Delay and Doppler Shift Estimation for OFDM-Based Radar-Radio (RadCom) System

Jian-Feng Gu, Jaber Moghaddasi, and Ke Wu
Poly-Grames Research Center & Center for Radio-frequency Electronics Research of Quebec (CREER),
Ecole Polytechique de Montreal,Montreal, QC, Canada H3T 1J4
Email: jian-feng-2.gu@polymtl.ca; jaber.moghaddasi@polymtl.ca; ke.wu@polymtl.ca

***Abstract*—The joint operation of radar sensing and wireless communication, namely RadCom system, yields a unique platform to meet the requirements of future intelligent transportation networks. Taking advantages of OFDM waveform, the range-Doppler coupling issues can be overcome for radar applications and complex equalization filter is no longer necessarily used to cope with frequency-selective fading channel because of multipath. This paper presents a technique for simultaneous estimation of the range and Doppler shift of targets using an OFDM-based Radcom system. Unlike the previous method based on fourier analysis to estimate the range and Doppler shift, we derive a subspace-based algorithm by applying a smoothing approach for joint estimation of range and Doppler shift without the pair matching problem. Compared to the previous method, our method is able to at least exhibit the following three advantages: 1) higher resolution for multiple targets, 2) less time-based data, and 3) avoidance of pair-matching techniques. Therefore, this method is more suitable for OFDM-based RadCom systems with high mobility and high data rate. Furthermore, the proposed algorithm is compared with the current counterpart with computer simulations.**

***Index Terms*—Radar, Radio, Joint Radar-Radio Technique, RadCom, OFDM, Range, Doppler shift, LS**

I. Introduction

The concept and technique of integrated radar sensing and wireless communication (RadCom, for short) systems into a unique platform has been proposed for future intelligent transportation systems [1], [2]. A meaningful hardware platform had never been demonstrated until a recent development of Time Division Duplex (TDD)-based transceiver architecture that was proposed in [2]. In this system, each time period is divided into four time slots equally, the first three slots are used to transmit a trapezoidal frequency-modulation continuous-wave (TFMCW) waveform to estimate range and Doppler shift caused by a high-speed movement of communication system and the last slot is arranged for enabling communication signal transmission and processing. Through such a simple time-domain scheme, the interference between radar and communication signals can be eliminated, and then parameter estimation using radar technique and data transmission with radio technique can be implemented separately and robustly. However, the TDD waveforms in such a RadCom system cannot satisfy increasing requirements of emerging high data rate transmission over mobile or wireless communication channels. Recently, Sturm et al. [3] have developed a OFDM-based RadCom system and proposed a so-called element-wise division technique to obtain the channel information and then a discrete-time Fourier transform (DFT) is used to obtain the range and Doppler shift independently. However, such a scheme presents at least three shortcomings when multiple targets exhibit relatively high speeds with each other: 1) a large number of OFDM symbols are needed to obtain a high frequency resolution; 2) the range resolution is constrained by the number of subcarriers and 3) a pair-matching method is needed to obtain the one-to-one relationship between range and Doppler shift.

To tackle these problems, we present in this work a subspace-based method for OFDM-based radar system to jointly estimate range and Doppler shift, which does not need to carry out the DFT algorithm for the range and Doppler shift estimation while multiple rotation invariant subarrays are constructed from the whole data which is estimated by normalized element-wise correlation. In this way, we can use the rotation invariance property and least squares (LS) or total LS approaches to obtain the ranges. Then, the matrix algebra operations are implemented to estimate the Doppler shift through the association with the estimated ranges. Therefore, the proposed method makes use of the association and fusion information to enjoy a better parameter estimate performance compared to the previous methods. In addition, our method does not need to carry out a two-dimensional pair-matching approach and accumulate a lot of OFDM symbols to obtain an accurate estimate for Doppler shifts, which both make our method more feasible in estimating the parameters of target with highly moving speed.

II. Signal Model for Radar Processing

An OFDM signal can be considered as the sum of multiple signal-carrier signals, say N subcarriers, with orthogonal carrier waveforms in the symbol duration T. Each subcarrier is modulated with different transmit data belonging to a QAM or PSK constellation. The transmitted time-domain signal within the M OFDM symbols is expressed by

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} s_m(n) e^{j2\pi n \triangle f t} p(t), \tag{1}$$

where $s_m(n)$ denotes the transmit data by the nth subcarrier of the mth symbol, $\triangle f = 1/T$ is the frequency spacing between subcarriers and $p(t)$ is the extended symbol duration window including T_{cp} the duration of cyclic prefix (CP) or guard interval, which is very important for OFDM system to eliminate intersymbol interference (ISI) due to the frequency-selective wireless channels. They are usually inserted between adjacent OFDM symbols. Note that since radar system receives the reflecting signals and self-communication signal as well, we assume in the following process that we have canceled the self-interference using the methods suggested in [4].

Suppose that there are K reflecting targets impinging on the radar system from distances $\{R_k\}_{k=1}^{K}$ with correspondingly relative velocity of $\{v_k\}_{k=1}^{K}$, each of which results in delay $\tau_k = \frac{2R_k}{c}$ and Doppler shift $f_{D,k} = \frac{2f_c v_k}{c}$, where c is the speed of light and f_c is the carrier frequency of OFDM signals. Herein, we assume that distances $\{R_k\}_{k=1}^{K}$ are distinct due to different fast moving targets. At the radar receiver of RadCom system, following matched filtering, symbol-rate sampling and discarding symbols falling in the cyclic prefix, the received time-domain signal of the mth OFDM symbol by N-point sampling before carrying out the discrete Fourier transform (DFT) can be expressed as

$$\begin{aligned} y_m(l) = &\sum_{k=1}^{K} \beta_k e^{j2\pi m \bar{f}_{D,k}} \sum_{n=0}^{N-1} s_m(n) e^{j2\pi n(m\bar{T} + \frac{\bar{T}}{N} l - \bar{\tau}_k)} \\ &+ w_m(l) \end{aligned} \tag{2}$$

where $w(l)$ is an additive circular symmetric complex white Gaussian noise (AWGN) with zero-mean and variance σ^2 and β_k is the attenuation of the kth target including path loss, reflection, and any processing gains, and $\bar{T} = \frac{T_{cp}+T}{T}$. To simplify the expressions, delay $\bar{\tau}_k \triangleq \frac{\tau_k}{T}$ and Doppler shift $\bar{f}_{D,k} \triangleq f_{D,k}(T_{cp}+T)$ are used in the following content and both of them are assumed to be constant within the M OFDM symbols. Then, it can easily be shown from (2) that the received signal after DFT can be obtained by

$$\hat{s}_m(n) = s_m(n) \sum_{k=1}^{K} \beta_k e^{j2\pi m \bar{f}_{D,k}} e^{-j2\pi n \bar{\tau}_k} + \hat{w}_m(n). \tag{3}$$

It is easy to verify that noise $\hat{w}_m(n)$ has the same distribution as $w_m(l)$ since the DFT is a linear transformation. From (3), we can see that there are the same OFDM signals among the different targets, let us normalize it using the known symbols $s_m(n)$ by element-wise correlation

$$\begin{aligned} \bar{s}_m(n) &\triangleq \frac{\hat{s}_m(n) * s_m^*(n)}{\|s_m(n)\|^2} \\ &= \sum_{k=1}^{K} \beta_k e^{j2\pi m \bar{f}_{D,k}} e^{-j2\pi n \bar{\tau}_k} + \bar{w}_m(n) \\ &\triangleq z_m(n) + \bar{w}_m(n), \quad n = 0, 1, \cdots, N-1, \end{aligned} \tag{4}$$

where $z_m(n)$ denotes the normalized complex gain of $\hat{s}_m(n)$, i.e., $\hat{s}_m(n) = z_m(n) s_m(n)$, and $\bar{w}_m(n) = \frac{\hat{w}_m(n) * s_m^*(n)}{\|s_m(n)\|^2}$, which is also a i.i.d Gaussian random variable with the same distribution as $w_m(l)$. From (4), we can see that the range-based phase information is changed with the subcarriers of OFDM system instead of the symbols while the Doppler shift-based phase information depends on the symbols provided that the range and Doppler shift are time-invariance during the whole period. Intuitively, DFT-based techniques [3] can be used to estimate the range and Doppler shift if there are enough symbols and subcarriers. Unfortunately, the range is changed rapidly when the targets are moving fast, the classical DFT techniques cannot be employed to obtain a high resolution estimation for multiple targets because of the fact that only few symbols can be exploited to guarantee the assumptions for (4). In addition, these DFT-based techniques estimate the range and Doppler shift independently and need the pair-matching methods to obtain the one-to-one relationship between the range and Doppler shift. In order to circumvent these shortcomings, a novel subspace-based algorithm for OFDM-based radar processing will be developed in the following sections.

III. Range and Doppler shift estimation

In this section, we will develop a method for jointly estimating the range and Doppler shift based on subspace-based technique. We will construct a data matrix using multiple Hankel submatrices and then the ranges of the reflecting targets are estimated by using the rotation invariance of the constructed data matrix. Since the multiple Hankel matrices are formed from the estimated ranges, an LS method is used to transfer a nonlinear, multidimensional Maximum Likelihood estimator to a linear estimator. In this way, we can exploit few symbols to estimate multiple reflecting targets and deal with the pair-matching problem without additional steps.

A. Range estimation based on rotation invariance

In an OFDM-based RadCom system the mth OFDM symbol consists of N subcarriers, where $N \gg 1$, we can make use of the rotation invariance structure of (4) in avoidance of additive noise to form a Hankel matrix of $(q+1) \times (p+1)$ dimensions as shown below

$$\mathbf{H}_m \triangleq \mathbf{A}_\tau D_m \mathbf{X}^T = \mathbf{a}_{f,m} \diamond \mathbf{A}_\tau D_\beta \mathbf{X}^T, \tag{5}$$

where $\mathbf{a}_{f,m} = [a_{f,m}^{(1)}, a_{f,m}^{(2)}, \cdots, a_{f,m}^{(K)}]$ is the phase offset in mth OFDM symbol due to the Doppler shift with $a_{f,m}^{(k)} = e^{j2\pi m \bar{f}_{D,k}}$, $D_\beta \triangleq diag\{\boldsymbol{\beta}\}$ with $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots \beta_K]^T$, and $D_m \triangleq diag\{\boldsymbol{\beta} \circ \mathbf{a}_{f,m}^T\}$, where $'\circ'$ and $'\diamond'$ denote Hadamard and Khatri-Rao products, superscript T is the transpose of matrix or vector, and 'diag' is a notation to transform a column vector to a diagonal matrix. We also denote $\mathbf{A}_\tau$ as the manifold matrix of the ranges which is a Vandermonde matrix of $(q+1) \times K$ dimensions

$$\mathbf{A}_\tau \triangleq [\mathbf{a}_{\tau,1}, \mathbf{a}_{\tau,2}, \cdots, \mathbf{a}_{\tau,K}], \tag{6}$$

and it is easy to verify that the matrix $\mathbf{X}$ is also a Vandermonde matrix with $(p+1) \times K$ dimensions. Note that since both $\mathbf{A}_\tau$ and $\mathbf{X}$ are Vandermonde matrices with the assumption for different ranges, $\mathbf{A}_\tau$ and $\mathbf{X}$ are of rank K provided that $q+1 \geq K$ and $p \geq K$, which implies that $\mathbf{H}_m$ is also of rank K. By collecting the whole $\{\mathbf{H}_m\}_{m=0}^{(M-1)}$, we can construct a new matrix as

$$\mathbf{H} \triangleq [\mathbf{H}_0^T, \mathbf{H}_1^T, \cdots, \mathbf{H}_{(M-1)}^T]^T = \mathbf{A}_f \diamond \mathbf{A}_\tau D_\beta \mathbf{X}^T, \quad (7)$$

where $\mathbf{A}_f = [\mathbf{a}_{f,1}^T, \mathbf{a}_{f,2}^T, \cdots, \mathbf{a}_{f,M}^T]^T$, and it is also easily to know that the rank of $\mathbf{H}$ is K. We get the estimate for $\mathbf{H}$ by adding the i.i.d Gaussian noise

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{W}, \quad (8)$$

where $\mathbf{W} = [\mathbf{W}_0^T, \mathbf{W}_1^T, \cdots, \mathbf{W}_{(M-1)}^T]^T$ in which $\mathbf{W}_m(i,j) = \bar{w}_m(i+j-2)$. From the assumption for $\bar{w}_m(n)$ in (4), we can obtain that $E[\mathbf{W}] = \mathbf{0}$ and $E[\mathbf{W}\mathbf{W}^H] = (p+1)\mathbf{I}_{M(q+1)}\sigma^2$, where E denotes a statistical expectation. Notice that there are many well documented methods [5] for estimating the $2K$ parameters of interest, namely $\{\bar{f}_{D,k}\}_{k=1}^K$ and $\{\bar{\tau}_k\}_{k=1}^K$, such as stochastic or deterministic maximum Likelihood (ML), weighted subspace fitting (WSF), and suboptimal MUSIC-based approaches. However, these algorithms are based on a two-dimensional parameter-searching method, which suffers from a high computational burden that prohibits an efficient application in fast time varying scenarios, especially real-time tracking for fast moving targets. Therefore, we will focus on more efficient approaches based on the ESPRIT and linear least squares techniques. From the rotation invariance structure of $\mathbf{A}_\tau$ in (6), we can easily find the following relationship that

$$\begin{aligned}\mathbf{A}_m^{(1)} &\triangleq \mathbf{A}_f \diamond \mathbf{A}_\tau(m(q+1)+1 : m(q+1)+q, :) \\ &= \mathbf{A}_f \diamond \mathbf{A}_\tau(m(q+1)+2 : (m+1)(q+1), :)D_\tau \\ &\triangleq \mathbf{A}_m^{(2)} D_\tau,\end{aligned} \quad (9)$$

where $D_\tau = [e^{j2\pi\bar{\tau}_1}, e^{j2\pi\bar{\tau}_2}, \cdots, e^{j2\pi\bar{\tau}_K}]$. If we let E_s be the matrix whose columns are the left singular vectors corresponding to the largest singular values of $\mathbf{H}$, then clearly it spans the signal subspace with the expression

$$E_s = \mathbf{A}_f \diamond \mathbf{A}_\tau T, \quad (10)$$

where T is any $K \times K$ nonsingular matrix. Similar to the definitions for $\mathbf{A}_m^{(i)}, i = 1, 2$, we can get the relationship

$$E_s^{(1)} = E_s^{(2)} \mathbf{\Phi}, \quad (11)$$

where $\mathbf{\Phi} = T^{-1} D_\tau T$, intuitively, we can obtain the K ranges by implementing the eigen decomposition of $\mathbf{\Phi}$. Since we should replace E_s by its known estimate $\hat{E}_s$ obtained from (8), the estimate $\hat{\mathbf{\Phi}}$ can be solved by the following minimization problem

$$\min_{\mathbf{\Phi}} = \|\hat{E}_s^{(1)} - \hat{E}_s^{(2)} \mathbf{\Phi}\|_F^2. \quad (12)$$

There are three LS-based methods to solve the above problem, such as classical LS, total least squares(TLS) and structured TLS [6]. Notice that although the structured TLS can obtain ML estimates of the parameter vector, it must suffer from the highest computational burden among them due to iterative steps without a closed-form solution [7]. The classical LS or TLS algorithms will be exploited in the following numerical simulations.

B. Doppler shift estimation based on least squares

Obviously, we can use some matrix operation for $\mathbf{H}$ to get a new matrix by $\mathbf{H}' = \mathbf{A}_\tau \diamond \mathbf{A}_f D_\beta \mathbf{X}^T$ [8], then using the similar steps for estimating the ranges to get the Doppler shifts. However, such a method cannot be applied in some cases, for example targets that are moving fast and some OFDM symbols that are missing. In the following, we will introduce a new method to estimate the parameters more robustly in complex scenarios.

From (6), we can see that the columns of $\mathbf{A}_\tau$ have a complex conjugate symmetric structure, that is

$$\mathbf{a}_{\tau,k} = e^{-j2\pi q\bar{\tau}_k} \mathbf{J}_{q+1} \mathbf{a}_{\tau,k}^*, k = 1, 2, \cdots, K \quad (13)$$

where $\mathbf{J}_a$ is the $a \times a$ permutation matrix with ones on the anti-diagonal, and the superscript $'*'$ denotes the complex conjugate. Making use of the property given by (13), the complex conjugate of the Hankel matrix in (5) can be expressed as

$$\mathbf{H}_m^* \triangleq \mathbf{J}_{q+1} \mathbf{A}_\tau D_{mb} \mathbf{X}^T \mathbf{J}_{p+1} \quad (14)$$

where $D_{mb} \triangleq D_m^* diag\{(\mathbf{A}_\tau^*(q+1,:) \circ \mathbf{X}^*(p+1,:))^T\}$ and $\mathbf{A}_\tau(a,:)$ denotes the ath row of $\mathbf{A}_\tau$ and the superscript $'H'$ is the conjugate transpose of matrix. It is easy to see that $\mathbf{A}_\tau(q+1,:) \circ \mathbf{X}(p+1,:) = \mathbf{A}_\tau(N,:) = [e^{-j2\pi(N-1)\bar{\tau}_1}, e^{-j2\pi(N-1)\bar{\tau}_2}, \cdots, e^{-j2\pi(N-1)\bar{\tau}_K}]$. Therefore, the backward version of (5) is defined as

$$\mathbf{H}_m^B \triangleq \mathbf{J}_{q+1} \mathbf{H}_m^* \mathbf{J}_{p+1} = \mathbf{A}_\tau D_{mb} \mathbf{X}^T, \quad (15)$$

and its estimate $\hat{\mathbf{H}}_m^B = \mathbf{H}_m^B + \mathbf{J}_{q+1} \mathbf{W}_m^H \mathbf{J}_{p+1}$. Let us define the Moore-Penrose right pseudoinverse of $\hat{\mathbf{H}}_m^B$ as

$$\lim_{p\to\infty} \hat{\mathbf{H}}_m^{B\dagger} = \lim_{p\to\infty} (\mathbf{X}^T)^\dagger D_{mb}^* {\mathbf{A}_\tau}^\dagger + \frac{\mathbf{J}_{p+1}\mathbf{W}_m\mathbf{J}_{q+1}}{(p+1)\sigma^2}, \quad (16)$$

where ${\mathbf{A}_\tau}^\dagger = (\mathbf{A}_\tau^H \mathbf{A}_\tau)^{-1} \mathbf{A}_\tau^H$ is the left pseudoinverse of $\mathbf{A}_\tau$ and $(\mathbf{X}^T)^\dagger = (\mathbf{X}^T \mathbf{X}^*)^{-1} \mathbf{X}^T$ is the right pseudoinverse of $\mathbf{X}^T$. Herein, we make use of the properties of the Moore-Penrose pseudoinverse [9] and the assumptions for additive noise and received signals, i.e., $\lim_{p\to\infty} \frac{1}{p+1}(\mathbf{H}_m^B \mathbf{J}_{p+1} \mathbf{W}_m^H \mathbf{J}_{q+1}) = \mathbf{0}$ and $\lim_{p\to\infty} \frac{1}{p+1}(\mathbf{J}_{p+1} \mathbf{W}_m^H \mathbf{J}_{q+1} \mathbf{J}_{q+1} \mathbf{W}_m \mathbf{J}_{p+1}) = \mathbf{I}_{q+1}\sigma^2$. Combining (8) and (16), we can obtain the following cross-correlations as

$$\begin{aligned}\mathbf{R}_{u,v}^B &\triangleq \lim_{p\to\infty} \hat{\mathbf{H}}_u \hat{\mathbf{H}}_v^{B\dagger} \quad (u, v = 0, 1, \cdots, M-1) \\ &\approx \mathbf{A}_\tau diag\{\boldsymbol{\beta}^{\mathbf{2}} \circ \mathbf{a}_{f,(u+v)}^T \circ \mathbf{A}_\tau(N,:)\} \mathbf{A}_\tau^\dagger,\end{aligned} \quad (17)$$

which implies that $\mathbf{R}_{u,v}^B \mathbf{a}_{\tau,k} = \mathbf{a}_{\tau,k} \lambda_{u,v}^{(k)}$, where $\mathbf{a}_{\tau,k}$ and $\lambda_{u,v}^{(k)} = (\beta_k)^2 e^{-j2\pi(N-1)\bar{\tau}_k} a_{f,u+v}^{(k)}$ are the kth eigenvector and eigenvalue of $\mathbf{R}_{u,v}^B$, respectively. Based on the above property, we can obtain the estimate of $\lambda_{u,v}^{(k)}$ by the corresponding estimated $\bar{\tau}_k$ using (12), and then construct the following correlations

$$r(u,v) = \lambda_{u,v}^{(k)} \lambda_{0,0}^{(k)*} = e^{j2\pi(u+v)\bar{f}_{D,k}} \tag{18}$$

to obtain the normalized Doppler shift as

$$\hat{\bar{f}}_{D,k}(u,v) = \arg[\hat{r}(u,v)]/(2\pi(u+v)). \tag{19}$$

Notice that the expression of (19) can be carried out when $(u+v)\bar{f}_{D,k} < 0.5$. Otherwise, we can first obtain the unambiguous angle of $r(u,v)$ by the techniques suggested in [10] then use (19) to obtain the estimates $\hat{\bar{f}}_{D,k}(u,v)$. To obtain the estimate of Doppler shift by exploiting (19) we shall compute the estimate of $\bar{f}_{D,k}$ by averaging the estimates $\hat{\bar{f}}_{D,k}(u,v)$ as

$$\hat{\bar{f}}_{D,k} = \frac{1}{(M^2-1)} \sum_{u=0}^{M-1} \sum_{v=0}^{M-1} \hat{\bar{f}}_{D,k}(u,v), \tag{20}$$

where $\hat{\bar{f}}_{D,k}(0,0) = 0$. From (20), we find that our proposed method can only use few OFDM symbols to estimate multiple Doppler shifts from multiple targets, which is impossible with the current approaches [3], [11].

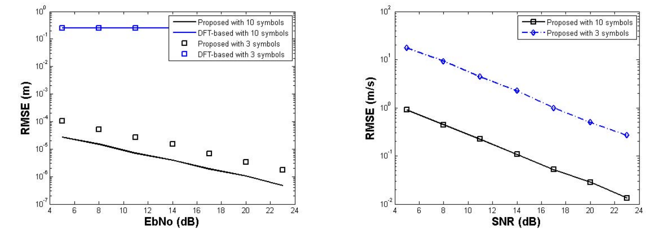
IV. Simulation Results

In this section, simulation results are presented to validate the proposed method and to illustrate its performance in MATLAB. The simulation parameters are listed in Table I.

TABLE I Simulation Parameters

Parameters	Valus
Carrier frequency (f_c)	5.9GHz
Bandwidth (f_c)	50MHz
Subcarrier spacing ($\triangle f$)	24.4kHz
Number of subcarriers (N)	2048
OFDM symbols (M)	3 or 10
Modulation scheme	64 QAM
Ranges (R_k)	15.5m and 20.5m(16.5m)
Velocities (v_k)	80m/s and 100m/s
Doppler shifts ($f_{D,k}$)	3.15kHz and 3.93kHz

For the purpose of a performance comparison, we evaluate the proposed method against the current DFT-based method suggested in [3] as shown in Fig.1. The range resolution of the DFT-based method proposed method is 3m, in this example, we will consider the ranges $15.5m$ and $20.5m$ to avoid the failure of the DFT-based method. Additionally, we set $q = 1023$ and $p = 1024$ for our proposed method, which is a tradeoff choice to achieve the optimal estimation for ranges and velocities. Notice that since we only use few OFDM symbols to estimate velocity, we only compare the ranges because the DFT-based method cannot work well for velocity estimation. From the figures, we can see that our proposed method can get a better estimate performance for the ranges without the resolution constraint of range estimation. The potential reasons lie in two aspects: one is that our method makes use of the association between subcarrier data and OFDM symbols and intends to obtain more information to improve the estimation performance, the other is that the



(a) (b)

Fig. 1. Range and velocity estimation performance w.r.t. EbN0 from 5dB to 15dB: (a) RMSE for R_1 (b) RMSE for $f_{D,1}$.

estimate accuracy and resolution are limited by the number of DFT points while our method is based on linear or nonlinear algebra matrix operation.

V. Conclusion

A highly efficient range-velocity estimation scheme using few OFDM symbols for RadCom system has been proposed and studied. Unlike the previous methods, our proposed method obtains the ranges through ESPRIT-like techniques while the Doppler shifts are estimated by the association between range and Doppler shift. In this way, we can improve the estimate performance of both ranges and velocity by taking advantage of the properties of OFDM system without the pair-matching problem. Furthermore, the simulations also verify that our proposed method can work well on few OFDM symbols and yields much better results than the current one.

References

[1] K. Mizui, M. Uchida, and M. Nakagawa, "Vehicle-to-vehicle communications and ranging system using spread spectrum techniques," *Electronics and Communications in Japan (Part I: Communications)*, vol. 79, no. 12, pp. 106–115, 1996.

[2] L. Han and K. Wu, "Multifunctional transceiver for future intelligent transportation systems," *IEEE Trans. Microw. Theory Tech.,*, vol. 59, no. 7, pp. 1879–1892, July 2011.

[3] C. Sturm and W. Wiesbeck, "Waveform design and signal processing aspects for fusion of wireless communications and radar sensing," *Proc. IEEE*, vol. 99, no. 7, pp. 1236–1259, 2011.

[4] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in *In Proceedings of the ACM SIGCOMM 2013*. ACM, Aug. 2013, pp. 375–386.

[5] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Optimum Array Processing*. Wiley-Interscience, 2004.

[6] I. Markovsky and S. Van Huffel, "Overview of total least-squares methods," *Signal processing*, vol. 87, no. 10, pp. 2283–2302, 2007.

[7] T. J. Abatzoglou, J. M. Mendel, and G. A. Harada, "The constrained total least squares technique and its applications to harmonic superresolution," *IEEE Trans. Signal Process.*, vol. 39, no. 5, pp. 1070–1087, 1991.

[8] N.-J. Li, J.-F. Gu, and P. Wei, "2-D DOA estimation via matrix partition and stacking technique," *EURASIP Journal on Advances in Signal Processing*, vol. 2009, 2009.

[9] D. A. Harville, *Matrix algebra from a statistician's perspective*. Springer, 2008.

[10] J.-F. Gu, S. C. Chan, W.-P. Zhu, and M. N. S. Swamy, "Joint DOA estimation and source signal tracking with Kalman filtering and regularized QRD RLS algorithm," *IEEE Trans. Circuits Syst. II: Exp. Briefs*, vol. 60, no. 1, pp. 46–50, 2013.

[11] C. R. Berger, B. Demissie, J. Heckenbach, P. Willett, and S. Zhou, "Signal processing for passive radar using OFDM waveforms," *IEEE J. Select. Topics Signal Process.,*, vol. 4, no. 1, pp. 226–238, Fed. 2010.