The chirp-based analog to information conversion in the LFM pulse compression radar

Peng-Fei Shi, Jian-Min Guo, Pin Lv and Hao Huan
School of Information and Electrics,
Beijing Institute of Technology, Beijing, China, 100081
e-mail: shipengfeibit@gmail.com, huanhao@bit.edu.cn

***Abstract*—The analog to information conversion (AIC) via random demodulation (RD) has suffered the performance loss in reducing the sampling rate of linear frequency modulation (LFM) pulse compression radar, especially for the local non-sparse targets in high resolution environment. In this paper, a novel AIC scheme via the determinate chirp mixing sequence is proposed to deal with this problem. The determinate chirp mixing sequence is given in two modes with fixed time duration and fixed bandwidth, respectively. Due to the spread spectrum as well as the local narrowband characteristics in undersampling condition, the determinate chirp mixing sequence guarantees the reconstruction performance of local non-sparse targets with low hardware consumption. Performance analysis and simulation results are provided to demonstrate the effectiveness of chirp mixing sequence in the restricted isometry property (RIP) and reconstruction performance of non-sparse targets for LFM pulse compression radar.**

***Index Terms*—Analog to information conversion, chirp mixing sequence, compressive sensing, LFM pulse compression radar.**

I. INTRODUCTION

The Analog to Information Conversion (AIC) via Random Demodulation (RD) has been proposed as a potential solution to overcome the resolution and performance limitations of high-speed Analogy to Digital Conversion(ADC) [1]. The idea was introduced to linear frequency modulation (LFM) pulse compression radar to reduce the sampling rate without reduction in waveform bandwidth [2]. However, in the receiver, the non-sparse targets in Fig. 1 destroy the reconstruction condition of compressive sensing, yielding the difficulty to use RD scheme in LFM pulse compression radar.

The Modulated Wideband Converter (MWC) was proposed as the generalized extension of RD scheme to deal with the broadband sparse signal [3][4]. This scheme is composed by a plurality of parallel RD structure which is efficient but complex in LFM pulse compression radar. To reduce the complexity of MWC, the compressive circulant matrix by cyclic shift of a special sequence with unit amplitude and random phase is proposed in [5]. However, a plurality of parallel RD structure is still vital for this scheme. Is it possible to reconstruct the local non-sparse targets with single RD scheme? The answer is given in the mixing sequence.

In the RD scheme, the action of mixing sequence is to guarantee the whole frequency domain information filled in the low frequency band as much as possible. However, the random PN sequence just makes the signal spectrum chaotic in the whole spectrum, rather than gathered in the low frequency band. The idea was first explored in the Nyquist Folder Receiver (NFR) [6], where the mixing sequence is generated by the non-uniform clock drivers to convert the frequency information of high-frequency signal into a low number of time-frequency information. However, the non-uniform clock drivers just limit its applications to some extent.

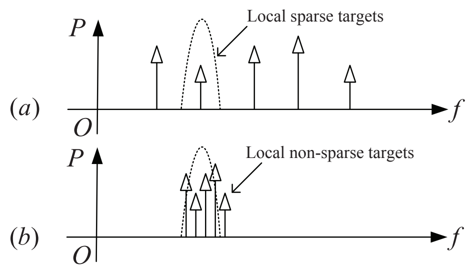

Fig. 1. The received targets after pulse compression: (a) local sparse targets; (b) local non-sparse targets.

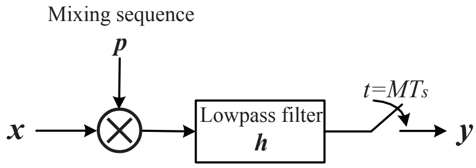

Fig. 2. The structure of analog to information conversion.

In this paper, a novel AIC scheme is proposed in LFM pulse compression radar, where the mixing sequence is the determinate chirp sequence rather than the pseudo-random PN sequence. Inspired by the structure of NFR, the chirp mixing sequence is designed in two modes with the same frequency response: one is the fixed time duration mode and the other is the fixed bandwidth duration mode. All of these methods are analyzed in local sparse targets and non-sparse targets, respectively.

II. THE CHIRP BASED AIC SCHEME

A. The AIC scheme via random demodulation

As shown in Fig. 2, the original signal $\boldsymbol{x} \in R^N$ has a K-sparse representation with respect to a basis $\boldsymbol{\Psi}$ if its transform $\boldsymbol{\alpha} \in R^N(\boldsymbol{x} = \boldsymbol{\Psi\alpha})$ contains at most K nonzero elements. Then, the signal is modulated by a pseudo-random PN sequence $\boldsymbol{p}$, so that it is not destroyed by the low-pass filter with the impulse response $\boldsymbol{h}$. Finally, the signal is sampled at rate of $1/M$ Nyquist sampling rate f_s, denoted as $\boldsymbol{y}$. The mathematical expression of the RD scheme is given as follows:

$$\boldsymbol{y} = \boldsymbol{HPx} = \boldsymbol{\Phi x} = \boldsymbol{A\alpha}, \tag{1}$$

where $A \in R^{\frac{N}{M}\times N} = \boldsymbol{\Phi\Psi} = \boldsymbol{HP\Psi}$ and $\boldsymbol{P}$ is a $N\times N$ matrix with the diagonal elements of $\boldsymbol{p}$ and $\boldsymbol{H}$ denotes the low pass filter with $1/M$ Nyquist sampling rate f_s that satisfies,

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{h} \end{bmatrix}_{\frac{N}{M}\times N} . \tag{2}$$

According to [2], the low-pass filter $\boldsymbol{h}$ is replaced with the integrator $\begin{bmatrix}1 & \cdots & 1\end{bmatrix}_{1\times M}$ for simplicity. Thus, the first step finds the sparse vector $\hat{\boldsymbol{\alpha}}$ by solving the following equation:

$$\min_{\hat{\boldsymbol{\alpha}}} \|\hat{\boldsymbol{\alpha}}\|_0 \qquad s.t. \quad \boldsymbol{A}\hat{\boldsymbol{\alpha}} = \boldsymbol{y}. \tag{3}$$

In order to solve the above l_0 problem, the restricted isometry property (RIP) was proposed in [7], where the matrix $\boldsymbol{A}$ satisfies RIP if there exists a restricted isometry constant $\delta_K, 0 < \delta_K < 1$, such that

$$(1-\delta_K)\|\boldsymbol{\alpha}\|_2^2 \le \|\boldsymbol{A\alpha}\|_2^2 \le (1+\delta_K)\|\boldsymbol{\alpha}\|_2^2 \tag{4}$$

However, this property is difficult to verify for a given matrix $\boldsymbol{A}$. Instead, another condition is the requirement that the measurement matrix $\boldsymbol{\Phi}$ must be incoherent with the sparsity basis $\boldsymbol{\Psi}$. This condition is easier to verify in practice than the RIP condition in [8]. The coherence μ between the two matrices is given as follows:

$$\mu(\boldsymbol{\Phi}, \boldsymbol{\Psi}) = \sqrt{N} \max_{i,j} \frac{|<\boldsymbol{\phi}_i, \boldsymbol{\psi}_j>|}{\|\boldsymbol{\phi}_i\|_2 \|\boldsymbol{\psi}_j\|_2} \tag{5}$$

where $\boldsymbol{\phi}_{i\in[1,\cdots,N/M]}$ and $\boldsymbol{\psi}_{j\in[1,\cdots,N]}$ represent the row vectors of $\boldsymbol{\Phi}$ and the column vectors of $\boldsymbol{\Psi}$, respectively. The coherence μ measures the maximum correlation between the two matrices. Since $\mu \in [1, \sqrt{N}]$, if $\mu(\boldsymbol{\Phi}, \boldsymbol{\Psi})$ is closer to 1, the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are incoherent.

B. The chirp mixing sequence

In traditional RD scheme, the pseudo-random PN sequence $\boldsymbol{p}$ must be alternated between values at or faster than the Nyquist frequency f_s of signal $\boldsymbol{x}$ to guarantee no distortion of the original signal's spectrum.

In the proposed AIC scheme, the chirp sequence is used to replace the pseudo-random PN sequence, where the mixing sequence is the determinate chirp sequence $\boldsymbol{c}$ rather than the random PN sequence $\boldsymbol{p}$ in Fig. 2. Compared with the pseudo-random PN sequence in (1), the chirp mixing sequence $\boldsymbol{c}$ is selected as the diagonal elements of $\boldsymbol{P}$,

$$\boldsymbol{c} = \exp(j\pi k\mu t^2), t \in [0, T_0] \tag{6}$$

where μ denotes the chirp rate and k is a controllable variable.

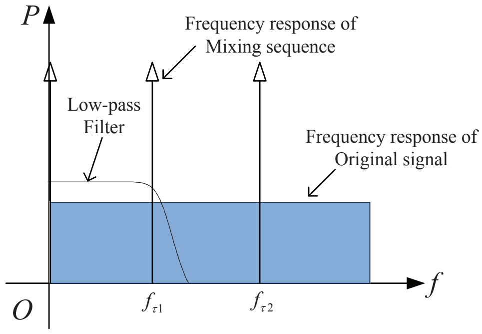

Fig. 3. Ideal frequency spectrum shift for low-pass filter.

III. THE SELECTION OF CHIRP MIXING SEQUENCE

In the AIC scheme, the purpose of the mixing sequence $\boldsymbol{p}$ is to guarantee the original signal $\boldsymbol{x}$ filled in low frequency band before the low-pass filter. The original signal $\boldsymbol{x}$ is multiplied by $\boldsymbol{p}$ in time domain, which is equivalent to the corresponding convolution in frequency domain.

According to the frequency convolution theorem, the convolution of the unit impulse response $\delta(\omega - \omega_\tau)$ with the frequency shift ω_τ and an arbitrary signal $X(\omega)$ is equivalent to the delay of the signal itself in the frequency domain,

$$X(\omega) * \delta(\omega - \omega_\tau) = X(\omega - \omega_\tau). \tag{7}$$

Since the original signal $\boldsymbol{x}$ is a wideband signal, the optimal choice for $\boldsymbol{x}$ should be a serial of unit impulse responses in Fig. 3. In frequency domain, different unit impulse response $\delta(\omega - \omega_\tau)$ shifts the corresponding bandwidth portion of $\boldsymbol{x}$ into the low-pass filter.

For the pseudo-random PN sequence is noise-like in frequency domain, it is difficult to shift high-frequency part of the signal $\boldsymbol{x}$ into the low-frequency band as respected. Meanwhile, the frequency-domain characteristic of undersampling chirp signal is approximately equivalent to a pulse train. Thus, the undersampling chirp signal is the so-called suitable mixing sequence in the AIC scheme.

Based on this conception, we propose two kinds of chirp mixing sequence: one is the same time duration with the pseudo-random PN sequence called fixed time duration mode; the other is the same bandwidth called fixed bandwidth mode. The detailed analysis is given in the LFM radar:

Let $\boldsymbol{x} = g_T(t)\exp(j\pi\mu t^2)$ be a chirp signal with a pulse repetition time (PRT) T_R, the sweep rate μ and bandwidth B, where $g_T(t)$ is the pulse shape function with the interval T.

A. Fixed time duration mode

For the fixed time duration mode, the received signal encoding with chirp mixing sequence is expressed as $\boldsymbol{x}' = \boldsymbol{xc} = \boldsymbol{x}\exp(j\pi k\mu t^2)$ with $t \in [0, T_R]$. In the receiver, the signal is sampled after the low-pass filter $\boldsymbol{h}$. Thus, the bandwidth B_{mixing} of the chirp mixing sequence $\boldsymbol{c}$ is decided by k,

$$B_{\text{mixing}} = k\mu \cdot T_R = \frac{kT_R}{T} \cdot B. \tag{8}$$

According to (8), the bandwidth of the chirp mixing sequence is larger than that of the original signal $\boldsymbol{x}$ if $k \geq T/T_R$. However, the condition $k \geq T/T_R$ only guarantees the completed reconstruction of targets with time delay less than kT_R. Thus, $k \geq 1$ is vital for the arbitrary targets to be reconstructed completely in the whole PRT T_R.

It is assumed that the sampling rate of $\boldsymbol{x}$ is $f_s = B$. Since the bandwidth of the chirp mixing signal $\boldsymbol{c}$ with fixed time duration mode is $kT_R/T \cdot B$, the sampling rate of $\boldsymbol{c}$ is in a state of undersampling. Thus, if $k \geq 1$, the frequency domain property of the chirp mixing sequence $\boldsymbol{c}$ with the sampling rate $f_s = B$ is a pulse train $\delta(\omega - \omega_\tau)$. If each part of original signal is included in the low-pass filter, the gap Δf between adjacent $\delta(\omega - \omega_\tau)$ should be less than the bandwidth of low-pass filter B/M. Since the mixing signal $\boldsymbol{xc}$ is sampled by $f_s = B$, then we have

$$\Delta f = \frac{1}{T_R} \cdot \frac{B_{\text{mixing}}}{f_s} = \frac{k}{T} \leq \frac{B}{M}. \tag{9}$$

B. Fixed bandwidth mode

In traditional RD scheme, the pseudo-random PN sequence $\boldsymbol{p}$ with the chip interval $1/B$ has the same bandwidth with the original signal $\boldsymbol{x}$. In the proposed AIC scheme via the chirp sequence, it is flexible to keep the same bandwidth by the chirp rate $\mu_k = k\mu$ and time duration T_{mixing},

$$T_{\text{mixing}} = \frac{B}{\mu_k} = \frac{T}{k}. \tag{10}$$

Then, similar to waveform of the Linear Frequency Modulation Continuous Wave (LFMCW) radar [9], the chirp mixing sequence is recycled across the whole PRT T_R, yielding $k \geq T/T_R$. Meanwhile, since the time-bandwidth product of original signal $\boldsymbol{x}$ is BT, the time-bandwidth product of the chirp signal BT_{mixing} should be no less than that of $\boldsymbol{x}$. Thus, the suitable range of value k is $k \geq 1$.

Since the time duration of the chirp mixing signal $\boldsymbol{c}$ with fixed bandwidth B is $T_{\text{mixing}} = T/k$, the gap Δf between adjacent $\delta(\omega - \omega_\tau)$ should be less than the bandwidth of low-pass filter B/M, given by

$$\Delta f = \frac{1}{T_{\text{mixing}}} = \frac{k}{T} \leq \frac{B}{M}, \tag{11}$$

Thus, the suitable range of value k for the chirp mixing sequence $\boldsymbol{c}$ is $1 \leq k \leq BT/M$.

IV. The Application of chirp mixing sequence

Since the reconstruction performance of AIC scheme is decided by both the distribution of received targets and the mixing sequence, the performance of the AIC scheme via chirp mixing sequence is analyzed in two kinds of received targets: one is the local sparse targets in Fig. 1(a); the other is the local non-sparse targets in Fig. 1(b).

TABLE I
RADAR SIGNAL SIMULATION.

Parameters	Values
Pulse Shape	Linear frequency modulation
Pulse Bandwidth	64 MHz
Pulse Duration	1 μs
Pulse Repetition Period	8 μs
Carrier Frequency	2 GHz
Sampling Frequency	16 MHz
Target Number	6
Channel Types	AWGN

A. Local sparse targets

The traditional AIC scheme via the pseudo-random PN sequence works well in the environments with the received local sparse targets in Fig. 1(a). If the received targets in $\boldsymbol{x}$ is stochastic, the received signal $\boldsymbol{x}' = \boldsymbol{xp}$ is also stochastic, which means that the randomness of the mixing sequence $\boldsymbol{p}$ is not vital. In this case, the determinate chirp mixing sequence is regarded as the similar spread spectrum sequence with the PN sequence. Thus, the chirp-based AIC scheme owns the similar performance with the AIC scheme via the pseudo-random PN or random Gaussian mixing sequence.

B. Local non-sparse targets

Since different targets are mapped to unique different time delay within the PRT T_R, the performance of AIC scheme is also decided by the number of local non-sparse targets in $\boldsymbol{x}$, together with the signal to noise rate (SNR) and mixing sequence. In traditional AIC scheme, the orthogonal matching pursuit (OMP) reconstruction algorithm is very sensitive with the local sparsity of received signal, which causes the difficulty in the local non-sparse targets in Fig. 1(b).

Fortunately, due to the local pulse characteristics of undersampling chirp signal in frequency domain, the chirp mixing sequence in Fig. 2 shifts the whole frequency domain information into the low-pass filter. Compared with the traditional random mixing sequence, such as pseudo-random PN and Gaussian sequence, the determinate chirp mixing sequence provides the controllable local narrow-band aliasing, which reduces the sensibility of reconstruction algorithm.

V. Numerical Simulations

In our simulations, we consider the received LFM pulse compression radar signal with multi-targets in zero-mean and unity variance white Gaussian noise. Two kinds of received multi-targets are given with the parameters depicted in table I: the random targets and the random continue targets. The comparison between chirp mixing sequence and traditional mixing sequence in AIC scheme is given in these cases. For each simulation, 10000 Monte-Carlo runs are performed.

In order to verify the RIP property of proposed chirp mixing sequence, Fig. 4 reports the obtained results for $N = 512$ and $M \in [1, 2, 4, 8]$ according to (5), where the measurement matrix $\boldsymbol{\Phi} = \boldsymbol{HP}$ is given in five kinds of mixing sequence:

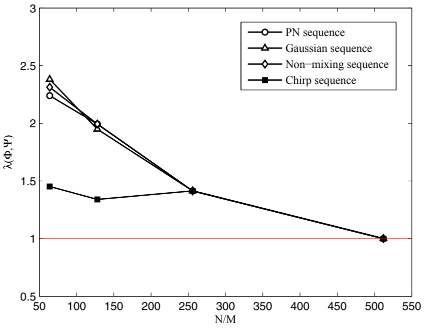

Fig. 4. Coherence between Φ and Ψ as a function of N/M, for $N = 512$ and $M \in [1, 2, 4, 8]$.

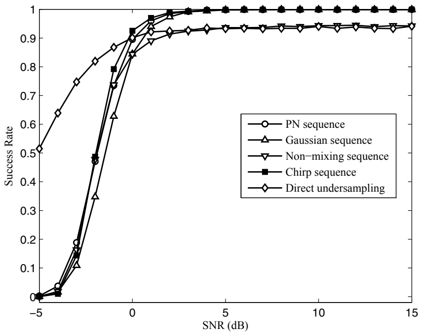

Fig. 5. Comparison of different mixing sequence for local sparse targets.

the pseudo-random PN sequence, the Gaussian sequence, the non-mixing sequence $[1, \cdots, 1]$ in [10], the chirp sequence and the direct undersampling sequence $[1, 0, \cdots, 0, 1, 0, \cdots]$. We obtain a lower μ with the chirp mixing sequence than that of the PN sequence, Gaussian sequence and non-mixing sequence, while it is higher than that of the direct undersampling sequence in $M = 4$. These results demonstrate that the chirp mixing sequence has low coherence between the measurement matrix $\mathbf{\Phi}$ and the sparse matrix $\mathbf{\Psi}$.

The reconstruction performance of AIC scheme via different mixing sequences are depicted in Fig. 5 with 6 local sparse targets and Fig. 6 with 6 local non-sparse targets. In Fig. 5, the AIC scheme via chirp sequence, pseudo-random PN sequence and Gaussian sequence share the similar reconstruction performance and reach the perfect reconstruction. Meanwhile, the non-mixing sequence and the direct undersampling sequence suffer the reconstruction bottleneck. If we consider the general 90% success rate, all of the methods work well, where the direct undersampling sequence is most efficient in low SNR.

The shortcoming of AIC scheme via the pseudo-random PN and Gaussian sequence is given in Fig. 6, where both of them suffer the reconstruction bottleneck. Meanwhile, the non-mixing sequence is failed to reconstruct the local non-sparse targets. Fortunately, the chirp sequence and the direct undersampling sequence could obtain the perfect reconstruction performance due to its particular impulse response in

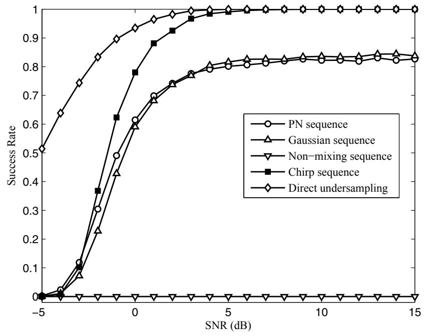

Fig. 6. Comparison of different mixing sequence for local non-sparse targets.

frequency domain. Thus, compared with the rest four kinds of mixing sequences, the chirp mixing sequence performs well in both the local sparse targets and non-sparse targets.

VI. Conclusion

In this paper, the deterministic chirp sequence is proposed to replace the pseudo-random PN and Gaussian sequence as the mixing sequence for the AIC scheme, which is used to deal with the local non-sparse targets in the LFM pulse compression radar. Compared with traditional mixing sequence, the chirp mixing sequence guarantee the reconstruction performance of both local sparse targets and non-sparse targets. Meanwhile, compared with the MWC scheme, the chirp-based AIC scheme is simple and practical with only one deterministic mixing sequence, rather than a serial of parallel AIC scheme via random demodulation.

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