# Symbol Alphabet Modifier for PAPR Reduction in OFDM Communications

Yuewen Wang and Ali N. Akansu
Department of Electrical and Computer Engineering
New Jersey Institute of Technology
Newark, NJ 07102, USA
Email: {YW46,Akansu}@NJIT.edu

***Abstract*—High Peak-to-Average Power Ratio (PAPR) is a major drawback of Orthogonal Frequency Division Multiplexing (OFDM) based communications systems. Many methods for PAPR reduction have been reported in the literature. They include popularly used Selective Mapping (SLM) and Partial Transmit Sequences (PTS) techniques. Although these two methods reduce PAPR quite well, their implementation cost is relatively high. In this paper, Symbol Alphabet Modifier Matrix (SAM) is proposed to jointly alter the phase and amplitude values of the original symbols in the alphabet. This framework utilizes only one FFT/IFFT operation pair for transmultiplexing of symbols without any side information. The merit of the proposed method to reduce PAPR for four cases without BER degradation along with its implementation simplicity is shown in the paper.**

***Index Terms*—PAPR; OFDM; Symbol Alphabet Modifier Matrix (SAM); SLM; Generalized DFT; FFT**

## I. Introduction

OFDM signal (frame) is a sum of orthogonal frequency modulated subcarriers (basis functions) weighted by corresponding symbol values picked from the alphabet of the chosen modulation technique [1], [2]. The resulting high peak-to-average power ratio of OFDM frame becomes a major deficiency of the system due to reduced power efficiency. Moreover, it causes signal distortion in non-linear modules of the system such as power amplifier (PA) and digital-to-analog converter (DAC).

Many PAPR reduction techniques have been reported in the literature [3]. These different methods include Clipping and Filtering [4], Coding Technique [5], Selected Mapping Technique (SLM) [6], [7], Partial Transmit Sequences (PTS) [8], [9], Tone Reservation (TR) [10], Tone Injection (TI) [11] and Active Constellation Extension (ACE) [12]. Among them, selected mapping (SLM) and partial transmit sequences (PTS) provide a good PAPR performance and have been successfully used in OFDM communications. The SLM and PTS techniques are similar in principle. They statistically aim to reduce the high PAPR by generating several candidate OFDM frames and pick the best one prior to transmission. Their main shortcoming is the implementation cost since multiple inverse fast-Fourier transform (IFFT) operations are required for each transmitted OFDM frame. Moreover, side information (SI) bearing the phase modification of the OFDM frame needs to be transmitted to the receiver error-free. The entire data block will be lost if SI is received with errors. As a consequence, it causes the degradation in BER performance.

Extensions of SLM and PTS were also proposed in order to lower computational cost or to eliminate SI by modifying the phase and/or amplitude of symbols in the original symbol alphabet (SA) [13], [14]. An SLM extension that eliminates SI by altering amplitude of data symbols, called A-SLM, is reported in [14] and included in PAPR and BER performance comparisons presented in Section IV.

A low-complexity PAPR reduction framework to jointly modify phase and amplitude of the original symbols in the alphabet of a digital modulation scheme such as $M$-PSK and $M$-QAM is proposed in this paper. An overview of the OFDM system and its PAPR is given in Section II. In Section III, the design procedure of the proposed technique is explained in detail for four cases. This framework utilizes only one IFFT/FFT operator pair for transmultiplexing of symbols without any SI bandwidth requirement. The merit of the method is shown through performance comparisons presented in section IV. Then, the concluding remarks are given.

## II. OFDM System and PAPR

An OFDM frame is generated by the multiplexing of independent symbol alphabets modulated with orthogonal frequency subcarriers. The incoming data bit stream is modulated into a sequence of symbols from the predefined symbol alphabet constellations such as $M$-ary Phase-Shift Keying ($M$-PSK) or $M$-ary Quadrature Amplitude Modulation ($M$-QAM). This populated symbol vector is denoted as $\underline{X} = [X(0), X(1), ..., X(N-1)]^T$, where $[\cdot]^T$ indicates the vector/matrix transpose operation. The baseband multicarrier signal is the summation of $N$ subcarriers weighted by symbols and expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi f_k t}, \quad 0 \le t \le N t_s. \qquad (1)$$

Subcarriers are orthogonal where $f_k = k\Delta f$ and $\Delta f = 1/Nt_s$, $t_s$ is the symbol period and $j = \sqrt{-1}$. Then, the

978-1-4673-9457-4/16/$31.00 ©2016 IEEE

discrete-time OFDM frame is the sampled version of (1) at the Nyquist rate $t = nt_s$ and written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}}, \quad n = 0, 1, ..., N-1. \quad (2)$$

Herein, the vector $\underline{x} = [x(0), x(1), ..., x(N-1)]^T$ denotes the OFDM signal in discrete time domain. The PAPR of an OFDM frame due to signal amplitudes fluctuation is defined as

$$\text{PAPR} = \frac{\max\limits_{n=0,1,...,N-1} |x(n)|^2}{E\left[|x(n)|^2\right]}, \quad (3)$$

where $E[\cdot]$ denotes the expectation operator.

The *complementary cumulative distribution function* (CCDF) is a commonly used measure to evaluate PAPR performance. The CCDF of the PAPR indicates the probability that the PAPR of a signal exceeds a given threshold, i.e. $\Pr\{\text{PAPR} > \text{PAPR}_0\}$.

## III. Symbol Alphabet Modifier Matrix

The design objective is to reduce the PAPR by minimizing the difference between the peak power and average power of the OFDM signal. Conceptually, in the ideal case, all components in the OFDM frame vector $\underline{x}$ having the same amplitude can make the PAPR to be 1 (0 dB) and thus the power difference is zero. Therefore, finding a proper symbol alphabet modifier along with the IFFT operation that generates the 'optimal' OFDM signal is addressed for such a design.

### A. Design Procedure

First, define an $N \times N$ matrix $C^{-1}$ assuming that can map a data symbol vector into an OFDM frame with constant amplitude. Here, superscript -1 is used to indicate the inverse matrix such that all matrices designed at the transmitter are identical to the denotation of the inverse DFT matrix $A_{DFT}^{-1}$ and also implied to be invertible. Hence, it is necessary to define the constraints for the design such that the matrix $C^{-1}$ must be invertible at the receiver and factorable to the $A_{DFT}^{-1}$ matrix which serves as the frequency selective orthogonal multiplexer for the OFDM systems [1], [2].

*1) :* Define the $N \times N$ transform matrix $C^{-1}$ consists of complex value elements as

$$C^{-1} = [c_n(k)] = \left[\alpha_{k,n} \cdot e^{j\varphi_n(k)}\right] \; k, n = 0, 1, ..., N-1, \quad (4)$$

where amplitudes of matrix elements are $\alpha_{k,n} \in \mathbb{R}^+$, $k$ and $n$ denote column and row indices of a matrix, respectively. Then, the OFDM frame in time domain is expressed as

$$|x(n)| = \left|\sum_{k=0}^{N-1} c_n(k) X(k)\right|. \quad (5)$$

By inspection, we force the equality of two arbitrary components in an OFDM frame as the following relationship

$$\begin{aligned} |x(m)| &= \left|e^{j\Delta\phi_{m,n}}\right| \cdot |x(m)| = |x(n)| \\ x(n) &= \sum_{k=0}^{N-1} c_m(k) e^{j\Delta\phi_{m,n}} \cdot X(k) \\ n, m &= 0, 1, ..., N-1, \; n \neq m, \end{aligned} \quad (6)$$

where $\Delta\phi_{m,n}$ denotes the phase difference between the $n^{th}$ and $m^{th}$ components of the OFDM frame vector. From (6), an intuitive design of the $n^{th}$ and $m^{th}$ rows of matrix $C^{-1}$ is given as follows

$$\begin{aligned} & c_n(k) = c_m(k) \cdot e^{j\Delta\phi_{m,n}}, \; {\scriptstyle n \neq m, \; n,m,k=0,1,...,N-1} \\ & \Rightarrow \alpha_{k,n} \cdot e^{j\varphi_n(k)} = \alpha_{k,m} \cdot e^{j\varphi_m(k)} \cdot e^{j\Delta\phi_{m,n}} \\ & \Rightarrow \alpha_{k,n} = \alpha_{k,m} = \alpha_k, \\ & \quad \varphi_n(k) = \varphi_m(k) + \Delta\phi_{m,n} = \varphi(k) + \Delta\phi_n, \end{aligned} \quad (7)$$

where $\Delta\phi_n = \Delta\phi_{0,n}$, $\varphi(k) = \varphi_0(k)$ is the phase of the $k^{th}$ element in the first row ($m = 0$) of the matrix $C^{-1}$.

Then, rewrite the matrix $C^{-1}$ as

$$C^{-1} = [c_n(k)] = \left[\alpha_k \cdot e^{j(\varphi(k)+\Delta\phi_n)}\right], \; {\scriptstyle k,n=0,1,...,N-1} \quad (8)$$

where $\alpha_k \in \mathbb{R}^+$, $\varphi(k)$ and $\Delta\phi_n \in [0, 2\pi)$, $\Delta\phi_0 = \Delta\phi_{0,0} = 0$.

However, this matrix is not invertible because the rank of such a matrix is 1 and the data symbol vector cannot be recovered at the receiver. In order to make the matrix $C^{-1}$ invertible, the diagonal elements of the matrix $C^{-1}$ is adjusted to be constant but non-unit amplitude $\alpha$, and the remaining elements to have another amplitude $\beta$ such that it has the full rank $N$. Moreover, if all rows or columns of the adjusted matrix are permutated, it still maintains the full rank property [16], and the PAPR of the OFDM frame vector also remains the same result. Accordingly, the modified matrix $C^{-1}$ can be designed in different permutation forms of the initial adjusted matrix that provides many possible transformation sets.

*2) :* Define a given permutation $\tilde{N}$ with $N$ elements that $\tilde{N} : \{0, 1, ..., N-1\} \to \{0, 1, ..., N-1\}$. For example, when $N = 4$ and the permutated order can be $\tilde{N} : \{2, 0, 3, 1\}$. The modified matrix that is invertible and can be employed as follows

$$\begin{aligned} \tilde{c}_n(k) &= \begin{cases} \alpha \cdot e^{j(\varphi(k)+\Delta\phi_n)} & k = \tilde{N}(n) \\ \beta \cdot e^{j(\varphi(k)+\Delta\phi_n)} & k \neq \tilde{N}(n) \end{cases} \\ &= \alpha^{I_{\tilde{N}(n)}(k)} \cdot \beta^{1-I_{\tilde{N}(n)}(k)} \cdot e^{j(\varphi(k)+\Delta\phi_n)}, \; {\scriptstyle k,n=0,1,...,N-1} \end{aligned} \quad (9)$$

where $\alpha \in (0, \infty)$, $\beta \in [0, \infty)$, $\alpha > \beta$, $\varphi(k)$ and $\Delta\phi_n \in [0, 2\pi)$, $\Delta\phi_0 = 0$, $\varphi(k) = \varphi(0) + A_k\pi$ for simplicity and $A_k \in \mathbb{Z}$.

$\mathrm{I}_n(k)$ is called *Indicator Function* [17], having the value 1 for element $k$ equals to element $n$ and the value 0 for element $k$ different than element $n$, which is defined as

$$\mathrm{I}_n(k) = \begin{cases} 1 & \text{if } k = n \\ 0 & \text{if } k \neq n \end{cases}. \quad (10)$$

To express the modified matrix $\tilde{C}^{-1}$ more intuitively, let the vector $\underline{\eta}^T = \left[e^{j\varphi(k)}\right]_{k=0}^{N-1}$, vector $\underline{\psi}^T = \left[e^{j\Delta\phi_n}\right]_{n=0}^{N-1}$, matrix $\Sigma_1 = diag\left(\underline{\eta}\right)$, $\Sigma_2 = diag\left(\underline{\psi}\right)$ and a real matrix can build up the matrix $\tilde{C}^{-1}$ as

$$\tilde{C}^{-1} = \Sigma_1 \Gamma \Sigma_2 = \underbrace{\begin{pmatrix} e^{j\varphi(0)} & & 0 \\ & \ddots & \\ 0 & & e^{j\varphi(N-1)} \end{pmatrix}}_{\Sigma_1} \times \underbrace{\begin{pmatrix} \beta & \cdots & \alpha & \cdots & \beta \\ \beta & \cdots & \cdots & \beta & \alpha \\ \beta & \alpha & \cdots & \cdots & \beta \\ \beta & \cdots & \cdots & \alpha & \beta \\ \alpha & \beta & \cdots & \cdots & \beta \end{pmatrix}}_{\Gamma} \times \underbrace{\begin{pmatrix} e^{j\Delta\phi_0} & & 0 \\ & \ddots & \\ 0 & & e^{j\Delta\phi_{N-1}} \end{pmatrix}}_{\Sigma_2}. \tag{11}$$

Some possible cases are invoked to build various matrices $\tilde{C}^{-1}$ by assigning different amplitude and phase values to the elements of the matrix, that is shown in part B.

*3) :* Now, $\tilde{C}^{-1}$ should be factorized into an inverse DFT matrix $A_{DFT}^{-1}$ and a matrix $B^{-1}$ since the OFDM system needs to be built up with utilization of the IFFT operator. The invertible matrix $B^{-1}$ is called *Symbol Alphabet Modifier matrix* (SAM) [15] and it is expressed as

$$\begin{aligned} \tilde{C}^{-1} &= A_{DFT}^{-1} \cdot B^{-1} \quad \tilde{C} \cdot \tilde{C}^{-1} = I \\ B^{-1} &= A_{DFT} \cdot \tilde{C}^{-1} \quad B \cdot B^{-1} = I, \end{aligned} \tag{12}$$

where $B^{-1}$ is derived from (9) and (12) as

$$\begin{aligned} B^{-1} &= \left[\tilde{b}_k\left(n\right)\right]_{SAM} \qquad k, n = 0, 1, ..., N-1 \\ \tilde{b}_k\left(n\right) &= \sum_{l=0}^{N-1} e^{-j\frac{2\pi kl}{N}} \cdot \tilde{c}_n\left(l\right) \\ &= \sum_{l=0}^{N-1} \alpha^{I_{\tilde{N}(n)}(l)} \cdot \beta^{1-I_{\tilde{N}(n)}(l)} \cdot e^{j\left(\varphi(l)+\Delta\phi_n-\frac{2\pi kl}{N}\right)}. \end{aligned} \tag{13}$$

One can normalize the power of the modified symbols close to the original ones before the IFFT operator by dividing with a normalization factor of Frobenius matrix norm as $\left\|B^{-1}\right\|_F \Big/ \sqrt{N}$. The SAM matrix $B^{-1}$ is rewritten as

$$B^{-1} = \left[\frac{\tilde{b}_k\left(n\right)}{\sqrt{\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\left|\tilde{b}_k\left(n\right)\right|^2 \Big/ N}}\right]_{\substack{SAM \\ k,n=0,1,\ldots,N-1}}. \tag{14}$$

The Fig.1 displays the block diagram of the OFDM system employed the proposed SAM method over additive white Gaussian noise (AWGN) and multipath fading channels.

### *B. Four Proposed Cases*

Here, some possible cases are pursued and investigated based on (9).

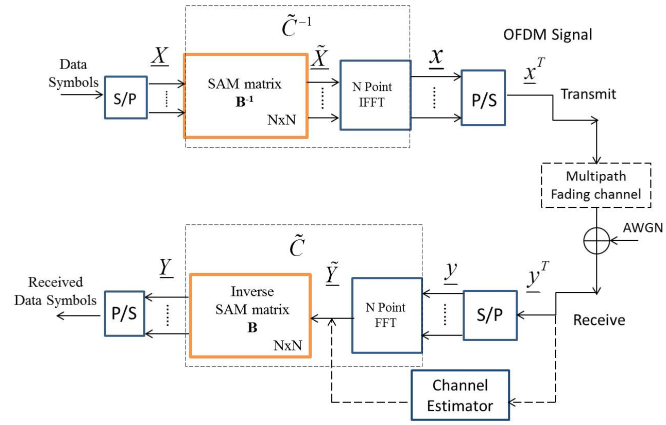



Fig. 1. Block diagram of the OFDM system with the proposed PAPR reduction method.

Case 1: In this case, the amplitudes $\alpha$ and $\beta$ are positive real numbers and $\alpha > \beta$, the matrix $\tilde{C}^{-1}$ can be normalized to $\tilde{c}_n\left(k\right) = \tilde{\alpha}^{I_{\tilde{N}(n)}(k)} \cdot e^{j(\varphi(k)+\Delta\phi_n)}$, here $\tilde{\alpha} = \alpha/\beta$. For readability, here we still use $\alpha$ instead of $\tilde{\alpha}$, and accordingly the SAM matrix is derived as

$$\tilde{b}_k\left(n\right) = \sum_{l=0}^{N-1} \alpha^{I_{\tilde{N}(n)}(l)} \cdot e^{j\left(\varphi(l)+\Delta\phi_n-\frac{2\pi kl}{N}\right)}. \tag{15}$$

The SAM matrix $B^{-1}$ in case 1 is an invertible but not orthogonal matrix such that it modifies the average power of the original OFDM signal. But the difference between modified power and original one is trivial when parameter $\alpha$ is large enough, where the numerical results tabulated in Table I have validate it. It is also noted that the performance of PAPR improves as the value of $\alpha$ increases.

Case 2: The parameter $\beta$ is zero, the matrix $\tilde{C}^{-1}$ becomes a constant modulus diagonal matrix and the value of parameter $\alpha$ can be normalized to 1. The SAM matrix becomes

$$\tilde{b}_k\left(n\right) = e^{-j\frac{2\pi k\tilde{N}(n)}{N}} \cdot e^{j\left(\varphi\left(\tilde{N}(n)\right)+\Delta\phi_n\right)}. \tag{16}$$

It is noted that case 2 is actually the ultimate form of case 1 when $\alpha$ goes to infinite large. The SAM matrix in this case becomes an orthogonal matrix that modifies the amplitude of original data symbols without alternating the average power of the OFDM signal.

Case 3: With the main focus of improving PAPR performance, it is sometimes compensated at the expense of increased BER, such as SLM and PTS technique require side information to be transmitted, along with the aforementioned amplitude modified SLM (A-SLM) method that increases signal power. As an extension of case 1, instead of using the sequence function with amplitude sequence of $[\alpha, \beta ..., \beta]$ in the previous design, a new sequence $\underline{S}$ denoted as basis sequence along with its cyclic shifted ones build up a new size of $N \times N$ matrix $\tilde{C}^{-1}$ namely, case 3. The matrix $\tilde{C}^{-1}$ is expressed as

$$\tilde{C}^{-1} = \begin{bmatrix} \underline{S} & \underline{S}^{(1)} & \cdots & \underline{S}^{(n)} & \cdots & \underline{S}^{(N-1)} \end{bmatrix}_{N\times N}^T. \tag{17}$$

Here, the superscript $(n)$, indicates the $n$ times right cyclic shift of the basis sequence $\underline{S}$.

At the receiver, after passing through DFT and inverse SAM matrix, the received data symbols are estimated as $\hat{X} = (BA_{DFT}) \cdot (A_{DFT}^{-1} B^{-1}) \cdot X + B \cdot W_0 = I \cdot X + B \cdot W_0$. $\tilde{C}^{-1}$ is to be an orthogonal matrix, thereby the noise term $BW_0$ has the same mean and variance as AWGN term $W_0$, which is $E\left[\left(W_0^H B^H\right)(BW_0)\right] = E\left[W_0^H W_0\right] = \sigma_{w_0}^2 I$, where $[\cdot]^H$ denotes the Hermitian operator. Accordingly, the basis sequence $\underline{S}$ is required to be a complex valued sequence having perfect periodic auto-correlation property. Furthermore, by looking at the previous design, it can be found that when strengthening a few data symbol values by parameter $\alpha$ and weakening others by parameter $\beta$, the power of OFDM frame is reduced into a smaller dynamic range compared to the power of original OFDM frame. When $\beta = 0$, the emphasis brings an optimal effect on PAPR reduction.

Therefore, case 3 is proposed to have the basis sequence that is defined by zero-tapping evenly as

$$\underline{S} = \left[ \underbrace{s_0, 0, ...,}_{N/L} \underbrace{s_1, 0, ...,}_{N/L} ..., \underbrace{s_{L-1}, 0, ...}_{N/L} \right]^T, \tag{18}$$

where the non-zero tapping sequence $\hat{\underline{S}} = [s_0, s_1, ..., s_{L-1}]^T$ is a perfect periodic auto-correlation sequence such as Zadoff-Chu sequence [18]. The $n^{th}$ row is generated as the $n$ times right shifted version of the basis sequence in (18).

Case 4: In case 3, the constant amplitude polynomial sequences having perfect periodic auto-correlation property cannot bring an outstanding peak power reduction for OFDM signals since the emphasis brought by the non-zero elements does not introduce enough impact on the data symbols. In view of this, to achieve a better PAPR performance in the new extensive framework of $\tilde{C}^{-1}$, the improved design for the basis sequence is proposed to have two non-zero components as

$$\underline{S} = \left[ s_0, \underbrace{0, ...,}_{N/2-1} s_1, \underbrace{0, ..., 0}_{N/2-1} \right]^T, \tag{19}$$

where $s_0$ and $s_1$ are non-constant amplitude complex values used to emphasis the data symbols, that is $s_0 = \alpha e^{j\theta_0}$ and $s_1 = \beta e^{j\theta_1}$. Such sequence having perfect auto-correlation yields $\theta_0 - \theta_1 = \{\pm\pi/2, \pm 3\pi/2\}$, $\theta_0$ and $\theta_1 \in [0, 2\pi)$. Due to the existing zero tapping inside the basis sequence, the periodic auto-correlation is always zero when the correlation shifting delay $m$ at the moment of $(m \bmod N/2) \neq 0$. The $n^{th}$ row used to be $n$ times cyclic shifted version of the basis one can be relaxed to have two different non-zero elements having perfect auto-correlation property. For $n = 0, 1, ..., N/2-1$, propose the $n^{th}$ row of the new matrix $\tilde{C}^{-1}$ to be

$$\underline{S}^{\langle n \rangle} = \left[ \underbrace{0, ..., 0,}_{n} \underbrace{s_0^{\langle n \rangle}, 0, ..., 0,}_{N/2} \underbrace{s_1^{\langle n \rangle}, 0, ..., 0}_{N/2-n} \right]^T, \tag{20}$$

TABLE I
AVERAGE POWER VARIATIONS OF SAM FOR FOUR CASES, PTS, SLM, A-SLM AND WHT FOR QPSK AND 16-QAM WITH $N = 64$

| $N = 64$ | Power Varied (QPSK) [dB] | Power Varied (16-QAM) [dB] |
|---|---|---|
| WHT | 0 | 0 |
| PTS | 0 | 0 |
| SLM | 0 | 0 |
| A-SLM | 2.5 | 4.2 |
| SAM$^1$ $\alpha > 10^3$ | $< 10^{-3}$ | $< 10^{-3}$ |
| SAM$^1$ $\alpha = 100$ | 0.03 | 0.1 |
| SAM$^2$ | 0 | 0 |
| SAM$^3$ | 0 | 0 |
| SAM$^4$ | 0 | 0 |

where the superscript $\langle n \rangle$ denotes the $n^{th}$ distinct sequence as the $n^{th}$ row of the matrix $\tilde{C}^{-1}$. For $n = N/2, ..., N-1$, the $n^{th}$ row sequence becomes the flipping version of the sequence at the row index of $n - N/2$ as

$$\underline{S}^{\langle n \rangle} = \left[ \underbrace{0, ..., 0,}_{n-N/2} \underbrace{s_1^{\langle n-N/2 \rangle}, 0, ..., 0,}_{N/2} \underbrace{s_0^{\langle n-N/2 \rangle}, 0, ..., 0}_{N-n} \right]^T, \tag{21}$$

where $s_0^{\langle n \rangle} = \alpha^{\langle n \rangle} e^{j\theta_0^{\langle n \rangle}}$ and $s_1^{\langle n \rangle} = e^{j\theta_1^{\langle n \rangle}}$, $\alpha^{\langle n \rangle} \in \mathbb{R}^+$. The phase difference between two must be $\theta_0^{\langle n \rangle} - \theta_1^{\langle n \rangle} = \{\pm\pi/2, \pm 3\pi/2\}$.

Herein, four cases are proposed for the SAM framework, The SAM matrix $B^{-1}$ in case 2, 3 and 4 is orthogonal such that it would not increase the power of OFDM frame. The PAPR performance of the OFDM signal generated in case 1 and case 4 are depending on the value of $\alpha$. Since the samples of the OFDM signal are populated as the sum of subcarriers weighted by data symbols, when $\alpha$ is larger, the data symbol emphasized by $\alpha$ will dominate the summation. Hence, less fluctuated signal amplitudes offer an improved PAPR performance.

## IV. PERFORMANCE COMPARISON

In this section, the performance of the proposed SAM method for four cases is compared with the SLM, PTS and A-SLM in terms of the PAPR and BER metrics. The Walsh-Hadamard Transform (WHT) that provides an orthogonal matrix prior to the IFFT for PAPR reduction [19] is also included in the comparisons. Table I tabulates the average power fluctuation (in dB) of the proposed SAM (superscripts such as 1 and 2 denote for case 1 and case 2 respectively), WHT, PTS, SLM, A-SLM methods for QPSK and 16-QAM with $N = 64$ subcarriers. The system complexities of OFDM employing various PAPR reduction methods considered in the paper are quantified with respect to the required number of multiplications and additions as tabulated in Table II. In PTS and SLM techniques, $U$ and $V$ IFFT operations are required. Besides, side information bits are used along with OFDM

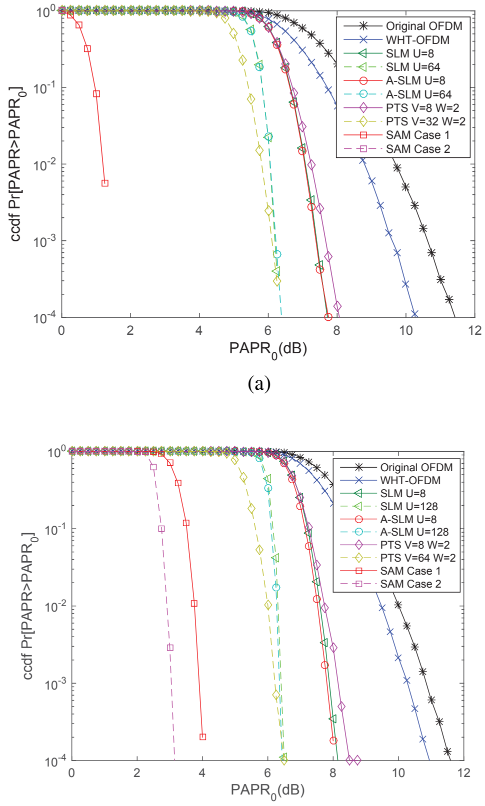



Fig. 2. PAPR performance of the proposed SAM in case 1 and case 2, WHT-OFDM, PTS, ordinary SLM, and A-SLM methods for (a) QPSK when $N = 128$, (b) 16-QAM when $N = 256$.

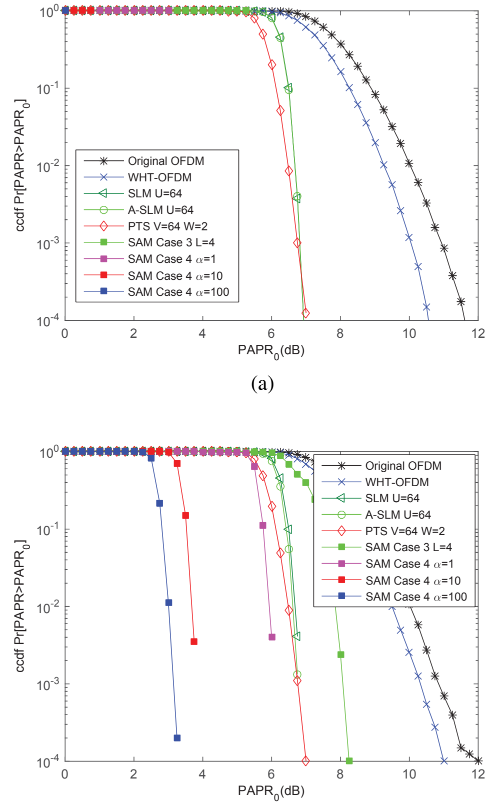



Fig. 3. PAPR performance of the proposed SAM in case 3 and case 4, WHT-OFDM, PTS, ordinary SLM, and A-SLM methods for (a) QPSK and (b) 16-QAM when $N = 256$.

TABLE II
SYSTEM COMPLEXITY OF SAM, PTS, SLM AND WHT METHODS FOR OFDM SYSTEMS

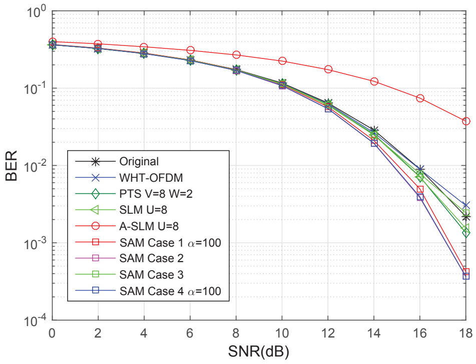
| | **# of Complex Multip.** | **# of Complex Add.** | **SI (Bit)** | **# of IFFTs** |
|---|---|---|---|---|
| Original | $\frac{N}{2}\log_2(N)$ | $N\log_2(N)$ | No | 1 |
| WHT | $\frac{N}{2}\log_2(N)$ $+N^2$ | $N\log_2(N)$ $+N(N-1)$ | No | 1 |
| PTS | $\frac{VN}{2}\log_2(N)$ $+VW^{V-1}$ | $VN\log_2(N)$ | Yes | $V$ |
| SLM | $\frac{UN}{2}\log_2(N)$ $+UN$ | $UN\log_2(N)$ | Yes | $U$ |
| SAM$^1$ | $N\log_2(N)$ $+N$ | $2N\log_2(N)$ $+2N-1$ | No | 1 |
| SAM$^2$ | $N\log_2(N)$ | $2N\log_2(N)$ | No | 1 |
| SAM$^3$ | $N\log_2(N)$ $+LN$ | $2N\log_2(N)$ $+N(L-1)$ | No | 1 |
| SAM$^4$ | $N\log_2(N)+2N$ | $2N\log_2(N)+N$ | No | 1 |

TABLE III
PAPR GAIN (DB) AT THE CCDF RATE OF $10^{-3}$

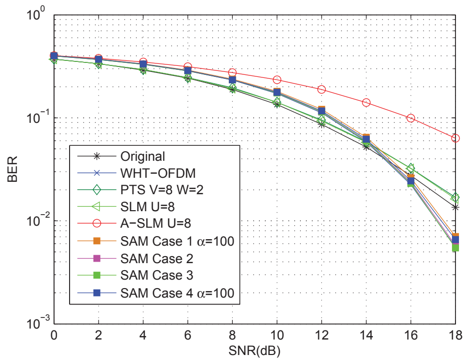
| $N = 256$ | **QPSK** | **16-QAM** |
|---|---|---|
| Original | 10.9 | 11.1 |
| WHT | 10.0 | 10.5 |
| PTS | 6.8 | 7.0 |
| SLM | 6.7 | 6.9 |
| A-SLM | 6.8 | 7.0 |
| SAM$^1$ | 1.2 | 3.9 |
| SAM$^2$ | 0 | 3.0 |
| SAM$^3$ | 5.2 | 8.8 |
| SAM$^4$ | 0.1 | 3.0 |

signal transmission to the receiver. On the other hand, the proposed SAM and WHT need only one pair of FFT/IFFT operations and no SI bits are required at the receiver. The computational complexity of the SAM method is calculated through (11) and (12). The proposed method has much lower computational complexity than the other methods.

Fig 2 (a) and (b) illustrate the PAPR performance of the proposed SAM in case 1 ($\alpha = 100$) and case 2, WHT-OFDM, PTS, ordinary SLM and A-SLM methods. For QPSK and with subcarriers $N = 128$, the numbers of possible OFDM candidate frames in the SLM and A-SLM are $U = 8$ and 64, $Smax = 12$, $D = 2.4$ as suggested in [14], while subblocks $V = 8$ and 32, phase coefficients $W = 2$ are used for PTS. Also $U = 8$ and 128, $Smax = 25$, $D = 4.4$, $V = 8$ and 64, $W = 2$ for 16-QAM and signal length $N = 256$ are used. Fig




(a)




(b)

Fig. 4. BER performance comparisons of the proposed SAM in 4 cases, WHTPTS, ordinary SLM, and A-SLM methods for 16-QAM and $N = 256$ over (a) AWGN channel and (b) multi-path fading channel.

3 (a) and (b) display the PAPR performance when the SAM is applied for case 3 and case 4. In case 3, the Zadoff-Chu sequence is employed with $L = 4$ in (18), and in case 4, $\alpha = [1, 10, 100]$ is taken for comparison.

Figure 4 (a) and (b) show the BER performance comparisons over the AWGN and multipath fading channels, respectively, where the multipath fading channel is assumed to be a three-path Rayleigh fading channel with equal power. The comparisons are given on the SAM for cases 1 and 2 in Fig. 5. Cases 3 and 4 are given in Fig. 6 along with WHT, PTS, SLM, A-SLM, and original OFDM signal. It is noted that in the BER performance of SLM and PTS, the SI is assumed to be transmitted error free such that no data loss caused by SI will be considered.

Table III compares the performance of the proposed SAM methods along with the other techniques with respect to the PAPR gain of CCDF at a given rate of $10^{-3}$, where $\alpha = 100$ in case 1 and case 4, $L = 4$ in case 3 with Zadoff-Chu sequence. It is shown that the proposed method for case 2 and case 4 reduced the PAPR significantly for various constellation scenarios. Meanwhile, case 1 and case 3 also improved the PAPR performance over other methods.

## V. Conclusion

In this paper, an efficient method called SAM that jointly modifies phase and amplitude values of symbols in the alphabet for PAPR reduction in the OFDM communications is proposed. Its superiority over the existing techniques with respect to system performance and computational complexity is shown.

## References

[1] A. N. Akansu, P. Duhamel, X. M. Lin and M. de Courville, “Orthogonal transmultiplexers in communication: a review,” *IEEE Trans. on Signal Processing*, Special Issue on Theory and Applications of Filter Banks and Wavelets, vol. 46, no. 4, pp. 979-995, Apr. 1998.

[2] A. N. Akansu and R. A. Haddad, *Multiresolution Signal Decomposition: Transforms, Subbands, and Wavelets*, Boston, Massachusetts: Academic Press, 1992.

[3] D. Lim, S. Heo, and J. No, “An overview of peak-to-average power ratio reduction schemes for OFDM Signals,” *Journal of Commun. and Netw.*, vol. 11, no. 3, pp. 229-239, Jun. 2009.

[4] H. Ochiai and H. Imai, “Performance of the deliberate clipping with adaptive symbol selection for strictly band-limited OFDM systems,” *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 11, pp. 2270-2277, Nov. 2000.

[5] T. Jiang and G. X. Zhu, “Complement block coding for reduction in peak-to-average power ratio of OFDM signals,” *IEEE Communications Magazine*, vol. 43, no. 9, pp. S17-S22, Sep. 2005.

[6] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, “Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping,” *IEE Electronic Letters*, vol. 32, no. 22, pp. 20562057, Oct. 1996.

[7] S. Y. Le Goff, K. K. Boon, C. C. Tsimenidis, and B. S. Sharif, “A novel selected mapping technique for PAPR reduction in OFDM systems,” *IEEE Trans. Comm.*, vol. 56, no. 11, pp. 1775-1779, Nov. 2008.

[8] S. H. Muller and J. B. Huber, “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” *Elect. Lett.*, vol. 33, no. 5, pp. 368-369, Feb. 1997.

[9] L. J. Cimini Jr and N. Sollenberger, “Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences,” *IEEE Communications Letters*, vol. 4, no. 3, pp. 86-88, Mar. 2000.

[10] M. Deumal, A. Behravan, and L. Pijoan, On cubic metric reduction in OFDM systems by tone reservation, IEEE Trans. Commun., vol. 59, no. 6, pp. 1612-1620, Jun. 2011.

[11] S. H. Han, J. M. Cioffi, and J. H. Lee, “Tone injection with hexagonal constellation for peak-to-average power ratio reduction in OFDM,” *IEEE Commun. Lett.*, vol. 10, no. 9, pp. 646-648, Sep. 2006.

[12] B. S. Krongold and D. L. Jones, “PAR reduction in OFDM via active constellation extension,” *IEEE Trans. Broadcast.*, vol. 49, no. 3, pp. 258-268, Sep. 2002.

[13] A. N. Akansu and H. Agirman-Tosun, “Generalized discrete Fourier transform with nonlinear phase,” *IEEE Trans. on Signal Processing*, vol. 58, no. 9, pp. 4547-4556, Sept. 2010.

[14] Y. C. Cho, S. H. Han, and J. H. Lee, “Selected mapping technique with novel phase sequences for PAPR reduction of an OFDM signal,” *IEEE 60th Vehi. Tech. Conf. (VTC2004-Fall)*, vol. 7, pp. 4781-4785, Sep. 2004.

[15] Y. W. Wang, A. N. Akansu, “Low-complexity peak-to-average power ratio reduction method for orthogonal frequency division multiplexing communications,” *IET Communications*, vol. 9, Oct. 2015.

[16] R. A. Brualdi, *Combinatorial Matrix Classes*, pp.19, Cambridge, United Kingdom: Cambridge University Press, 2006.

[17] G. B. Folland, *Real Analysis: Modern Techniques and Their Applications.* John Wiley and Sons Press, 1999.

[18] R. L. Frank, S. A. Zadoff and R. Heimiller, “Phase shift pulse codes with good periodic correlation properties,” *IRE Transactions on Information Theory*, vol. 8, no. 6, pp.381-382, Oct. 1961.

[19] K. M. Yew, M. Drieberg, V. Jeoti, “On PAPR reduction in OFDM systems: a technique using normalized complex Hadamard transform,” *TENCON 2005 IEEE Region 10*, pp.21-24, Nov. 2005.