A New Proposed Peak-to-Average Power Reduction Parameter to Evaluate SLM and PTS as OFDM PAPR Reduction Schemes

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Abstract—**Two techniques of the most known ones for peak to average power ratio (PAPR) reduction of orthogonal frequency division multiplexing (OFDM) are selected mapping (SLM) and partial transmit sequence (PTS). Both schemes were proposed as distortion less PAPR reduction algorithms. However, it has been an argument to prove which scheme is the most efficient. In this paper, we propose a new PAPR reduction efficiency parameter which will be applied on the complementary cumulative distribution function (CCDF) of each technique to compare the results. We show as well how the performance of the system reacts when increasing the probability of getting high PAPR values. Using the proposed efficiency formula, we show that PTS system performance improves when increasing the probability, whereas the SLM system performance gets impaired when increasing the probability within the same range.**

Keywords—Orthogonal frequency division multiplexing (OFDM); Peak to average power ratio (PAPR); Complementary cumulative distribution function (CCDF); Selected mapping (SLM); Partial transmit sequences(PTS).

I. INTRODUCTION

OFDM is a multi carrier modulation technique which has been recently widely used in different communication systems especially the ones with high data rates [1]. OFDM has become so popular nowadays due to its flexible and efficient management of inter- symbol interference (ISI). In addition, OFDM offers high spectral efficiency as a result of multicarrier orthogonality aspect. Such system aspects would improve overall system performance and communication link quality [1]. However, OFDM has a major drawback which is the high PAPR. Having a system with high PAPR will force the power amplifier to work in the non-linear region where the power conversion is inefficient which affects, consequently, the battery life in the mobile communications devices. This inefficient power conversion causes power growth as well resulting in even higher amplitude peaks [2]. Since the impact of high PAPR is severe on the system performance, many literatures have been published to focus on developing modified algorithms with low PAPR. Overviews of PAPR reduction techniques are covered in [3, 4]. In our paper we are focusing on two of these PAPR reduction techniques: selected mapping (SLM) and partial transmit sequences (PTS). SLM and PTS were published for the first time in [5] and [6], respectively. Since SLM and PTS were published, questions and discussions have raised to find out which one is the more practical, less complicated, and most efficient at the same time. Therefore, many comparison publications have been made for that purpose. Our area of interest in this paper is to develop a criterion to evaluate both SLM and PTS system performances based on their CCDF responses. In order to evaluate SLM and PTS techniques, we should cover the theory and mechanism used mostly to evaluate PAPR reduction in any PAPR reduction system. OFDM divides the information stream, needed to be sent, into N individual sub-streams. These N sub-streams are sent via L sub-channels with significantly lower data rate at each sub-channel. The frequencies of those N sub-channels are orthogonal allowing them to overlap without any interference. Reducing data rate at these sub-channels and overlapping transmission frequencies mean the system would have lower ISI and less occupied bandwidth satisfying the desired communication quality criterion. However, when adding these sub-streams up together to form the time domain OFDM signals, the PAPR problem takes place significantly. OFDM transmitter and receiver block diagram are shown in Fig. 1 and 2 respectively.For a baseband OFDM signal:

$$\mathrm{x(t)} = \left(\frac{1}{\sqrt{N}}\right)\sum_{n=0}^{N-1} \mathrm{Xn}\, e^{j2\pi n\Delta ft}, 0 \le t \le NT \qquad (1)$$

Where N is number of blocks andΔfissubcarrier spacing.Then,

$$PAPR = \frac{Peak\ Power}{Average\ Power} = \frac{\max\limits_{0\le t\le NT} |x(t)|^2}{\frac{1}{NT}\int_0^{\mathrm{NT}} |x(t)|^2 dt} \quad (2)$$

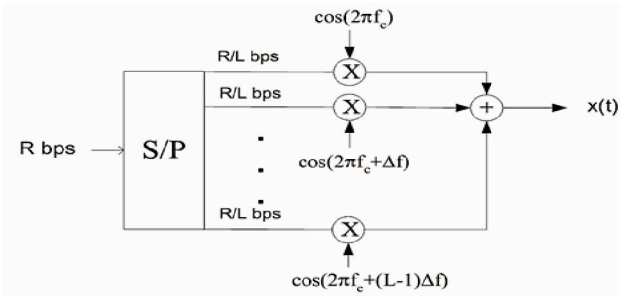

Fig.1. OFDM transmitter [7].

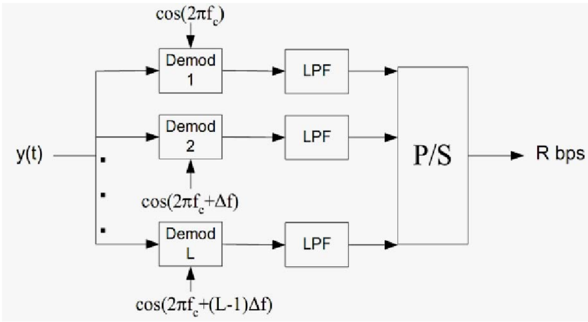

Fig.2.OFDM receiver [7].

To evaluate a PAPR reduction of such system, the complementary cumulative distribution function (CCDF) is used. CCDF is a graphical representation which shows how much the probability that the system exceeds certain PAPR values [1]. The CCDF mathematical expression is calculated through the cumulative distribution function (CDF) in [8] as the following:

$$F(z) = 1 - \exp(z) \quad (3)$$

To derive the CCDF from the CDF function, the following set of equations is applied:

$$P(PAPR > z) = 1 - P(PAPR \leq z)$$

$$= 1 - \mathrm{F(z)}^N$$

$$= 1 - (1 - \exp(-z))^N \quad (4)$$

Where, N = subcarriers number.

Fig. 3, for example, shows the CCDFs of two original and modified OFDM signals with number of subcarriers 256 and 1024. It is shown in fig 3 that 0.01 percent of unmodified OFDM data blocks with 1024 subcarriers have PAPR of 12.4 dB. After the modification is implemented for the same OFDM signal with the same number of subcarriers, 0.01 of the modified OFDM data blocks have PAPR of 9.1dB

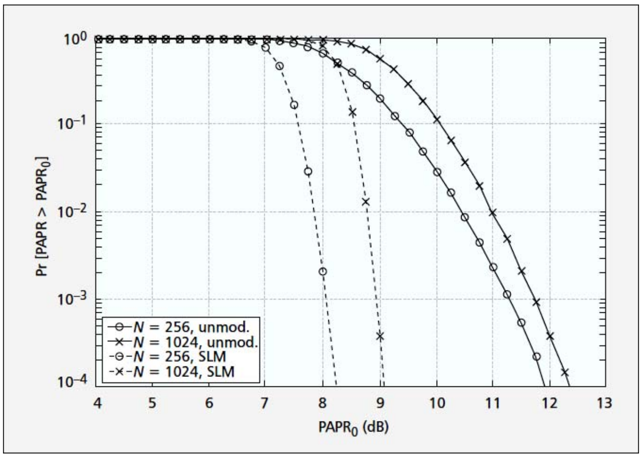

Fig.3. CCDFs of PAPR of two OFDM signals after being modified using SLM scheme [1].

II. SELECTED MAPPING

As a distortion-less OFDM PAPR reduction method, SLM was proposed first in [5]. The theory behind SLM is to represent the data block at the transmitter by different data blocks which all contain the same information as the original data blocks do. These new data blocks result after multiplying the original data block by a sequence of phases generated at the transmitter. Then the criterion of which data block among others should be selected for transmission is to choose the one which gives the lowest PAPR [8, 9]. SLM block diagram is shown in Fig. 4.

The SLM scenario is summarized as the following:

1. Thetransmitter generates F unique phase sequences whose length is L

$$P^{(f)} = [P_{f,0}, P_{f,1}, \dots, P_{f,L-1}] \quad (5)$$

2. The original data block is multiplied by these F phase sequences to generate F unique representations of the original data block.

$$M^{(f)} = [M_0 P_{f,0}, M_1 P_{f,1}, \dots, M_{L-1} P_{f,L-1}\ ,\ \mathrm{f} = 1, 2, \dots \mathrm{F} \quad (6)$$

3. The inversediscrete Fourier transform (IDFT) is applied on each of these modified data blocks as it is shown in Fig. 4. The result after IDFT is simply F data blocks in time domain each one has length of L as follows:

$$m^{(f)}(\mathrm{t}) = (1/\sqrt{L}) \sum_{n=0}^{L-1} \mathrm{Mn}\ P_{f,n} e^{j2\pi n \Delta f t} \quad (7)$$

Where, $0 \leq \mathrm{t} \leq \mathrm{LT}$, f = 1, 2,… F.

4. Finally,the modified data block which gives the lowest PAPR is selected for transmission. At the receiver side, in order to retrieve the original data, side information should be sent from the transmitter to inform the receiver which phase sequence has been selected for transmission [1]. Proposed SLM algorithm without side information was proposed in [10].

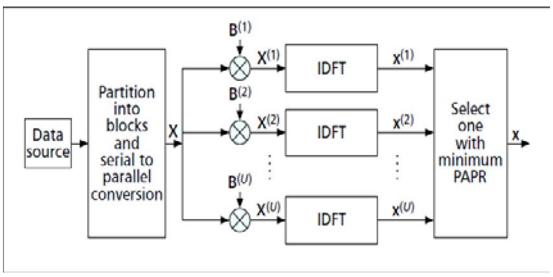

Fig.4. SLM system block diagram [1].

III. PARTIAL TRANSMIT SEQUENCES

According to the theory of PTS, which was first presented in [5], the data block is divided into sub-blocks. Subcarriers of each those sub-blocks should be deliberate to a phase factor. Those phase factors were designed in such way so that PAPR is minimized when recombining sub-blocks to form the main data block a gain. The PTS block diagram is shown in Fig. 5. The PTS scenario supported with mathematical expressions is summarized in the following steps: [11]

1. The inputdata block X is divided and separated into M sub-blocks,

$$X_m = [X_{m,0}, X_{m,1}, \dots, X_{m,L-1}], \text{ m=1, 2,... M} \quad (8)$$

That means if we recombine these sub-blocks, we would get the original data block X as the following,

$$\sum_{m=1}^{M} X_m = \text{X} \quad (9)$$

2. The second stepis to convert the sub-blocks to the time domain using inverse fast Fourier transform (IFFT) to form the signal x_m from X_m as the following:

$$x_m = [x_{m,0}, x_{m,1}, \dots, x_{m,L-1}], \text{ m=1, 2,... M} \quad (10)$$

3. To the purpose ofminimizing PAPR, each sub-block in time domain is rotated by the phase factor

$$b = [b_0, b_1, \dots, b_{M-1}] \text{ , where } b_m = e^{j\emptyset}, 0 \le \emptyset < 2\pi \quad (11)$$

4. The last step isto add all the sub-blocks up to form the final time domain signal which is

$$X'(b) = \sum_{m=1}^{M} b_m X_m \quad (12)$$

Or, $X^{'}(b) = [X_0^{'}(b), X_1^{'}(b), \dots, X_{NL-1}^{'}(b)]$ (13)

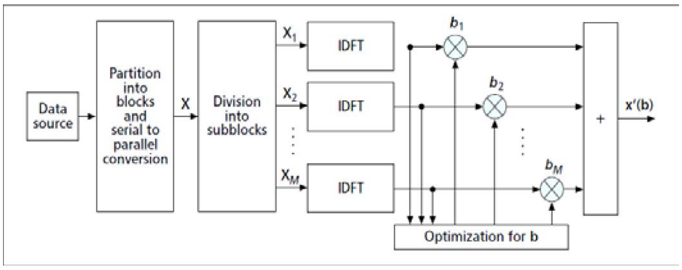

Fig.5. PTS system block diagram [1].

Since PTS suggests dividing the input data block into M sub-blocks and applying IFFT on each one of these sub-blocks, PTS procedure requires M times IFFT operations for each data block. Therefore, phase selection and complexity of PTS increase when M increases.

IV. REDUCTION EFFICIENCIES OF SLM ANDPTS

An OFDM system is simulated under the following specifications: 10,000 OFDM symbols, 64 subcarriers, oversampling factor of 4, 16 OFDM candidates, and QPSK modulation scheme. The simulation results of SLM and PTS algorithms are shown in Fig. 6 and 7, respectively, where the CCDFs are plotted for both schemes. In the previous comparison of SLM and PTS, It has been proven that PTS is more complex than the SLM is in most cases [12]. Far away from the complexity metric used in the previous comparisons [12, 13, 14], we develop a PAPR reduction efficiency factor in this paper based on CCDF information for both SLM and PTS. Form CCDF information, we calculate the PAPR reduction at certain standard probabilities which are 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1}. At these probabilities we get the corresponding PAPR values for both SLM and PTS PAPR reduction systems. These PAPR values extracted from both systems will be compared to those values extracted at the same standard probabilities from the original OFDM signal. Comparing original OFDM PAPRs with those from SLM and PTS response we can calculate PAPR reduction values at the each of these standard probabilities. Therefore, the PAPR reduction is:

$$\text{OFDM PAPR Reduction=Original PAPR – modified PAPR} \quad (14)$$

Both the original and modified PAPRs should be at the same probability point.

Finally, the proposed formula for the PAPR Reduction Efficiency is:

$$PAPR\ Reduction\ Efficiency = \frac{OFDM\ PAPR\ Reduction}{Original\ \ OFDM\ PAPR} \quad (15)$$

V. COMPARISON AND DISCUSSION OF SIMULATION RESULTS

Table I shows the original PAPRs, modified PAPRs, and PAPR reduction efficiencies calculated using equations 14 and 15.

TABLE 1: PAPR VALUES OF ORIGINAL OFDM, SLM, PTS, AND THEIR EFFICIENCIES AT STANDARD PROBABILITIES

Probability of (PAPR > PAPRo)	PAPR of original OFDM	SLM PAPR	SLM PAPR reduction efficiency	PTS PAPR	PTS PAPR reduction efficiency
10^{-4}	11.2217	8.3165	25.8895	6.7586	39.7721
10^{-3}	10.6217	8.0165	24.5275	6.4566	39.2128
10^{-2}	9.8217	7.6165	22.4527	5.8566	40.3704
10^{-1}	8.6217	7.0165	18.6186	4.8566	43.6696

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After applying the proposed two formulas, 14 and 15 on both SLM and PTS CCDF's, we have come up with the results shown in Fig. 8. Looking at the results shown in table 1 and

Fig. 8, interestingbehavior can be seen. The PAPR reduction efficiency of PTS algorithm improves with having the probabilityof ($PAPR > PAPR_{\circ}$)increased from 10^{-4} to 10^{-1}, while the PAPR reduction efficiency of SLM algorithm gets impaired with having the probability increased within the same range.

Both SLM and PTS algorithms were applied to the same OFDM signal and tested for PAPR reduction under the same circumstances.

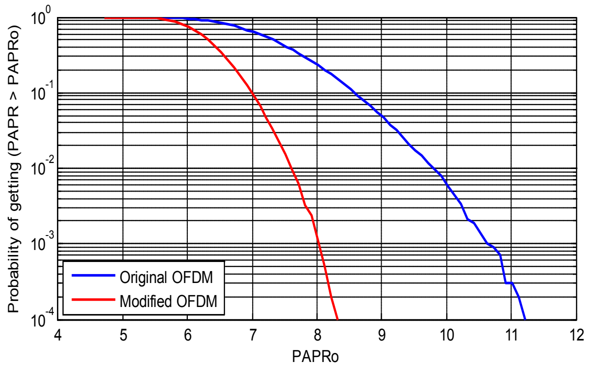

Fig.6. CCDFs of SLM for OFDM PAPR reduction

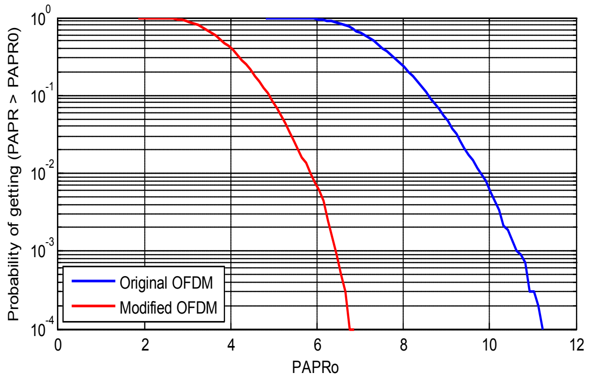

Fig.7. CCDFs of PTS for OFDM PAPR reduction

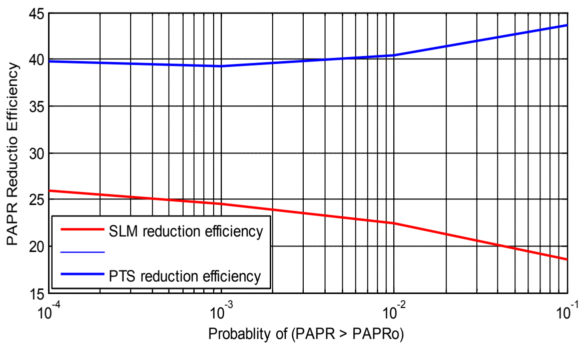

Fig.8. SLM and PTS PAPR reduction efficiencies versus probability of ($PAPR > PAPR_{\circ}$)

VI. CONCLUSION

It has been always a controversial topic to evaluate SLM and PTS algorithms for OFDM PAPR reduction. Literature publications have shown that PTS PAPR reduction system is more complex than SLM PAPR reduction system is. In this paper, after simulating both PAPR reduction schemes SLM and PTS, a proposed way is implemented to evaluate both SLM and PTS techniques from the angle of the system efficiency when increasing the probability of getting high PAPR values. Results have shown that PTS technique overweighs SLM technique when increasing the probability of having ($PAPR > PAPR_{\circ}$).

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