Iterative-Grouping and Image PTS for PAPR Reduction in OFDM System

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***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) which is widely used transmission technique for all 4G communication systems faces a major issue of Peak to average power ratio (PAPR).Partial Transmit Sequence (PTS)is the most preferred technique for the reduction of PAPR. But it involves complex searching algorithms for finding the most optimal combinations of OFDM signals. Increased complexity with any increase in the number of sub-blocks is a major drawback of PTS. In this paper, Iterative-Grouping and image-PTS (IGI-PTS) technique is proposed which mainly focuses on reducing the computational complexity involved in search of optimal phase factors. It is combination of two basic grouping and imaging techniques and further using iterations to simplify the searching process when the numbers of sub-blocks are in significantly high.**

I. INTRODUCTION

THIS paper proposes a modified Partial transmit sequence scheme which decreases the computational complexity. For increased demand of high rate data transmission 4G wireless communication systems were devised. OFDM[1] is the implementation technique which divides the main data stream into a number of data streams which are transmitted simultaneously over different sub-carriers. The sub-carriers in the OFDM system are independent of each other that is they are orthogonal to each other. The high rate data streams are transmitted over the sub-carriers. As the sub-carriers are independent of each other and also due to attachment of a cyclic prefix, ISI is prevented amongst the sub-carriers. These characteristics make OFDM desirable for wireless applications demanding high data rate transmission. Although it possesses one major drawback is high Peak to average power ratio (PAPR). High PAPR brings on OFDM signal distortion in the nonlinear region of high power amplifier (HPA)and high bit error rate(BER).

Partial Transmit Sequence [2], [3] is the most widely used technique for reducing PAPR in OFDM system. It has its own drawback of high complexity. While searching for an optimal set of signals offering low PAPR, PTS uses complex search algorithm which results in increased number of computations. Total computations further increase with any increase in number of sub-blocks. Apart from PTS there have been many schemes introduced to overcome the drawback of PAPR such as Amplitude Clipping and Filtering[10],Coding[8],constellation extension[11], Selected Mapping[3][9], Interleaving[3]. Several modifications have been done to reduce the complexity of PTS[4-5].

In this paper, an Iterative-grouping and image partial transmit sequence (IGI-PTS) scheme has been proposed which is developed on the basis of grouping phase weighting (GPW) [6]and reduced complexity computational partial transmit sequence (RCC-PTS) [7] and further enhances it using iterations .In GPW, all the sub-blocks are split into several groups, and each group can obtain its own sub-candidate sequences by using the same set of phase weighting factors; then, sub-candidate sequences from different groups are combined to generate all the OFDM candidate sequences. In RCC-PTS, a low complexity phase weighting process is implemented, and the key point is that the inherent relationship between phase weighting sequences is considered, which results in simplifying the computation for candidate signals. The process after combining these two schemes is shown to be implemented in iterations for system with larger number of sub-blocks.

II. OFDM SYSTEM AND PAPR PROBLEM

The main concept in OFDM is orthogonality of sub-carriers. The orthogonality allows simultaneous transmission on a lot of sub carriers in a tight frequency space without interference from each other. The OFDM signal with N number of sub-carriers is represented as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j2\pi f_k T} \tag{1}$$

Where X_k, k=0,1,2…N-1 are input symbols modulated by any of the suitable modulation schemes. The PAPR is the relation between the maximum power of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. When all the points achieve the maximum value simultaneously, this will cause the output envelope to suddenly shoot up which causes a 'peak' in the output envelope. Due to the presence of large number of independently modulated carriers in OFDM system, the peak value can be very high as compared to average value of whole system.

$$PAPR = \frac{\max_{n=1}^{N}\left(|z_n|^2\right)}{\frac{1}{N}\sum_{n=1}^{N}|z_n|^2} \tag{2}$$

Where: $\max_{n=1}^{N}\left(|Z_n|^2\right)$ is the peak signal power.

$\frac{1}{N}\sum_{n=1}^{N}|z_n|^2$ is the average signal power.

III. Conventional PTS

PTS is one of the most preferred techniques for PAPR reduction due to easier implementation. In the PTS technique, an input data block of N symbols is partitioned into disjoint sub-blocks. The subcarriers in each sub-block are weighted by a phase factor for that sub-block. The phase factors are selected such that the PAPR of the combined signal is minimized.
The partial transmit sequence (PTS) technique partitions an input data block of N symbols into V disjoint sub blocks as follows:

$$X = [X^0, X^1, X^2, ... X^{v-1}] \tag{3}$$

Where X^i are the sub-blocks that are consecutively located and also are of equal size.Scrambling (rotating its phase independently) is applied to each sub-block in the PTS technique. Then each partitioned sub-block is multiplied by a corresponding complex phase factor:

$$b^v = e^{j\phi V} \tag{4}$$

V=1,2,….V are the sub-blocks. Subsequently taking its IFFT to yield,

$$x = IFFT\{\sum_{v=1}^{V} b^v X^v\} = \sum_{v=1}^{V} b^v * IFFT\{X^v\} = \sum_{v=1}^{V} b^v X^v \tag{5}$$

In general, the selection of the phase factors is limited to a set of elements to reduce the search complexity.To optimize rest of *V-1* sub-blocks for each input sequence, overall W^{V-1} combinations are to be analyzed to select candidate with minimum PAPR.Each candidate requires *(V-1)* complex addition and multiplication .So total number of complex multiplication and additions are $W^{(V-1)} * (V-1)$ each.

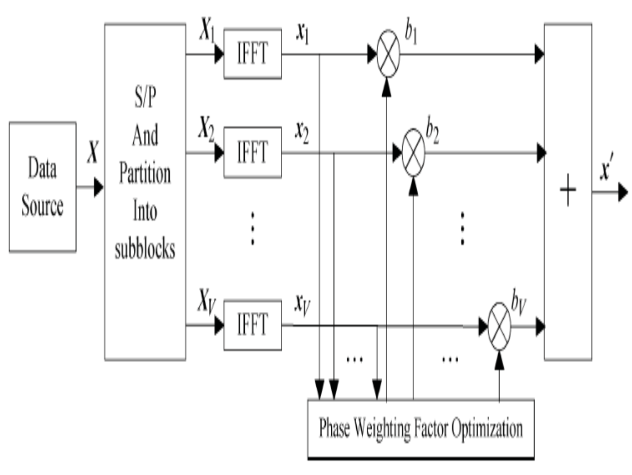

Fig 1: Block diagram of Conventional PTS

IV. Iterative-Grouping And Image Pts

An OFDM signal which is represented as,

$$x = \sum_{i=1}^{i=V} b_i x_i \tag{7}$$

It can be broken down to so that sub-block forms different groups and phase weighing process can be implemented for each group separately. For example, for V=5 and W=2 with phase weighing factors as {1,-1}, the sub-blocks can be divided to form three groups. Phase weighting sequences for each group will be as follows:

1st group:- $[1,1]^T \ [1,-1]^T$

2nd group:- $[1,1]^T \ [1,-1]^T \ [-1,1]^T \ [-1,-1]^T$

3rd group:- $[1]^T$

The sub-blocks will be multiplied with combination of phase factors {1,-1} as shown above and the equations for the so obtained groups are as follows,

$G_{11} = X_1 + X_2$ Number of additions= 1,

$G_{12} = X_1 - X_2$ Number of multiplication =1,

$G_{21} = X_3 + X_4$

$G_{22} = X_3 - X_4$ Number of additions= 1,

$G_{23} = -X_3 + X_4$ Number of multiplication=2,

$G_{24} = -X_3 - X_4$

$G_{31} = X_5$ Number of additions= 0,

Number of multiplication=0,

(8)

The equations above belong to all the 3 groups formed using 5 sub-blocks with the third group having only one sub-block. The output OFDM signals are given as follows,

$$Y_1 = G_{11} + G_{21} + G_{31} = X_1 + X_2 + X_3 + X_4 + X_5$$
$$Y_2 = G_{11} + G_{21} - G_{31} = X_1 + X_2 + X_3 + X_4 - X_5$$
$$Y_3 = G_{11} + G_{22} + G_{31} = X_1 + X_2 + X_3 - X_4 + X_5$$
$$Y_4 = G_{11} + G_{22} - G_{31} = X_1 + X_2 + X_3 - X_4 - X_5$$

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$$Y_{13} = G_{12} + G_{23} + G_{31} = X_1 - X_2 - X_3 + X_4 + X_5$$
$$Y_{14} = G_{12} + G_{23} - G_{31} = X_1 - X_2 - X_3 + X_4 - X_5$$
$$Y_{15} = G_{12} + G_{24} + G_{31} = X_1 - X_2 - X_3 - X_4 + X_5$$

$$Y_{16} = G_{12} + G_{24} - G_{31} = X_1 - X_2 - X_3 - X_4 - X_5 \quad (9)$$

For all the above equations,
Number of additions= 2,

Further it is observed that the group equations in (8) bears relation of opposite sign which is,

$$G_{24} = -G_{21}$$
$$G_{23} = -G_{22}$$

These output OFDM signals can further be reduced by using this concept,

$$\begin{aligned}
Y_1 &= G_{11} + G_{21} + G_{31} = X_1 + X_2 + X_3 + X_4 + X_5\\
Y_2 &= G_{11} + G_{21} - G_{31} = X_1 + X_2 + X_3 + X_4 - X_5\\
Y_3 &= G_{11} + G_{22} + G_{31} = X_1 + X_2 + X_3 - X_4 + X_5\\
Y_4 &= G_{11} + G_{22} - G_{31} = X_1 + X_2 + X_3 - X_4 - X_5\\
Y_5 &= G_{11} + G_{23} + G_{31} = X_1 + X_2 - X_3 + X_4 + X_5 = G_{11} - G_{22} + G_{31}\\
Y_6 &= G_{11} + G_{23} - G_{31} = X_1 + X_2 - X_3 + X_4 - X_5 = G_{11} - G_{22} - G_{31}\\
Y_7 &= G_{11} + G_{24} + G_{31} = X_1 + X_2 - X_3 - X_4 + X_5 = G_{11} - G_{21} + G_{31}\\
Y_8 &= G_{11} + G_{24} - G_{31} = X_1 + X_2 - X_3 - X_4 - X_5 = G_{11} - G_{21} - G_{31}\\
Y_9 &= G_{12} + G_{21} + G_{31} = X_1 - X_2 + X_3 + X_4 + X_5\\
Y_{10} &= G_{12} + G_{21} - G_{31} = X_1 - X_2 + X_3 + X_4 - X_5\\
Y_{11} &= G_{12} + G_{22} + G_{31} = X_1 - X_2 + X_3 - X_4 + X_5\\
Y_{12} &= G_{12} + G_{22} - G_{31} = X_1 - X_2 + X_3 - X_4 - X_5\\
Y_{13} &= G_{12} + G_{23} + G_{31} = X_1 - X_2 - X_3 + X_4 + X_5 = G_{12} - G_{22} + G_{31}\\
Y_{14} &= G_{12} + G_{23} - G_{31} = X_1 - X_2 - X_3 + X_4 - X_5 = G_{12} - G_{22} - G_{31}\\
Y_{15} &= G_{12} + G_{24} + G_{31} = X_1 - X_2 - X_3 - X_4 + X_5 = G_{12} - G_{21} + G_{31}\\
Y_{16} &= G_{12} + G_{24} - G_{31} = X_1 - X_2 - X_3 - X_4 - X_5 = G_{12} - G_{21} - G_{31}
\end{aligned} \quad (10)$$

This particular stage is what we call as first iteration. In order to reduce the computations further we can move to the second iteration where we take the following grouping,

$$\begin{aligned}
S_{11} &= G_{21} + G_{31}\\
S_{12} &= G_{21} - G_{31}\\
S_{21} &= G_{22} + G_{31}\\
S_{22} &= G_{22} - G_{31}
\end{aligned} \quad (11)$$

Since this case contains 3 groups i.e. odd number of groups we have to spare G_{11} while grouping. This grouping will further help in reducing computations as follows

$$\begin{aligned}
Y_1 &= G_{11} + S_{11}\\
Y_2 &= G_{11} + S_{12}\\
Y_3 &= G_{11} + S_{21}\\
Y_4 &= G_{11} + S_{22}\\
Y_5 &= G_{11} - S_{22}\\
Y_6 &= G_{11} - S_{21}\\
Y_7 &= G_{11} - S_{12}\\
Y_8 &= G_{11} - S_{11}\\
Y_9 &= G_{12} + S_{11}\\
Y_{10} &= G_{12} + S_{12}\\
Y_{11} &= G_{12} + S_{21}\\
Y_{12} &= G_{12} + S_{22}\\
Y_{13} &= G_{12} - S_{22}\\
Y_{14} &= G_{12} - S_{21}\\
Y_{15} &= G_{12} - S_{12}\\
Y_{16} &= G_{12} - S_{11}
\end{aligned} \quad (12)$$

For all the above equations,

Number of additions=1

A. *For Original PTS*

Number of additions=64
Number of Multiplications=64

B. *For Proposed PTS scheme*

Number of additions=24
Number of multiplications=6

For addition,

$$CCRR = \left\{1 - \frac{24}{64}\right\} \times 100 = 62.5\%$$

For multiplication,

$$CCRR = \left\{1 - \frac{6}{64}\right\} \times 100 = 90.625\%$$

The proposed scheme shows improvement in CCRR in both the cases. In a similar way it can be applied for V=8 and W=2. The sub-block will be multiplied with combination of phase factors {1,-1} and the equations for the groups so obtained are as follows,

$$\left.\begin{aligned}
G_{11} &= X_1 + X_2\\
G_{12} &= X_1 - X_2
\end{aligned}\right\} \text{Number of additions= 1, Number of multiplication= 1,}$$

,

$$\left.\begin{aligned}
G_{21} &= X_3 + X_4\\
G_{22} &= X_3 - X_4\\
G_{23} &= -X_3 + X_4\\
G_{24} &= -X_3 - X_4\\
G_{31} &= X_5 + X_6\\
G_{32} &= X_5 - X_6\\
G_{33} &= -X_5 + X_6
\end{aligned}\right\} \text{Number of additions= 1, Number of Multiplication=2,}$$

$$\left.\begin{array}{ll} G_{34}=-X_5-X_6 & \\ G_{41}=X_7+X_8 & \text{Number of additions= 1,} \\ G_{42}=X_7-X_8 & \text{Number of Multiplication=2} \\ G_{43}=-X_7+X_8 & \\ G_{44}=-X_7-X_8 & , \end{array}\right\} \tag{13}$$

the group equations in (13) bear relation of opposite sign which is,

$$\begin{aligned} G_{31} &= -G_{34} \\ G_{32} &= -G_{33} \\ G_{41} &= -G_{44} \\ G_{42} &= -G_{43} \end{aligned} \tag{14}$$

These output OFDM signals can further be reduced by using this concept,

$$\begin{aligned}
Y_1 &= G_{11}+G_{21}+G_{31}+G_{41} \\
&= X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8 \\[1em]
Y_2 &= G_{11}+G_{21}+G_{31}+G_{42} \\
&= X_1+X_2+X_3+X_4+X_5+X_6+X_7-X_8 \\[1em]
Y_3 &= G_{11}+G_{21}+G_{31}+G_{43} \\
&= X_1+X_2+X_3+X_4+X_5+X_6-X_7+X_8 \\
&= G_{11}+G_{21}+G_{31}-G_{42} \\[1em]
Y_4 &= G_{11}+G_{21}+G_{31}+G_{44} \\
&= X_1+X_2+X_3+X_4+X_5+X_6-X_7-X_8 \\
&= G_{11}+G_{21}+G_{31}-G_{41} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
Y_{125} &= G_{12}+G_{24}+G_{34}+G_{41} \\
&= X_1-X_2-X_3-X_4-X_5-X_6+X_7+X_8 \\
&= G_{12}-G_{21}-G_{31}+G_{41} \\[1em]
Y_{126} &= G_{12}+G_{24}+G_{34}+G_{42} \\
&= X_1-X_2-X_3-X_4-X_5-X_6+X_7-X_8 \\
&= G_{12}-G_{21}-G_{31}+G_{42} \\[1em]
Y_{127} &= G_{12}+G_{24}+G_{34}+G_{43} \\
&= X_1-X_2-X_3-X_4-X_5-X_6-X_7+X_8 \\
&= G_{12}-G_{21}-G_{31}-G_{42} \\[1em]
Y_{128} &= G_{12}+G_{24}+G_{34}+G_{44} \\
&= X_1-X_2-X_3-X_4-X_5-X_6-X_7-X_8 \\
&= G_{12}-G_{21}-G_{31}-G_{41}
\end{aligned} \tag{15}$$

The equations above belong to the first iteration, further they can be reduced by second iteration. The groups in second iteration will be,

$$\begin{array}{ll}
S_{11}=G_{11}+G_{21} & S_{31}=G_{31}+G_{41} \\
S_{12}=G_{11}-G_{21} & S_{32}=G_{31}-G_{41} \\
S_{13}=G_{11}+G_{22} & S_{33}=G_{31}+G_{42} \\
S_{14}=G_{11}-G_{22} & S_{34}=G_{31}-G_{42} \\
S_{21}=G_{12}+G_{21} & S_{41}=G_{32}+G_{41} \\
S_{22}=G_{12}-G_{21} & S_{42}=G_{32}-G_{41} \\
S_{23}=G_{12}+G_{22} & S_{43}=G_{32}+G_{42} \\
S_{24}=G_{12}-G_{22} & S_{44}=G_{32}-G_{42}
\end{array} \tag{16}$$

Since this case contains 4 groups i.e. even number of groups we have to include G_{11} while grouping. This grouping will further help in reducing computations as follows,

$$\left.\begin{array}{ll}
Y_1 = S_{11}+S_{31} & \\
Y_2 = S_{11}+S_{33} & , \\
Y_3 = S_{11}+S_{34} & \\
Y_4 = S_{11}+S_{32} & \text{Number of Additions=1,} \\
\ldots\ldots\ldots\ldots & \\
Y_{125} = S_{22}-S_{32} & \\
Y_{126} = S_{22}-S_{34} & \\
Y_{127} = S_{22}-S_{33} & \\
Y_{128} = S_{22}-S_{31} &
\end{array}\right\} \tag{17}$$

A. For Original PTS

Number of additions = 896
Number of multiplications = 896

B. For Proposed Scheme

Number of additions = 152
Number of multiplications = 14

For addition,

$$CCRR = \left\{1-\frac{82}{384}\right\}\times 100 = 78.6\%$$

For multiplication,

$$CCRR = \left\{1-\frac{14}{384}\right\}\times 100 = 98.43\%$$

The reduction in computational complexity in new scheme is done in terms of CCRR. Values of CCRR are given below in Table 1.

V. PERFORMANCE ANALYSIS

The reduction in computational complexity in new scheme is done in terms of CCRR. Values of CCRR are given below in Table 1.

Table 1 CCRR Comparison of proposed scheme

SUB BLOCKS AND PHASE VECTORS	CCRR FOR MULTIPLICATION	CCRR FOR ADDITION
V=5,W=2	90.625	62.5
V=6,W=2	93.75	71.25
V=7,W=2	97.39	78.6
V=8,W=2	98.43	83.03

The CCRR values for multiplication and addition show appreciable improvement which shows that the scheme has provided better results than the conventional PTS scheme.

As far as Multiplicative CCRR is concerned for weight set {1,-1} there is significant reduction in terms of computational complexity. And as the number of sub-blocks increase the CCRR gets improvised. For example, for V=5sub blocks the CCRR is reduced by a margin of 62.5% and for subsequent increase in block size V = 6, 7, 8 the addition complexity is reduced by 71.25,78.6 and 83.03% respectively, which is very inspiring. CCRR for multiplications, the values are in the range of 90.62 to 98.43% So, this scheme overall works better as the system becomes larger and larger.

VI. CONCLUSION

In this paper, Iterative grouping and image partial transmit sequence (IGI-PTS) scheme is proposed for PAPR reduction of OFDM signals. The scheme is developed on the basis of concepts of grouping and opposite signs amongst phase factors. Application of Iterations further help in decrementing the computational complexity existing in original PTS and its variants. The CCRR values tabulated in the table 2 clearly shows drastic decrease in computation complexity without any reduction in PTS candidate signals.

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