The Parameter Setting Problem of Signal OFDM-LFM for MIMO Radar

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***Abstract*—Multiple Input Multiple Output (MIMO) radar is a new radar technique, which transmits orthogonal signal at the transmitter and forms wide beam. In this paper, we select the Orthogonal Frequency Division Multi LFM signal as the orthogonal waveform. In this model, the bandwidth of single LFM and the frequency interval are important. Only the situation of equi-interval frequency is studied here. Assume the bandwidth is fixed, the selection of frequency interval affects not only the range resolution and processing speed, but also other performance of radar. In this paper, we investigate the relationship of the two parameters, also the concepts of range sidelobe are introduced. Then we demonstrated the random initial phase can suppress the second-class sidelobe. In the end, we draw the conclusion that by appropriate parameter setting, the OFDM-LFM waveform can has good performance in MIMO radar.**

I. INTRODUCTION

MIMO radar can be viewed as the development of phased array radar. On transmit, the array will be divided into low gain elements (or subarrays), each transmitting orthogonal waveform. So it has wide low gain transmitting beam. On receiver, the received signal is processed through a bank of digital beams in one or more direction. MIMO radar has many merits, mitigating the interception range, enhancing the detection ability of weak target in clutter, also detection of the low velocity target in clutter [1].

Applying Orthogonal Frequency Division Multi LFM signal waveform into MIMO radar is a good idea. The signal of each element (or subarray) is transmitted through different sub carrier wave. Each signal occupies different frequency segment, just like the frequency stepped signal[2], and the frequency step between the adjacent two sub carrier waves is relative fixed we discuss here. In the latter signal processing, the signal component of different transmitting channels can be separated through a group of matched filters. What's more, the effect of wide bandwidth can also be received.

II. THE SIGNAL PROCESSING FLOW

The signal after transmission in the space, then through digital beam forming, is processed by matched filtering. Then through equivalent transmit beam forming, the component of the different transmitter can be added. In the following, moving target indicator and moving target detection is needed in clutter environment. In the end, the CFAR detection is used to judge if target exists.

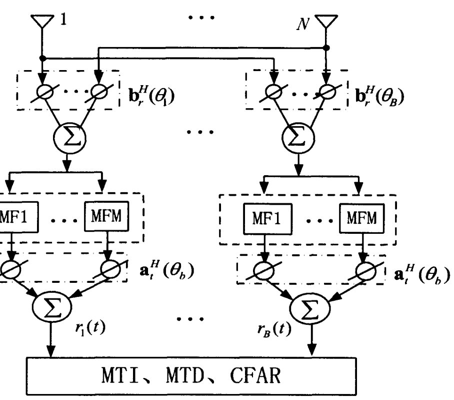

Figure 1. The signal processing flow of MIMO radar (assume there are M transmitting channels and N receive antennas, and B receiving beams formed in space)

III. THE SIGNAL CHARACTERISTIC

The transmitting signal model OFDM-LFM we discussed here is frequency stepped LFM actually.

Assume there are M transmitting channels, and each channel transmits a single LFM signal with bandwidth B_s, with frequency interval f_Δ between two channels adjacent to make the signal from different channels orthogonal. So the signal of kth transmitter is

$$s_k(t) = u_k(t)e^{j2\pi f_0 t}, \quad 1 \le k \le M \tag{1}$$

Where

$$u_k(t) = \frac{1}{\sqrt{T_p}} rect(\frac{t}{T_p}) e^{j2\pi(kf_\Delta t + \frac{1}{2}\mu t^2)} e^{j\phi_k}, \quad 1 \le k \le M \tag{2}$$

Where f_0 is the carrier frequency of the signal on first

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transmitting channel; T_p is the signal duration; $\mu = B_s / T_p$ is the modulating slope; f_Δ is the frequency interval, here we assume the intervals between the arbitrary two adjacent transmitters are the same; ϕ_k is the initial phase of kth transmitting signal.

It can be easily shown that when $f_\Delta T_p = n$ is hold, the orthogonal of channels is satisfied, that is to say

$$\int_0^{T_p} s_m(t) s_n^*(t) dt = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases} \tag{3}$$

When the condition (formula(3)) is met, and the single LFM bandwidth is fixed, how to select the frequency interval is a key problem. So we should study under which condition, the radar can has good performance.

During signal processing, we study matched filtering here, because the weight factor of receiving beam and that of equivalent transmitting beam can be counteracted. The signal on the receiver contains all the components of the orthogonal signal from the transmitter. And the matched filters are a bank of filters due to all the transmitting signals. When the signal at each individual receiver is processed through these matched filters, it means each signal component from each transmitter through all different match filters. Then output of the each component is added by the weight factor which is produced by the equivalent transmitting beam forming (see). We can easily gain the output of the matched filter of the single LFM signal $u_1(t)$, rewrite below[3]

$$u_1(t) = \left| \frac{\sin\{\pi[f_d + \mu(t-\tau)][T_p - |t-\tau|]\}}{\pi[f_d + \mu(t-\tau)]T_p} \right| \tag{4}$$

We can view the above formula (4)as one component processed through matched filter which is made of the same signal component .whereas different matched filters are different from the one in the frequency step. As the above description shows, the synthetic output of matched filtering is equivalent to the adding effect of the output of a bank matched filters. So the synthetic output is the unite effect of different signal components can be expressed as

$$r(t) = \left| \frac{\sin\{\pi[f_d + \mu(t-\tau)][T_p - |t-\tau|]\}}{\pi[f_d + \mu(t-\tau)]T_p} \right| \left| \sum_{m=1}^{M} e^{j(m-1)2\pi f_\Delta (t-\tau)} \right| \tag{5}$$

So the synthetic output can be expressed by the form of the Sinc function multiplied by adding of geometric proportion, which is also the periodic discrete Sinc function, described as below

$$u_2(t) = \left| \sum_{m=1}^{M} e^{j(m-1)2\pi f_\Delta (t-\tau)} \right| = \left| \frac{\sin[M\pi f_\Delta (t-\tau)]}{\sin[\pi f_\Delta (t-\tau)]} \right| \tag{6}$$

It can be easily shown that the shape of the output waveform is mostly decided by $u_1(t)$, $u_2(t)$ .when $f_d = 0$, the -4dB width of $u_1(t)$ is

$$\Delta T_1 = 1 / B_s \tag{7}$$

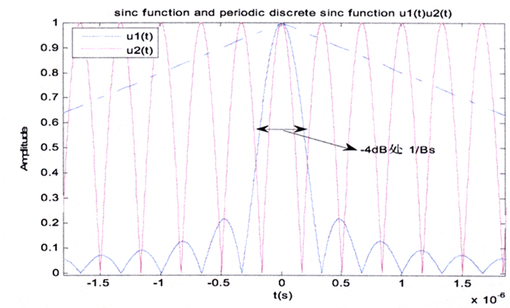
$$\Delta R_1 = c / 2B_s \tag{8}$$

$u_2(t)$ is periodic with period

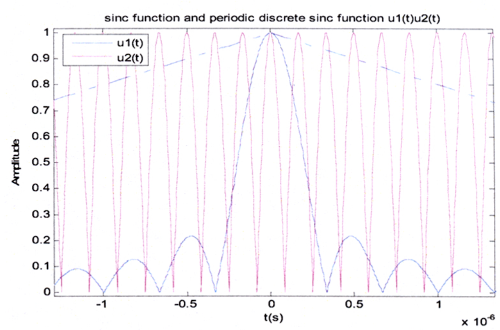
$$\Delta T_2 = 1 / f_\Delta \tag{9}$$

$$\Delta R_2 = c / 2f_\Delta \tag{10}$$

The relationship of $u_1(t)$, $u_2(t)$ and the parameters B_s , f_Δ is showed as below



(a) $f_\Delta = B_s$ the entire amplified



(b) $f_\Delta > B_s$ the figure amplified

Figure 2 The relationship of sinc function, the periodic discrete sinc function and the bandwidth , the frequency step

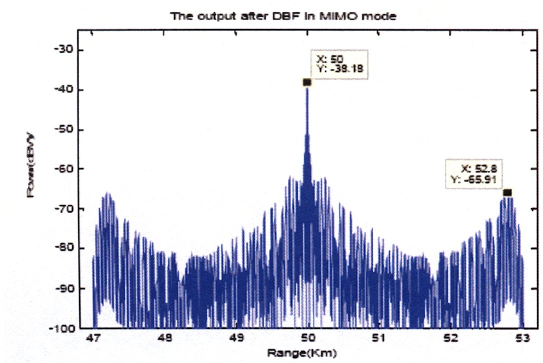
IV. THE RELATIONSHIP OF THE SYNTHESIZED OUTPUT SIGNAL AND THE TWO PARAMETERS

Here we discuss three cases about relationship of the synthesized output and the two parameters, and assume the LFM bandwidth is fixed here.

A. The Frequency Step Equal to the LFM Bandwidth ($f_{\Delta} = B_s$)

From the above description, we can easily see that in this case, the positions of the peaks except the central peak of $u_2(t)$, are just the same as the positions of the zeros of $u_1(t)$. Assume the radar parameters are: the LFM bandwidth $B_s = 3\text{MHz}$; the pulsewidth $T_p = 20\mu s$ the number of transmitters $M = 4$; a target position $R = 50\text{Km}$; the sampling frequency $f_s = 60\text{MHz}$;

When frequency step $f_{\Delta} = 3\text{MHz}$ the synthesized output signal is



(a) the entire figure

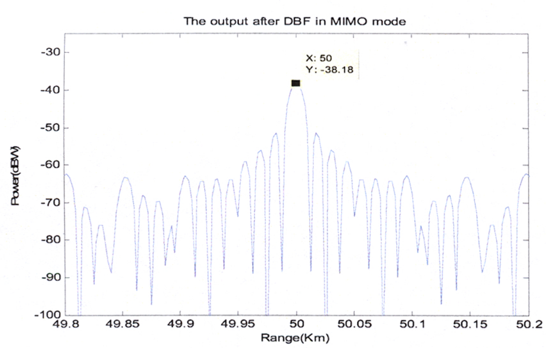


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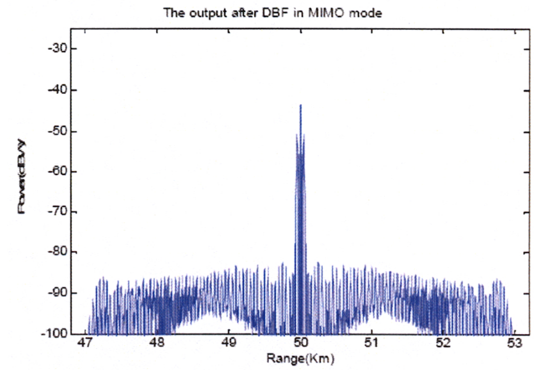
Figure 3 The synthesized output signal with the bandwidth equaled to the frequency step

When the matched filter is added hamming window, the processed result is



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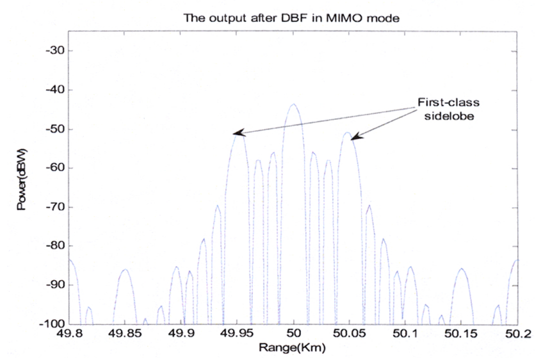


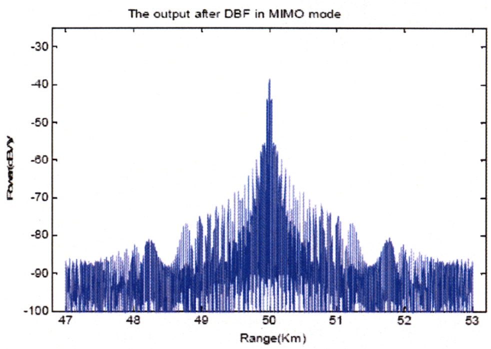
(b) the figure amplified

Figure 4 The synthesized output signal with the bandwidth equaled to the frequency step (adding hamming window)

In Figure 4 (b), there are two peaks except the main peak, actually, it is because that after adding hamming window, the mainlobe become wider, so the other two peaks of $u_2(t)$ come into the mainobe, and partition the mainlobe. We call the two peaks are first-class sidelobe.

B. The Frequency Step Bigger Than the LFM Bandwidth ($f_{\Delta} > B_s$)

Similar to above adding window case, in this case, the synthesized output signal has two first-class range sidelobes. It is because that the positions of the peaks of $u_2(t)$ become closer, and more than one peak come into the mainlobe of $u_1(t)$, causing the mainlobe parted. Assume the radar parameters are the same with above simulation example, except frequency step $f_{\Delta} = 5\text{MHz}$ and the synthesized output signal is



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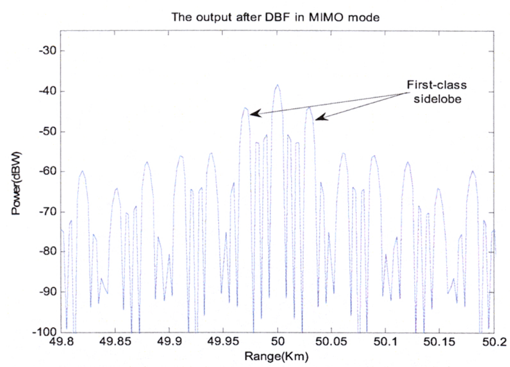


(b) the figure amplified

Figure 5 The synthesized output signal with the frequency step bigger than the bandwidth

We can easily get the positions of the first-class sidelobes in the MIMO radar system (formula (9)and (10)), as follows

TABLE 1 The positions of first-class sidelobes

f_Δ	**Time** $1/f_\Delta$	**Range** $c/2f_\Delta$
3M	-	-
4M	0.25 μs	37.5m
5M	0.2 μs	30m
6M	0.167 μs	25m

C. The Ffrequency Step Smaller Than the LFM Bandwidth ($f_\Delta < B_s$)

It is need to be pointed that, other signal component have effect on the output of the matched filtering, which can be equivalent to correlation between different signals. We can draw a conclusion from paper [4] that when the frequency step is bigger than the bandwidth, the correlation can be ignored. But when smaller, it has influence to the synthesized output signal. The correlation can be expressed as

$$u_1'(t)=\left|\frac{\sin\{\pi[f_d+\mu(t-\tau)+nf_\Delta][T_p-|t-\tau|]\}}{\pi[f_d+\mu(t-\tau)+nf_\Delta]T_p}\right|, n=1,\cdots,M-1 \quad (11)$$

Where n is the number of steps. When $\mu(t-\tau)+nf_\Delta+f_d=0$, the positions of the correlation can be gained

$$t=\tau+\frac{T_P}{B}[nf_\Delta+f_d] \quad (12)$$

The positions have relation to the number of steps n, and the interval between two positions are a constant $\Delta t=T_P f_\Delta/B_s$, also the interval between the main peak and the biggest correlation peak is

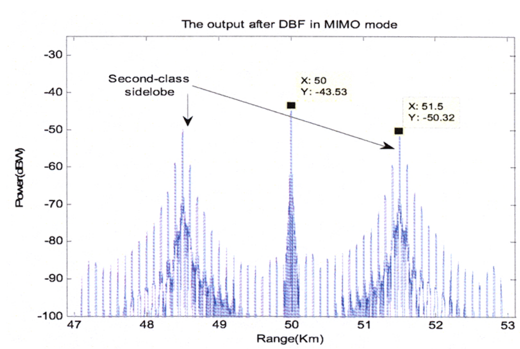
$$\Delta t_1=\frac{T_P}{B}f_\Delta \quad (13)$$

Actually the correlation peak modulated by the periodic discrete sinc function caused sidelobes, which we call them second-class sidelobe. Here we also give the positions of second-sidelobe (formula (13))under different situations:

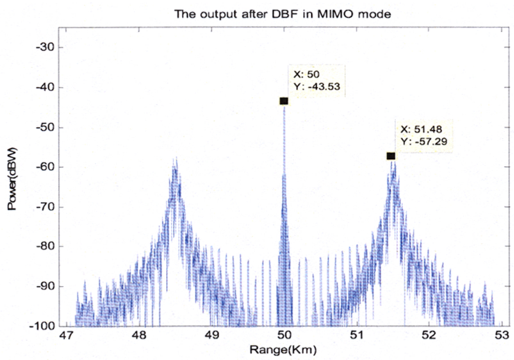
TABLE 2 positions of second-class sidelobes

f_Δ	**Time** $\tau_\Delta=\frac{f_\Delta}{\mu}$	**Range** $c\tau_\Delta/2=\frac{cf_\Delta}{2\mu}$
1M	$(1/3)T_P$	1km
1.5M	$(1/2)T_P$	1.5km
2M	$(2/3)T_P$	2km
3M	-	-

Assume the radar parameters are the same with above simulation example, except frequency step $f_\Delta=1.5$MHz the synthesized output signal is



(a) with zero initial phase



(b) with non-zero initial phase

Figure 6 The synthesized output signal with the frequency step smaller than the bandwidth (adding hamming window)

TABLE 3 The initial random phases in Figure 6(b)

2.6955	5.6235	5.1208	1.3572
6.2204	0.7917	3.8013	1.5017
2.1991	3.1227	5.6235	1.8598
2.7206	4.2914	0.3016	6.0004

We can see when applying the initial random phase into the transmitting signal, the second sidelobe has been mitigated 7dB (Figure 6). If we adopt GA (gene algorithm)to optimize the random phases, better sidelobe suppression can be gained.

V. tHE DOPPLER FREQUENCY INFLUENCE TO THE SYNTHESIZED OUTPUT SIGNAL

As we know, the mainlobe of the output of matched filter will walk when the Doppler frequency is big. Then the other peaks of the periodic discrete sinc will come into the mainlobe and cause the first-class sidelobe. Assume the Doppler frequency is f_d, then the distance mainlobe walks will be

$$\Delta t = -\frac{f_d}{B_s} T_p \tag{14}$$

So we can calculate the maximal Doppler frequency under which limit to ensure no first-class sidelobes or they are very small. When $\frac{f_d}{B_s} T_p < \frac{1}{B_s}$ that is to say

$$f_d < \frac{1}{T_p} \tag{15}$$

We ignore the influence of the first sidelobes.

VI. tHE CONCLUSION

From the analysis above, we can easily draw the conclusion that the first-class and second-class sidelobe can't be eliminated. When we select the frequency step smaller than single LFM bandwidth, we can ensure no first-class sidelobes, and adopt the random initial phases to mitigate the second-class sidelobe. But the suppression effect is finite, especially in searching situation, the high sidelobe would cover up the weak target, casuing many false alarms. So we suggest select the parameters due to the radar modes as below.

In searching case, there is no need to achieve high range resolution, we select frequency step bigger than single LFM bandwidth, allow the first-class sidelobe appearing. In this case, the two sidelobes and the mainlobe should be viewed as one target, and process according to narrowband mode. In tracking situation, the maily position of the target and the second-class sidelobes have been gained, so we select $f_\Delta < \alpha B_s$, $\alpha < 1$, allowing second-class sidelobe to exist, and process the signal around target only. Meanwhile in this case, the high range resolution can be received, with total bandwidth $B_s + (M-1)f_\Delta$. So we can draw the conclusion that by choosing appropriate parameter, the OFDM-LFM waveform can have good performance in MIMO radar.

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