Pulse Shaping Approach to PAPR Reduction for Multiuser OFDM Systems

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***Abstract*—Multiuser orthogonal frequency division multiplexing (MU-OFDM) communication systems have higher system efficiency compared to single user OFDM systems. However, the high peak-to-average power ratio (PAPR) of the MU-OFDM signal is considered as one of the major drawbacks of the MU-OFDM systems. This paper proposes to use computationally efficient optimisation approach to design a set of orthogonal pulse shaping waveforms to reduce the PAPR of the MU-OFDM signal. Autocorrelation and cross-correlation of the pulse shaping waveforms are specified in the constraints in order to minimise intersymbol interference (ISI) and co-channel interference (CCI). Methods to solve the optimal waveform set design problem are presented. Numerical results illustrate that the designed set of pulse shaping waveforms is efficient in reducing the PAPR of the OFDM signal in multiuser communication systems.**

I. Introduction

In recent years, OFDM modulation technique has been adopted and implemented by many wireless communication standards due to its high bandwidth efficiency and robustness against multipath fading. One of the main drawbacks of the OFDM systems is that the transmitted signal often exhibits a high PAPR. The PAPR of the OFDM transmitted signal determines the power efficiency of the high power amplifier (HPA). Therefore, reducing the PAPR of the OFDM signal is essential as power efficiency in wireless communication relates to coverage range, power consumption and the size of the terminals.

Multiuser OFDM is a promising technique for high downlink capacities in mobile communication systems. In multiuser communication systems, the OFDM bandwidth is shared among multiple users. It is critical to design the OFDM subcarriers in such a way that users do not interfere with each other to avoid ISI [1]. Previous work in [2] proposed to design different precoder for different user for MU-OFDM. However, the proposed design [2] does not consider PAPR reduction of the OFDM signal. Recently, a method in [3] is proposed to establish user independence and to reduce the PAPR of the MU-OFDM signal but the results only show marginal PAPR reduction.

This paper proposes to use computationally efficient optimisation approach to design a set of pulse shaping waveforms for reducing the PAPR of the MU-OFDM signal. Designing of a set of pulse shaping waveforms is fundamentally different from designing a shaping pulse for a single user OFDM system because cross-correlations between the different pulse shaping waveforms have to be taken into consideration. Therefore, the optimisation design problem such as in [6] cannot be applied to solve the problem of designing a set of pulse shaping waveforms.

This paper is structured as follows. Section II presents the OFDM system model for multiuser communication. Section III introduces the pulse shaping filter design for MU-OFDM signal and computationally efficient method is presented to solve the pulse shaping waveform set design problem. Numerical results are presented in Section IV to demonstrate the effectiveness of the designed set of pulse shaping waveforms in reducing the PAPR of the MU-OFDM signal. Finally, concluding remarks are drawn in Section V.

II. System Model

A. MU-OFDM System Model without Pulse Shaping

Consider that U users utilise the OFDM system with N subcarriers. The number of subcarriers allocated for each user is $N_u = \frac{N}{U}$. It is assumed that each user has N_u number of data streams mapped to the allocated subcarriers as illustrated in Fig.1. The baseband equivalent of the OFDM transmitted signal for the u^{th} user is given by

$$\hat{s}^u(t) = \sum_{k=0}^{N'_u} Z_k^u e^{j2\pi kt/T}, \quad 0 \leq t < T \tag{1}$$

where $N'_u = (u-1)N_u + m$ and $m = 0, 1, 2, ...N_u - 1$. The subcarrier allocation can be on a fixed basis or on a dynamic basis [4][5]. It is assumed that the subcarriers allocation method is fixed.

B. MU-OFDM System Model with Pulse Shaping

When a single user OFDM signal is subjected to pulse shaping, the baseband equivalent of the OFDM transmitted signal is given by

$$\hat{s}(t) = \sum_{k=0}^{N-1} Z_k p_k(t) e^{j2\pi kt/T}, 0 \leq t \leq T \tag{2}$$

where $p_k(t)$ is the shaping pulse at the k^{th} subcarrier with a duration of T.

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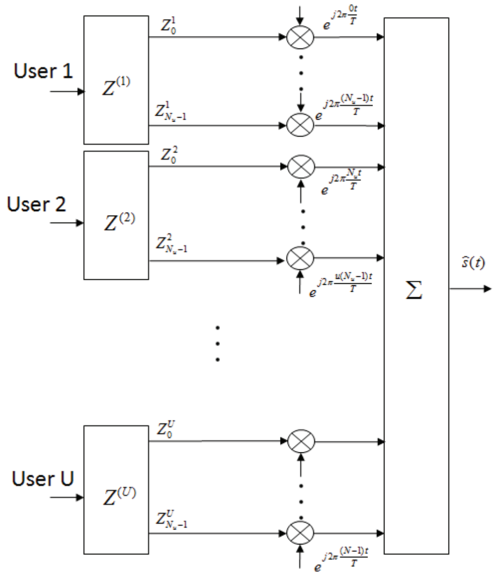

Fig. 1. Block diagram of multiuser OFDM system

For multiuser communication systems, U users share the N subcarriers OFDM system. The number of subcarriers allocated for each user is $N_u = \frac{N}{U}$ and the baseband equivalent MU-OFDM signal with pulse shaping can be expressed as

$$s(t) = \sum_{u=1}^{U} s^u(t), 0 \leq t \leq T \tag{3}$$

where

$$s^u(t) = \sum_{k=0}^{N'_u} Z_k^u p_k^u(t) e^{j2\pi kt/T} \tag{4}$$

where $N'_u = (u-1)N_u + m$, $m = 0, 1, 2, ...N_u - 1$ and $p_k^u(t)$ is a pulse shaping waveform at the k^{th} subcarrier for the u^{th} user.

Pulse shaping in the frequency domain can be regarded as a precoding process. The block diagram of the precoded multiuser OFDM (PMU-OFDM) is illustrated in Fig.2. The baseband equivalent of the PMU-OFDM transmitted signal at the u^{th} user can be expressed as

$$s^u(t) = \sum_{k=0}^{N'_u - 1} d^u e^{j2\pi kt/T}, \quad 0 \leq t < T \tag{5}$$

where $N'_u = (u-1)N_u + m$, $m = 0, 1, 2, ...N_u - 1$ and

$$d_k^u = P_k^u(e^{j\omega}) Z_k^u \tag{6}$$

To reduce the PAPR of the MU-OFDM transmitted signal, the proposed approach is to design one principle pulse for each individual user and the different shaping pulses for all the subcarriers belonging to this particular user can be generated by cyclic shifting of this principle pulse. Let $P_0^u(e^{j\omega})$ denotes the frequency response of the principle pulse for u^{th} user, then

$$P_k^u(e^{j\omega}) = P_0(e^{j\omega}) e^{-j\omega k}, k = 1, ..., N'_u - 1 \tag{7}$$

where $N'_u = (u-1)N_u + m$, $m = 0, 1, 2, ...N_u - 1$.

In practical implementations, (7) is often discretised and the discretised pulse can be expressed as

$$P_k^u = P_0^u e^{-j2\pi \frac{ik}{N_u}} \tag{8}$$

where $i = 0, 1, ..., L-1$, $L = (1+\beta)N_u$ and $k = 1, ..., N'_u - 1$.

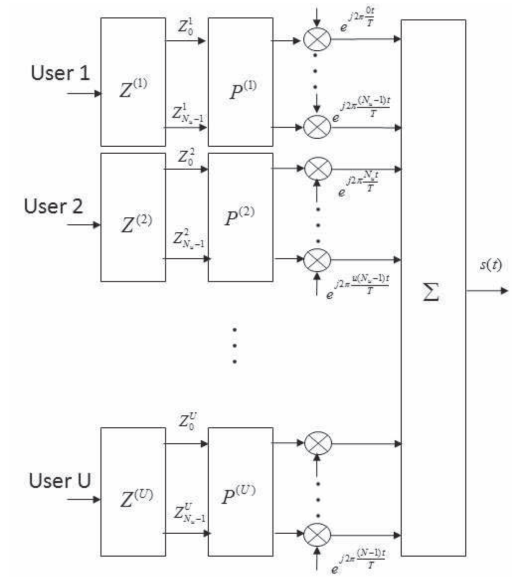

Fig. 2. Block diagram of PMU-OFDM

III. Pulse Shaping Waveform Set Design for MU-OFDM System

The design of a set of pulse shaping waveforms can be generalised as the design of a set of filters with specified constraints in time and frequency domain [7][2]. The objective of the design problem is to minimise the stopband energy of the designed pulse shaping waveforms and to maintain the spectral shaping efficiency. Constraints are introduced in the problem formulation in order to minimise ISI and ICI. All of the pulse shaping waveforms in the set are specified to have low autocorrelation value at nonzero lags and low cross-correlation value at all lags for low ISI and ICI, respectively.

A. Problem Formulation

For any given u, consider the following finite impulse response (FIR) filter $P_u(e^{j\omega})$

$$P_u(e^{j\omega}) = \sum_{k=0}^{M-1} h_u(k) \phi_k(e^{j\omega}) = \boldsymbol{h_u}^T \boldsymbol{\phi}(e^{j\omega}) \tag{9}$$

where

$$\boldsymbol{h_u} = [h_u(0), h_u(1), \ldots h_u(M-1)]^T$$
$$\boldsymbol{\phi} = [\phi_0, \phi_1, \ldots, \phi_{M-1}]^T$$

where

$$\phi_k(e^{j\omega}) = e^{-j\omega k} \quad k = 0, 1, 2, \ldots \tag{10}$$

The pulse shaping waveform set design problem, denoted as Problem (PU), can be stated as follows.

Problem(PU). Design a set of U filters $P_U = \{P_1(e^{j\omega}), P_2(e^{j\omega}), ..., P_U(e^{j\omega})\}$ which solves the following constrained optimisation problem

$$\min_{h_u} \max_{U} \max_{\omega_p \in \Omega_p} \Big\{ ||P_u(e^{j\omega_p})|^2 - |D(\omega_p)|^2| + \frac{\gamma}{\pi} \int_{\Omega_s} |P_u(e^{j\omega})|^2 d\omega \Big\} \quad (11)$$

for $u = 1, 2, \ldots, U$ and subject to the following autocorrelation constraint

$$|R_{ac}(m)| = |\sum_{i=0}^{M-1} h_u(i) h_u(i - mD)| \le \eta_1 \quad (12)$$

where $m = 1, 2, ...$ and also the cross-correlation constraint

$$|R_{cc}(m)| = |\sum_{i=0}^{M-1} h_n(i) h_u(i - mD)| \le \eta_2 \quad (13)$$

for $n \neq u$, $n = 1, 2, \ldots, U$, $u = 1, 2, \ldots, U$ and $m = 0, \pm 1, \pm 2, \ldots$, where $D(\omega)$ is a prescribed desired frequency response (usually the frequency response of an ideal filter), Ω_p the passband frequency set, Ω_s the stopband frequency set, D a positive integer representing the number of samples per symbol interval and γ a weighting parameter chosen by the designer.

Remarks: (a) The first part of the objective function is to shape every filter to a desired spectral shape (as prescribed by $D(\omega)$). The second part of the objective function is to minimise the stopband energy leakage. The parameter γ in the objective function is chosen by the designer to reflect the designer's preference for better fitting (small γ) or lower stopband energy leakage (larger γ) of the designed set of pulse shaping waveforms. $D(\omega)$ is often selected to be the frequency response of a Nyquist pulse. One of the main reasons to use a Nyquist pulse is to allow the implementation of the matched filter at the receiver. (b) In constraint (12) and (13), η_1 and η_2 are small positive value in order to maintain a low ISI and low ICI, respectively.

B. Problem Conversion

After some algebraic manipulations (see Appendix), the optimisation problem (PU) can be converted to the following simplified optimisation problem (PUb) in which both the objective function and the contraints are quadratic of the parameters to be optimised

Problem(PUb). Find $h_u(u = 1, 2, ..., U)$ which solve the following optimisation problem

$$\min_{h_u} \max_{u} \max_{\omega_p \in \Omega_p} \Big\{ ||h_u^T Q(\omega) h_u| - |D(\omega_p)|^2| + \frac{\gamma}{\pi} \int_{\Omega_s} Q_s(\omega) d\omega \Big\} \quad (14)$$

for $u = 1, 2, ..., U$ subject to

$$|h_u^T Q_m h_u| \le \eta_1 \quad (15)$$

for $2 \le u \le U$ and $m = 1, 2, ...$ and

$$|h_n^T G_m h_u| \le \eta_2 \quad (16)$$

where $n \neq u$, $n = 1, 2, \ldots, U$, $u = 1, 2, \ldots, U$, $m = 0, \pm 1, \pm 2, \ldots$, $G_m = Q_m$ for $m > 0$ and $G_m = Q_m^T$ for $m \le 0$.

Remarks: Instead of solving problem (PUb) to obtain $P_1(e^{j\omega})$, $P_2(e^{j\omega})$, ..., $P_U(e^{j\omega})$ simultaneously, in our numerical computation, iterative technique is used to design $P_u(e^{j\omega})$ in a sequential manner. First, problem (PUb) is solved for $P_1(e^{j\omega})$ subject to autocorrelation constraint (15). Then, by using $P_1(e^{j\omega})$, the optimisation problem (PUb) is solved for $P_2(e^{j\omega})$, $P_3(e^{j\omega})$, $P_4(e^{j\omega})$, etc one by one in an iterative fashion with both autocorrelation constraint (15) and cross-correlation constraint (16).

IV. Numerical Results

In this section, numerical results are presented to demonstrate the effectiveness of the designed set of pulse shaping waveforms in reducing the PAPR of the MU-OFDM signal. Fig.3 and Fig. 4 illustrate the spectra and the impulse responses of a set of four designed pulse shape waveforms. The ideal frequency response $D(\omega)$ is the raised cosine filter with $\beta = 0.2$. Other design parameters are $D = 4$, $M = 48$, $\eta_1 = 0.001$, $\eta_2 = 0.001$ and $N = 64$. The autocorrelation is set at low value at $m \neq 0$ ($|R_{ac}(m)| \le 0.001$) and the cross-correlation is set at low value at $m = 0, \pm 1, \pm 2, \ldots$ ($|R_{cc}(m)| \le 0.001$) to minimise ISI and CCI.

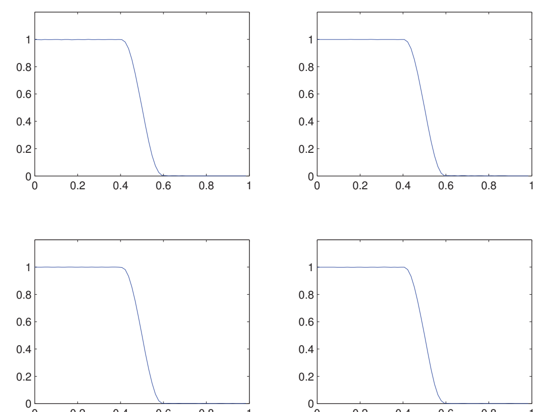

Fig. 3. Frequency spectrum of the four designed set of filters

Fig.5 illustrates the CCDF of the PAPR of the PMU-OFDM for $U = 2$. It shows that the designed set of pulse shaping waveforms reduces the PAPR by approximately 2 dB for $N = 64$, $N = 128$ and $N = 256$.

Fig.6 illustrates that the designed set of pulse shaping waveforms reduces the PAPR for different numbers of subcarriers. For $N = 128$, the PAPR can be reduced by approximately 2 dB for $U = 2$ and 3 dB for $U = 4$. For $U = 4$ and $N = 256$, the PAPR can be reduced by approximately 2 dB.

For further investigation, Fig.7 compares the CCDF of the

PAPR of MU-OFDM for $U = 2$, $U = 4$ and $U = 8$ for $N = 256$. For $U = 8$ the PAPR of the OFDM signal can be reduced by approximately 2 dB.

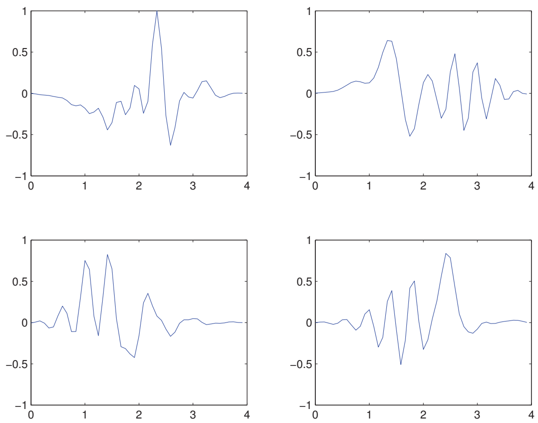

Fig. 4. Impulse response of the four designed set of pulse shaping filters

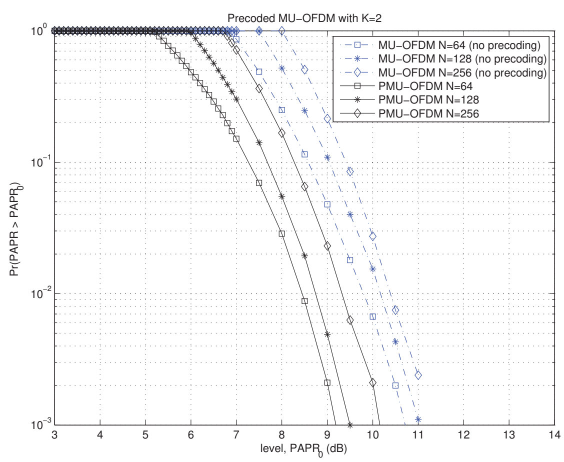

Fig. 5. PAPR for PMU-OFDM with U=2

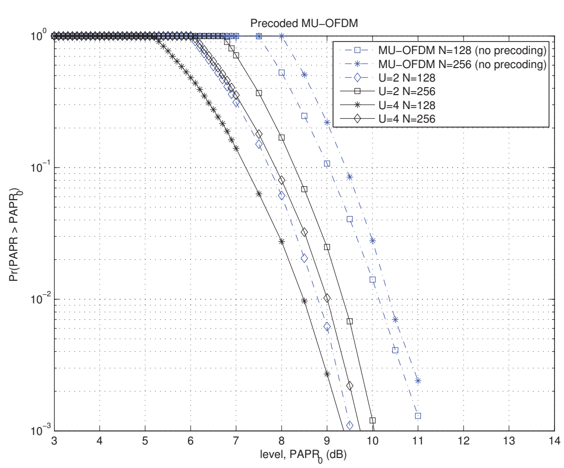

Fig. 6. PAPR for PMU-OFDM with U=2 and U=4

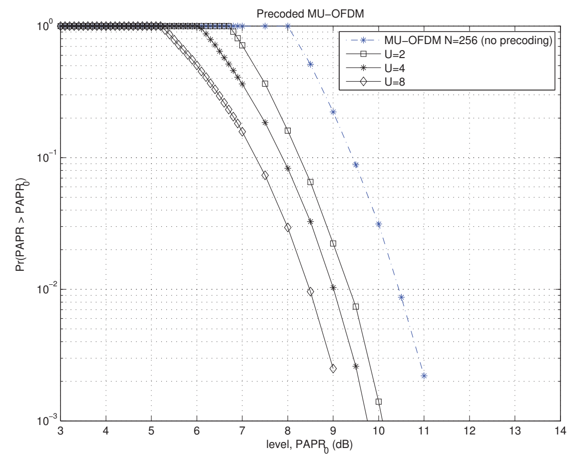

Fig. 7. PAPR for PMU-OFDM for N=256

V. Concluding Remarks

In this paper, we have proposed to design a set of pulse shaping waveforms using an optimisation approach to reduce the PAPR of the MU-OFDM signal. The pulse shaping waveform set design problem was formulated as a constrained minimax optimisation problem with both autocorrelation and cross-correlation constraints. The design problem was then simplified and solved effectively by iterative techniques. Numerical results demonstrated the effectiveness of the designed pulse shaping waveform set in reducing the PAPR of the OFDM signal in multiuser communication systems

VI. Appendix: Problem Simplification

In this section, the objective function and the constraint of Problem (PU) are simplified.

It is assumed that the matched filter is used and the channel is considered flat, the magnitude of the overall transfer function can be expressed as

$$|P_u(e^{j\omega})|^2 = h_u^T Q(\omega) h_u \tag{17}$$

where $h_u = [h_u(0), h_u(1), ..., h_u(M-1))$ and

$$Q(\omega) = e(\omega)e(\omega)^T \tag{18}$$

The magnitude error in the passband can be expressed as

$$\xi_p = ||h_u^T Q(\omega) h_u| - |D(\omega_p)|^2| \quad \omega_p \in \Omega_p \tag{19}$$

The energy error in the stopband can be written as

$$\xi_s = h_u^T Q_s(\omega) h_u \tag{20}$$

where Q_s is a matrix of NxN defined by

$$Q_s = \frac{1}{\pi} \int_{\Omega_s} Q(\omega) d\omega \tag{21}$$

In order to simplify constraints (12) and (13), the following matrix is defined as

$$Q_m = \begin{bmatrix} 0_{NU} & 0_m \\ I_m & 0_{NU}^T \end{bmatrix} \tag{22}$$

where I_m represents an $(N-mD)\times(N-mD)$ identity matrix, 0_m represents a $(mD)\times(mD)$ zero matrix, 0_{NU} represents an $(mD)\times(N-mD)$ zero matrix. The autocorrelation constraints in (12) can be expressed as

$$|R_{ac}(m)| = |h_u^T Q_m h_u| \leq \eta \tag{23}$$

and the crosscorrelation constraints in (13) can be expressed as

$$|R_{cc}(m)| = |h_n^T G_m h_u| \leq \eta \tag{24}$$

where $G_m = Q_m$ for $m > 0$ and $G_m = Q_m^T$ for $m \leq 0$.

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