CE-SLM-Clipping

A Union Algorithm of PAPR Reduction for OFDM systems

Clipping algorithm 裁剪算法
Cross Entropy，交叉熵
Selective mapping 选择性映射

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***Abstract* — Clipping algorithm is an effective method to reduce the Peak-to-Average Power Ratio for OFDM systems, but it introduces nonlinear distortions and causes the error rate of the system rising at the same time. To solve this drawback of clipping algorithm, this paper introduces CE-SLM algorithm to clipping, in other words when the PAPR of signal is greater than the threshold, and CE-SLM algorithm will be applied. CE-SLM algorithm uses the Cross Entropy (CE) to optimize the symbol sequence in SLM algorithm in order to reduce PAPR. This process doesn't introduce nonlinear distortions and can reduce the error rate of the system. Computer simulation results show that the union algorithm can reduce the error rate of the system under the condition of keeping the same PAPR reduction effect with clipping algorithm.**

A.非线性失真 B.误码率升高 CE-SLM 非线性失真

Keywords- OFDM; PAPR; Cross Entropy; clipping

夹戴，别上

I. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is a multi-carrier modulation technique [1], whose principle is to divide the channel into multiple orthogonal sub-channels so that the signal can be transmitted in each sub-channel. Therefore, OFDM has the advantages of high spectrum efficiency and strong ability to resist multipath fading. It has been widely used in ADSL, DAB, DVB, HDTV and WLAN, and it has become one of the core technology of 4G. However, OFDM has some obvious drawbacks. One of the major problems of the OFDM system is that OFDM signal has higher peak to average power ratio (PAPR) because OFDM signal is the sum of many narrowband signals in the time domain. The high PAPR of OFDM signal has higher linear demand to A/D, D/A converters and high power amplifier in OFDM systems. Furthermore, high PAPR signals can cause the nonlinear distortion easily which result in inter symbol interference, and finally cause the decline of bit error rate (BER) performance in OFDM system. Therefore, the high PAPR of an OFDM signal need to be solved urgently [2-3].

At present, the method of reducing PAPR of the OFDM system is roughly divided into three types [4]: pre-distortion technology, coding technology and probability technology. (1) Clipping algorithm is one of pre-distortion technology and is very simple and practical. But it can unavoidably introduce in-band noise and out-of-band interference, and also makes the performance of the system worse because it directly intercepts the signal which exceeds the threshold. (2) Selective mapping (SLM) is one of probability technology. SLM algorithm needs to obtain point multiplication of the random M phase sequence and the original OFDM signal, and then chooses the frame with minimum PAPR to transfer after making IFFT transformation on the M independent OFDM signal which has the same information. SLM can reduce the PAPR of OFDM system without distortion, but it increases the complexity of the system because of multiple IFFT transformation. (3) The CE-SLM algorithm in reference [5] optimize the symbol sequence in SLM by introducing the Cross Entropy (CE), thus reduce the PAPR of the OFDM system without distortion.

SLM ⇒ CE-SLM

To solve the problem that the clipping algorithm can bring non-linear distortion, this paper will introduce the CE-SLM algorithm to the clipping algorithm. That means that the symbol sequence will be optimized by using CE algorithm and point multiply with original signal until the PAPR meet the requirements of OFDM system, instead of been directly intercepted when it exceeds the limit threshold. The union algorithm presented in this paper can reduce the PAPR in an acceptable range without introducing nonlinear distortion. Computer simulation results show the union algorithm can reduce the error rate of the system under the condition of keeping the same PAPR reduction effect with clipping algorithm.

II. PAPR AND CE - SLM ALGORITHM

2.1. PAPR of the OFDM system

Xk为调制后的数据

OFDM signal is the sum of N independent signals, it can be expressed as.

不太理解?

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_k e^{j2\pi nk/JN}\qquad n=0,1,...,N-1 \tag{1}$$

Among them, X_k is the data after modulation and N is the number of sub-carrier. The ratio of the maximum peak power to average power is called PAPR, and it can be expressed as

$$PAPR=10\log_{10}\frac{\max\left(|x_n|^2\right)}{E\left(|x_n|^2\right)} \tag{2}$$

dB

Where $\max|x(n)|^2$ represents the maximum peak power, $E\left[|x(n)|^2\right]$ means average power, and dB is the unit of PAPR.

Usually, the PAPR of OFDM system is measured by the complementary cumulative distribution function (CCDF). CCDF is defined as follow.

互补累积分布函数

$$\begin{aligned}P(PAPR>z)&=1-P(PAPR\le z)\\&=1-\left(1-\exp(-z)\right)^N\end{aligned} \tag{3}$$

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z不变, $N\uparrow$, $\Rightarrow$ CCDF$\uparrow$

门槛，阈值，下限

We can know from formula (3): if the threshold z doesn't change, increasing the subcarrier number N means that the CCDF value will increase and the probability exceeding the threshold z will increase.

概率

2.2. CE - SLM algorithm

迭代算法

CE algorithm is an iterative algorithm to select the optimal sequence and is used to estimate the probability of rare events by Rubinstein in reference [5] for the first time. On the other hand, CE algorithm is a kind of effective method to solve the combinatorial optimization problem of rare event. In SLM algorithm, the symbol selection sequence $c=[c_0,...,c_{N-1}]$ is optimized by using CE algorithm [6], and $\hat{X}=[X_0c_0,...,X_{N-1}c_{N-1}]$ can be obtained by multiplying the optimal symbol selection sequence with the original signal. Let $L(c)=\max|\hat{x}(n;c)|^2$, then the problem of reducing the PAPR can be translated into the following combinatorial optimization problem [7-11]:

组合

组合的

峰值最小化：

$$\boxed{\begin{aligned}&\min_c L(c)\\ &subject\ to: c\in\{1,-1\}^N\end{aligned}} \tag{4}$$

Let $d\in\{0,1\}^N$, then $c=1-2d$, where the elements in sequence d obey the binomial distribution and can be expressed as follow

二项式分布

$$\boxed{\begin{aligned}&P(d_k=1)=p_k\\ &P(d_k=0)=1-p_k\ \ k=0,...,N-1\end{aligned}} \tag{5}$$

The probability density function is defined as:

概率密度函数

$$f(d;p)=\prod_{k=0}^{N-1}p_k^{d_k}(1-p_k)^{1-d_k} \tag{6}$$

The optimal probability P^* can be generated from following formula (7)

$L(d_i)$是什么?

$$\boxed{\frac{1}{n}\sum_{i=1}^{n}I_{\{L(d_i)\le\gamma\}}\nabla\ln f(d_i;p)=0} \tag{7}$$

Where $I_{\{x\le\gamma\}}=\begin{cases}1 & x\le\gamma\\ 0 & otherwise\end{cases}$, $\gamma=L_{n_s}$, n_s is decided by sample number n and coefficient $n_s=\lceil\rho n\rceil$, $\lceil\bullet\rceil$, means rounding up, i.e. choosing the minimum integer which is bigger than the number.

向上取整

The partial derivative of formula (6) is (OK)

偏导数

$$\boxed{\frac{\partial\ln f(d;p)}{\partial p_k}=\frac{d_k}{p_k}-\frac{1-d_k}{1-p_k}} \tag{8}$$

取ln, 纵和
对于p_k, i求导为0,
$i=k$时,
$\ln f(d_k;p_k)$
$=d_k\ln p_k+(1-d_k)\ln(1-p_k)$
求导有
$\frac{\partial\ln f(d;p)}{\partial p_k}=\frac{d_k}{p_k}-\frac{1-d_k}{1-p_k}$

Then the updating formula of the probability P can be obtained

概率

$$\boxed{p_k^*=\frac{\sum_{i=1}^{n}I_{\{L(d_i)\le\gamma\}}d_{ik}}{\sum_{i=1}^{n}I_{\{L(d_i)\le\gamma\}}}} \tag{9}$$

And d_{ik} is the kth element of d_i.

In CE-SLM algorithm, the probability P can be updated by formula (9) until optimal symbol sequence c^* is obtained. By this way, the PAPR of the OFDM system can be reduced.

联合算法

III. The Theory of the Union Algorithm

3.1. The original clipping algorithm

原始的截剪算法

The theory of the clipping algorithm [11] is that the amplitude of a signal will be set as the threshold if the amplitude exceeds the threshold; if it isn't bigger than the threshold, any processing will not be done. This theory can be expressed as

$$x_k=\begin{cases}x_k & |x_k|<A\\ Ae^{j\phi(x_k)} & |x_k|\ge A\end{cases} \tag{10}$$

不处理

处理

Among them, A is the clipping algorithm threshold, and $\phi(x_k)$ denotes the phase of the signal. The threshold A of the clipping algorithm is decided by the clipping ratio (CR), it is expressed as follow

$$\boxed{CR=\frac{A}{\sqrt{P_{in}}}} \tag{11}$$

$\Rightarrow A=CR\cdot\sqrt{P_{in}}$

P_{in} 平均功率

Where P_{in} denotes the average power of the signal before clipping.

From the theory of the clipping algorithm, we can see that the signal which exceeds the limit threshold is directly intercepted in the clipping algorithm. Although it can reduce the PAPR of OFDM system obviously, it introduces the nonlinear distortion at the same time and decrease the BER performance of system.

3.2. The implementation of the union Algorithm

To solve the problem that the clipping algorithm can introduce non-linear distortion, this paper will introduce the thought of CE-SLM algorithm to the clipping algorithm. That is, when the PAPR of the signal is bigger than the PAPR allowed by the system, CE algorithm be used to optimize the symbol sequence of the SLM algorithm until PAPR of the signal meet the system requirements; when the PAPR of the signal is smaller than $PAPR\_th$, then it won't be disposed.

Now, the maximum PAPR of the OFDM systems is limited to $PAPR\_th$. The flow chart of union algorithm is shown in figure 1.

Step 1: Initialize the parameters: Let $PAPR\_th=7dB$, $P_0=(1,0.5,0.5,...,0.5)$, $\rho=0.17$, and $n=30$;

被处理

Step 2: Calculate the PAPR of the signal: If $PAPR$ is smaller than $PAPR\_th$, then it won't be disposed; if $PAPR$ is bigger than $PAPR\_th$, then it will be disposed by the CE-SLM algorithm;

Step 3: Generate n samples of d_i according to P, calculate $L(d_i)$ and get ascending sequence $\{L_0,...,L_{N-1}\}$;

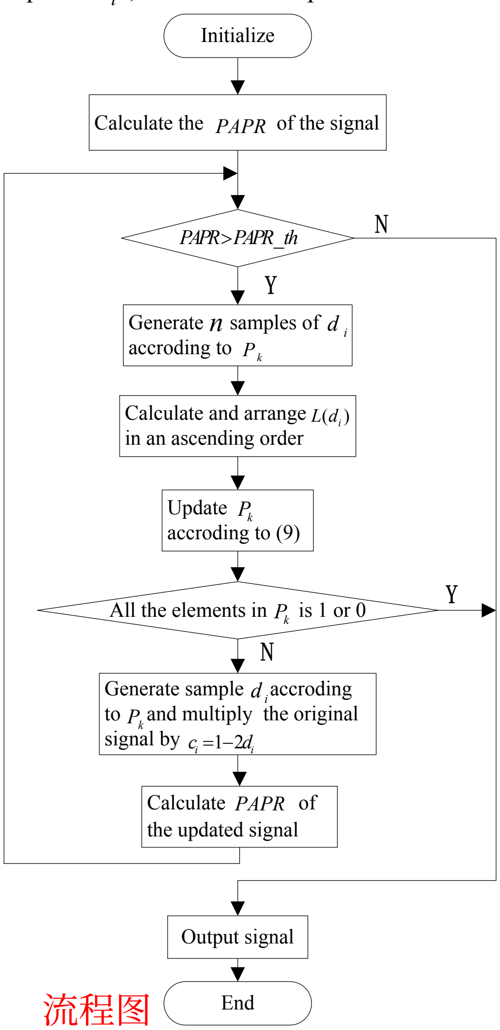
通分: 同乘 $p_k(1-p_k)$ 有: $(1-p_k)d_k-(1-d_k)p_k=d_k-p_kd_k-p_k+p_kd_k=d_k-p_k$

公式7: $\Rightarrow \frac{1}{n}\sum_{i=1}^{n}I_{\{L(d_i)\le\gamma\}}[d_{ik}-p_k]=0\Rightarrow$ (消去$\frac{1}{n}$, 同乘n) $\Rightarrow p_k^*=\frac{\sum I_{\{L(d_i)\le\gamma\}}d_{ik}}{\sum I_{\{L(d_i)\le\gamma\}}}$

Step 4: Set $\gamma = L_{n_s}$ and update p_k according to formula (9). If all the elements in p_k are 1 or 0, then record the symbol sequence c_i ,output the signal and end loop, otherwise go to step 5;

Step 5: Generate d_i according to the updated signal p_k , then let $c_i = 1 - 2d_i$ and multiply the original signal with c_i ;

Step 6: Calculate the PAPR of the updated signal. If PAPR is bigger than $PAPR\_th$, then go to step 3; if PAPR is smaller than $PAPR\_th$, output the signal, record the symbol sequence c_i , then end the loop.



流程图

Figure 1.The flow chart of the union algorithm

In order to contrast and analyze two algorithms in the following simulation, the choosing method of threshold in clipping algorithm is shown as follows when PAPR is limited to $PAPR\_th$.

The below formula can be derived easily from formula (2)

$$\max\left(|x_n|^2\right) = E\left(|x_n|^2\right)\cdot 10^{PAPR/10} \tag{12}$$

, then the signal peak power can be shown as follows

$$Peak\_th = E\left(|x_n|^2\right)\cdot 10^{PAPR\_th/10} \tag{13}$$

Thus, the clipping algorithm can be expressed as

$$x_k = \begin{cases} x_k & |x_k| \le \sqrt{Peak\_th} \\ sqrt(Peak\_th)e^{j\phi(x_k)} & |x_k| > \sqrt{Peak\_th} \end{cases} \tag{14}$$

Where $\sqrt{Peak\_th}$ denotes the clipping threshold. The performance of PAPR and BER of the union algorithm and clipping algorithm expressed by (14) will be presented in the next section.

3.3. Simulation Experiment

算法对比 { clipping algor / union algor

In order to verify the effectiveness of the union algorithm, this paper compares the clipping algorithm with the union algorithm by means of MATLAB simulation.

First, the effect of reducing PAPR by the clipping algorithm and the union algorithm is compared in this paper. The simulation parameters are set as follows: the sub-carrier number $N = 128$, the numbers of OFDM signal is 1000, modulation technique is QPSK, and sampling number generated by probability P is 30. Let $PAPR\_th = 7dB$, then $L_{n_s} = 5$. In Figure 2, the CCDF curve of the clipping algorithm and the union algorithm are shown.

From Figure 2, we can know that the PAPR of the signal can be reduced to the range required by the system by using the threshold in this paper. When setting $PAPR\_th = 7dB$, the PAPR of the signal are reduced directly down to 7dB by the conditional clipping algorithm according to (14). From the CCDF curve, we can see that the union algorithm also can reduce the PAPR of the signal down to 7dB. The effect of reducing PAPR is almost the same with the clipping algorithm. But because the union algorithm in this paper uses the CE-SLM to reduce the PAPR of the signal without distortion, instead of directly intercepting the signal whose PAPR is bigger than the permission of OFDM system, the union algorithm can avoid effectively nonlinear distortions when reducing the PAPR of the signal.

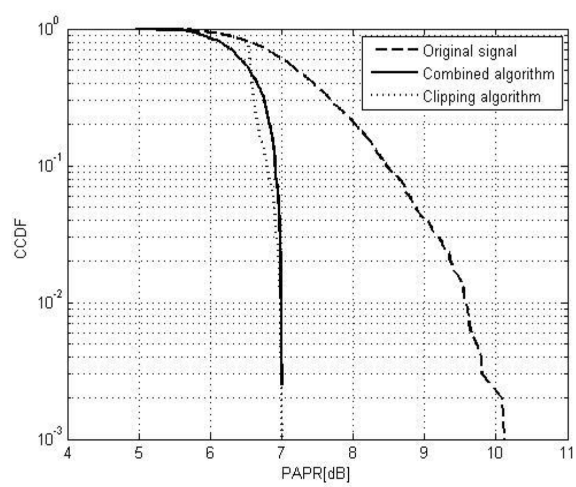


Figure 2.The CCDF curve of different algorithms

In order to verify error performance of the union algorithm, the BER of the two algorithms is compared in this paper. The simulation parameters are set as follows: the sub-carrier number $N = 128$, the numbers of OFDM signal is 10000, modulation technique is QPSK, and sampling number generated by probability P is 30. Let $\rho = 0.17$, $L_{n_s} = 5$ and $PAPR\_th = 7dB$. Fig.3 shows the error rate curve of the clipping algorithm and the union algorithm.

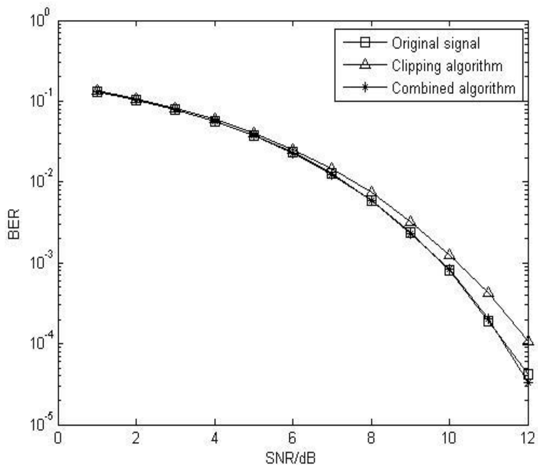


Figure 3.The error rate curve of different algorithms

Form the fig.3, we can see that the error rate of the clipping algorithm is significantly higher than the error rate of the original signal, and the error rate curve of the union algorithm and the error rate curve of the original signal almost coincide when the signal-to-noise ratio is low. It means the union algorithm don't introduce the nonlinear distortion when reducing the PAPR of the OFDM signal effectively. Compared with the clipping algorithms, the union algorithm in this paper reduces the BER of the OFDM system significantly.

IV. CONCLUSIONS

A union algorithm which combines the clipping algorithm with the CE-SLM algorithm is presented by this paper. When the PAPR of the signal is bigger than the PAPR that the OFDM system allows, CE-SLM algorithm is used directly to reduce the PAPR of the signal. By using this way, the problem of non-linear distortion in the clipping algorithm can be solved. The union algorithm in this paper doesn't introduce the non-linear distortion, so it reduces the BER of the system more effectively.

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将OFDM的PAPR算法
应用到OFDM-LFM上.