LFM Signal Parameter Estimation based on Nuttall Window Energy Barycenter Correction Method

DUAN Hao-nan , ZHU Li , SUN Feng
Department of Detection and Control Engineering, Nanjing University of Science and Technology
Institute of Electro-Optical
Nanjing, China
805909466@qq.com

***Abstract*—LFM signal is widely used in the domain of radar, sonar and communication. Both reliability and real-time is important for parameter estimation. The Fractional Fourier Transform exists problem of large calculation and bad real-time performance. To solve the problem, we use the Energy Barycenter Correction Method based on Nuttall window , combining with the idea of descending dimension. This method accomplish the task of turning two-dimensional search into one-dimensional search and get accurate estimates of frequency modulation ratio. At last, we use the method of the Fractional Fourier Transform to estimate Initial Frequency. So the real-time performance can be ensured, and the estimation accuracy can be improved. The simulation results show the validity and accuracy of the method, and the method has certain engineering application value.**

Keyword—Energy Barycenter Correction Method; Nuttall window; The Fractional Fourier Transform; parameter estimation; descending dimension

I. INTRODUCTION

LFM signal plays an important role in the field of modern radar, sonar, communication and so on.So the detection and parameter estimation of LFM signal is very meaningful. There are many kinds of algorithms to estimate the parameter, such as Short Time Fourier Transform (STFT), Wigner Vile Distribution(WVD), Fractional Fourier Transformation(FrFT) [6].However, STFT's Time frequency distribution clustering will rapid decrease as signal to noise ratio(SNR) increases. WVD has a better clustering than STFT, but the inevitable cross term, which appears in the process of application, becomes the major drawback of the WVD. FrFT have much more better clustering than WVD, and it will get accurate estimates of parameter with a low SNR. However, the computational complexity of the algorithm is large[3].

Aiming at the above-mentioned problems, the paper put forward a algorithm of EBCM which using Nuttall window, combining with the method of descending dimension .Ensure real-time performance and improve accuracy to satisfied the need of project..

II. THE METHOD OF DESCENDING DIMENSION

Discrete form of Single component LFM signal with noise can be expressed as:

$$\begin{aligned} x(t) &= s(t) + w(t) \\ &= a_0 \exp(j\theta_0 + j2\pi f_0 t + j\pi k_0 t^2) + w(t) \end{aligned} \tag{1}$$

where a_0 represents amplitude of signal, θ_0 is initial phase, f_0 is initial frequency, k_0 is frequency modulation ratio; $w(t)$ is a white Gaussian noise and mean 0, variance $2\sigma^2$.

Signal $x(t)'s$ FrFT is defined as[4]:

$$X_\alpha(u) = F^p[x(t)] = \int_{-\infty}^{\infty} x(t) K_\alpha(t,u)\, dt \tag{2}$$

where p is FrFT order $\alpha = p\pi/2$, $K_\alpha(t,u)$ is FrFT transformation kernel:

$$K_\alpha(t,u) = \begin{cases} \sqrt{\dfrac{1-j\cot\alpha}{2\pi}} \cdot \exp(j\dfrac{t^2+u^2}{2}\cot\alpha \\ -tu\csc\alpha) & ,\alpha \neq \mathrm{n}\pi \\ \delta(t-u) & ,\alpha \neq 2\mathrm{n}\pi \\ \delta(t+u) & ,\alpha \neq 2\mathrm{n}(2\mathrm{n}\pm 1)\pi \end{cases} \tag{3}$$

where n is integer.

FrFT uses rotation angle α as the variable, form two dimensional distribution of (α, u), And form a good energy aggregation spectrum[12] as shown in Fig.1. That is general thought of LFM signal detection and parameter estimation.

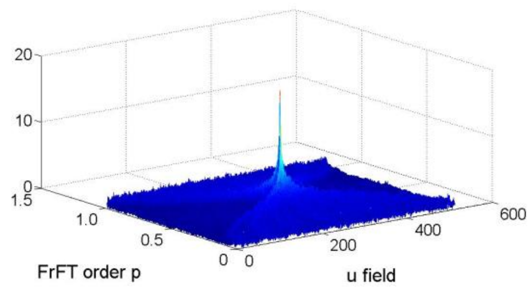

Fig.1 FrFT energy aggregation spectrum

Obvious peak of the spectrum can be seen in Fig.1. To realize LFM signal detection and parameter estimation, we carried out two-dimensional search for peak point. And compute initial phase and initial frequency by formulas[5]

$$\begin{cases} \hat{k}_0 = -\cot(\hat{p}_0\pi/2), \\ \hat{f}_0 = \hat{u}_0 \csc(\hat{p}_0\pi/2), \end{cases} \tag{4}$$

To get parameter estimation, two-dimensional search is necessary. The accurate estimation brings large computation[1]. To solve the problem, combining with the method of descending dimension[8], let the LFM signal delay multiplication:

$$\begin{aligned} R_x(\tau) &= x(t)x^*(t-\tau) = a_0{}^2 \exp\{j[2\pi k_0 \tau t \\ &+(2\pi f_0\tau - \pi k_0\tau^2)]\} + s(t)n^*(t-\tau) \\ &+ s^*(t-\tau)n(t) + n(t)n^*(t-\tau) \end{aligned} \tag{5}$$

where first item of formula as a complex sine signal, rest of all as noise. So we can get $R_x(\tau)'s$ carrier frequency f_R by FFT. And use formula

$$\hat{k}_{0'} = f_R / \tau \tag{6}$$

to get frequency modulation ratio estimate $\hat{k}_{0'}$, back derive the $\hat{p}_0$ by (4). Determine a relatively small search-range variable r_0, FrFT in the range $[\hat{p}_0 - r_0, \hat{p}_0 + r_0]$ to reduce the computation in two-dimensional search.

III. Energy Barycenter Correction Method Based on Nuttall Window

A. Energy Barycenter Correction Method

Energy barycenter characteristic of window function is the basic principle of EBCM. When n takes the infinity, energy barycenter of window function is located at coordinate origin. The accuracy is related to the type of window function and the number of N [11].

Hanning window is used by Traditional EBCM, and Hanning window's discrete spectrum energy center is infinite approximate to coordinate origin. We can use several large power spectra line to compute precise center coordinate of main lobe.

When n takes the infinity, Rectangular window, Hanning window, Blackman-Harris window meet EBCM condition

General formula for correcting frequency is

$$x_0 = \frac{\sum_{i=-n}^{n} Y_i \cdot (m+i) \cdot f_s / N}{\sum_{i=-n}^{n} Y_i} \qquad \text{n=0,1,...,}\infty \tag{7}$$

where x_0 is center of main lobe, m is index, Y_i is the power spectrum value of line i.

B. Nuttall Window

Nuttall is a kind of cosine combined window[2]. The representation of time domain is

$$w_N(n) = \sum_{m=0}^{M-1} (-1)^m b_m \cos(\frac{2\pi n \cdot m}{N}) \tag{8}$$

where M is the number of items of window function, b_m is coefficient of Nuttall window, $n = 0,1,...,N-1$

Nuttall window's spectrum function expression is

$$\begin{aligned} &W_N(w) = \\ &\sum_{m=0}^{M-1} (-1)^m \frac{b_m}{2} [W_R(w - \frac{2\pi}{N} m) + W_R(w + \frac{2\pi}{N} m)] \end{aligned} \tag{9}$$

where $W_R(w)$ is spectrum function of rectangular window.

Energy barycenter deduction of Nuttall window:

Let $y(x)$ be Nuttall window's spectrum module function,

$$y(x) = \sum_{m=0}^{M-1} \frac{b_m}{2} [W_R(x - \frac{2\pi}{N} m) + W_R(x + \frac{2\pi}{N} m)] \tag{10}$$

Let $G(x)$ be power spectrum function,

$$\begin{aligned} G(x) &= y^2(x) \\ &= (\sum_{m=0}^{M-1} \frac{b_m}{2} [W_R(x - \frac{2\pi}{N} m) + W_R(x + \frac{2\pi}{N} m)])^2 \end{aligned} \tag{11}$$

By condition of EBCM which the energy barycenter of window function is located at the coordinate origin [7], that means,

$\sum_{i=-n}^{n} G(x+i) \cdot (x+i) = 0$,according to the (11),

$$\begin{aligned} &\sum_{i=-n}^{n} G(x+i) \cdot (x+i) \\ &= \sum_{i=-n}^{n} \left[(\sum_{m=0}^{M-1} \frac{b_m}{2} [W_R(x+i - \frac{2\pi}{N} m) + W_R(x+i + \frac{2\pi}{N} m)])^2 \right] \\ &\cdot (x+i) \\ &= \sum_{i=-n}^{n} \sum_{m=0}^{M-1} \frac{b_m^2}{4} [W_R^2(x+i - \frac{2\pi}{N} m) \\ &+ 2W_R(x+i - \frac{2\pi}{N} m) W_R(x+i + \frac{2\pi}{N} m) \\ &+ W_R^2(x+i + \frac{2\pi}{N} m)] \cdot (x+i) \end{aligned} \tag{12}$$

Because $N \gg m$,

$2W_R(x+i-\frac{2\pi}{N}m)W_R(x+i+\frac{2\pi}{N}m)$ can be approximated by: $\frac{2\pi}{N}m \to 0$

So (12) can be simplified as:

$$\sum_{i=-n}^{n} G(x+i)\cdot(x+i) = \sum_{i=-n}^{n}\sum_{m=0}^{M-1}\frac{b_m^2}{4}$$
$$[\frac{W_R^2(x+i-\frac{2\pi}{N}m)\cdot(x+i-\frac{2\pi}{N}m)\cdot(x+i)^{\frac{1}{2}}}{x+i-\frac{2\pi}{N}m} + \frac{2W_R(x+i)W_R(x+i)\cdot(x+i)}{(x+i)^{\frac{1}{2}}} + \frac{W_R^2(x+i+\frac{2\pi}{N}m)\cdot(x+i+\frac{2\pi}{N}m)\cdot(x+i)^{\frac{1}{2}}}{x+i+\frac{2\pi}{N}m}] \quad (13)$$

the energy barycenter of rectangular window function is located at the coordinate origin, therefore

$$\sum_{i=-n}^{n} W_R^2(x+i)\cdot(x+i) = 0 \quad (14)$$

so,

$$\sum_{i=-n}^{n} G(x+i)\cdot(x+i) = 0 \quad (15)$$

It turns out that Nuttall window meet the condition of EBCM.

Attenuating speed and peak of side lobe is important criterion to measure the side lobe characteristics[9]. The side lobe characteristics of some windows[10] are listed in Table.1

Table.1 The side lobe characteristics of some windows

window	peak	Attenuating speed
Hanning	-31	18
Blackman	-58	18
Blackman-Harris	-92	6
3 1 Order Nuttall	-64	18
3 3 Order Nuttall	-47	30
3 minimum sidelobe Nuttall	-71	6
4 1 Order Nuttall	-93	18
4 1 Order Nuttall	-83	30
4 5 Order Nuttall	-61	42
4 minimum sidelobe Nuttall	-98	6

4 5 Order Nuttall window has better side lobe attenuating speed(42dB)and lower peak of side lobe(-61dB) than Hanning window. So the paper adopt EBCM based on Nuttall window.

C. Algorithm process of LFM signal parameter estimation

1) Delay multiplication for $x(n)$ and get $R_x(\tau)$.
2) Adopt EBCM for and get .Compute precise by (6).
3) Get k_0 by step.2 and back derive the FrFT order $\hat{p}_0$.
4) If the high precision is not necessary, we can use $\hat{p}_0$ directly to search for the peak point and compute the initial phase f_0; If necessary, take the interval $[\hat{p}_0 - r_0, \hat{p}_0 + r_0]$,which r_0 is small enough, carry out two-dimensional peak search by the step size *r* to get the precise initial phase.

IV. SIMULATION ANALYSIS

In order to verify the performance of the Nuttall window, take 50 points between two frequency indexes to compute RMSE(Root Mean Square Error). Result of EBCM based on Nuttall window, Hannning window, and Blackman-Harris window are shown in Fig.2.

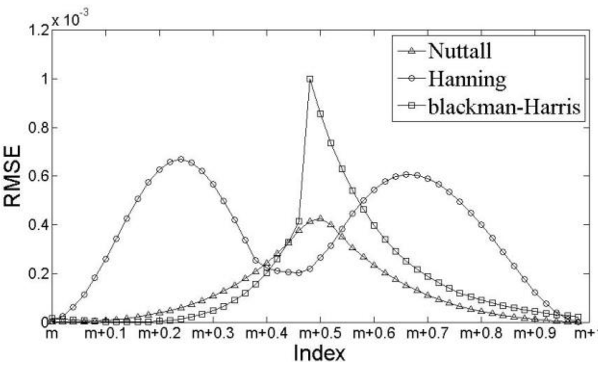

Fig.2 RMSE of EBCM based on Nuttall window, Hanning window and Blackman-Harris window

As Fig.2 illustrates, EBCM based on Nuttall window has a little worse RMSE than Hanning window only at the center of two frequency indexes. And at rest of the region has a much better RMSE. The Blackman-Harris window on the overall performance is not as good as the Nuttall window. Therefore, EBCM based on Nuttall window should have a more accurate estimates. The feasibility of EBCM based on Nuttall window is verified.

Frequency estimation of $R_x(\tau)$ has a direct effect on the performance of estimation of frequency modulation ratio. To verify this conclusion, set $x(n)$ as:

Length of signal sample $N = 2000$, initial frequency $f_0 = 343.3Hz$, frequency modulation ratio $k_0 = 1kHz/s$, amplitude of signal $a_0 = 1.0V$, initial phase $\theta_0 = 0$.

Comparing proposed method、traditional EBCM and FFT estimate parameter of $x(n)$.We estimate the frequency modulation ratio k_0 first. Take different SNR and compute the RMSE as shown in Fig.3

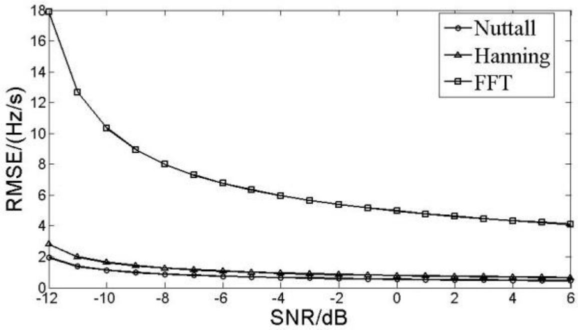

Fig.3 RMSE of k_0

As Fig.3 shows, The performance of EBCM based on Nuttall window is better than traditional EBCM, and at low SNR condition, parameter estimation is still reliable. Estimation by FFT directly have a much more larger RMSE than EBCM. Especially when SNR is below negative 6dB, estimation performance is significantly decreased. So we can not use this method to estimate parameter combining with FrFT.

In order to estimate initial frequency, take different SNR as well, let search-range variable $r_0 = 0.001$ and compute the RMSE, as shown in Fig.4.Zoom out the range $[-10,-5]dB$, get Fig.5.

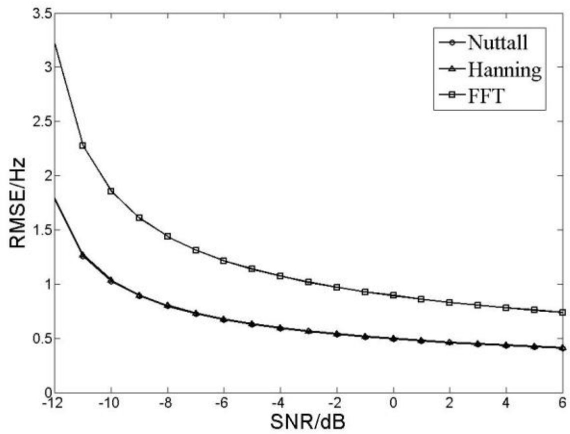

Fig.4 RMSE of f_0

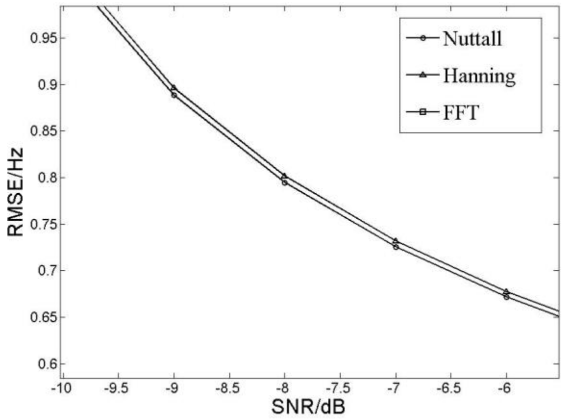

Fig.5 Zoom out

As Fig.4 illustrates, FFT has a large error than EBCM. From Fig.5 we know, the performance of EBCM based on Nuttall window is better than traditional EBCM, but not by much.

Let $SNR = -4dB$,the error and computation time are listed in Table.2

Table.2 simulation results 1

	f_0/Hz	Error1	k_0/Hz/s	Error2	time/s
FFT	343.972	0.672	1001.069	1.069	0.095749
Traditional EBCM	343.949	0.349	999.649	0.350	0.109355
EBCM based on Nuttall	343.946	0.346	999.699	0.300	0.115878

If the high precision is not necessary, we can ignore the search-range variable, and don't make peak search, the error and computation time are listed in Table.3

Table.3 simulation results 2

	f_0/Hz	Error1	k_0/Hz/s	Error2	time/s
FFT	344.773	1.173	1004.000	4.000	0.005749
Traditional EBCM	344.000	0.485	1000.700	0.700	0.006470
EBCM based on Nuttall	343.997	0.435	1000.397	0.397	0.008335

Table.2 shows that the LFM signal estimation of EBCM based on Nuttall window is more accurate than that of traditional EBCM and FFT for initial frequency and frequency modulation ratio: for initial frequency, comparing with FFT, the error is reduced about 48.4%, comparing with traditional EBCM, The error is reduced about 0.7%; for frequency modulation ratio, comparing with FFT, the error is reduced about 71.9%, comparing with traditional EBCM, The error is reduced about 14.2%, and the computation time isn't significantly improved than use FrFT directly. As Table.3 shows, if the high precision is not necessary, two-dimensional search process can be omitted. The EBCM based on Nuttall window is more accurate than the other two methods, especially in the estimation of frequency modulation ratio, the error was significantly lower than FFT. It can guarantee the accuracy and further reduce the amount of computation, ensure the real-time performance.

V. CONCLUSIONS

The paper deduct the discrete spectrum energy center of Nuttall window is infinite approximate to coordinate origin , turns out that Nuttall window meet the condition of EBCM. Combining with the side lobe characteristics of Nuttall window, effectively improve the estimation accuracy of frequency modulation ratio and the initial frequency of LFM signals. We make sure the search-range variable is small enough, to avoid the huge amount of computation which generated by FrFT peak search. The simulation result shows that we can effectively improve the accuracy of estimation under a condition of acceptable computation time increases and the method has certain engineering application value.

REFERENCES

[1]. P.P. Du, B. Chen. "Detection and Parameter Estimation of LFM Signal Based on FrFT". Radio Engineering of China, vol. 5, issue 43, pp.26-28,2013.

[2]. H. Wen, Z.S. Teng, C.C. Li, J.D. He, S.S. Wei. "Harmonic analysis algorithm based on Nuttall window and its application in power measurement". Chinese Journal of Scientific Instrument, vol. 9, issue 30, pp. 1823-1828, 2009.

[3]. C. Candan, M. A. Kutay, H. M. Ozaktas. The "Discrete fractional Fourier transform". IEEE Transactions on Signal Processing , vol. 5, pp. 1329-1337,2000.

[4]. X.H. Zhao, R. Tao, Y. Wang. "A Fast Algorithm for Fractional Fourier Transform with Zooming-in Ability". 2008 4th International Conference on Wireless Communications, Networking and Mobile Computing, vol. 10, pp. 1-4,2008.

[5]. X.H. Zhao, B. Deng, R. Tao." Dimensional Normalization in the Digital Computation of the Fractional Fourier Transform". Transactions of Beijing Institute of Technology, vol. 25, issue 4 pp.360-364,2005.

[6]. J.J. Gao, F.L. Su. "A new cross-range scaling algorithm based on FrFT". IEEE 10th International Conference on Signal Processing Proceedings, vol. 10, pp. 2043-2046, 2010.

[7]. K. Ding L.Q. Jiang "Energy barycenter Correction Method for Discrete Spectrum ". Journal of Vibration Engineering, vol. 14, issue 3, pp. 212-217,2001.

[8]. H.N. Duan, L. Zhu. "Improved Algorithm for Parameter Estimation of LFM Signal Based on FrFT-Interpolation Method ".Journal of Microwaves, pp. 212-217,2015.

[9]. C.S Zheng, K. Ding, Z.J. Yang. "Noise influence on frequency estimation accuracy from Energy barycenter Correction Method for discrete spectrum". Information and Automation vol. 6, pp. 1477-1481,2009.

[10]. G.Z. Chen, L.D. Chen, Z.F. Cai. "Harmonic analysis based on Nuttall window interpolation FFT". Electric Power Automation Equipment，vol. 4, issue 31, pp.27-31,2011.

[11]. F. Sun, L. Zhu, C. Zhang, Z. Liu. "The Key Ranging Algorithm for Millimeter Wave LFMCW Radar". Journal of Microwaves, pp. 197-202., 2015.

[12]. L. Ma, J.D. Zhu, Y.J. Zhao. "Adaptive Detection and Parameter Estimation of LFMCW Signal in Periodic FrFT Domain". Journal of Signal Processing, vol. 1, pp. 103-110,2015.