A Study of SLM PAPR Reduction of OFDM Signals without Side Information

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***Abstract*—Selected mapping (SLM) is a well-known technique to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM). SLM generally requires the side information for each OFDM symbol, which leads to decrease the transmitting efficiency. In addition, false side information detection degrades the bit error rate performance significantly. In this paper, we propose a novel SLM scheme without any side information by devising a phase rotation and simple scrambler detection in the receiver. Computer simulation results show that the proposed scheme achieves excellent performances in terms of both PAPR reduction and BER.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been applied for various wireless applications due to high spectral efficiency and excellent anti-multipath fading capability. However, a large peak-to-average power ratio (PAPR) is a major disadvantage of OFDM, and thus various PAPR reduction techniques have been proposed for OFDM systems, such as clipping [1], block coding [2], partial transmit sequence (PTS) [3], and selected mapping (SLM) [4]. Among these methods, SLM has been well investigated because of its simple implementation and good PAPR reduction performance.

In SLM, a transmitted data is input into multiple scramblers and apply a different phase rotation in each independent scrambler. After the IFFT, the OFDM symbol with the lowest PAPR is selected and transmitted. Side information which scrambler is selected must generally be transmitted for each symbol to recover the data, which leads to decrease the transmitting efficiency. Moreover, false side information detection causes the whole data symbol lost. Hence, it requires heavy forward error correction (FEC), which decreases the transmitting efficiency. To solve this problem, several techniques without sending side information have already been proposed such as [6]-[8]. In this paper, we propose a novel SLM scheme without side information by devising a phase rotation and simple scrambler detection in the receiver.

The paper is organized as follows: In Section II, we briefly explain about OFDM and PAPR, and a conventional scheme. In Section III, the proposed SLM model is presented. In Section IV, we present simulation results for the proposed scheme. Finally, we summarize and conclude the paper in Section V.

II. CONVENTIONAL SCHEME

A. OFDM and PAPR

An OFDM signal $s(t)$ is represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t} \quad (0 \le t < T_s) \tag{1}$$

where N is the number of subcarriers, T_s is the duration of the OFDM symbol, and $\Delta f = 1/T_s$ is the subcarrier spacing. The PAPR of the OFDM signal is expressed as

$$\text{PAPR} = \frac{\max_{0 \le t \le T_s} |s(t)|^2}{E[|s(t)|^2]} \tag{2}$$

B. Conventional Scheme

Fig. 1 shows the conventional SLM scheme. In Fig. 1, transmitted data is input to the mapper and a transmitted symbol $\boldsymbol{X} = [X_0, X_1, \cdots, X_{N-1}]$ is generated in the mapper. Then the transmitted symbol $\boldsymbol{X}$ is input to U scramblers. In each scrambler, the transmitted symbol $\boldsymbol{X}$ is scrambled, resulting in a following scrambled symbol $\boldsymbol{X}^u$:

$$\boldsymbol{X}^u = \boldsymbol{X}\boldsymbol{P}^u \tag{3}$$

where

$$\boldsymbol{P}^u = \text{diag}\left[p_0^u, p_1^u, \cdots, p_{N-1}^u\right] \tag{4}$$

is a scramble matrix and

$$p_n^u \in \{\pm 1, \pm j\} \qquad (u = 0, 1, \cdots, N-1) \tag{5}$$

Each scrambled symbol $\boldsymbol{X}^u$ is modulated into an OFDM symbol $\boldsymbol{x}^u$ in the OFDM modulator. Then the OFDM symbol with the lowest PAPR among all $\boldsymbol{x}^u$ is selected and transmitted. If the symbol $\boldsymbol{x}^m$ is transmitted, the selected number m is also transmitted as a side information. In this scheme, $\log_2 U$ bits are required for the side information to recover the data in the receiver. This causes a decrease of transmission efficiency.

In the receiver, the OFDM demodulation is performed first. A demodulated symbol $\hat{\boldsymbol{X}}$ is descrambled based on a received side information $\hat{m}$, resulting in a received symbol $\boldsymbol{Y}$:

$$\boldsymbol{Y} = \hat{\boldsymbol{X}}\boldsymbol{P}^{\hat{m}*} \tag{6}$$

After the data decision, a received data is finally output. In the conventional scheme, if the received side information $\hat{m}$ is incorrect, the demodulated symbol $\boldsymbol{Y}$ is incorrectly descrambled. This causes a significant BER degradation.

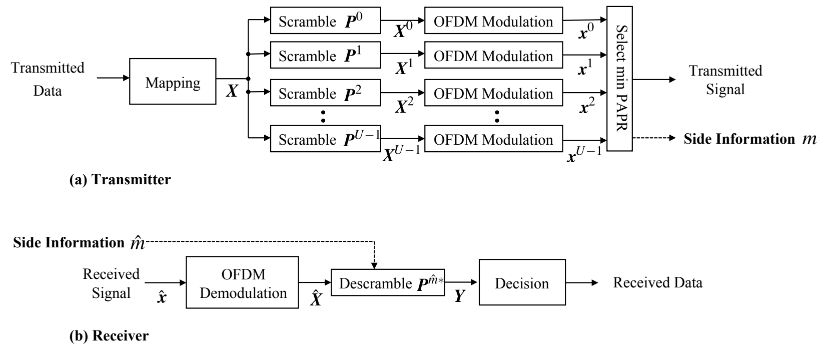

Fig. 1. Conventional SLM with side information

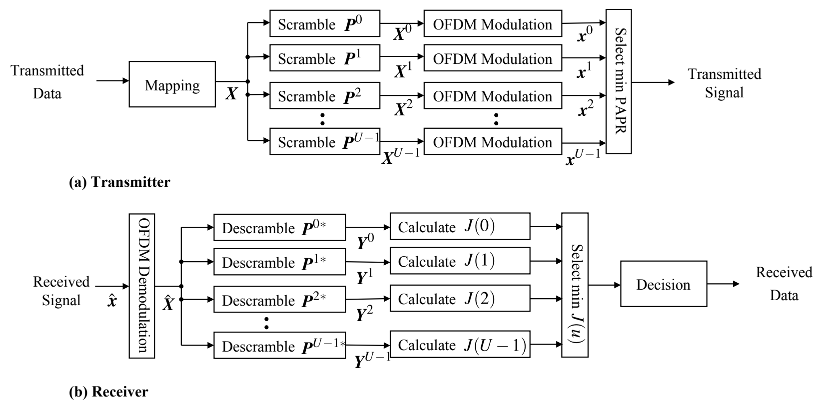

Fig. 2. Proposed SLM Model

III. Proposed Scheme

Fig. 2 shows the proposed scheme. In the transmitter, transmitted data is input to the signal mapper and a transmitted symbol $\boldsymbol{X} = [X_0, X_1, \cdots, X_{N-1}]$ is generated. Then the transmitted symbol $\boldsymbol{X}$ is input to U scramblers simultaneously. In the u-th $(U = 0, 1, \cdots, U-1)$ scrambler, the transmitted symbol $\boldsymbol{X}$ is scrambled, resulting in a following scrambled symbol $\boldsymbol{X}^u$:

$$\boldsymbol{X}^u = \boldsymbol{X}\boldsymbol{Q}^u \tag{7}$$

where

$$\boldsymbol{Q}^u = \mathrm{diag}\left[e^{j\phi q_0^u}, e^{j\phi q_1^u}, \cdots, e^{j\phi q_{N-1}^u}\right] \tag{8}$$

is the u-th scramble matrix, $\phi \in [0, 2\pi)$ is a rotation phase, and $q_i^u \in \{0, 1\}$. The rotation phase ϕ is decided according to the primary modulation (signal mapping). Hereafter, the sequence $\{q_n^u\}$ is called the u-th scrambler sequence. In the OFDM modulators, each scrambled symbol $\boldsymbol{X}^u$ is modulated into U OFDM symbols $\boldsymbol{x}^u$. Then, the OFDM symbol with the lowest PAPR among all $\boldsymbol{x}^u$ is selected for transmission.

In the receiver, the OFDM demodulation is performed first. Then a demodulated symbol $\hat{\boldsymbol{X}}$ is input to U descramblers simultaneously. In the u-th $U = 0, 1, \cdots, U-1$ descrambler, the demodulated symbol $\hat{\boldsymbol{X}}$ is descrambled, resulting in the following u-th candidate symbol $\boldsymbol{Y}^u$:

$$\boldsymbol{Y}^u = \hat{\boldsymbol{X}}\boldsymbol{P}^{u*} \tag{9}$$

After descrambling, by calculating an evaluation function depending on the phase rotation, we can estimate which scrambled symbol is selected and transmitted in the transmitter. Let θ_i^u denote a phase of the subcarrier of the u-th candidate symbol $\boldsymbol{Y}^u$. In the case of quaternary phase-shift keying (QPSK) mapping, the evaluation function $J(u)$ is defined by

$$J(u) = \sum_{i=0}^{N-1} \left| \left(\theta_i^u \bmod \frac{\pi}{2}\right) - \left(\phi \bmod \frac{\pi}{2}\right) \right| \tag{10}$$

Fig. 3 (a)-(d) shows an example for calculating $J(u)$. The received signal phase is assumed to be θ_i^u as shown in Fig. 3 (a). First, in Fig. 3 (b), the received signal phase θ_i^u is normalized into a first quadrant by performing $(\theta_i^u \mathrm{mod}\ \pi/2)$. Next, the phase difference from a correct signal is calculated by subtracting $(\phi \quad \bmod \pi/2)$ shown in Fig. 3 (c) and its absolute value shown in Fig. 3 (d) expresses an absolute phase error.

Thus, the evaluation function $J(u)$ expresses the total absolute phase error from the correct mapping. In other word, the evaluation function $J(u)$ is the phase likelihood. Therefore, it is expected that the evaluation function takes a minimum value when the candidate symbol is correctly descrambled. That is, by selecting the candidate symbol with the smallest evaluation function, we can estimate the transmitted scrambled symbol without any side information. What is more, the evaluation function can realize only adder and subtracter, resulting in a

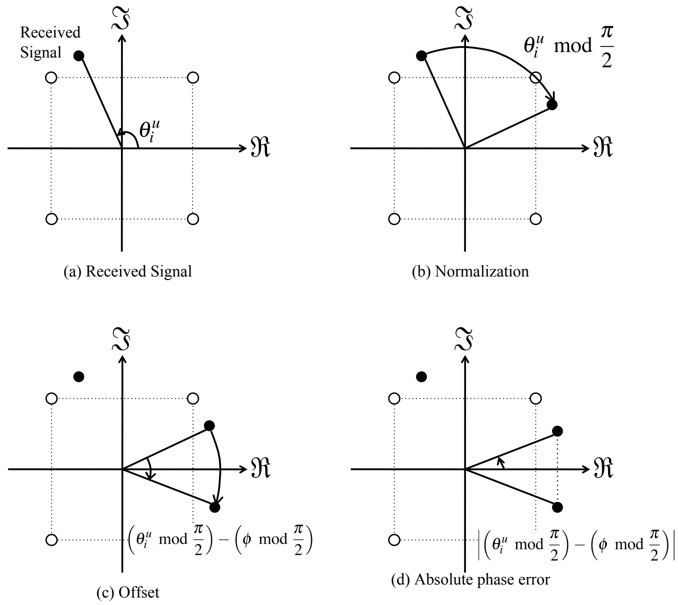

Fig. 3. Example of the evaluation function $J(n)$

TABLE I
SIMULATION CONDITIONS

Modulation (mapping)	QPSK
Number of subcarriers N	64, 128, 512, 1024
Scramble sequence q_i^u	Gold code
Oversampling factor	4
Channel model	AWGN

simple implementation. After the data decision, a received data is finally output.

IV. COMPUTER SIMULATION

In this section, we evaluate the performance of the proposed SLM scheme for QPSK mapping under the condition shown in Table. I.

A. Optimal rotation phase

We define a 'false scrambler detection' as the case that $J(m)$ does not take the minimum value in the receiver though the OFDM symbol $\boldsymbol{x}^m$ is selected in the transmitter. If the false scrambler detection occurs, the incorrect descrambled symbol is output and the BER performance is degraded. Both a probability of the false scrambler detection P_f and the performance of the PAPR reduction depend on the rotation phase ϕ. In the following, we decide an optimal rotation phase ϕ_{opt} in the sense of minimizing P_f and maximizing the PAPR reduction performance. First, Fig. 4 shows a rotation phase dependence of P_f, for $\phi \in [-\pi, \pi]$. The system parameters are: $N = 64, E_b/N_0 = 1.0$ dB, AWGN channel, and QPSK mapping. As can be seen from Fig. 4, the probability of the false scrambler detection depends on the phase rotation ϕ and takes the minimum one when ϕ equals to $\pm\pi/4, \pm 3\pi/4$.

Next, Fig. 5 shows a rotation phase dependence of a complementary cumulative distribution function (CCDF) of

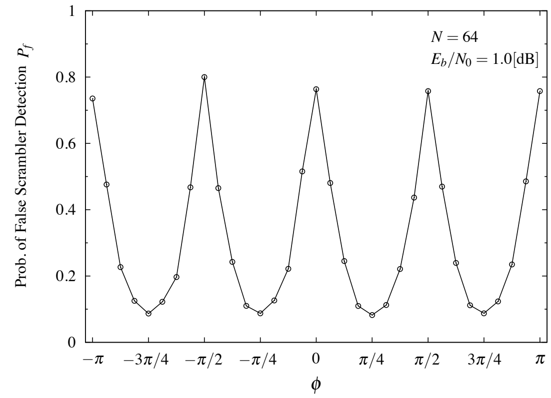

Fig. 4. Rotation phase dependence of P_f

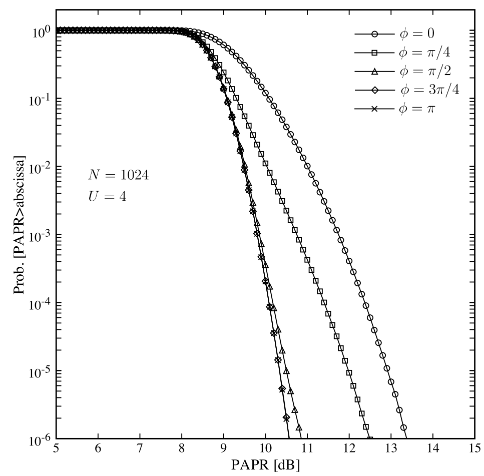

Fig. 5. Rotation phase dependence of CCDF

PAPR for $N = 1024$ and $U = 4$ with the phase rotation $\phi = 0, \pi/4, \pi/2, 3\pi/4, \pi$. From Fig. 5, a good PAPR performance is obtained when $|\phi| \geq 3\pi/4$.

From these results, we conclude that $\phi_{opt} = \pm 3\pi/4$ for the QPSK mapping.

B. PAPR reduction performance

Hereafter, we fix the rotation phase ϕ at its optimal value $3\pi/4$. Fig. 6 shows the CCDF of PAPR for $N = 1024$ with $U = 1, 2, \cdots, 5$. From Fig. 6, the PAPR reduction performance of the proposed scheme is identical with that of the conventional scheme.

C. False scrambler detection probability and BER performance

Fig. 7 shows the probability of the false scrambler detection P_f, for $N = 64, 128, 512, 1024$ and $U = 4$. From Fig. 7, it is confirmed that the proposed scheme achieves the good scrambler detection probability for $N = 512, 1024$. Even for $N = 64, 128$, the proposed scheme performs well at $E_b/N_0 \geq 5$ dB.

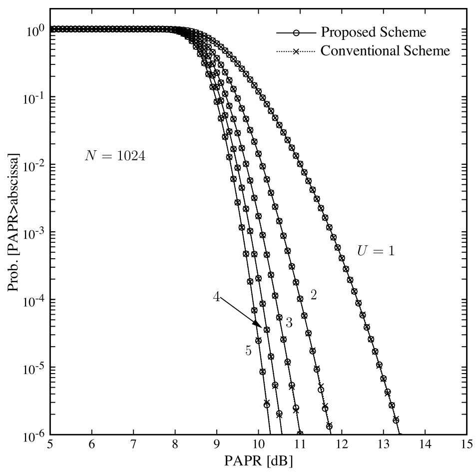

Fig. 6. CCDF of PAPR

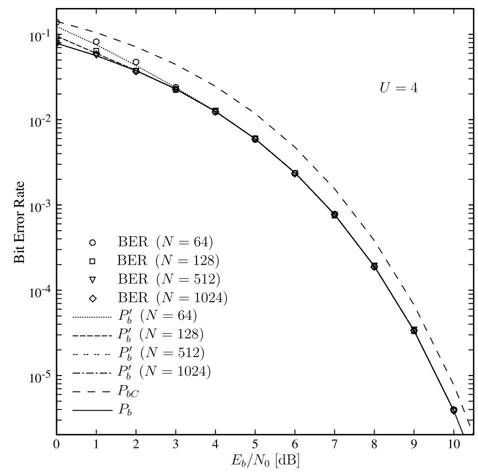

Fig. 8. BER performance of the proposed scheme

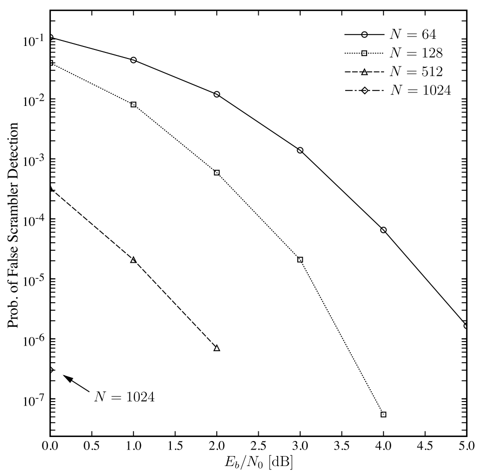

Fig. 7. Probability of the false scrambler detection, P_f

Finally, we consider the BER performance of the proposed scheme. As mentioned above, the false scrambler detection degrades the BER performance. The BER P_b' considering the false scrambler detection probability P_f can be expressed as

$$P_b' = P_b(1 - P_f) + \frac{1}{2}P_f \tag{11}$$

where P_b is the theoretical BER without the false scrambler detection.

Fig. 8 shows the BER performance of the proposed scheme and the BER P_b' obtained by Eq. (11), for $N = 64, 128, 512, 1024$ and $U = 4$. Because the conventional scheme requires side information of two bits at $U = 4$, the BER of the conventional scheme can be expressed as

$$P_{bC} = P_b(1 - P_b)^2 + \frac{1}{2}(2 - P_b)P_b \tag{12}$$

From Fig. 8, the proposed scheme is expected to get better BER performance than the conventional one. Also, it is confirmed that the BER of the proposed scheme agrees well with P_b' and the false scrambler detection does not influence the BER performance at $E_b/N_0 \geq 3$ dB.

V. Conclusion

In this paper, we propose the novel SLM scheme without sending explicit side information. The proposed scheme defines the arbitrary phase rotation according to the primary modulation in the scrambler and the evaluation function corresponding to the phase rotation in the receiver. The evaluation function that enables to detect the scrambler can realize only adder and subtracter, resulting in a simple implementation. Finally, results of the computer simulation show that the proposed scheme achieves an excellent performance in terms of both BER and PAPR reduction.

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