Short Paper: A Numerical Comparison of Chirp Sequence versus OFDM Radar Waveforms

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***Abstract*—The radar waveforms OFDM and chirp sequence are numerically compared in terms of receiver operating characteristic, accuracy in range and Doppler estimation and necessary signal bandwidth. By means of Monte Carlo simulations, fairly equal performace is shown, which supports a previous analytical result by the authors, which revealed that both signals yield the same baseband signal under narrowband assumption and negligible range-doppler-coupling in the chirp sequence waveform. However, as unambiguous range and Doppler get large, chirp sequence needs prohibitively more bandwidth than OFDM.**

I. Introduction

For today's radar applications, especially in the field of advanced driver assistance systems, powerful waveforms (WFs) are needed, which provide accurate and unambiguous measurements of range and velocity even of weak targets, while at the same time eliminating ghost targets. Furthermore they should not be demanding in terms of signal processing in order to be implemented in small low cost sensors with low power consumption. Two promising WFs are Orthogonal Frequency Division Multiplexing (OFDM) [1] known from communications and chirp sequence (CS) [2]. In [3], the authors showed, how these two WFs have to be parametrized in order to deliver the same range and Doppler resolution as well as the same unambigous range and Doppler. Using this parametrization, analytical derivation of the baseband (BB) signals revealed, that both radar WFs yield the same BB signal if the chirps frequency gradient is sufficiently steep, such that range-doppler-coupling (RDC) can be neglected and if the signals are narrowband. In that case, both WFs can be processed using the same two-dimensional (2D) FFT processing, which is why the same performance can be assumed [3]. The focus of this paper is to numerically support these analytic findings.

II. The Waveforms OFDM and Chirp Sequence

Both WFs at a center frequency f_c are briefly summarized and it is shown that proper processing yields the same BB signals. For transceiver block diagrams, see [1], [2] and [3].

A. OFDM Radar

An OFDM symbol is formed by dividing the used frequency band of bandwidth B into N individually modulated subcarriers, which are mutually orthogonal if the subcarrier distance is $\Delta f = {}^{U}\!/_{T}$, where $U \in \mathbb{N}$ and T is the OFDM symbol duration. The time domain samples spaced by $T_\mathrm{S} = {}^{U}\!/_{B}$ can be efficiently calculated using an IFFT. In order to avoid inter-symbol-interference, a cyclic prefix of length $T_\mathrm{G} \geq {}^{2R_\mathrm{max}}\!/_{c}$, where R_max is the maximum target distance and c the speed of light, is added at the beginning of each OFDM symbol as guard interval. For each radar measurement, a whole OFDM frame consisting of M OFDM symbols is processed in order to achieve a desired doppler resolution of $\Delta v = \frac{c}{2 f_\mathrm{c} M T_\mathrm{O}}$, with $T_\mathrm{O} = T + T_\mathrm{G}$. The transmitted symbols within one frame compose a matrix $\mathbf{F}_\mathrm{tx} \in \mathcal{A}^{N\times M}$ with the modulation alphabet $\mathcal{A}$, where rows represent subcarriers and columns OFDM symbols [1]. Assuming a single target at range R with a relative radial velocity v_r, the received echo is an attenuated (by $b < 1$), time- (by $\tau = \frac{2R}{c}$) and frequency-shifted (by $f_\mathrm{D} = \frac{2 f_\mathrm{c} v_\mathrm{r}}{c}$) version of the transmitted signal. After discarding the cyclic prefix, an FFT yields the received symbols $(\mathbf{F}_\mathrm{rx})_{n,m}$. Division of the received by their corresponding transmitted symbols yields the matrix $\mathbf{F}$ used for radar processing [1]. Its elements are

$$\begin{aligned}(\mathbf{F})_{n,m} &= (\mathbf{F}_\mathrm{rx})_{n,m} (\mathbf{F}_\mathrm{tx})^{-1}_{n,m} \\ &= b \exp^{j2\pi f_\mathrm{D} T_\mathrm{O} m} \exp^{-j2\pi (n\Delta f)\tau} \exp^{-j2\pi f_\mathrm{c}\tau}. \end{aligned} \quad (1)$$

B. Chirp Sequence Radar

In CS, L consecutive linear chirps of duration T_c are transmitted and downconverted to BB by mixing with the corresponding received echoes, a process called stretch processing [4]. This yields a BB signal, which posseses a constant beat frequency f_B if target kinematic parameters are stationary. Due to the maximum echo delay $T_\mathrm{ol} = {}^{2R_\mathrm{max}}\!/_{c}$, the useful signal period reduces to $T_\mathrm{c} - T_\mathrm{ol}$, since from the end of the received echo, for a maximum time of T_ol, the echo chirp would elsewise be mixed with the next transmitted chirp yielding a different beat frequency. Thus, in order to achieve a desired range resolution $\Delta R = {}^{c}\!/_{2B}$, the chirp bandwidth has to be

$$B_\mathrm{c} = B + B_\mathrm{ol} = B T_\mathrm{c} \left(T_\mathrm{c} - T_\mathrm{ol}\right)^{-1}. \quad (2)$$

The BB signal sampled at rate $f_\mathrm{S,c} = \frac{K}{T_\mathrm{c} - T_\mathrm{ol}}$ can be arranged into a $K \times L$ matrix [3] with elements, assuming a single echo,

$$\begin{aligned}(\mathbf{M})_{k,l} \approx\ & b \exp^{-j2\pi f_\mathrm{c}\tau} \exp^{j2\pi f_\mathrm{D} T_\mathrm{c} l} \\ & \cdot \exp^{-j2\pi k B (T_\mathrm{c} - T_\mathrm{ol})^{-1} T_\mathrm{S,c}\tau} \exp^{j2\pi k f_\mathrm{D} T_\mathrm{S,c}}, \end{aligned} \quad (3)$$

where the approximation in (3) is due to neglegting the residual video phase [4], which is feasible for narrowband signals ($B_\mathrm{c} \ll f_\mathrm{c}$) [2]. The last exponential in (3) represents the RDC, which can be neglected for very steep frequency gradients, because then $f_\mathrm{D} T_\mathrm{S,c}$ will be very small and the resulting phase will almost not change within T_c.

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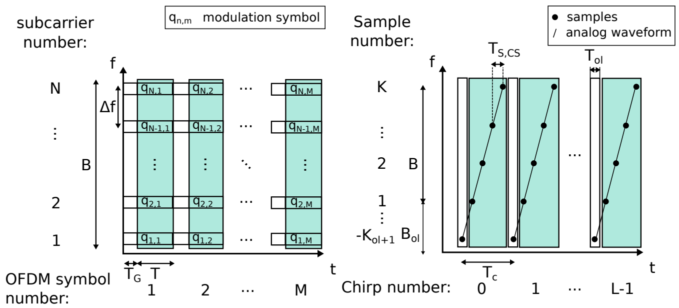

Fig. 1. Both waveforms and according parameters in the time-frequency plane.

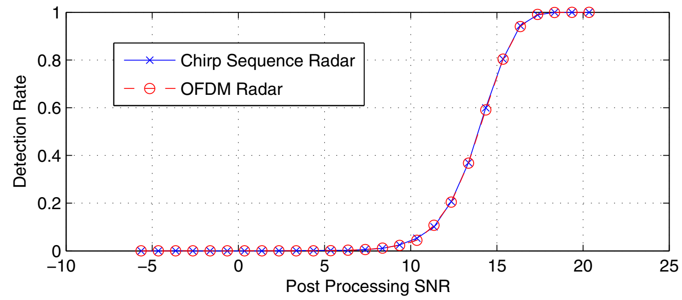

Fig. 3. Receiver Operating Characteristic at a false alarm probability of 10^{-6}.

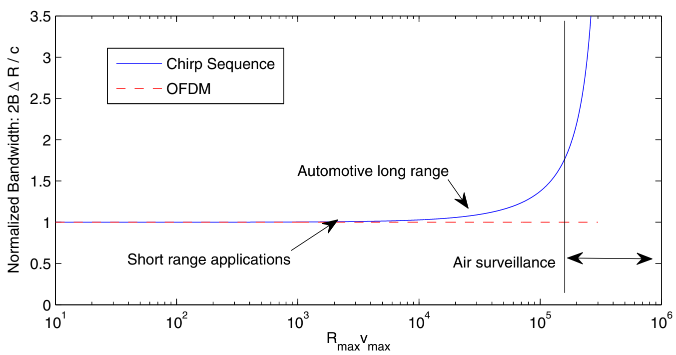

Fig. 2. Necessary signal bandwidth B_{s} relative to useful bandwidth.

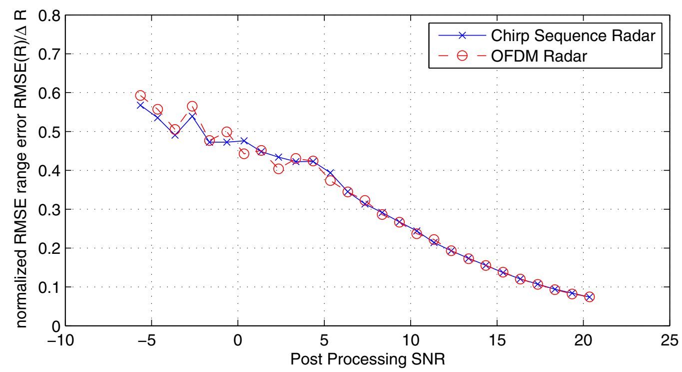

Fig. 4. Normalized root mean square error of range estimation.

C. Analytical Comparison and Proposed Processing

In [3], it has been pointed out, that, if $K = N, L = M, T_{\mathrm{c}} = T_{\mathrm{O}}, T_{\mathrm{G}} = T_{\mathrm{ol}}$ and $T_{\mathrm{S}} = T_{\mathrm{S,c}}$, then both WFs achieve the same range and Doppler resolution and the same unambigous range and Doppler. If furthermore, the RDC in (3) can be neglected, then (3) = (1) and radar processing consists of threshold detection in a 2D periodogram, which is calculated by performing a 2D FFT of the matrices $\mathbf{F}$ and $\mathbf{M}$, respectively. The WF parameters are visually summarized in fig. 1. Further details on parametrization are discussed in [3]. Merely the additional bandwidth needed in CS compared to OFDM in order to achieve the same ΔR shall be pointed out: since the used signal bandwith in OFDM radar is B, a range resolution of $\Delta R = {}^{c}/_{2B}$ can be achieved, whereas in CS, the necessary bandwidth is given by (2). The normalized bandwidths are depicted in fig. 2 as functions of $v_{\max} R_{\max}$, where $v_{\max}$ is the maximum unambiguous velocity. As $v_{\max} R_{\max}$ gets large like in air surveillance, the excess bandwith gets prohibitive. For automotive long range radar it is roughly 15% and for short range radar it is negligible.

III. Numerical Results

To numerically compare both WFs, which were parametrized as in II-C, 10^5 Monte Carlo runs using a Neyman-Pearson detector have been conducted. In each run, a single slow moving target was randomly placed within one resolution cell. Before calculating the zero padded (factor: 3) 2D periodogram, a hamming window was applied [1]. For CS, RDC compensation was used. As seen in fig. 3, the detection performance for both WFs is almost identical: at a false alarm probability of 10^{-6}, both achieve 95 % detection rate at 16 dB post processing SNR. The normalized root mean square error of estimated range, which is shown in fig. 4, is also fairly similar for both waveforms. The same behaviour was observed for the normalized velocity estimation error.

IV. Conclusion

Monte Carlo simulations showed that the radar waveforms chirp sequence and OFDM perfom nearly identical in narrow band scenarios. This supports a previous finding of the authors which stated that the baseband signals of both waveforms are identical if range-doppler-coupling can be neglected. Furthermore it was revealed that CS needs more bandwidth in order to achieve the same range resolution as OFDM when the product of maximum unambiguous range and Doppler is large. This meaningful finding can help in choosing the proper waveform for a given application.

References

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