An Improved OFDM Chirp Waveform Scheme for GMTI in Clutter Environment

Jiahua Zhu*, Chongyi Fan, Pengzheng Lei, Xiaotao Huang, Guofu Zhu

National University of Defense Technology, School of Electronic Science and Engineering

Yanwachi Main Street, No.47, 410073, Changsha, Hunan, P.R.China

email: zhujiahua1019@hotmail.com

Abstract: *Orthogonal frequency division multiplexing (OFDM) chirp waveform is a new kind of waveform scheme that combine the OFDM principle with the linear frequency modulation (LFM) waveform. In this paper, the OFDM chirp waveform generation model is improved to the condition with arbitrary numbers of orthogonal basic signals, and the detection performance under the energy detector (ED) of this waveform in clutter environment is also analyzed. The simulation results proved that the OFDM chirp waveform performs better than the traditional OFDM and LFM waveforms in clutter environment.*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) chirp waveform is a new kind of waveform scheme proposed in radar field, which combines the linear frequency modulation (LFM) waveform with the OFDM principle. LFM waveform is one of the mostly used scheme in ground moving target indication (GMTI) with the following qualities[1-3]. Firstly, it has constant envelope in time domain, which leads to a high efficiency of the transmitter modules of a phased-array antenna. Secondly, its spectrum approximates to rectangular shape with the increase of time-bandwidth product, which could maximize the spectral efficiency and signal-to-noise ratio (SNR). Thirdly, it can further exploit the linear frequency-time characteristics for easier signal processing. On the other hand, the OFDM waveform is introduced recently since it can solve the range-Doppler coupling problem caused by LFM waveform. This waveform is orthogonal in frequency domain[4-5], so it can accomplish the transmission and reception in multi-channels by just one antenna and obtain much more information from targets, which means it has the potential to save the cost as well as enhance the target detection performance. But one major problem of OFDM waveform is the fast variation of its signal envelope. Therefore, the OFDM chirp waveform is put forwarded to solve this problem.

The OFDM chirp waveform was first proposed by Dr. Wen-Qin Wang[6-7] and Dr. Jung-Hyo Kim[1,8] respectively at the year 2012 and they all illustrated it has almost constant modulus, large time-bandwidth and good performance on correlation. In 2013, Dr. Ming-Yang Zhou[9] improved the OFDM chirp waveform generation model using two orthogonal basic signals given by Dr. Kim to three orthogonal basic signals and proved its better performance on the ambiguity function.

However, the previous researches mainly discuss the processing techniques of multiple-input multiple-output (MIMO) synthetic aperture radar (SAR) systems. Few achievement about target detection based on OFDM chirp waveform was published before.

Since the OFDM chirp waveform combines the orthogonality of subcarriers and intrinsic characteristics of traditional LFM waveform[1], it could be promising to have better performance than both the LFM and the OFDM waveform on moving target detection.

This paper further improved the OFDM chirp waveform generation model to arbitrary numbers of orthogonal basic signals in theory. Moreover, the OFDM chirp waveform is proved to have good target detection performance in clutter environment, which is better than the traditional OFDM and chirp waveforms. The remainder sections of this paper are organized as follows. Section 2 makes a brief introduction of the OFDM chirp waveform modulation and extends its waveform generation model to arbitrary numbers of orthogonal basic signals. Based on the Energy Detector (ED), Section 3 lays emphasis on the target detection performance of OFDM chirp waveform in Rayleigh clutter. Then, some simulations and comparison analyses are given in Section 4, proving that in the clutter environment, the OFDM chirp waveform could perform better than LFM and OFDM waveform on moving target detection. Finally, the paper is concluded in Section 5.

2. Signal Description and Modeling

Based on the researches made by Dr. Jung-Hyo Kim and Dr. Ming-Yang Zhou, an OFDM chirp waveform generation model using three orthogonal basic signals was built. But actually, this model could be further improved to the condition with arbitrary numbers of orthogonal basic signals.

Assume that the input sequence is $S[\overline{p}]$ with N discrete spectral components in spectrum, which are separated by $M\Delta f$ as shown in Fig. 1. Interleave $(M-1)N$ zeros in the input sequence $S[\overline{p}]$ and we can get $\boldsymbol{S}_1[p]$ in Fig. 1. Then, we could shift $\boldsymbol{S}_1[p]$ by $\Delta f, 2\Delta f, ..., (M-1)\Delta f$ to obtain $\boldsymbol{S}_2[p], \boldsymbol{S}_3[p], ..., \boldsymbol{S}_M[p]$ respectively. The OFDM modulation could be obtained by transforming those data sequences into the time domain through the MN point inverse discrete Fourier transform (IDFT). Therefore, the M OFDM modulators have the same number of orthogonal subcarrier, and we could get M OFDM modulators with each orthogonal subcarrier that are mutually shifted by Δf. Notice that although both modulators contain MN subcarriers, there are only N useful subcarriers that carry the input data.

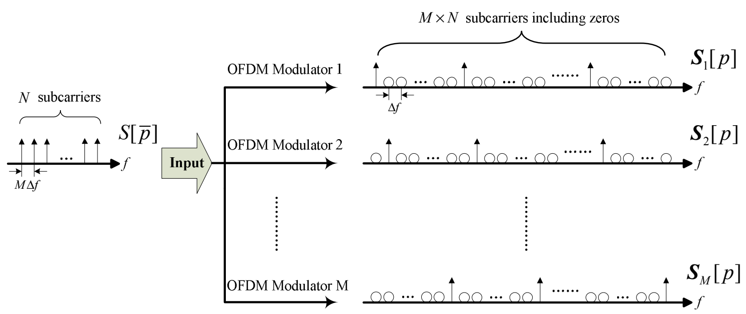

Fig. 1 OFDM chirp waveform generation scheme

In Fig. 1, the arrows represent subcarriers and the circles represent insert zeros. We first use an LFM signal spectrum as the input of each OFDM modulator, which could be given as

$$S[\overline{p}] = \mathcal{F}\{s[n]\} = \mathcal{F}\{\exp[j\pi k(nT_s)^2]\} \tag{1}$$

where $s[n]$ is the N-point discrete time samples of a complex LFM signal, k denotes the chirp rate, $T_{\rm s}$ means the sampling interval and $\mathcal{F}\{\cdot\}$ is the Fourier transform operator. Then, we can generate M input data sequence with zeros interleaved as follows using $S[\overline{p}]$.

$$\boldsymbol{S}_1[p] = [S[0],0,0,...,0,0,S[1],0,0,...,0,0,\ldots,S[N-1],0,0,...,0,0] \quad (2)$$

$$\boldsymbol{S}_2[p] = [0,S[0],0,0,...,0,0,S[1],0,0,...,0,\ldots,0,S[N-1],0,0,...,0] \quad (3)$$

$$\cdots\cdots\cdots\cdots$$

$$\boldsymbol{S}_M[p] = [0,0,...,0,0,S[0],0,0,...,0,0,S[1],\ldots,0,0,...,0,0,S[N-1]] \quad (4)$$

where $p = 0,1,...,MN-1$. Then using the method aforementioned, we can express $s_1[n]$ as a repetition of $s[n]$ over the length of MN.

$$s_1[n] = s[n]\text{rect}\left[\frac{n}{N}\right] + s[n-N]\text{rect}\left[\frac{n-N}{N}\right] + ... + s[n-(M-1)N]\text{rect}\left[\frac{n-(M-1)N}{N}\right] \quad (5)$$

In the same way, the other $M-1$ signals could be described through a phase shift

$$s_2[n] = s_1[n]\exp\left(jn2\pi\frac{1}{MN}\right) \quad (6)$$

$$s_3[n] = s_1[n]\exp\left(jn2\pi\frac{2}{MN}\right) \quad (7)$$

$$\cdots\cdots\cdots\cdots$$

$$s_M[n] = s_1[n]\exp\left(jn2\pi\frac{M-1}{MN}\right) \quad (8)$$

As a result, the generation model of OFDM chirp waveform with arbitrary numbers of orthogonal basic signals is deduced in this section. Without loss of generality, we then use $M = 3$ as an example in next sections to discuss its target detection performance considering the computation burden and processing precision. Because with the increase of M, the subcarrier numbers of each OFDM modulator would correspondingly increase M times. Although it could bring a higher processing precision, it also aggravates the burden of computing devices and needs higher cost.

3. Target Detection

In this section, the ED detector is used to analyze the target detection performance in clutter environment. Given a radar detection scene with one moving point target in clutter and background noise, then we could construct the detection problem to choose between two possible hypotheses as follows:

$$\begin{cases} \mathcal{H}_0 : \boldsymbol{y} = \boldsymbol{c} + \boldsymbol{w} \\ \mathcal{H}_1 : \boldsymbol{y} = \boldsymbol{s} \otimes \boldsymbol{h}_{\rm t} + \boldsymbol{c} + \boldsymbol{w} \end{cases} \quad (9)$$

where $\boldsymbol{y}$, $\boldsymbol{s}$ and $\boldsymbol{c}$ is the receiving echo wave, the transmitting signal and the clutter of each OFDM modulator respectively, "$\otimes$" is the convolution operator, $\boldsymbol{h}_{\rm t}$ is the target channel response, $\boldsymbol{w} \sim \mathbb{C}\mathcal{N}(0,\sigma_{\rm n}^2)$ is the complex white Gaussian noise and $\sigma_{\rm n}^2$ is its variance. All parameters in (9) are expressed in vector forms. Besides, the clutter in the region of interests is set as follows.

$$\boldsymbol{c} = \sqrt{\boldsymbol{c}_1 + \boldsymbol{c}_2} \tag{10}$$

$$\boldsymbol{c}_1 \sim \mathbb{C}\mathcal{N}(0, \sigma_{\mathrm{c}}^2) \tag{11}$$

$$\boldsymbol{c}_2 \sim \mathbb{C}\mathcal{N}(0, \sigma_{\mathrm{c}}^2) \tag{12}$$

Therefore, the amplitude of clutter would satisfy Rayleigh distribution.

The likelihood ratio of ED should compute the total received signal energy across the operating frequency range[10], and according to the Neyman-Pearson (NP) criterion[11], the likelihood ratio function could be given as the quotient of the data probability density functions (PDFs) $p(\boldsymbol{y} \mid \mathcal{H}_1)$ and $p(\boldsymbol{y} \mid \mathcal{H}_0)$ under the conditions $\mathcal{H}_1$ and $\mathcal{H}_0$.

$$L(\boldsymbol{y}) = \frac{p(\boldsymbol{y} \mid \mathcal{H}_1)}{p(\boldsymbol{y} \mid \mathcal{H}_0)} = \frac{\sum_{n=0}^{MN-1} |\boldsymbol{y}[n]|^2}{\sum_{n=0}^{MN-1} |\boldsymbol{c}[n] + \boldsymbol{w}[n]|^2} > \gamma \tag{13}$$

To calculate the $P_{\mathrm{FA,ED}}$ and $P_{\mathrm{D,ED}}$, the Monte Carlo method[11] is introduced since the analytical expression of test statistics $L(\boldsymbol{y})$ cannot be obtained.

4. Simulations and Performance Analysis

In order to demonstrate the target detection performance of OFDM chirp waveform, we make some simulations and compare the results with the traditional LFM and OFDM waveforms.

In the radar target detection scene, there is a point moving target with Rayleigh clutter and background Gaussian noise. Besides, some global parameters for each waveform are given in Table 1.

Table 1 Global parameters of simulations

Parameters		Value	Unit
Operating frequency f_{c}		1	GHz
Bandwidth B		200	MHz
Pulse number		10000	—
Pulse repetition frequency		500	Hz
Frequency sampling rate f_{s}		$2B$	MHz
Pulse length T_{P}		8.5	μs
Moving point target	Original distance to radar	1000	m
	Initial speed v_{t}	10	m/s
Discrete time sample number N		$f_{\mathrm{s}} \cdot T_{\mathrm{P}}$	—

To satisfy the orthogonal quality of the OFDM, T_{P} should be no less than M/B and here we set $T_{\mathrm{P}} = 8.5\mu\mathrm{s}$, $M = 3$ without loss of generality; Δf should be no less than $1/T_{\mathrm{P}}$ and here we set $\Delta f = 66.7\mathrm{MHz}$. Besides, the energy of each waveform is normalized to 1 under the same energy level of noise.

Then, the receiver operating characteristic (ROC) curves of three waveforms are shown in Fig. 2 when the signal-to-clutter ratio (SCR) is $-20\mathrm{dB}$ and SNR is $-10\mathrm{dB}$.

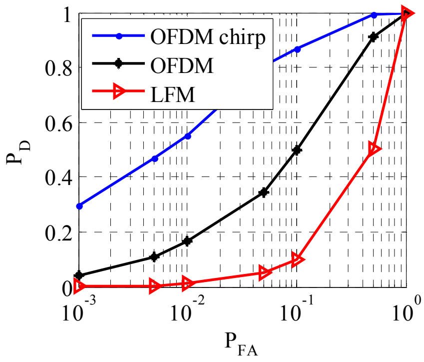

Fig. 2 ROC curves of the OFDM chirp, OFDM and LFM waveforms

From Fig. 2 it is apparently that the OFDM chirp waveform performs better than the other two waveforms. More specifically, the P_{D} of the OFDM chirp waveform is about 0.31 when $P_{\mathrm{FA}} = 10^{-3}$, while the counterparts of OFDM and LFM waveforms are only 0.05 and 0.01, respectively. As we know, the OFDM chirp waveform has much more orthogonal subcarriers than LFM waveform, thus more target information could be exploited for target detection algorithms and higher detection probability could be obtained. Besides, using chirp signals as the OFDM symbols can suppress the cross-correlation components better than the conventional OFDM waveform, since LFM signal is less sensitive to the time and frequency synchronization. Therefore, the OFDM chirp waveform provides higher detection probability in comparison with that of OFDM waveform.

Besides, fix $SNR = -10\mathrm{dB}$, the detection performance of these three waveforms at different SCRs when $P_{\mathrm{FA}} = 10^{-3}$ is shown in Fig. 3.

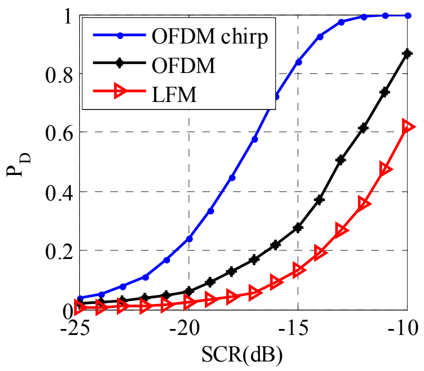

Fig. 3 Detection Probability at different SCRs for the OFDM chirp, OFDM and LFM waveforms

As is illustrated in Fig. 3, the OFDM chirp waveform has about 4.8dB and 6.7dB performance gain over the traditional OFDM and LFM waveforms when $P_{\mathrm{D}} = 0.6$, which further proved its better target detection performance.

5. Conclusion

This paper exploits an improved OFDM chirp waveform scheme for ground moving target detection in Rayleigh clutter environment. We use the ED to prove the better performance of OFDM chirp waveform and also analyze its performance gain compared to the traditional OFDM and LFM waveforms.

In our future work, the detection scene will be extended to more complex and practical conditions such as considering the multipath effect to further evaluate the performance of OFDM chirp waveform.

6. Acknowledgment

This work was supported in part by the National Natural Science Foundation of China under Grant 61271441 and in part by the University Innovation Foundation for Postgraduate of National University of Defense Technology (NUDT) under Grant S140402.

References:

[1] J. H. Kim, M. Younis, A. Moreira and W. Wiesbeck, "A novel OFDM chirp waveform scheme for use of multiple transmitters in SAR," *IEEE Geoscience and Remote Sensing Letters*, vol. 10, no. 3, pp. 568-572, May 2013.

[2] J. C. Curlander and R. N. McDonough, *Synthetic Aperture Radar Systems and Signal Processing*. U.S.: Wiley, 1991.

[3] M. A. Richards, *Fundamentals of radar signal processing*, U.S.: McGraw-Hill Companies, 2005.

[4] 2014. M. Morelli, C. J. Kuo and M. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): a tutorial review," *Proceedings of the IEEE*, vol. 95, no. 7, pp. 1394-1427, Jul. 2007.

[5] J. H. Zhu, C. Y. Fan, X. T. Huang and Z. M. Zhou, "Ground moving target detection using airborne radar based on OFDM waveform," *14th International Radar Symposium*, Dresden, Germany, 19-21th Jun. 2013: pp. 929-934.

[6] W. Q. Wang, "OFDM waveform diversity design for MIMO SAR imaging," *2012 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, Munich, Germany, 22-27th Jul. 2012: pp. 2090-2093.

[7] W. Q. Wang, "Mitigating range ambiguities in high PRF SAR with OFDM waveform diversity," *IEEE Geoscience and Remote Sensing Letters*, vol. 10, no. 1, pp. 101-105, Jan. 2013.

[8] J. H. Kim, "Multiple-Input Multiple-Output Synthetic Aperture Radar (SAR) for Multimidal Operation," Karlsruhe: Karlsruhe Institute of Technology, Dec. 2011.

[9] M. Y. Zhou, H. Y. Liu, W. Z. Liu and Y. M. Gu, "An improved OFDM chirp waveform design," *2013 IEEE 5th International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications (MAPE)*, Chengdu, China, 29-31th Oct. 2013: pp. 478-482.

[10] Y. W. Jin, J. M. F. Moura, Y. Jiang, D. D. Stancil and A. G. Cepni, "Time Reversal Detection in Clutter: Additional Experimental Results," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 140-154, Jan. 2011.

[11] M. S. Kay, *Fundamentals of statistical signal processing, Volume II*: *Detection theory*, U.S.: Prentice Hall PTR, 1998.