An Enhanced TKM-TR Method for PAPR Reduction of OFDM Signals with Peak Regrowth and Peak Residual Reduced

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***Abstract*—In this paper, the authors propose an enhanced TKM-TR method to reduce the peak-to-average power ratio (PAPR) of the OFDM systems. The proposed method uses distinct sign vectors to generate a set of different scaling vectors in the TKM-TR, thus alternative OFDM signals are generated and the one with the lowest PAPR is transmitted. Compared to the original method, simulation results show that the peak regrowth and peak residual are fairly reduced and the proposed method yields more PAPR reduction with a slightly computational complexity reduced in the simulations.**

Keywords-orthogonal frequency division multiplexing (OFDM); peak-to-average power ratio (PAPR); TKM-TR; peak regrowth; peak residual

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has drawn much attention in today's communication systems, due to its high spectral efficiency, flexible implementation and robustness to multipath fading. However, a major problem of OFDM signal is its high peak-to-average power ratio (PAPR), the high PAPR requires a large power back-off for the signal sending to the power amplifier (PA), which leads to great power inefficiency [1].

In recent years, there are different kinds of methods proposed to mitigate the PAPR [2], in which, the tone reservation (TR) scheme is one of the most popular technique. The model of TR scheme was proposed by Tellado, which can be formulated as a quadratically constrained quadratic program (QCQP) optimization program. To solve the QCQP problem with low complexity, Tellado proposed the SCR-TR by adopting gradient algorithm [3], however, the SCR-TR's convergence speed is slow. Then, many improved TR schemes are developed to accelerate the convergence speed and reduce the computational complexity [4]-[7], among these, the TKM-TR proposed in [7] is a high performance and low complexity scheme, which constructs the time-domain kernel matrix by applying multiple time-domain kernels in one iteration, and then generates the peak reduction signals. But it suffers peak residual and peak regrowth, which are the common problem of most TR schemes, thus causing some performance degradation.

In this paper, we proposed a performance enhanced TKM-TR by generating alternative OFDM signals and choosing the one with the lowest PAPR, which reduces the peak regrowth and peak residual. The following part is organized as follows. The model of the OFDM system and the tone reservation are reviewed in Section II. Section III presents the proposed the enhanced TKM-TR method. And followed the simulation analysis in Section IV. In Section V, the conclusion is given.

II. SYSTEM MODEL

For N sub-carriers, the OFDM signal x_n can be generated as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/LN},\ n = 0,1,\dots,LN-1 \tag{1}$$

where $\boldsymbol{X} = \left[X_0, X_1, \dots, X_{N-1}\right]^T$ is the input data symbols, L is the oversampling rate, and $L \geq 4$ is required to get an accurate approximation of the continuous-time PAPR [8].

The PAPR of the OFDM signal is obtained as

$$\mathrm{PAPR} = \frac{\max\limits_{0\leq n\leq \mathrm{LN}-1} \left|x_n\right|^2}{E\left[\left|x_n\right|^2\right]} \tag{2}$$

The tone reservation reserves N_r sub-carriers as the peak reduction tones (PRT) to carry the frequency-domain peak reduction signal $\boldsymbol{C}$, and the data symbols are loaded on the rest $N - N_r$ sub-carriers.

Then the time-domain OFDM signal with TR technique applied is

$$\hat{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{c} = IDFT\left(\boldsymbol{X} + \boldsymbol{C}\right) \tag{3}$$

where $\boldsymbol{c}$ is the corresponding time-domain peak reduction signal.

And the $\boldsymbol{C}$ is reserved just on the PRT set, i.e.,

$$X_k + C_k = \begin{cases} X_k, & k \in \mathcal{R}^c \\ C_k, & k \in \mathcal{R} \end{cases} \tag{4}$$

where $\mathcal{R} = \left\{\mathrm{i}_0, i_1, \dots, i_{N_r-1}\right\}$ is the position of the PRT set and $\mathcal{R}^c$ is the corresponding complementary set.

Finding an optimal peak reduction signal $\boldsymbol{c}$ or $\boldsymbol{C}$ is a QCQP problem and computational complexity to find the optimal solution is proved to be $O\left(N_r LN^2\right)$ in [3], which is highly computational consumption, so in general, some sub-optimal algorithms are proposed to solve the problem.

III. PROPOSED ENHANCED-TKM-TR SCHEME

A. The TKM-TR Scheme

The TKM-TR scheme is a low complexity sub-optimal iterative clipping and compensating technique. It exploits the time-domain kernel to construct time-domain kernel matrix, then the matrix is scaled by a scaling vector, which is generated by clipping the signal and solving a modified optimization problem, and the peak reduction signals for the OFDM frame are obtained.

The time-domain kernel is defined as

$$\boldsymbol{p} = \left[p_0, p_1, \ldots, p_{LN-1} \right]^T = IDFT\left(\boldsymbol{P}\right) \tag{5}$$

where $\boldsymbol{P} = \left[P_0, P_1, ..., P_{N-1} \right]^T$ is the frequency-domain kernel, which is

$$P_k = \begin{cases} 0, & k \in \mathcal{R}^c \\ 1, & k \in \mathcal{R} \end{cases} \tag{6}$$

The TKM-TR scheme firstly sets a clipping ratio CR, then gets the clipping threshold $A = \sqrt{CR \cdot \sigma^2}$ to the input OFDM signals, where σ^2 is the average power of the input OFDM signal, then the clipping noise is

$$f_n = \begin{cases} x_n - Ae^{j\arg(x_n)}, & |x_n| > A \\ 0, & |x_n| \le A \end{cases} \tag{7}$$

The peak reduced signal of TKM-TR is iteratively updated as

$$\hat{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{M}_P \boldsymbol{\beta} \tag{8}$$

where $\boldsymbol{M}_P = \left[\boldsymbol{p}_{s_0}, \boldsymbol{p}_{s_1}, ..., \boldsymbol{p}_{s_{M-1}} \right]_{LN \times M}$ is time-domain kernel matrix, which is constructed by circularly shifting $\boldsymbol{p}$ to the right by the clipping noise positions $\mathcal{S} = \left\{ s_0, s_1, ..., s_{M-1} \right\}$, and s_i is the location where $\left| f_n \right| > 0$. $\boldsymbol{\beta} = \left[\beta_0, \beta_1, ..., \beta_{M-1} \right]^T$ is the scaling vector for each column of $\boldsymbol{M}_P$, and $\beta_i = \left| f_{s_i} \right| / amp_p$, with $amp_p = \max\left(\left| p_0, p_1, \ldots, p_{LN-1} \right| \right)$.

For the peak signal at location s_i, The scaling vector β_i can be explicated as scaling the corresponding time-domain kernel to the same amplitude as the clipping noise.

The TKM-TR only ensures the peak reduction at the locations $\mathcal{S}$, but inter-kernel interference and the influence on other sample locations where the amplitude below A are not taken into consideration, which would cause peak residual at $\mathcal{S}$ and peak regrowth at other sample locations.

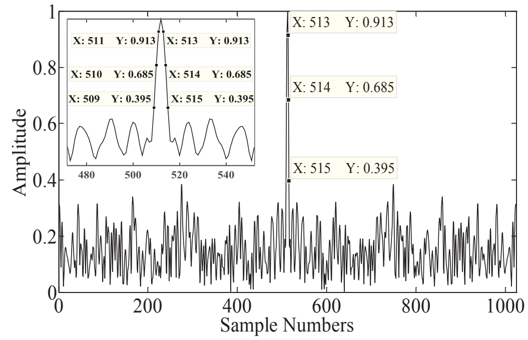

Figure 1. The 512 samples shifted time-domain kernel with 32 randomly reserved tones in total 256 sub-carriers and 4 times oversampling.

B. Proposed Scheme

Fig. 1 shows the amplitude of the time-domain kernel right circularly shifted for 512 samples with 32 randomly reserved tones in total 256 sub-carriers with 4 times oversampling. The kernel is axial symmetry by its axis main pulse, and by numerical calculations, we can observe that the amplitude of the first, second and third side-pulse is $sd_1 = 0.913$, $sd_2 = 0.685$, and $sd_3 = 0.395$, respectively, when we normalized the amplitude of the main pulse to 1.

In each iteration, the more the kernels are, the higher the probability of peak residual and peak regrowth is. Moreover, the consecutive peak locations contribute more to these effects. From the observation of the time-domain kernel and the time-domain OFDM signal, we get the result that for the consecutive peak locations $s_{i_1}, s_{i_2}, ..., s_{i_n}$ $\left(n = 2,3,4\right)$ with the corresponding scaling factors $\beta_{i_1}, \beta_{i_2}, ..., \beta_{i_n}$ where $\max\left(\beta_{i_1}, \beta_{i_2}, ..., \beta_{i_n}\right) = \beta_j$, when the following rules are satisfied, the corresponding scaling factors can be set to be 0, i.e., the time-domain kernel at these locations can be omitted and the peak reduction signals at these locations are generated by some other kernels.

Rule 1: if $\beta_j / \beta_{j\pm1} > sd_1$, then $\beta_{j\pm1} = 0$;

Rule 2: if $\beta_j / \beta_{j\pm2} > sd_2$, then $\beta_{j\pm2} = 0$;

Rule 3: if $\beta_j / \beta_{j\pm3} > sd_3$, then $\beta_{j\pm3} = 0$;

Through the simulation of TKM-TR when 10^5 OFDM blocks are generated using $N = 256$ with 32 reserved tones, the oversampling rate $L = 4$, and 4 iterations, we find that the probability of consecutive peak locations is high, the average probability that the signal satisfies the Rules 1~3 in each iteration is 0.34.

From the analysis above, we can get the conclusion that for the peak location $\mathcal{S}$ with M entries, the peak reduction signals can be generated by less than M time-domain kernels, not exactly M in the TKM-TR method.

Since the location of the peak and the peak regrowth of the OFDM signals are randomly distributed, we propose an enhanced-TKM-TR method from the conclusion above. The main idea of the enhanced-TKM-TR is as follows.

Define V distinct sign vector $\boldsymbol{E}^{(v)} = \left[E_0^{(v)}, E_1^{(v)}, ..., E_{M-1}^{(v)} \right]$ with $E_i^{(v)} = \{0,1\}$, $i = 0,1,...,M-1$, $v = 0,1,...,V-1$, and $Prob\left(E_i^{(v)} = 0\right) = \alpha$. In each iteration in the TKM-TR, the scaling vector $\boldsymbol{\beta}$ is multiplied with V vectors $E^{(v)}$, resulting in a set of different scaling vectors with components

$$\beta_i^{(v)} = \beta_i \cdot E_i^{(v)},\ i = 0,1,...\mathrm{M}-1,\ v = 0,1,...,V-1 \tag{9}$$

Then, V different peak reduced signals are generated as (10) and the one with the lowest PAPR is selected for transmission.

$$\begin{aligned} \hat{\boldsymbol{x}}^{(v)} &= \boldsymbol{x} - \boldsymbol{M}_P \boldsymbol{\beta}^{(v)} \\ &= \boldsymbol{x} - \left[\boldsymbol{p}_{s_0}, \boldsymbol{p}_{s_1}, ..., \boldsymbol{p}_{s_{M-1}} \right] \cdot \left[\beta_0 E_0^{(v)}, \beta_1 E_1^{(v)}, ..., \beta_{M-1} E_{M-1}^{(v)} \right]^T \\ &= \boldsymbol{x} - \left[\boldsymbol{p}_{s_0}\beta_0, \boldsymbol{p}_{s_1}\beta_1, ..., \boldsymbol{p}_{s_{M-1}}\beta_{M-1} \right] \cdot \boldsymbol{E}^{(v)} \end{aligned} \tag{10}$$

The flow diagram of the proposed enhanced-TKM-TR algorithm is showed as Fig. 2.

Since the value of $\boldsymbol{E}^{(v)}$ is 1 or 0, there is no need for extra multiplications when calculating (10) compared with the original TKM-TR. We compare the computational complexity of real multiplications and real divisions with the same criterion in [4] and [7], and we also ignore the complexity of the regular part for a TR algorithm. So the complexity of the referred TKM-TR method is

$$\begin{aligned} MUL_{\text{TKM}} &= K_1\left(3\bar{M}_1 + 2LN\bar{M}_1\right) \\ DIV_{\text{TKM}} &= K_1\left(2\bar{M}_1\right) \end{aligned} \tag{11}$$

where K_1 is the iteration number and $\bar{M}_1$ is the average value of the length of set $\mathcal{S}$ in TKM-TR, which is a random variable and can be obtained through numerical simulation.

The computational difference between the enhanced-TKM-TR and the original one is computing the PAPR of the updated peak reduced signal $\hat{\boldsymbol{x}}$ in the iterations. The original TKM-TR only needs to calculate a single one, while the enhanced-TKM-TR needs to calculate V different signals, which requires $2(V-1)LN$ extra real multiplications and $(V-1)$ extra real divisions. So we can get the complexity of the enhanced-TKM-TR as

$$\begin{aligned} MUL_{\text{enhanced-TKM}} &= K_2\left[3\bar{M}_2 + 2LN\bar{M}_2 + 2(V-1)LN\right] \\ DIV_{\text{enhanced-TKM}} &= K_2\left(2\bar{M}_2 + V - 1\right) \end{aligned} \tag{12}$$

where the definitions of K_2 and $\bar{M}_2$ are the same as above.

The numerical comparison of the computational complexity will be presented in the next section.

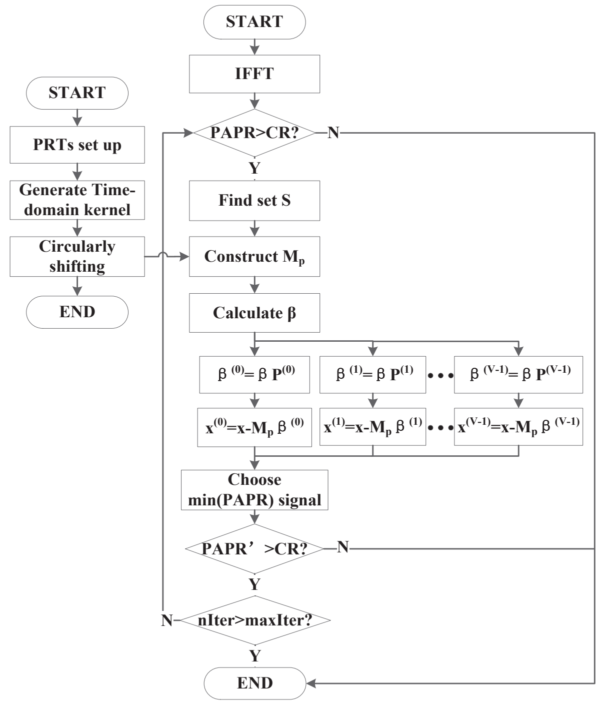

Figure 2. The flow diagram of the enhanced-TKM-TR.

IV. SIMULATION RESULTS

The simulations are conducted with the same simulation parameters as the TKM-TR in [7]. Which is, the OFDM system with $N = 256$ subcarriers, 32 randomly reserved tones and 16-QAM mapping applied, the oversampling rate is set to be 4.

Fig. 3 shows the PAPR reduction performance of the TKM-TR and the enhanced algorithm for total 4 iterations, the clipping ratio is $CR = 6.5dB$ and $V = 4$ sign vectors are generated randomly with $\alpha = 0.4$, and α is not sensitive when $0.2 < \alpha < 0.7$. The PAPR reduction of the TKM-TR is 2.12, 3.39, 4.26, and 4.58dB, respectively, for 1-4 iterations and $CCDF = 10^{-3}$. As for the enhanced-TKM-TR, the PAPR reduction is 2.85, 3.82, 4.30, and 4.52dB, respectively.

The PAPR performance for the 1st and 2nd iteration is improved greatly, which means the peak residual and peak regrowth are reduced. And for the 4th iteration, the performance of the enhanced one is slightly degenerated when the PAPR approaching to the threshold, the reason is that the enhanced TKM-TR method could influence the TKM-TR optimization outcomes when setting some scaling vectors to 0.

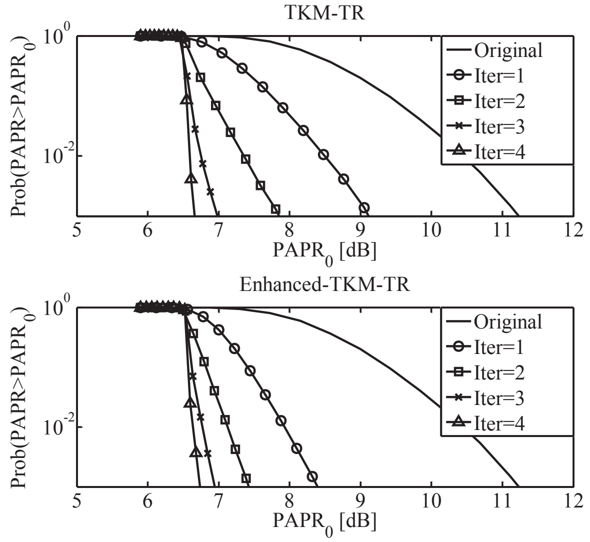

Figure 3. PAPR reduced by the TKM-TR (up) and the enhanced-TKM-TR (down) for 1-4 iterations with CR=6.5dB and V=4.

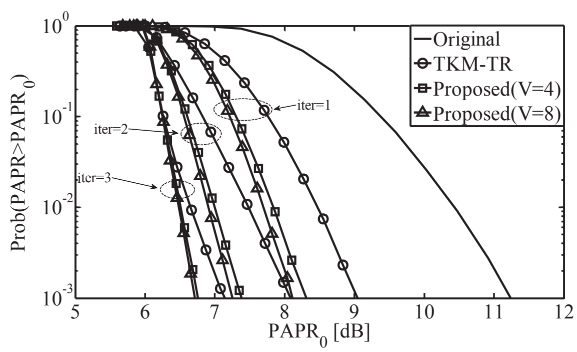

Figure 4. PAPR reduced by the TKM-TR and the enhanced-TKM-TR for 1-3 iterations with CR=6 dB and V=4,8.

Fig. 4 compares the PAPR performance when different parameter V is employed with the clipping ratio $CR = 6dB$ and $\alpha = 0.3$. The larger the V, the better the performance. When $V = 8$, the PAPR is 0.20, 0.12, and 0.05dB improved compared to $V = 4$, for 1, 2, and 3 iterations, respectively.

The clipping ratio in fig.3 and fig.4 is different. We know that when the clipping ratio is lowered, there would be more PAPR reduction in each iteration, but the length of $\mathcal{S}$ would be larger and the convergence rate slower, also, it will lead to more power increasing. So the computational complexity would be higher when the clipping ratio CR is lower. To set a clipping ratio appropriately depends on the system requirements and simulation of the algorithm, so we just present a reasonable clipping ratio for our simulations.

From Fig. 4 we can tell that the PAPR performance of the enhanced-TKM-TR with parameter $V = 8$ for 1 iteration and TKM-TR for 2 iterations is almost the same, i.e., 8.13dB for the enhanced TKM-TR and 8.12dB for the original one. And the $\bar{M}$ we got from the statistical results of the simulation is 14.5 and 18.2, for the enhanced TKM-TR with 1 iteration and the TKM-TR with 2 iterations, respectively. So, using (12), we get the complexity of real multiplications and real divisions of the two algorithms as Table I. The comparison in Table I shows that the complexity of the enhanced algorithm is slightly better than the original one when achieving the same PAPR performance, which means that the enhanced TKM-TR is practical and efficient.

TABLE I. REAL MULTIPLICATIONS AND REAL DIVISIONS COMPLEXITY COMPARISON OF DIFFERENT SCHEMES

	Real Multiplications	Real Divisions
TKM-TR	57838	57
Enhanced-TKM-TR	52280	44

V. CONCLUSION

In this paper, an enhanced TKM-TR method has been proposed to suppress the peak residual and peak regrowth in the TKM-TR method, thus improving the PAPR performance of the OFDM signals. Qualitative and quantitative analysis have been conducted and the simulations show that the PAPR reduction performance of the enhanced-TKM-TR outperforms the original TKM-TR with a slightly better complexity performance. So the enhanced TKM-TR reduces the required iteration number when achieving the same PAPR, which means it can be applied to the OFDM systems that a relatively small iteration number is a necessary condition.

REFERENCES

[1] Simon Litsyn, *Peak power control in multicarrier communications*. New York: Cambridge University Press, 2007.

[2] Y. Rahmatallah, and S. Mohan. "Peak-to-average power ratio reduction in OFDM systems: A survey and taxonomy," *IEEE Commun Surveys & Tutorials*, vol. 15, issue. 4, pp. 1567–1592, Nov. 2013.

[3] J. Tellado, Multicarrier modulation with low PAR : applications to DSL and wireless. New York: Kluwer Academic Publishers, 2000.

[4] L. Wang and C. Tellambura, Analysis of clipping noise and tone reservation algorithms for peak reduction in OFDM systems, *IEEE Trans. Veh. Technol.*, vol. 57, no. 3, pp. 1675–1694, May 2008.

[5] Y. Wang, W. Chen, and C. Tellambura, Genetic algorithm based nearly optimal peak reduction tone set selection for adaptive amplitude clipping PAPR reduction, *IEEE Trans. Broadcast.*, vol. 58, no. 3, pp. 462–471, Sept. 2012.

[6] J. Hou, J. Ge, and F. Gong, Tone Reservation Technique Based on Peak-Windowing Residual Noise for PAPR Reduction in OFDM Systems, *IEEE Trans. Veh. Technol.*, vol. 64, no. 11, pp. 5373–5378, May 2008.

[7] P. Yu, and S. Jin, A Low Complexity Tone Reservation Scheme Based on Time-domain Kernel Matrix for PAPR Reduction in OFDM Systems, *IEEE Trans. Broadcast*., vol. 61, no. 4, pp. 710–716, Dec. 2015.

[8] C. Tellambura, Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers, *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 185–187, May 2001.