Pilot Assignment for PTS-OFDM with Channel Estimation for PAPR Reduction

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***Abstract*—The partial transmit sequence (PTS) technique is an effective method for the peak-to-average power ratio (PAPR) reduction of orthogonal frequency division multiplexing (OFDM) signals. Recently, applying channel estimation to PTS-OFDM with different pilot assignment schemes were proposed for removing the requirement of transmitting the side information (SI) to the receiving ends. This paper proposes a novel fixed end pilot assignment (FEPA) scheme which always assigns pilots at both ends of each subblock in PTS-OFDM, and thus avoids the requirement of linear extrapolation in channel estimation. PTS-OFDM with the proposed method for channel estimation may achieve the same bit error rate performance (BER) as that of PTS-OFDM with perfect SI. Simulation results show that the novel pilot assignment scheme keeps the same performance as the traditional PTS but provides better performance in the PAPR reduction and BER compared to other pilot assignment schemes.**

Keywords— OFDM; PAPR; PTS; side information; channel estimation; pilot assignment

I. INTRODUCTION

OFDM technique is attractive in wireless communication with high-speed data transmission for its effectiveness against the frequency selective fading channel. However, one of the major drawbacks of OFDM system is the high peak-to-average power ratio (PAPR) of the transmit signals as a result of the large dynamic range of OFDM output signals. High PAPR OFDM signals will reduce the efficiency of the power amplifier and increase the complexity of the analog-to-digital and digital-to-analog converters. Many proposals have been provided to cope with the high PAPR problem of OFDM signals, such as amplitude clipping [1], tone reservation (TR) [2], active constellation extension (ACE) [3], selected mapping method (SLM) [4], and partial transmit sequence (PTS) [5], [6]. The general PTS (T-PTS) technique is effective in the PAPR reduction of OFDM signals at the price of transmitting the side information (SI) to the receiving end. Note that SI only represents the phase factor of the optimal combined signal and will bring about a diminution in data rate. To manage this problem, a channel estimation method using pilots for PTS-OFDM was proposed in [7]. Denoting the virtual channel frequency response as the combination of the channel frequency response and the optimal phase factor from the general PTS technique, this estimation method simply utilized the comb type pilot assignment to recover original data symbols. In this scheme, some pilots at the edge of subblocks must apply the linear extrapolation for channel estimation. The information for channel estimation using linear extrapolation is not as accurate as that using linear interpolation. Hence, the bit-error-rate (BER) performance of the system due to estimation errors from linear extrapolation is much worse than that from linear interpolation. Although the novel comb type pilot assignment scheme was proposed for improving the BER performance, the use of linear extrapolations is still inevitable for some pilots [7-9].

This paper proposed the fixed end pilot assignment (FEPA) scheme for channel estimation in PTS-OFDM systems for PAPR reduction. In this novel scheme, pilots are assigned fixedly to both ends of each subblock. The remaining pilot tones are inserted with equal spacing. Hence, estimates of the frequency channel response on data symbols are obtained by linear interpolations for channel estimation [9]. Simulations were conducted such that the BER performance and PAPR reduction of the PTS-OFDM system with channel estimation using the proposed FEPA scheme were evaluated and compared to general PTS-OFDM and that with different pilot assignment schemes for channel estimation. Our simulation results showed that the proposed method is effective in PAPR reduction and can achieve more accurate channel estimations.

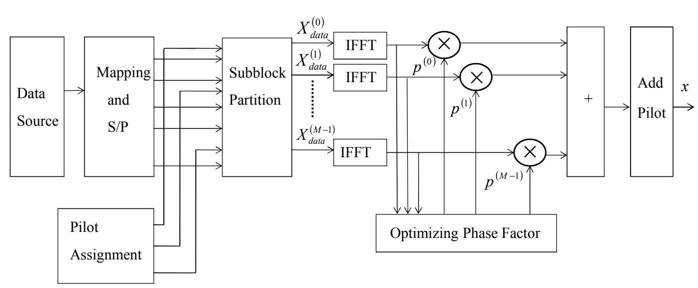

Fig. 1. Block diagram of PTS-OFDM with channel estimation

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II. SYSTEM MODEL

A. PTS-OFDM with Channel Estimation Scheme

In an OFDM system with N subcarriers, the discrete-time transmitted OFDM signal is given by

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \qquad 0 \le n \le N-1, \tag{1}$$

where N is the number of subcarriers and $X_k, k = 0,1,\cdots,N-1$, are input data symbols modulated by PSK or QAM. The PAPR of the transmitted OFDM signal in dB is defined as

$$PAPR \overset{\Delta}{=} 10 \cdot \log_{10}\left(\frac{\max|x_n|^2}{E\left[|x_n|^2\right]}\right), \tag{2}$$

where $E[\cdot]$ denotes the expected value operator.

Fig. 1 shows the block diagram of the partial transmit sequence (PTS) scheme with channel estimation using assigned pilots. In this scheme, the pilot tones are inserted to estimate the frequency response of the fading channel. The pilot tone $\mathbf{X}_{pilot}$ stays unchanged and the real data symbol $\mathbf{X}_{data}$ is partitioned into M disjoint subblocks for searching the minimum PAPR values.

Several subblock partition techniques are commonly used, including the adjacent partition, interleaved partition and random partition. In general, the random partition method has the best effects in PAPR reduction performance compared to the other two partition techniques [5]. However, it is too complicated to implement. Recently, some pseudo-random methods were proposed for reducing complexity and keeping PAPR reduction performance the same as that from random partition method [10-12].

B. Phase Factor Search forMinimum PAPR of PTS-OFDM

After the IFFT operation, the time domain signal of $\mathbf{X}_{data}^{(m)}$, $m = 0,1,\cdots,M-1$, is the partial transmit sequence, written as $\mathbf{x}_{data}^{(m)} = \left[x_{data,0}^{(m)}, x_{data,1}^{(m)},\cdots,x_{data,N-1}^{(m)}\right]$, $m = 0,1,\cdots,M-1$. Each partial transmit sequence $\mathbf{x}_{data}^{(m)}$ is reformed by multiplying a phase factor $p^{(m)}$ and then the time domain signal $\mathbf{x}$ is assembled by added together these combined sequences, expressed by

$$\mathbf{x} = \sum_{m=1}^{M} p^{(m)} \cdot \mathbf{x}_{data}^{(m)}, \tag{3}$$

where $p^{(m)} \in \Gamma$. Here $\Gamma = \left\{e^{j2\pi\omega/\Omega} \middle| \omega = 0,1,\cdots,\Omega-1\right\}$ is the set of allowed phase factors and Ω is the number of allowed phase factors. In search of $M-1$ phase factors for minimum PAPR of the transmitted signals must be performed. Therefore, a total of $\Omega^{(M-1)}$ searches are required to find the optimum set of phase factors.

Searching for the optimal phase factor to obtain OFDM signals with the lowest PAPR value will improves PAPR performance of the PTS-OFDM system. The objective of phase optimization is to select a set of optimal values of $\{p^{(m)}, m = 1,\ldots,M\}$ such that the PAPR of OFDM signals is smallest. By comparing the PAPR values of different phase factors is performed by the algorithm as follows.

$$\Psi = \underset{\{p^{(1)},p^{2},\ldots p^{(M)}\}}{\arg\min} \left(\max_{0\le n\le N} \left| \sum_{m=1}^{M} p^{(m)} \cdot \mathbf{x}_{data}^{(m)} \right) \right| \right), \tag{4}$$

where $\Psi = \{p^{(1)*}, p^{(2)*},\ldots, p^{M*}\}$ are the judgment conditions when the function takes the minimum value [11]. Obviously, the complexity of search optimization increases exponentially with the number of subblocks.

Finally, the combined signal with the lowest PAPR is chosen and transmitted. To recover real data symbol $\mathbf{X}_{data}$, channel estimation must be performed at the receiving end in the PTS-OFDM with channel estimation scheme.

III. PILOT ASSIGNMENT SCHEMES FOR CHANNEL ESTIMATION

A. Fixed End Pilot Assignment (FEPA) Scheme

For improving channel estimation errors caused by linear extrapolation, a novel pilot assignment method is proposed. This method assures the start and end positions of the pilot sequence are fixed in each subblock.

Let $\{p_i\}_{i=0}^{N_p-1}$ denote the desired position sequence, where N_p is the total number of pilots and p_i denotes as the position of the i-th pilot. The constraint on the positions of pilots is that both ends of each sub-block must have pilot tones. Let the start position $p_0 = l_1$ and the next position $p_1 = l_1 + l_2$. We define the position assignment rule as $p_{i+2} = p_i + l_2 + l_3$ for $i = 0,1,\cdots,N_p$ -1, where l_2 and l_3 are the interval between the adjacent positions of pilot tones respectively and N_p is the number of pilot tones of OFDM block. The rule implies the following recursive relation with three parameters l_1, l_2, and l_3, as follows:

$$\begin{aligned} p_2 &= l_1 + l_2 + l_3 \\ p_3 &= l_1 + 2l_2 + l_3 \quad . \\ p_4 &= l_1 + 2l_2 + 2l_3 \end{aligned} \tag{5}$$

Hence, the general recursive relation is given by

$$\begin{cases} p_{i+2} = p_i + l_2 + l_3 \\ p_{i+1} = p_{i-1} + l_2 + l_3 \end{cases}, \tag{6}$$

This is a linear homogeneous relation of degree 3. To solve for the equations, the associated characteristic equation is given as the linear recurrence $t^3 - t^2 - t + 1$, where t denotes as the polynomial variable. This characteristic equation has tree roots of 1, 1, and -1. Hence the explicit formula of the general solution to the recurrence can be written as $p_i = \mathrm{A}(1)^i + i\mathrm{B}(1)^i + \mathrm{C}(-1)^i$ for $i = 0,1,\ldots,N_p - 1$. Using this formula and initial conditions in (5), we have $l_1 = \mathrm{A} + \mathrm{B} - \mathrm{C}$, $l_1 + l_2 = \mathrm{A} + 2\mathrm{B} + \mathrm{C}$, and $l_1 + l_2 + l_3 = \mathrm{A} + 3\mathrm{B} - \mathrm{C}$. Solving for A, B, C, the explicit formula of the pilot position sequence is obtain as

$$p_i = \left(l_1 - \frac{l_2 + 3l_3}{4}\right) + i\left(\frac{l_2 + l_3}{2}\right) + \left(\frac{l_2 - l_3}{4}\right)(-1)^i . \tag{7}$$

Let N denote the block number of OFDM symbols, N_p the number of pilot tones, and M the number of partitioned sub-blocks. In the proposed FEPA pilot assignment method, the position of the assigned pilot must be expressed in terms of these parameters for channel estimation schemes. We now link l_1, l_2, and l_3 in (7) with the three parameter by observing the following assignment rules:

1) Define s the start position of the pilot tones in each subblock. Denote $\overline{d}$ as the interval between the start position and the mid position and $\underline{d}$ as the interval between the mid position and the end position. By defining the two parameters, $\Gamma_N \equiv N / M$ and $\Gamma_{N_p} \equiv N_p / M$, for $1 \le m \le M$, it follows that $s = \Gamma_N(m-1) + 1$, $\overline{d} = \left\lceil (\Gamma_N - 1)/(\Gamma_{N_p} - 1) \right\rceil$, and $\underline{d} = \left\lfloor (\Gamma_N - 1)/(\Gamma_{N_p} - 1) \right\rfloor$, where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ respectively denote the ceiling and floor functions.

2) Two discriminants are established as follows:

$$\Delta_1 = 1 + \overline{d}\left(\left\lceil \frac{\Gamma_{N_p} - 1}{2} \right\rceil\right) + \underline{d}\left(\left\lfloor \frac{\Gamma_{N_p} - 1}{2} \right\rfloor\right) \tag{8}$$

and

$$\Delta_2 = 1 + \overline{d}\left(\left\lfloor \frac{\Gamma_{N_p} - 1}{2} \right\rfloor\right) + \underline{d}\left(\left\lceil \frac{\Gamma_{N_p} - 1}{2} \right\rceil\right), \tag{9}$$

According to the results in (8) and (9), we set $l_2 = \overline{d}$ and $l_3 = \underline{d}$ for $\Delta_1 = \Gamma_N$ or $l_2 = \underline{d}$ and $l_3 = \overline{d}$ for $\Delta_2 = \Gamma_N$.

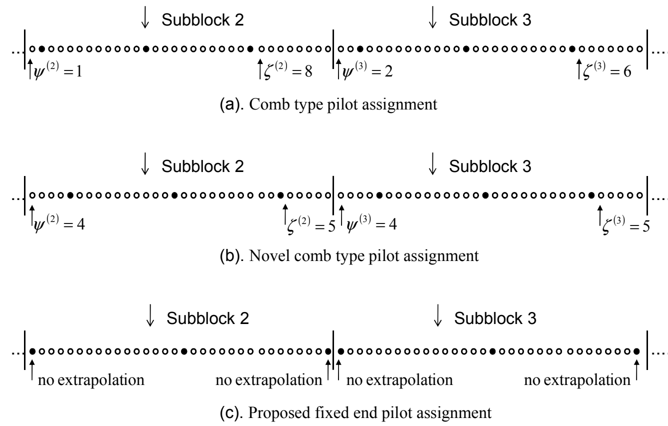

Fig. 2. Different pilot assignment methods for $N = 256$, $N_p = 24$, $k \in [32,63]$, and $M = 8$.

3) Substituting the values of s, $\underline{d}$, and $\overline{d}$ into (7), the explicit formula of the proposed fixed end pilot assignment can be further expressed as follows:

For $M = 1$ with $s = 1$,

$$\begin{aligned} p_i &= \left(s - \frac{\underline{d} + 3\overline{d}}{4}\right) + i\left(\frac{\underline{d} + \overline{d}}{2}\right) + \left(\frac{\underline{d} - \overline{d}}{4}\right)(-1)^i \quad . \\ i &= 1,2,\ldots,N_p \end{aligned} \tag{10}$$

For $M > 1$ and $m = 1,\cdots,M$ with $s = (m-1)\Gamma_{N_p} + 1$,

$$\begin{aligned} p_k &= \left(s - \frac{\underline{d} + 3\overline{d}}{4}\right) + k \cdot \left(\frac{\underline{d} + \overline{d}}{2}\right) + \left(\frac{\underline{d} - \overline{d}}{4}\right)(-1)^k \quad , \\ k &= (m-1)\Gamma_{N_p} + 1, (m-1)\Gamma_{N_p} + 2, \cdots, m\Gamma_{N_p} \end{aligned} \tag{11}$$

where k represents the true position index of the pilot sequence in the m-th subblock. Note that the value of s varies with the subblocks.

Different pilot assignment schemes are displayed in Fig. 2. In comparison with the comb-type and novel comb-type methods [7], we can see that the FEPA method does not use linear extrapolation in channel estimation and hence more accurate channel estimation can be attained. Applying the proposed pilot tones assignment to the CE-PTS scheme, better BER performance can be achieved.

B. Channel Estimation for PTS Using the FEPA Scheme

In this section, we applying the proposed FEPA scheme to PTS-OFDM with channel estimation. First, the pilot tone is inserted to estimate frequency response of the fading channel in the input data block. Let $\delta = \{ j_0, j_1, \cdots, j_{k,} \cdots, j_{N_p - 1} \}$ be the set of the pilot tone positions, where j_k specifies the position of the pilots. It follows that δ^c, as the complementary set of δ in the set $\mathrm{I} = \{0,1,\cdots,N-1\}$, represents the position set of data symbols. The input data block $\mathbf{X}$ is partitioned into M disjoint subblocks $X^{(m)} = [X_0^{(m)}, X_1^{(m)}, \cdots, X_k^{(m)} \cdots, X_{N-1}^{(m)}]$, $m = 0,1,\cdots,M-1$. Here the input data symbol $X_k^{(m)}$ is further defined as

$$X_k^{(m)} = X_{data,k} + X_{pilot,k} = \begin{cases} X_{data,k}, & k \in \delta^c \\ X_{pilot,k}, & k \in \delta \end{cases}. \qquad (12)$$

Denote $x' = [x'_0, x'_1, \cdots, x'_{N\text{-}1}]$ as the time domain signal containing the optimal phase factors $p'^{(m)}\ (0 \le m \le M\text{-}1)$ for the minimum PAPR. The transmitted signal x' in time domain can be written as

$$x' = \sum_{m=1}^{M} p'^{(m)} x^{(m)}. \qquad (13)$$

After performing FFT on the received signal at the receiving end, the signal in terms of the transform can be expressed as

$$R'_l = X'_l H_l + W_l, \ \ l = 0,1,\cdots,N-1, \qquad (14)$$

where H_l is the frequency channel response of the fading channel, W_l is the Fourier transform of the corresponding additive Gaussian noise. The received signal R'_l is partitioned into M disjoint subblocks $R'^{(m)} = \left[R_0'^{(m)}, R_1'^{(m)}, \cdots, R_{N\text{-}1}'^{(m)}\right]$, $m = 0,1,\cdots,M-1$, using the same partition scheme as that at the transmitting end. Thus $R_i'^{(m)}$ can be written as

$$R_i'^{(m)} = p'^{(m)} X_i H_i + W_i, \quad m = 0,1,\cdots,M-1, \qquad (15)$$

where $mN/M \le i < (m+1)N/M$ is the index of the subcarrier in the m-th subblock, H_i is the frequency channel response of the fading channel, W_i represents the additive noise in frequency domain.

Denote $H'_i = p'^{(m)} H_i$ as the virtual frequency channel response, (15) can be rewritten as

$$R_i'^{(m)} = \begin{cases} X_{pilot,i} H'_{pilot,\,i} + W_i, \ i \in \delta \\ X_{datd,\,i} H'_{data,\,i} + W_i, \ i \in \delta^c \end{cases}, \qquad (16)$$

where $H'_{pilot,i}$ and $H'_{data,i}$ are the frequency channel response on the pilot tone and the real data symbol, respectively. Since the value and positions of $X_{pilot,i}$ are known at the receiver, $\hat{H}'_{pilot,o}$ is defined as

$$\hat{H}'_{pilot,i} = \frac{R_i'^{(m)}}{X_{pilot,i}}, \quad i \in \delta, \qquad (17)$$

where $\hat{H}'_{pilot,i}$ is the estimation of $H'_{pilot,i}$. Then, denote $\hat{H}'_{data,i}$ as the estimation of $H'_{data,i}$, which can be obtained by linear interpolation for $j_{(mN_{pilot}/M)} < i < j_{((m+1)N_{pilot}/M)-1}$. Finally, the estimated data $\hat{X}_{data,i}$ at the receiver is obtained by

$$\hat{X}_{data,i} = \frac{R_i'^{(m)}}{\hat{\tilde{H}}_{data,i}}, \quad i \in \delta^c. \qquad (18)$$

Therefore, the PTS-OFDM with channel estimation method could recover the real data symbol X_{data} without the SI. The system with the proposed FEPA scheme only uses linear interpolation for channel estimation, which will have better BER performance in comparison with other schemes.

TABLE I. COMPUTER SIMULATION PARAMETERS

parameter	value
Number of subcarriers	256
Number of pilot tones	24,8
Number of subblocks	4,8
Modulation	QPSK
Cyclic prefix	16 T
Maximum multipath delay	10 T , 22 T

IV. SIMULATION RESULTS

The criteria for performance evaluation considered here are the complementary cumulative distribution function (CCDF) as a function of the PAPR distribution when the PTS scheme is used, where the CCDF is the probability of PAPR of OFDM signals over threshold PAPR_0 . To generate CCDFs of the PAPR, 100,000 OFDM blocks are generated randomly for each simulation. Parameters of OFDM signals used in the simulations for PAPR reduction and channel estimation are shown in Table I. For simplicity, the number of allowed phase factors is set by $\Omega = 2$. Thus the set of phase factor is $p^{(m)} \in \{1, -1\}$.

A. Channel Estimation Using Different Pilot Assignment Schemes

Fig. 3 and Fig. 4 show the CCDFs of the PAPR for general PTS-OFDM (T-PTS) and PTS-OFDM using various pilot assignments for channel estimation with the number of subblocks $M = 4$ and $M = 8$, respectively. It is seen that better PAPR reduction performance is achieved with increase of subblock number M . We can see from Fig. 4 that the proposed scheme has almost the same PAPR performance compared to the general PTS and PTS with channel estimation using the Comb-type and Novel-Comb-type pilot assignment schemes in [7]. Note that our scheme does not require transmitting side information (SI) to the receiving end.

The BER performance of general PTS-OFDM and PTS-OFDM using the different assignment schemes in multipath fading channels for channel estimation with the number of subblocks $M = 4$ and $M = 8$ are shown in Fig. 5 and Fig. 6, respectively. For $M = 8$ in Fig. 6, we can see that the PTS-OFDM using the proposed pilot assignment scheme has the same BER performance as that of the general PTS-OFDM system with perfect side information. In compared with other pilot assignment schemes, however, its BER performance is better than that using the Comb-type and Novel-Comb-type pilot assignment schemes.

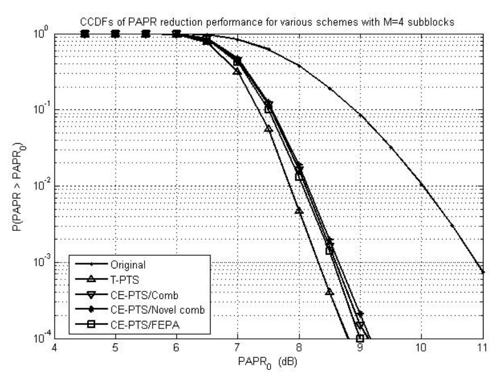

Fig. 3. CCDF of the PAPR of PTS-OFDM with $M = 4$ subblocks using various pilot assignment schemes for channel estimation.

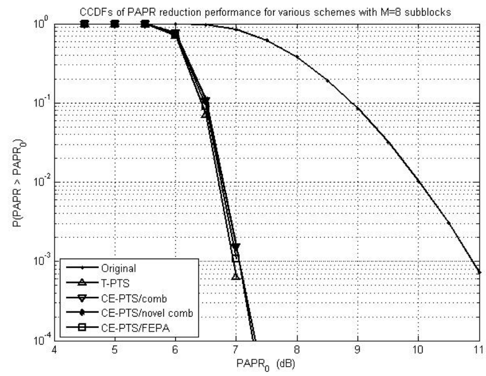

Fig. 4. CCDF of the PAPR of PTS-OFDM with $M = 8$ subblocks using various pilot assignment schemes for channel estimation.

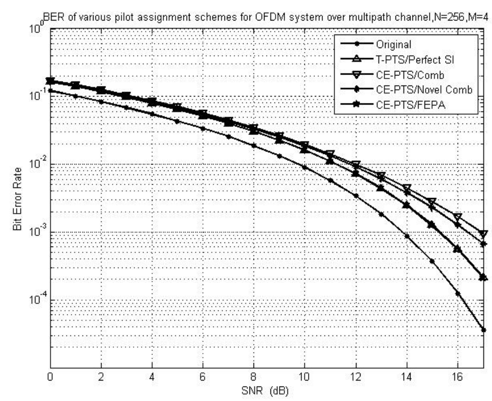

Fig. 5. BER performance of PTS-OFDM in multipath channels with $M = 4$ subblocks using various pilot assignment schemes for channel estimation.

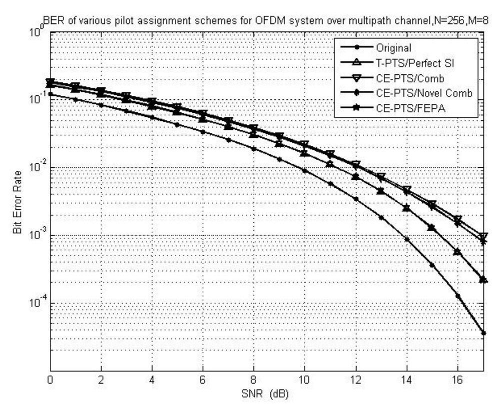

Fig. 6. BER performance of PTS-OFDM in multipath channels with $M = 8$ subblocks using various pilot assignment schemes for channel estimation.

B. Channel Estimation Using Different Numbers of Pilots

In this section, the pilot assignment with different numbers of pilot tones for channel estimation is discussed. Fig. 7 shows the BER performance curves of the general PTS-OFDM and PTS-OFDM using various pilot assignments for channel estimation in multipath channels for the case of $N_p = 8$ pilots. It can be seen obviously that, with perfect SI as the optimal phase factor, the general PTS-OFDM has better BER performance than that with channel estimation. Due to limited number of pilots used for the channel estimation in multipath channels, the BER performance of PTS-OFDM using the proposed scheme is affected, but not far from that of the general PTS-OFDM system with perfect SI. However, it still outperforms that using the Comb-type and the Novel-Comb-type pilot assignment schemes in the BER performance.

By increasing the number of pilots to $N_p = 24$, Fig. 8 shows the BER performance curves of the general PTS-OFDM and PTS-OFDM using various pilot assignments for channel estimation in multipath channels. PTS-OFDM using the proposed scheme has the same BER performance as that of the general PTS-OFDM in multipath channel and has better performance compared to that using the Comb-type and the Novel-Comb-type pilot assignment schemes. Obviously, increasing the number of pilots for channel estimation will improve the BER performance of PTS-OFDM systems.

V. Conclusions

This paper proposed a novel fixed end pilot assignment (FEPA) scheme for channel estimation to improve BER performance and reduce PAPR values of PTS-OFDM systems without transmitting SI to the receiving end. With the proposed method, every subblock has pilots assigned in both ends and, linear extrapolation for the estimation is avoided. The proposed method can further reduce estimation errors. Our simulations show that the PTS-OFDM system using the proposed pilot assignment scheme for channel estimation has the same performance as the traditional PTS but better performance both in PAPR reduction and bit error rates compared with that using the Comb-type and Novel-Comb-type pilot assignment schemes.

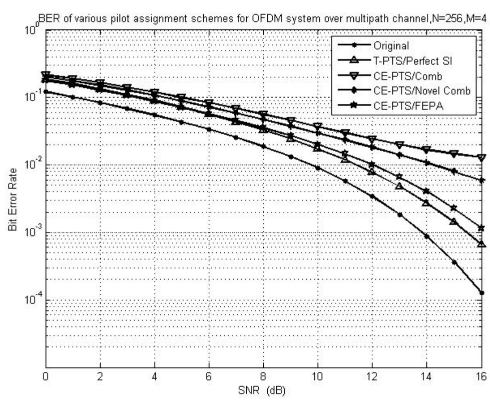

Fig. 7. BER performance of PTS-OFDM in multipath channels with $M = 4$ subblocks and $N_p = 8$ pilots using various pilot assignment schemes for channel estimation.

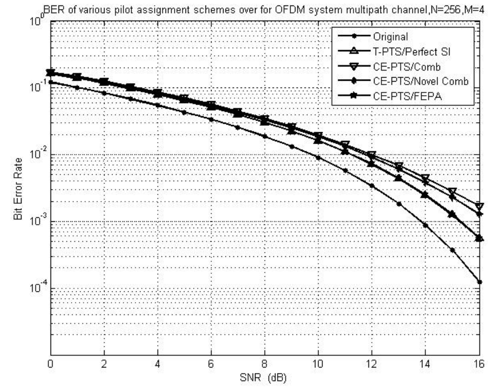

Fig. 8. BER performance of PTS-OFDM in multipath channels with $M = 4$ subblocks and $N_p = 24$ pilots using various pilot assignment schemes for channel estimation.

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