Constant Envelope DCT- and FFT-based OFDM Systems with Continuous Phase Chirp Modulation over Fading Channels

Rayan H. Alsisi and Raveendra K. Rao
Innovation Centre for Information Engineering (ICIE), Faculty of Engineering
Department of Electrical and Computer Engineering The University of Western Ontario
London, Ontario, N6A 5B9, Canada
Email:{alsisi, rrao}@uwo.ca

***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) systems with Continuous Phase Chirp Modulation (CPCM) are considered. In these systems, this modulation is used to realize constant envelope OFDM signals. Both Discrete Cosine Transform (DCT)- and Fast Fourier Transform (FFT)-based OFDM systems are considered and their Bit Error Rate (BER) performances over AWGN and fading channels are presented. Expressions for BER over Ricean and Rayleigh channels are derived and illustrated as function of modulation parameters. Results indicate that use of DCT instead of FFT in an OFDM system with CPCM results in better BER performance.**

***Keywords*—BER analysis, DCT, FFT, OFDM, PM, Rayleigh Fading Channel, Ricean Fading Channel**

I. Introduction

OFDM is a multicarrier technique popularly employed for wireless digital communication. However, the transmitted signal in an OFDM system has high amplitude fluctuations, which results in large value of PAPR making it sensitve to nonlinear distortion caused by the power amplifier in the transmitter. Recently, FFT- and DCT-based OFDM systems with phase modulation have been considered for mitigation of high value PAPR [1], [2], [3], [4]. Phase modulation achieves constant envelope signal with 0 dB PAPR. This allows power amplifier to operate close to saturation level thereby achieving maximum power efficiency.

CPCM is used for data communication due to its attractive properties such as anti-eavesdropping, anti-interference and low-Doppler sensitivity. The transmitted signal has much greater bandwidth than the information signal. CPCM can be used to enhance BER performance of OFDM systems and at the same time it assures 0 dB PAPR of transmitted signals [5], [6].

A new CE-OFDM-CPCM system is proposed that has the following advantages: 1) power efficiency, 2) secure communication, 3) enhanced bandwidth efficiency, and 4) enhanced BER performance. In general DCT-based OFDM system has several advantages over corresponding FFT-based system [3], [7]: 1) it uses real arithmetic, and 2) it requires half the bandwidth of FFT-based OFDM system. The intent is to examine both DCT- and FFT-based systems with CPCM. These will be referred to as CE-DCT-OFDM-CPCM and CE-FFT-OFDM-CPCM systems. BER performances of these systems are examined over AWGN channel and then over slowly varying fading channels.

II. OFDM Signals

The expression for continuous-time of baseband FFT-based OFDM signal is:

$$x_{FFT}(t) = \sum_{n=0}^{N-1} X_n \, e^{j2\pi nt/T}, n = 0, \cdots, N-1 \tag{1}$$

where $X_0, X_1, \cdots, X_{N-1}$ are N independent data symbols obtained from an arbitrary modulation constellation. The discrete-time representation of (1) [13] is given by:

$$x_{FFT}(k) = \sum_{n=0}^{N-1} X_n \, e^{j2\pi kn/N} \tag{2}$$

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and the corresponding frequency-domain signal is:

$$X_{FFT}(n) = \frac{1}{N}\sum_{k=0}^{N-1} x_k \; e^{-j2\pi kn/N} \tag{3}$$

As an alternative to FFT-based OFDM system [3], [7], DCT-based OFDM system can be considered. The DCT-based OFDM signal is given by:

$$x_{DCT}(t) = \sum_{n=0}^{N-1} X_n \; \cos(\pi nt/T) \tag{4}$$

The equivalent representation of (3) [7] is:

$$x_{DCT}(k) = \sqrt{\frac{2}{N}}\sum_{n=0}^{N-1} X_n \;\; B_n \cos(\pi n(2k+1)/2N) \tag{5}$$

where

$$B_n = \begin{cases} \sqrt{\frac{1}{2}} & n = 0 \\ 1 & n = 1, \cdots, N-1 \end{cases} \tag{6}$$

and the corresponding frequency-domain signal is:

$$X_{DCT}(n) = \sqrt{\frac{2}{N}}\sum_{k=0}^{N-1} x_k \;\; B_k \cos(\pi n(2k+1)/2N) \tag{7}$$

The expression for CE-OFDM-CPCM signal is:

$$s\left(t\right) = A_c \cos\left(2\pi f_c t + \phi(t) + \phi_0\right), \; 0 \le t \le T \tag{8}$$

where A_c and f_c are the carrier amplitude and carrier frequency of the signal, ϕ_0 is the starting phase assumed to be zero for coherent transmission, and $\phi(t)$ is given by:

$$\phi(t) = x_k g\left(t - kT_s\right) + \pi q \sum_{i=1}^{k-1} x_i, \quad kT_s \le t \le \left(k+1\right)T_s \tag{9}$$

where the symbol duration is T_s, and $x_k, k = 0, \cdots, N-1$, are the real-valued OFDM signal representing the message signal. x_k is either $x_{FFT}(k)$ or $x_{DCT}(k)$. The phase function $g(t)$ is given by

$$g(t) = \begin{cases} 0 & t \le 0, \; t > T_s \\ 2\pi \int_0^t f(\tau)d\tau, & 0 \le t \le T_s \\ \pi q = \pi(h-w) & t = T_s \end{cases} \tag{10}$$

where $f(t)$ is the frequency function and for CPCM:

$$f(t) = \begin{cases} 0 & t \le 0 \; t > T_s \\ \frac{h}{2T_s} - \frac{w}{T_s^2}t & 0 \le t \le T_s \end{cases} \tag{11}$$

Using (11) in (10), we get

$$g(t) \begin{cases} 0 & t \le 0, \; t > T_s \\ \pi\left[h\frac{t}{T_s} - w\left(\frac{t}{T_s}\right)^2\right] & 0 \le t \le Ts \\ \pi q = \pi\left(h-w\right) & t = T_s \end{cases} \tag{12}$$

where the h and w are dimensionless parameters. Since $q = (h - w)$, the set (h, w) is chosen to be the independent signal modulation parameters.

In FFT-based OFDM system, real-valued message signal (x_k) can obtained by using a conjugate symmetric data vector, $[0, X_0, \cdots, X_{N-1}, 0, X^*_{N-1}, \cdots, X^*_0]$, as input to the IFFT block [2] and in the DCT-based system the modulating signal (x_k) are real. [3]. The bandwidth of $s(t)$ for both of systems is determined by $max(2h_f, 2)W \; Hz$ [9], where W is the bandwidth of message signal. For the the FFT-based system $W = N/T \; Hz$ and for the DCT-based system $W = N/2T \; Hz$ [3], [8]. The block diagram of CE-OFDM-CPCM system is shown in Fig. 1.

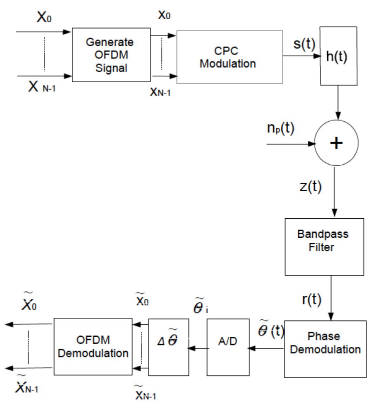

Fig. 1: CE-OFDM-CPCM system diagram

III. BER PERFORMANCE OVER AWGN CHANNEL

With reference to Fig. 1, the received signal $s(t) + n_p(t)$, where $n_p(t)$ is zero mean additive white Gaussian noise with PSP of $N_0/2$, is fed to an ideal BPF whose output is:

$$r(t) = s(t) + n(t) \tag{13}$$

where

$$n(t) = n_I(t)cos2\pi f_c t - n_Q(t)sin2\pi f_c t \tag{14}$$

In polar form (14) becomes:

$$n(t) = x(t)cos[2\pi f_c t + \Psi] \tag{15}$$

with

$$x(t) = \sqrt{[n_I(t)^2 + n_Q(t)^2]} \tag{16}$$

and

$$\Psi = tan^{-1}(n_Q(t)/n_I(t)) \tag{17}$$

The the phase demodulator output is then given by [9], [10], [11]:

$$\theta(t) = \phi(t) + \Phi(t) \tag{18}$$

where $\Phi(t)$ is the noise signal and its average power is $\frac{2WN_0}{A_c^2}$ The signal is then fed to the A/D converter to obtain JN samples over $0 \leq t \leq T$, with J the oversampling factor. That is,

$$\theta[i] = \theta(t)\mid_{t=\frac{i}{J}T_s},\ i = 0, \cdots, JN-1 \tag{19}$$

The phase difference at the receiver is then cosnidered and is given by [3]:

$$\Delta\tilde{\theta} = \tilde{\theta}_{i+1} - \tilde{\theta}_i \tag{20}$$

where

$$\Delta\tilde{\theta} = (\phi_{i+1} - \phi_i) + (\Phi_{i+1} - \Phi_i) \tag{21}$$

The estimate of the signal $\tilde{x_k}$ can be shown to be given by:

$$\tilde{x_k} = \frac{1}{q\pi}\sum_{l=0}^{J-1}\left(\phi_{(Jk+1)+l} - \phi_{(Jk)+l}\right) + \left(\Phi_{(Jk+1)+l} - \Phi_{(Jk)+l}\right) \tag{22}$$

Equation (23) can be written as:

$$\tilde{x_k} = x_k + \frac{1}{q\pi}\left(\Phi_{J(k+1)} - \Phi_{(Jk)}\right) \tag{23}$$

$\tilde{x_k}$ are then fed to the OFDM demodulator to obtained the estimate of infomation symbols $\tilde{X_n} = X_n + w_n$, where w_n has zero mean and its variance can be shown to be:

$$E\left[w_n^2\right] \approx \frac{4WN_0}{q^2\pi^2A_c^2} \tag{24}$$

The BER is related to Symbol Error Rate by [12], [13]:

$$BER \approx \frac{SER}{\log_2(M)} \tag{25}$$

Thus, the BER for CE-OFDM with CPCM system is given by:

$$BER \approx \left(\frac{2(M-1)}{M\log_2(M)}\right) Q\left(\pi(h-w)\sqrt{I\frac{E_b}{N_0}}\right) \tag{26}$$

where $I = 3\frac{\log_2(M))}{M^2-1}$ and $I = \frac{3}{2}\frac{\log_2(M))}{M^2-1}$ for DCT- and FFT-based systems, respectively. Thus, the BER given by (26) is a function of (h, w), modulation parameters, E_b/N_0, signal-to-noise ratio, and M, the number of amplitude levels in the mapper/modulator. The modulation parameters h and w represent the peak-to-peak frequency deviation divided by the symbol rate and the frequency sweep width divided by the symbol rate.

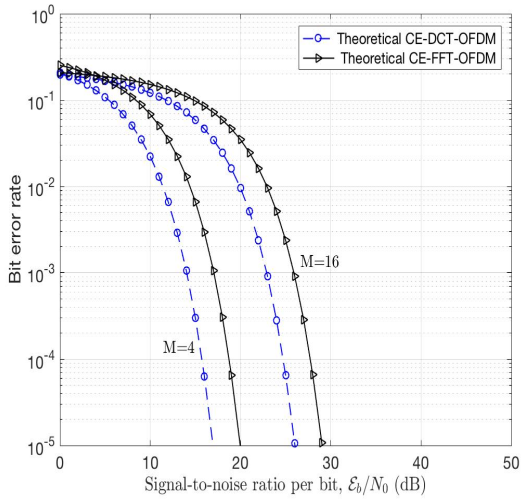

Fig. 2: BER Vs. E_b/N_0 of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for ($h = 0.4, w = 0.1$) over AWGN channel, as a function of M.

The error probability performances of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for $M = 4$, and 16 are illustrated in Fig. 2. BER decreases as M decreases for fixed value of modulation parameters. It is observed that at BER of 10^{-4} E_b/N_0 required for CE-DCT-OFDM system is 3 dB less than that required for CE-FFT-OFDM system.

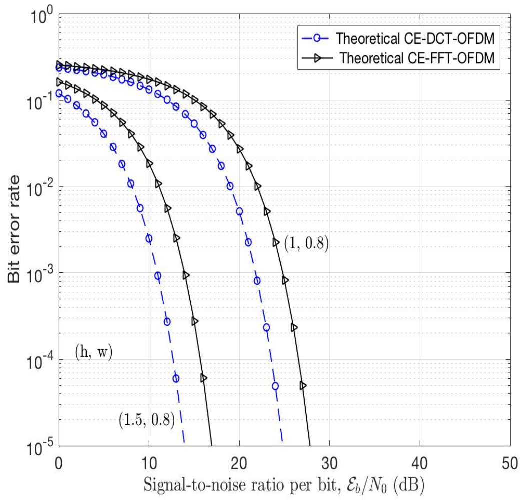

Fig. 3: Error probability performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for $M = 8$ and $w = 0.8$ over AWGN channel, as a function of h.

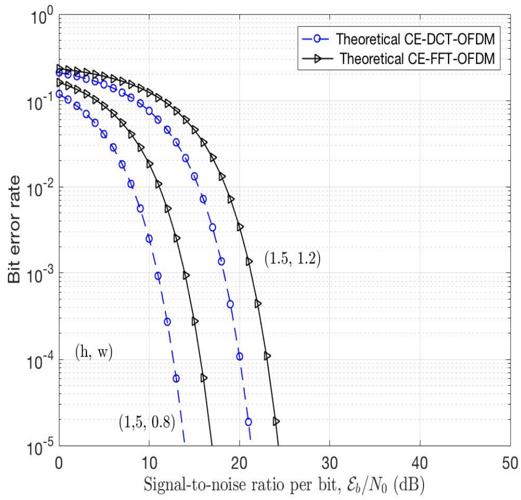

Fig. 4: Error probability Performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for $M = 8$ and $h = 1.5$ over AWGN channel, as a function of w.

The effects of modulation parameters, (h, w), on error probability for CE-FFT-OFDM and CE-DCT-OFDM systems are plotted in Fig.3 and 4. These plots results are shown for $M = 4$ as a function of (h, w). It is noted from these figures that error probability decreases as h increases and w decreases.

IV. BER PERFORMANCE OVER FADING CHANNEL

The performance of CE-OFDM systems with CPCM over Rayleigh and Rice fading channels can be derived assuming that the fading is flat. The received signal is given by:

$$r(t) = h(t) * s(t) + n(t) \quad (27)$$

where $h(t) = \alpha\delta(t)$. The bit error probability (P_b) of CE-OFDM-CPCM system can be evaluated using (26) and the Probability Density Function (PDF) of γ. That is,

$$P_b = \int_0^\infty P_b(\gamma)p_\gamma(\gamma)d\gamma \quad (28)$$

where $\gamma = \alpha^2 E_b/N_0$ is the instantaneous SNR per bit, and $\bar{\gamma} = \Omega E_b/N_0$ is the average SNR per bit. $\Omega = E\{\alpha^2\}$, where α is the fading amplitude. The Gaussian Q-function in (26) can be alternatively written as:

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} exp\left(-\frac{z^2}{2sin^2(\phi)}\right)d\phi \quad (29)$$

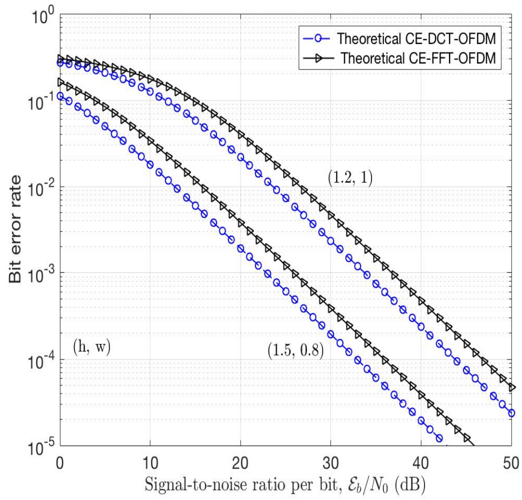

Fig. 5: Error probability performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for Rayleigh fading, as a function of h and w.

A. Rayleigh Fading Channel

For a Rayleigh channel, γ is distributed exponentially. That is, [14]:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} exp\left(\frac{\gamma}{\bar{\gamma}}\right), \gamma \geq 0 \tag{30}$$

Substituting $P_b(\gamma)$ from (26), and $p_\gamma(\gamma)$ given by (30) in (28), we get

$$P_b = \left(\frac{M-1}{M\log_2(M)}\right)\left[1 - \sqrt{\frac{1}{1+\frac{1}{I\pi^2(h-w)^2\bar{\gamma}/2}}}\right] \tag{31}$$

Fig. 5 shows the BER performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for $M = 4$ over Rayleigh flat channel. It is noted that BER decreases as h increases and w decreases for a fixed SNR. For example at BER of 10^{-4} the SNR required for $(h = 1.5, w = 0.8)$ is 10 dB less than that required for $(h = 1.2, w = 1)$. Moreover, it is observed that CE-DCT-OFDM-CPCM system performs better than CE-FFT-OFDM-CPCM system; for example at BER of 10^{-4}, and $(h = 1.5, w = 0.8)$, E_b/N_0 required for CE-DCT-OFDM-CPCM system is 3 dB less than that required for CE-FFT-OFDM-CPCM system.

B. Ricean Fading Channel

Rician channel often has one strong direct line-of-sight (LOS) component and many random weaker components, and γ follows a noncentral chi-square distribution given by [14]:

$$p_\gamma(\gamma) = \frac{(1+K)\,e^{-K}}{\bar{\gamma}} exp\left[-\frac{(1+K)\,\gamma}{\bar{\gamma}}\right] I_0\left[2\sqrt{\frac{(K+K^2)\,\gamma}{\bar{\gamma}}}\right], \gamma \geq 0 \tag{32}$$

Substituting (26) and (32) in (28), we get

$$P_b = \frac{2}{\pi}\left(\frac{M-1}{M\log_2(M)}\right) \int_0^{\pi/2} \frac{(1+K)sin^2(\phi)}{(1+K)sin^2(\phi) + I\pi^2(h-w)^2\bar{\gamma}/2} exp\left[-\frac{KI\pi^2(h-w)^2\bar{\gamma}/2}{(1+K)sin^2(\phi)+I\pi^2(h-w)^2\bar{\gamma}/2}\right] d\phi \tag{33}$$

Fig. 6 depicts BER of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for $K = 10$ dB and $M = 8$ as a function of (h, w). At BER as h increases and w decreases, for example at BER of 10^{-4} the SNR required for $(h = 1.5, w = 0.8)$ is 11 dB less than that required for $(h = 1.2, w = 1)$. Also, it is observed that CE-DCT-OFDM-CPCM system performs better than CE-FFT-OFDM-CPCM system; for example at BER of 10^{-4}, and $(h = 1.2, w = 1,)$, E_b/N_0 required for CE-DCT-OFDM-CPCM is 4 dB less than that required for CE-FFT-OFDM-CPCM.

Fig. 7 illustrates the effect of K on BER. These results are obtained for CE-DCT-OFDM and CE-FFT-OFDM systems with 8-PAM mapper for Ricean flat channel for fixed value of (h, w). It is evident from the figure that the BER decreases as K increases.

V. CONCLUSION

A description of constant envelope DCT- and FFT-based OFDM system with CPCM is given. CPCM transforms the OFDM signal to have 0 dB PAPR, suited for nonlinear amplification. Performance analysis of this system over AWGN and flat fading channels is examined and closed-form expressions for BER have been obtained. Improved BER performance is seen with increased value of h and decreasing value of w at fixed SNR. The result shows that BER of CE-DCT-OFDM-CPCM system is better than CE-FFT-OFDM-CPCM system. It is also observed that BER performance can be controlled by varying h, w, and M.

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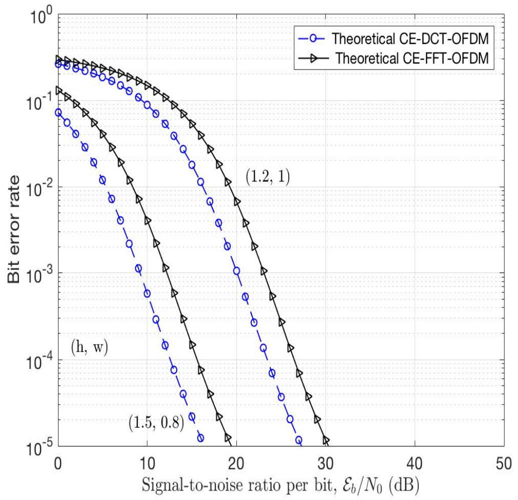

Fig. 6: Error probability performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for 8-PAM mapper for Ricean fading, as a function of h and w.

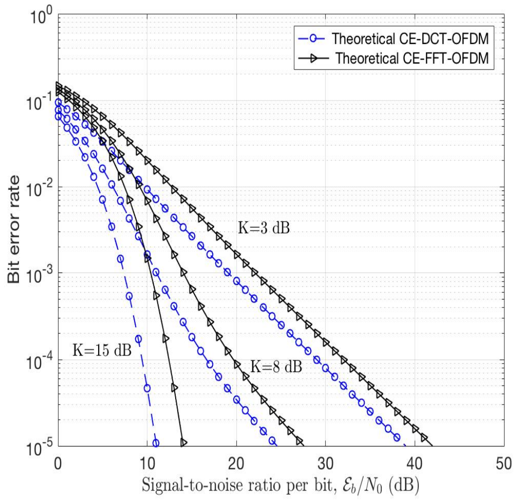

Fig. 7: Error probability performance of CE-DCT-OFDM and CE-FFT-OFDM systems with CPCM for 8-PAM mapper for Ricean fading, as a function of K.

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