

Simultaneous Radar Detection and Communications Performance with Clutter Mitigation

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Abstract—We analyze the performance of a joint radar-communications receiver performing target detection while simultaneously decoding a message from an in-band communications user. We assume that there is clutter in the environment and that the joint receiver performs basic clutter mitigation. Inner bounds on the performance of the joint radar-communications receiver are then formulated. Bounds on performance of the joint system are measured in terms of data information rate for communications and area under the receiver operating characteristic (ROC) curve for radar.

Index Terms—Joint Radar-Communications, Radar Information Theory, Detection, Clutter, Performance Bounds

I. INTRODUCTION

Radar users may increasingly be forced to coexist with in-band communications users as RF convergence in desirable bands continues to escalate [1]. The presence of one system is traditionally viewed as interference by the other and both systems' performances are degraded. However, cooperation or co-design approaches can potentially utilize the presence of radar and communications systems to mutually increase the performance of both systems. Estimation rate was previously derived to aid system designers by measuring radar estimation in information units for comparison with multi-access communications systems [1–3]. This derivation assumes detection has already occurred. In this paper, we discuss radar detection performance in the context of cooperative radar and communications. We derive bounds on the performance of a joint radar-communications system that is simultaneously trying to decode a communications message from an in-band user. The derivation presented in this paper is an extension of the communications water-filling inner bound discussed in Reference [1]. Joint radar-communications performance is measured in data rate for communications and area under the ROC curve for radar detection.

A. Background

In References [1–9], the estimation rate is defined and used to score a radar user against a communications user in the

same frequency band. This is beneficial for deriving joint performance bounds, as it measures and compares radar tracking estimation in information units with data rate limitations for the communications users on the same scale of spectral efficiency: bits/second/Hz. However, these works assume the detection process has already taken place. While Reference [4] incorporates a binary detection distribution, the effect of communications is ignored. The estimation rate accounts for the presence of in-band communications systems to formulate inner bounds on joint performance, but only after assuming detection is complete [1].

References [10, 11] investigate the application of information theory to improve radar system performance. In those works, the premise that maximizing signal-to-noise ratio (SNR) does not, in general, maximize information was introduced. Using information theory, a new type of receiver, the *a posteriori* radar receiver, is developed that maximizes the quantity of information given by the *a posteriori* distribution of a target parameter.

In Reference [12], waveform optimization for detection and target information extraction are considered. The waveform was designed to maximize the mutual information between the target parameter and measurements obtained from the receiver. The maximization of this mutual information was shown to improve the radar system performance in terms of target classification ability or average probability of error. Matched illumination is used in References [13, 14] to design a target detection system that maximizes the target SNR (which corresponds to maximizing the probability of detection in gaussian noise and interference). Reference [15] uses information theory to develop an expression for radar capacity which, in combination with traditional communications capacity, can be used to measure the total capacity of a joint radar-communications network. A Neyman-Pearson based cooperative metric is developed in Reference [16] that captures both radar detection performance and communications data rate in a joint cost metric. Reference [17] considers a spectrum sharing scheme for radar and communications systems which utilize the same orthogonal frequency-division multiplexing (OFDM) waveform for transmission (communications) and environment illumination (radar). A communications capacity vs radar detection probability performance bound is also provided. Finally, Reference [18] investigates utilizing the energy from communications users to improve a radar user's probability of detection. In this scenario, the radar waveform is optimized to function with the in-band communications system operating as the primary user.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Office of Naval Research or the U.S. Government.

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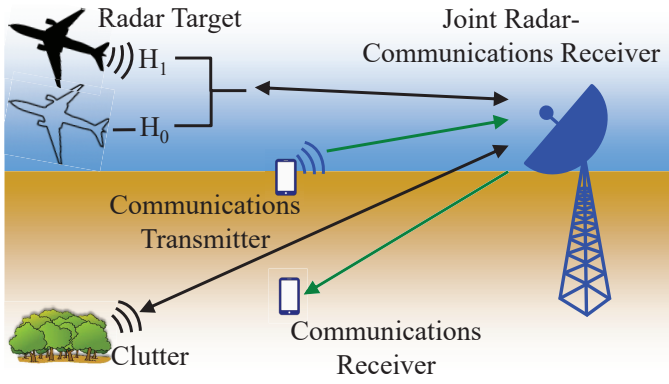


Fig. 1. Top level diagram of problem description. Environment contains radar clutter, an in-band communications user and, possibly a radar target. The joint radar-communications receiver is capable of simultaneous target detection and communications decoding. A target is illuminated along with clutter in the alternative hypothesis. If this is true, the reflected energy arrives at the joint radar-communications receiver. Otherwise, only clutter energy is seen at the joint radar-communications receiver. A communications signal is input to the system through the antenna sidelobe. The superposition of the radar signal and the communications signal is perturbed by thermal receiver noise and clutter.

B. Contributions

We consider a scenario in which a communications user operates while an in-band radar user searches for a potentially nonexistent target. We assume that clutter is present, and that the clutter scatterers are static over some N_p radar pulses and have no intrinsic clutter motion (ICM). The clutter can be estimated in the absence of a radar target during the first coherent processing interval (CPI) (N_p pulses). These estimates can then be used to mitigate clutter in the next CPI, in which a target may or may not be present. This estimation and mitigation of clutter is performed via one-dimensional coherent change detection [19], which assumes that the clutter estimates from the previous CPI is a good estimate for clutter in the current CPI.

In this paper, we assume the joint radar-communications receiver can simultaneously perform radar target detection and decode a communications signal. The inner bounds on performance are found by considering this ideal receiver in different scenarios and deriving performance bounds on radar and communications systems respectively. The simulation scenario considered in this paper is depicted in Figure 1.

The principle contributions of this paper are as follows:

- Establish the joint access radar detection problem
- Extend previously derived bounds on performance to account for radar detection
- Present examples of joint bounds in terms of communications data rate and area under radar detection ROC curves

This paper is organized as follows. In Section II, we characterize the clutter model and discuss clutter mitigation. A table of notation is also provided. In Section III, we derive bounds on the performance of the joint radar-communications system. We also discuss successive interference cancellation (SIC), a technique which enables simultaneous target detection

and communications decoding. In Section IV, we present examples of these performance bounds evaluated over a set of parameters. Finally, in Section V, we present a summary of results obtained in this paper.

II. CLUTTER MODELING AND MITIGATION

In this section, we present the model used to characterize clutter and the techniques used to mitigate clutter. We assume that there are multiple resolvable clutter scatterers. We first look at clutter in each range cell (post-matched filtering for radar) and then calculate the total pre-matched filtering clutter return observed by the joint radar-communications system receiver. The residual clutter power remaining after clutter mitigation is assumed to be Gaussian and is treated as an additional additive noise term. The clutter model and associated processing techniques used here were first discussed in Reference [7]. A table of significant notation employed in this paper is provided in Table I.

TABLE I
SURVEY OF NOTATION

Variable	Description
$\langle \cdot \rangle$	Expectation
$\ \cdot \ $	L2-norm or absolute value
B	Full bandwidth of the system
B_{rms}	Root-mean-squared radar bandwidth
$x(t)$	Unit-variance transmitted radar signal
P_{rad}	Radar power
τ_m	Time delay to m^{th} scatterer
a_m	Complex combined antenna, cross-section, and propagation gain for m^{th} scatterer
T	Radar pulse duration
δ	Radar duty factor
γ	Radar spectral shape parameter
σ_{resi}^2	Clutter residual power
$r(t)$	Unit-variance transmitted communications signal
P_{com}	Total communications power
b	Combined antenna gain and communications propagation loss (amplitude)
$n(t)$	Receiver thermal noise
σ_{noise}^2	Thermal noise power
k_B	Boltzmann constant
T_{temp}	Absolute temperature
$\sigma_{int+n+resi}^2$	Interference plus thermal noise plus clutter residual for communications receiver
$\sigma_{\tau,proc}^2$	Variance of range fluctuation process
α	Bandwidth splitting parameter for sub-band splitting
β	Power fraction used by communications only sub-band

A. Modeling and Mitigation

We assume that each clutter scatterer is static (with no ICM) over some N_p radar pulses such that the radar return over this time period is highly correlated (approximately constant). With these assumptions, we resolve the scatterers into appropriate range cells and perform maximum likelihood amplitude estimation for clutter returns from the first CPI. Since the clutter is static over a certain number of pulses, we can perform estimation over multiple pulses, obtaining very accurate clutter estimates. We assume that the estimation performance achieves the Cramér-Rao lower bound (CRLB) for amplitude estimation.

These range and amplitude estimates of clutter are used to mitigate the clutter return in the next CPI. Since we assume

that clutter estimation achieves the CRLB, the clutter residual in each range cell has power given by the CRLB for amplitude estimation which, for noise power $\sigma_{\text{noise}}^2 = k_B T_{\text{temp}} B$, is given by [7, 20],

$$\sigma_{\text{resi}}^2 = \frac{k_B T_{\text{temp}} B}{n_s} = \frac{k_B T_{\text{temp}} B}{N_p T B}, \quad (1)$$

where n_s is the total number of independent samples in the period of integration. As seen in References [7, 21], this minimum error (residual) is a zero mean complex Gaussian, $n_{\text{resi}}(t)$, with variance given by Equation (1).

Matched filtering doesn't change the complex Gaussian nature of the clutter residual but a factor of $n_s = N_p T B$ is applied to the variance. The radar return signal after clutter mitigation for L clutter scatterers is

$$z(t) = \sum_{m=1}^L \frac{1}{\sqrt{N_p T B}} n_{\text{resi}}(t) + n(t). \quad (2)$$

Thus, $z(t) \sim \mathcal{CN}(0, \sigma_{n+\text{resi}}^2)$, where $\sigma_{n+\text{resi}}^2 = \sigma_{\text{noise}}^2 + \frac{L \sigma_{\text{resi}}^2}{N_p T B}$.

Performing clutter mitigation in this manner requires that the clutter estimates obtained from the previous CPI remain good estimates for the current CPI, which is an optimistic assumption. If less accurate clutter estimates are used, the resulting clutter residual is larger and the system performance is further degraded.

The estimation and mitigation of clutter can be performed using one-dimensional coherent change detection [19], which ascertains when the estimation and mitigation CPIs begin and end. Clutter is then mitigated by subtracting the total power of the first CPI from the second.

1) *Importance of Clutter:* Inclusion of clutter is necessary because of a subtlety in the problem formulation. Because both systems share the same spectrum allocation, each system pushes for more spectral access. For a communications system, an increase in spectral access directly improves performance by increasing the data rate. For a radar system, increased spectral access improves radar estimation but not radar detection. For example, a radar waveform with an impulse-like spectrum can perform target detection, but has poor ranging resolution and degrades time-delay estimation performance (which traditionally succeeds radar detection).

The introduction of clutter couples radar detection performance with spectral allocation. By increasing spectral allocation for the radar system, ranging resolution is improved, resulting in smaller range cells. As a result of smaller range cells, the number of scatterers in the target range cell reduces, thereby reducing the clutter residual power after mitigation and improving radar detection performance.

III. JOINT RADAR-COMMUNICATIONS PERFORMANCE

In this section, we investigate joint radar-communications system performance and discuss SIC. We show how radar detection performance is measured and derive false alarm and detection probabilities for the joint radar-communications system. A derivation of the communications water-filling inner bound on joint system performance is also provided.

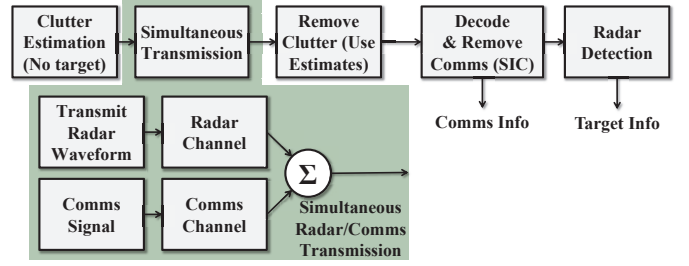


Fig. 2. Joint radar-communications system block diagram showcasing clutter mitigation and simultaneous communications signal decoding and target detection. Since the communications system can be adversely affected by clutter, clutter mitigation has to be done before any communications signal decoding. Additionally, the clutter cancellation residual further degrades the channel, adversely impacting both the communications and radar performances.

A. Successive Interference Cancellation (SIC)

At the receiver, we employ a process called SIC [1], an algorithm that takes advantage of the target tracking ability of the joint radar-communications system to ensure that communications signal decoding and radar detection can be done cooperatively. We assume that the target range is known to within some process variation (target tracking) and a predicted radar return is generated using this knowledge of the target. The predicted return is then subtracted from the received signal, and the resultant signal is then used to decode the communications signal and similarly remove it. Radar target estimation can then be done without communications interference. The process variation can be modeled as a zero-mean Gaussian distribution $n_{\tau_m, \text{proc}}$ with variance $\sigma_{\tau_m, \text{proc}}^2 = \langle \|n_{\tau_m, \text{proc}}\|^2 \rangle$. [1, 2].

By employing SIC at the receiver, the received signal is corrupted by thermal noise as well as returns from all L clutter scatterers, which has an adverse effect on communications performance. Hence, we have to perform clutter mitigation first. In doing so, communications performance is affected by the total clutter residual. However, for radar detection performance, since matched filtering is already done and the environment has been resolved into range cells, the radar will only see clutter residue from the current range cell in which target detection is being performed. A block diagram of the joint radar-communications system in this scenario is shown in Figure 2.

For N targets and L clutter scatterers, a communications receiver employing SIC has an interference plus noise plus clutter residual variance given by [1, 2]

$$\sigma_{\text{int}+\text{n}+\text{resi}}^2 = P_{\text{rad}} \left(\sum_{m=1}^N \|a_m\|^2 (2\pi)^2 B_{\text{rms}}^2 \sigma_{\tau_m, \text{proc}}^2 \right) + \sigma_{\text{noise}}^2 + \frac{L \sigma_{\text{resi}}^2}{N_p T B}, \quad (3)$$

where B_{rms} results from Parseval's theorem [20] and is extracted from bandwidth B as $\gamma^2 B^2 = (2\pi)^2 B_{\text{rms}}^2$. γ is the scaling constant between B and B_{rms} times 2π [1].

B. Receiver Operating Characteristic Curves for Radar Detection

The ROC curves for radar detection in a joint radar-communications system are derived for the aforementioned scenario. Since SIC is employed at the receiver, the received signal, $z(t)$, contains only the radar return and thermal noise. We perform matched filtering and measure the total energy in each range cell for each CPI. Assuming H_0 is the null hypothesis,

$$\begin{aligned} H_0 : z(t) &= n_{\text{resi}}(t) + n(t) \\ H_1 : z(t) &= a \sqrt{N_p T B P_{\text{rad}}} x(t - \tau) + n_{\text{resi}}(t) + n(t), \end{aligned} \quad (4)$$

where $\sqrt{N_p T B}$ is present due to the coherent integration factor from matched filtering. Equation (4) implies that the radar return energy from a range cell is drawn from either a complex central chi-squared distribution of one degree under H_0 ($P_{\|z(t)\|^2|H_0}(q) = P_{\chi^2}^C(q, N_p, \sigma_{\text{noise}}^2 + \frac{\sigma_{\text{resi}}^2}{N_p T B})$) or a complex non-central chi-squared distribution of one degree under H_1 ($P_{\|z(t)\|^2|H_1}(q) = P_{\chi_{\text{nc}}^2}^C(q, \sigma_{\text{noise}}^2 + \frac{\sigma_{\text{resi}}^2}{N_p T B}, N_p T B a^2 P_{\text{rad}})$) [20, 22].

Hence, at detection threshold η , the resulting false-alarm probability is given by [20]

$$P_{\text{FA}} = 1 - \frac{1}{\Gamma(N_p)} \gamma \left(N_p, \frac{\eta}{\sigma_{\text{noise}}^2 + \frac{\sigma_{\text{resi}}^2}{N_p T B}} \right), \quad (5)$$

and the detection probability is given by [20]

$$P_{\text{D}} = Q_1 \left(\sqrt{\frac{2 N_p T B a^2 P_{\text{rad}}}{\sigma_{\text{noise}}^2 + \frac{\sigma_{\text{resi}}^2}{N_p T B}}}, \sqrt{\frac{2 \eta}{\sigma_{\text{noise}}^2 + \frac{\sigma_{\text{resi}}^2}{N_p T B}}} \right), \quad (6)$$

where $Q_1(\cdot, \cdot)$ is the Marcum Q-function [20].

C. Communications Water-filling Bound

An inner bound is derived by considering a scenario in which we partition the total bandwidth according to some α into two sub-bands; one sub-band for communications only and the other sub-band for both radar and communications. The water-filling algorithm is used to distribute the total communications power between the two sub-bands [1]. Water-filling optimizes the power and rate allocation between multiple channels [20, 23].

Given some bandwidth separation α ,

$$B = B_{\text{com}} + B_{\text{mix}}, B_{\text{com}} = \alpha B, B_{\text{mix}} = (1 - \alpha) B, \quad (7)$$

water-filling [23] is utilized to optimize the communications power utilization, β , between sub-bands. For an optimized power allocation parameter β , the communications only rate bound is given by [2, 7]

$$R_{\text{CO}} \leq \alpha B \log_2 \left[1 + \frac{\beta P_{\text{com}} b^2}{k_B T_{\text{temp}} \alpha B} \right], \quad (8)$$

and the mixed use communications rate bound is given by [2, 7]

$$R_{\text{MU}} \leq (1 - \alpha) B \log_2 \left[1 + \frac{b^2 (1 - \beta) P_{\text{com}}}{\sigma_{\text{int+n}}^2} \right], \quad (9)$$

where $\sigma_{\text{int+n}}^2$ is given by Equation (3).

The subsequent false alarm probability and probability of detection are given by Equations (5) and (6)

IV. EXAMPLES

In this section, through Figures 3 and 4, we present example performance bounds by evaluating the results of Section III for a set of example parameters. The parameter set used in this section is displayed in Table II.

TABLE II
PARAMETERS FOR EXAMPLE PERFORMANCE BOUND

Parameter	Value
Bandwidth	5 MHz
Center Frequency	3 GHz
Effective Temperature	1000 K
Communications Range	10 km
Communications Power	100 W
Communications Antenna Gain	0 dBi
Communications Receiver Sidelobe Gain	10 dBi
Radar Target Range	80 km
Radar Antenna Gain	30 dBi
Radar Power	1000 W
Target Cross Section	10 m ²
Target Process Standard Deviation	100 m
Time-Bandwidth Product	100
Number of Clutter Scatterers	10000
Total Pulses Integrated	10, 5, 1

The evaluated communications water-filling bound on performance is shown in Figure 3 for different values of number of radar pulses integrated (N_p). The communications performance, measured by data rate, and the detection performance, measured by area under the ROC curves, are compared against each other as the bandwidth splitting parameter α is swept from 0 to 1. For values of α closer to 1, more bandwidth is allocated for communications only, resulting in improved communications performance and lower radar detection performance. When α is 1, the radar does not have any spectral allocation and cannot illuminate the environment, resulting in the detector making random guesses for target detection. This is highlighted by an area under the ROC curve of 0.5, which is characteristic of a random guess detector, when α is 1. Similarly, for values of α closer to 0, more bandwidth is allocated for mixed use, resulting in improved detection performance and communications performance approaching the SIC communications rate. Additionally, target detection performance also depends on the number of integrated radar pulses, N_p . An increase in N_p results in improved target detection performance, characterized by an area under the ROC curve closer to 1, as seen in Figure 3. The sample evaluation presented in this section uses a simplistic model for clutter. Under more complicated clutter models, the clutter residual is larger and adversely affects communications performance. As shown in Reference [7], introducing a small amount of

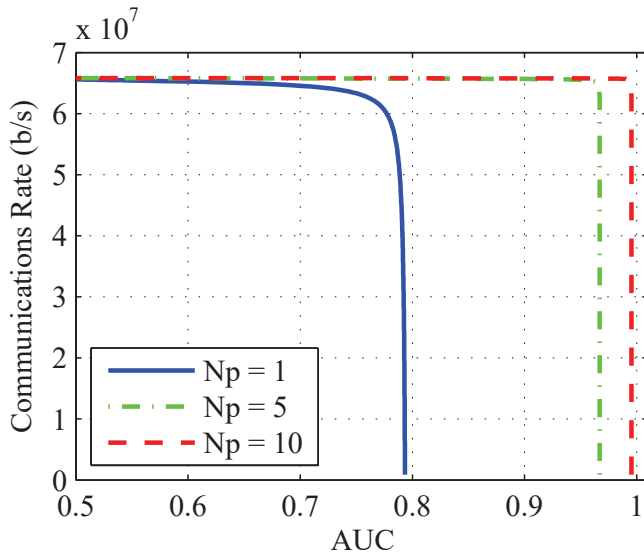


Fig. 3. The evaluated communications water-filling bound on performance is shown. The figure shows the communications data rate and detection area under the ROC curves as α , the bandwidth splitting parameter, is swept from 0 to 1. We see that as α increases, more bandwidth is assigned to communications only, hence communications performance increases and radar performance decreases. The opposite effect on bandwidth happens as α

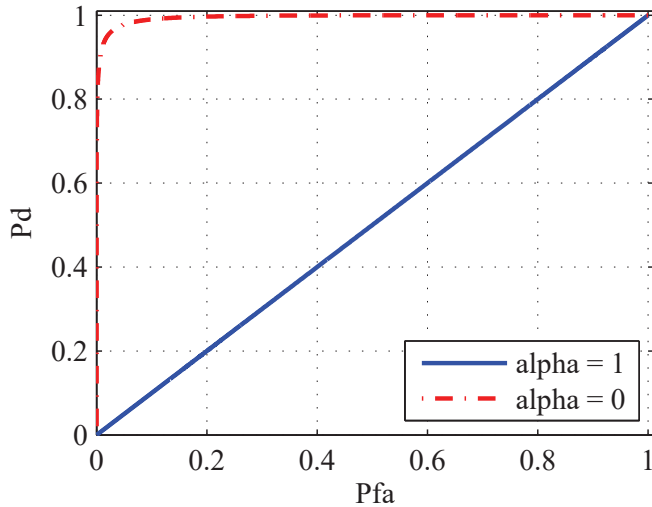


Fig. 4. Detection performance for different values of α and $N_p = 10$. Detection performance here is highlighted through ROC curves for different values of α . As expected, we see that detection performance improves as α moves from 1 to 0. When α is 1, the radar cannot illuminate the environment and the detector has to make random guesses for target detection, resulting in a ROC curve given by a straight line, which is characteristic of a random guess detector.

ICM has a significant negative impact on communications performance.

In Figure 4, we more closely evaluate the relationship between detection performance and spectral allocation assigned for mixed use (i.e. spectrum allocated for radar). In this figure, detection performance is captured via ROC curves. As expected, the detection performance improves as α moves from 1 to 0. When α is 1, the radar cannot illuminate the environment and the detector has to make random guesses for target detection, resulting in a ROC curve given by a straight line, which is characteristic of a random guess detector.

V. CONCLUSION

In this paper, we investigate the radar detection performance for a system employing a cooperative radar and communications signaling scheme. We assume that static clutter is present in the environment and employ clutter mitigation at the joint radar-communications system receiver. We then extend the previously derived communications water-filling bound on joint system performance to include radar detection and evaluate this bound for a set of example parameters. Communications performance is measured in data rate and radar detection performance is measured in area under the ROC curve.

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