Analysis of quantization metrics for PAPR evaluation in OFDM systems

Martha Cecilia Paredes Paredes*, David Mejía Navarrete*, Diego Fernando Puga Placencia†

*Departamento de Electrónica Telecomunicaciones y Redes de Información,
Escuela Politécnica Nacional, Ladrón de Guevara, E11-253, Quito-Ecuador
Email: {cecilia.paredes, david.mejia}@epn.edu.ec

†Jefatura Gestión Técnica de Equipos Terminales y Materiales
Corporación Nacional de Telecomunicaciones, 9 de Octubre N24-113 y Cordero, Quito -Ecuador
Email: diego.puga@cnt.gob.ec

***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) has become the popular modulation technique in high speed wireless communications. It has several advantages over other techniques, such as high data rate, strong immunity to multipath and high spectral efficiency. However, it could have high peaks in the transmitted signal; this is the well-known Peak-to-Average Power Ratio (PAPR) problem. It is very important to deal with PAPR reduction in OFDM systems to avoid signal degradation. Currently, the PAPR problem is an active area of research and in this paper we discuss about the PAPR problem and the main metrics that are used to measure the PAPR in any OFDM system. Moreover their advantages and disadvantages have been enumerated in order to provide the readers the actual situation of the metric to cuantizate the PAPR problem.**

***Index Terms*—Crest factor, cubic metric, OFDM, PAPR, peak power reduction**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission widely used in wireless communication systems. The actual interest in this technique is mainly due to the recent advances in digital signal processing technology. Several international standards are making use of OFDM for high-speed wireless communications, such as Digital Television (DTV), Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T), Brazilian Digital Television System (ISDB-T), standards for Wireless Local Area Networks (WLAN), standards for Wireless Metropolitan Area Networks (WMAN), and standards for mobile communications such as Long Term Evolution (LTE). For wireless applications, an OFDM-based system can be of interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation can be realized using Fast Fourier Transform (FFT) techniques [1].

However, OFDM signal at the transmitted side suffers from high instantaneous power with respect to its average power. This is the well-known Peak-to-Average Power Ratio (PAPR) problem. These high peaks introduce a serious degradation in performance when the signal passes through a non-linear High Power Amplifier (HPA) [2]. The non-linearity of the HPA leads to in-band distortion, which increases Bit Error Rate (BER), and out-of-band radiation, which causes adjacent channel interference [2].

In the literature, there are several proposals to overcome the well-known PAPR problem, such as clipping and Active Constellation Extension (ACE), Partial Transmit Sequence (PTS), SeLected Mapping (SLM), Simple Amplitude Predistortion (SAP), Orthogonal Pilot Sequences (OPS), Simple Amplitude Predistortion aided by Orthogonal Pilot Sequences (OPS-SAP), etc. (see [1], [3] and references therein). The peak reduction obtained by all these techniques can be measured using some metrics such as PAPR, Crest Factor (CF) and Cubic Metric (CM). However, a performance comparison of each metric is missing in the literature.

In this paper, we describe the PAPR problem and we explain each metric to measure the PAPR problem in OFDM system. The performance of each metric is analyzed through simulations. The remainder of this paper is organized as follows. Section II introduces the OFDM signal model and the PAPR definition. In Section III, major existing metrics available to evaluate peaks are discussed. In Section IV the metrics are evaluated and compared through simulations. Finally, the conclusions are drawn in Section V.

II. OFDM systems and PAPR definition

The first OFDM system did not employ digital modulation/demodulation and the signal were generated with a bank of analog modulators. The continuous-time model can be considered as an ideal system which is in practice nowadays digitally synthesized. From the continuous-time model, the discrete-time is derived. For this reason, in this paper we expose the discrete-time OFDM model.

An OFDM system consists of splitting the available bandwidth BW [Hz] into N subchannels (called subcarriers) of equal bandwidth. The subcarriers are placed with a separation between them of $\Delta f = BW/N$ [Hz] [1]. The set of frequencies assigned to the subcarriers, considering f_0 the lower frequency of the available band, is given by the next equation:

$$f_k = f_0 + \frac{BW}{N}k = f_0 + \Delta f k, \quad k = \{0, \ldots, N-1\} \quad (1)$$

To accomplish the orthogonality, the length of an OFDM-symbol is given by $T = 1/\Delta f = N/BW$ [s] or a multiple of

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it. For the discrete-time model the signal should be sampled, considering $T = NT_s = N/BW$, where T_s represents the sampling period of each OFDM-symbol, which is sampled N times [1].

In the discrete-time model, OFDM transmitted signal is efficiently implemented by an Inverse Discrete Fourier Transform (IDFT) operation and the demodulation is realized by DFT [4]. In Fig. 1, the OFDM transmitter is presented, which is easily implemented via IDFT. Then, the time-domain transmitted signal for the ℓth OFDM-symbol $\mathbf{b}^\ell = \left[b^\ell(0), \ldots, b^\ell(N-1)\right]$ is given by:

$$b^\ell\left[n\right] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a^\ell\left(k\right) e^{j2\pi kn/N}, \quad 0 \le n < N-1 \qquad (2)$$

where k and n are the frequency and time indices respectively, $a^\ell\left(k\right)$ is the complex data symbol transmitted over the kth subcarrier, $k = \{0, \ldots, N-1\}$ and $\mathbf{a}^\ell = \left[a^\ell(0), \ldots, a^\ell(N-1)\right]$.

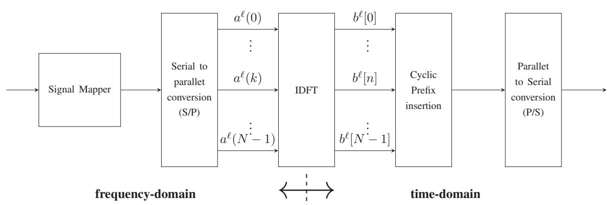


Fig. 1: Discrete-time OFDM transmitter with IDFT

To obtain a high spectral efficiency, the frequency response of the subchannels in OFDM are overlapped and orthogonal. However, it is desirable to maintain this orthogonality, even though the signal passes through a time-dispersive channel, because if orthogonality is lost there will be problems in the received signal. In [5], to combat Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI) a guard space between symbols is used, this guard space consists of a copy of the last part of the OFDM symbol which is prepended of the transmitted symbol, called the Cyclic Prefix (CP). Detailed discussion of CP are beyond the scope of this work and will not be discussed further.

A. *OFDM Signal Statistics*

If a discrete-baseband OFDM symbol, defined by equation (2), is considered as the sum of N independent signals modulated in each subcarrier, where each one of the complex symbols in frequency-domain $\mathbf{a}^\ell = \left[a^\ell(0), \ldots, a^\ell(N-1)\right]$, are considered as independent, identically distributed (i.i.d.) complex Gaussian random variables, with zero mean and variance equal to unity, then, the samples $\mathbf{b}^\ell = \left[b^\ell(0), \ldots, b^\ell(N-1)\right]$ are also i.i.d. Gaussian random variables, with zero mean and variance equal to unity [6].

If N is sufficiently large ($N > 32$), based on the Central Limit Theorem (CLT) [7], the real part of the signal $\mathcal{R}\left\{b^\ell[n]\right\}$ and its imaginary part $\mathcal{I}\left\{b^\ell[n]\right\}$ in the time-domain are distributed according to a Gaussian function $\mathcal{N}(0, \sigma^2)$, while its envelope $|b^\ell[n]|$ is distributed according to a Rayleigh distribution [1], whose Probability Distribution Function (PDF) is given by [8],

$$R = |\mathbf{b}| \sim f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \qquad (3)$$

where σ^2 represents the variance of real and imaginary components of $b^\ell[n]$.

Fig. 2a shows the real and imaginary part, as well as the envelope of the OFDM signal $b^\ell[n]$ in time-domain; while 2b shows the corresponding distributions of each one. Also, Fig. 2 shows that occasionally peaks appear compared to the average of the signal over time.

Thus, due to CTL, a small percentage of the frequency-domain samples $b^\ell[n]$ will have very large magnitudes (as shown in Fig. 2b). This results in the well-known PAPR problem of an OFDM system.

B. *The PAPR problem*

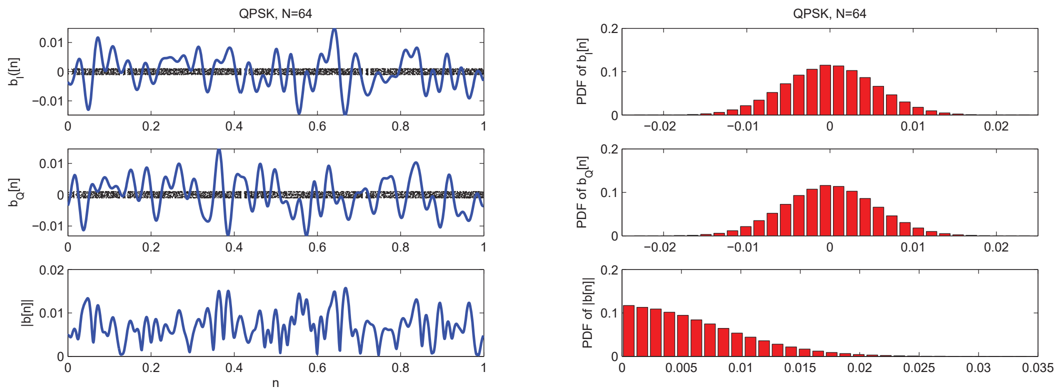
The major obstacle in the transmitter side of any OFDM system, occurs when the signal comes to have very high peak instantaneous power with respect to its average power, due to the possibility that the subcarriers can be aggregated with identical phase in a certain time, thus the time-domain signal achieves a very important contribution of power [6], [9]. While it is highly unlikely that a match in phase occurs in a majority number of subcarriers, the probability that a high peak appears is sufficient to trigger large fluctuations in the envelope of the OFDM signal [6].

High signal peaks have led certain hardware components such as Digital to Analog Converters (DAC) and HPA to be below their efficient performance behavior [6], [10].

Power efficiency in wireless and mobile communications is necessary because it allows to have a better coverage area, there are savings in power consumption, which means smaller terminal designs, etc. Therefore, it is important that HPA operation is efficient in power, but this is affected by the high peaks of the time-domain OFDM signal passing through it, producing both in-band and out-of-band distortion problems, which results in a degradation of system performance [2].

To obtain enough transmission power, most HPAs operate near the saturation point, as illustrated in Fig. 3, to take advantage of its peak performance. Furthermore, in practical systems the HPA is a limited device, both in power and very sensitive to variation in the amplitude of the signal. Therefore, the OFDM signal with high fluctuations in its envelop can cause saturation on the device, producing in-band distortion, that results in several changes on the received signal, such as rotation, attenuation and shift, increasing the BER [1], [2], [9] and also producing out-of-band radiation, which causes adjacent channel interference [1], [2], [9].

The most efficient operating point for a HPA is at the saturation level. However, high peaks encountered in OFDM signals can drive the nonlinear HPA into saturation. Therefore, Input Back-Off (IBO) is required to shift the operating point to the left as shown in Fig. 3. The IBO factor is defined as the



(a) time-domain signal

(b) PDF of the signal

Fig. 2: Characterization of time-domain OFDM signal with $N = 64$ subcarriers using QPSK modulation

ratio between saturation power of HPA and average power of the input signal. The conventional solution to the high peak problem is to backoff the operating point of the nonlinear HPA, but although simple, this approach usually causes a significant power efficiency penalty, or a high value of HPA, increasing the cost of the terminal. Many methods have been

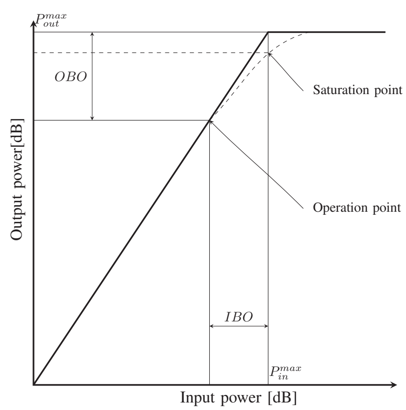


Fig. 3: Characterization of HPA response

implemented in software, and there are several proposals in the literature (see [1], [3] and references therein) to suppress the power peaks in OFDM before the time-domain signal passes through HPA. Each PAPR reduction technique is evaluated using a metric. These metrics are discussed in the next section.

III. Quantization Metrics to Evaluate Peaks on OFDM Signals

To assess the effects produced by high peak power of the OFDM signals, several quantization metrics have been defined, such as: Peak-to-Average Power Ratio (PAPR), Crest Factor (CF) and Cubic Metric (CM). The most widely metric used is PAPR, so the problem of power peaks is also known as the problem of PAPR in OFDM systems.

A. Peak-to-Average Power Ratio (PAPR)

The Peak-to-Average Power Ratio, known as PAPR, and denoted by χ^ℓ is the most often used metric to quantify the variations of signal envelope.

The mathematical definition of PAPR of a discrete-time signal $\mathbf{b}^\ell = \left[b^\ell(0), \ldots, b^\ell(N-1)\right]$ is given by the following expression [6]:

$$\chi^\ell = \text{PAPR}\left\{\mathbf{b}^\ell\right\} = \frac{\max\left|b^\ell\left[n\right]\right|^2}{E\left[\left|b^\ell\left[n\right]\right|^2\right]}, \quad 0 \leq n < N-1 \tag{4}$$

where, $\max\left|b^\ell\left[n\right]\right|^2$ denotes the maximum instantaneous power, $E\left[\left|b^\ell\left[n\right]\right|^2\right]$ is the average power of the signal, $E\left[\cdot\right]$ is the mathematical expectation and $|\cdot|$ is the module operation.

A mathematically equivalent form to define the PAPR is given by [6]:

$$\chi^\ell = \text{PAPR}\left\{\mathbf{b}^\ell\right\} = \frac{\parallel \mathbf{b}^\ell \parallel_\infty^2}{E\left[\parallel \mathbf{b}^\ell \parallel_2^2\right]/N}, \tag{5}$$

where, $\mathbf{b}^\ell = \left[b^\ell[0], \ldots, b^\ell[N-1]\right]$ is a vector whit $1 \times N$ elements that collects the time-domain samples and $\parallel \cdot \parallel_p$ denotes the $p-$norm.

B. Crest Factor (CF)

Unlike PAPR, CF denoted by $\mathcal{F}_C$ measures the high peaks in terms of its envelope. CF can indicates how extreme peaks are in a signal, and is defined as the square root of the PAPR [1]:

$$\mathcal{F}_C = \sqrt{\chi^\ell} \tag{6}$$

where, χ^ℓ is the PAPR for the ℓth time-domain OFDM symbol.

C. Cubic Metric (CM)

Although PAPR is the classical and most widely used metric to quantify the envelope fluctuations, another metric known as Cubic Metric (CM) has been proposed and adopted by the third Generation Partnership Project (3GPP). The motivation behind the CM lies in the fact that a major part of the distortion introduced by the non-linearity of the HPA is due to the third order intermodulation product, which can be expressed as the convolution of the signal and the third order non-linearity of the HPA model. While PAPR considers only the main peak of power, CM accounts for the secondary peaks of power that affect the HPA performance due to the cubic term in the HPA gain characteristic function [3]. CM is mathematical defined by [11]:

$$\mathcal{C}_M = \frac{\mathcal{C}_{M_{\text{net}}} - \mathcal{C}_{M_{\text{ref}}}}{K} \text{ [dB]} \tag{7}$$

where, $\mathcal{C}_{M_{\text{ref}}}$ is a reference level of the wideband code-division multiple-access voice reference signal, K is an empirical factor, and $\mathcal{C}_{M_{\text{net}}}$ is the raw cubic metric defined as:

$$\mathcal{C}_{M_{\text{net}}}\{b^{\ell}[n]\} = 20\log_{10}\left(\sqrt{E\left\{\left(\frac{|b^{\ell}[n]|}{\sqrt{E\{b^{\ell}[n]\}}}\right)^3\right\}}\right) \text{ [dB]} \tag{8}$$

For example, for LTE in downlink $\mathcal{C}_{M_{\text{ref}}} = 1.52$ [dB] and $K = 1.56$ [12].

D. Complementary Cumulative Distribution Function of the Metrics

Due to PAPR, CF and CM are random variables, the most common way to evaluate these metrics performance is through the Complementary Cumulative Distribution Function (CCDF), which determines the probability that PAPR, CF or CM of a certain OFDM-symbol goes beyond a fixed threshold (χ_0).

Thus, the CCDF of PAPR can be written as [2]:

$$\text{CCDF}\left(\chi^{\ell}\right) = Pr\left(\chi^{\ell} > \chi_0\right) = 1 - \left(1 - e^{-\chi_0^2}\right)^N. \tag{9}$$

In a similar way, the CCDF of CF and CM can be obtained from equation (9), where χ^{ℓ} can be replaced by $\mathcal{F}_C$ and $\mathcal{C}_{M_{\text{ref}}}$ respectively.

While Crest Factor is used to present amplitude values, and Cubic Metric is used in LTE systems. Amplitude values are interested when characteristics of the HPA are considered. There are other metrics that have been proposed, such as the ones presented in [13].

IV. Simulation Results

In this section, we provide the performance results for each analyzed metric, in terms of CCDF (*i.e.* CCDFs of PAPR, CF and MC). The results are obtained through Matlab simulations by averaging over 10^4 randomly generated OFDM-symbols, QPSK, modulations, and different numbers of subcarriers $N = \{64, 128, 256, 512, 1024\}$ are considered.

Figs. 4, 5 and 6 show the statistical behavior of PAPR, CF and MC respectively for OFDM systems, without any PAPR reduction technique, using QPSK and $N = \{64, 128, 256, 512, 1024\}$ subcarriers. In these figures, it can be seen that curves of CCDF of PAPR, CF and MC present similar results, also the value of PAPR, CF and MC exceeds a certain threshold that increases as the value of N increases, because with a higher amount of subcarriers it is more likely that these subcarriers could be added with same phases. For instant, the probability that the PAPR for an OFDM system with $N = 1024$ and QPSK exceeds by approximately 11.5 dB, is only 10^{-3}.

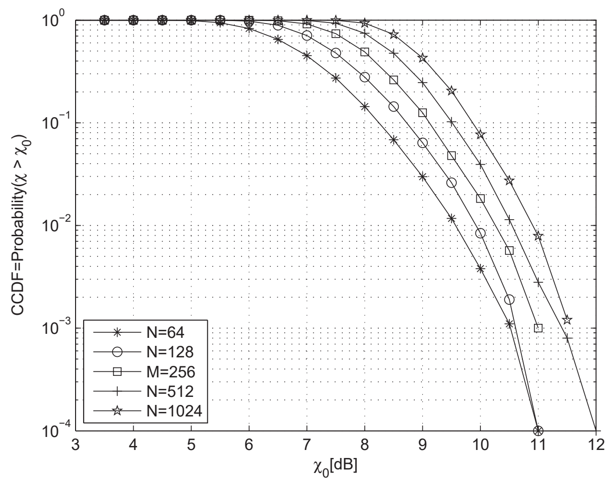


Fig. 4: CCDF of PAPR for OFDM system with $N = \{64, 128, 256, 512, 1024\}$ subcarriers and QPSK modulation

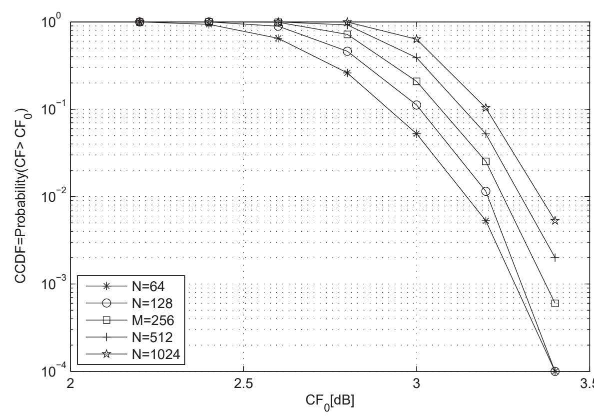


Fig. 5: CCDF of CF for OFDM system with $N = \{64, 128, 256, 512, 1024\}$ subcarriers and QPSK modulation

In order to evaluate the metrics, we applied an OPS-SAP (Simple Amplitude Predistortion aided by Orthogonal Pilot Sequences) PAPR reduction technique, which it is a very promising scheme for PAPR reduction in wireless communications [9], [14]. This OPS-SAP scheme is a Constellation Extension (CE) based technique, which intelligently plays with the outer constellation points of the frequency-domain signal in order to minimize the PAPR. OPS-SAP has a two-step architecture [9], [14]. The first step carries out an Orthogonal Pilot Sequences (OPS) scheme, where an appropriately orthogonal pilot sequence $p_m[n], m \in 1, \ldots, M$ is inserted to obtain the lowest PAPR from the available pilot sequence set. Then, the second step consists on apply the Simple Amplitude Predistortion

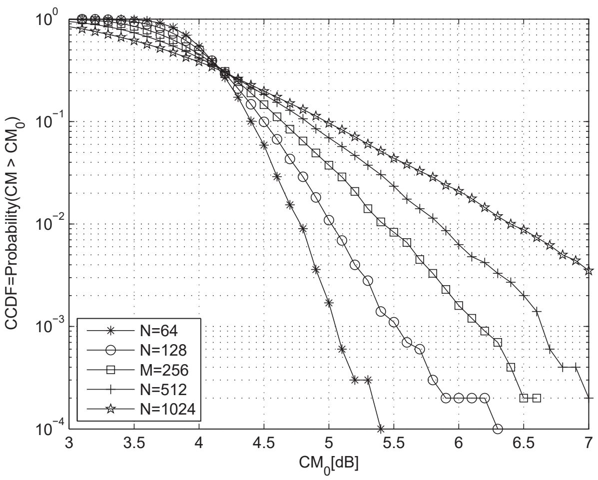


Fig. 6: CCDF of CM for OFDM system with $N = \{64, 128, 256, 512, 1024\}$ subcarriers and QPSK modulation

(SAP) algorithm, which consists in chose the best symbol in the frequency-domain that contributes to achieve a lower PAPR, and then, these symbols are extended with a constant scaling factor α. In this case, SAP algorithm is applied over both pilots and data symbols, allowing constellation extension of N complex symbols.

Thus, in Fig. 7, 8 and 9 the performance of OPS-SAP scheme is analyzed in terms of CCDF of PAPR, CF and CM, respectively. The curves with OPS-SAP technique are compared with the results obtained in a OFDM system without any PAPR reduction technique (labelled "Original"), and using OPS and SAP schemes. For one hand, in Fig. 7, the OPS-SAP algorithm represents a reduction in PAPR of approximately 3 dB, 2 dB and 1.2 dB at a probability of 10^{-3} with respect to "Original", OPS and SAP schemes, respectively. For the other hand, in Fig. 8, the OPS-SAP algorithm represents a reduction in CF of approximately 0,6 dB, at a probability of 10^{-3} with respect to "Original". Finally, in terms of CM, OPS-SAP presents a gain of approximately 3,5 dB, at a probability of 10^{-3} with respect to "Original'.

V. Conclusions

This paper describes one of the main problems of OFDM systems, the problem of high peak power. When an OFDM signal with high peaks passes through a power amplifier, this device, sensitive to variations of the signal, will saturate causing distortions in the signal, which degrades system performance. To quantify the PAPR in OFDM systems currently there are three metrics: PAPR, Crest Factor and Cubic Metric. These metrics, which are considered random variables, can be presented using CCDFs. The CCDF of PAPR is most often used in the literature to determine the behavior of PAPR problem, while Crest Factor is used to present amplitude values, and Cubic Metric is used in LTE systems.

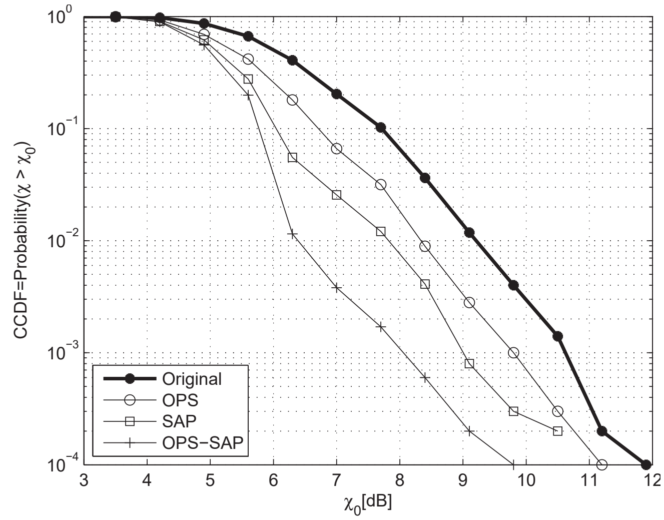


Fig. 7: CCDF of PAPR when OPS-SAP techniques are applied to an OFDM system with $N = 64$ subcarriers and QPSK modulation compared to "Original".

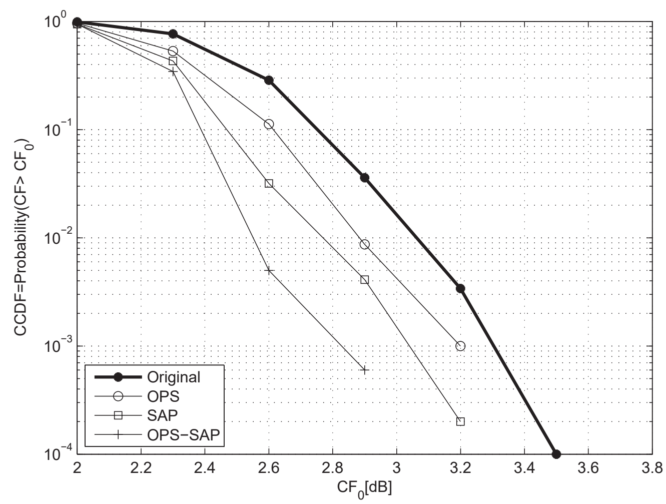


Fig. 8: CCDF of CF when OPS-SAP techniques are applied to an OFDM system with $N = 64$ subcarriers and QPSK modulation compared to "Original".

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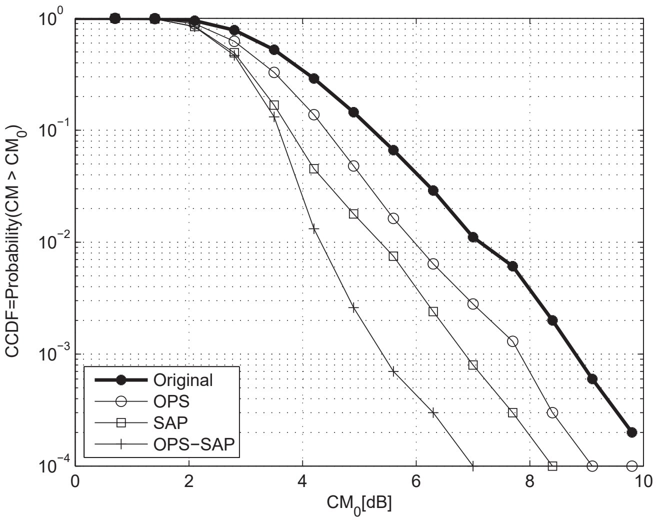

Fig. 9: CCDF of CM when OPS-SAP techniques are applied to an OFDM system with $N = 64$ and QPSK modulation compared to “Original”.

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