An Improved OFDM Chirp Waveform Design

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) chirp waveform has good correlation and time-bandwidth product performance, but it offers only two orthogonal signals. In this paper, we propose an improved OFDM chirp waveform based on traditional linear frequency modulation waveform as well as using OFDM to conduct the multi-carrier modulation, which can be used for multiple-input multiple-output (MIMO)radar imaging. We also analyzed the performance of the designed OFDM Chirp signals with correlation function and ambiguity function. It is shown that the designed OFDM chirp waveform has satisfactory performance for MIMO radar imaging.**

***Keywords*-OFDM, OFDM Chirp, waveform diversity, waveform design, linear frequency modulation, ambiguity function.**

I. INTRODUCTION

Although multiple-input multiple-output (MIMO) radar concept has been proposed for several years, waveform diversity design is still the most important and challenging issue in realizing a practical MIMO SAR system. In MIMO radar systems, each transmit subantenna transmit a unique waveform, orthogonal to the waveforms transmitted by the other subantennas. To obtain a high range resolution, the autocorrelation of the transmitted waveform should be close to an impulse function. Several methods have been proposed to achieve this goal. A typical suggestion was to use Barker codes. However, the signals designed with the Barker codes often have only a single carrier frequency.

Consequently, its spectrum is almost similar to a Sinc function, but the bandwidth efficiency is low [1]. So, the signaling schemes with good range-Doppler properties are needed. One polyphase code for netted radars was presented in [2], and optimized in [3]. One kind of Costas pulses with the so called property was developed in [4]. The Costas signals have good range-Doppler properties, but each chip contains only one frequency. Thus, it is a crucial task to design waveforms for MIMO radars.

This paper considers mainly orthogonal frequency division multiplexing (OFDM)-based waveform diversity design. OFDM was originally developed as a wideband communication modulation technique, where the wideband is divided into multiple orthogonal narrowband subchannels. The use of OFDM signal in radar systems was proposed firstly by N. Levenon in 2000 [5]. The primary disadvantage of using OFDM in wireless communication lies in that time and frequency synchronization is crucial to ensure subcarrier orthogonality; however, sensitivity to time and frequency synchronization is beneficial for radar systems because radar receiver usually uses a stored version of the transmit signal to measure time-delay and frequency offset between the transmitted signal and the received signal to derive the target parameters [6], [7]. For these reasons, OFDM based radar signals have received considerable attention in recent years [8], [9]. Specially, Dr.Jung-Hyo Kim[14] proposes a OFDM Chirp waveform design method, which has good correlation and time-bandwidth product performance. However, the method is suitable only for designing two orthogonal signals. To overcome this disadvantage, in this paper we extend it to design three orthogonal OFDM chirp signals.

The remaining sections of this paper are organized as follows. Section II makes a brief introduction of traditional LFM and OFDM modulation technique. Section III presents the improved OFDM chirp waveform, Next, the correlation and ambiguity function performance of the designed waveform are analyzed in Section IV. Finally, this paper is concluded in Section V.

II. LFM AND OFDM BASICS

Linear frequency modulation (LFM) is one of the most popular and widely applied pulse compression signal which broaden the bandwidth size by linear modulating on frequency of signals.

The definition of LFM according to[5]is:

$$x(t) = \cos(\pi \frac{\beta}{\tau} t^2) \qquad (0 \le t \le \tau) \tag{1}$$

In this equation, τ means pulse width and β refers to maximum bandwidth range. There are two kinds of LFM signals, the so-called up-chirp and down-chirp signal. We mainly make example of the up-chirp signal and the complex form of the signal is:

$$x(t) = e^{j\pi\beta\frac{t^2}{\tau}} = e^{j\theta(t)} \qquad (0 \le t \le \tau) \tag{2}$$

τ is proportional to the distance resolution. The higher the resolution is, the shorter the pulse width must be.

We describe basic OFDM. OFDM is a spread-spectrum transmission technique where the signal is comprised of multiple carriers. The technique is based on the carriers constituting the signal being mathematically orthogonal, as a result of the uniform frequency spacing in between the carriers. The OFDM-modulated signal can be represented by:

$$S_n(t)=\sum_{k=0}^{N-1} S_{n,k} e^{j2\pi k\Delta f t}, \qquad 0 \le t \le T_s \tag{3}$$

Where T_s, Δf and N are the symbol duration, the sub-channel space, and the number of sub-channels of OFDM signals, respectively.

A cyclic prefix(CP) or guard interval is critical for OFDM to avoid interblock interference(IBI) caused by the delay spread of wireless channels. They are usually inserted between adjacent OFDM blocks.

III. IMPROVIED OFDM-CHIRP WAVEFORM DESIGN

A. Design Scheme

The basic idea behind the proposed method is to exploit the orthogonality of discrete frequency components. This means that the orthogonality of the waveforms is independent on the types of input sequences.

Based on this idea, Dr.Jung-Hyo Kim used two OFDMs to modulate the same input sequence and resulted in two different sequences. However, this method offers only two orthogonal signals. To overcome this disadvantage, we extend this method to design three OFDM Chirp signals. Figure 1 illustrates the design scheme..

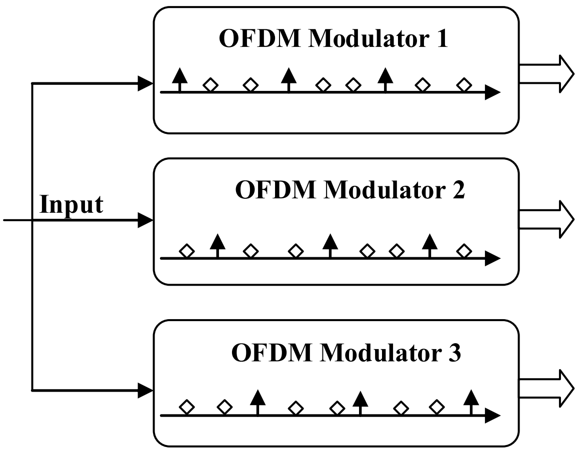

Figure 1. system scheme

In this scheme, arrows means subcarriers and diamond means the inserted zeros.

According to the form ,the three OFDM Modulators have the same number of orthogonal subcarrier ,each subcarrier is divided by Δf and shifted Δf with other signals. Because of the orthogonality of the subcarrier, the same input will result in three different receive signals and separate easily by receivers. And hence easing the demodulation and processing of the OFDM-Chirp signal.

B. Equation and Realization

We suppose that the length of the input data is N, and N subcarrier are required in the receiver. The three OFDM waveform signals are superposed and therefore return to a receiver. Hence, the received signal contains total 3N subcarrier components from both waveforms, so that DFT with the length N fails to demodulate the signal. Thus, the modulator(IDFT) and the demodulator(DFT) should both have 3N points to satisfied the requirement..

The following is the equation of the signal:

$$\mathbf{S}_1[p]\begin{cases}\exp(-j\pi\dfrac{(p.\Delta f)^2}{K_r}) & \dfrac{p}{3} \quad is \quad \text{int}\,eger\\ 0 & else\end{cases} \tag{4}$$

Use the same method, we derive the second signal:

$$\mathbf{S}_2[p]\begin{cases}\exp(-j\pi\dfrac{((p-1).\Delta f)^2}{K_r}) & \dfrac{p-1}{3} \quad is \quad \text{int}\,eger\\ 0 & else\end{cases} \tag{5}$$

And the third signal:

$$\mathbf{S}_3[p]\begin{cases}\exp(-j\pi\dfrac{((p-2).\Delta f)^2}{K_r}) & \dfrac{p-2}{3} \quad is \quad \text{int}\,eger\\ 0 & else\end{cases} \tag{6}$$

As we all known, the frequency domain form is the Fourier Transform of the time domain. So, we can get the signals in different form

$$S[\bar{p}]=\mathscr{F}\{\mathrm{s[n]}\} \tag{7}$$

573 According to the form ,s[n] denotes the discrete time samples of a complex chirp signal with length of N. Using the equation 2.4, we generate three input date sequence by the zero interleaving and shift as follows:

$$\mathbf{S}_1[p]=\{S[0],0,0,S[1],0,0,S[2]\cdots S[N-1],0,0\} \tag{8}$$

$$\mathbf{S}_2[p]=\{0,S[0],0,0,S[1],0,0\cdots 0,S[N-1],\ 0\} \tag{9}$$

And

$$\mathbf{S}_3[p]=\{0,0,\ S[0],0,0,S[1],0,\cdots 0,0,\ S[N-1]\} \tag{10}$$

Where p=0,1,2...3N-1.Both data sequences contain total 3N components, respectively.

Next, we list the discrete form of the three waveforms in time domain :

$$s_1[n] = s[n]rect[\frac{n}{N}] + s[n-N]rect[\frac{n-N}{N}] + s[n-2N]rect[\frac{n-2N}{N}] \tag{11}$$

So the second signal should be phase shift:

$$s_2[n] = (s[n]rect[\frac{n}{N}] + s[n-N]rect[\frac{n-N}{N}] + s[n-2N]rect[\frac{n-2N}{N}])\exp(jn\frac{2\pi}{3N})$$

Simplify this equation as:

$$s_2[n] = s_2[n]\exp(jn\frac{2\pi}{3N}) \tag{12}$$

And in the same way, we can easily get the third signal:

$$s_3[n] = s_1[n]\exp(jn\frac{4\pi}{3N}) \tag{13}$$

These modulated OFDM waveforms are converted to analog forms by DAC. So, we conduct simulation of the three data sequence and get the real part and imaginary part of the three signals in time domain :And make comparison among them:

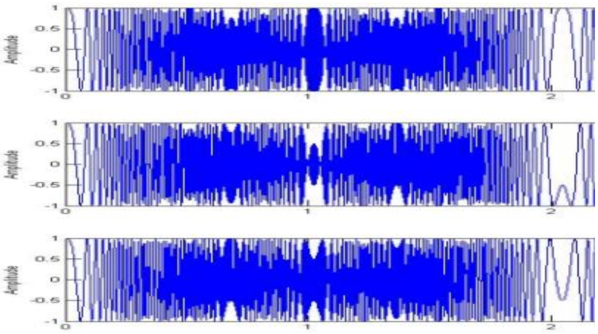

Figure 2. real part of three signals in time domain

At the same time, we discuss the imaginary part of the three signals and make a comparison, the pictures are as followings:

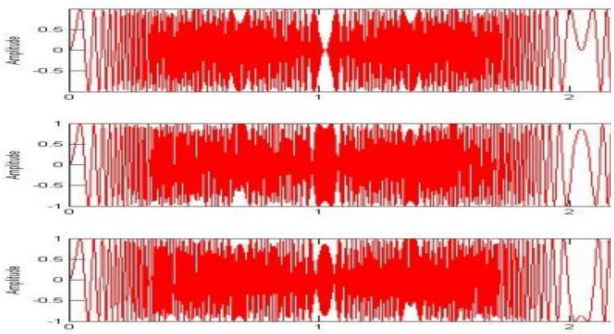

Figure 3. imaginary part of three signals in time domain

The three forms of OFDM Chirp signal waveforms are in 50 MHz bandwidth and N=1024.

From the two pictures we clearly know that the three signal was shift by offset Δf and the phase change of the three waveform was shift by 60 .Compared with signal s1(t), signal s2(t) and signal s3(t), we can easily find the phase shift among the three signals. In the next part, we will prove the orthogonality of the OFDM Chirp signals and better performance of the three signals.

IV. CORRELATION AND PERFORMANCE ANALYSIS

In this part, we will analysis the two waveform in time domain and then conduct the correlation analysis to prove and evaluation the performance of the OFDM Chirp signals.

A. Time Domain Generation

According to the relationship we proposed before $\Delta f = 1/2NT_s$,the waveform can be rewritten as follows:

$$s_1(t) = s(t)rect(\frac{t}{T_p}) + s(t-T_p)rect(\frac{t-T_p}{T_p}) + s(t-2T_p)rect(\frac{t-2T_p}{2T_p}) \tag{14}$$

Also ,according to the shift, we come out with the second signal:

$$s_2(t) = s_1(t)\exp(\frac{4}{3}j\pi\Delta ft) \tag{15}$$

And the third:

$$s_3(t) = s_1(t)\exp(\frac{8}{3}j\pi\Delta ft) \tag{16}$$

in these equation,

$$s(t) = \exp(j\pi K_r t^2),$$

and we can easier conduct correlation analysis in the next part.

B. Correlation Analysis

The correlation analysis is one of the most important part in the design and analysis of the radar new waveform.

Autocorrelation focus on the comparison of the signal with itself and conduct convolution calculate. The equation as follows:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)dt \tag{17}$$

Cross-correlation means correlation with two different signals to make sure the correlation level and the equation as follows:

$$R_{xy}(\tau)=\int_{-\infty}^{\infty}x(t)y^{*}(t-\tau)dt=\int_{-\infty}^{\infty}x(t+\tau)y^{*}(t)dt \quad (18)$$

In this paper, we calculate the sum of the three signals and make correlation with three signals respectively. Suppose the sum of the three signal is r(t), and r(t) can be re-written as:

$$r(t)=s_1(t)+s_2(t)+s_3(t) \quad (19)$$

Then the result of the three signals with r(t) and ideal OFDM chirp signal are plotted in Fig.4 and Fig.5 respectively.

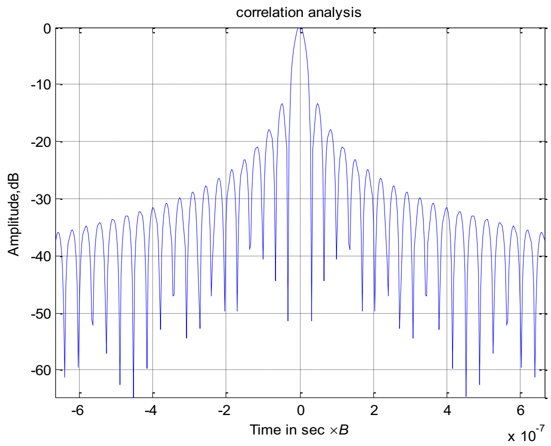

Figure 4. Correlation result of ideal signals

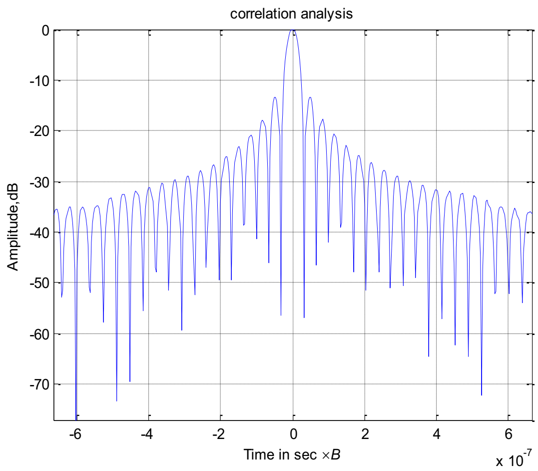

Figure 5. Correlation result of the three signals

As depicted in the Fig.4 and Fig.5, the correlation analysis describe the the three OFDM signal and the ideal OFDM chirp signal. We find that the correlation picture is identical to the ideal correlation result which proves the orthogonality of the OFDM Chirp signals. The correlation result shows that the The peak sidelobe ration approached to 13.2 dB which correspondent to the ideal OFDM chirp signal. While the orthogonality is provided by the orthogonal subcarriers, this is the most significant concept of the present novel OFDM waveform scheme. From the result, we clearly know that we get the perfect waveforms which have orthogonality and constant envelop at the same time. So, the signals will have better performance in the Doppler condition in the application of SAR.

C. Ambiguity Function:

The ambiguity function(AF) represents the time response of a filter matched to a given finite energy signal when the signal is received with a delay τ and a Doppler shift υ relative to the nominal values(zeros) expected by the filter. The AF definition followed in this paper is:

$$\left|\chi(\tau,\upsilon)\right|=\left|\int_{-\infty}^{\infty}u(t)u^{*}(t+\tau)\exp(j2\pi\upsilon t)dt\right| \quad (20)$$

Where u is the complex envelope of the signal. A positive υ implies a target moving toward the radar. Positive τ implies a target farther from the radar than the reference(τ =0) position.The ambiguity function is a major tool for studying and analyzing radar signals. It will serve us extensively in the following pictures, the following two pictures shows the Ambiguity Function of the ideal OFDM chirp signal and the three OFDM chirp signals respectively:

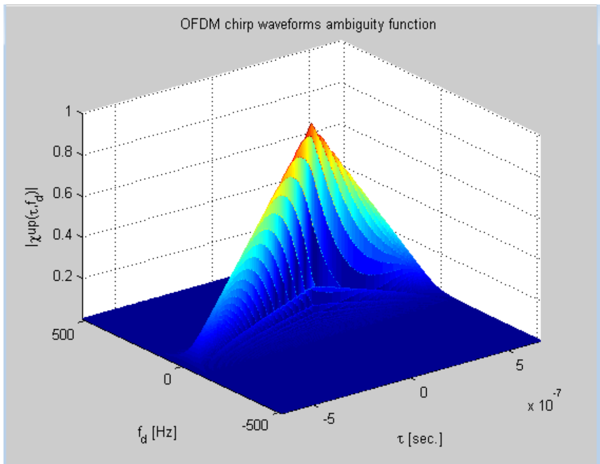

Figure 6. Ambiguity Function of ideal signals

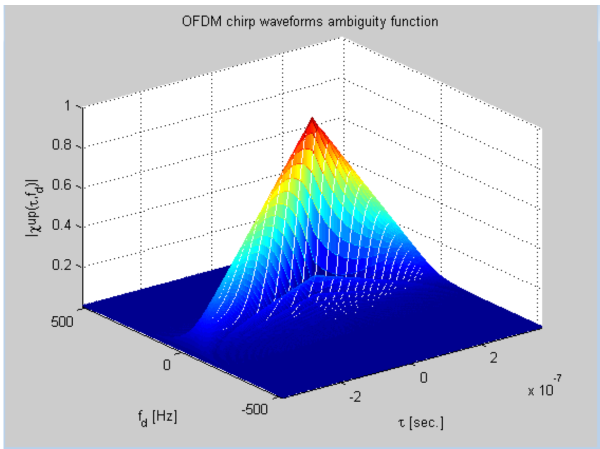

Figure 7. Ambiguity Function of the three signals

Compared with Fig.6,we find that the Ambiguity Function of three OFDM chirp signals has the nearly the same performance with the three OFDM chirp signals. We can clearly know that the three signals are robust constructed and will have strong performance on the Doppler Effect situations. Besides, three signals will provide more subchannles and higher velocity which can be used in the future high velocity data transmission and detection. So, our proposed OFDM Chirp signals are well developed and will have a good application in the future work.

V. CONCLUSION

In this paper, we propose an improved method to design three OFDM Chirp signals based on traditional linear frequency modulation signal and OFDM technique. In this way, the designed OFDM chirp waveforms preserve their orthogonality, large time-bandwidth product, and good envolope at the same time, which are valuable for developing practical MIMO radar systems. In subsequent work, we plan to optimize the OFDM chirp waveform with mathematical optimization tool.Besides, using the method we proposed, we can further create more radar signal waveforms and get a higher transmission velocity and better detective resolution and range.This paper was greatly helped by Prof Wen-Qin Wang.It's his innovation and creative ideas leading us to create the novle method for radar waveform design.Thanks again for his kind help.

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