Effect of Clutter on Joint Radar-Communications System Performance Inner Bounds

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***Abstract*—We analyze the effects of clutter on inner bounds for the performance of a joint radar-communications system. Radar returns from clutter are often characterized by a randomly fluctuating cross section. Hence, statistical methods must be employed to model the clutter and its cross-section. In this paper we consider two clutter models and analyze their effect on the inner bounds on performance of a joint radar-communications system. Bounds on performance of the joint system are measured in terms of data information rate for communications, and radar estimation information rate for the radar.**

***Index Terms*—Radar-Communications Co-existence, Clutter, SSPARC**

I. Introduction

Clutter is a term used to describe radar returns from scatterers that are present in the environment but not considered targets, like vegetation, trees, buildings etc. Clutter causes significant performance degradation for radar systems and as a result, many techniques have been developed to mitigate or eliminate its effect on radar performance. In this paper, we conduct a preliminary analysis on the effect of clutter on the inner bounds on performance of a joint radar-communications system designed for radar-communications co-existence. More specifically, we explore the effect of clutter on fundamental radar and communications coexistence performance bounds introduced in [1].

A. Background

It is worth noting that in our efforts presented here, we focus on the radar estimation performance rather than radar detection considered in [2]–[6]. More specifically, the work presented here is focused on the estimation of a dynamic target parameter, time delay or target range, from the received target return. The performance of the radar system is measured in terms of the estimation rate.

The work presented in [2], [3] investigated the application of information theory to improve radar system performance developing the *a posteriori* radar receiver that maximizes the quantity of information (instead of output SNR). In [4], it is shown that radar performance can be improved by designing radar waveforms that maximize the mutual information between the target parameter and the received measurements.

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The performance gains of applying space-time adaptive processing techniques in radar systems suffering from clutter interference have been discussed in [7], [8]. In this paper, we opt to apply simplified processing techniques instead of the more complex space-time adaptive processing techniques to deal with clutter interference so as to better understand the effects of clutter on joint radar-communications system performance.

B. Contributions

We consider two simple models for clutter in order to study its effects clutter on joint radar-communications system performance. In the first model, we assume that each clutter scatterer is static and slowly-fluctuating. In the second model, we assume that each clutter scatterer is static but has some small internal clutter motion (ICM) [8], which induces a narrow doppler spread in the clutter power spectrum. We eliminate clutter returns by employing simple radar processing techniques and evaluate the inner bounds on joint radar-communications system performance.

In this paper, we assume the joint radar-communications receiver can simultaneously perform radar target parameter estimation and decode a communications signal. The inner bounds on performance are found by considering this idealized receiver in different scenarios and deriving bounds for the estimation and data rates for the radar and communications systems respectively.

This paper is organized as follows. In Section II, we present the two models used to characterize clutter. In Section III, we introduce the idea of a received signal with a predicted radar return signal suppressed and also introduce the radar estimation information rate, a metric used to measure the performance of the radar system. In Section IV, we evaluate several inner bounds on the performance of the joint radar-communications system. Finally, in Section V, we present a summary of results obtained in this paper.

II. Clutter Models

In this section, we will present the two models we will use to describe clutter and the techniques used to eliminate clutter under each model. We assume that all clutter scatterers are resolvable. We first look at clutter in each range cell (post-matched filtering for radar) and then calculate the total pre-matched filtering clutter return observed by the joint radar-communications system receiver. As mentioned earlier, the clutter elimination methods presented here are not standard clutter elimination techniques, but rather simplified methods employed so we can better study the effects of clutter on joint radar-communications system performance. We present a table of significant notation that will be employed in this paper in Table I.

TABLE I
SURVEY OF NOTATION

Variable	Description
$\langle\cdot\rangle$	Expectation
$\|\cdot\|$	L2-norm or absolute value
B	Full bandwidth of the system
B_{rms}	Root-mean-squared radar bandwidth
$x(t)$	Unit-variance transmitted radar signal
P_{rad}	Radar power
τ_m	Time delay to m^{th} scatterer
$\tau_m^{(k)}$	k^{th} observation of time delay for m^{th} scatterer
a_m	Complex combined antenna, cross-section, and propagation gain for m^{th} scatterer
T	Radar pulse duration
δ	Radar duty factor
γ	Radar spectral shape parameter
σ^2_{resi}	Clutter Residual Power
$r(t)$	Unit-variance transmitted communications signal
P_{com}	Total communications power
b	Combined antenna gain and communications propagation loss (amplitude)
σ^2_{noise}	Thermal noise power
k_B	Boltzmann constant
T_{temp}	Absolute temperature
$\sigma^2_{\text{int+n+resi}}$	Interference plus thermal noise plus clutter residual for communications receiver
$\sigma^2_{\tau,\text{proc}}$	Variance of range fluctuation process
B_{com}	Communications only sub-band
B_{mix}	Mixed radar and communications sub-band

A. Static, Slowly Fluctuating Clutter

In this model, we assume the clutter is static but slowly fluctuating such that the radar return over N_p radar pulses is highly correlated (approximately constant). We assume that the clutter return is constant over the same N_p radar pulses for each scatterer, but there can be other models where the clutter return is constant for a different number of pulses for each scatterer. For L resolvable clutter scatterers, the radar return, $z(t)$, is given by

$$z(t) = \sqrt{P_{\text{rad}}}\sum_{m=1}^{L} a_{\text{m}}\, x(t-\tau_{\text{m}}) + n(t), \tag{1}$$

where $n(t)$ is the zero-mean thermal noise drawn from a complex Gaussian distribution with variance $\sigma^2_{\text{noise}} = k_B\, T_{\text{temp}}\, B$.

With these assumptions, we perform matched filtering on the received signal and resolve each scatterer into a separate range cell. We perform maximum likelihood estimation for the clutter return amplitude across each range cell, and use the estimate to eliminate the clutter return. Since the clutter return for each range cell is constant across a certain number of pulses, we can perform amplitude estimation over multiple pulses, moving the amplitude estimate closer to the true value. Eliminating clutter with this highly accurate estimate will leave behind only a tiny residual of clutter in each range cell. This residual can be interpreted as the minimum error between the true value of amplitude and its estimate. As seen in [9], this minimum error (residual) is a zero mean complex Gaussian, $n_{\text{resi}}(t)$, with variance given by the the Cramér Rao lower bound for amplitude estimation [10] which, for noise power $\sigma^2_{\text{noise}} = k_B\, T_{\text{temp}}\, B$, is given by,

$$\sigma^2_{\text{resi}} = \frac{k_B\, T_{\text{temp}}\, B}{n_s} = \frac{k_B\, T_{\text{temp}}\, B}{N_p T B}, \tag{2}$$

where n_s is the total number of independent samples in the period of integration.

Thus, after clutter elimination, each range cell will have a zero mean complex gaussian residue with variance given by Equation (2). Noting that matched filtering doesn't change the complex Gaussian nature of the clutter residual but adds a factor of $n_s = N_p\, T B$ to the variance, the radar return signal after clutter elimination is

$$z(t) = \sum_{m=1}^{L} \frac{1}{\sqrt{N_p\, T B}}\, n_{\text{resi}}(t) + n(t). \tag{3}$$

Thus, we see that $z(t) \sim \mathcal{CN}(0, \sigma^2_{\text{n+resi}})$, where $\sigma^2_{\text{n+resi}} = \sigma^2_{\text{noise}} + \frac{L\,\sigma^2_{\text{resi}}}{N_p\, T B}$.

B. Static Clutter with Intrinsic Clutter Motion (ICM)

Similar to the previous model, we assume that the clutter is static over N_p radar pulses. However, in this model, we say that the clutter also has some small intrinsic motion (ICM) which causes the clutter returns to randomly fluctuate over each pulse. For each clutter scatterer, the ICM, which is not very large, induces a narrow doppler spread, centered around zero doppler, in the clutter power spectrum. We model the clutter power spectrum as a very narrow Gaussian function with standard deviation σ_{f_D}, obtained empirically from real world data, centered around zero doppler [11].

Since the doppler spread is very narrow, a three-tap model, with each tap located in a different doppler cell and the center tap located at zero doppler, captures the Gaussian clutter power spectrum. Each tap in this configuration represents the average clutter power in the corresponding doppler cell. Due to the fluctuations in clutter being random, we can model the distribution of the complex amplitude of the clutter return at each doppler tap as zero mean complex Gaussian with variance given by the clutter power spectrum at each tap.

In order to eliminate clutter from the radar return signal, we perform simple doppler processing. Once again, we note that the doppler processing discussed in this section is not standard doppler processing but a greatly simplified method implemented so we can better study the effects of clutter on system performance. The time average of the radar return signal over N_p pulses (DFT at zero frequency) will give an estimate of the clutter spectrum at zero doppler. Using this estimate, we eliminate the clutter response at zero doppler for each clutter scatterer, with only a small residual remaining. Similar to the previous scenario, this residual will have a zero mean complex Gaussian distribution with variance given by

Equation (2). Since this process only eliminates the clutter at the zero doppler tap, the clutter at the non-zero doppler taps caused by ICM still remain, resulting in a model mismatch.

Thus, after clutter elimination, each range cell with a clutter scatterer present will have a zero mean complex Gaussian residue with variance given by

$$\sigma^2_{\mathrm{resi}} = \frac{k_B\, T_{\mathrm{temp}}\, B}{N_p\, TB} + \sigma^2_{-1^{\mathrm{th}}\mathrm{DopplerTap}} + \sigma^2_{+1^{\mathrm{th}}\mathrm{DopplerTap}}. \quad (4)$$

As noted previously, since matched filtering only adds a factor of $n_s = N_p\, TB$ to the variance of the Gaussian residual, we see that, for L clutter scatterers, $z(t) \sim \mathcal{CN}(0, \sigma^2_{\mathrm{n+resi}})$, where $\sigma^2_{\mathrm{n+resi}} = \sigma^2_{\mathrm{noise}} + \frac{L\,\sigma^2_{\mathrm{resi}}}{N_p\, TB}$.

III. Joint Radar-Communications Receiver

In this section, we consider the joint radar-communications system complex baseband received signal, $z(t)$, for a multiple-access communications and radar return channel. We assume that the residual of the unpredicted radar return is modeled well by a Gaussian distribution and that target range is known up to some Gaussian random process variation which is within one over the bandwidth. We consider only the portion of time during which the radar return overlaps with the communications signal.

A. Received Signal with Predicted Radar Return Suppressed

In order to mitigate interference caused by the radar signal and improve the performance of the communications system, we employ a process called Successive Interference Cancellation (SIC) [1] at the receiver. We assume that the target range is known up to some process variation (target tracking) and a predicted radar return is generated using this knowledge of the target range and subtracted from the received signal. This received signal with suppressed predicted radar return is used to decode and remove the communications signal from the received signal. Radar target estimation can then be done without communications interference. This approach is only useful if the error in delay is smaller that $1/B$.

Additionally, we assume that the clutter elimination process discussed in Section II has taken place, and as a result the received signal will be corrupted with be the sum of thermal noise and clutter residual for L clutter scatterers, $n_{\mathrm{n+resi}}(t)$, which is given by Equation (3). Thus, the pre-matched filtering clutter residual from all range cells, $L\,\frac{n_{\mathrm{resi}}(t)}{\sqrt{N_p\, TB}}$, acts as interference from the communications receiver's perspective, when decoding the communications signal. On the other hand, it is assumed that post-matched filtering target detection is already done and estimation is performed only in range cells in which a target has been detected. Hence for radar estimation, the radar will only see the post-matched filtering clutter residue for a single range cell, $n_{\mathrm{resi}}(t)$.

For N targets and L clutter scatterers, the received signal with the predicted radar return suppressed and clutter eliminated is given by

$$\begin{aligned}\tilde{z}_{\mathrm{com}}(t) &= \sqrt{P_{\mathrm{com}}}\, b\, r(t) + n_{\mathrm{n+resi}}(t) \\ &\quad + \sqrt{P_{\mathrm{rad}}} \sum_{m=1}^{N} a_m [x(t-\tau_m) - x(t-\tau_{m,\mathrm{pre}})]\,. \end{aligned} \quad (5)$$

As stated earlier, we have some knowledge of the target delay (or target range), up to some fluctuation in the return due to an underlying target random process. This fluctuation is modeled by a zero-mean Gaussian distribution $n_{\tau_{\mathrm{m}},\mathrm{proc}}$. During the k^{th} observation, the delay for the m^{th} target will be $\tau_m^{(k)} = \tau_{m,\mathrm{pre}}^{(k)} + n_{\tau_{\mathrm{m}},\mathrm{proc}}$, where $\tau_{m,\mathrm{pre}}$ is the predicted target delay. The variance of the range fluctuation process is $\sigma^2_{\tau_{\mathrm{m}},\mathrm{proc}} = \left\langle \|n_{\tau_{\mathrm{m}},\mathrm{proc}}\|^2 \right\rangle$. Utilizing the same derivation as in [1], a communications receiver employing SIC will have an interference plus noise plus clutter residual variance given by [1]:

$$\begin{aligned}\sigma^2_{\mathrm{int+n+resi}} &= P_{\mathrm{rad}} \left(\sum_{m=1}^{N} \|a_m\|^2 \,(2\pi)^2\, B^2_{\mathrm{rms}}\, \sigma^2_{\tau_{\mathrm{m}},\mathrm{proc}} \right) \\ &\quad + \sigma^2_{\mathrm{noise}} + \frac{L\,\sigma^2_{\mathrm{resi}}}{N_p\, TB}, \end{aligned} \quad (6)$$

where f is frequency, $X(f)$ is the frequency response of $x(t)$, and B_{rms} comes from employing Parseval's theorem [10]. B_{rms} is extracted from bandwidth B as $\gamma^2\, B^2 = (2\pi)^2\, B^2_{\mathrm{rms}}$, where the value γ is the scaling constant between B and B_{rms} times 2π [1].

B. Radar Estimation Rate

Here we present a parametrization of the radar in terms of information rate, the 'radar estimation rate'. The radar estimation rate, first introduced in [1], can be thought of as the maximum rate at which information about the target (encoded in bits) can be transmitted by the target (on radar illumination) to the radar receiver. The estimation information rate incorporates the insights of the rate distortion theory but emphasizes the symmetry with the communications bound. The estimation rate is given by the mutual information between the target and the radar.

We construct this information rate by considering the entropy of a random parameter being estimated and the entropy of the estimation uncertainty of that parameter [1].

Assuming both process uncertainty and estimation uncertainty are Gaussian, we see that the radar estimation information rate for time delay estimation is bounded explicitly as [1]

$$\begin{aligned}R_{\mathrm{est}} &\le \sum_m \frac{\delta}{2T} \log_2\left(1 + \frac{\sigma^2_{\tau_{\mathrm{m}},\mathrm{proc}}}{\sigma^2_{\tau_{\mathrm{m}},\mathrm{est}}}\right) \\ &= \frac{1}{2} \sum_m B \\ &\qquad \log_2 \left[1 + \frac{2\sigma^2_{\tau,\mathrm{proc}}\, \gamma^2\, N_p\, B^2\, (TB)^2\, \|a_m\|^2\, P_{\mathrm{rad}}}{TB\, k_B\, T_{\mathrm{temp}}\, B + \sigma^2_{\mathrm{resi}}} \right]^{\delta/(TB)} . \end{aligned} \quad (7)$$

It is worth noting, that by employing this time delay estimation entropy in the rate bound, it is assumed that the estimator achieves the Cramér-Rao performance. If the error variance is larger, then the rate bound is lowered.

IV. Inner Bounds on Joint Radar-Communications System Performance

In this section, we derive inner bounds on the performance of the joint radar-communications system. As mentioned earlier, performance is measured in in terms of data information rate for the communications system and estimation information rate for the radar system. To simplify the discussion, we consider only a single radar target with delay τ and gain-propagation-cross-section product a and L resolvable clutter scatterers. We also assume that clutter elimination has taken place.

A. Successive Interference Cancellation (SIC) Bound

In this section, we take a look at the technique of SIC that was mentioned in Section III-A. After suppressing the radar return and decoding and removing the communications signal from the observed waveform, radar parameter estimation can be done free of communications interference. The resulting inner bound on performance is called the SIC bound.

If $R_{\rm est} \approx 0$, it is as if there is no radar interference (and as a result no clutter residual) and the communications data rate will be,

$$R_{\rm com} \leq B\,\log_2\!\left(1 + \frac{b^2\,P_{\rm com}}{k_B\,T_{\rm temp}\,B}\right) . \tag{8}$$

If $R_{\rm com}$ is sufficiently low for a given transmit power, then the receiver can decode and remove the communications signal from the observed waveform, enabling interference free radar parameter estimation. The interference plus noise plus clutter residual from the communications receiver's perspective, $\sigma^2_{\rm int+n+resi}$, is described by Equation (6), and the communications data rate will be,

$$R_{\rm com} \leq B\,\log_2\!\left[1 + \frac{b^2\,P_{com}}{\sigma^2_{\rm int+n+resi}}\right] , \tag{9}$$

In this regime, the corresponding estimation rate bound $R_{\rm est}$ is given by Equation (7). The SIC inner bound is given by the convex hull [12] between points given by Equations (8),(9) and (7).

B. Communications Water-filling Bound

In this section, we consider a scenario in which the total bandwidth is split into two sub-bands, one sub-band for communications only and the other sub-band for both radar and communications. It is not necessary that the sub-bands will be of equal bandwidth. We use water-filling to distribute the total communications power between the two sub-bands [1]. Water-filling optimizes the power and rate allocation between multiple channels [10], [13]. The mixed use channel operates at the SIC rate vertex defined by Equations (7) and (9).

Given some bandwidth separation α,

$$B = B_{\rm com} + B_{\rm mix}\,,\; B_{\rm com} = \alpha\,B\,,\; B_{\rm mix} = (1-\alpha)\,B\,, \tag{10}$$

then we optimize the power utilization, β, between sub-bands,

$$P_{\rm com} = P_{\rm CO} + P_{\rm MU}\,,\; P_{\rm CO} = \beta\,P_{\rm com}\,,\; P_{\rm MU} = (1-\beta)\,P_{\rm com}\,. \tag{11}$$

There are two effective channels

$$\mu_{\rm com} = \frac{b^2}{k_B\,T_{\rm temp}\,B_{\rm com}}\,,\; \mu_{\rm mix} = \frac{b^2}{\sigma^2_{\rm int+n+resi}}\,, \tag{12}$$

$$\sigma^2_{\rm int+n+resi} = \|a\|^2\,P_{\rm rad}\,\gamma^2\,B^2_{\rm mix}\,\sigma^2_{\rm proc} + k_B\,T_{\rm temp}\,B_{\rm mix} + \frac{L\,\sigma^2_{\rm resi}}{N_p\,TB}\,. \tag{13}$$

The first for the communications only channel, and the second for the mixed use channel. We apply the water-filling result derived in [1] and see that the optimal power distribution (β) between the two sub-channels is given by:

$$\beta = \alpha + \frac{1}{P_{\rm com}}\left(\frac{\alpha-1}{\mu_{\rm com}} + \frac{\alpha}{\mu_{\rm mix}}\right);\quad \text{when }\; P_{\rm com} \geq \frac{\alpha}{(1-\alpha)\,\mu_{\rm mix}} - \frac{1}{\mu_{\rm com}}\,. \tag{14}$$

The resulting communications rate bound in the communications-only sub-band is given by

$$R_{\rm CO} \leq \alpha\,B\log_2\left[1 + \frac{\beta\,P_{\rm com}\,b^2}{k_B\,T_{\rm temp}\,\alpha\,B}\right] . \tag{15}$$

The mixed use communications rate inner bound is given by

$$R_{\rm MU} \leq (1-\alpha)\,B\,\log_2\left[1 + \frac{b^2\,(1-\beta)\,P_{\rm com}}{\sigma^2_{\rm int+n}}\right], \tag{16}$$

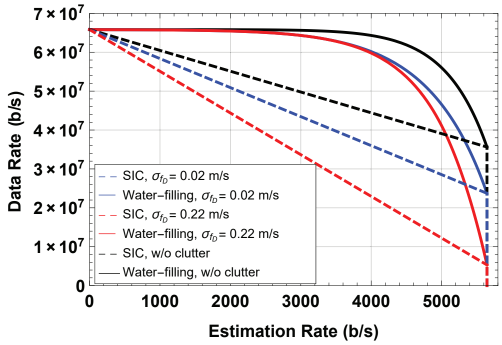
where $\sigma^2_{\rm int+n}$ is given by Equation (13). The corresponding radar estimation rate inner bound is then given by

$$R_{\rm est} \leq \frac{\delta}{2\,T}\log_2\!\left(1 + \frac{2\sigma^2_{proc}\gamma^2\,N_p\,B^4_{\rm mix}\,T\,\|a\|^2 P_{\rm rad}}{TB_{\rm mix}\,k_B\,T_{\rm temp}B_{\rm mix} + \sigma^2_{\rm resi}}\right) . \tag{17}$$

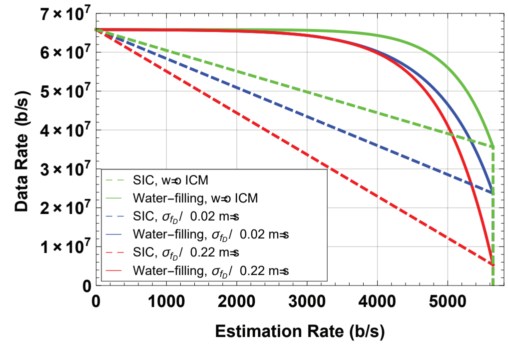
C. Example

In Figure 1, we compare the inner bounds on performance in the presence of different types of clutter described in Section II. The parameters used in the example are displayed in Table II. It is assumed that the communications signal is received through an antenna sidelobe. It should be noted that the bounds presented in this section are not fundamental limits on performance, and much tighter bounds can be achieved by applying more effective methods of clutter elimination.

In Figure 1, the SIC bounds are represented by dashed lines and the water-filling bounds by solid lines. In Figure 1a, we compare the inner bounds of performance in the absence of clutter, shown in black, to the inner bounds on performance in the presence of clutter with large ICM, shown in red, and small ICM, shown in blue. In Figure 1b, we compare the inner bounds of performance in the presence of clutter with large



(a) Performance Bounds with Clutter Absent Vs. Performance Bounds with Clutter with Large and Small ICM Present



(b) Performance Bounds with Clutter with Small ICM Vs. Clutter with Large ICM Vs. Clutter with no ICM

Fig. 1. Data Rate and Estimation Rate Bounds for Parameters in Table II

ICM, shown in red, small ICM, shown in blue, and no ICM, shown in green.

As seen from Figures 1a and 1b, there is almost no change in performance in the presence clutter without ICM when compared to performance in the absence of clutter. This is expected since static, slowly-fluctuating clutter can easily be estimated accurately and eliminated, leaving behind a negligible amount of residual. However, the presence of clutter with ICM can impose a significant degradation on performance from both a communications and radar system's perspective. Whenever the communications and radar signals overlap, communications performance degrades significantly since the total clutter residual acts as interference from the communications receivers perspective. Furthermore, since radar estimation is done after target detection, the clutter residual from only the current range cell effects estimation performance and as a result, radar performance degrades by a smaller amount when compared to communications performance. Additionally,as seen from Table II, the radar target is far away, causing the clutter residual present in the target range cell to be negligible. Hence, the radar performance undergoes almost no degradation. However, radar performance would degrade significantly for closer radar targets. Finally, we see that clutter with large ICM will cause more degradation in performance when compared to the effect of clutter with small ICM on performance.

V. Conclusion

In this paper, we analyzed the performance of a joint radar-communications performance bound in the presence of two different types of clutter; clutter with ICM and clutter without ICM. We saw that the inner bounds on system performance undergo significant degradation due to the presence of clutter with ICM, whereas adding clutter without ICM has very little effect. Due to the unavoidable nature of clutter for radar systems, the need for more robust and effective methods for elimination of clutter with motion are needed to prevent any loss in performance for a joint radar-communications system.

TABLE II
Parameters for Example Performance Bound

Parameter	Value
Bandwidth	5 MHz
Center Frequency	3 GHz
Temperature	1000 K
Communications Range	10 km
Communications Power	100 W
Communications Antenna Gain	0 dBi
Communications Receiver Side-lobe Gain	10 dBi
Radar Target Range	80 km
Radar Antenna Gain	30 dBi
Radar Power	100 kW
Target Cross Section	10 m^2
Target Process Standard Deviation	100 m
Time-Bandwidth Product	100
Number of Clutter Scatterers	120
Total Pulses Integrated	480
Doppler Spread Standard Deviation (Small ICM)	0.4 Hz
Doppler Spread Standard Deviation (Large ICM)	4.4 Hz

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