A Combined PTS-Companding Scheme for PAPR Reduction in OFDM System

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***Abstract*—One of the main disadvantages of Orthogonal Frequency Division Multiplexing (OFDM) communication system is its high peak-to-average power ratio (PAPR). A PAPR reduction scheme for OFDM system which combines the partial transmit sequence (PTS) and the companding transformation is proposed. It takes advantage of the PTS, which does not cause signal distortion with linear transform, and the companding transformation, which is simple and direct. The simulation results show that the new scheme improves the PAPR performance about 0.5dB and 6dB respectively compared to the PTS and companding transformation, and ensures the bit error rate (BER) and transmission rate at the same time.**

Keywords*—OFDM; PAPR; PTS; Non-linear companding transformation

I. INTRODUCTION

OFDM is widely used technique for multicarrier modulation [1]. It improves not only the immunity to the inter-symbol interference (ISI) and multi-path interference, but also the spectral efficiency. However, the signal of a multicarrier transmission system is the superposition of a series of signals of its sub-channels, so the OFDM signal may have a higher PAPR if they are in the same phase. In order to avoid signal distortion, the power amplifier in the system should operate linearly within a wide range.

Recently, there are three methods usually used to reduce the PAPR of OFDM signals: the pre-distortion technique, the encoding method and the probability method [2]. With the pre-distortion technique, the signals are transformed by nonlinear processing in advance, then the high-power signals with high peaks are pre-distorted. The pre-distortion technique is simple and direct, but the nonlinear transformation introduces in-band noise and out-band interference. Among pre-distortion techniques, the companding method can reach a comparatively lower BER [3]. The encoding method adopts variable encoding schemes to produce different code groups, and then selects the code group with smaller PAPR as the OFDM symbol to transmit data. But the encoding method is the most complex and it is tedious to encode and decode. Moreover, the information rate drops so quickly that it is only suitable for small sub-carriers. The probability method [4] aims at reducing the occurrence probability of the peak signals, but it does not guarantee PAPR reduction. Moreover it may introduce a certain amount of information redundancy. Two representative kinds of probability methods are the partial transmit sequence (PTS) and the selective mapping (SLM). Both of them reduce the probability of in-phase superposition by changing the phase distribution of the original signals. They can reduce the PAPR of signals effectively without distortion.

The paper is organized as following. Section II presents the theoretical analysis, and describes the definition of PAPR, the principle of traditional PTS and μ-law Companding transformation. The proposed PTS-Companding methods is presented in section III. The simulation results and conclusion are given in section IV and V respectively.

II. THEORETICAL ANALYSIS

A. PAPR

PAPR is the ratio between the maximum power and the average power of the signal in one symbol period. The PAPR of OFDM system is defined as [5]

$$\mathrm{PAPR}(dB) = 10\log_{10}\frac{\max\{|x_n|^2\}}{E\{|x_n|^2\}} \tag{1}$$

where x_n represents the discrete OFDM signals after N-point Inverse Fast Fourier Transform (IFFT), that is

$$x_n = \frac{1}{\sqrt{N}}\sum_{\mathrm{k}=0}^{N-1} X_k e^{j2\pi nk/N} \tag{2}$$

where N denotes the number of sub-carriers, and $X_k(0 \le \mathrm{k} \le N-1)$ denotes the k-th complex modulated symbol in a block of N information symbols.

The complementary cumulative distribution function (CCDF), which indicates the probability that the PAPR is above a certain threshold, is generally used to measure the PAPR reduction capability of a communication system.

B. Traditional PTS

The key to the probability method is linear transformation $Y_n = A_n X_n + B_n (1 \le n \le N)$, where X_n is the original input data before IFFT in frequency domain, and Y_n is the output after linear transformation [1]. The aim of the probability method is to find the N-point vector A and B, which can make occurrence probability of the peak values of the transmission signals after IFFT smaller. The PTS method firstly sets the vector B to zero,

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and then select an appropriate vector as A to reduce the occurrence probability of the peak signals. Also the amplitude of the vector A is limited to the unit amplitude, that is, the linear transformation of PTS [6,7] is only phase rotation.

In the PTS-OFDM system, there are three ways of data partitioning: adjacent partition, interleaved partition and random division. It has been proved that the random division is the best and the interleaved partition is the worst when it comes to the PAPR performance, however the interleaved partition is the least complex. Besides, the number of groups V and the number of phase factors P also affect the PAPR in a PTS system. The more V and P, the better the PAPR, but at the cost of the system complexity.

C. *μ-law Companding Transformation*

The companding transformation mostly compresses the high power signals and amplifies the low power signals with non-uniform quantization functions, such as the μ-law and A-law companding functions, to sustain the average power. It can reduce the PAPR and enhance the anti-disturbance ability of the low power signals. It is required that, at the transmitter, the signals after orthogonal modulation in time domain are transformed with the companding transformation that is less computationally intensive compared with PTS. At the receiver, the signals are restored by the corresponding inverse companding transformation before demodulation.

The μ-law companding transformation [8] is a mainstream one. It takes advantage of a nonlinear transform function based on the μ-law non-uniform quantization in speech processing. It must be noted that this technique introduces in-band noise and out-band interference at the receiver because the inverse companding transformation amplifies the large signals as well as their noise, although the small signals along with their noise are attenuated. It leads to a worse BER [9].

III. PROPOSED SCHEME OF PTS-COMPANDING

In this paper, a combined scheme combining the PTS and companding transformation is proposed. Firstly, the signals are transformed with PTS to reduce occurrence probability of most high peak signals. Then the signals go through the companding transformation to be further dealt with the remaining high peak signals. The system architecture is shown Fig. 1.

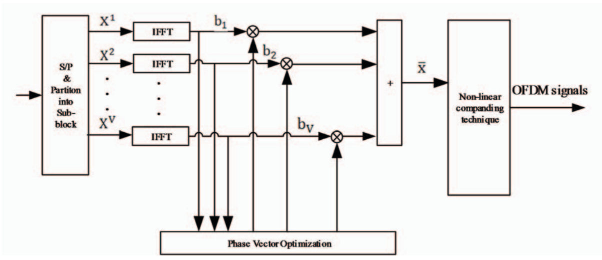

Fig. 1. system structure of the PTS-Companding

In Fig .1, the input data X are divided into V sub-blocks with a length N, represented by $\{X^v, v=1,2,\ldots,V\}$. The sub-blocks should be ensured not to overlap each other, no matter how they are partitioned. Then

$$X = \sum_{v=1}^{V} X^v \tag{3}$$

Each data block is multiplied by its phase factor b_v respectively after N-point IFFT, and the phase factors of the different data blocks are statistically independent. It indicates that the V phase factors which the rotation vector contains are statistically independent.

$$x = \sum_{v=1}^{V} \left(b_v \cdot IFFT(X_v)\right) \tag{4}$$

Phase vector optimization is to choose the appropriated weighted-coefficients $[b_1, b_2, \cdots, b_V]$ to make the peak values of the powers in formula (4) to be minimum, therefore the optimization of weighted-coefficients should meet the equation written as

$$\{b_1, b_2, \cdots, b_V\} = \underset{\{b_1, b_2, \cdots, b_V\}}{\arg\min} \left(\max_{1 \le n \le N} |x|^2 \right) \tag{5}$$

where argmin(.) represents the condition that make the function achieve the minimum value. After the above operation, we adopt the μ-law companding transformation to process the signal x further, making the PAPR better. Then the signal now becomes x′.

$$x' = \frac{Ax \ln\left(1 + \mu \dfrac{|x|}{A}\right)}{|x| \ln(1+\mu)} \tag{6}$$

where μ is the comapanding coefficient, and A is a normalization constant. We take A as max(|x|).

At the receiver, the received signal y after serial-to-parallel conversion is transformed with the μ′-law inverse companding transformation.

$$y' = \frac{A' y}{\mu' |y|} \left\{ \exp\left(\frac{|y| \ln(1+\mu')}{A'} \right) - 1 \right\} \tag{7}$$

where μ′ is the inverse companding coefficient, and A′ is the normalization constant. We take A′ as max(|y|).

Then the signals y′ are divided into V sub-vectors y^v of length N in sequence. With the carrying side information $[b_1, b_2, \cdots, b_V]$, the sub-vectors y^v are divided by their rotation factor b_v respectively, then go through N-point FFT. Finally the data blocks are summed up as shown in formula (8).

$$Y = \sum_{v=1}^{V} FFT\left(y^v / b_v\right) \tag{8}$$

The combined scheme can improve PAPR performance, but the BER and transmission rate are maintained. The method of reducing the BER is discussed in part 1), and how to ensure the transmission rate is presented in part 2).

1) Optimization of the inverse companding coefficient μ′. In order to improve the poor BER performance caused by the companding transformation, a companding coefficient μ′ of the receiver smaller than that of the transmitter [10], namely, μ′=Kμ (K<1) should be chosen to reduce the noise gain of the large signals and ensure a certain BER.

2) Simple search of the phase space. The PTS selects an appropriate rotation vector to decrease the occurrence probability of peak signals. The number of all the phase combinations $\{b_1,b_2,b_3,\cdots b_V\}$ that the exhaustive search needs to traverse is P^V, where V is the group number and P is the phase factor number. So the more the V and P, the more complex the system.

In order to reduce the amount of computation, the simple search should be adopted when the number of phase combinations is large. Firstly we set all the weighted coefficient b_v=1(v=1,2,3,…,V) and let b_1 traverse every vector in the phase space to select the optimal one such that the PAPR reduction is the best recently. Then we select the optimal values for the remaining weighted coefficients b_v (v=2,3,...,V) similarly to complete an iterative search. With the same method the weighted coefficient optimization repeats till the required iterative search number is reached. Generally the iterative search repeats 3 times and the PAPR performance does not improves significantly after 3 repetition. Although this simple search affects the PAPR reduction, it greatly enhances the transmission rate when P or V is large.

IV. Simulation And Analysis

The proposed scheme is extensively simulated by MATLAB with the parameters shown in Table I. The PAPR and BER performance of the new scheme is compared with the traditional PTS and μ-law companding transformation. A wave audio signal with size of 11KB is used as the input source in simulation.

TABLE I. SIMULATION PARAMETERS

Method	Parameters			
PTS-Companding	sub-carrier number N			64
	multipath channel	delay	d1	6
			d2	10
		attenuation factor	a1	0.32
			a2	0.28
	data partitioning			random division
	group number V			4
	phase factor number P			2
	companding coefficient μ			3
	inverse companding scale K			0.4
	simple search			No
PTS	the same as the PTS-Companding			
Companding Technique	the same as the PTS-Companding			
SLM	group number V			4
	phase factor number P			4
tone reservation (TR)	reserved carrier number			8
	attenuation factor			0.5
	iteration time			10
Clipping and Filtering	clipping ratio (CR)			4
	clipping cycle time			4

TABLE II. SIMULATION RESULTS

Method	Performance	
	BER(%)	Elapsed Time(s)
Original	0.09	0.61
PTS	0.09	0.86
Companding Technique	0.14	0.21
Companding Technique/μ=81[a]	5.99	0.22
SLM	0.05	0.37
TR	0.12	15.31
Clipping and Filtering	0.89	0.23
PTS-Companding	0.13	1.22
PTS-Companding/μ=81	5.31	1.23
PTS-Companding/K=0.7	0.18	1.21
PTS-Companding/K=1	0.25	1.22
PTS-Companding/V=2	0.15	0.59
PTS-Companding/V=8	0.15	8.23
PTS-Companding/P=4	0.15	7.18
PTS-Companding/P=8	0.13	103.11
PTS-Companding/P=4 simple search	0.17	2.09
PTS-Companding/P=8 simple search	0.15	3.24

[a] The parameters are consistent with Table I unless otherwise indicated.

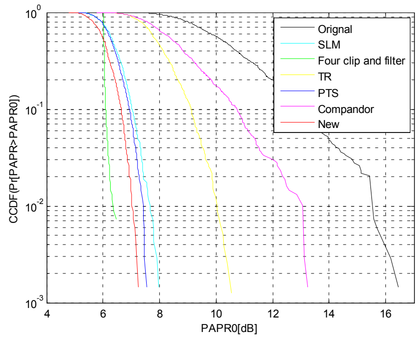

Fig. 2. PAPR performance comparison of several algorithms

Fig. 2 shows the comparison result of the PAPR performance between the new scheme and the traditional ones. The BER and elapsed time are summarized in Table II. As shown in Fig. 2, the PAPR differences between the new method and the clipping, SLM, TR, PTS, companding transformation are +0.9dB, -0.8dB, -3.2dB, -0.45dB, -6dB respectively when the probability is of 10^{-2}. Although the PAPR performance of the clipping is better, its BER is up to 6 times higher. When the new scheme P is adjusted to 4, the CCDF distribution curve is very close to that of the clipping, and the BER of the new scheme is only increased by 0.02%. Table II shows that the new scheme can ensure a certain BER and transmission rate.

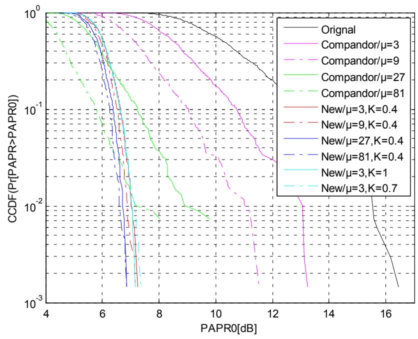

Fig. 3. PAPR performance comparison under different μ

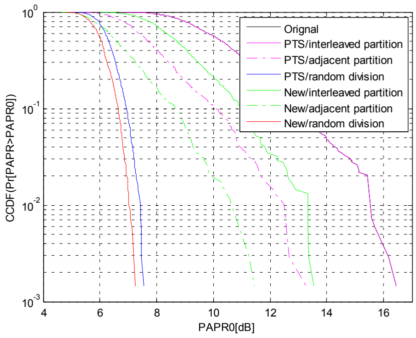

Fig. 4. PAPR performance comparison under different grouping methods

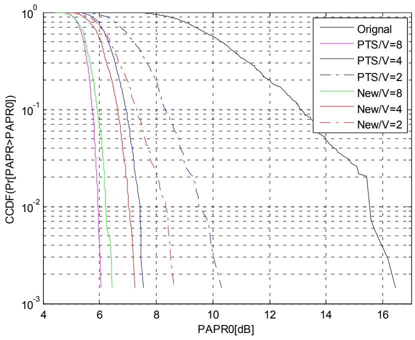

Fig. 5. PAPR performance comparison under different group numbers

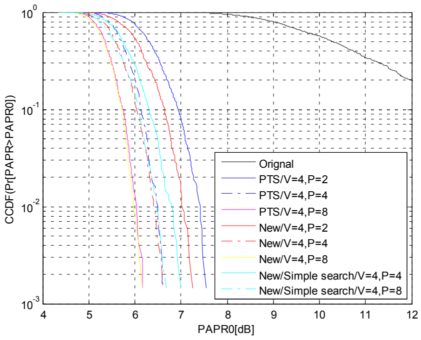

Fig. 6. PAPR performance comparison under different phase numbers

Fig. 3 shows the PAPR performance in the four cases where the μ are 3, 9, 27 and 81 respectively. For the traditional μ-law companding transformation, the PAPR performance gets better with increasing of μ but at the cost of the signal quality. When μ is 81, the BER is 5.99%. For the new scheme, there is a positive correlation between the μ and the PAPR performance generally, but the PAPR reduction is not so obvious. Its BER is 5.31% when μ is 81 with which signal distortion is as bad as the companding transformation. In addition, as shown in Fig. 3, when μ is 3 and the inverse companding scale K is 0.4, 0.7 and 1, the BER is 0.13%, 0.18% and 0.25% respectively although the PAPR does not change much, which indicates that a smaller inverse companding coefficient μ′ at the receiver can improve the signal quality.

Fig. 4-6 shows the PAPR due to the parameters involved in the PTS, that is, the ways of data partitioning, the group number V and the phase factor number P. The results shown in Fig. 4 prove that the best PAPR performance can be obtained by random division for the new scheme and PTS. As we can see from Fig. 5, the new scheme can reduce the PAPR of OFDM signals effectively, and the more the group number V, the better PAPR, but at the cost of the system complexity. The elapse time are 8.23s, 1.22s and 0.59s respectively when the V are 8, 4 and 2.

In Fig. 6, we change the phase factor number P for comparison. When P gets larger, there are more choices for the weighted coefficients and the PAPR gets better, but at the cost of the transmission rate of the system. Because the more the V or P, the more the combinations, and the more the time to traverse the combinations for finding the optimal solution. However, when we adopt the simple search, the PAPR reduction weakens. The elapsed time of the exhaustive search is 7.18s and 103.11s respectively when V=4/P=4 and V=4/P=8, while the elapsed time of the simple search are 2.09s and 3.24s respectively. There is a trade-off between the complexity and PAPR performance.

V. Conclusion

This paper proposes combined PTS-Companding methods to improve the PAPR in OFDM system. The proposed scheme is proved by extensive MATLAB simulation. The PAPR performance improves around 0.5dB and 6dB respectively when compared to the traditional PTS and μ-law companding transformation with the same parameters, and the new scheme also ensures the BER and transmission rate.

References

[1] Li Li. Principle, Application and Simulation of MIMO-OFDM System [M]. Beijing: China Machine Press, 2014.

[2] S. Luo, F. Yan. Research and Comparison on Technology of PAPR Reducing for OFDM [J]. Shanxi Electronic Technology, 2010 (1): 44-46.

[3] X. Wang, T. Hung, C. S. Ng. Reduction of peak-to-average power ratio of OFDM system using a companding technique [J]. IEEE Trans. on Broadcasting, 1999, 45(3): 303-307.

[4] A. S. Mohamad, Abdel Halim Zekry, Fatama Newagy. A combined PTS-SLM Scheme for PAPR Reduction in Multicarrier Systems. Global High Tech Congress on Electronics (GHTCE), 2013 IEEE, pp.146-150.

[5] H. Ochiai, H. lmai. On the distribution of the peak-to-average power ratio in OFDM signals [J]. IEEE Transactions on Communications, 2001, 49(2):282-289.

[6] Jianping Wang, Ying Guo, Xianwei Zhou. PTS-Clipping Method to Reduce the PAPR in ROF-OFDM system [J]. IEEE Transactions on Consumer Electronics, vol. 55, no. 2, pp.356-359, MAY 2009.

[7] Yuzhong Jiao, Xin'an Wang, Xuewen Ni, Ying Xu, Xuejiao Liu. A Novel PTS by Using the Scheme of Peaks Counteraction [J]. Microelectronics and Computer, 2010,27 (7): 35-39.

[8] Tao Jiang, Yang Yang, Yonghua Song. Exponential companding technique for PAPR reduction in OFDM systems [J]. IEEE Transactions on Broadcasting, 2005, 51(2): 244-248.

[9] Hewei Yu. Effect of Companding Arithmetic for Reducing PAPR on Symbol Error Rate of an OFDM System [J]. Telecommunication Engineering, 2006,46 (5): 34-39.

[10] M. Friese. Multicarrier Modulation with Low Peak-to-Average Power Ratio [J]. Electronics Letters, 1996, 32(8):713-714.