Low-Complexity PAPR Reduction Technique for OFDM Systems Using Biased Subcarriers
Technique de réduction à faible complexité du PAPR pour des systèmes utilisant des sous-porteuses OFDM biaisées

Muhammad Ajmal Khan, *Student Member, IEEE*, and Raveendra K. Rao, *Senior Member, IEEE*

***Abstract*— A peak-to-average power ratio (PAPR) reduction technique using biased subcarriers is proposed and investigated. A known time-domain reference sample (D_{ref}) is used to bias the subcarriers at the transmitter, and the same bias is used at the receiver to recover the sequence of original subcarrier samples. A closed-form analytical expression for complementary cumulative distribution function for PAPR has been derived and is illustrated as a function of the introduced bias. The effectiveness of the proposed technique is evaluated both analytically and numerically. Analytical and simulation results confirm that significant reduction in PAPR can be achieved. For example, it is shown that nearly 9.45-dB reduction in 0.1% PAPR can be achieved for a 16-QAM orthogonal frequency division multiplexing system with 1024 subcarriers. Numerical results show that the average bit error rate performance of the proposed system does not degrade relative to the original system. It is found that the proposed technique has the lowest complexity among the various available techniques for PAPR reduction.**

***Résumé*— La technique de réduction qui se base sur le rapport "valeur de crête/valeur moyenne" (PAPR) en utilisant des sous-porteuses biaisées est proposée et étudiée dans cet article. Un échantillon de référence connu dans le domaine temporel (Dref) est utilisé pour polariser les sous-porteuses à l'émetteur, et la même polarisation est utilisée au niveau du récepteur pour récupérer la séquence d'échantillons de sous-porteuse d'origine. Une expression analytique pour la fonction de distribution cumulative complémentaire pour PAPR a été dérivée et est illustrée. L'efficacité de la technique proposée est évaluée à la fois analytique et numérique. Les résultats d'analyse et de simulation confirment que la réduction significative PAPR peut être obtenue. Par exemple, il est démontré que la réduction de près de 9.45 dB à 0.1% PAPR peut être réalisée pour un système de multiplexage par répartition orthogonale de la fréquence 16-QAM avec 1024 sous-porteuses. Les résultats numériques montrent que la performance de taux d'erreur binaire moyen du système proposé ne se dégrade pas par rapport au système d'origine. On constate que la technique proposée a la complexité la plus faible parmi les différentes techniques disponibles pour la réduction du PAPR.**

***Index Terms*— Biased subcarriers, complementary cumulative distribution function (CCDF), complexity, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

MODERN wireless communication systems are aimed at providing high-speed data transmission to support Internet, high quality multimedia, and high definition streaming videos. Orthogonal frequency division multiplexing (OFDM), a multicarrier transmission format, has emerged as a key technology to achieve high data rates as well as to increase the reliability of communication system in harsh wireless environment. OFDM has been widely adopted by many wireless and wireline communication standards, such as Third Generation Partnership Project (3GPP), Long-Term Evolution Advanced (LTE-A), wireless local area networks [(WLANs) IEEE 802.11a and IEEE 802.11g], worldwide interoperability for microwave access [(WiMAX) IEEE 802.16], digital audio broadcasting (DAB), digital video broadcasting (DVB), European HIPERLAN/2, and digital subscriber line (DSL) [1], [2].

OFDM divides the data into several parallel and orthogonal streams or subchannels (also known as subcarriers), and each subcarrier has a constant amplitude. However, when these subcarriers are summed up, the resulting OFDM signal amplitude fluctuates over a wide range of values. For an N subcarrier OFDM system, the peak power can theoretically be N times larger than the average power [3]. The ratio of the

Manuscript received October 25, 2014; revised July 2, 2015; accepted October 10, 2015. Date of current version February 1, 2016. This work was supported by the Natural Sciences and Engineering Research Council–Canada Graduate Scholarship (NSERC-CGS) of Canada. *(Corresponding author: Muhammad Ajmal Khan.)*

The authors are with the Department of Electrical and Computer Engineering, The University of Western Ontario, London, ON N6A 5B9, Canada (e-mail: mkhan386@uwo.ca; rrao@uwo.ca).

Associate Editor managing this paper's review: Peng Hu.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/CJECE.2015.2492919

peak power to the average power is referred to as the peak-to-average power ratio (PAPR). The OFDM signal with high PAPR requires a power amplifier with a large linear range capable of accommodating the signal. However, a practical power amplifier has a limited linear region beyond which it saturates to a maximum output level. Thus, efficient schemes are needed that can reduce the occurrence of large signal peaks at the input of the power amplifier to minimize the detrimental effects of nonlinear distortions, without sacrificing the power efficiency.

A number of techniques have been proposed in the literature to mitigate the problem of PAPR [1], [2], such as clipping and filtering [4], [5], companding [6], [7], tone reservation (TR) [8], [9], tone injection (TI) [10], partial transmit sequence (PTS) [11], [12], selective mapping (SLM) [13], [14], constellation shaping [15], and coding-based schemes [16]. Clipping and filtering is considered to be the simplest PAPR reduction technique [4], which limits the peak envelope of the input signal to a predetermined level, otherwise passes the signal without change [17]. Filtering the clipped signal helps to reduce the out-of-band distortion but leads peak power regrowth [18]. Various techniques to minimize the harmful effects of clipping and filtering have been proposed but are unable to maintain the error performance [2]. Companding transforms have low computational complexity and simple implementation but come with the price of increased error rate [19], [20]. In the SLM technique, a set of different OFDM symbols is generated, all representing the same information as the original OFDM symbol, then the one with the lowest PAPR is chosen for transmission [13], [21]. Information about the selected symbol is required to be transmitted to the receiver as side information, which reduces the data rate. Another disadvantage of SLM is its high computational complexity due to the symbol selection process [1]. In the PTS technique, an input data block of length N is partitioned into a number of disjoint subblocks and then weighted by a phase factor, which is selected to minimize the PAPR of the combined signal [11], [22]. The PTS technique has similar disadvantages as that of the SLM technique. In the TI technique, the constellation size is increased so that each of the points in the original basic constellation is mapped into several other points in the expanded constellation [10], [23], which helps to reduce PAPR but with increased transmission power and computational complexity. In the TR technique, a subset of tones is reserved, which carries no data information. A time-domain signal that is added to the original time-domain signal is obtained through optimization to reduce PAPR [8], [9]. The TR technique suffers from reduced data rate, increased transmit power, and high computational complexity. Coding can also be used to mitigate the problem of high PAPR [24], [25], but it reduces data rate and increases the computational complexity. Thus, each PAPR reduction technique has its own disadvantages and complexity issues in an OFDM system.

In this paper, a low complexity and simple-to-implement PAPR reduction technique using biased subcarrier is proposed for an OFDM system, and it requires no side information to be communicated to the receiver. A chosen time-domain signal sample (D_{ref}) is used as the reference for carrying out the modification process at the transmitter, and the same reference signal is employed at the receiver to remove the modification to recover the original samples. The computational complexity of the proposed technique is minimal compared with several techniques available in the literature, due to simple operations involved at both the transmitter and the receiver. An analytical closed-form expression for the complementary cumulative distribution function (CCDF) of PAPR for the proposed technique has been derived, and extensive computer simulations have been carried out to illustrate the performance of the technique. Furthermore, error performances of the OFDM system with and without the proposed technique have been evaluated numerically.

This paper is organized as follows. Section II describes the OFDM system model that uses the proposed PAPR reduction technique. Section III provides mathematical analysis and a closed-form expression for the CCDF of the proposed technique. An analysis of computational complexity is presented in Section IV, and a comparison of complexity with other techniques is also presented. Analytical and simulation results are presented and discussed in Section V. The proposed PAPR reduction technique is compared with well-known PAPR reduction techniques in Section VI. Finally, the conclusion is drawn in Section VII.

II. Proposed OFDM System Model

The block diagram of the OFDM system with the proposed PAPR reduction technique is shown in Fig. 1. The input data are assumed to be a sequence of binary digits from an equally likely and statistically independent source. These binary digits are then mapped to some constellation using a digital modulation scheme, such as M-ary quadrature amplitude modulation (M-QAM) or M-ary phase-shift keying. A serial-to-parallel (S/P) converter is used to store N constellation points over an OFDM symbol interval. In order to obtain time-domain signal samples, the signal is passed through N-point inverse fast Fourier transform (IFFT) with L oversampling factor. It is known that the PAPR of the discrete-time version is not precise, and hence, an oversampling factor of $L = 4$ has been used to provide sufficiently accurate PAPR values [26]. The L-time oversampled time-domain signal samples can be represented as [27], [28]

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi nk/NL}, \quad 0 \le k \le NL-1. \tag{1}$$

This leads to the discrete-time PAPR (γ) expression, given by

$$\gamma = \max_{0\le k\le NL-1} \frac{|x_k|^2}{\mathbb{E}[|x_k|^2]} \tag{2}$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator. The proposed modifier is applied to the time-domain signal samples, and the difference between the time-domain signal samples and D_{ref} is computed before the parallel-to-serial (P/S) converter.

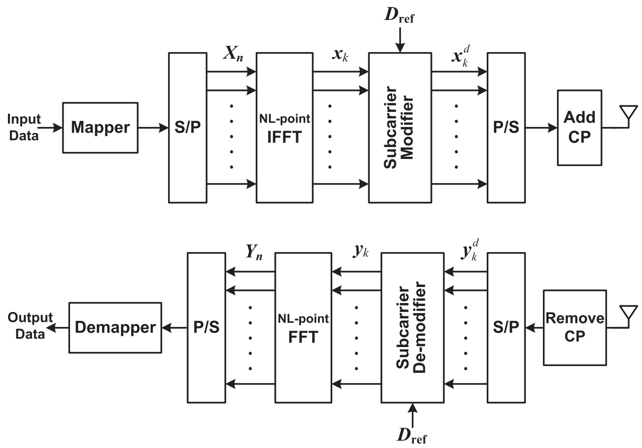

Fig. 1. Proposed OFDM baseband system model.

Mathematically, this can be written as

$$x_k^d = D_{\text{ref}} - x_k. \tag{3}$$

Then, new PAPR (γ^d) using the proposed technique can be computed using

$$\gamma^d = \max_{0 \le k \le NL-1} \frac{|x_k^d|^2}{\mathbb{E}\big[|x_k^d|^2\big]}. \tag{4}$$

The cyclic prefix [(CP) or guard interval] is added after P/S converter in an OFDM system. It is significant to note that CP has no effect in the PAPR analysis [3], and thus, it is not considered for the PAPR analysis. At the receiver, the CP is removed, and then, the signal is passed through S/P converter. The inverse operation is performed at the receiver to recover the original subcarrier samples, and then, a demodifier is applied after S/P converter, as shown in Fig. 1. The demodifier performs the mathematical operation on the time-domain signal samples and is represented as

$$y_k = D_{\text{ref}} - y_k^d. \tag{5}$$

In this paper, the reference sample is assumed equal to $D_{\text{ref}} = \delta_r + j\delta_i$, where δ_r and δ_i are the real and imaginary components, respectively, and take any arbitrary real values. Thus, reference sample D_{ref} can be chosen arbitrarily and is discussed in detail in Section V.

III. CCDF OF PAPR FOR THE PROPOSED TECHNIQUE

The CCDF is a metric used to illustrate the statistical behavior of PAPR in an OFDM system. The CCDF denotes the probability of the event that the PAPR of an OFDM symbol exceeds a specified value of PAPR, i.e., PAPR_0 or γ_0. To find the CCDF, we obtain the mean and variances of the unmodified and modified time-domain signal samples and then apply the central limit theorem. Thus, the real and imaginary parts of the unmodified time-domain signal samples $\{x_k\}$ can be approximated to be statistically independent Gaussian random variables, each with a mean of 0 and a variance of 0.5 [28].

The modified time-domain signal samples $\{x_k^d\}$, obtained using (3), are complex Gaussian random variables. Thus

$$x_k^r = \frac{x_k^d}{\sqrt{\mathbb{E}\big[|x_k^d|^2\big]}} \tag{6}$$

is a complex Gaussian random variable and substituting (6) into (4), the PAPR of the proposed technique can be written as

$$\gamma^d = \max_{0 \le k \le NL-1} |x_k^r|^2. \tag{7}$$

The CCDF can thus be written mathematically as

$$\begin{aligned} F(\gamma_0) &= \text{Prob}\{\gamma^d > \gamma_0\} \\ &= 1 - \text{Prob}\{\gamma^d \le \gamma_0\}. \end{aligned} \tag{8}$$

The probability of the event that the PAPR, for an N subcarrier OFDM system is below a threshold (γ_0), is the probability that all the N samples are below this threshold. Since the N samples $\{x_k^d\}$ are mutually independent, this probability can be represented as

$$\begin{aligned} \text{Prob}\{\gamma^d \le \gamma_0\} &= \prod_{k=0}^{N-1} \text{Prob}\big\{|x_k^r|^2 \le \gamma_0\big\} \\ &= \big[\text{Prob}\big\{|x_k^r|^2 \le \gamma_0\big\}\big]^N. \end{aligned} \tag{9}$$

Substituting (9) into (8), the CCDF can be written as

$$F(\gamma_0) = 1 - \big[\text{Prob}\big\{|x_k^r|^2 \le \gamma_0\big\}\big]^N. \tag{10}$$

To accurately estimate the PAPR, an *ad hoc* parameter α is used in the exponent, and thus, (10) can be written as

$$F(\gamma_0) = 1 - \big[\text{Prob}\big\{|x_k^r|^2 \le \gamma_0\big\}\big]^{\alpha N} \tag{11}$$

where $\alpha = 2.8$ is a practical approximation [29].

As x_k^r is complex Gaussian random variable, its amplitude $|x_k^r|$ follows Rayleigh distribution, and consequently, its energy $|x_k^r|^2$ becomes a noncentral chi-squared distribution [30] with two degrees of freedom with a noncentrality parameter λ. The probability of x_k^r using noncentral chi-squared distribution can be written as

$$\text{Prob}\big\{|x_k^r|^2 \le \gamma_0\big\} = 1 - Q_1(\sqrt{\lambda}, \sqrt{\chi}) \tag{12}$$

where Q_1 is the first-order Marcum Q-function [31] and can be computed through table lookup, while λ and χ are related to the mean and variance of x_k^r. Substituting (12) into (11), the CCDF is given by

$$F(\gamma_0) = 1 - [1 - Q_1(\sqrt{\lambda}, \sqrt{\chi})]^{\alpha N}. \tag{13}$$

For real components, we obtain

$$\mathbb{E}\big[\big(\Re\{x_k^d\}\big)^2\big] = \mathbb{E}[(\Re\{D_{\text{ref}} - x_k\})^2] = \delta_r^2 + \frac{1}{2} \tag{14}$$

and for imaginary components

$$\mathbb{E}\big[\big(\Im\{x_k^d\}\big)^2\big] = \delta_i^2 + \frac{1}{2} \tag{15}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and imaginary components, respectively.

Using (14), we find the mean of real components of x_k^r given by

$$\mu_r = \mathbb{E}\big[\Re\{x_k^r\}\big] = \frac{\delta_r}{\sqrt{\delta_r^2 + \frac{1}{2}}}. \tag{16}$$

Similarly, the mean of imaginary components of x_k^r can be shown to be given by

$$\mu_i = \frac{\delta_i}{\sqrt{\delta_i^2 + \frac{1}{2}}}. \tag{17}$$

The variances of real and imaginary components of x_k^r can be shown to be given by

$$\sigma_r^2 = \mathbb{E}\big[\big(\Re\{x_k^r\}\big)^2\big] - \big(\mathbb{E}\big[\Re\{x_k^r\}\big]\big)^2 = \frac{1}{2\delta_r^2 + 1} \tag{18}$$

and

$$\sigma_i^2 = \frac{1}{2\delta_i^2 + 1}. \tag{19}$$

Using the mean and variance of x_k^r, λ and χ of noncentral chi-squared distribution are given by

$$\lambda = 2\big(\delta_r^2 + \delta_i^2\big) \tag{20}$$

and

$$\chi = 2\big(\delta_r^2 + \delta_i^2 + 1\big)\gamma_0. \tag{21}$$

Substituting λ from (20) and χ from (21) into (13), we obtain a closed-form expression for the CCDF given by

$$F(\gamma_0) = 1 - \left[1 - Q_1\left(\sqrt{2\big(\delta_r^2 + \delta_i^2\big)},\ \sqrt{2\big(\delta_r^2 + \delta_i^2 + 1\big)\gamma_0}\right)\right]^{\alpha N}. \tag{22}$$

It is important to note that the CCDF of PAPR is a function of γ_0, δ_r, δ_i, $\alpha = 2.8$, and N.

If the real and imaginary components of the reference sample are equal (i.e., $\delta = \delta_r = \delta_i$ and $D_{\text{ref}} = \delta + j\delta$), then (22) reduces to

$$F(\gamma_0) = 1 - \left[1 - Q_1\left(2\delta, \sqrt{2(2\delta^2 + 1)\gamma_0}\right)\right]^{\alpha N}. \tag{23}$$

IV. Complexity Analysis of the Proposed Technique

Computational complexity is an important factor in analyzing any PAPR reduction technique. In general, better is the PAPR reduction capability, higher will be its computational complexity. In this section, we quantify the computational complexity of the proposed technique and also compare it with the complexities of the other well-known PAPR reduction techniques. The complexity is quantified by the number of real additions and multiplications, because the computational complexities of division and subtraction operations are nearly equivalent to those of multiplication and addition, respectively. In addition, division is treated as multiplication and subtraction as addition. It is noted that a complex multiplication operation is equivalent to four real multiplications and two real additions, and a complex addition is equivalent to two real additions.

TABLE I

Computational Complexities of PAPR Reduction Techniques

Technique	Computational Complexity
Clipping & Filtering	$4NF + 2N$ Multiplications $4NF + 2N$ Additions
Selective Mapping	$2MN(1 + \log_2 N) + M$ Multiplications $3MN(1 + \log_2 N) + M(N-1) - 1$ Additions
Partial Transmit Sequence	$2MN\log_2 N + 2N + 1$ Multiplications $3MN\log_2 N + (M-1)[2N(M+1) - 1]$ Additions
Proposed Technique	$4N$ Additions

In the proposed technique, no multiplication or division operation is required; only subtraction operations are carried out at the modifier and the demodifier. Since the subtraction operations are performed for N OFDM subcarriers at the modifier, as shown in (3), and each complex subtraction operation is equivalent to two real additions, thus a total of N complex subtraction operations is equivalent to $2N$ real additions. Similarly, N complex subtraction operations are performed at the demodifier, as shown in (5), which is equivalent to $2N$ real additions. Therefore, the computational complexity of the proposed technique can be quantified as equal to $4N$ real additions.

Next, we compare the computational complexity of the proposed technique with other available techniques, as shown in Table I, where F is the length of the finite impulse response filter used for filtering process in the clipping and filtering technique and M is the number of subblocks used in the PTS technique. The computational complexities of the available PAPR reduction techniques have been reported in [1]. It is noticed that the clipping and filtering technique is known as the least complex technique with $4NF+2N$ multiplications and $4NF + 2N$ additions [1], which is far greater than the complexity of the proposed technique. Thus, it is evident that the proposed technique has the lowest complexity among several available techniques for PAPR reduction.

V. Numerical Results

In this section, the PAPR and bit error rate (BER) performances of the proposed technique using computer simulations and derived analytical expression are presented. All computer simulations have been carried out in MATLAB environment. To illustrate the performance, an N subcarrier OFDM system with M-QAM modulation is considered. For an accurate estimation of PAPR, the signal is oversampled by a factor of 4 ($L = 4$), and the 10^6 random OFDM blocks have been generated to obtain the numerical results. The numerical results are illustrated using the CCDF, and PAPR is measured with and without the proposed PAPR reduction technique.

First, δ_i of the reference sample $D_{\text{ref}} = \delta_r + j\delta_i$ is kept constant ($\delta_i = 1$), and δ_r is varied to obtain the CCDFs for

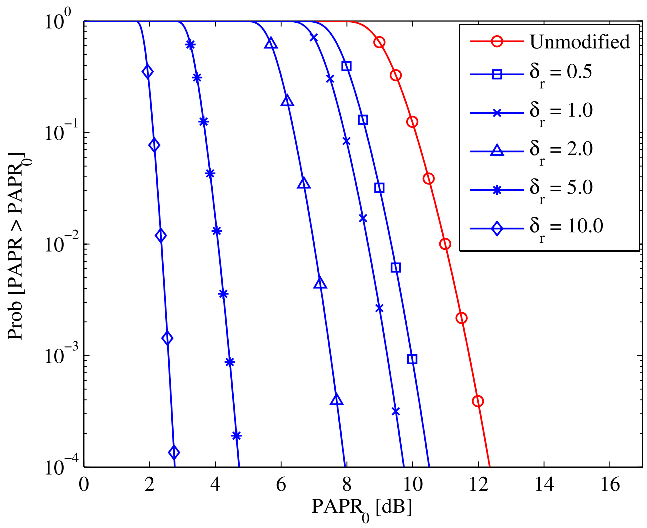

Fig. 2. CCDFs of PAPR with different values of δ_r for 1024-subcarrier OFDM system with 64-QAM modulation and $\delta_i = 1$.

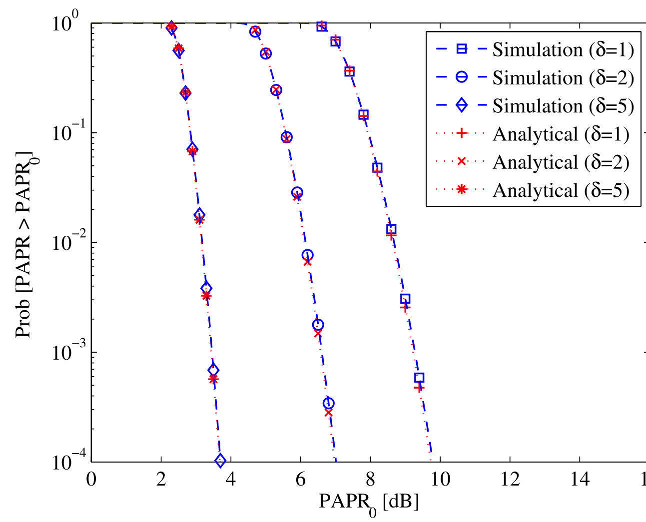

Fig. 4. Comparison of simulation and analytical CCDFs for 1024-subcarrier OFDM system with 16-QAM modulation.

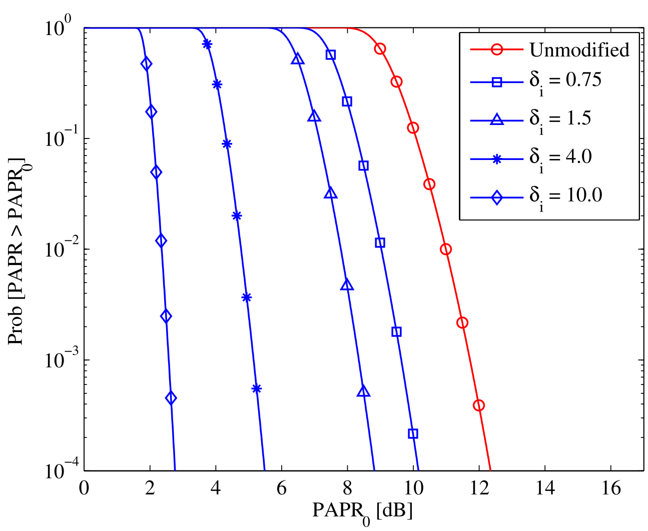

Fig. 3. CCDFs of PAPR with different values of δ_i for 1024-subcarrier OFDM system with 64-QAM modulation and $\delta_r = 1$.

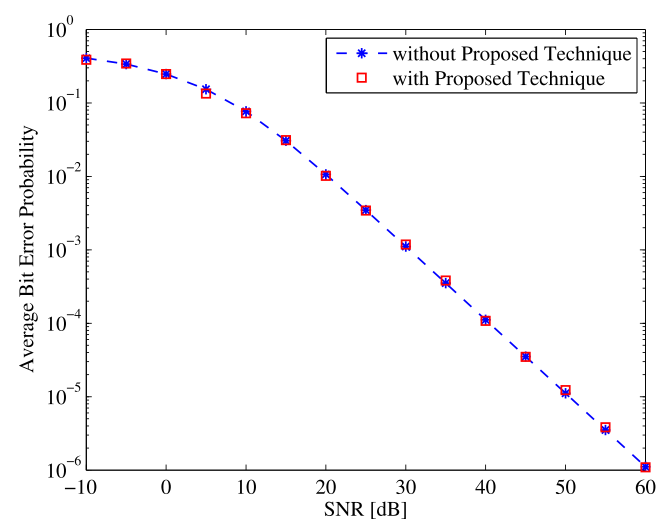

Fig. 5. Error performance using the proposed technique for 256-subcarrier OFDM system with 64-QAM modulation over Rayleigh fading channel.

a 1024-subcarrier OFDM system with 64-QAM modulation, as shown in Fig. 2. The CCDF of the unmodified OFDM system is also shown in Fig. 2 for comparison. It is shown that the unmodified OFDM system has a PAPR, which exceeds 11.7 dB for less than 0.1% of the blocks. With the proposed technique, 0.1% PAPR reduces to 10 dB at $\delta_r = 0.5$, resulting in 1.7-dB reduction, provided δ_i is kept constant at 1. When δ_r increases to 1, 2, 5, and 10, 0.1% PAPR reduces to 9.2, 7.5, 4.4, and 2.6 dB, resulting in 2.5-, 4.2-, 7.3-, and 9.1-dB reductions, respectively.

Fig. 3 shows the CCDFs of the proposed technique as a function of δ_i by keeping δ_r fixed at 1 for 1024-subcarrier OFDM system with 64-QAM modulation. It is observed that the proposed technique reduces 0.1% PAPR to 9.6, 8.3, 5.1, and 2.6 dB at $\delta_i = 0.75$, 1.5, 4, and 10, compared with 11.7 dB for the unmodified OFDM system, which results in 2.1-, 3.4-, 6.6-, and 9.1-dB reductions.

Fig. 4 compares the computer simulation results with the analytical results obtained using the closed-form expression derived in (23). Fig. 4 shows the CCDFs of PAPR of the proposed technique using $\delta = 1$, 2, and 5 for a 1024-subcarrier OFDM system with 16-QAM modulation. It is significant to note that the simulation results match with those of the analytical results, thus validating our mathematical derivations.

To portray the effect of the proposed PAPR reduction technique on the error performance of the system, the average bit error probability of the system was evaluated using Monte Carlo simulation and is shown in Fig. 5. An OFDM system with 256 subcarriers and 64-QAM modulation was considered in this simulation over a Rayleigh fading channel under the assumption of perfect channel estimation. The simulations were carried out for the unmodified OFDM system as well as for the proposed system with $\delta_r = \delta_i = 0.75$.

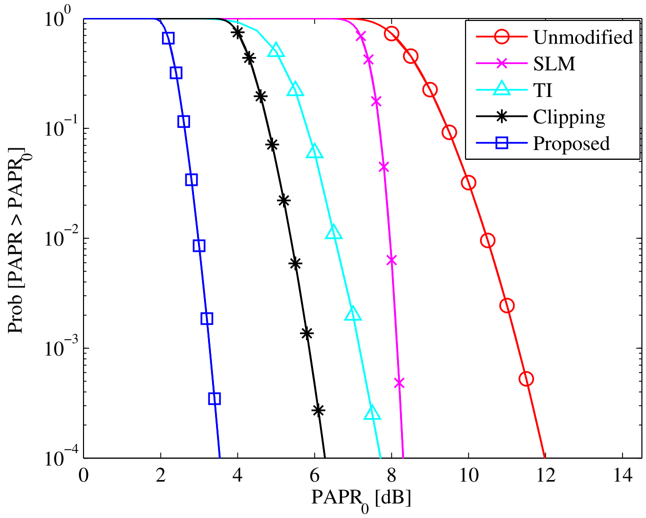

Fig. 6. Comparison of the CCDFs for different PAPR reduction techniques for 256-subcarrier OFDM system with 16-QAM modulation.

TABLE II
COMPARISON OF PAPR REDUCTION TECHNIQUES

PAPR Reduction Technique	BER Degrade	Data Rate Loss	Computational Complexity
Clipping	Yes	No	Low
SLM	No	Yes	High
PTS	No	Yes	High
TI	No	No	High
TR	No	Yes	High
Proposed	No	No	Low

It is important to note that δ_r and δ_i do not affect the error performance but with the expense of $\delta_r^2 + \delta_i^2$ more transmit power. It is observed that the bit error probability of the OFDM system does not change after applying the proposed modification for the reduction of PAPR in the system.

Fig. 6 compares the CCDFs of different PAPR reduction techniques for an OFDM system with 256 subcarriers and 16-QAM modulation. It is evident from the figure that all the PAPR reduction techniques mitigate the PAPR largely; however, their corresponding error performances, data rate losses, and computational complexities are different, which is discussed in Section VI. For example, the clipping technique can further improve the PAPR performance by reducing its clipping level, which will severely degrade the BER performance of the system. Similarly, the PAPR performance using the TI technique can be further improved but at the cost of huge computational complexity. It is observed that the proposed technique provides significant PAPR reduction compared with SLM, TI, and clipping PAPR reduction techniques.

VI. COMPARISON WITH OTHER PAPR REDUCTION TECHNIQUES

In this section, we compare our proposed technique with other well-known PAPR reduction techniques in terms of BER, data rate, and computational complexity. A summary of comparison is presented in Table II, which is based on the discussion in Section I and [1]. It is apparent from Section IV that the proposed technique has very low computational complexity. It is also clear from Section V that the proposed technique does not degrade error performance. It is also observed that the proposed technique does not reduce data rate as no side information is required to be transmitted. Thus, the proposed technique offers superior performance factors relative to the other PAPR reduction techniques.

VII. CONCLUSION

This paper proposed a low complexity and easy-to-implement PAPR reduction technique, which incorporated a modifier and a demodifier at the transmitter and the receiver, respectively. Due to simple mathematical and signal operations at the transmitter and at the receiver, the computational complexity of the proposed technique requires only $4N$ real additions, which is the lowest among all the available PAPR reduction techniques. A closed-form mathematical expression for the CCDF of PAPR has been derived for the proposed technique. Extensive computer simulations for the CCDF have shown that the simulation results have close agreement with the analytical results, derived in this paper. Furthermore, the error performance of the OFDM system with and without the proposed technique is the same. Thus, the proposed technique proffers low complexity, full spectral efficiency, and full data rate without signal distortion and requires no side information to be transmitted.

Due to the low computational complexity of the proposed PAPR reduction technique, it can be implemented using field-programmable gate array (FPGA), which is a good future extension of this paper.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments and feedback.

REFERENCES

[1] Y. Rahmatallah and S. Mohan, "Peak-to-average power ratio reduction in OFDM systems: A survey and taxonomy," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 4, pp. 1567–1592, Nov. 2013.

[2] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Commun.*, vol. 12, no. 2, pp. 56–65, Apr. 2005.

[3] M.-O. Pun, M. Morelli, and C.-C. J. Kuo, *Multi-Carrier Techniques for Broadband Wireless Communications*. London, U.K.: Imperial College Press, 2007.

[4] R. O'Neill and L. B. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," in *Proc. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, vol. 1. Sep. 1995, pp. 71–75.

[5] J. Armstrong, "Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering," *Electron. Lett.*, vol. 38, no. 5, pp. 246–247, Feb. 2002.

[6] X. Wang, T. T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using a companding technique," *IEEE Trans. Broadcast.*, vol. 45, no. 3, pp. 303–307, Sep. 1999.

[7] X. Wang, T. T. Tjhung, and Y. Wu, "On the SER and spectral analyses of A-law companded multicarrier modulation," *IEEE Trans. Veh. Technol.*, vol. 52, no. 5, pp. 1408–1412, Sep. 2003.

[8] J. Tellado and J. M. Cioffi, "Peak power reduction for multicarrier transmission," in *Proc. IEEE Global Commun. Conf.*, Nov. 1998, pp. 5–9.

[9] H. Li, T. Jiang, and Y. Zhou, "An improved tone reservation scheme with fast convergence for PAPR reduction in OFDM systems," *IEEE Trans. Broadcast.*, vol. 57, no. 4, pp. 902–906, Dec. 2011.

[10] J. Tellado-Mourelo, "Peak to average power ratio reduction for multicarrier modulation," Ph.D. dissertation, Dept. Elect. Eng., Stanford Univ., Stanford, CA, USA, 1999.

[11] S. H. Müller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electron. Lett.*, vol. 33, no. 5, pp. 368–369, Feb. 1997.

[12] Y.-J. Cho, J.-S. No, and D.-J. Shin, "A new low-complexity PTS scheme based on successive local search using sequences," *IEEE Commun. Lett.*, vol. 16, no. 9, pp. 1470–1473, Sep. 2012.

[13] R. W. Bäuml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *Electron. Lett.*, vol. 32, no. 22, pp. 2056–2057, Oct. 1996.

[14] H.-S. Joo, S.-J. Heo, H.-B. Jeon, J.-S. No, and D.-J. Shin, "A new blind SLM scheme with low decoding complexity for OFDM systems," *IEEE Trans. Broadcast.*, vol. 58, no. 4, pp. 669–676, Dec. 2012.

[15] A. Aggarwal and T. H. Meng, "Minimizing the peak-to-average power ratio of OFDM signals using convex optimization," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 3099–3110, Aug. 2006.

[16] M.-J. Hao and C.-H. Lai, "Precoding for PAPR reduction of OFDM signals with minimum error probability," *IEEE Trans. Broadcast.*, vol. 56, no. 1, pp. 120–128, Mar. 2010.

[17] J. Heiskala and J. Terry, *OFDM Wireless LANs: A Theoretical and Practical Guide*. Carmel, IN, USA: Sams Publishing, 2002.

[18] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 89–101, Jan. 2002.

[19] Y. Rahmatallah, N. Bouaynaya, and S. Mohan, "On the performance of linear and nonlinear companding transforms in OFDM systems," in *Proc. Wireless Telecommun. Symp.*, Apr. 2011, pp. 1–5.

[20] Y. Rahmatallah, N. Bouaynaya, and S. Mohan, "Bit error rate performance of linear companding transforms for PAPR reduction in OFDM systems," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2011, pp. 1–5.

[21] S. H. Müller and J. B. Huber, "A comparison of peak power reduction schemes for OFDM," in *Proc. IEEE Global Telecommun. Conf.*, vol. 1. Nov. 1997, pp. 1–5.

[22] W. S. Ho, A. S. Madhukumar, and F. Chin, "Peak-to-average power reduction using partial transmit sequences: A suboptimal approach based on dual layered phase sequencing," *IEEE Trans. Broadcast.*, vol. 49, no. 2, pp. 225–231, Jun. 2003.

[23] S. H. Han, J. M. Cioffi, and J. H. Lee, "On the use of hexagonal constellation for peak-to-average power ratio reduction of an ODFM signal," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 781–786, Mar. 2008.

[24] S. B. Slimane, "Reducing the peak-to-average power ratio of OFDM signals through precoding," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, pp. 686–695, Mar. 2007.

[25] T. Jiang and X. Li, "Using fountain codes to control the peak-to-average power ratio of OFDM signals," *IEEE Trans. Veh. Technol.*, vol. 59, no. 8, pp. 3779–3785, Oct. 2010.

[26] C. Tellambura, "Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers," *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 185–187, May 2001.

[27] R. Van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Norwood, MA, USA: Artech House, 2000.

[28] G. Li and G. L. Stüber, Eds., *Orthogonal Frequency Division Multiplexing for Wireless Communications*. New York, NY, USA: Springer-Verlag, 2006.

[29] R. van Nee and A. de Wild, "Reducing the peak-to-average power ratio of OFDM," in *Proc. IEEE Veh. Technol. Conf.*, vol. 3. May 1998, pp. 2072–2076.

[30] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*. New York, NY, USA: McGraw-Hill, 2002.

[31] A. H. Nuttall, "Some integrals involving the Q_M function (Corresp.)," *IEEE Trans. Inf. Theory*, vol. 21, no. 1, pp. 95–96, Jan. 1975.

Muhammad Ajmal Khan (S'14) received the B.E. (Hons.) degree in electrical engineering from the NED University of Engineering and Technology, Karachi, Pakistan, in 1997, and the M.S. degree in electrical engineering from King Fahd University (KFUPM), Dhahran, Saudi Arabia, in 2001. He is currently pursuing the Ph.D. degree in wireless communications and data networks with the Department of Electrical and Computer Engineering, The University of Western Ontario (UWO), London, ON, Canada.

In 2001, he joined KFUPM as a Lecturer, where he has been involved in teaching and research activities. His current research interests include wireless communications and security, in particular, wireless security, physical layer security, cooperative diversity, cognitive radio networks, device-to-device communications, massive MIMO and multiuser MIMO, OFDM and PAPR, spread spectrum systems, and engineering education.

Mr. Khan received the Distinguished Service Award and Best Service Award from KFUPM, and Gold Medal Awards from the NED University of Engineering and Technology and Siemens Engineering Company, Ltd., Karachi. He also received prestigious scholarships and awards during his doctoral studies, including the Natural Sciences and Research Council of Canada–Alexander Graham Bell Canada Graduate Scholarship, the Ontario Graduate Scholarship, the Teaching Assistant Award, the Outstanding Research Contribution Scholarship, and the Academic Achievement Scholarship.

Raveendra K. Rao (SM'01) received the B.E. degree in electronics and communications from the National Institute of Engineering, University of Mysore, Mysore, India, in 1980, and the M.Tech. degree in communication and radar and the Ph.D. degree in electrical engineering from IIT Delhi, New Delhi, India, in 1982 and 1988, respectively.

He was a Tenured Faculty Member with the Delhi Institute of Technology, New Delhi, from 1988 to 1999. He founded the Communication and Signal Processing Laboratory, Delhi Institute of Technology, and was in charge of the Communication and Signal Processing Laboratory from 1989 to 1999 and he served in various capacities, including the Chair of the Electronics and Communications Engineering Department and a member of the Board of Governors. From 1996 to 1998, he was the Chair of the Electronic Department and the Deputy Principal of the Caledonian College of Engineering (affiliated to the Glasgow Caledonian College of Engineering, Glasgow, U.K.), Muscat, Oman. He has served on the engineering faculties of Mysore, Roorkee, and Delhi Universities in India, and the University of Ontario Institute of Technology, Oshawa, ON, Canada. Since 1999, he has been with the Department of Electrical and Computer Engineering, The University of Western Ontario (UWO), London, ON, Canada, where he is currently a Tenured Faculty Member. He played major roles in the establishment and development of Electronic and Communication Engineering Laboratories with the Caledonian College of Engineering. He is the Founding Member of the Innovation Centre for Information Engineering at UWO. He has authored over 50 technical papers in journals and international conferences. He has supervised more than 30 master's and Ph.D. students. His current research interests include digital communications, wireless networks, statistical signal processing, and multiuser and multicarrier communication systems.

Dr. Rao is a fellow of the Institution of Electronics and Telecommunication Engineers (IETE), and a Registered Professional Engineer in the province of Ontario, Canada. He received the IEEE AES/COM Chapter India Award for the contributions in the area of communications in 1985, the Young Scientist of the International Union of Radio Science Award in 1987, and the Student Journal Award of IETE for best paper published in 1994. He was nominated for the prestigious Vikas Jyothi Award, India, in 1993. He is listed in Asia's *Who's Who of Men and Women of Achievement*.