

LOW PEAK-TO-AVERAGE RATIO OFDM CHIRP WAVEFORM DIVERSITY DESIGN

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ABSTRACT

Large time-bandwidth product waveform diversity design is a challenging topic in multiple-input multiple-output radar high-resolution imaging because existing methods usually can generate only two large time-bandwidth product waveforms. This paper proposes a new low peak-to-average ratio (PAR) orthogonal frequency division multiplexing chirp waveform diversity design through randomly subchirp modulation. This method can easily yield over two orthogonal large time-bandwidth product waveforms. More waveforms means that more degrees-of-freedom can be obtained for the system. The waveform performance is evaluated by the ambiguity function. It is shown that the designed waveform has the superiorities of a large time-bandwidth product which means high range resolution and low transmit power are allowed for the system, almost constant time-domain and frequency-domain modulus, low PAR and no range-Doppler coupling response in tracking moving targets.

Index Terms— Waveform diversity design, MIMO radar, OFDM waveform, chirp diverse waveform, ambiguity function.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar waveform diversity design has received much attention. However, existing methods [1] often assume some prior knowledge of the impulse response of the target and use this information to optimize the waveforms. The waveform used in actual MIMO radars should have a wide bandwidth, so as to obtain a high resolution. Moreover, a high average transmit power is required for the transmitted waveforms which means a large time-bandwidth is required. Certainly, the waveforms should also have good ambiguity function characteristics

such as Doppler resolution and matched filtering sidelobe performance. For these requirements, most of the existing waveforms are not suitable for MIMO radars due to their small time-bandwidth product and are difficult to implement in practice. For instance, the Costas pulses-based waveforms [2] have good range-Doppler properties, but each chip contains only one frequency.

Orthogonal frequency division multiplexing (OFDM) is a popular choice in communication and radar systems [3]. It is because OFDM-like waveforms have been shown to be suitable for radar applications and the feasibility of integrating communication functions in radar networks has also been explored. It is pointed out in [4] that OFDM-coded radar signals are comparable with classical chirp signals and experiences no range-Doppler coupling.

Up to now, the most commonly used MIMO radar waveform design is based on the up- and down-chirp modulation [5], but it offers only two orthogonal waveforms. From a practical point of view we believe that MIMO radar should use chirp-based waveforms due to their desirable properties including high range resolution, constant modulus, implementation simplicity, and low ambiguity function sidelobes. Recently, we have extended conventional OFDM signal to the OFDM chirp waveform [6] by using chirp frequency instead of sinusoid frequency, but its peak-to-average ratio (PAR) performance may not be satisfactory. In [7], a novel OFDM chirp waveform design scheme is proposed for MIMO radar, but it offers only two orthogonal waveforms. Moreover, it will generate grating lobes due to the sparse sampling.

In a MIMO radar more antennas means more degrees-of-freedom can be obtained for the system, but more orthogonal waveforms are required. To generate more orthogonal waveforms, this paper proposes a practical OFDM chirp waveform diversity design with randomly subchirp modulation. Relative to our previous work [6], this paper adaptively designs the OFDM chirp waveform with jointly randomly subchirp modulation, whereas in [6] the waveform is manually designed and consequently it is difficult to design over three orthogonal waveforms.

The remaining sections are organized as follows. The

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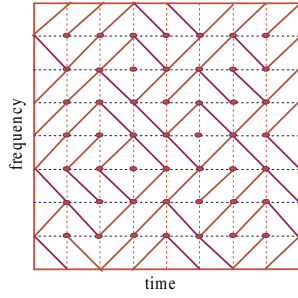


Fig. 1. Illustration of OFDM chirp basis.

OFDM chirp waveform with randomly subchirp modulation is proposed in Section 2, followed by the ambiguity function analysis and numerical results in Section 3. Finally, conclusions are drawn in Section 4.

2. OFDM CHIRP WAVEFORM DIVERSITY DESIGN

An important quantity in waveform diversity design is the relative level between the auto-correlation and cross-correlation of the waveforms. We have concluded in [8] that using the chirp waveforms occupy adjacent starting frequency with inverse chirp rate or nonoverlapping frequency bands can yield a satisfactory suppression of the cross-correlation component. Therefore, we use the chirp signals occupy adjacent starting frequency with inverse chirp rate or nonoverlapping frequency bands as the OFDM chirp basis. Figure 1 illustrates an example OFDM chirp basis, where the number of subchirps is only used for illustration purpose.

The hybrid-chirp basis signals are used by each subcarrier frequency interval at the same time. It can be represented by an $N_c N_s$ matrix. The N_c rows represents the subcarriers and the N_s columns are the chips. This matrix can also be viewed as an $N_c N_s$ -element chirp sequence, where each element has a duration T_b , demultiplexed onto one of the N_c subcarriers such that each subcarrier has N_s subchirp signals. The OFDM chirp pulse duration is determined as $T_p = N_s T_b$.

The cross-correlation between any two subcarriers during each subchirp duration is expressed as

$$\frac{1}{T_b} \int_0^{T_b} [\exp(j2\pi f_{m,n}t)] [\exp(j2\pi f_{m,n'}t)]^* dt \quad (1)$$

$$= \text{sinc}[(f_{m,n} - f_{m,n'})T_b]$$

where $*$ is the conjugate operator, $m = 1, 2, \dots, N_s, n = 1, 2, \dots, N_c, f_{m,n}$ is the subcarrier starting frequency for the m th chip and n th subcarrier, and n' is an integer different from n . To make the two subcarriers orthogonal in time, namely $\text{sinc}[\pi(f_{m,n} - f_{m,n'})T_b] = 0$, the subcarrier frequency $f_{m,n}$ should be separated by

$$f_{m,n} - f_{m,n'} = \frac{k}{T_b}, \quad k = 1, 2, \dots \quad (2)$$

where k is an arbitrary integer.

Correspondingly, the Fourier transform of each subcarrier over the duration of a subchirp is

$$\frac{1}{\sqrt{|k_{r_{m,n}}|}} \exp\left(j\frac{\pi}{4} \text{sgn}[k_{r_{m,n}}]\right) \exp\left(-j\pi \frac{(f - k/T_b)^2}{k_{r_{m,n}}}\right) \quad (3)$$

where $k_{r_{m,n}}$ is the chirp rate for the m th chip and n th subcarrier, $\text{sgn}[\cdot]$ is the signum function and f is the frequency variable. It is clear that (3) is different from the conventional OFDM spectrum:

$$T_b \text{sinc}\left[\left(f - \frac{k}{T_b}\right) T_b\right] \quad (4)$$

Since $\text{sinc}(fT_b - k)$ has a first null at the inverse of the signal duration T_b , the spectra of the subcarriers will be overlapped, but the subcarrier signals are mutually orthogonal.

For notation simplicity, we express the the OFDM chirp basis signals as a general matrix function:

$$\Phi = \begin{bmatrix} \phi(0,0) & \phi(0,1) & \dots & \phi(0,N_c-1) \\ \phi(1,0) & \phi(1,1) & \dots & \phi(1,N_c-1) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(N_s-1,0) & \phi(N_s-1,1) & \dots & \phi(N_s-1,N_c-1) \end{bmatrix} \quad (5)$$

where $m = 0, 1, \dots, N_s - 1, \phi(m, n), n = 0, 1, \dots, N_c - 1$, denotes the subchirp signal in the (m, n) entry of the OFDM chirp basis. The $\phi(m, n)$ can be expressed as

$$\phi(m, n) = u(t - mT_b) \exp(j2\pi f_{m,n}(t - mT_b)) \times \exp\left\{j\pi k_{r_{m,n}}(t - mT_b)^2\right\} \quad (6)$$

where is $k_{r_{m,n}}$ the chirp rate. Mathematically, the OFDM chirp basis of (5) can be rewritten as

$$s(t) = \text{sum}\{\Phi(\cdot)\} \quad (7)$$

where $\text{sum}\{\Phi(\cdot)\}$ is defined as the sum of all elements in Φ .

Orthogonal OFDM chirp waveforms are generated by using a random matrix to modulate the OFDM chirp basis as follows:

$$\mathbf{X} = \mathbf{R} \odot \Phi \quad (8)$$

where \mathbf{R} is an $M \times N$ random binary matrix, where each of its entries independently takes the value 0 or 1, and \odot is the Hadamard (element-wise) product. Here 1 means that the corresponding subchirp basis is chosen for the waveform while 0 means that the corresponding subchirp basis is not used in the waveform. The modulated OFDM chirp waveform can then be expressed as the sum of all elements of the matrix \mathbf{X} :

$$x(t) = \text{sum}\{\mathbf{X}(\cdot)\} \quad (9)$$

In doing so, multiple orthogonal OFDM chirp waveforms can be generated. The maximum allowable number of the OFDM chirp waveforms, L_{\max} , is

$$L_{\max} = N_c \quad (10)$$

To reduce the PAR of the OFDM chirp waveform in the frequency domain, we should minimize the difference of the sum of each column elements. That is,

$$\text{minimize } |\text{sum}\{\mathbf{R}(m_1, :)\} - \text{sum}\{\mathbf{R}(m_2, :)\}| \quad (11)$$

$$\{m_1, m_2\} \in [1, 2, \dots, N_s], m_1 \neq m_2$$

where $\text{sum}\{\mathbf{R}(m, :)\}$ is defined as the sum of the m th column elements of \mathbf{R} . Meanwhile, the PAR of the OFDM chirp waveform in time domain can be ensured by

$$\text{minimize } |\text{sum}\{\mathbf{R}(:, n_1)\} - \text{sum}\{\mathbf{R}(:, n_2)\}| \quad (12)$$

$$\{n_1, n_2\} \in [1, 2, \dots, N_c], n_1 \neq n_2$$

where $\text{sum}\{\mathbf{R}(:, n)\}$ is defined as the sum of the n th row elements of \mathbf{R} .

3. SIMULATION RESULTS

According to (2), $\Delta f = 50$ MHz, $T_b = 1 \mu s$ and $N_s = N_c = 8$ are designed for the OFDM hybrid-chirp basis. Assume that we want to design the OFDM chirp waveforms with bandwidth $B_r = 400$ MHz and pulse duration $T_p = 8 \mu s$. Figure 2 illustrates four possible implementations, where the waveforms *a1* and *a2* use only one basis signal in each subchirp duration and the waveforms *b1* and *b2* simultaneously use two basis signals in each subchirp duration.

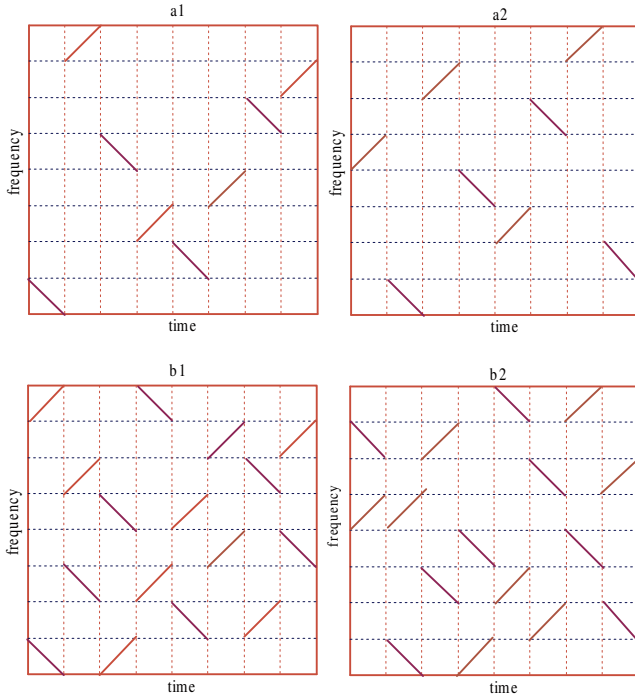


Fig. 2. Four OFDM chirp waveform examples.

Their corresponding time-domain (real-part of the complex signal) and frequency-domain representations are shown

in Figures 3 and 4, respectively. An almost constant time domain waveform modulus can be obtained for the complex OFDM chirp waveforms. That is to say, the waveforms have low PAR in time domain. It is also observed that the OFDM chirp waveforms have good PAR performance and almost constant modulus, which are desired for radio frequency (RF) hardware transmitter. This is rather different from standard OFDM waveforms that have poor PAR performance. Although the spectra are not uniform across the bandwidth like conventional chirp signal, the bandwidth is covered with no visible gaps. It can thus be concluded that the proposed method can design a large time-bandwidth product OFDM chirp waveform with good PAR performance.

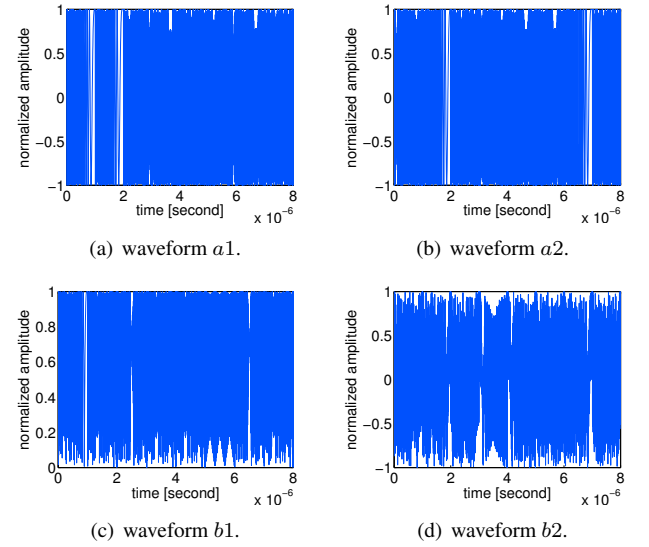


Fig. 3. Time-domain representations.

To further see what happens when the targets are moving, we have to correlate the emitted pulse with a copy shifted in range and Doppler frequency. This leads to the definition of the cross-ambiguity function between $x_k(t)$ and $x_{k'}(t)$

$$\chi_{x_k x_{k'}}(t_d, f_d) = \int x_k(t) x_{k'}^*(t - t_d) \exp(j2\pi f_d t) dt \quad (13)$$

where t_d is the time delay and f_d is the Doppler frequency. In particular, when $x_k(t) = x_{k'}(t)$, (13) is simplified to the self-ambiguity function:

$$\chi_{x_k x_k}(t_d, f_d) = \int x_k(t) x_k^*(t - t_d) \exp(j2\pi f_d t) dt \quad (14)$$

This ambiguity function evaluated at $(t_d, f_d) = (0, 0)$ is equal to the matched filtering output that is matched perfectly to the signal reflected from the target of interest.

Figure 5 compares the self-ambiguity functions (*a1* and *b1*) and cross-ambiguity functions of the OFDM chirp waveform pulses. Different from conventional chirp waveform

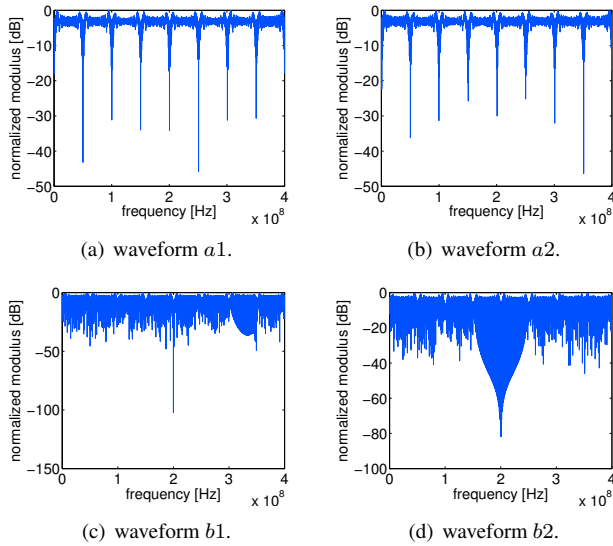


Fig. 4. Frequency-domain representations.

which is subject to range-Doppler coupling, the OFDM chirp waveforms have no range-Doppler coupling effects. Range-Doppler coupling will introduce an increment in time delay of the radar returns and this effect cannot be distinguished from the range changes. As a result, the OFDM chirp waveform has an advantage in tracking moving targets. As the value of $\chi_{x_k x_k}(0, 0)$ represents the matched filtering output without any mismatch errors, the sharper the function $\chi_{x_k x_k}(t_d, f_d)$, the better range resolution and azimuth (Doppler) resolution can be obtained for the radar system. Obviously, the designed OFDM chirp waveform has a satisfactory self-ambiguity function performance in range resolution and Doppler frequency resolution without range-Doppler coupling.

4. CONCLUSION

This paper proposes a low PAR OFDM chirp waveform diversity design for MIMO radars. The waveform performance is investigated by the ambiguity functions. Simulation results show that the designed waveform has the advantages of high range resolution, large time-bandwidth product, low PAR in both time and domains, and no range-Doppler coupling. More importantly, this method can easily yield over three orthogonal waveforms with a large time-bandwidth product.

5. REFERENCES

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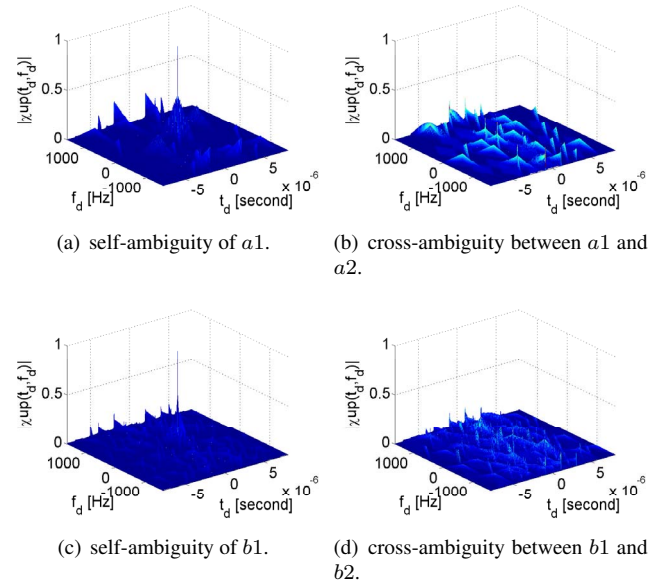


Fig. 5. Self-ambiguity and cross-ambiguity functions.

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