Combination Sine PS Method for PAPR Reduction in OFDM Systems

Lei Xu
Space Star Technology Co., Ltd.
Beijing, China
xulei_rai@163.com

Linhai Gu
Space Star Technology Co., Ltd.
Beijing, China
215718571@qq.com

Jiangchun Liu
Space Star Technology Co., Ltd.
Beijing, China
jiangchun_liu@163.com

***Abstract*—The pulse shaping (PS) is an effective and flexible technique for reducing the peak to average power ratio(PAPR) of orthogonal frequency division multiplexing (OFDM) system. However, different PS waveforms result in different probabilities of reducing the PAPR of OFDM signals. A combination sine pulse shaping (CS-PS) technique is proposed in this paper. The CS-PS waveform contains a linear term and three sine terms. Simulation results show that the CS-PS can get better performance of reducing the PAPR compared to conventional PS method.**

Keywords—Orthogonal Frequency Division Multiplexing; pulse shaping; peak to average power ration

I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) can meet higher data transfer rate in wireless communication. It offers high spectral efficiency and greater immunity to multipath fading. Thus, it is implemented in various wireless communication standards such as WIMAX (IEEE802.16a/e), WLAN (IEEE802.11a/g), WMAN (Wireless Metropolitan Area Networks) and fourth generation (4G) mobile radio communication systems. However, it have a high peak-to-average power ratio (PAPR), which has seriously nonlinear distortion and the transmission signal frequency spectrum proliferation in the non-linear launch channel [1].

In order to reduce the high PAPR, several methods have been proposed. PAPR reduction techniques can mainly be classified to three classes: signal pre-distortion technologies, coding technologies and probabilistic technologies. The signal pre-distortion technologies include the clipping method [2]. The clipping technique is the simplest reduction PAPR method, but it leads both in-band distortion and out-of-band distortion. The coding technologies have a block coding [3]. The block coding does not cause signal distortion, while its calculation is complicated, and it may produce redundant data. The third class technology such as selective mapping (SLM), partial transmit sequence (PTS) and pulse shaping (PS), is a class of effective method without distortion[4]. Among these methods, the PS method is considered as one of promising methods which can achieve better PAPR performance with smaller side information. The most commonly used Nyquist pulse is the raised-cosine pulse [4]. A improved raised-cosine pulse has been proposed in [5]. The flipped-exponential pulse is proposed in [6]. However, different PS waveforms result in different probabilities of reducing the PAPR of OFDM signals. Seeking a perfect pulse shaping waveforms for PAPR reduction of the OFDM signals is the key to research the PS technology.

Considering different pulse shaping waveforms resulting in different probabilities of reducing the PAPR of OFDM signals, this paper presents a combination sine pulse shaping (CS-PS) technique. The CS-PS can improve PAPR performance of OFDM system.

II. ANALYSIS OF THE PAPR FOR OFDM SIGNAL

Taking the N subcarriers, which are chosen to be orthogonal, the complex envelope OFDM signal is the superimposition of modulated subcarriers by IFFT operation. The mathematical expression is

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s(k) \exp(j2\pi kn/N) \quad n = 0,1,2,\cdots,N-1 \tag{1}$$

where $j = \sqrt{-1}$, and $s(k)$ is the data symbol transmitted on the k -th subcarrier.

The PAPR is the ratio period between the maximum instantaneous power $|x(n)|^2$ and its average power $\mathrm{E}[|x(n)|^2]$. The PAPR of OFDM signals $x(n)$ can be defined as

$$\text{PAPR}=10\lg \frac{\max\left\{|x(n)|^2\right\}}{\mathrm{E}\left\{|x(n)|^2\right\}} \tag{2}$$

The PAPR performance of the OFDM signal is analyzed by using the complementary cumulative distribution function (CCDF), because the rayleigh distribution of PAPR is stochastic in nature. The CCDF is defined as the probability of PAPR of the OFDM signal exceeding a threshold value PAPR0, which can be written as follows

$$\begin{aligned} CCDF(N, PAPR0) &= \Pr\{PAPR > PAPR0\} \\ &= 1 - \Pr\{PAPR \le PAPR0\} \\ &= 1 - \left(1 - \exp(-PAPR0)\right)^N \end{aligned} \tag{3}$$

The CCDF curve denotes the statistical distribution of the OFDM signal power exceeding a given power value. The X axis denotes dB of the signal peak power exceeding the average power value. The Y axis represents ratio of the signal peak power exceeding a given power value of the X axis.

978-1-5090-3025-5/17/$31.00 ©2017 IEEE

III. PS FOR REDUCTION PAPR

The OFDM transmitter block diagram of the PS technology for reduction PAPR is illustrated in Fig. 1.

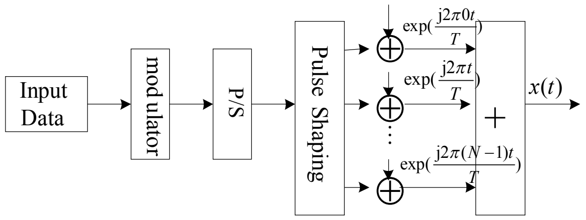

Fig.1 Block diagram of PS for reduction PAPR

Considering the OFDM system with N orthogonal subcarriers and 16 Quadrature Amplitude Modulation (QAM), the basedband signal was converted into N channel parallel streams. Each stream in the data block will modulate one of N subcarriers. The OFDM signal is the superimposition of modulated subcarriers using IFFT operation. Thus, the complex envelope of N subcarrier OFDM with PS can be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}p_k(t)s(k)\exp(j2\pi f_k t) \quad 0\le t\le T \tag{4}$$

where $p_k(t)$ is the pulse shaping function, and f_k is the subcarrier frequency of k-th subcarrier, that is, $f_k=k/T$.

The PS is an efficient method for reduction PAPR. The $p_k(t)$ must meet the following conditions [4]

a) $\int_0^T |p_k(t)|^2\,dt=T$

b) $p_k(t)=0$, $|t-T/2|>T/2$

c) $P_k(f)=0$, $|f-B|>B(1+\beta)$

d) $\int_0^T p_m(t)p^*{}_n(t)\exp(j2\pi(f_m-f_n)dt$

$$=\begin{cases}1 & m=n\\ 0 & m\neq n\end{cases} \quad (m,n=0,1,2,\cdots,N-1) \tag{5}$$

where $P_k(f)$ is the frequency response of $p_k(t)$, T is the transmission symbol period, $B=1/2T_s$ is the Nyquist frequency, T_s is Nyquist sample frequency, that is, $T_s=T/N$, β is coefficient related to N, that is, $0<\beta<1$, $(\cdot)^*$ represents the conjugate.

The cross-correlation function between sampling values of OFDM signal can be written as

$$R_{1,2}(t_1,t_2)=\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\mathrm{E}[(s(m)s^*(n)]p_m(t)p^*{}_n(t) \cdot\exp(j2\pi(mt_1-nt_2) \tag{6}$$

where $\mathrm{E}[\cdot]$ is the expectation operator.

The above equation shows that the cross-correlation function depends on the time waveforms of the different carriers. Thus, a proper selection of the subcarrier time waveforms will increase the cross-correlation function and decrease the PAPR of OFDM signals.

Due to having no intersymbol interference (ISI) characteristics, the Nyquist PS has been widely applied in the OFDM system. The $p(kT_s)$ is satisfied with the following relation

$$p(kT_s)=\begin{cases}1, & k=0\\ 0, & \text{otherwise}\end{cases} \tag{7}$$

By (2) and (7), we can know that the PAPR can be written as follows

$$\mathrm{PAPR}_{\max}=\frac{1}{N}\max\left(\sum_{k=0}^{N-1}|p_k(t)|\right)^2 \le\frac{1}{N}\left(\sum_{k=0}^{N-1}\max|p_k(t)|\right)^2 =N \quad 0\le t\le T \tag{8}$$

All of meeting the (5) demands of the PS $p(t)$ can be used to reducing the PAPR of the OFDM signals. When using a rectangular pulse, the maximum PAPR is the number of subcarriers N.

Since the PS $p(t)$ is a time limited periodic waveform, the exponential Fourier series can be expressed as

$$p(t)=\sum_{k=-L}^{N+L-1}C_k\exp(j2\pi kt/T) \quad 0\le t\le T \tag{9}$$

where, $L=[N\alpha]/2$, and α is the roll-off factor, C_k is the Fourier coefficient of $p(t)$, and is given by

$$C_k=\frac{1}{T}\int_0^T p(t)\exp(-j2\pi kt/T) \tag{10}$$

According to $p(t)$, we can know that the subcarrier $p_k(t)$ can be written as follows

$$p_k(t)=\sum_{k=-L}^{N+L-1}C_k\exp(-j2\pi nk/N)\exp(j2\pi(k-n)t/T) \tag{11}$$

By (3) and (10), $x(t)$ can be written as follows

$$\begin{aligned}x(t)&=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}p_n(t)s(n)\exp\left(j2\pi\frac{n}{T}t\right)\\&=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}s(n)\sum_{k=-L}^{N+L-1}C_k\exp\left(-j2\pi\frac{nk}{T}\right)\exp\left(j2\pi\frac{k}{T}t\right)\\&=\frac{1}{\sqrt{N}}\sum_{k=-L}^{N+L-1}Y(k)\exp\left(j2\pi\frac{k}{T}t\right)=\mathrm{IFFT}(Y)\end{aligned} \tag{12}$$

where $Y(k)=\sum_{n=0}^{N-1}s(n)C_k\exp\left(-j2\pi\frac{nk}{T}\right)$.

Let $\mathbf{X}=[s(0),s(1),\cdots,s(n)]^T$ is the vector of data symbols. $\mathrm{p}_i=C_i[1,\exp\left(-j2\pi\frac{i}{N}\right),\cdots,\exp\left(-j2\pi i\frac{N-1}{N}\right)]^T$ and $\mathbf{P}=[\mathrm{p}_{-L},$

$\mathrm{p}_{-L+1},\cdots,\mathrm{p}_{N+L-1}]^T$, $(\cdot)^T$ represents the matrix transpose. Thus

$$\mathbf{Y}=\mathbf{PX}=[Y(0),Y(1),\cdots,Y(n)] \tag{13}$$

To keep the orthogonality property between the different subcarriers in OFDM system, the precoding matrix $\mathbf{P}$ is satisfied with the following relation [5]

$$\mathbf{PP}^H=\mathbf{I} \tag{14}$$

where $(\cdot)^H$ represents the matrix conjugate, and $\mathbf{I}$ is $N\times N$ identity matrix.

IV. Combination Sine Pulse

The PS can reduce the PAPR of the OFDM signals. The most popular PS is the raised-cosine pulse (RC), then its frequency can be written as [4]

$$H_{RC}(f)=\begin{cases}1 & 0\le f\le B(1-\alpha)\\ \dfrac{1}{2}\left\{1+\cos\left(\dfrac{\pi}{2B\alpha}(f-B(1-\alpha))\right)\right\} & B(1-\alpha)\le f\le B(1+\alpha)\\ 0 & B(1+\alpha)\le f\end{cases} \tag{15}$$

where α is the roll-off factor, $B=1/2T_s$ is the Nyquist frequency, and T_s is Nyquist sample frequency. Its impulse response can be written as

$$h_{RC}(t)=\sin\mathrm{c}(\frac{t}{T_s})\frac{\cos(2\pi\alpha\, t/T_s}{1-(2\alpha t/T_s)^2} \tag{16}$$

In the Nyquist filter design, it define a unified model for a class of PS [5], and is given by

$$H(f)=\begin{cases}1 & |f|\le B(1-\alpha)\\ G\big((B(1-\alpha)-|f|)\big) & B(1-\alpha)<|f|\le B\\ 1-G\big((B(1+\alpha)-|f|)\big) & B<|f|\le B(1+\alpha)\\ 0 & B(1+\alpha)<|f|\end{cases} \tag{17}$$

where $G(f)$ is a function satisfying $G(0)=1$.

The flipped-exponential pulse (FE), $G(f)=\exp(f)$, has been proposed in [5] based on [6]. The frequency response and impulse response of the FE are respectively expressed as

$$H_{FE}(f)=\begin{cases}1 & |f|\le B(1-\alpha)\\ \exp\{\lambda[B(1-\alpha)-|f|]\} & B(1-\alpha)<|f|\le B\\ 1-\exp\{\lambda[B(1+\alpha)-|f|]\} & B<|f|\le B(1+\alpha)\\ 0 & B(1+\alpha)<|f|\end{cases} \tag{18}$$

$$h_{FE}(t)=\frac{1}{T_s}\sin\mathrm{c}(\frac{1}{T_s})\frac{4\lambda\pi t\sin(\pi\alpha\, t/T_s)+2\lambda^2\cos(\pi\alpha\, t/T_s)-\lambda^2}{(2\pi t)^2+\lambda^2} \tag{19}$$

where $\lambda=\ln 2/\alpha B$.

The geometric principle of constructing the Nyquist PS is the frequency response waveform with piecewise linear and odd symmetry [6] in Fig. 2. The frequency response retains a convex shape in the interval $B(1-\alpha)<|f|\le B$ and the concave shape in $B<|f|\le B(1+\alpha)$. This method can transfer some energy to the high frequency spectral range.

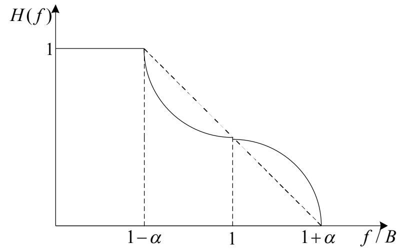

Fig.2 The principle of onstructing the Nyquist PS

From Fig. 2, the frequency response function $H(f)$ consists of linear function and the odd function. Thus, this paper presents a combination sine pulse (CS). The CS $H(f)$ is given by

$$H(f)=\begin{cases}0 & |f|\le B(1-\alpha)\\ G(f) & B(1-\alpha)<|f|\le B(1+\alpha)\\ 1 & |f|>B(1+\alpha)\end{cases} \tag{20}$$

where $G(f)$ can be written as follows

$$G(f)=a\frac{B(1+\alpha)-|f|}{2\alpha B}+b\sin\left(\frac{B(1-\alpha)-|f|}{\alpha}\pi\right)+c\sin\left(3\pi\frac{B(1-\alpha)-|f|}{\alpha}\right)+d\sin\left(5\pi\frac{B(1-\alpha)-|f|}{\alpha}\right)$$

where a, b, c and d are the coefficients of $G(f)$. The above equation shows that $G(f)$ contains a linear term that stretches from $B(1-\alpha)<|f|\le B$ to $B<|f|\le B(1+\alpha)$ and three sine terms of amplitudes b, c and d. The period of the first sine function encompasses the transition range form $B(1-\alpha)<|f|\le B$ to $B<|f|\le B(1+\alpha)$ and its phase is π, in order to exhibit the required concave shape. The second sine function and the third sine function are third harmonic and quintuple harmonic of the first one, which were used to modify the first term. The higher order harmonic is much smaller than third harmonic and quintuple harmonic, this paper only considers third harmonic and quintuple harmonic.

In order to the CS can transfer some energy to the high frequency spectral range, the linear term of amplitude is $a=1$. How much energy transfer mainly depends on the first sine terms of amplitude b. The smaller b is, the smaller energy transfer. But the b is too big, the energy transfer decreases. The c and d value depend on the b. This paper take $b=0.15$,

$c = 0.005$ and $d = 0.001$.

The frequency responses of the three PS with roll-off factor $\alpha = 0.22$ in Fig.3. From the Fig.3, the CS can transfer energy form $B(1-\alpha)<|f| \le B$ to $B<|f| \le B(1+\alpha)$. Compared with FE pulse shaping (FE-PS) and RC pulse shaping (RC-PS), CS pulse shaping (CS-PS) can get a better displacement of energy within the transition band form $B(1-\alpha)<|f| \le B$ to $B<|f| \le B(1+\alpha)$, which improve the PAPR suppression performance for the OFDM system.

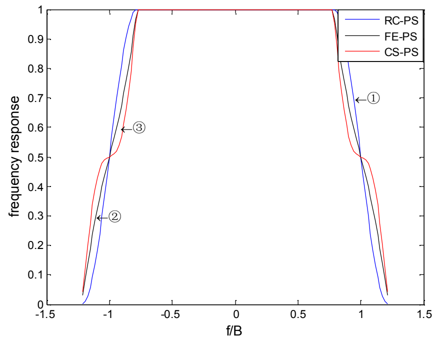

Fig.3 Frequency responses of three PS

V. SIMULATION RESULTS

In this section, the simulation analysis is used to evaluate the PAPR reduction capability with proposed scheme. The following parameters have been used for the MATLAB simulation of the reduction of PAPR in OFDM system. In the present work, we have considered 16 quadrature amplitude modulation (16-QAM) on 1024 subcarriers.

Fig. 4 shows the comparison of complementary cumulative distribution function (CCDF) for the conventional OFDM, clipping, PTS, SLM, and combination sine pulse shaping (CS-PS) precoding based on OFDM. At the CCDF of 10^{-1}, compared with without PAPR reduction, clipping, PTS and SLM, the reduction in PAPR is 3 dB, 1.5 dB, 1.2 dB and 1 dB for CS-PS in OFDM system.

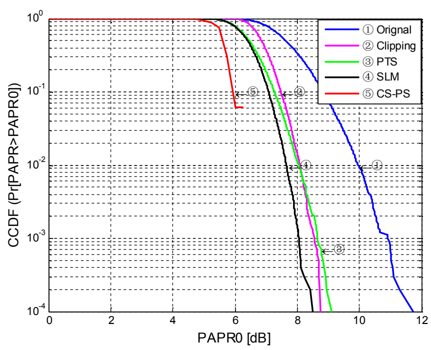

Fig.4 Comparisons of CCDF with diffierent reduction PAPR technique

Fig.5 shows the comparison of the CCDF of the PAPR performance of the OFDM system for conventional OFDM, raised-cosine pulse shaping (RC-PS), flipped-exponential pulse shaping (FE-PS), and CS-PS based on OFDM. The roll-off factor is set $\alpha = 0.2$. From the Fig. 5 , we can observe that the CCDF of the CS-PS is better than that of the RC-PS and FE-PS. From these results, compared with without PAPR reduction, RC-PS and FE-PS, the reduction in PAPR is 3.8 dB, 0.3 dB and 0.1 dB for CS-PS at the CCDF of 10^{-2}.

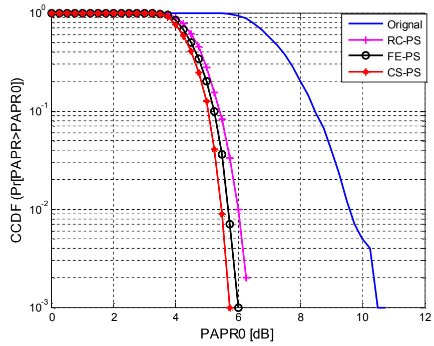

Fig.5 Comparisons of CCDF with three PS technique

Fig.6 shows the comparison of the CCDF of the PAPR performance of the OFDM system using CS-PS for different roll-off factors. From these results, with the increasing of α, the value of reduction PAPR is deceased.

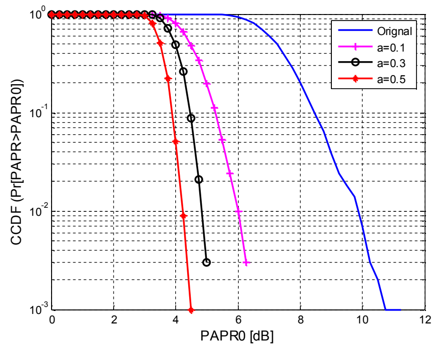

Fig.6 PAPR reduction performance of the CS-PS with

VI. CONCLUSION

In this paper, we proposed a combination sine pulse shaping (CS-PS) technique, which contains a linear term and three sine terms. Simulation results show that, the PAPR reduction performance of the CS-PS is improved compared with clipping, PTS, SLM, RC-PS and FE-PS.

REFERENCES

[1] E. Hong and D. Har, “Peak-to-Average Power Ratio Reduction in OFDM Systems Using All-Pass Filters”, IEEE Transactions On Broadcasting, vol. 56, n. 1, Mar. 2010.

[2] T. Tang, “The block coding used in OFDM system to reduce the PAPR,” in *WiCOM*, Sept, 2006, pp. 1-4.

[3] A. Goel. P. Gupta and M. Agrawal, “SER analysis of PTS based techniques for PAPR reduction in OFDM systems,” *Digital Signal Process Rev J*, vol. 23, n. 1, pp.368-369, Jan. 2013.

[4] D. W. Lim, J. S. No, C. W. Lim and H. Chung,“A new SLM OFDM scheme with low complexity for PAPR reduction,” *IEEE Signal Process Lett*, vol. 12, n. 2, pp. 93-96, Feb. 2005.

[5] P. Siming, Y. Z, Gang, Y. H. Shen and J. Wei, “Peak-to-average power ratio reduction for pulse-shaping OFDM systems,” in *WCSP*, Oct, 2015, pp. 1-5.

[6] T. Sultana and S. A. Shawkat, “Employment of pulse shaping techniques for efficient PAPR reduction in OFDM system,” in *ICCIT*, Dec, 2014.