

# Doppler Estimation in an OFDM Joint Radar and Communication System

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**Abstract**—In this paper a processing algorithm that allows for estimating the velocity of multiple reflecting objects with standard OFDM communication signals is discussed. This algorithm does not require any specific coding of the transmit data. The technique can be used in combination with a range estimation algorithm in order to implement active radar sensing functions into a communication system for vehicular applications. Simulation and measurement results which prove the operability of the developed algorithm are presented.

**Index Terms**—Doppler, OFDM, radar.

## I. INTRODUCTION

In the current technological development the radio frequency front-end architectures used in radar and digital communication technology are becoming more and more similar. In both applications more and more functions that have traditionally been accomplished by hardware components are now being replaced by digital signal processing algorithms. Moreover, today's digital communication systems use frequencies in the microwave range for transmission, which are close to the frequency ranges traditionally used for radar applications. This technological advancement opens the possibility for the implementation of joint radar and communication systems that are able to support both applications on one single platform while utilizing a common transmit signal. A typical application area for such systems would be in the intelligent transportation networks, which require the ability of inter-vehicle communication as well as reliable environment sensing.

Recently, OFDM signals have gained a lot of attraction for the purposes mentioned. This is motivated by two facts. Firstly, the most current released communications standards, e.g. IEEE 802.11p, employ OFDM signals [1]. Secondly, in the radar community recently OFDM signals have attracted general interest because their suitability for radar applications has been proven [2]. Hence, OFDM signals seem to be the ideal basis for joint radar and communication implementations.

Besides the range measurement, the capability of Doppler measurement is also an important feature of radar systems. In particular regarding vehicular applications, the availability of velocity information is an essential requirement. In the investigation of OFDM radar concepts up to now, little attention has been paid to Doppler estimation. A method to compute the Doppler from OFDM signals, independent of the coding applied to the multi-carriers, has been presented in

[3]. This method however is correlation based thus having the drawback of higher computing efforts and high Doppler ambiguities.

In this paper, a Doppler processing scheme for the OFDM signal is presented. This scheme operates regardless of the transmitted signal information and coding by processing the symbols that compose the OFDM symbols directly instead of processing the baseband signals. Therefore the algorithm can be applied in combination with the transmission of arbitrary user data and is able to resolve multiple reflecting objects with a high dynamic range and low sidelobe levels.

In Section II, the OFDM joint radar and communication system (RadCom) concept will be discussed, along with the mathematical description of the OFDM signal, range and Doppler processing. A parameterization strategy taking into account the communication and radar systems aspects on the Doppler is detailed in Section III. Section IV shows the simulation results of the Doppler processing scheme and finally in Section V the measurement verification is presented.

## II. OFDM RADCOM CONCEPT

The OFDM transmit signal consists of parallel orthogonal subcarriers, each modulated with data. The resulting time domain signal is expressed by

$$x(t) = \sum_{\mu=0}^{M-1} \sum_{n=0}^{N-1} D(\mu N + n) \exp(j2\pi f_n t), \quad 0 \leq t \leq T \quad (1)$$

with  $N$  denoting the number of subcarriers used,  $M$ , the number of consecutive symbols evaluated,  $f_n$ , the individual subcarrier frequency,  $T$ , the OFDM symbol duration, and  $\{D(n)\}$ , called the 'complex modulation symbol', is the arbitrary data modulated with a discrete phase modulation technique e.g. phase-shift keying (PSK). Interference between the subcarriers is avoided based on the condition of orthogonality given by

$$f_n = n\Delta f = \frac{n}{T}, \quad n = 0, \dots, N-1 \quad (2)$$

In the presence of a reflecting object at the distance  $R$  from the RadCom platform with the relative velocity of  $v_{rel}$  which

results in the Doppler frequency of  $f_D$ , the receive OFDM symbol in time domain becomes

$$y(t) = \sum_{\mu=0}^{M-1} \sum_{n=0}^{N-1} D_r(\mu, n) \exp(j2\pi f_n t) \quad (3)$$

where

$$D_r(\mu, n) = D(\mu, n) \exp\left(-j2\pi f_n \frac{2R}{c_0}\right) \exp(j2\pi f_D t) \quad (4)$$

Based on (4), it can be seen that the distortions due to the channel is fully contained in the received complex modulation symbols  $\{D_r(n)\}$ , which is obtained at the receiver at the output of the OFDM demultiplexer prior to channel equalization and decoding. Thus comparing the transmitted signal  $\{D(n)\}$  with the soft-side time-variant received signal  $\{D_r(n)\}$  would yield the frequency domain channel transfer function. This is computed by simply performing an element-wise division

$$I_{div}(\mu, n) = \frac{D_r(\mu, n)}{D(\mu, n)} \quad (5)$$

In this manner, the acquisition of the range and Doppler profiles will be independent of the payload data.

#### A. Range Processing

For an object at the distance  $R$  from the radar, all subcarriers within the same reflected OFDM symbol will experience a linear amount of phase shift equivalent to two times the time length taken to travel the distance  $R$ . Assuming that the object is stationary, the corresponding channel transfer function is

$$I_{div}(n) = \exp\left(-j2\pi n \Delta f \frac{2R}{c_0}\right) \quad (6)$$

The channel impulse response containing the range profile of the object can then be determined by taking an inverse discrete Fourier transform (IDFT) of  $\{I_{div}(n)\}$

$$\begin{aligned} h(p) &= \text{IDFT}(\{I_{div}(n)\}) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} I_{div}(n) \exp\left(j2\pi \frac{n}{N} p\right), \quad p = 0, \dots, N-1 \end{aligned} \quad (7)$$

#### B. Doppler Processing

Unlike with communication signals, the reflected radar signal of an object moving with a relative velocity of  $v_{rel}$  will experience twice the amount of Doppler shift according to

$$f_D = \frac{2 v_{rel}}{\lambda} \quad (8)$$

where  $\lambda = c_0/f_c$ , with  $c_0$  being the speed of light and  $f_c$ , the carrier frequency.

This causes a phase shift of  $2\pi\mu f_D T_{sym}$  on every subcarrier of the  $\mu$ -th OFDM symbol, with  $T_{sym}$  being the total duration

of the transmitted symbol. It can be assumed that the Doppler affects all subcarriers by the same amount since the system bandwidth is much smaller than the carrier frequency. Thus for an object having a non-zero relative velocity to the radar, the corresponding time-varying channel transfer function (due to the Doppler only) is

$$I_{div}(\mu) = \exp(j2\pi f_D \mu T_{sym}), \quad 0 \leq \mu \leq M-1 \quad (9)$$

By taking the discrete Fourier transform (DFT) through the time axis the Doppler term can be estimated.

$$\begin{aligned} h(q) &= \text{DFT}(\{I_{div}(\mu)\}) \\ &= \frac{1}{M} \sum_{\mu=0}^{M-1} I_{div}(\mu) \exp\left(-j2\pi \frac{\mu}{M} q\right), \quad q = 0, \dots, M-1 \end{aligned} \quad (10)$$

### III. SYSTEM PARAMETERIZATION

There exists two major constraints pertaining the joint operation of the RadCom; the first is the subcarrier spacing and the second is the cyclic prefix (CP) length. The subcarrier spacing is limited by the Doppler frequency which has the potential to shift the alignment of the subcarriers thus destroying their orthogonality. Assuming a maximum relative velocity for typical traffic scenarios to be  $v_{rel} = 200$  km/h = 55.6 m/s, according to (8) this would result in the maximum Doppler shift of  $f_{D,max} = 8.9$  kHz for the carrier frequency  $f_c = 24$  GHz. Following a rule-of-thumb, it can be assumed that the subcarrier spacing of  $\Delta f > 10 f_{D,max}$  will ensure that the orthogonality remains.

To avoid inter-symbol interference (ISI), each elementary OFDM symbol is prepended with a prefix containing a repetition of some of its last values (thus rendering the time domain symbol 'cyclic'). This CP duration  $T_{CP}$  is governed by the *maximum excess delay* which is the maximum time difference between the arrival of the first and last propagation path in a multipath environment. Assuming that due to the high attenuation of the scattering process the maximum detectable distance is 200 m and taking into account that the reflected signal to the radar has to travel twice the distance, we obtain a time duration of 1.33  $\mu$ s which corresponds to the maximum delay difference between the propagation path of 400 m. Hence, setting  $T_{CP} > 1.33 \mu$ s would ensure that ISI is avoided.

As such, in order to obtain round numbers, the elementary symbol length of  $T = 11 \mu$ s was chosen which is equivalent to  $\Delta f = 90.909$  kHz. The CP length is chosen to be  $T_{CP} = 1/8 T = 1.375 \mu$ s, resulting in the total transmitted OFDM symbol duration of  $T_{sym} = T + T_{CP} = 12.375 \mu$ s.

#### A. Doppler resolution

The unambiguous Doppler frequency is related to the symbol duration  $T_{sym}$  and can be expressed by

$$v_{max} = \frac{\lambda}{2T_{sym}} \quad (11)$$

TABLE I  
OFDM SYSTEM PARAMETERS

Symbol	Parameter	Value
$f_c$	Carrier frequency	24 GHz
$N$	Number of subcarriers	1024
$\Delta f$	Subcarrier spacing	90.909 kHz
$T$	Elementary OFDM symbol duration	11 $\mu$ s
$T_{CP}$	Cyclic prefix duration	1.375 $\mu$ s
$T_{sym}$	Transmit OFDM symbol duration	12.375 $\mu$ s
$B$	Total signal bandwidth	93.1 MHz
$\Delta r$	Range resolution	1.61 m
$r_{max}$	Maximum unambiguous range	1650 m
$v_{max}$	Maximum unambiguous velocity	$\pm 252.5$ m/s
$M$	Number of evaluated symbols	256
$\Delta v$	Velocity resolution	1.97 m/s

Substituting the parameter values,  $v_{max} = 505$  m/s is obtained. Since Doppler can be both positive and negative, it should rather be expressed as  $v_{max} = \pm 252.5$  m/s. This then corresponds to around  $\pm 910$  km/h which is more than required for an automotive application.

The Doppler resolution is dependent on the number of evaluated symbols  $M$ , and amounts to

$$\Delta f_D = \frac{1}{MT_{sym}} \quad (12)$$

or in terms of velocity resolution, taking into account that twice the Doppler of the relative velocity occurs for a reflected wave

$$\Delta v = \frac{\lambda}{2MT_{sym}} \quad (13)$$

In principle, evaluating a greater number of OFDM symbols would give a finer velocity resolution. This is however impractical as moving objects must remain within one range resolution cell during the evaluation. Hence, by evaluating over  $M = 256$ , with the duration of 3.17 ms, an object traveling at the maximum unambiguous velocity would have traveled only 0.8 m, which is still within the resolution cell size of 1.61 m. With this, the velocity resolution becomes  $\Delta v = 1.97$  m/s or 7.1 km/h, guaranteeing an appropriate performance for practical automotive applications. All system parameters are summarized in Table I.

#### IV. SIMULATION RESULTS

For the verification of the range and Doppler processing presented in Section II, a simulation of two point-scatterers has been implemented in MATLAB. The simulation model comprises a transmitter, receiver and a point-scatterer channel model utilizing the parameters in Table I. The point-scatterer channel model computes the distance, velocity, phase and

attenuation for every predefined point scatterers and is able to support an arbitrary number of them. At the receiver, the received baseband signal is processed with the algorithm as described in (6) and (9) without prior channel equalization and decision.

Two identical point scatterers placed at  $R = 20$  m from the radar with the respective relative velocities of  $v_1 = 0$  m/s and  $v_2 = 7$  m/s were set in the simulation. At the FFT processing, a Hamming window is applied to minimize the sidelobes. The resulting radar image is as shown in Fig. 1. The two objects are clearly separable in range and Doppler and the sidelobes only occur due to the FFT processing. Fig. 2 shows a clearer picture of the objects possessing the same normalized reflected power (the cut of the radar image at  $R = 20$  m), versus their relative velocities of 0 m/s and 7 m/s.

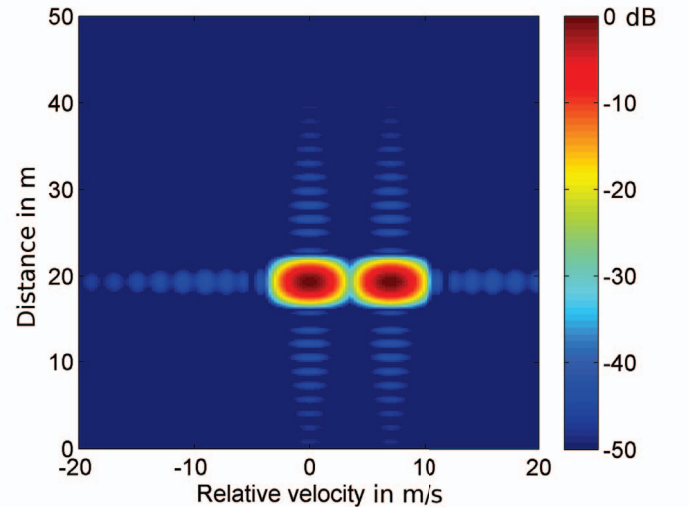


Fig. 1. Simulation: Radar image of 2 point scatterers

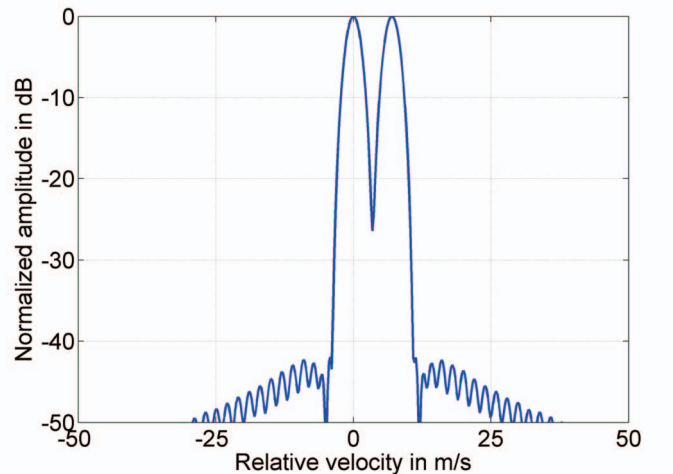


Fig. 2. The 2 point scatterers at  $R = 20$  m

## V. MEASUREMENT VERIFICATION

In verifying the simulation results, a measurement scenario emulating the simulation scenario has also been done. The measurement setup is as shown in Fig. 3. A stationary corner reflector with the radar cross section (RCS) of  $\sigma = 16.3 \text{ dBm}^2$  and a car moving at 25 km/h (7 m/s) are located 20 m away from the radar at the time of measurement. The resulting radar image is as shown in Fig. 4. Fig. 5 depicts the normalized reflected power from the stationary corner reflector and the moving car at  $R = 20 \text{ m}$ .



Fig. 3. Measurement scenario

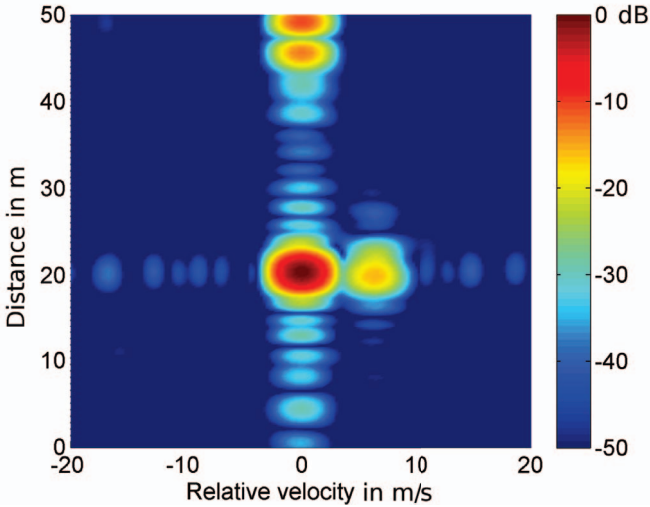


Fig. 4. Radar image of the measurement scenario

Thus it can be seen that the measured result corresponds highly with the simulated result. Although the reflection from the car is approximately 15 dB weaker than the reflection of the corner reflector, it is nevertheless sufficient to be distinguished in the radar image. Also seen in the figure are other reflecting objects which are the result of the metal road signs behind the car and reflections from the ground. Hence this demonstrates the capability of the processing algorithm in resolving multiple reflecting objects. The detailed

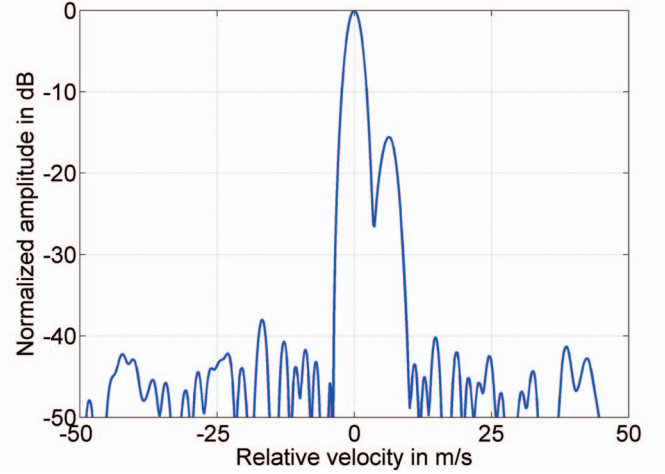


Fig. 5. Measured result at  $R = 20 \text{ m}$

measurement setup for performance verification can be found in [4].

## VI. CONCLUSION

A Doppler estimation algorithm has been presented and discussed in this paper. The algorithm allows for the estimation of velocities from multiple reflecting objects and can be used together with the described range estimation algorithm. It has also been shown that the algorithm functions irrespective of the coding used on the transmit signal due to its operation on the modulation symbols instead of on the baseband signals. A theoretical parameterization study has also been discussed in which suitable values have been derived for the operation of both the radar and the communication function for the typical application area of car-to-car communication. Finally, the simulation results and the corresponding measurement verification results have shown that this approach achieves a high dynamic range, limited only by the sidelobes from the Fourier Transformation which can be improved with a Hamming window.

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