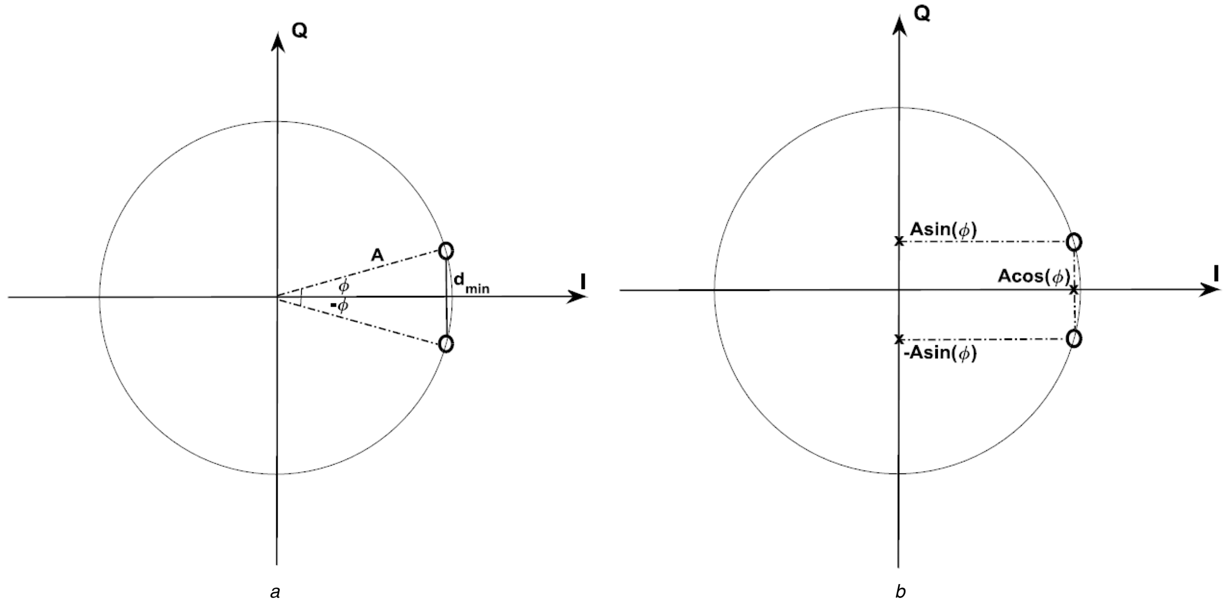

Fig. 1 *BRPSK modulation*
(a) BRPSK, ***(b)*** I-Q projection of BRPSK

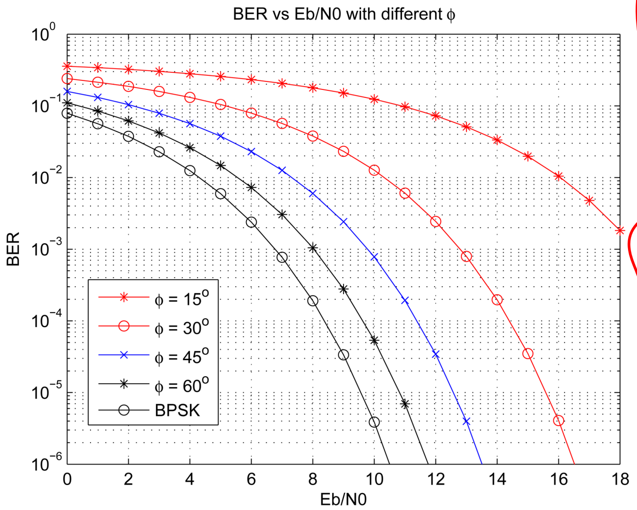

Fig. 2 *BER of BRPSK* 误码率

$$
\begin{aligned}
s(t) &= \mathrm{Re}\left[\sum_{i=0}^{N-1} p(t - iT_b)\tilde{x}(t)\cdot \mathrm{e}^{\mathrm{j}\theta_i}\right] \\
&= \mathrm{Re}\left[\sum_{i=0}^{N-1} p(t - iT_b)A_c \mathrm{e}^{\mathrm{j}2\pi f_0 t}\cdot \mathrm{e}^{\mathrm{j}2\pi(k/2)t^2}\cdot \mathrm{e}^{\mathrm{j}\theta_i}\cdot \mathrm{e}^{\mathrm{j}\phi_I}\right] \\
&= \sum_{i=0}^{N-1} p(t - iT_b)\cdot A_c\cos\left(2\pi f_0 t + 2\pi\frac{k}{2}t^2 + \theta_i + \phi_I\right) \\
&= \sum_{i=0}^{N-1} p(t - iT_b)\cdot A_c\cos\left(2\pi f_0 t + 2\pi\frac{k}{2}t^2 + b_i\cdot\phi + \phi_I\right)
\end{aligned} \tag{8}
$$

where $\tilde{x}(t) = \mathrm{e}^{\mathrm{j}(2\pi f_0 t + 2\pi(k/2)t^2 + \phi_I)}$ is the complex envelope of the LFM chirp waveform.

$b_i \in \{1, -1\}$

Note that the phase offset being introduced to the ith data symbol now is $\theta_i = b_i\cdot\phi$, hence if $b_i = 1$, phase ϕ is introduced, and if $b_i = -1$, phase $-\phi$ is introduced. Evidently, the conventional BPSK can be considered as a special case of the BRPSK with $\phi = 90^\circ$ (with a constant phase offset of $\pi/2$).

Because the reduced phase shift keying modulation uses smaller phase difference, the constellations points have a much smaller distance (d_{min}). As a direct result, this new modulation scheme has poorer BER performance compared with the original BPSK. The BER of original BPSK in AWGN channel is determined by:

BPSK:
$$
Q\left(\frac{d_{min}/2}{\sigma}\right) = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{9}
$$

where $A = \sqrt{E_b}$, $N_0/2$ is the power spectral density (PSD) of the additive white Gaussian noise, and E_b is the bit energy.

On the other hand, the BER of BRPSK is

BRPSK:
$$
Q\left(\frac{d_{min}/2}{\sigma}\right) = Q\left(\frac{A\sin(\phi)}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b\sin^2(\phi)}{N_0}}\right) \tag{10}
$$

Comparing (9) and (10), it is clear that the $\sin^2(\phi)$ term significantly reduces the BER performance of BRPSK. For example, when $\phi = 15^\circ$, $\sin^2(\phi) = 0.0670$ which corresponds to -11.74 dB. Hence, the BER performance of such a binary reduced PSK requires 11.74 dB in additional SNR to achieve the same BER performance of original BPSK. However, because of the very high SNR of the linear chirp radar signal, this performance loss is totally tolerable. Fig. 2 shows the BER versus E_b/N_0 curves of binary reduced PSK modulations with different phase ϕ. As shown in Fig. 2, there is an 11.74 dB difference in the BER performance of a binary reduced PSK with $\phi = 15^\circ$ than that of a conventional BPSK (where $\phi = 90^\circ$).

It is important and interesting to note that when we project the BRPSK constellation to its in-phase component and quadrature component (Fig. 1*b*), it is evident that only the quadrature component contains the digital data. Hence, the optimum receiver can be implemented easily by a matched filter only considering the quadrature component of the received signal.

3.2 Spread spectrum LFM chirp with BRPSK modulation

While the reduced PSK modulated LFM chirp has some performance loss, this loss can be easily compensated by coupling with pseudo-noise code spreading spectrum as well. For example, a pseudo-noise code with code length M larger than 15 (11.76 dB) will be sufficient to compensate for the performance loss of reduced PSK modulation with ϕ of 15°.

10lg15=11.76dB

Mathematically, the transmitted spread spectrum LFM chirp signal with BRPSK modulation is:

$$
\begin{aligned}
s(t) &= \sum_{i=0}^{P-1}\sum_{k=0}^{M-1} p_c(t - iT_b - kT_c)\cdot A_c\cos\left(2\pi f_0 t + 2\pi\frac{k}{2}t^2 + \theta_{i,k} + \phi_I\right)\\
&= \sum_{i=0}^{P-1}\sum_{k=0}^{M-1} p_c(t - iT_b - kT_c)\cdot A_c\cos\left(2\pi f_0 t + 2\pi\frac{k}{2}t^2 + b_i c_k \phi + \phi_I\right)
\end{aligned}
\tag{11}
$$

Compared with (7), it is evident that the only difference between the spread spectrum LFM chirp signal with BRPSK modulation and the PN-chirp signal is that $\theta_{i,k}$ is no longer 0 or π but ϕ or $-\phi$.

Similarly, the optimum receiver for spread spectrum BRPSK modulated LFM chirp signal can be easily implemented by a matched filter only considering the quadrature component of the received signal.

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3.3 Symbol duration determination

With the required E_b/N_0 value established, the required symbol duration T_b can be determined. To maximise the data rate, it is desirable to use the smallest symbol duration. The LFM signal is a constant envelope waveform hence the energy in one bit can be expressed as

$$E_b = \frac{V^2}{2}T_b = P_R T_b \tag{12}$$

where V is the magnitude of the received LFM waveform at the designated communication receiver, and P_R is the bit power at the receiver relative to a 1 Ω resistance. Therefore, the BER of the proposed system with BRPSK corresponds to:

$$Q\left(\sqrt{\frac{2E_b \sin^2(\phi)}{N_0}}\right) = Q\left(\sqrt{\frac{2P_R T_b \sin^2(\phi)}{N_0}}\right) \tag{13}$$

Since in most radar systems transmitting LFM waveforms, the practice is to drive the power amplifier into the saturation regime and keep the transmitted power constant, we can use the phase term *phi* and the bit duration T_b to control the desired BER.

To achieve enough bit energy at the receiver, the symbol duration must extend over a set period of time. Therefore, the energy expression for a single *transmitted* bit can be rewritten as

$$E_T = P_T T_b \tag{14}$$

The effect of distance on the received energy must also be considered since radar energy falls of as a square of the distance. Unlike the radar pulse, the communication message is received and processed at the receiver so the one-way radar range equation, or Friis equation, can be used:

$$E_R = \frac{E_T G_T A_e}{4\pi r^2 L} \tag{15}$$

where E_R is the received energy, G_T is the transmitter aperture gain, r is the radar to communication receiver distance, and L represents other losses. Further, the effective antenna aperture can be expressed as

$$A_e = \frac{G_R \lambda^2}{4\pi} \tag{16}$$

where G_R is the receiver aperture gain and λ is the signal wavelength. Incorporating both A_e and (14) for the bit energy, E_T, into the one way range equation allows rewriting of the bit energy, E_b, at the receiver as

Ep ET

$$E_b(T_b, r) = \frac{P_T T_b G_T G_R \lambda^2}{(4\pi r)^2 L} = \left[\frac{P_T G_T G_R}{(4\pi)^2 L}\right]\left(\frac{T_b}{r^2}\right) \tag{17}$$

The first bracketed term in the equation is determined by the performance characteristics of the radar transmitter while the second parenthetical term determines the energy of the communication message pulse at the receiver as a function of symbol duration and range.

Message bit detection and probability of misidentifying a message bit depends on signal-to-noise ratio. For message recovery the bit energy to noise PSD is crucial in ensuring an acceptable BER. For thermal noise, the noise PSD, N_0, can be expressed as

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$$N_0 = \kappa T_0 F \tag{18}$$

where F is the receiver noise factor, κ is Boltzmanns constant ($1.3806488 \times 10^{-23}$ m^{2}kgs^{-2}K^{-1}) and T_0 is the antenna aperture noise temperature in degrees Kelvin (normally assumed to be 290°).

Rewriting the energy equation in terms of required bit SNR and sample length:

$$\frac{E_b}{N_0} = \frac{P_T T_b G_T G_R \lambda^2}{(4\pi r)^2 L \kappa T_0 F} \geq [\mathrm{SNR}]_{\mathrm{req}} \tag{19}$$

$$T_b \geq \frac{[\mathrm{SNR}]_{\mathrm{req}}(4\pi r)^2 L \kappa T_0 F}{P_T G_T G_R \lambda^2} \tag{20}$$

根据[SNR]req
得出 Tb,min

This constraint provides a lower limit on the symbol duration required to achieve a bit energy level that ensures an adequate BER at the receiver.

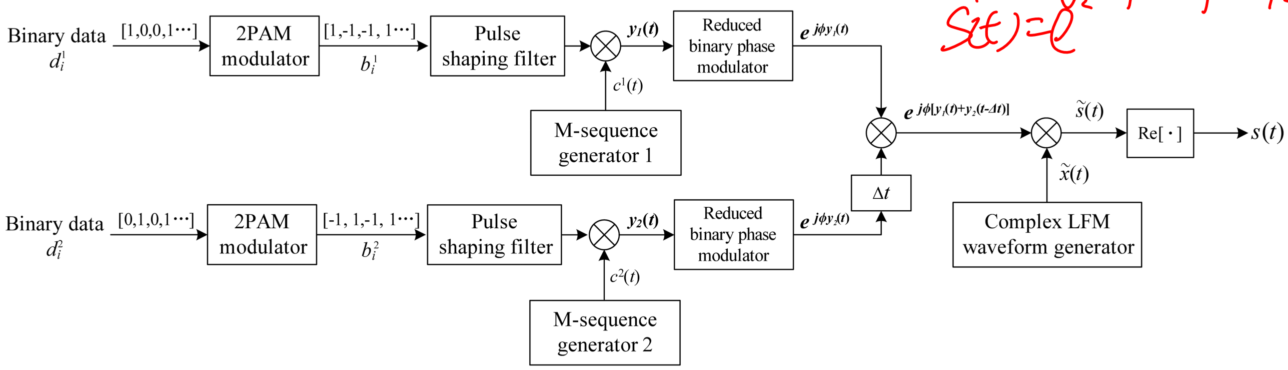
4 Channelised LFM chirp with preferred pair of M-sequences

So far we have only modulated one data stream onto the LFM chirp waveform. To increase the data rate, or to allow multiple access, we now propose to modulate two data streams simultaneously onto the LFM chirp. This is achieved by employing a preferred pair of M-sequences. 同时调制2个数据流

M-sequences are pseudorandom binary sequences that can be produced by the use of binary linear feedback shift register (LFSR) systems using modulo-2 arithmetic and an *xor* function for addition. More conventionally, M-sequences are produced using primitive polynomial functions of degree m with a binary-valued, (0, 1), seed of the same length [9]. The primitive polynomial (or LFSR implementation) generates a repeating binary sequence of length $2^m - 1$ [10].

The primary benefit of M-sequences for the mixed modulated waveform lies in their length and correlation properties. Individual M-sequences exhibit high auto-correlation values while minimising out-of-phase (e.g. $\tau \neq 0$) auto-correlation values. Further, 'preferred pairs' of M-sequences exhibit excellent cross-correlation between the paired bit sequences. Since the preferred M-sequences pairs are selected so as to minimise cross-correlation between the overlapping bit sequence, a channelised approach can be used to simultaneously transmit two bit sequences. Fig. 3 shows the transmitter of such a channelised system.

In such a channelised system, two distinct digital messages are modulated onto the LFM chirp waveform. The first data stream d_n^1 is converted into a 2PAM data stream, then multiplied with the first M-sequence $c^1(t)$. The resulting signal $y_1(t)$ is then used to create the phase modulation signal $\mathrm{e}^{\mathrm{j}\phi y_1(t)}$. The second data stream goes through a similar process to create another phase modulation signal $\mathrm{e}^{\mathrm{j}\phi y_1(t-\Delta T)}$. Note that these two data streams are not necessarily synchronised, hence there might exist a time offset $\Delta T = QT_c$ between the two data streams. Next, the two phase modulation signals are multiplied together to create the complex baseband



$\tilde{S}(t)=e^{j[2\pi f_0 t+\pi\mu t^2+\phi[y_1(t)+y_2(t-\Delta\tau)]]}$

Fig. 3 *Channelised LFM chirp with 2 bit streams*

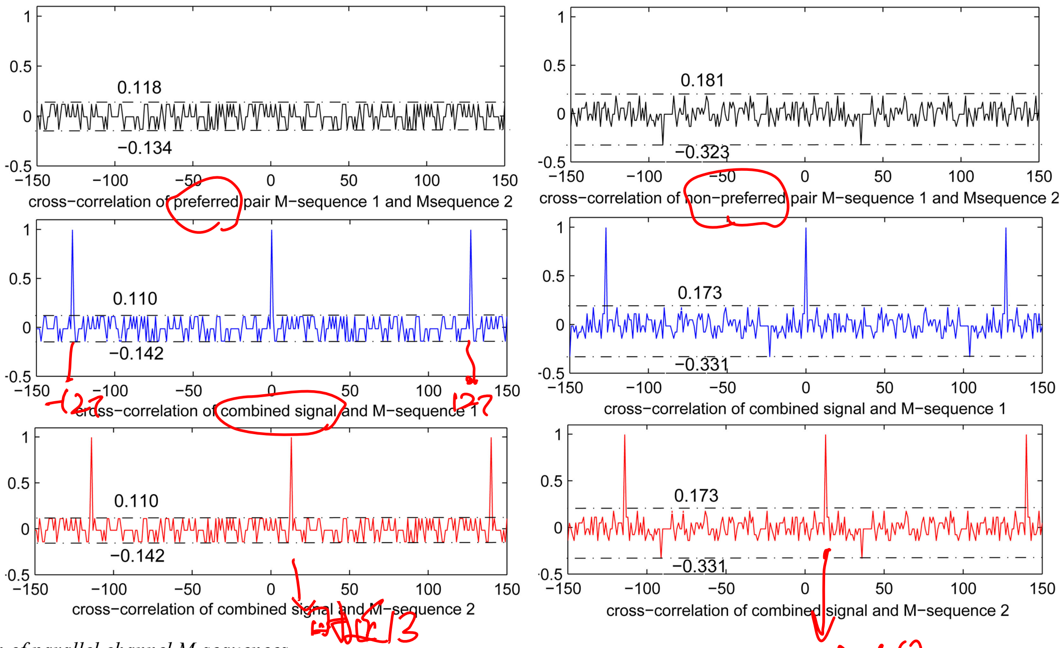


Fig. 4 *Correlation of parallel channel M-sequences*

(a) Cross-correlation of preferred pair M-sequences, *(b)* Cross-correlation of non-preferred pair M-sequences

signal $e^{j\phi(y_1(t)+y_2(t-\Delta T))}$ to be multiplied with the complex envelope of LFM waveform. Mathematically, the transmitted signal is

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$$
\begin{aligned}
s(t) &= \mathrm{Re}\left[e^{j\phi(y_1(t)+y_2(t-\Delta T))}\tilde{x}(t)\right] \\
&= \sum_{i=0}^{P-1}\sum_{k=0}^{M-1} p_c(t-iT_b-kT_c)\cdot A_c\cos \\
&\quad \left(2\pi f_0 t + 2\pi\frac{k}{2}t^2 + (b_i^1c_k^1 + b_i^2c_{k-Q}^2)\phi + \phi_I\right)
\end{aligned} \tag{21}
$$

M=127

where b_i^1 is the ith information bit of data stream 1, c_k^1 is the kth chip of the first M-sequence $c^1(t)$, and b_i^2 corresponds to the second data stream, while c_k^2 is the kth chip of the second M-sequence $c^2(t)$.

To illustrate the ability to recover each channel from the combined message vector, two M-sequences of period 2^7-1 were generated using primitive polynomials of x^7+x+1 and x^7+x^3+1 with common initial seeds of [00000001]. These two M-sequences are a preferred pair with minimised cross-correlation. The resulting M-sequence basic vectors have length of 127 chips. Random data bits are encoded using each M-sequence with a time delay of 13 chips between the channels to ensure the matched filter is actually de-correlating the correct message bit. The two M-sequences are added and a matched filter for each M-sequences is used to process the combined signal. The result of the normalised cross-correlation of the two M-sequence basis vectors, as well as the normalised cross-correlation of each M-sequence filter with the message vector are shown in Fig. 4*a*. As can be seen the normalised cross-correlation between the two messages is low as compared with the in-phase correlation result of the lower two plots. Also clearly shown is the time delay between the two data streams.

As a comparison, we show in Fig. 4*b* the same normalised cross-correlation result of two M-sequences which are not preferred pair. Specifically, polynomials of $x^7+x^6+x^3+x+1$ and x^7+x^3+1 are used to generate two M-sequences. The same time delay of 13 chips are introduced between the two data streams. It is clear that the non-preferred pair of M-sequences exhibit higher sidelobes in the normalised cross-correlations, leading to higher interference and poorer performances.

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5 Performance of radar waveform

The underlying assumption in integrating an in-band message into a radar pulse is that the radar performance will not be significantly impacted by the addition of short duration phase discrete into the LFM signal. Two possible metrics of performance are the signal autocorrelation function and the PSD plot.

The autocorrelation plots are useful to determine how the radar receiver's matched filter processes the received signal. Normally the received signal is correlated with a copy of the transmitted signal to determine the maximum position which, in turn, corresponds to the position of the target in the returned pulse. The PSD provides a measure of how much energy is outside the main bandwidth of the pulse. For a LFM signal, which is the case for this paper, the majority of the spectral energy is in the bandwidth of the LFM signal. However, adding short duration phase discretes causes energy to occur outside the main lobe of the PSD. Since energy must be conserved, these sidelobes represent energy that is not

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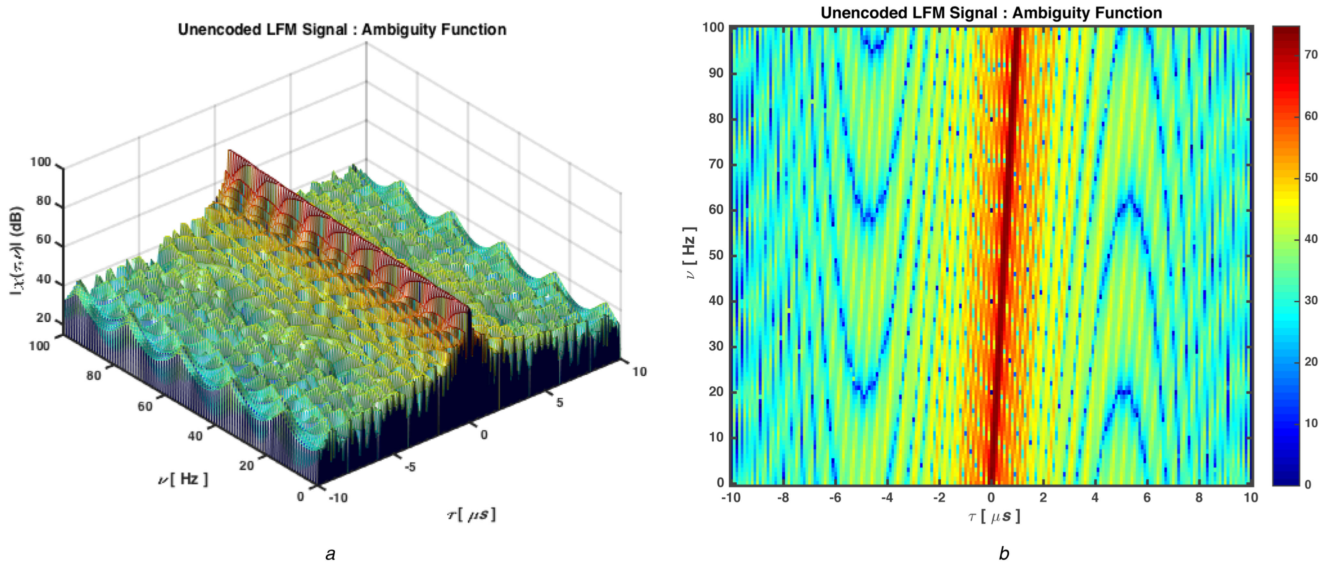


Fig. 5 *AF of unmodulated LFM chirp signal*
(a) LFM AF plot, ***(b)*** LFM AF plot – top view

available in the main lobe for radar processing. However, a more complete metric that encompasses the autocorrelation and the PSD is the radar AF. 本质上

The AF is essentially the Fourier transform of the cross-correlation of the radar pulse with its matched filter. The radar AF provides a measure of the response of a matched filter to a finite energy signal in the presence of a time delay, τ, and Doppler frequency shift, ν. The periodic AF is well known in the literature and can be expressed as [11]: $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$$|\chi(\tau,\nu)| = \left| \int_0^T u(t)u^*(t-\tau)e^{2\pi j\nu t}\,dt \right| \quad (22)$$

where the variables τ and ν represent the time delay and Doppler-shifted frequency of the returned signal, respectively.

The ideal (and hypothetical) AF has a ‘thumbtack’ appearance at the origin (e.g. $\tau = 0$ and $\nu = 0$) for a Dirac delta impulse signal. In this situation the radar can provide accurate target position as long as there is no delay or Doppler frequency shift. More realistically, radars need to have some tolerance to time delays and Doppler shifts so the ideal ‘thumbtack’ response is not necessarily a desired matched filter response.

To illustrate the effectiveness of the proposed scheme, numerical simulations have been performed using a LFM chirp pulse with the following parameters (Table 1).

It is important to note that to implement the proposed system, the sampling rate is much higher than the Nyquist sampling rate for the radar waveform. The pulse width and sample rate provide a digitised signal of 5001 samples that can be used to encode a message sequence. The plots that follow illustrate the autocorrelation, PSD and AF response, respectively, for the basic unmodulated LFM pulse. These provide a baseline for comparison with the LFM chirp pulses that have embedded communication messages.

For the modulated LFM chirp pulse, the carrier is assumed to be a C-band radar with a 10 kW peak power operating at 5 GHz. The overall gain of the transmitter antenna and receiver is assumed to be 2000 with 50 dB of attenuation due to transmitter, receiver, measurement, and processing losses. In addition, a BER of 10^{-5}

Table 1 LFM pulse parameters

Pulse width (PW)	10 μs
Bandwidth (BW)	5 MHz
Center frequency (f_C)	2.5 MHz
Sampling frequency (f_S)	5×10^8 Hz
Signal gain, A	1

was desired with a maximum operating range of 500 km (3̃00NM). A series of numerical simulations were done using (20) to determine the minimum symbol duration for various phase discrete values from ±15° through a traditional BPSK of ±90°. As expected the most stressing was for ±15° of phase shift with a minimum sample length of 45 samples. At the other extreme, the use of ±45° or ±90° phase shifts required significantly less samples to achieve the required BER – on the order of 1.

Fig. 5 is the AF for the unmodulated LFM signal. The plot illustrates good accuracy for determining target position in time but allows some flexibility for frequency error due to Doppler effects. As a comparison, Fig. 6 shows the AF of a LFM signal with an embedded BPSK modulated communication message (using phase changes of ±90°) to encode a ‘1’ or ‘0’. As Fig. 6 clearly shows, while time delay remains the same, Doppler tolerance has greatly diminished, and the sidelobes are higher for most of the (range, Doppler) domain.

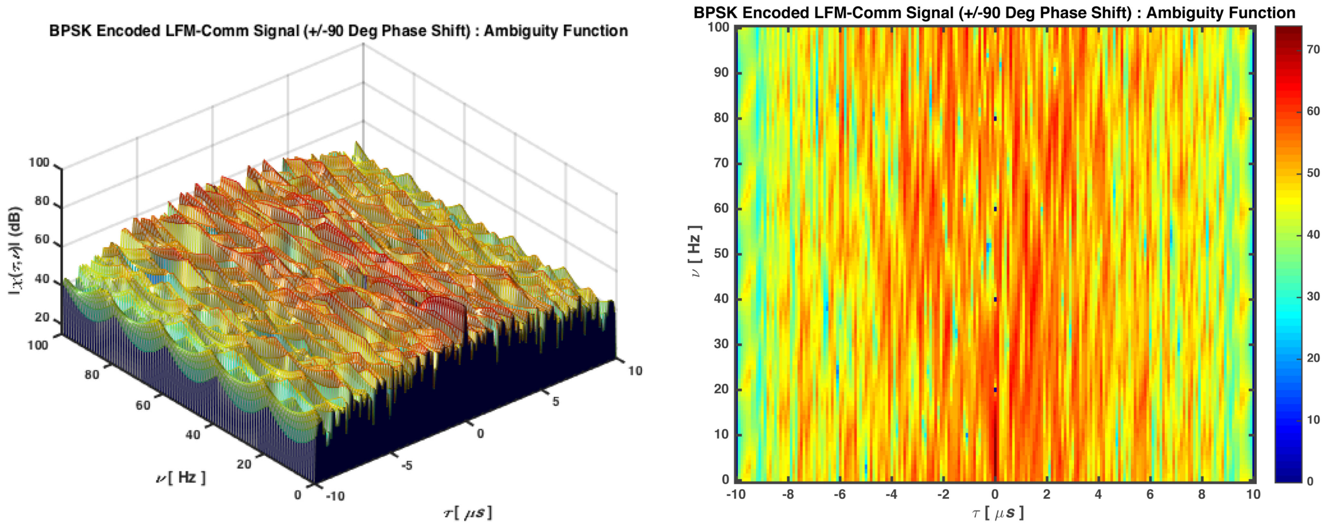
The effect of incorporating phase discretes into the LFM pulse is equivalent to adding time varying Doppler effects to the initial LFM signal. In essence, each bit sequence is a series of angular phase shifts which correspond to changes in the time domain.

Fig. 7*a* depicts the PSD and autocorrelation of the same mixed LFM-Comm signal employing ±90° constant phase shift for message encoding. The PSD plot clearly shows the increased bandwidth caused by the use of large phase angle changes. Further, the autocorrelation plot exhibits fairly high sidelobes beyond the first sidelobe.

The combination of the poor AF performance (when compared with the baseline LFM ambiguity plot) and the wider bandwidth of the PSD may not be acceptable radar performance. For these reasons we have explored the use of new reduced PSK modulated LFM chirp waveforms. Fig. 7*b* provides a view of the autocorrelation (i.e. zero-Doppler cut of the AF) and pulse PSD. In the upper figure the PSD of the M-sequence encoded pulse is in blue while the PSD of the unencoded, basic LFM pulse is shown in red. As can be seen while additional energy is distributed in the sidelobes the autocorrelation of the communication pulse is consistent with what one would expect for an LFM matched filter with a high in-band ($\tau = 0$) autocorrelation value and low out of band correlation.

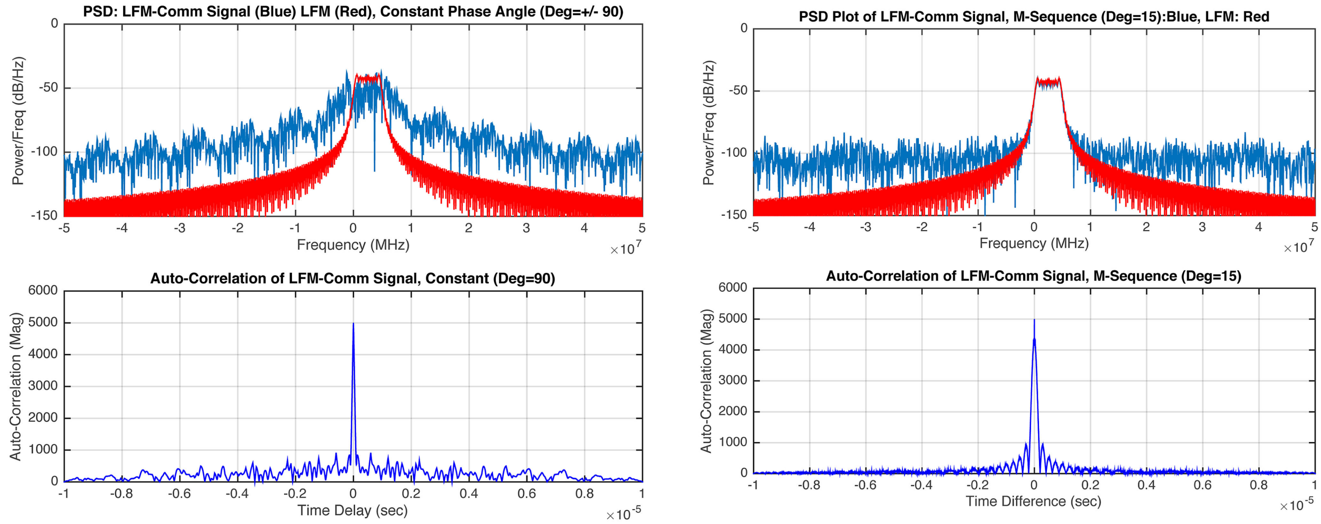
The following plots show the case of our proposed BRPSK modulated LFM chirp waveform with channelised M-sequences with $\phi = 15^\circ$. To ensure adequate BER for unforeseen errors at low phase angles, a 6-bit preferred primitive polynomial was chosen to provide a bit-sample length of 63. As previously mentioned, the radar pulse in this example consists of 5001 samples. Based on the bit sequence length 79 bits can be encoded for each M-sequence, thus providing 158 bits of information per pulse.

$5001 \div 63 \approx 79.38 \Rightarrow 79 \times 2 = 158$
$63 \times 79 = 4977$
$4977 + 13 = 4990$



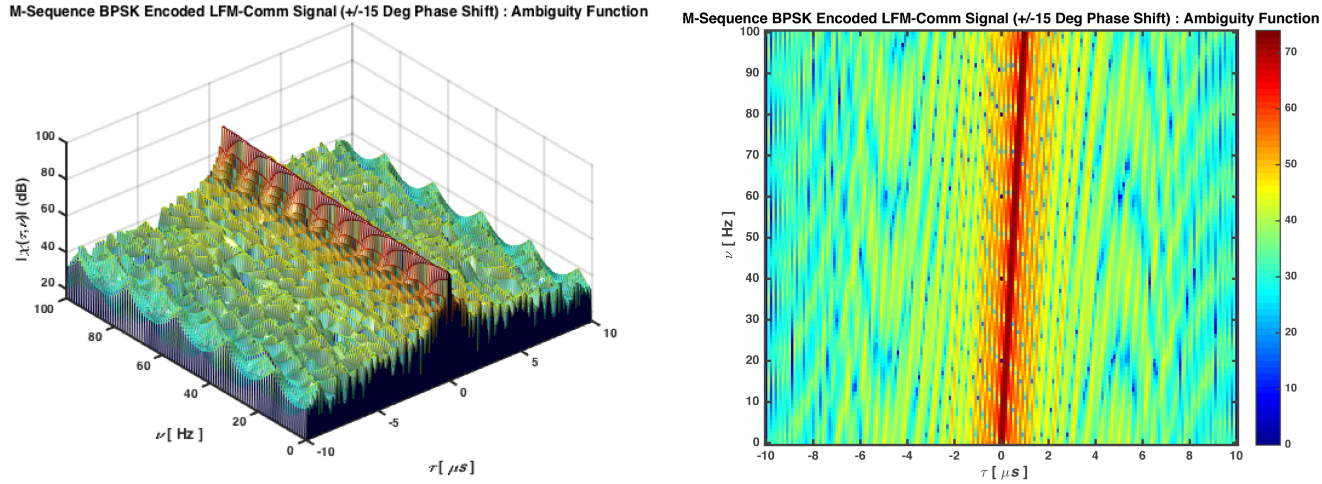
a *b*

Fig. 6 *AF of BPSK modulated LFM chirp signal: ±90° phase shift*
(a) BPSK modulated LFM AF plot, ***(b)*** BPSK modulated LFM AF plot – top view



a *b*

Fig. 7 *PSD/autocorrelation comparison of LFM chirp (red), BPSK modulated LFM chirp, and BRPSK modulated LFM chirp*
(a) LFM chirp vs BPSK modulated LFM chirp ($\phi = 90$), ***(b)*** LFM chirp vs BRPSK modulated LFM chirp ($\phi = 15$)



a *b*

Fig. 8 *AF of reduced BPSK modulated LFM chirp signal with two bit streams ($\phi = 15^{\circ}$)*
(a) BRPSK modulated LFM AF plot, ***(b)*** BRPSK modulated LFM AF plot – top view

The effect of using BRPSK modulation can be clearly seen in the radar ambiguity plots for channelised LFM chirp signal of two bit streams shown in Fig. 8. In the plot the use of smaller phase difference preserves the Doppler tolerance of the LFM radar pulse.

6 Conclusion

In this paper, we have developed a mixed-modulated LFM chirp waveform to serve radar and communication functions simultaneously. Specifically, we have proposed a novel modulation scheme called reduced binary phase shift keying (RBPSK). Exploiting the RBPSK modulation and spread spectrum, a reliable communication message is embedded inside the radar pulse without sacrificing the radar performances. Furthermore, using a preferred pair of M-sequences, a channelised transmission with two data streams is developed to enhance the data rate of the communication.

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