# A Novel Measurement Matrix Optimization Method for Radar Sparse Imaging with OFDM-LFM Signals

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Abstract—Compressed Sensing (CS) has been widely used in radar imaging field to reduce the data amount. The measurement matrix has direct effect on the degree of dimension reduction and the quality of target image. However, the measurement matrix is usually chosen as random Gaussian matrix or local Fourier matrix, and the influence from target characteristics to the measurement matrix optimization has not been considered. In this paper, focuses on the OFDM-LFM signals, a novel measurement matrix optimization method for radar sparse imaging is proposed. In this method, genetic algorithm is used to implement the measurement matrix optimization by equaling the measurement matrix to the chromosome. And then the satisfied imaging result can be achieved with minimal measurement dimension by using the obtained optimal measurement matrix. Some simulation results illustrate the effectiveness of the proposed method.

 ${\it Index Terms} \hbox{$-$OFDM-LFM$ signals, measurement matrix optimization, genetic algorithm.}$ 

#### I. INTRODUCTION

With the development of radar signal processing technology, the requirement of imaging resolution has been increased significantly. The high sampling rate based on Nyquist sampling theory and the large data amount poses great challenge to the hardware designation and the memory space of signal processing system. However, the compressed sensing (CS) theory can recover a sparse or compressible signal from far fewer measurements than what the Nyquist sampling theory claimed with high probability, by exploiting the signal sparsity [1-3]. Therefore, the compressed sensing (CS) theory has been widely used in radar imaging field, which can reduce the required measurements drastically [4-7]. The CS theory is introduced into high resolution range profile synthesis through sparse frequency waveforms and the full-resolution image is reconstructed. A sparse imaging algorithm based on CS is proposed to estimate the locations of the scattering centers from a very limited number of measurements [8].

However, in most ISAR imaging method based on compressed sensing, the measurement matrix is usually chosen

as random Gaussian matrix or local Fourier matrix, and the observation dimension is determined by experience. The existing methods have not considered the influence from target characteristics to the measurement matrix optimization. Hence, we intend to find a measurement matrix optimization method based on target characteristics. The genetic algorithm is a kind of evolutionary algorithm, and the optimal solution or the suboptimal solution will be obtained after iterations [9]. The measurement matrix optimization can be treated as an optimization problem of the measurement matrix dimension and structure. Therefore the measurement matrix optimization can be implemented based on the genetic algorithm.

As is known, the orthogonal frequency division multiplexing-linear frequency modulation (OFDM-LFM) signal has been applied in radar imaging as a kind of broadbandwidth radar signal [10]. Therefore, a measurement matrix optimization method for sparse ISAR imaging with OFDM-LFM signals based on genetic algorithm is proposed in this paper. With this method, the measurement matrix dimension and structure optimization can be implemented under a given image quality requirement. Meanwhile, the SBL algorithm is chosen for signal reconstruction. Also, the noise variance estimation is obtained by signal subspace decomposition, and the estimated value is applied to SBL algorithm, which can avoid the iterative calculation of noise variance during the signal reconstruction. As a result, the computation of SBL algorithm can be reduced effectively.

This paper is organized as follows. A measurement matrix optimization method based on genetic algorithm is proposed in Section 2. The improved sparse Bayesian learning algorithm is illustrated in Section 3. Simulations are presented in Section 4 to validate the effectiveness of the proposed method, and some conclusions are made in the last section.

# II. MEASUREMENT MATRIX OPTIMIZATION BASED ON GENETIC ALGORITHM

Assuming the radar is located at the point O in the (X,Y) coordinates. For simplicity, the target area can be divided into

 $U \times V$  grids. Then, the distance from each grid to the reference point P is marked as  $R_{\Delta w}, w = 1, 2, ...UV$ . The transmitted signal contains N subpulses.

After the dechirp processing, the *i*-th sub-pulse signal can be represented as:

$$s_c(i,t) = \sum_{w=1}^{UV} \rho_w \cdot \exp(-j2\pi\mu \frac{2R_{\Delta w}}{c}t) \cdot \exp(-j2\pi(f_c + i\Delta f) \cdot \frac{2R_{\Delta w}}{c}) \cdot \exp(j\pi\mu (\frac{2R_{\Delta w}}{c})^2) + n_i(t')$$
(1)

where  $\rho_w$  is the reflection coefficient of the w-th grid,  $f_c$  is the initial carrier frequency,  $\Delta f$  is the carrier frequency interval between each two adjacent subpulses,  $\mu$  is the chirp rate, and  $n_i$  is the noise of the i-th sub-pulse.

For the OFDM-LFM radar system, high resolution ISAR imaging method with spectrum-sparse signal has been described clearly in [10], but it does not consider the measurement matrix optimization based on target features.

In fact, the optimal measurement matrix for different targets is of great difference. Therefore, a measurement matrix optimization method based on the genetic algorithm is proposed in this paper, which considers the target and the signal characteristics influence. With this method, the satisfied imaging result can be obtained with minimal measurement dimension. The concrete steps of measurement matrix optimization based on genetic algorithm can be described as follows:

## (1) Setting the population and coding scheme

The measurement matrix  $\hat{g}$  is equivalent to chromosome (Namely the binary string) to set a population. Assume that there are L chromosomes in the population, and each chromosome contains N genes. The values of each gene is 0 or 1 at random, where "0" indicates this frequency point will not be used and "1" indicates the frequency point will be used.

## (2) Duplication

In each generation, the L(1-GGAP) optimal individuals will be copied to the next generation directly, where GGAP is the generation gap value and it should be  $GGAP \in (0,1)$ .

## (3) Selection

The roulette wheel selection method is used in this paper. The objective function value is defined as  $f = \|\hat{g}\|_1$ , where  $\|\cdot\|_1$  indicates the 1- norm operation. The ultimate goal is to make the objective function minimal under the condition of the imaging quality requirements.

Here, the fitness value FitnV of each individual is estimated according to the objective function value. In other words, the individual will have a bigger fitness value when its

objective function value is smaller. However, for those individuals whose imaging results can't reach the image quality, their fitness value should be reduced be 0.1 times of its original value.

#### (4) Crossover

Here, the single-point crossover operator is used. In the individual coding string, a crossing point is set with the crossover probability at random, and then part chromosomes of the two paired individuals to be exchanged at the crossing point. The execution process is as follows:

Step1: Select two individuals randomly and make them paired.

Step2: For each pair of paired individuals, the location after one amelogenin locus is set to be the crossing point at random.

Step3: For each pair of paired individuals, part chromosomes of the two individuals to be exchanged at the crossing point according to the crossover probability, thus to produce two new individuals.

#### (5) Mutation

Here, the basic bit mutation is used. Making mutation operation on the value of one amelogenin locus set randomly in the individual coding string with mutation probability. The execution process is as follows:

Step1: Each amelogenin locus of the individual is specified to be mutation point with mutation probability.

Step2: For each specified mutation point, making inversion operation on its value to generate a new individual in the next generation.

Based on the above series of operations, it will converge to the optimal chromosome through MAXGEN iterations, where MAXGEN is the supreme genetic algebra, and the optimal chromosome is the optimal measurement matrix.

# III. IMPROVED SPARSE BAYESIAN LEARNING ALGORITHM BASED ON SIGNAL SUBSPACE DECOMPOSITION

Sparse Bayesian learning (SBL) method adopts the multi probability distribution as the sparse priori of the unknown coefficients, so it is not easy to fall into the local minimum and shows superior performance in sparse signal reconstruction algorithms. However, SBL algorithm requires the iterative calculation of noise variance during the signal reconstruction

[11]. If the estimation of  $\sigma^2$  can be known aprior, the computation of SBL algorithm can be reduced significantly. Hence, the signal subspace decomposition method is utilized to obtain the noise variance estimation in this paper.

Let  $S_C$  be the matrix form of the signal  $s_c(i,t)$  shown as Eq.(1), then it can be written as:

$$S_C = A \cdot S + n \tag{2}$$

where

$$S_i(t') = \rho_i \cdot \exp(-j2\pi\mu \frac{2R_{\Delta i}}{c}t') \exp(-j2\pi f_c \cdot \frac{2R_{\Delta i}}{c}) \exp(-j\pi\mu (\frac{2R_{\Delta i}}{c})^2)$$

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \exp(-j2\pi\Delta f \cdot \frac{2R_{\Delta 1}}{c}) & \exp(-j2\pi\Delta f \cdot \frac{2R_{\Delta 2}}{c}) & \cdots & \exp(-j2\pi\Delta f \cdot \frac{2R_{\Delta UV}}{c}) \\ \vdots & \vdots & \vdots & \vdots \\ \exp(-j2\pi N\Delta f \cdot \frac{2R_{\Delta 1}}{c}) & \exp(-j2\pi N\Delta f \cdot \frac{2R_{\Delta 2}}{c}) & \cdots & \exp(-j2\pi N\Delta f \cdot \frac{2R_{\Delta UV}}{c}) \end{bmatrix}, S = \begin{bmatrix} S_1(t') \\ S_2(t') \\ \vdots \\ S_{UV}(t') \end{bmatrix}$$

Since the signal and noise are uncorrelated, the covariance matrix of the column data in  $S_C$  can be decomposed into two parts corresponding to the signal and noise respectively. With the eigenvalue decomposition of the covariance matrix, the noise variance estimation  $\sigma^2$  can be obtained by computing the mean value of those small eigenvalues.

#### IV. SIMULATIONS

To validate the effectiveness of the measurement matrix optimization method proposed in this paper, simulations are given in this section.

The distribution of the target scatterers is shown in Fig. 1. The parameters setting of OFDM-LFM signals are as follows. The carrier frequency is  $f_c = 10 \, \mathrm{GHz}$ , the the carrier frequency interval is  $\Delta f = 4.6875 \, \mathrm{MHz}$ , sub-pulse number is N = 64, and  $PRF = 250 \, \mathrm{Hz}$ . The distance between the target and radar is 10km at the beginning of imaging. The target is with movement speed of 300m/s along the direction parallel to the baseline, and the azimuthal coherent accumulation time is 1s. White Gaussian noise with  $\sigma^2 = 5$  are added into the echo signal. Without compressed sensing, target imaging obtained with the full-bandwidth waveforms is as shown in Fig. 2.

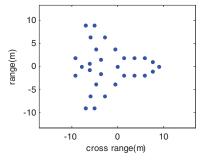


Fig. 1 Geometry of the target

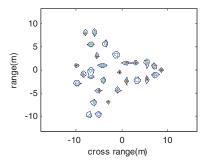


Fig.2 Target imaging with full-bandwidth waveforms

With the eigenvalue decomposition of the covariance matrix of echo signal, the noise variance estimation can be obtained which is  $\hat{\sigma}^2 = 4.85$ , and the error is only 3%.

Under compressed sensing framework, the measurement matrix optimization can be implemented based on genetic algorithm. The SBL algorithm is used for signal reconstruction, and the iteration estimation of noise variance is no longer needed. Just applying  $\hat{\sigma}^2 = 4.85$  to the signal reconstruction process, we can obtain satisfied reconstruction result. The parameters in genetic algorithm are set as follows. The individuals number in the population is 100, each chromosome is composed of 64-bit binary number, crossover probability is 0.7, mutation probability is 0.05, generation gap value GGAP=0.9, the threshold of penalty factor is 0.85 (that is, the imaging quality requirement is 0.85), and the supreme genetic algebra MAXGEN=500.

The dimension of the obtained optimal measurement matrix is 26, and Fig. 3 shows the target image using this optimal measurement matrix. If choosing a random part unit matrix with 26 dimensions as the measurement matrix, the imaging result is shown as Fig. 4.

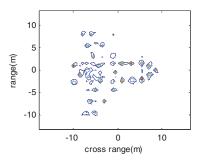


Fig. 3 Target imaging with optimal matrix

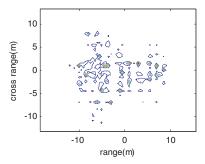


Fig.4 Target imaging with random part unit matrix

Obviously, the imaging result shown in Fig.3 is far better than that in Fig.4. We can see that with the same dimension, the optimal measurement matrix obtained via the proposed method can achieve a better imaging result compared with the random measurement matrix. Also, the noise variance estimation based on subspace decomposition can decrease the computation of SBL algorithm effectively. Thus, the real-time performance of the measurement matrix optimization method proposed can be improved significantly.

#### V. CONCLUSION

Focus on OFDM-LFM signals, a measurement matrix optimization method based on genetic algorithm is proposed in this paper, in which, the influence from target characteristics to the measurement matrix optimization is taken full consideration, and the optimal measurement matrix can be obtained with genetic algorithm. As a result, the satisfied imaging result can be achieved with minimal measurement dimension with the obtained optimal measurement matrix. Also, the subspace decomposition method is used for noise

variance estimation, thus the computation load of the proposed algorithm is decreased significantly.

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