Achieving Real-Time Efficiency for Adaptive Radar Pulse Compression

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Abstract—**The Adaptive Pulse Compression (APC) algorithm has been shown to effectively suppress the range sidelobes resulting from large scatterers to the level of the noise thereby unmasking nearby small scatterers and yielding substantial sensitivity improvement over deterministic filtering techniques. However, implementation in a real-time system is limited by the relatively high computational complexity of the algorithm. As such, this paper considers dimensionality reduction techniques as a means to reduce this computational cost in order to enable real-time operation. In general, numerous different methods exist to decompose the Minimum Mean-Square Error (MMSE) framework upon which the APC algorithm is based. In this paper dimensionality reduction is achieved either by segregating contiguous blocks of the received signal samples or by decimating the received signal. Each of the two approaches then facilitate the approximation of the original MMSE cost function of APC by the sum of lower dimension MMSE cost functions thus yielding filters that require a lower computational cost to implement. The reduced dimension algorithms, collectively denoted as the Fast Adaptive Pulse Compression (FAPC) algorithm, substantially reduce the computational cost of APC while still maintaining much of its performance benefit.**

I. Introduction

Pulse compression allows a long modulated transmit pulse to achieve the range resolution of a short pulse while providing adequate receive energy for detection. For a solitary point scatterer in white noise, the SNR of the pulse compressed output is maximized by the matched filter [1]. However, in a more realistic scenario containing multiple scatterers of varying magnitude, the range sidelobes induced by a large scatterer can mask the presence of nearby small scatterers thus limiting radar sensitivity.

Recently, the Adaptive Pulse Compression (APC) algorithm [2] was developed via a Minimum Mean-Square Error (MMSE) formulation in order to adaptively suppress the range sidelobes from large scatterers thereby unmasking nearby small scatterers. The elimination of range sidelobes is achieved by adaptively estimating a pulse compression receive filter for each individual range cell using the relative power estimates of surrounding range cells that is obtained from initial matched filtering or a previous adaptive stage. For a given range cell the respective filter places nulls at the relative range offsets corresponding to nearby large scatterers thus suppressing the range sidelobes that are induced by the large scatterers. The APC algorithm has been shown [2] to be superior to standard matched filtering, mismatched filtering [3], and Least-Squares estimation [4].

The computational cost of the APC algorithm limits its use in most current real-time systems. Following standard matched filtering, APC utilizes the resulting range profile estimate to adaptively determine the MMSE receive filter for each individual range cell. This filter simultaneously maximizes SNR and suppresses the range sidelobe interference from nearby scatterers thus resulting in a substantially more accurate estimate of the range cell complex amplitude. This process may be repeated as desired though it has been found that a single adaptive stage significantly reduces range sidelobes and three adaptive stages removes virtually all sidelobe interference resulting from numerous exceedingly large scatterers. The computational cost of APC per adaptive stage is quadratic in the length N (in samples) of the transmit waveform [2]. In this paper, dimensionality reduction techniques are employed to reduce the computational cost of APC. From a conceptual standpoint, similar endeavors have been considered for Space-Time Adaptive Processing (STAP) which must also contend with the cost of adapting on a high dimensionality space [5]. By applying dimensionality reduction via a segmented approximation to the original MMSE cost function, the Fast Adaptive Pulse Compression (FAPC) algorithm is developed. While numerous possible variations of FAPC exist, we shall consider two particular embodiments.

The Contiguous FAPC algorithm is obtained by segregating each set of N received signal samples into M blocks of contiguous samples. A different adaptive receive filter is then determined for each block. Likewise, the Decimation FAPC algorithm is obtained by decimating the N received signal samples into M blocks with a distinct adaptive filter determined for each block. These embodiments of the FAPC algorithm enable efficient implementation schemes resulting in as much as an order of magnitude reduction in the computation cost of adaptive pulse compression while still maintaining much of the performance benefit. In addition, Contiguous FAPC is observed to provide improved robustness to Doppler mismatch due to its adaptivity over a time span shorter than the full temporal extent of the transmitted waveform.

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II. Received Signal Model

A. Full-Dimensionality Model

The received radar return at the ℓ^{th} range cell can be defined as

$$y(\ell) = \mathbf{x}^T(\ell)\,\mathbf{s} + v(\ell) \tag{1}$$

for $\ell = 0, \cdots, L+N-2$, where $\mathbf{s} = [s_0\ s_1\ \cdots\ s_{N-1}]^T$ is the length N transmit waveform, $\mathbf{x}(\ell) = [x(\ell)\ x(\ell-1)\ \cdots\ x(\ell-N+1)]^T$ is the portion of the range profile that the transmitted waveform $\mathbf{s}$ convolves with at delay ℓ, $v(\ell)$ is additive noise, $(\bullet)^T$ is the transpose operation, and L is the number of range cells of interest. Collecting N consecutive samples of the received radar return signal, the full-dimension received signal model can be expressed as

$$\mathbf{y}(\ell) = \mathbf{X}^T(\ell)\,\mathbf{s} + \mathbf{v}(\ell) \tag{2}$$

where $\mathbf{v}(\ell) = [v(\ell)\ v(\ell+1)\ \cdots\ v(\ell+N-1)]^T$ and

$$\mathbf{X}(\ell) = [\mathbf{x}(\ell)\ \ \mathbf{x}(\ell+1)\ \cdots\ \mathbf{x}(\ell+N-1)] = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+N-1) \\ x(\ell-1) & x(\ell) & \cdots & x(\ell+N-2) \\ \vdots & \vdots & \ddots & \vdots \\ x(\ell-N+1) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix}. \tag{3}$$

This full-dimension signal model will be referred to when deriving the two different reduced-dimensionality models.

B. Reduced-Dimensionality Models

The reduced-dimension signal models are formed by subdividing the full-dimension model. This is accomplished by segregating the N received signal samples of $\mathbf{y}(\ell)$ into M segments of length $N/M = K$ using either contiguous blocking or decimation.

The decimated model of the received signal is constructed by decimating the full-dimension received signal from (2) as

$$\tilde{\mathbf{y}}_{D,m}(\ell) = [y(\ell+m)\ \ y(\ell+M+m)\ \cdots\ y(\ell+N-2M+m)\ \ y(\ell+N-M+m)]^T \tag{4}$$

for $m = 0, 1, \cdots, M-1$. Thus, the matrix $\mathbf{X}(\ell)$ from (3) is partitioned into M sub-matrices of dimension $N \times K$ as

$$\tilde{\mathbf{X}}_{D,m}(\ell) = [\mathbf{x}(\ell+m)\ \ \mathbf{x}(\ell+M+m)\ \cdots\ \mathbf{x}(\ell+N-2M+m)\ \ \mathbf{x}(\ell+N-M+m)] \tag{5}$$

for $m = 0, 1, \cdots, M-1$. The noise vector $\mathbf{v}(\ell)$ is likewise segmented as

$$\tilde{\mathbf{v}}_{D,m}(\ell) = [v(\ell+m)\ \ v(\ell+M+m)\ \cdots\ v(\ell+N-2M+m)\ \ v(\ell+N-M+m)]^T \tag{6}$$

thus producing the decimated received signal model

$$\tilde{\mathbf{y}}_{D,m}(\ell) = \tilde{\mathbf{X}}^T_{D,m}(\ell)\,\mathbf{s} + \tilde{\mathbf{v}}_{D,m}(\ell). \tag{7}$$

In a similar manner the contiguous blocking model of the received signal is constructed by segmenting the full-dimension received signal from (2) as

$$\tilde{\mathbf{y}}_{C,m}(\ell) = [y(\ell+Km)\ \ y(\ell+Km+1)\ \cdots\ y(\ell+Km+K-2)\ \ y(\ell+Km+K-1)]^T \tag{8}$$

for $m = 0, 1, \cdots, M-1$. As such, the matrix $\mathbf{X}(\ell)$ from (3) is partitioned into M sub-matrices of dimension $N \times K$ as

$$\tilde{\mathbf{X}}_{C,m}(\ell) = [\mathbf{x}(\ell+Km)\ \ \mathbf{x}(\ell+Km+1)\ \cdots\ \mathbf{x}(\ell+Km+K-2)\ \ \mathbf{x}(\ell+Km+K-1)] \tag{9}$$

for $m = 0, 1, \cdots, M-1$. The contiguously blocked noise vector is similarly segmented as

$$\tilde{\mathbf{v}}_{C,m}(\ell) = [v(\ell+Km)\ \ v(\ell+Km+1)\ \cdots\ v(\ell+Km+K-2)\ \ v(\ell+Km+K-1)]^T \tag{10}$$

thereby yielding the contiguously blocked received signal model

$$\tilde{\mathbf{y}}_{C,m}(\ell) = \tilde{\mathbf{X}}^T_{C,m}(\ell)\,\mathbf{s} + \tilde{\mathbf{v}}_{C,m}(\ell). \tag{11}$$

The two reduced-dimension received signal models of (7) and (11) will be employed to derive two particular embodiments of the Fast Adaptive Pulse Compress (FAPC) algorithm.

III. Fast Adaptive Pulse Compression

The Minimum Mean-Square Error (MMSE) cost function employed for the original APC algorithm [2] is

$$J(\ell) = E\left[\left|x(\ell) - \mathbf{w}^H(\ell)\,\mathbf{y}(\ell)\right|^2\right] \tag{12}$$

for $\ell = 0, 1, \cdots, L-1$, where $\mathbf{w}(\ell)$ is the N-dimensional MMSE pulse compression filter used to estimate $x(\ell)$ and $E[\bullet]$ is the expectation operator.

For the two embodiments of the FAPC method, the original MMSE cost function of (12) is approximated as the sum of M components where each component is the MMSE cost function for a length $K = \frac{N}{M}$ segment of the MMSE filter. The aggregate approximate cost function is thus

$$\tilde{J}(\ell) = \sum_{m=0}^{M-1} E\left[\left|\frac{1}{M}x(\ell) - \tilde{\mathbf{w}}_m^H(\ell)\tilde{\mathbf{y}}_m(\ell)\right|^2\right] \tag{13}$$

where $\tilde{\mathbf{w}}_m(\ell)$ is the m^{th} K-length segment (according to either decimation or contiguous blocking) of the N-length piecewise MMSE filter $\tilde{\mathbf{w}}(\ell)$ and $\tilde{\mathbf{y}}_m(\ell)$ is the m^{th} K-length segment of the N-length $\mathbf{y}(\ell)$ also according to either decimation or contiguous blocking.

Minimization of the piecewise MMSE cost function of (13) yields the piecewise MMSE pulse compression filter $\tilde{\mathbf{w}}(\ell)$, the m^{th} K-length segment of which is denoted as

$$\tilde{\mathbf{w}}_m(\ell) = \left(E\left[\tilde{\mathbf{y}}_m(\ell)\,\tilde{\mathbf{y}}_m^H(\ell)\right]\right)^{-1} E\left[\frac{1}{M}\tilde{\mathbf{y}}_m(\ell)\,x^*(\ell)\right] \tag{14}$$

for $m = 0,1,\cdots,M-1$, where $(\bullet)^*$ is the complex conjugate operation. The following details the particular form realized by (14) when employing decimation and contiguous blocking, respectively.

A. Decimation FAPC

For the decimation embodiment of FAPC the m^{th} filter segment $\tilde{\mathbf{w}}_m(\ell)$ in (14) is given by

$$\tilde{\mathbf{w}}_m(\ell) = \left[w_m \;\; w_{M+m} \;\; w_{2M+m} \;\; \cdots \;\; w_{N-2M+m} \;\; w_{N-M+m}\right]^T \tag{15}$$

for $m = 0,1,\cdots,M-1$. For example, with $N = 12$ and $M = 3$ so that $K = 12/3 = 4$, the Decimation FAPC filter is depicted in Fig. 1.

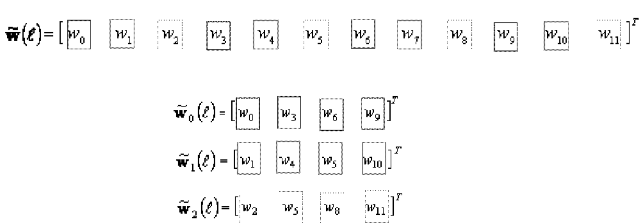

Fig. 1 Coefficient allocation for the Decimation FAPC filter

Substituting the decimated received signal model $\tilde{\mathbf{y}}_m(\ell)$, from (7), into (13) and assuming that the range cells are, in general, uncorrelated with one another and also uncorrelated with the noise, the decimated filter segments are obtained as

$$\tilde{\mathbf{w}}_m(\ell) = \frac{1}{M}\rho(\ell)\left(\tilde{\mathbf{C}}_m(\ell) + \tilde{\mathbf{R}}_m\right)^{-1}\mathbf{s}_m \tag{16}$$

where $\rho(\ell) = E\left[\left|x(\ell)\right|^2\right]$ is the expected power of $x(\ell)$, $\tilde{\mathbf{R}}_m = E\left[\tilde{\mathbf{v}}_{D,m}(\ell)\,\tilde{\mathbf{v}}_{D,m}^H(\ell)\right]$ is the decimated noise covariance matrix based on (6), and the decimated signal correlation matrix $\tilde{\mathbf{C}}_m(\ell)$ is

$$\tilde{\mathbf{C}}_m(\ell) = \sum_{i=0}^{M-1}\sum_{k=-K+1}^{K-1}\rho(\ell + Mk - i + m)\,\mathbf{s}_{i,k}\mathbf{s}_{i,k}^H \tag{17}$$

where $\mathbf{s}_{i,k}$ is the i^{th} decimated segment of waveform $\mathbf{s}$ delay shifted by k, *e.g.* for $N = 6$ and $M = 2$ so that $K = 6/2 = 3$ Fig. 2 illustrates the decomposition of $\mathbf{s}$ into its decimated and delay-shifted components.

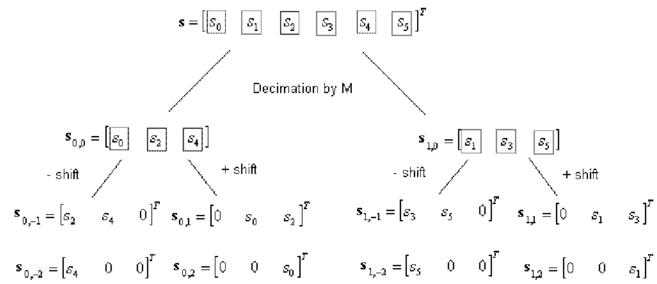

Fig. 2 Decimated decomposition of transmit waveform

B. Contiguous FAPC

For the contiguous blocking version of FAPC the m^{th} filter segment $\tilde{\mathbf{w}}_m(\ell)$ in (14) is given by

$$\tilde{\mathbf{w}}_m(\ell) = \left[w_{Km} \;\; w_{Km+1} \;\; w_{Km+2} \;\; \cdots \;\; w_{Km+K-2} \;\; w_{Km+K-1}\right]^T \tag{18}$$

for $m = 0,1,\cdots,M-1$. For example, given $N = 12$, $M = 3$ so that $K = 12/3 = 4$, the Contiguous FAPC filter is shown in Fig. 3.

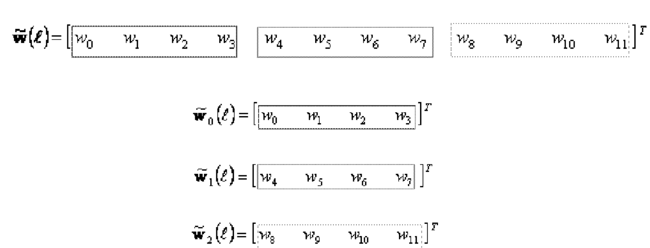

Fig. 3 Coefficient allocation for the Contiguous FAPC filter

Substituting the contiguous received signal model $\tilde{\mathbf{y}}_m(\ell)$, from (11), into (13) and assuming again that the range cells are, in general, uncorrelated with one another and also uncorrelated with the noise, the contiguously blocked filter segments are given as

$$\tilde{\mathbf{w}}_m(\ell) = \frac{1}{M}\rho(\ell)\left(\tilde{\mathbf{C}}_m(\ell) + \tilde{\mathbf{R}}_m\right)^{-1}\mathbf{s}_m \tag{19}$$

for $m = 0, 1, \cdots, M-1$, where $\rho(\ell) = E\left[\left|x(\ell)\right|^2\right]$ is the expected power of $x(\ell)$, $\tilde{\mathbf{R}}_m = E\left[\tilde{\mathbf{v}}_{C,m}(\ell)\,\tilde{\mathbf{v}}_{C,m}^H(\ell)\right]$ is the contiguously blocked noise covariance matrix based on (10), and the contiguously blocked signal correlation matrix is

$$\tilde{\mathbf{C}}_m(\ell) = \sum_{k=-N+1}^{K-1} \rho(\ell + k + Km)\,\mathbf{s}_k\mathbf{s}_k^H \tag{20}$$

where $\mathbf{s}_k$ is the k^{th} delay shift of the K-length contiguously blocked segment of the transmitted waveform $\mathbf{s}$, *e.g.* for $N = 6$ and $K = 3$ Fig. 4 illustrates the decomposition of $\mathbf{s}$ into its contiguous delay-shifted components.

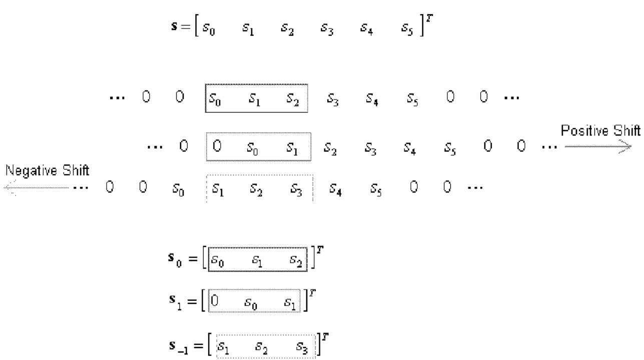

Fig. 4 Contiguous decomposition of transmit waveform

IV. IMPLEMENTATION

The reduced dimensionality afforded by FAPC can markedly reduce the computational complexity relative to the full-dimension APC algorithm while maintaining much of the performance benefit. Each of the M reduced-dimension matrices $\left(\tilde{\mathbf{C}}_m(\ell) + \tilde{\mathbf{R}}_m\right)$ are $K \times K$ and thus incur much lower computational cost to invert when compared to the $N \times N$ matrix $\left(\mathbf{C}(\ell) + \mathbf{R}\right)$ in the original APC algorithm [2]. A fast matrix update can also be used to reduce the computation of the matrix inverse in the same manner as the original APC algorithm [2]. When this fast update is utilized, APC incurs an average computational cost of $O\left(4N^2\right)$ complex operations per adaptive stage for each range cell. Furthermore, the FAPC algorithm can be implemented such that only a single reduced-dimension correlation matrix requires updating at each successive range cell where the remaining (M–1) matrices were previously determined to estimate a different portion of the piecewise filter for a prior range cell and therefore can simply be replaced with the previous matrices. It can be shown that when the fast update and matrix replacement are utilized as well as some moderate parallel processing the average computational cost per adaptive stage for FAPC is $O\left(4K^2 + N\right)$ for each range cell. Figure 5 illustrates the computational efficiency that can be achieved using FAPC compared to the original APC and the standard matched filter. As a general rule of thumb, setting $M \approx \sqrt{N}$ yields an order of magnitude reduction in the computational cost. However, as M approaches the filter length N, the performance of FAPC degrades to that of the matched filter.

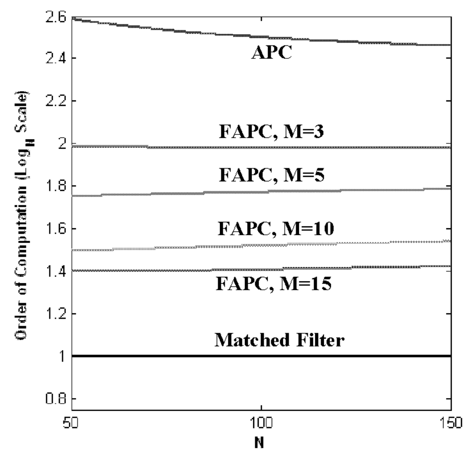

Fig. 5 Average computational cost per range cell vs N

V. SIMULATION RESULTS

The performance of the FAPC algorithm is demonstrated by comparing it with the matched filter and the original APC algorithm for three simulated scenarios. The first case consists of a small target masked in the range sidelobes of a large target. The second case consists of a large moving target masking a small target. Finally, the third case consists of both large and small targets distributed randomly in range with randomly assigned Doppler shifts.

The transmitted waveform utilized for all three cases is the length N = 60 polyphase Lewis-Kretschmer P3 code [6] with M=4. The ground truth range profiles consist of point targets in noise which is modeled as zero-mean complex Gaussian. For all three scenarios, the APC and FAPC algorithms employ an initialization stage (matched filtering) followed by three adaptive stages.

For the first case, a large target masking a small target, the small target has an SNR of 20 dB (after pulse compression) with the large target 40 dB higher. The results for this case are illustrated in Figs. 6 and 7 for the decimation and contiguous blocking versions of FAPC, respectively. The large range sidelobes induced by the matched filter mask the small target. The APC and FAPC algorithms are able to drive the range sidelobes from the large target into the noise thus resolving the smaller target. In terms of MSE performance, the matched filter yields −33 dB while APC, Decimation FAPC, and Contiguous FAPC all achieve −59 dB. Note that for very small values of the complex amplitude estimates both versions of FAPC tend to drive the estimate to zero because they are only approximations to the true MMSE cost function. For this reason a lower bound has been instituted as in [2] so as to prevent matrix ill-conditioning that may otherwise occur.

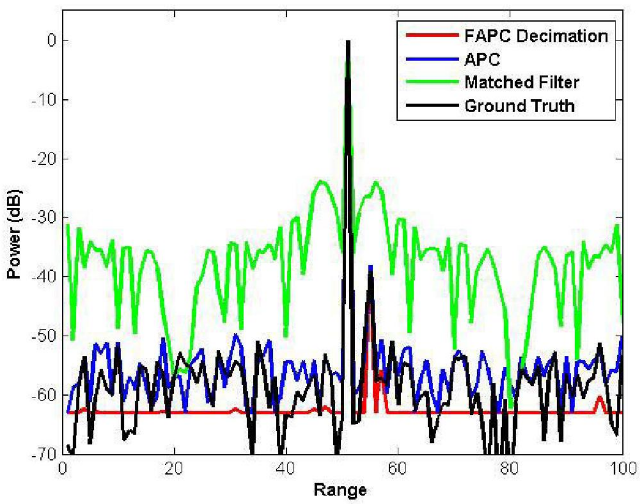

Fig. 6. Large target masking small target (Decimation)

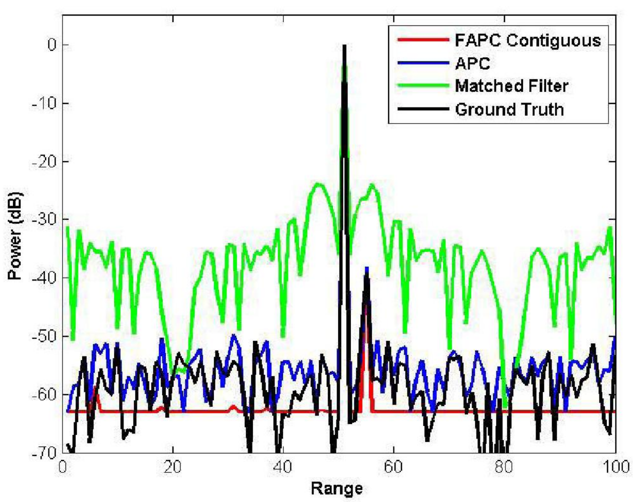

Fig. 7. Large target masking small target (Contiguous)

The second scenario is similar to the first except the large target possesses significant radial motion thereby inducing Doppler mismatch. As in the previous example, the smaller has an SNR of 20 dB with the large target is 40 dB higher. The large target has a Doppler shift analogous to that induced by a radial velocity of Mach 2 illuminated by a 1μs pulse from an S-band radar. The results are shown in Figs. 8 and 9 for the two embodiments of FAPC. From Fig. 8, APC and Decimation FAPC algorithms show slight performance degradation relative to the previous case due to Doppler mismatch that manifests as spreading around the large target. In Fig. 9 Contiguous FAPC is found to be more tolerant to Doppler mismatch due to filter adaptation on a shorter time span (over each *K*-length contiguous block). The matched filter again yields −33 dB in terms of MSE with APC and Decimation FAPC achieving −46 dB and Contiguous FAPC attaining −47 dB.

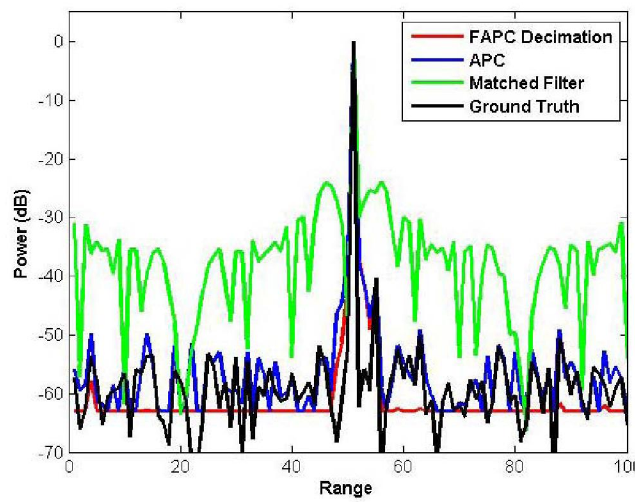

Fig. 8. Large moving target masking small target (Decimation)

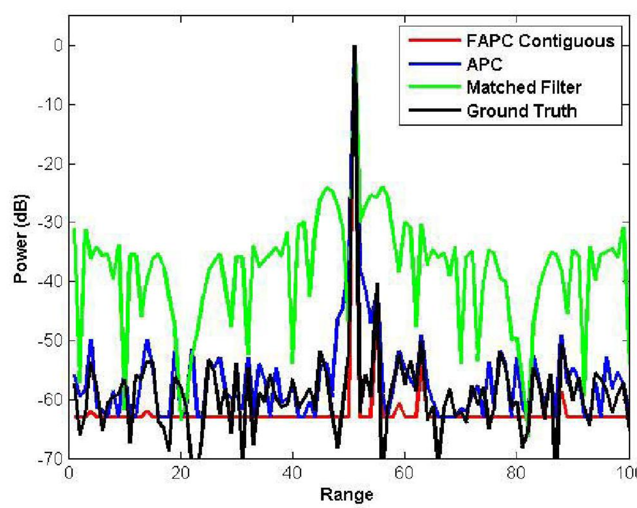

Fig. 9. Large moving target masking small target (Contiguous)

The final case consists of numerous targets randomly distributed in range with greatly varying powers and randomly assigned Doppler shifts. The targets possess Doppler shifts over the length of the waveform that are randomly chosen from a uniform distribution with a maximum analogous to a Mach 3 target illuminated by a 1μs pulse at S-band. Figures 10 and 11 illustrate the results for the decimation and contiguous versions of FAPC, respectively. The matched filter exhibits the usual range sidelobes masking the smaller targets. In contrast APC and the two versions of the FAPC algorithms uncover nearly all of the masked targets. For this case the matched filter yields −28.32 dB while APC, Decimation FAPC, and Contiguous FAPC result in −44 dB, −43 dB, and −44 dB, respectively.

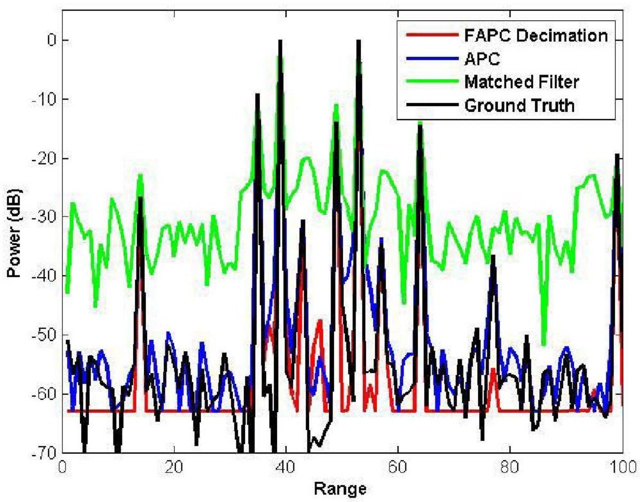

Fig. 10. Dense moving target scenario (Decimation)

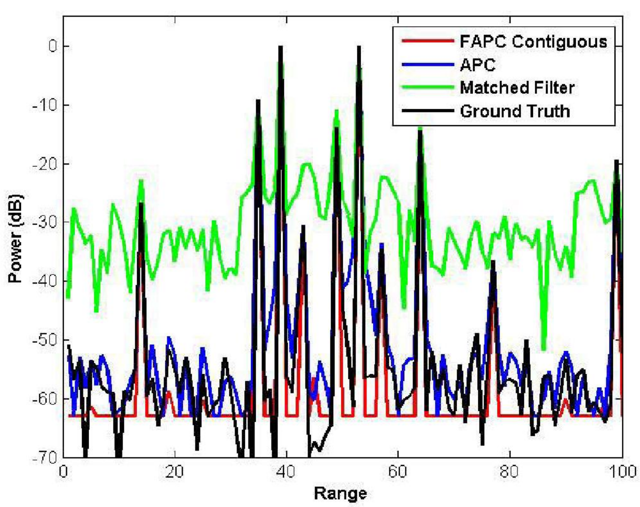

Fig. 11. Dense moving target scenario (Contiguous)

Conclusions

The original Adaptive Pulse Compression (APC) algorithm is capable of almost complete range sidelobe mitigation though the computational cost limits its use in most current real-time systems. Fast APC (FAPC) is presented whereby reduced dimensionality techniques, namely decimation and contiguous blocking, are used to reduce the computation of adaptive pulse compression while maintaining much of the performance benefit. In addition, the contiguous blocking embodiment of FAPC exhibits higher tolerance to Doppler mismatch than the original APC algorithm.

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