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Generalized OFDM-LFM Waveform Design and Analysis for Multistatic Airborne Radar

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Abstract—**In recent years,multistatic radar systems are extensively used for detection process, as it includes multiple transmitters and multiple receivers spaced at different locations. Due to multiple transmitted signals by multiple transmitters, waveform design with a large time-bandwidth product, low cross-correlation interferences, and a low peak–average power ratio, is a challenging task. Even thoughmany waveform design techniques exist,they lack high time-bandwidth product, which is one of the primary requirements for multistatic airborne radars. In this paper to meet these requirements, in the present study an orthogonal frequency division multiplexing waveform (OFDM) with linear frequency modulated (LFM) signal is designed which has a high time-bandwidth product.This method jointly utilizes the effectiveness of conventional LFM and OFDM signals, to achieve spectral efficiency through multi carrier modulation as well as better range and Doppler resolution. A generalized formulation of OFDM-LFM waveform is presented and the performance is analyzed in time and frequency domains in terms of various ambiguity function parameters.**

Keywords-OFDM; linear Frequency Modulation (LFM); Ambiguity Function (AF);pulse compression

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I. INTRODUCTION

Presently, multistatic radar systems are widely used for the detection and tracking of stealth targets due to their efficient detection ability and increased coverage area. It employs multiple transmitters and receivers having the capability of transmitting arbitrary and diverse signals at each transmitter which needs enormous performance improvements and the challenge of waveform diversity design. In the design of a radar system it is desirable to have a substantially good resolution in time and Doppler along with large value of time-bandwidth product. Range resolution can be achieved by short duration pulse but it shows an adverse effect in Doppler resolution. For better Dopper resolution, long duration pulse is effective whereas, range resolution degrades with long duration pulses. In order to resolve this trade off, researchers had developed a technique called pulse compression by which the properties of a short duration pulse can be achieved by compressing a long duration pulse [1]. Different authors have discussed several pulse compression techniques like linear frequency modulation (LFM), Barker code, Binary phase codes, Polyphase codes, Costas codes, Nonlinear Frequency Modulation etc. [2], 脉冲压缩技术 among which linear frequency modulation proposed by R. H. Dickie in 1945, is the most popular one. In the LFM,the instantaneous frequency of the signal increases linearly over time, that widens the signal bandwidth which results in an increase in the time-bandwidth product. Due to large bandwidth LFM signal provides a better range resolution. The spectrum 频谱 of the 外形 LFM signal approaches a rectangular shape as the time-bandwidth product increases so that its spectral efficiency 频谱效率 and signal-to-noiseratio (SNR) is maximized [3]. But the major limitations of the LFM signal are the complexity in achieving adequate pulse-to-pulse diversity [4] and the range Doppler coupling [5]. In Barker code pulse compression technique, which is obtained as a sequence of finite N values of +1 and −1 with the ideal autocorrelation property, the phase of the carrier signal is altered according to the code sequence [6]. The problem associated with the Barker code pulse compression is that, its spectrum is almost similar to the sinc function, which results in a low bandwidth efficiency. The Costas code pulse compression technique has a very good range Doppler property but the chirp contains a single pulse.

In 2000 N. Levanon investigated a new set of signals which can achieve high resolution in both delay and Doppler without the reduction of pulse duration, known as the orthogonal frequency division multiplexing (OFDM) radar signals [4]. The primary disadvantage of using OFDM in wireless communication lies in the fact that time and frequency synchronization is crucial to ensure subcarrier orthogonality. However, sensitivity to time and frequency synchronization is beneficial for radar systems because radar receiver usually uses a stored version of the transmitted signal to measure time-delay and frequency offset between the transmitted signal and the received signal to derive the target parameters [7]. Due to this motivation OFDM based radar signals have engrossed a sustainable attention in recent years. J. P. Strakla et al presented wideband phased array radar architecture for radar applications where the multi carrier OFDM is used to achieve pulse-to-pulse diversity. D. Garmatyuk et al in

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2009 in their paper described the application of wideband OFDM systems for radar systems [5]. M. A. Sebt et al proposed an OFDM radar signal by optimizing the shape of the ambiguity function using the least square and finally an algorithm is proposed to lower the peak to average power ratio [4, 7]. C. Sturm et al discussed the limitations of OFDM waveforms in radar applications and proposed an approach for the range processing in OFDM radars.

Even though OFDM radar signals have advantages, they have some sort of limitations. The high side lobes and time varying nature of the OFDM signals makes the detection of OFDM signals more complex for the matched filtering process. Radars are usually placed inside airplanes or satellites, thus a high average transmit power is required because of the long distance between the radar-platform to the targets. Hence, the waveforms should have a large time-bandwidth product [3]. Therefore there is a need to design an OFDM waveform with large time bandwidth product.

①In 2012 W. Q. Wang proposed an orthogonal frequency division multiplexing radar waveform with a large time bandwidth product [3]②J.H. Kim in 2013 presented an OFDM based new waveform technique for the use of multiple transmitters in synthetic aperture radar (SAR) data acquisition [7]. In his paper two waveforms modulated by two orthogonal subcarrier sets that are mutually shifted by Δf are used where a LFM signal spectrum is used for the OFDM modulation. As an improved version of the OFDM chirp of this paper a three waveform based OFDM modulation is proposed by Z. Mingyang et al in 2013 [8]. In 2014 S. Cheng et al proposed a four waveform OFDM chirp signal with the waveform analysis that gives a high time bandwidth product.

In this paper, a generalized formulation of OFDM-LFM waveforms is presented. The time and frequency domain representations are analyzed for different waveforms. Ambiguity Function for different waveforms are presented. The trade-off between the side lobe magnitudes and the time - bandwidth product are examined through simulation.

The rest of the sections in this paper are organized as follows①section II describes the LFM pulse compression, OFDM radar signal and the OFDM-LFM waveform design approach②In section III the frequency domain, time domain OFDM-LFM waveform design are explained and the ambiguity function of the OFDM-LFM waveform with different LFM signals are analyzed.The simulation results and analysis are also provided in section III③Finally,section IV provides the conclusion and future work.

II. LFM Pulse Compression and OFDM

Linear frequency modulation is a popular pulse compression technique for radarsystems due to its high range and Doppler resolution characteristics. The frequency of the signal increases (up-chirp) or decreases (down-chirp) linearly over the pulse duration. The technique of applying a different chirp rate for each pulse is known as chirp diversity [1]. Let us consider a transmitted signal with frequency f_1 with a frequency sweep rate of k, then the instantaneous phase of the chirp (LFM) signal is represented as [9]

相位: $$\emptyset(t) = 2\pi\left(f_1 t + \frac{1}{2}kt^2\right) \tag{1}$$

By taking the derivative of the phase function the instantaneous frequency of the signal is obtained as

$$f(t) = \frac{d}{dt}\left(f_1 t + \frac{1}{2}kt^2\right) = f_1 + kt \tag{2}$$

Where, the chirp rate (k) is defined by the ratio of the bandwidth (B) to the chirp duration (T_p) given by $k = \frac{B}{T_p}$

Neglecting the amplitude and carrier frequency terms, a chirp signal can be represented by:

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$$x(t) = exp\left(j2\pi\left(f_1 t + \frac{1}{2}kt^2\right)\right) \tag{3}$$

A. OFDM Radar signal:

OFDM is a spread-spectrum transmission technique, where the signal is comprised of multiple carriers, which are transmitted over a single transmission path. Each subcarrier carries a small portion of the entire signal. The OFDM signals can be generated by choosing any one of the modulation scheme such as①binary phase-shift keying (BPSK),②quadrature phase-shift keying (QPSK), or ③quadrature amplitude modulation (QAM) that contains a certain number of bits per symbol [10].

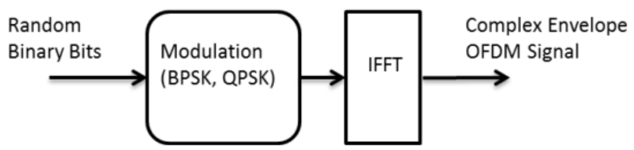

Fig. 1. Block diagram of OFDM System

The complex envelope representation of a single OFDM transmitted symbol $p(t)$ of bandwidth B and pulse duration T_p is represented as

$$p(t) = a(t)\sum_{k=0}^{N-1} s(k)\exp(j2\pi k\,\Delta f\,t) \tag{4}$$

Where N represents the total number of subcarriers, $\Delta f = \frac{1}{T_p}$ is the spacing between the adjacent subcarriers. $s(k) = [s_0 s_1 \ldots\ldots\ s_{N-1}]$is the modulation symbols.

$$a(t) = \begin{cases} 1, & 0 \le t \le T_p \\ 0, & otherwise \end{cases} \tag{5}$$

Even though the subcarriers overlap, they do not interfere with each other due to orthogonality between them.

If the OFDM signal $p(t)$is sampled at a sampling interval of $T_s = \frac{T_P}{N}$, the discrete time OFDM signal is obtained that is represented by

$$p(n) = p(t = nT_s) = \sum_{k=0}^{N-1} s(k) \exp\,(j2\pi k\ \Delta f\ n\ T_s) \quad (6)$$

Where $n \in [0, 1, 2 \ldots \ldots \ldots N-1]$ is the discrete sequence corresponding to the sampled values. The spectrum of the OFDM signal can be obtained by performing the DFT of $p(n)$, given by

$$P\left(\frac{k}{NT_s}\right) = T_s \sum_{k=0}^{N-1} p(n) \exp\left(-j2\pi k \frac{n}{N}\right) \quad (7)$$

The number of subcarriers N and spacing between subcarriers Δf are related to the bandwidth by

$$B = N\Delta f \quad (8)$$

where $\Delta f = 1/NT_s$. So the bandwidth in equation (8) will be

$$B = \frac{1}{T_s} \quad (9)$$

The symbol duration T_P of a single OFDM symbol is expressed as$T_P = NT_s$. Therefore, increasing the number of subcarriers and keeping $T_s$as a constant, the OFDM signal duration increases.

B. OFDM-LFM waveform design

The basic OFDM chirp waveform is to exploit the orthogonality of discrete frequency components. This means that the orthogonality of the designed waveforms is independent on the input sequences [3]. With this concept J. H. Kim [7] implemented two OFDMs to modulate the same input sequence that results in two different sequences. Subsequently authors in [11] designed four OFDM chirp waveform. The OFDM chirp waveform exploits both the orthogonality of subcarriers and intrinsic characteristics of traditional chirp waveform. In this scheme, the frequency utilization increases as it exploits full bandwidth for eachwaveform [3].

Let us consider an OFDM modulation process which is carried out with two LFM waveforms. Assume that the input sequence (spectrum) is $X(p)$with N discrete spectral components which are separated by $2\Delta f$. In the first step, the input signal $X(p)$ is interleaved by N zeros, which forms a new data sequence$X_1(p)$. Then, the interleaved data sequence $X_1(p)$ is shifted by Δf to get another sequence$X_2(p)$. These data sequences are transformed into the time domain by the 2N-point inverse discrete Fourier transform (IDFT), which is the OFDM modulation [1]. As a result, we obtain two waveforms modulated by two orthogonal subcarrier sets that are mutually shifted by Δf. Their demodulation is performed by 2N-point DFT. Both the sets contain 2N subcarriers but use only N subcarriers to carry the input data [7, 3].

C. Generalised OFDM-LFM signal modelling

The input sequence $X(p)$ has N discrete spectral components separated by $K(\Delta f/2)$. This input sequence $X(p)$ interleaved by $K-1$ zeros that gives rise to $X_1(p)$, where $K = 1,2,3,4, \ldots . N-1$. Then $X_1(p)$ is shifted by $K(\Delta f/2)$ $(i.e. \Delta f/2, \Delta f, 3\Delta f/2 \ldots . . (N-1)\Delta f/2)$ that produces the signal sequences $X_2(p), X_3(p) \ldots . X_K(p)$ respectively. By performing a KN-point IDFT, all the K-sequences are transformed to a time domain signal. By this process, we get K orthogonal waveforms modulated by distinct subcarrier sets that are shifted by $K(\Delta f/2)$. It should be emphasized that all the sets contain KN subcarriers but each uses only N subcarriers.

As mentioned above in this work we will consider the spectrum of LFM signal for the OFDM modulation. The LFM signal in discrete form is given by:

$$x(m) = \exp(j\pi k(mT_{ss})^2) \quad (10)$$

Where $m = 0, 1, 2, \ldots . . N-1. T_{ss}$is the sampling interval, k is the chirp rate i.e. $k = \frac{B}{T_p}$, B is the bandwidth and T_p is the chirp duration. The Fourier transform of the LFM signal is:

$$X(p) = \square[x(m)] = \square\{x(m)\}$$
$$= F\{\exp(j\pi k(mT_{ss})^2)\} \ (11)$$

Where Fdenotes the Fourier Transform operator and $x(m)$denotes the discrete time sample of the chirp signal of length N. The data sequences are interleaved by zeros and shifted and the K input data sequence are generated as follows:

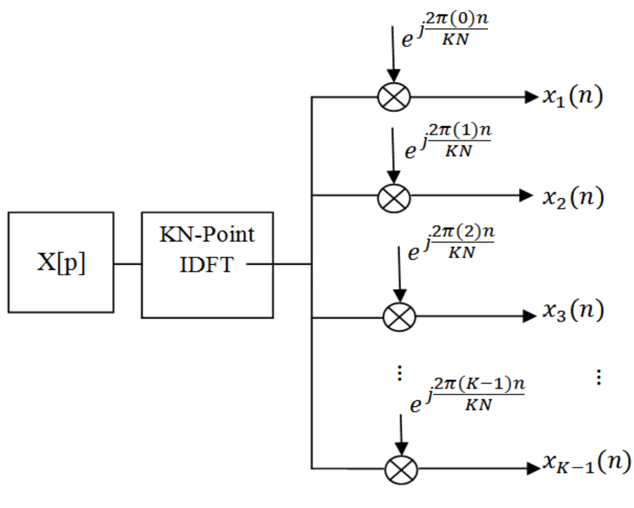

Fig. 2. Block diagram to generate K generalized OFDM-LFM waveform

$$X_1[p] = [x[0], 0_1, . . 0_K, x[1], 0_1, 0_2 \ldots . 0_K, x[2], 0_1 . . 0_K] \quad (12)$$

$$X_2[p] = [0_1, x[0] . . 0_K, 0_1, x[1], 0_2 \ldots . 0_K, 0_1, x[2], . . 0_K] \quad (13)$$

Similarly

$$X_K[p] = \left[0_1 \ldots 0_K, x[0], 0_1, \ldots . 0_K, x[1], 0_1, . . 0_K, x[2]\right] \quad (14)$$

By performing the KN-point IDFT the time domain sequence $x_1[n]$ is represented as

$$x_1[n] = \frac{1}{2}\Bigg\{x[n]rect\left[\frac{n}{N}\right] + x[n-N]rect\left[\frac{n-N}{N}\right] + x[n-2N]rect\left[\frac{n-2N}{N}\right] + \cdots + x[n-(k-1)N]rect\left[\frac{n-(K-1)N}{N}\right]\Bigg\} \quad (15)$$

Where $n = 0,1,2, \ldots . KN-1$

As we know that the K orthogonal subcarrier sets are mutually shifted by $K\Delta f/2$, the other sequenses are derived from the $x_1[n]$, that is represented as

$$x_i[n] = x_1[n]\, exp\left(j\frac{(i-1)2\pi n}{KN}\right) \quad (16)$$
$$i = 2,3,4 \ldots K$$

The OFDM-LFM waveforms are obtained using (15) and (16) and are illustrated in Fig. 2. The continuous time representations of the sequences are expressed by considering $\Delta f = \frac{1}{NT_{ss}}$, as

$$x_1(t) = \frac{1}{2}\Bigg\{x(t)rect\left[\frac{t}{T_p}\right] + x(t-T_p)rect\left[\frac{t-T_p}{T_p}\right] + x(t-2T_p)rect\left[\frac{t-2T_p}{T_p}\right] + \cdots + x(t - KT_p)rect\left[\frac{t-KT_p}{T_p}\right]\Bigg\} \quad (17)$$

The other signals are proceed from the $x_1(t)$ i.e. given by

$$x_i(t) = x_1(t)\, exp\left(j\pi\frac{it}{2T_p}\right) \quad (18)$$

The zero interpolation makes the peakpower of the waveform to be reduced. Two adjacent waveforms are distinguishable by the sub-carrier frequency offset $K\Delta f/2$and have a$\pi/2$phase difference at $t = T_p$.

III. OFDM-LFM SPECTRUM AND AMBIGUITY FUNCTION ANALYSIS

A. *Frequency Domain Analysis*

The time domain representations of the OFDM-LFM waveforms for two-LFM signal is shown in Fig. 3 and six-

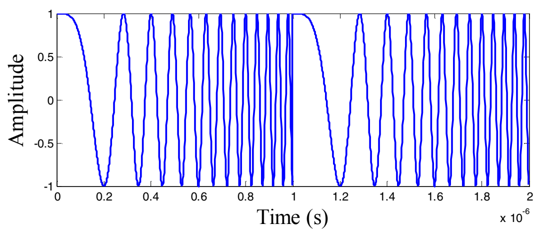

Fig. 3. OFDM-LFM waveform with two LFM signals

LFM signal is shown in Fig. 4. All the waveforms are chirp-like signals and hence should show similar frequency characteristics. According to Parseval's theorem [7], the total energy of the input sequence is conserved and they will occupy samefrequencybandwidth as shown in Fig. 5.The spectrum of a single LFM is rectangular in shape with increased time bandwidth product [1]. From Fig. 5, it can be inferred that the spectrum of the OFDM-LFM waveforms with two LFM signals, four LFM signals and six LFM signals are similar and rectangular in shape that makes the time bandwidth product large compared to a LFM signal.

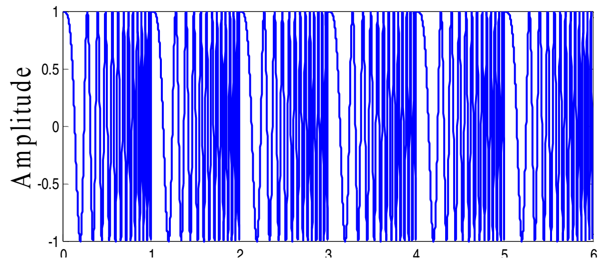

Fig. 4: OFDM-LFM waveform with six LFM signals

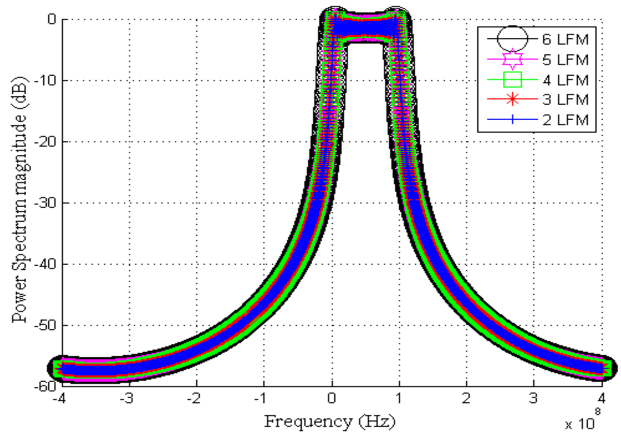

Fig. 5: Combined spectrum of LFM signals in an OFDM-LFM waveform

B. *Ambiguity Function:*

Ambiguity Function (AF) is a major tool for analyzing radar signals. It represents the time response of the matched filter when a signal is received with a delay τ and Doppler shift v relative to the nomial values expected by the filter [11]. In a radar system the AF is defined by [1]

$$|\chi(\tau,v)| \triangleq \left|\int_{-\infty}^{\infty} x(t)x^{*}(t+\tau)e^{j2\pi v t}dt\right| \qquad (19)$$

Where $*$ denotes the conjugate and τ is the temporal mismatch between the filter and the signal (delay), v is the Doppler shift. The value $|\chi(0,0)|$ represents the matched filter output when the delay and Doppler are not present. Therefore the sharper the function arround (0,0) the better the Doppler and range resolution. The design of the OFDM-LFM waveform includes, zero padding in frequency domain that leads to periodicity in the time domain.

An OFDM chirp signal with N subcarriers and K number of LFM signals each with duration T_c is represented in [2] as

$$x(t) = \sum_{n=0}^{N}\sum_{k=0}^{K} u(t-kT_c) \times exp\left[j2\pi f_{n,k}(t-kT_c)\right] \times exp\left[j\pi k_{r_{n,k}}(t-kT_c)^2\right] \qquad (20)$$

Where $f_{n,k}$ and $k_{r_{n,k}}$ denotes the frequency and the chirp rate of the subcarriers. The OFDM pulse durationT_p should satisfy$K.T_c \ll T_p$. By substituting (20) in (19) we get

$$|\chi(\tau,v)| = \left|\sum_{n=0}^{N}\sum_{k=0}^{K} u(t-kT_c)u(t+\tau-kT_c) \times exp\left[j2\pi f_{n,k}(t-kT_c)\right] \times exp\left[-j2\pi f_{n,k}(t+\tau-kT_c)\right] \times exp\left[j\pi k_{r_{n,k}}(t-kT_c)^2\right] \times exp\left[-j\pi k_{r_{n,k}}(t+\tau-kT_c)^2\right] \times exp[jvt]dt\right| \qquad (21)$$

Fig. 6 and Fig. 7 shows the AF plot of the OFDM-LFM signal with two and six LFM signals respectively. Each LFM signal produces corresponding sidelobes in the OFDM-LFM waveform. So the total number of side lobes for K LFM signals is given by $K*2-1$ due to the symmetry of the waveforms. As the number of LFM signal increases, the zero padding also increases by a factor of $K$with increase in the side lobe peak. The parameters used for simulation are, number of subcarriers (N) =2048, $T_p = 4\mu s$ and B=10 MHz. The matched filter output of waveform with different LFM signals are shown in Fig. 8. From Fig. 8, it can be inferred that the number of side lobes and its magnitude increases with increase in the number of LFM signals.

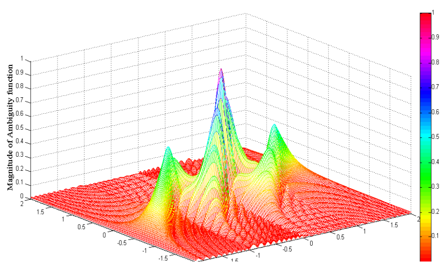

Fig. 6: Ambiguity function of OFDM-LFM with 2 LFM

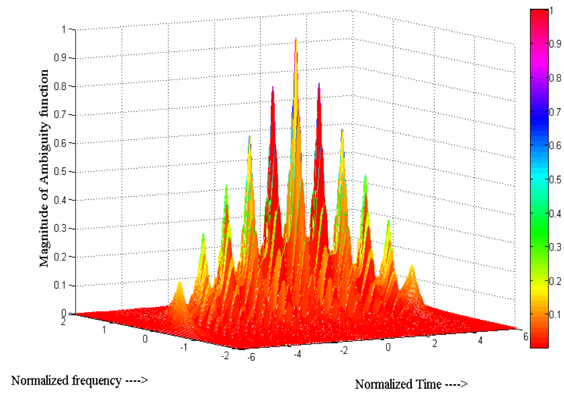

Fig. 7: Ambiguity function of OFDM-LFM with 6 LFM

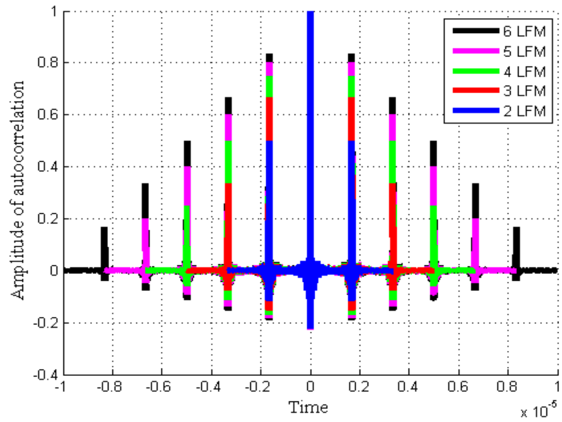

Fig. 8: Matched filter output of waveform with different LFM signals

According to the tolerance and sensitivity level for different applications, different threshold values have to be considered. The first and second sidelobe magnitude for different LFM signals in OFDM-LFM waveform shown in Fig.9. It can be inferred that as the number of LFM signals in the waveform increases from two to six, the first side lobe increases from -6 dB to -1 dB. The time-bandwidth product for different number of LFM signals are shown in Fig. 10. As the number of signals increases in the waveform, the time-bandwidth product increases linearly but with an increase in the sidelobe level as shown in Fig.9 and Fig. 10. Hence, there is a tradeoff between the time bandwidth product and the peak sidelobe level for the design of a better range and Doppler resolution radar waveform

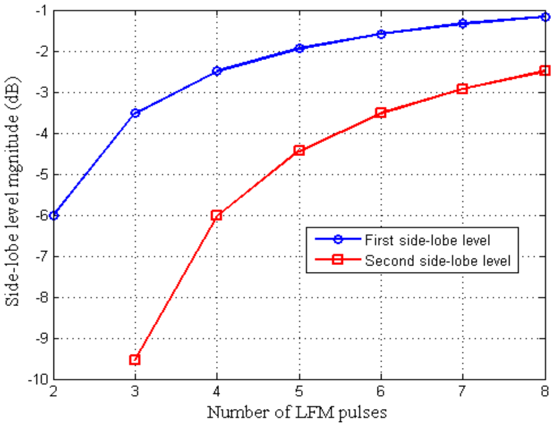

Fig. 9: First and second side lobe level magnitude of the waveform with different LFM signals

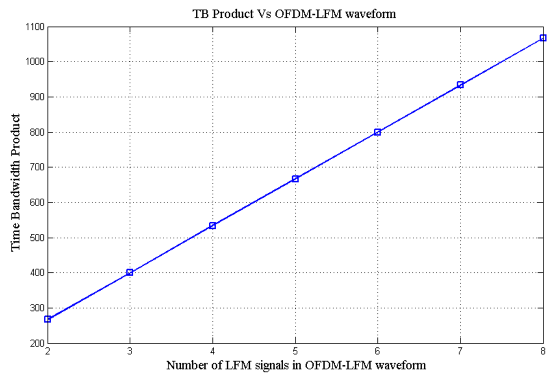

Fig. 10: Time bandwidth product versus the OFDM-LFM waveform with different LFM signals

IV. CONCLUSION

A generalized OFDM-LFM waveform design approach with K number of LFM signals is proposed in this paper. The simulation result shows the two and six orthogonal chirp waveform with large time bandwidth product and good envelope without degrading the orthogonality. In light of the present analysis on generalized OFDM-LFM systems, tradeoff between the time bandwidth product and the peak side lobe level has been highlighted. As the number of signals in the waveform increases, the performance increases in terms of time-bandwidth product but the performance of the detection process became less effective due to the increase in the side lobe levels. Mathematical optimization techniques can be applied to the OFDM-LFM waveforms so that the sidelobes are minimized in the presence of more LFM signals, which may be considered as a future study.

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未来研究点：利用数学优化技术 降低OFDM-LFM的旁瓣

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