Spectrum Maps for Cognition and Co-Existence of Communication and Radar Systems

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***Abstract*—An essential part of cognitive radios and radars is to build situation awareness of the radio spectrum. The state of the time-space-frequency varying spectrum may be presented as spatio-temporal spectrum maps that may be used to better utilize and share spectral resources among radar and communication systems, manage interference and avoid jamming. RF measurements made using multiple sensors in distinct locations are crucial in creating such spectrum maps. However, one also needs to know the state of the spectrum between the sensor locations and capture the dynamic behavior of spectrum in order to facilitate agile use of waveforms, interference management and co-existence of radar and communication systems. Advanced methods stemming from spatial statistics are developed for constructing and maintaining spectrum maps. The performance of the methods is studied in simulation.**

I. Introduction

Crowded radio spectrum is becoming a serious problem as more and more radio systems are developed to operate in spectrum bands with favorable propagation properties. This is partly caused by the rigid regulation of radio spectrum which does not allow flexible use of underutilized spectral resources. In order to deal with these problems, significant changes in spectrum regulation are being envisioned such that spectrum sharing and co-existence of different radio systems are allowed in large chunks of radio spectrum. Moreover, radio systems that are able to use idle spectrum in a flexible manner are being developed, for example, cognitive radio systems and joint radar-communication systems. Smart and dynamic spectrum sharing allows for utilizing the scarce spectral resources efficiently [1]. Awareness of the state of radio spectrum is a key enabler for efficient and agile spectrum use and co-existence of different radio systems. Acquiring such awareness is a challenging task because the state of the spectrum varies depending on time, frequency and location. Spectrum sensing and related inference about the state of the spectrum are necessary in order to build the awareness of radar spectrum environment. Co-operative sensing using multiple spatially distributed sensors makes the sensing more reliable through spatial diversity and captures the state of the spectrum in a broader geographical area.

A convenient way to represent spectrum awareness is spectrum cartography [2]–[5]. The goal is to estimate the spectrum state not just in a distinct measurement point at certain time instance but create a spectrum map which describes the state of spectrum at any location. This information may be utilized by secondary users, other radio systems sharing the same spectrum, or for surveillance purposes. In radar systems, the information could be used for optimizing the transmitted waveforms, allocation of power and sensor selection.

The basic idea is to use the measured values, for example the received signal strength (RSS) observed in many different sensor locations and then estimate or interpolate the RSS values over the whole region of interest. Spectrum cartography based on spatial interpolation has earlier been exploited in cognitive radios. In [2], [3] the well-know Kriging technique from geostatistics has been used. A method using Orthogonal Matching Pursuit method for the purpose is proposed in [6]. The time-dependent nature of the wireless communication radio environment has been considered in [7], where Kriged Kalman method was proposed. A space-frequency domain method for estimating spectral maps was proposed in [5].

The use of spectral maps has been considered mainly in the context of wireless communications, in particular, cognitive radios. However, adaptive and cognitive radar systems could significantly benefit from spectral maps, for example in mission planning, avoiding unintentional and intentional interference, radar resource allocation as well as in spectrum sharing in joint radar-communications systems. In radar systems, the use of spectrum cartography is largely unexplored. There are some significant differences how radars use the spectrum in comparison to communication systems. Depending on the radar task, radars may form very narrow beams in order to illuminate targets. Significantly higher transmit powers are used as well. Radars may also scan the operational environment using a rotating antenna system which introduces a periodic variation in the state of the spectrum. Some of the radars may be on a moving platform. Moreover, jamming and clutter can be time-frequency-location varying. Consequently, the spectrum map in the vicinity of a radar will be highly dynamic. Some recent studies on wireless communications in a shared spectrum environment between radar and communication systems, e.g. [8], [9] address the problem of accessing radar spectrum temporal holes for data transmission purposes. The basic idea is to estimate the radar scanning parameters and then adapt the communication parameters such that the spectrum opportunities are exploited and the transmission does not overlap with radar. This will lead to more efficient spectrum usage.

In this paper we develop a method for generating spectral maps for situations where communication and radar systems co-exist in same frequency. A novel algorithm is proposed to estimate how state of the spectrum evolves dynamically as function of time and location. The observations are made by multiple distributed radars or radio frequency sensors in distinct locations. The method stems from both spatial Kriging estimation method and parametric modeling. The output of the algorithm is a spatio-temporal spectral map which captures the periodic scanning patterns of radar beams and estimates the underlying static spectral map as well.

The rest of the paper is organized as follows. In Section II the system model is presented. In Section III a brief overview of spectrum cartography and spatial estimation using different variants of the Kriging method is provided. In Section IV simulation examples on generating the dynamic spectrum maps using the proposed algorithm are shown.

II. System model

In this paper we consider a system where communication and radar transmitters share the same frequency band at same time and same geographical area. An example, where two scanning radars and two omnidirectionally transmitting wireless communication transceivers co-exist in the same neighborhood, is shown in Figure 1. We assume the radars are scanning in full 360 degree circular or narrower sector mode, that implies periodicity which can be identified. However, we do not assume any a priori knowledge of the scanning parameters. Furthermore, we do not assume any prior knowledge of any wireless transmitters in the system.

In Figure 1 the sensor network nodes used to estimate the spectral maps are drawn with dots. The sensors drawn completely in red are intelligent sensors, for example surveillance sensors. The white dots with red borders indicate locations of simple spectrum sensors. We assume that the locations and timing of the estimates for all sensors are known. Furthermore, we assume all sensors can measure RSSI, and the intelligent sensors are assumed to provide also estimates of the angle of arrival (AoA). Timing information is assumed to be attached to the RSSI and AoA estimates, but all other parameters are estimated from these values. The distributed sensor network transmits the acquired measurements or local processing results to a fusion center. We also assume the fusion center has knowledge of the location of the sensors.

We assume the following propagation conditions: there is no shadowing in the radio propagation paths. Hence at any point $\mathbf{x}$, at any time t the received power from one transmitter is modeled in simplified manner as:

$$P_{rb}(t, \mathbf{x}) = \tilde{P}_b \frac{1}{d_{bx}^{\alpha}} I(t, b, \mathbf{x}), \tag{1}$$

where the d_{bx} is the distance from bth transmitter to the point $\mathbf{x}$ and α is the path-loss exponent. The $\tilde{P}_b$ denotes a constant, which includes e.g. the unknown transmit power and antenna gains. $I(t, b, \mathbf{x})$ is the indicator function, which is either 0 or 1 depending on whether the beam of radar transmitter b illuminates at time t or not. The received signal strength indicator (RSSI) at point point $\mathbf{x}$ is denoted as:

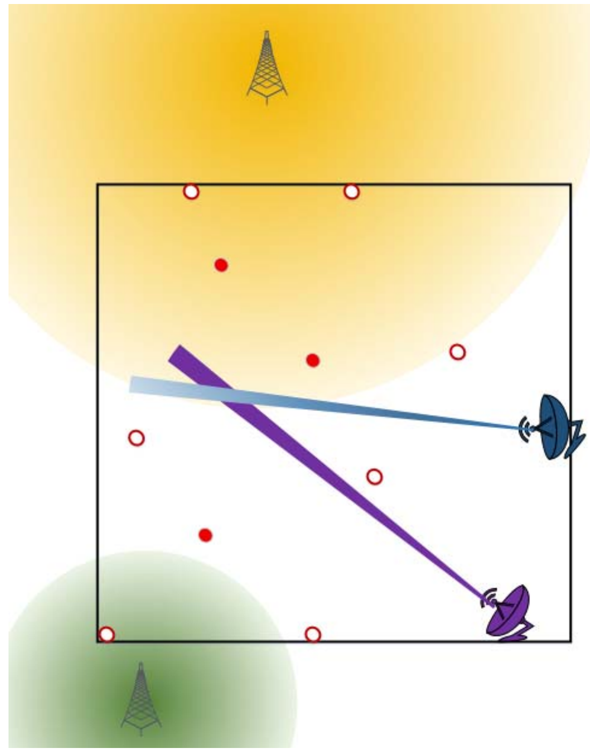

Fig. 1. Overview of the system model. Two wireless transceivers and two radars are sharing the same bandwidth and neighborhood as shown. The solid red dots are intelligent surveillance sensors, and the white dots with red border are simple spectrum sensors with less processing capability. The black square is the area of interest.

$$Z(t, \mathbf{x}) = P_r(t, \mathbf{x}) + n_t, \tag{2}$$

where t is the time, n_t is the noise term and P_r is the sum of received signal powers from all transmitters. The RSSI measurement at a sensor located at known position $\mathbf{x_i}$ and known time t_k is denoted as $\hat{Z}(k, i)$. Here i denotes the sensor index and k the time index.

In Figure 2 are shown examples of spatial spectrum maps with an assumptions of knowledge of all transmitter parameters and idealistic propagation model. White Gaussian noise is added to received signal at each location. The maximum coverage of each transmitter is limited based on antenna heights due to curvature of the earth. In order to visualize the temporal variation due to the scanning radars two different time instances are drawn. The time difference is one second, while the scanning sector for both radars is 90°, the cycle time is two seconds and the beamwidth is 1°.

III. Spectrum cartography and estimation

In order to create a spectrum map spectrum cartography (SC) can be used. Several different methods have been proposed for SC in the context of cognitive radios [2], [3], [5]–[7]. In most papers the unknown spatial values are estimated

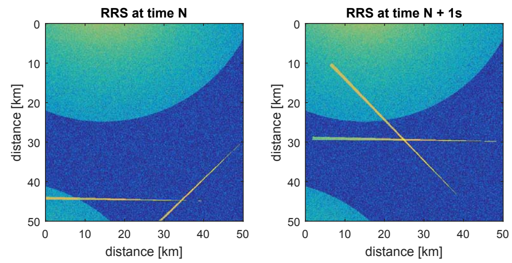

Fig. 2. RSSI calculated with the model with two omnidirectional transmitters and two scanning radars. The two Figures highlight the rapidly changing RSSI due to clockwise scanning radars.

via interpolation based on measurements provided by sensor network. Alternatively, the measurements may be used to estimate model parameters needed in constructing a spatial map. Temporal variation and dynamic modeling of spectrum maps is largely unexplored. In case of cognitive radio systems, spectrum map updating using Kalman filtering is proposed in [7]. However, as stated earlier the statistical and directional properties of radar transmissions and used power levels differ from wireless communications, hence this approach is not well suited to our case.

Capturing both the temporal and spatial variation in the state of the spectrum is a challenging task. Especially, in a system where radar signal introduces short temporal and spatial peaks in the spectrum. In order to cope with both static and dynamic parts of propagation environment we decouple the problem to two stages. We will first generate the spectral map using spatial Kriging interpolation in the same spirit as in cognitive radios. It will be enhanced with a dynamic component capturing the periodic scanning and other dynamic behavior of the radars.

A. Kriging interpolation

Kriging interpolation is a spatial statistics based interpolation method which takes spatial correlations into account. The underlying stochastic model assumes stationarity, at least locally. The three most commonly used kriging methods are simple, ordinary and universal kriging, denoted as SK, OK and UK [10], [11]. They are all based on knowledge (or estimate) of the spatial structure of second order statistic, i.e. covariance or variogram, and a set of known values at known locations. In our case, acquired measurements at distinct sensor locations are used. The unknown values at any other location are estimated as linear combinations of the measured values as follows:

$$\hat{Z}(\mathbf{x}) = \sum_{i=1}^{N} \lambda(i) \hat{Z}(\mathbf{x}_i). \tag{3}$$

Here $\lambda(i)$ are the Kriging weights and $\hat{Z}(\mathbf{x}_i)$ are the measurements at N known locations. These measurements are modeled as spatially dependent random variables (RV) denoted by $Z(\mathbf{x})$, where $\mathbf{x}$ is the location. Furthermore, $Z(\mathbf{x})$ can be decomposed into

$$Z(\mathbf{x}) = \mu(\mathbf{x}) + R(\mathbf{x}), \tag{4}$$

where $\mu(\mathbf{x})$ is the mean and $R(\mathbf{x})$ is a residual component, which is assumed to be stationary RV with zero mean. The different kriging methods model $\mu(\mathbf{x})$ differently. SK assumes the mean to be constant and known. OK assumes the mean is unknown but locally constant. In UK the mean is not constant even locally. It is assumed that there is a trend, hence the mean is not constant locally. The local trend is modeled as follows:

$$\mu(\mathbf{x}) = \sum_{l=0}^{L} \beta(l) f_l(\mathbf{x}). \tag{5}$$

Here $f_l(\mathbf{x})$ are known trend functions and $\beta(l)$ are weights to be defined. Typically $f_l(\mathbf{x})$ are low order polynomials of the location coordinates $\mathbf{x}$.

The spatial correlation is modeled with a semivariogram, defined as follows:

$$\gamma(h) = \frac{1}{2} E[\{Z(x+h) - Z(x)\}^2], \tag{6}$$

where h is the distance of two points. It can be defined based on measurements, or by deterministic functions that describe the spatial relationship based on the distance h between the points. In this paper we us commonly used spherical variogram model [3], [12]:

$$\gamma(h) = [\frac{3}{2}\frac{h}{a} - \frac{1}{2}(\frac{h}{a})^3], \text{if } \ h < a, \tag{7}$$

where a is the range parameter. It defines the maximum distance where two values are still correlated. Other well known variogram models are linear, exponential and Gaussian models.

With these definitions the Kriging weights, $\lambda(i)$, and the trend weights $\beta(l)$ are derived jointly by minimizing the estimation variance of the mean at known sensor positions. The unbiasedness of the estimator is obtained by setting constraint for the weights $\sum_i \lambda(i) = 1$. In this study we have used the UK method, whereas OK was used in [2], [3] in the context of cognitive radio.

B. RSSI measurements and stationary

In a radar system, such as the one considered in this paper, the measured RSSI values depend both on time instance and location of the sensors. Moreover, rapid changes in RSSI may occur. Hence, the assumption of stationarity typically employed in Kriging is clearly violated. As stated earlier we proposed to decouple the construction of the spectrum map to a dynamic and a static part. The spatial Kriging interpolation is applied to the static part that is assumed to remain stationary over the observation period.

Typical spectrum sensors simply average the RSSI measurements over a given period of time. In cognitive radios this might work, but with strong radar beams the mean values do not represent the static state. Furthermore, the mean is dependent on selection of the averaging time window length and might change noticeably from one period to the next. Intuitively, the median RSSI values are the ones that are more suitable choice in this case. However, median is more computationally demanding and may require buffering. If we use a threshold to find the temporal peaks, we can also estimate the static mean without the peak values. Hence, we can avoid the more complex median estimation. A simple algorithm is given in Table 1. An online implementation may be provided or the processing can be done for one measurement period. The threshold is set based on mean and variance of the measurements in the previous period, so no buffering is needed. Since the threshold is adaptive, the number of samples, K_s, in the averaging may change from one measurement period to the next. The mean could also be further averaged over several measurement periods e.g. by using a moving average filter.

The obtained average RSSI values, denoted as m_s in the algorithm, are further used as inputs for Kriging interpolation. The measurements from all the sensors are needed, hence the measurements need to be transmitted to a fusion center. Alternatively, they can only send the information of the change in the mean m_s. Additionally, the timing information of the peak RSSI values can be utilized in the fusion center for example for localization purposes or characterizing the periodicity of the scanning.

Algorithm 1 RSSI estimation

initialize
 mean rssi : $m_0 = \frac{1}{K}\sum_{k=1}^{T} \text{rssi}_k$
 variance rssi: $v_0 = \frac{1}{K}[\sum_{k=1}^{K} \text{rssi}_k^2] - m_0^2$
 define threshold : $\text{thr}_0 = m_0 + 2 * v_0$
Estimate for time steps $s = 1, 2, ...$
 $R_s = 0$; $S_s = 0$; $K_s = 0$;
 for t=1 : K
  if $\text{rssi}_s < \text{thr}_{s-1}$
   $R_s = R_{s-1} + \text{rssi}_k$ and $K_s = K_{s-1} + 1$
   $S_s = S_{s-1} + \text{rssi}_k^2$
 update mean rssi : $m_s = \frac{1}{K_s} R_s$
 update var rssi : $v_s = \frac{1}{K_s} S_s - m_s^2$
 update new threshold : $\text{thr}_s = m_s + 2 * v_s$

C. Identifying the radar parameters

In order to identify the radar operating parameters, we assume, there are a few intelligent surveillance sensors in our sensor network. They can send either raw measurements or other useful information to fusion center in order to estimate radar locations, power levels, periodicity and timing. In our simulations we have used measurement of RSSI, timing and angle of arrival (AoA) to estimate radar locations, cycle, angular scanning velocity and relative transmitted power. We assume that at least two of the intelligent sensors are in the scanning sector of each radar. Hence, radar locations can be estimated using triangulation. The time difference of arrival estimation is used to determine the periodicities and to check that the location candidates make sense.

The most challenging problem is to define the scanning sector, even though the location and periodicity have been successfully estimated. We can estimate from the angular velocity and the cycle period, how wide is the scanning sector. Furthermore, since we know that the radar beam illuminates at least two sensors we can estimate initial sector bounds, i.e. angles where the radar scans for sure. However, we don't exactly know the minimum or maximum scanning angles. We can narrow the possible scanning sector, by checking all timing information about the large RSSI values from all spectral sensors. If they match with the timing of a radar beam, we re-estimate the minimum and maximum scanning angles. In Figure 3 is shown an example to emphasize this situation. On left subfigure are the three surveillance sensors and radar location shown. From measurements we can find out where (and when) the radar beam is pointing while it is in sector between the black lines. Based on this minimum and maximun angles for the scan are set. But we also know that one cycle corresponds to larger sector. In the middle subfigure we have used the RSSI peak timing information of the spectral sensors to widen the sector. The right subfigure shows there is still an uncertainty region of the scanning sector left. Basically it means that for a period of time we don't know for sure if the radar is scanning the upper or the lower sector. How wide this sector is depends on the locations of the sensors and the accuracy of the estimates.

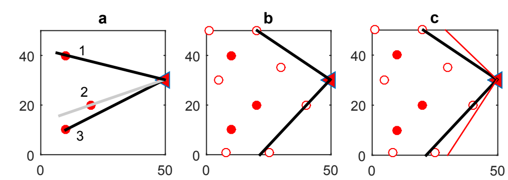

Fig. 3. Estimation of the radar scanning sector. In Figure a) only the intelligent sensors are used to estimate the scanning sector. In b) all sensors help to define the scanning sector. Based on the timing information the uncertainty in sectors is indicated with red lines.

Once we have estimated the radar parameters we can predict where the radar beam is at any time instant k. This can be done by evaluating the parametric model given in section II.

D. Constructing a spectrum map

The spatio-temporal spectrum map at time instant k is constructed by combining the static and dynamic parts. First, the static part is obtained by Universal Kriging method, see equation (3). As inputs to the interpolation we propose to use the RSSI measurement from all sensors, see algorithm 1. The interpolated map is denoted by $\mathcal{M}_{ik}$. The dynamic part of the spectrum map is obtained from the parametric model separately for each found radar. These radar maps are denoted as $\mathcal{M}_{rk}$, where r is the radar index and must be updated separately for each time step k.

In order to combine the static and dynamic part at time index k we select in each spatial location the maximum from all the maps. I.e. from $\mathcal{M}_{rk}$ and $\mathcal{M}_{ik}$. Since the static map does not change rapidly we can do the construction in two stages. The static map is interpolated less frequently where as the dynamic part updated in a regular basis. Whereas, the static part interpolation can be triggered only if the RSSI measurements indicate the static behavior has changed sufficiently.

Algorithm 2 for constructing a composite spectrum map $\mathcal{M}_k$ is described below.

Algorithm 2 Spectrum map estimation

Slow temporal updates - once each estimation interval δ_t
  measure RSSI over time window δ_t from all sensors
    if the RSSI values differ from previous
      re-interpolate the RSSI values, $\mathcal{M}_i$
  update the scanning radar parameters
    position, power, range, cycle, min, max angles
    define certain and uncertain regions
Fast temporal update every ϵ_t
  for each scanning radar r estimate $\mathcal{M}_r$
    in case of uncertain angle, define two angles
Combine radar maps to interpolated map
  $\mathcal{M}_k$ = max($\mathcal{M}_i$, max($\mathcal{M}_r$))

IV. SIMULATION RESULTS

We have studied the performance of the algorithm in simulation using the scenario shown in Figure 1. The following radar parameters are used: The scanning sector for both radars is $90°$, and the beamwidth is $1°$. The radars are focusing their operation in angular directions of $180°$ and $225°$, if $0°$ is defined to point to right. Each scan lasts 2 seconds. The transmission powers of the two radars are 8 dB stronger than the transmission powers of the omnidirectionally transmitting communication transmitters. The path-loss exponent is assumed to be $\alpha = 2$.

Total of ten sensors are placed in a distributed manner in the geographical area of interest. The measurement period for RSSI is $\delta_t = 3$ seconds. Unless otherwise stated, the algorithm 1 is used estimate the static mean from RSSI measurements. The location, cycle, timing and power of the radars are estimated based on the three intelligent surveillance sensors (red bullets). The scanning sector estimate is improved by using all ten available sensors.

In the simulations, Kriging interpolation uses data only from the 7 closest sensors. Universal Kriging is used with first and second order trend polynomials of the sensor position. We also computed results with ordinary Kriging for comparison. The spherical variogram model of equation (7) was used with range parameter 50 km. For the Kriging interpolation a Matlab toolbox MGSTAT [12] has been used.

First, in Figure 4, is shown how different RSSI measurement influence the static spectrum map estimated with the Kriging interpolation. Results are shown with three different RSSI estimates. In the left subfigure, simple mean RSSI without any modifications is used. In the middle subfigure, the proposed RSSI estimation approach is used and in the right subfigure median RSSI is used. These Figures clearly show how the strong radar signals decrease the accuracy of the spectrum map, unless the dynamic component is captured separately from the measurements.

Next, in Figure 5, a comparison of the ordinary kriging (OK) and universal kriging (UK) methods are shown. Only

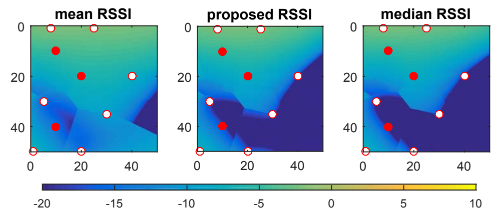

Fig. 4. Interpolating spectrum map with Universal kirging (UK) with spherical variogram and $N = 7$ know sensors. The estimated RSSI values are averaged over 3 seconds. Left subfigure RSSI is simple mean, in the center subfigure proposed RSSI estimator and in the right subfigure median RSSI.

the proposed RSSI value was used in these simulations. For reference, in the right subfigure spectrum map from the model with known parameters are shown. The Figures show clearly that taking the trend into account improves the performance. Naturally, if the number of sensors is increased, the performance of both methods will improve since the RSSI field is observed using more dense sampling. Eventually, with dense enough network the performance of OK becomes quite similar with the performance of UK.

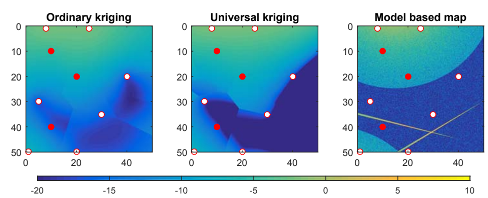

Fig. 5. Creating a spectrum map by interpolating with two different kriging methods. The proposed RSSI averaged over 3 second was used.

In Figure 6, is plotted a fully estimated spatio-temporal spectrum map for one time instant. For comparison, the model based map with known parameters is shown in the left subfigure. As seen, there is a small drift in the angle of the lower scanning radar. This is actually due to estimation error in locating the lower radar. The position of the other radar (in $[50, 30]$ km) was more accurately estimated. This is expected, since the intelligent sensors, plotted in red, are more conveniently located for estimating the location of that radar. Obviously there may be time-difference-of-arrival or other advanced emitter localization tools available, and consequently accurate location information.

In the next Figure 7, are shown three different time instances of the estimated spectral map. This Figure is shown to point out the uncertainty in identifying the bounds of the angular interval for scanning, which might occur if the sensor network is not dense enough. As a results, the instantaneous map seems to have two beams from one radar. This indicates the estimated angle is not known for sure. An example is seen in the middle subfigure 7, where the lower radar seems to have two beams. Only one of them is actually true, but with this sensor setup

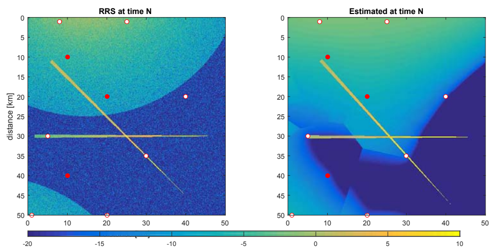

Fig. 6. On the left, a spectral map calculated from a model with known parameters and on the right estimated spectral map for the same time instant.

it was not possible to estimate which of the beams is the true beam. In the right subfigure, both radars' scanning angles have two options, hence total four beams are drawn.

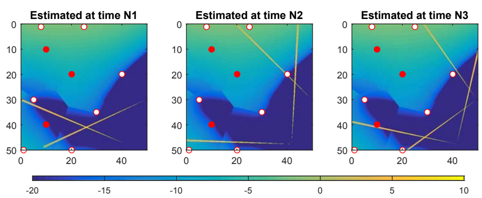

Fig. 7. The uncertainty of the scanning angles of the two radars in three different timesteps. In the left subfigure both the scanning angles are known accurately, in the middle subfigure the scanning angle of one radar is subject to uncertainty. Hence there are plotted two possible scanning angles. At time $N3$ the scanning angle of both radars is unclear, hence there are total four beams seen in the spectral map.

In Figure 8 is illustrated the contribution of a dynamic component caused by a radar to the spatio-temporal spectrum map. In the left subfigure, the spectrum map is constructed with standard measurements without thresholding. In the middle subfigure, the measurements have captured the dynamic behavior of the radar and removed it from the static spectral map. Finally, in the right subfigure, the static map is combined with dynamic map estimates from the two radar found by the algorithm.

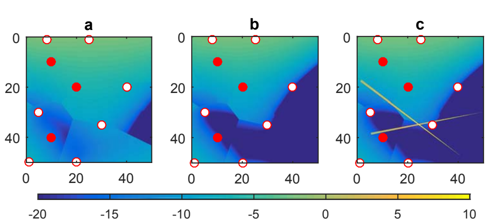

Fig. 8. The subfigure a) shows the situation if dynamics are not taken into account while constructing the spectral map. In the middle, subfigure b), static spectral map has been constructed with proposed RSSI estimation method to ignore the dynamic radar impact. The c) the composite spectral map which captures both dynamic and static behavior.

V. Conclusion

In this paper we developed a method for generating spectral maps for situations where communication and radar systems co-exist in same frequency. The method captures the dynamic behavior of the spectrum with scanning radars. We have combined spatial Kriging interpolation method with dynamic parametric model in order to capture rapid changes in the spectrum. Presented preliminary results obtained in simulations are promising. The simple method issued here was capable of characterizing state of the dynamic time and space varying spectrum usage over time and space. However, more simulations in various scenarios are needed. In our future work we will further develop techniques for capturing the dynamic component and find methods exploiting the identified spectral opportunities.

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