Complexity Analysis of SLM PAPR Reduction Schemes in Wireless OFDM System

Anuj Singal
Dept. of Electronics and Communication Engg.
Guru Jambeshwar University of Sci. & Tech.
Hisar, India
anuj.singal08@gmail.com

Deepak Kedia
Dept. of Electronics and Communication Engg.
Guru Jambeshwar University of Sci. & Tech.
Hisar, India
kedia29@gmail.com

***Abstract*-Orthogonal Frequency Division Multiplexing (OFDM) is an attractive and emerging multi-carrier modulation technique with high spectral efficiency, robust reliability and immune to multipath fading. However, High PAPR is one of the major challenging issues in the OFDM system which leads to power inefficiency and signal distortion. To solve this problem, various PAPR reduction techniques exist like SLM, PTS, PRC, Envelope Scaling, etc. The performance of selected mapping (SLM) techniques is good in case of PAPR reduction, but it requires a large number of inverse fast Fourier transform (IFFT) for generating the transmitting signal and due to this computational complexity of the system increases. So, we analyse here the computational complexity of various SLM PAPR reduction techniques. It is noticed that SLM with additive mapping approach reduces the computational complexity without BER degradation as compared with the conventional SLM. Also, computational complexity for M-QAM modulation is less as compared to M-PSK modulation.**

Keywords- OFDM, IFFT, PTS, PRC, PAPR, SLM.

I. Introduction

Orthogonal Frequency Division Multiplexing is a multi-carrier modulation technique being used in many latest wireless communications standards such as WIMAX, IEEE 802.11, IEEE 802.15.3a, IEEE 802.16, Long Term Evolution (LTE), etc. [1]. It supports high frequency spectrum efficiency, less interference, robust reliability and immune to the multipath fading in the frequency selective channel fading environments. At the transmitter, when an OFDM modulated signal passes through nonlinear devices like high power amplifier (HPA) then sometimes instantaneous output power may increase so large in comparison to the average power of the system. This is known as PAPR. Due to this it causes significant inter modulation interference, distortion and undesirable out-of-band radiation. In the OFDM system, one of the most challenging problems is high PAPR [2].

In recent year, several techniques have been proposed such as clipping, interleaving, block coding, companding, tone reservation (TR), tone injection (TI), peak reduction carrier (PRC), selected mapping (SLM), partial transmit sequence (PTS), etc to mitigate the high PAPR in the OFDM system [3]. Among these PAPR reduction techniques, SLM scheme is one of the most efficient techniques for PAPR reduction in the OFDM system. In the SLM schemes, several alternative phase sequences that are statistically independent are generated. Finally, the sequence which has the minimum PAPR among them is selected for the transmission. The PAPR reduction possibility increases with increasing the number of alternative phase sequences. But the computational complexity increases as well since it requires a large number of inverse fast Fourier transform (IFFT) for generating the transmission signals [4]. In past, several schemes of SLM have been proposed and analyzed to reduce this computational complexity. In [5], the linear combinations of alternative signal sequences are used for generation of additional alternative signal sequences as compared to conventional SLM. So, it helps in reducing the computational complexity without degrade the PAPR reduction performance as compared to the conventional SLM scheme. In the low complexity scheme [6] and [7], alternative signal sequences are generated by using the partially selective mapping technique or at the intermediate stage of FFT. In [8], SLM scheme with discrete Fourier transform (DFT)-shaping is modified with the help of windowed and pre-computed sparse matrix. Further, one of the alternative signal sequences which have lower PAPR is selected. At last, DFT-shaping scheme is used only on to the selected sequence but it requires additional memory and computations at the receiver.

In this paper, complexity of various SLM PAPR reduction schemes in wireless OFDM system is

978-1-4673-9939-5/16/$31.00 ©2016 IEEE

analyzed. The computational complexity is considerably reduced without degrade BER, SNR and PAPR reduction performance as compared to conventional SLM OFDM system.

This paper has been organized as follows. Section II gives an overview of OFDM system. Section III presents selective mapping in brief. The complexity analysis of various SLM PAPR reduction schemes is reviewed in Section IV. The simulation results are examined in section V. Finally, the conclusions are drawn in section VI.

II. OFDM (Orthogonal Frequency Division Multiplexing)

OFDM is a multi-carrier modulation technique in which the data symbols modulate with parallel and regularly spaced sub-carriers. The sub-carriers should have minimum frequency separation for maintaining the orthogonality in the time domain waveforms. Also, the carrier frequencies required to the satisfied the following requirement for maintaining the orthogonality of the carriers:-

$$f_k = f_0 + \frac{K}{T_s}, K = 1, 2, \ldots\ldots . N-1. \quad (1)$$

Where T_s = OFDM symbol duration, K = an integer, f_k = frequency of k_{th} carrier, f_0 = fundamental frequency [9]. The block model of OFDM transmission system is shown in Fig1.

First of all, the input serial data is processed by Channel Encoder at the transmitting end. The Channel encoder provides forward error correction or to reduce the probability of error, which occurred due to the channel effect at the receiver. Further it passes through the Frequency or Time Interleaver. By Interleaving, resistance to frequency-selective channel conditions called fading increases. Now, modulation of data sequence is taking place by using the different modulation schemes such as BPSK, QPSK QAM, etc. The data symbols are converted from serial to parallel in N different sub streams. Further, each sub stream will IFFT modulated with a separate carrier by passing through the IFFT modulation block.

Now, PAPR of system is reduced by SLM PAPR reduction techniques. To eliminate the inter block interference (IBI) and the inter symbol interference (ICI), a cyclic prefix is inserted. Now, parallel data is converted into the serial data. The serial data (OFDM symbol) modulates at a high-frequency carrier and passed it to the RF modulation stage. In the last, the RF modulated signal is transmitted to the receiver using the transmit antennas [9, 10].

The baseband discrete time OFDM signal can be expressed as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi nk/N}, 0 \le n < N \quad (2)$$

Here, N is number of subcarriers, A is an input symbol sequence, and n is discrete time index. Also, equation 2 can be expressed in N x N Hermitian matrix form as

$$a = Q^H A \quad (3)$$

Where, $Q_{k,n} = \frac{1}{\sqrt{N}} e^{-j2\pi nk/n}$

III. Selective Mapping (SLM) Techniques

To reduce the PAPR, various SLM PAPR reduction techniques exist like Conventional SLM, SLM with Additive mapping etc. A brief discussion of these PAPR reduction techniques is as follows:

a) *Conventional SLM [11]:-* In the Conventional SLM technique, *U* phase sequences is generated that is $\gamma^{(u)} = [\gamma_0^{(u)}, \gamma_1^{(u)} \ldots\ldots\ldots\ldots . \gamma_{JN-1}^{(u)}]^T$, where $\gamma_k^{(u)} = exp^{j\phi_k^{(u)}}$, $\phi_k^{(u)} \epsilon [0, 2\pi]$, k = 0, 1, 2..... *JN*-1, u = 1, 2, 3, *U*. Further, the input data X= $[X_0, X_1, X_2 \ldots\ldots X_{JN-1}]^T$ is multiplied with each one of the *U* various phase sequence $\gamma^{(u)}$ which provides a set of *U* different data blocks $R^{(u)} = [X_0\gamma_0^{(u)}, X_1\gamma_1^{(u)} \ldots\ldots X_{JN-1}\gamma_{JN-1}^{(u)}]^T$, u = 1, 2, 3, *U*. Now, all *U* alternative data blocks are converted into time domain with the help of IFFT. Finally, the signal which has minimum PAPR is selected for transmission as shown in Fig. 2.

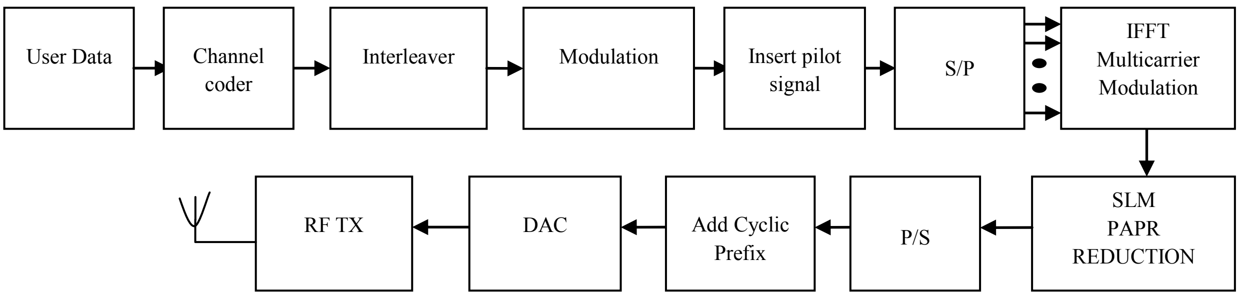

Fig. 1:- OFDM Transmitter [9]

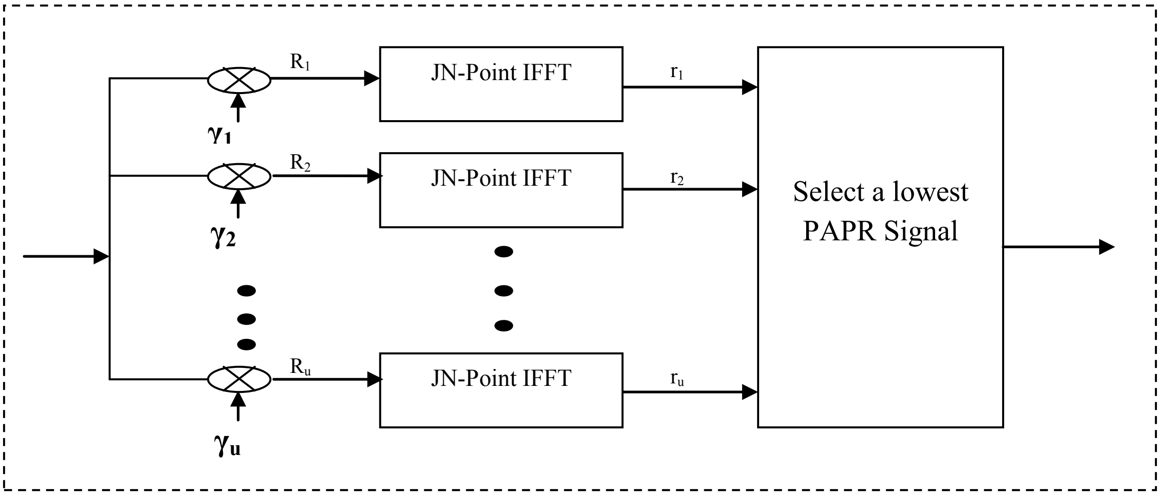

Fig. 2:- Block Diagram of the SLM approach [11]

b) *SLM with Additive mapping [12]:* -In SLM with additive mapping scheme, additive mapping sequence is used to generate alternate symbol sequence by simply combining the mapping signal sequence with OFDM signal sequence in time domain. It reduces the computational complexity without BER degradation as compared to conventional SLM. In SLM with additive mapping scheme, additive mapping sequence can be expressed as:-

$$X_k^{(u)} = A_k + D_k^{(u)} \qquad (4)$$

$$d = Q^H D \qquad (5)$$

Here,$D_k^{(u)} = [D_0^{(u)}, D_1^{(u)} \ldots \ldots \ldots D_{N-1}^{(u)}]^T$, $0 \leq u < U$, are called additive mapping sequence. Also, $D = D_I + jD_Q$, where D_I and D_Q is in-phase and quardrature component of D respectively. These additive mapping sequence $D^{(u)}$ is determined with the help of phase sequence $P^{(u)}$ and input symbol sequence A. The additive mapping signal sequence when all element of $P^{(u)}$ is -1 is defined as

$$d^{(-1)} = Q^H D^{(-1)} = Q^H D_I^{(-1)} + jQ^H D_Q^{(-1)} \qquad (6)$$

Similarly, the lth additive mapping signal sequence for $P_k^{(l)}$ is defined as

$$d^{(l)} = Q^H D^{(l)} = Q^H D_I^{(l)} + jQ^H D_Q^{(l)} \qquad (7)$$

So, we can generate 16 sequences for M-QAM for single phase using a, $d_I^{(-1)}$, $d_Q^{(-1)}$,$d_I^{(l)}$ and $d_Q^{(l)}$ as shown in Table1:

Table1:- Additive mapping alternative sequences [12]

U	$x^{(u)}$	a	$m^{(u)}$			
			$d_I^{(-1)}$	$jd_Q^{(-1)}$	$d_I^{(l)}$	$jd_Q^{(l)}$
0	$x^{(0)}$	+				
1	$x^{(1)}$	+	+			
2	$x^{(2)}$	+		+		
3	$x^{(3)}$	+	+	+		
4	$x^{(4)}$	+			+	
5	$x^{(5)}$	+				+
6	$x^{(6)}$	+			+	+
7	$x^{(7)}$	+	+		-	
8	$x^{(8)}$	+	+			+
9	$x^{(9)}$	+	+		-	+
10	$x^{(10)}$	+		+	+	
11	$x^{(11)}$	+		+		-
12	$x^{(12)}$	+		+	+	-
13	$x^{(13)}$	+	+	+	-	
14	$x^{(14)}$	+	+	+		-
15	$x^{(15)}$	+	+	+	-	-

IV. Complexity Analysis of OFDM System

It is known that number of complex multiplication and addition required is N x N and N x (N-1) respectively for direct computation of N-point DFT. So a brief introduction of complexity of various PAPR reduction techniques like Conventional SLM, SLM with Additive mapping, etc. is explained below:-

a. *Conventional SLM [11]:*- In the conventional SLM, U no. of phase sequences is used for generating the transmitting signal. So, it required U number of IIFT for U phase sequences. It is known that no. of complex multiplication and addition required is $\mathrm{JN}\log_2\mathrm{JN}$ and $\frac{\mathrm{JN}}{2}\log_2\mathrm{JN}$ respectively for computation of JN-point DFT using FFT algorithm. Hence, no. of complex multiplication and addition required is $\mathrm{UJN}\log_2\mathrm{JN}$ and $\mathrm{U}\frac{\mathrm{JN}}{2}\log_2\mathrm{JN}$ respectively for the conventional SLM since it required U number of IIFT.
b. *SLM with Additive mapping [12]:* - In SLM with additive mapping with M-QAM, 16 sequences are generated for single phase with the help of a, $d_I^{(-1)}$, $d_Q^{(-1)}$,$d_I^{(l)}$ and $d_Q^{(l)}$ as shown in Table1. It is know that one complex IFFT for a and two real IFFT for $d_I^{(-1)}$, $d_Q^{(-1)}$ are required to generate first four alternative sequence. Similarly two real IFFT for $d_I^{(l)}$, $d_Q^{(l)}$ are required to generate for last 12 alternative sequence as shown in Table1. It is assumed that the computational complexity of two real JN-point IFFT is equivalent to that of one complex JN- point IFFT and JN-2 complex additions. So, two complex JN-point IFFT and JN-2 complex addition is required to generate first four alternative sequence. Also, one complex JN-point IFFT and JN-2 complex addition is required to generate for last 12 alternative sequences. In the Table1, it is noted that number of phase sequence U is equivalent to 4+12V for M-QAM. Its mean first four sequence remain same and last 12 alternative sequences changes for each phase. So, total no. of complex multiplication required is

$$2\frac{\mathrm{JN}}{2}\log_2\mathrm{JN} + \mathrm{V}\frac{\mathrm{JN}}{2}\log_2(\mathrm{JN})$$

$$= \left(2 + \left\lceil\frac{\mathrm{U}-4}{12}\right\rceil\right)\frac{\mathrm{JN}}{2}\log_2(\mathrm{JN}) \quad (8)$$

Similarly, total no. of complex addition required is

$$2\,\mathrm{JN}\log_2(\mathrm{JN}) + \mathrm{V}\,\mathrm{JN}\log_2(\mathrm{JN})$$

$$+(\mathrm{V}+1)\,(\mathrm{JN}-2) + (12\mathrm{V}+3)\,\mathrm{JN}$$

$$= \left(2 + \left\lceil\frac{\mathrm{U}-4}{12}\right\rceil\right)\mathrm{JN}\log_2(\mathrm{JN})$$

$$+\left(13\left\lceil\frac{13\mathrm{U}-4}{12}\right\rceil + 4\right)\mathrm{JN} - 2\left(\left\lceil\frac{\mathrm{U}-4}{12}\right\rceil + 1\right) (9)$$

In equation 9 the four terms are considered due to the following reasons:-

i. It is known that 1 complex IFFT for a and two real IFFT for $d_I^{(-1)}$, $d_Q^{(-1)}$ are required to generate first four alternative sequence. So two complex JN-point IFFT addition is required for this and it is equivalent to $2\mathrm{JN}\log_2(\mathrm{JN})$. Similarly $\mathrm{VJN}\log_2(\mathrm{JN})$ no. of complex addition is required for last 12 alternative sequences for V phase.
ii. $(\mathrm{V}+1)(\mathrm{JN}-2)$ Complex addition required during computation of complex multiplication.
iii. 3JN Complex addition and 12 JN Complex addition is required for addition of first four sequences and last 12 alternative sequences respectively for single phase. So total number of (3+12V) JN complex addition is required.

But In SLM with additive mapping with M-PSK only 4 sequences can be generated for single phase with the help of a, $d_I^{(-1)}$, $d_Q^{(-1)}$,$d_I^{(l)}$ and $d_Q^{(l)}$. So, total no. of required complex multiplication is

$$\frac{\mathrm{JN}}{2}\log_2(\mathrm{JN}) + \mathrm{V}\frac{\mathrm{JN}}{2}\log_2(\mathrm{JN})$$

$$= \left(1 + \left\lceil\frac{\mathrm{U}-1}{3}\right\rceil\right)\frac{\mathrm{JN}}{2}\log_2(\mathrm{JN}) \quad (10)$$

Similarly total no. of required complex addition is

$$\mathrm{JN}\log_2(\mathrm{JN}) + \mathrm{VJN}\log_2(\mathrm{JN}) + \mathrm{V}(\mathrm{JN}-2) + 3\mathrm{VJN}$$

$$= (1 + [\frac{\mathrm{U}-1}{3}])\mathrm{JN}\log_2(\mathrm{JN})$$

$$+4\left\lceil\frac{\mathrm{U}-1}{3}\right\rceil\mathrm{JN} - 2\left\lceil\frac{\mathrm{U}-1}{3}\right\rceil \quad (11)$$

Lastly, complexity analysis of various PAPR reduction techniques like Conventional SLM, SLM with Additive mapping, etc. is summarized in the Table 2 below:-

Table 2:- Computational Complexity Analysis of Conventional SLM and SLM with Additive Mapping PAPR Reduction Scheme [12]

	Total number of complex multiplications	Total number of complex additions
Conventional SLM	$\frac{U}{2}JN\log_2(JN)$	$UJN\log_2(JN)$
SLM with additive mapping for M-QAM	$\left[2+\frac{U-4}{12}\right]\left[\frac{JN}{2}\log_2(JN)\right]$	$\left[\left(2+\frac{U-4}{12}\right)(JN\log_2 JN)\right]+\left[\left(13\left(\frac{13U-4}{12}\right)+4\right)JN\right]-\left[2\left(\frac{U-4}{12}\right)+1\right]$
SLM with additive mapping for M-PSK	$\left[1+\frac{U-1}{3}\right]\left[\frac{JN}{2}\log_2(JN)\right]$	$\left[\left(1+\frac{U-1}{3}\right)(JN\log_2 JN)\right]+\left[4\left(\frac{U-1}{3}\right)JN\right]-\left[2\left(\frac{U-1}{3}\right)\right]$

Here, U is the total number of alternative-signal sequences, N is number of subcarrier and J is the oversampling factor. In JN-point IFFT no. of complex additions and multiplications are $JN\log_2(JN)$and $\frac{JN}{2}\log_2(JN)$ respectively. Also, it is assumed that the computational-complexity of two real-additions is equivalent to the one complex addition during complexity analysis.

V. Results

In this section, complexity analysis of various PAPR reduction techniques is presented in the form of fig. 3 and fig. 4 respectively. Here, both fig. 3 and fig. 4 compare the computational complexity of the conventional SLM, SLM with additive mapping scheme for M-QAM and M-PSK. In the fig. 3, number of complex addition compared with respect to number of phase sequences U, where U=4, 7, 10, 13…., N =512 and J=4. It is noticed that number of complex additions for SLM with additive mapping schemes reduces without BER degradation as compared to conventional SLM. Also, when we compared the complexity of SLM with Additive mapping for M-QAM and M-PSK, it is observed that number of complex additions for M-QAM modulation is less as compared to M-PSK modulation. Similarly number of complex multiplications is compared with respect to number of phase sequences U in the fig. 4.

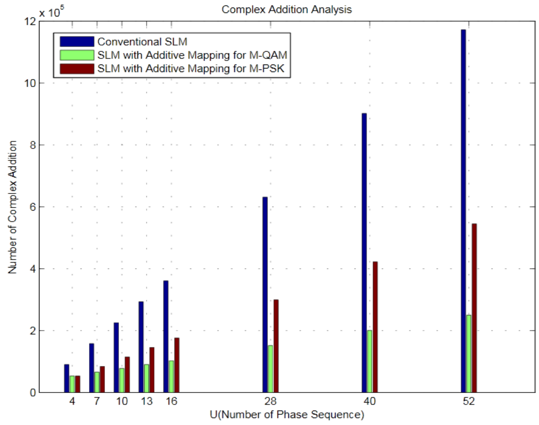

Fig. 3:- Comparison of Complex Addition Analysis for Conventional SLM, SLM with Additive Mapping for M-QAM and M-PSK (N=512, J=4).

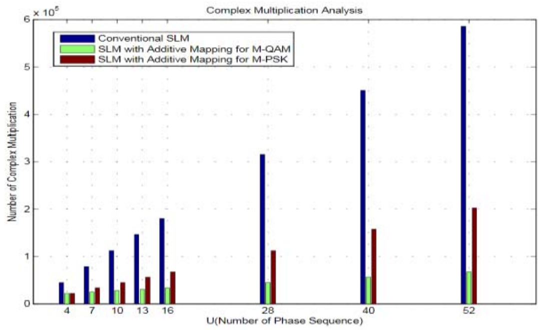

Fig. 4:- Comparison of Complex Multiplication Analysis for conventional SLM, SLM with Additive Mapping for M-QAM and M-PSK (N=512, J=4).

VI. Conclusion

In this paper, complexity analysis of various PAPR reduction techniques like Conventional SLM, SLM with Additive mapping for M-QAM and M-PSK etc is presented. In SLM with additive mapping scheme, alternate symbol sequence is generated by simply adding mapping-signal sequence to an OFDM signal sequence in the time domain. It reduces the computational complexity without BER degradation as compared to conventional SLM. Also, when we compared the complexity of SLM with Additive mapping for M-QAM and M-PSK, it is observed that computational complexity for M-QAM modulation is less as compared to M-PSK modulation.

References

[1]. Houshou Chen, Hsinying Liang, *"Combined Selective Mapping and Binary Cyclic Codes for PAPR Reduction in OFDM Systems," IEEE Transactions on Wireless Comm.*, vol.6, no.10, pp. 3524-3528,Oct. 2007.

[2]. Seung Hee Han, Jae Hong Lee "*An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multi-carrier Transmission", IEEE Transactions on Wireless Comm.*, vol.12, no. 2, pp. 56-65, 2005.

[3]. Anuj Singal, Deepak Kedia, *"Design issues and challenges in MIMO-OFDM system," IUP Journal of Telecomm.* vol.6, no.1, Oct. 2014.

[4]. Jing, Jinkuan, Xin and Bin Wang, "*Low-complexity PAPR Reduction with Low Computational Complexity for OFDM Systems," Proceedings of Elsevier on the Medical Physics and Biomedical Engineering*, vol. 33, pp. 1401–1407, 2012.

[5]. Seok-Joong Heo, Hyung-Suk Noh, Jong-Seon No, and Dong-Joon Shin, *"A modified SLM scheme with low complexity for PAPR reduction of OFDM systems," IEEE Trans. on the Broadcast.*, vol. 53, no. 4, pp. 804–808, Dec. 2007.

[6]. Dae Woon Lim, Jong Seon No, Chi Woo Lim, and Habong Chung, *"A new SLM OFDM scheme with low complexity for PAPR reduction," IEEE Signal Processing Letter,* vol. 12, no. 2, pp. 93–96, Feb. 2005.

[7]. Ghassemi A. and Gulliver T. A., *"Partial selective mapping OFDM with low complexity IFFTs," IEEE Communication Letter,* vol. 12, no. 1, pp. 4–6, Jan. 2008.

[8]. Chih-Y. Hsu and H.-H. Chao, *"Novel SLM scheme with low-complexity for PAPR reduction in OFDM system," IEICE Trans. on Fundamentals.,* vol. E91-A, no. 7, pp. 1689–1696, Jul. 2008.

[9]. Hwi Jeong, Hyung Kong, *"Performance analysis of CDMA-OFDM system via cooperative communication in wireless fading environment", Proceedings of IEEE on strategic technology* ,Ulsan, vol. 6, no. 3, pp. 176-181, Sep. 2006.

[10]. Taewon H. , Gang Wu and Chenyang Yang., *"OFDM and its wireless applications: A survey," IEEE Transaction on Vehicular Tech.,* vol. 58, no. 4, pp. 1673-94, May 2009.

[11]. Chin-Liang Wang and Yuan Ouyang, "*Low-complexity selected mapping schemes for peak-to-average power ratio reduction in OFDM systems," IEEE Transaction on Signal Processing*, vol. 53, no. 12, pp. 4652–4660,Dec. 2005.

[12]. Jong-Seon No, Hyun-Bae Jeon and Dong-Joon Shin,, "*A Low-Complexity SLM Scheme Using Additive Mapping Sequences for PAPR Reduction of OFDM Signals,*" *IEEE Transaction on Broadcasting*, vol. 57, no. 4, pp. 866-875, Dec. 2011.