High-Resolution Radar Detection in Interference and Nonhomogeneous Noise

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***Abstract*—This letter addresses the problem of high-resolution radar detection in interference and nonhomogeneous noise. The target signal and interference lie in two linearly independent known subspaces, but with unknown coordinate. The noise is modeled by a compound-Gaussian process with unknown covariance matrix and random texture. According to the two-step generalized likelihood ratio test-based design approach, we derive a distributed target detector. Numerical examples show that the proposed detector can provide better detection performance than their counterparts in nonhomogeneous environments.**

***Index Terms*—Compound-gaussian process, generalized likelihood ratio test, high-resolution radar, subspace interference.**

I. Introduction

ADAPTIVE detection of a distributed target in high-resolution radar systems has received considerable attention in recent years. Experimental evidence in high-resolution radar [1],[2] shows that the homogeneous Gaussian assumption is no longer met in practice. Thus, more appropriate models to depict the nonhomogeneity are needed. As a viable model to depict the non-Gaussian noise in nonhomogeneous environments, compound-Gaussian process is demonstrated by theoretical arguments [3], and is verified by the statistical analysis of experimental data [1],[2]. In compound-Gaussian model, clutter returns can be seemed as the product of a complex, possibly correlated Gaussian process (referred to as speckle), and a nonnegative component (referred to as spiky), which exhibits much longer decorrelation time than the former. Under this framework, various algorithms for distributed target detection are proposed. Assuming that the noise can be clustered into groups of cells sharing the same noise power, Conte *et al.* [4] successfully derive the generalized likelihood ratio test (GLRT) detector. Then, the same problem with Rao and Wald tests is discussed in [5]. The distributed target detection in compound-Gaussian noise is realized by exploiting order statistic theory [6]. Performance analysis of distributed target detection in compound-Gaussian clutter is studied in [7].

In practical radar systems, the interference usually exists due to electronic counter measure systems or civil broad casting systems. The detection of a point-like target in interference and white Gaussian noise is addressed in [8]. The target signal and interference lie in two known linearly independent subspaces but with unknown coordinates. The problem in [8] is extended for a distributed target in [9]. The detection of a point-like target in the interference and Gaussian noise with unknown covariance matrix is discussed in [10]. Recently, the invariance theory is used to detect a point-like target in homogeneous noise and subspace interference [11]–[13]. In [14], the detection problem is solved by the method of sieves with the assumption that the subspace interference exists both in the test and training data. In contrast, the subspace interference is assumed to only lie in the test data [15], and the GLRT and two-step GLRT are devised to detect a distributed target. The corresponding Rao tests are proposed in [16]. A distributed target detector in interference is explored in [17], where the useful signal components are aligned with an unknown direction constrained to a given subspace. Note that all the above studies for distributed target detection with interference assume the noise is homogeneous. However, this assumption is often invalid for the high-resolution radar detection, as stated above.

We consider herein the problem of high-resolution radar detection in interference and nonhomogeneous noise. The interference is characterized by the subspace model, and the noise is compound-Gaussian with inverse Gamma texture. To design an effective distributed target detector, we adopt the two-step GLRT criterion. First, the test statistic with known covariance matrix is derived. Then, the estimate of covariance matrix is substituted into the test statistic to obtain the final detector. The comparison with its competitors highlights that the proposed detector can achieve significant enhancement of detection performance in nonhomogeneous environments.

Notations: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the conjugate, transpose, and conjugate transpose, respectively. $\mathrm{tr}(\cdot)$ indicates the trace of a matrix, $|\cdot|$ denotes the determinant of a square matrix, and $||\cdot||$ is the modulus of a complex number. The notation "$\propto$" denotes "be proportional to."

II. Problem Formulation

Assume that data are collected from N sensors and the target distributes across L successive range cells. Under H_1 hypothesis, the primary data vector $\mathbf{x}_l \in \mathbb{C}^{N\times 1}$ is expressed as

$$\mathbf{x}_l = \mathbf{s}_l + \mathbf{i}_l + \mathbf{n}_l, , l = 1, \dots, L \tag{1}$$

where

1) $\mathbf{s}_l = \mathbf{H}\mathbf{a}_l$. Here, $\mathbf{H} \in \mathbb{C}^{N\times r}$ is a known full-rank matrix and $\mathbf{a}_l \in \mathbb{C}^{r\times 1}$ denotes the unknown signal coordinate. In other words, the target signal belongs to the subspace spanned by the columns of $\mathbf{H}$.

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2) $\mathbf{i}_l = \mathbf{J}\mathbf{b}_l$ denotes the interference. The columns of the full-rank known matrix $\mathbf{J} \in \mathbb{C}^{N\times q}$ span the subspace where the interference lies, and $\mathbf{b}_l \in \mathbb{C}^{q\times 1}$ is the unknown coordinate vector. Note that it is necessary to assume that the subspace spanned by the columns of $\mathbf{J}$ and that spanned by the columns of $\mathbf{H}$ are linearly independent. Moreover, we suppose that $q + r \le N$.
3) $\mathbf{n}_l$ denotes the noise vector. In high-resolution radar, it follows compound-Gaussian process that is interpreted as the product of two independent components

$$\mathbf{n}_l = \sqrt{\tau_l}\mathbf{g}_l, l = 1, \ldots, L \tag{2}$$

where $\mathbf{g}_l$ (referred to as the speckle component) is a complex Gaussian random vector with zero-mean and covariance matrix $\mathbf{M}$, and τ_l (referred to as the texture component) is a positive random variable that represents the local power level. τ_k follows the inverse Gamma distribution with the probability density function

$$f(\tau_l) = \frac{\beta_l^{\alpha_l}}{\Gamma(\alpha_l)}\tau_l{}^{-(\alpha_l+1)}e^{-\beta_l/\tau_l} \tag{3}$$

where $\Gamma(\cdot)$ is the Gamma function, andα_l and β_l are the shape parameter and scale parameter, respectively. α_l and β_l are assumed known in advance or accurately estimated.

Note that the covariance matrix $\mathbf{M}$ is unknown. A set of training data, $\mathbf{x}_k = \mathbf{n}_k, k = 1, \ldots, K$, is required to estimate the unknown covariance matrix.

In summary, the detection problem of a distributed target in the noise and interference can be formulated as follows:

$$H_0 : \begin{cases} \mathbf{x}_l = \mathbf{J}\mathbf{b}_l + \mathbf{n}_l, l = 1, \ldots, L \\ \mathbf{x}_k = \mathbf{n}_k, k = 1, \ldots, K \end{cases}$$

$$H_1 : \begin{cases} \mathbf{x}_l = \mathbf{H}\mathbf{a}_l + \mathbf{J}\mathbf{b}_l + \mathbf{n}_l, l = 1, \ldots, L \\ \mathbf{x}_k = \mathbf{n}_k, k = 1, \ldots, K. \end{cases} \tag{4}$$

The following notations are used for convenience: $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_L]$, $\mathbf{A} = [\mathbf{a}_1, , \ldots, \mathbf{a}_L]$, $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_L]$, and $\tau = [\tau_1, \ldots, \tau_L]$.

III. Proposed Detector

A. Design of the Detector

We resort to the two-step procedure to devise the detector. First, the test statistic under the assumption of known covariance matrix is derived. Then, the estimate of covariance matrix is substituted into the test statistic to obtain the final detector. The GLRT with known $\mathbf{M}$ for the problem (4) is expressed as

$$\frac{\max\limits_{\mathbf{A}}\max\limits_{\mathbf{B}} \int f(\mathbf{X}|\mathbf{A},\mathbf{B},\tau)f(\tau)d\tau}{\max\limits_{\mathbf{B}} \int f(\mathbf{X}|\mathbf{B},\tau)f(\tau)d\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \xi \tag{5}$$

where

$$f(\mathbf{X}|\mathbf{B},\tau) = \prod_{l=1}^{L}\frac{1}{(\pi\tau_l)^N|\mathbf{M}|} \times \exp\Big[-\frac{(\mathbf{x}_l - \mathbf{J}\mathbf{b}_l)^H\mathbf{M}^{-1}(\mathbf{x}_l - \mathbf{J}\mathbf{b}_l)}{\tau_l}\Big] \tag{6}$$

$$f(\mathbf{X}|\mathbf{A},\mathbf{B},\tau) = \prod_{l=1}^{L}\frac{1}{(\pi\tau_l)^N|\mathbf{M}|} \times \exp\Big[-\frac{(\mathbf{x}_l - \mathbf{J}\mathbf{b}_l - \mathbf{H}\mathbf{a}_l)^H\mathbf{M}^{-1}(\mathbf{x}_l - \mathbf{J}\mathbf{b}_l - \mathbf{H}\mathbf{a}_l)}{\tau_l}\Big] \tag{7}$$

and ξ is the threshold.

Denote $\mathbf{U} = [\mathbf{H}\ \mathbf{J}] \in \mathbb{C}^{N\times(q+r)}$ and $\mathbf{c}_l = [\mathbf{a}_l^T\ \mathbf{b}_l^T]^T \in \mathbb{C}^{(q+r)\times 1}$. Let $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_l, \ldots, \mathbf{c}_L]$. Then, (7) can be rewritten as

$$f(\mathbf{X}|\mathbf{C},\tau) = \prod_{l=1}^{L}\frac{1}{(\pi\tau_l)^N|\mathbf{M}|} \times \exp\Big[-\frac{(\mathbf{x}_l - \mathbf{U}\mathbf{c}_l)^H\mathbf{M}^{-1}(\mathbf{x}_l - \mathbf{U}\mathbf{c}_l)}{\tau_k}\Big]. \tag{8}$$

For each $\mathbf{b}_l$ in $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_l, \ldots, \mathbf{b}_L]$, the estimation $\hat{\mathbf{b}}_l$ of $\mathbf{b}_l$ is obtained by nulling the derivative of ln $f(\mathbf{X}|\mathbf{B},\tau)$ with respect to $\mathbf{b}_l$. Thus, we have

$$\hat{\mathbf{b}}_l = (\mathbf{J}^H\mathbf{M}^{-1}\mathbf{J})^{-1}(\mathbf{J}^H\mathbf{M}^{-1}\mathbf{x}_l). \tag{9}$$

Similarly, nulling the derivative of ln$f(\mathbf{X}|\mathbf{C},\tau)$ with respect to $\mathbf{c}_k$ yields

$$\hat{\mathbf{c}}_l = (\mathbf{U}^H\mathbf{M}^{-1}\mathbf{U})^{-1}(\mathbf{U}^H\mathbf{M}^{-1}\mathbf{x}_l). \tag{10}$$

Substituting $\hat{\mathbf{b}}_l$ and $\hat{\mathbf{c}}_l$ into (6) and (8), respectively, we get

$$f(\mathbf{X}|\hat{\mathbf{B}},\tau) = \prod_{l=1}^{L}\frac{1}{(\pi\tau_l)^N|\mathbf{M}|}\exp\left(-\frac{\tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{J}}}^{\perp}\tilde{\mathbf{x}}_l}{\tau_l}\right) \tag{11}$$

and

$$f(\mathbf{X}|\hat{\mathbf{C}},\tau) = \prod_{l=1}^{L}\frac{1}{(\pi\tau_l)^N|\mathbf{M}|}\exp\left(-\frac{\tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{U}}}^{\perp}\tilde{\mathbf{x}}_l}{\tau_l}\right) \tag{12}$$

where

$$\mathbf{P}_{\tilde{\mathbf{J}}}^{\perp} = \mathbf{I}_N - \mathbf{P}_{\tilde{\mathbf{J}}}, \quad \mathbf{P}_{\tilde{\mathbf{U}}}^{\perp} = \mathbf{I}_N - \mathbf{P}_{\tilde{\mathbf{U}}} \tag{13}$$

with

$$\begin{aligned} \mathbf{P}_{\tilde{\mathbf{J}}} &= \widetilde{\mathbf{J}}(\widetilde{\mathbf{J}}^H\widetilde{\mathbf{J}})^{-1}\widetilde{\mathbf{J}}^H, \mathbf{P}_{\tilde{\mathbf{U}}} = \widetilde{\mathbf{U}}(\widetilde{\mathbf{U}}^H\widetilde{\mathbf{U}})^{-1}\widetilde{\mathbf{U}}^H \\ \widetilde{\mathbf{J}} &= \mathbf{M}^{-1/2}\mathbf{J}, \widetilde{\mathbf{U}} = \mathbf{M}^{-1/2}\mathbf{U}, \tilde{\mathbf{x}}_l = \mathbf{M}^{-1/2}\mathbf{x}_l. \end{aligned} \tag{14}$$

Integrating the product of (11) and $f(\tau)$ with respect to τ results in

$$\begin{aligned} &\int f(\mathbf{X}|\hat{\mathbf{B}},\tau)f(\tau)d\tau \\ &= \int \prod_{l=1}^{L}\frac{\pi^{-N}\beta_l^{\alpha_l}}{\Gamma(\alpha_l)(\tau_l)^{N+\alpha_l+1}|\mathbf{M}|}\exp\left\{-\frac{\beta_l + \tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{J}}}^{\perp}\tilde{\mathbf{x}}_l}{\tau_l}\right\}d\tau \\ &\propto \prod_{l=1}^{L}\frac{\beta_l^{\alpha_l}[\beta_l + \tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{J}}}^{\perp}\tilde{\mathbf{x}}_l]^{-(\alpha_l+N)}}{\Gamma(\alpha_l)|\mathbf{M}|}. \end{aligned} \tag{15}$$

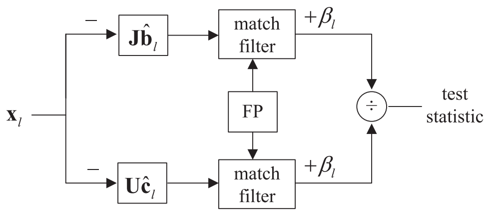

Fig. 1. Detector diagram.

Similarly, integrating the product of (12) and $f(\tau)$ with respect to τ yields

$$\int f(\mathbf{X}|\hat{\mathbf{C}},\tau)f(\tau)d\tau \propto \prod_{l=1}^{L}\frac{\beta_l^{\alpha_l}[\beta_l+\tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{U}}}^{\perp}\tilde{\mathbf{x}}_l]^{-(\alpha_l+N)}}{\Gamma(\alpha_l)|\mathbf{M}|}. \tag{16}$$

Substituting (15) and (16) into (5) results in

$$\prod_{l=1}^{L}\frac{\beta_l+\tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{J}}}^{\perp}\tilde{\mathbf{x}}_l}{\beta_l+\tilde{\mathbf{x}}_l^H\mathbf{P}_{\tilde{\mathbf{U}}}^{\perp}\tilde{\mathbf{x}}_l}\underset{H_0}{\overset{H_1}{\gtrless}}\xi. \tag{17}$$

In compound-Gaussian noise, the fixed point (FP) estimate is a widely used for the covariance matrix estimation [18], which is

$$\hat{\mathbf{M}}=\frac{N}{K}\sum_{k=1}^{K}\frac{\mathbf{x}_k\mathbf{x}_k^{\dagger}}{\mathbf{x}_k^{\dagger}\hat{\mathbf{M}}^{-1}\mathbf{x}_k}. \tag{18}$$

The existence and uniqueness of the solution to the above equation have been proven in [19] , and it can be obtained by a recursive procedure [18],[19]. Therefore, replacing $\mathbf{M}$ in (17) by the FP estimate $\hat{\mathbf{M}}$ yields the proposed detector

$$\prod_{l=1}^{L}\frac{\beta_l+\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{J}}}}^{\perp}\hat{\tilde{\mathbf{x}}}_l}{\beta_l+\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{U}}}}^{\perp}\hat{\tilde{\mathbf{x}}}_l}\underset{H_0}{\overset{H_1}{\gtrless}}\xi \tag{19}$$

where $\hat{\tilde{\mathbf{J}}}=\hat{\mathbf{M}}^{-1/2}\mathbf{J}$, $\hat{\tilde{\mathbf{U}}}=\hat{\mathbf{M}}^{-1/2}\mathbf{U}$, and $\hat{\tilde{\mathbf{x}}}_l=\hat{\mathbf{M}}^{-1/2}\mathbf{x}_l$.

In addition, we can rewrite (19) as another form

$$\prod_{l=1}^{L}\frac{\beta_l+(\mathbf{x}_l-\mathbf{J}\hat{\mathbf{b}}_l)^H\hat{\mathbf{M}}^{-1}(\mathbf{x}_l-\mathbf{J}\hat{\mathbf{b}}_l)}{\beta_l+(\mathbf{x}_l-\mathbf{U}\hat{\mathbf{c}}_l)^H\hat{\mathbf{M}}^{-1}(\mathbf{x}_l-\mathbf{U}\hat{\mathbf{c}}_l)}\underset{H_0}{\overset{H_1}{\gtrless}}\xi. \tag{20}$$

From (20), the proposed detector is independent of the shape parameter α_l. However, α_l influences the performance of this detector through the test data, training data, and the FP estimate. This will be analyzed in Section IV.

Additionally, it can be seen that the main procedures of this detector are: the received data are first passed through the subtraction operator to eliminate the interference; then, it is passed through the match filter with the FP covariance matrix. The detector diagram is shown in Fig. 1.

B. CFAR Analysis

Now we show that the proposed detector ensures the asymptotic constant false alarm rate (CFAR) property. By CFAR, we mean that under H_0 hypothesis, the statistical distribution of the detector is independent of the covariance matrix.

First, (19) can be rewritten as

$$\prod_{l=1}^{L}\frac{\beta_l+\hat{\tilde{\mathbf{x}}}_l^H\hat{\tilde{\mathbf{x}}}_l-\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{J}}}}\hat{\tilde{\mathbf{x}}}_l}{\beta_l+\hat{\tilde{\mathbf{x}}}_l^H\hat{\tilde{\mathbf{x}}}_l-\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{U}}}}\hat{\tilde{\mathbf{x}}}_l}\underset{H_0}{\overset{H_1}{\gtrless}}\xi. \tag{21}$$

Denote $\overline{\mathbf{M}}=\mathbf{M}^{-\frac{1}{2}}\hat{\mathbf{M}}\mathbf{M}^{-\frac{1}{2}}$ and $\tilde{\mathbf{g}}_l=\mathbf{M}^{-1/2}\mathbf{g}_l$. Then, we obtain

$$\hat{\tilde{\mathbf{x}}}_l^H\hat{\tilde{\mathbf{x}}}_l=\tau_l\tilde{\mathbf{g}}_l^H\overline{\mathbf{M}}^{-1}\tilde{\mathbf{g}}_l. \tag{22}$$

Under H_0 hypothesis, $\tilde{\mathbf{g}}_l$ is distributed as a zero-mean complex circular Gaussian distribution, with a covariance matrix $\mathbf{I}$. The statistical properties of the FP estimator indicate that $\overline{\mathbf{M}}$ is the unique estimator of the identity matrix [19]. Therefore, $\hat{\tilde{\mathbf{x}}}_l^H\hat{\tilde{\mathbf{x}}}_l$, under H_0, is independent of $\mathbf{M}$.

Let $\mathbf{y}_l=\mathbf{V}^H\tilde{\mathbf{g}}_l$, $\mathbf{R}=\mathbf{V}^H\overline{\mathbf{M}}\mathbf{V}$, and $\mathbf{V}$ are an unitary matrix partitioned as $\mathbf{V}=[\mathbf{J}_{//},\mathbf{V}_2]$, where $\mathbf{J}_{//}=\widetilde{\mathbf{J}}(\widetilde{\mathbf{J}}^H\widetilde{\mathbf{J}})^{-\frac{1}{2}}$. Here $\mathbf{V}_2$ is a semiunitary matrix such that $\mathbf{V}_2^H\mathbf{J}_{//}=\mathbf{0}_{(N-q)\times q}$ and $\mathbf{V}_2^H\mathbf{V}_2=\mathbf{I}_{(N-q)\times(N-q)}$. Thus, we have

$$\begin{aligned}\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{J}}}}\hat{\tilde{\mathbf{x}}}_l&=\tau_l\mathbf{y}_l^H\mathbf{R}^{-1}\mathbf{V}^H\widetilde{\mathbf{J}}(\widetilde{\mathbf{J}}^H\mathbf{V}\mathbf{R}^{-1}\mathbf{V}^H\widetilde{\mathbf{J}})^{-1}\widetilde{\mathbf{J}}^H\mathbf{V}\mathbf{R}^{-1}\mathbf{y}_l\\&=\tau_l\mathbf{y}_l^H\mathbf{R}^{-1}\mathbf{V}^H\mathbf{J}_{//}(\mathbf{J}_{//}^H\mathbf{V}\mathbf{R}^{-1}\mathbf{V}^H\mathbf{J}_{//})^{-1}\mathbf{J}_{//}^H\mathbf{V}\mathbf{R}^{-1}\mathbf{y}_l.\end{aligned} \tag{23}$$

Note that, under H_0 hypothesis, $\mathbf{y}_l$ and $\mathbf{R}$ are statistically equivalent to $\tilde{\mathbf{g}}_l$ and $\overline{\mathbf{M}}$, respectively.

Using the partitioned matrix inversion formula [20, p. 363] and $\mathbf{V}^H\mathbf{J}_{//}=[\mathbf{I}_q,\mathbf{0}_{q\times(N-q)}]^T=\mathbf{E}_q$, we rewrite (23) as

$$\begin{aligned}&\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{J}}}}\hat{\tilde{\mathbf{x}}}_l\\&=\tau_l\mathbf{y}_l^H\mathbf{R}^{-1}\mathbf{E}_q(\mathbf{R}_{11}-\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{R}_{21})\mathbf{E}_q^T\mathbf{R}^{-1}\mathbf{y}_l\\&=\tau_l(\mathbf{y}_{l,1}-\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{y}_{l,2})^H(\mathbf{R}_{11}-\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{R}_{21})^{-1}\\&\quad\times(\mathbf{y}_{l,1}-\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{y}_{l,2})\end{aligned} \tag{24}$$

where

$$\mathbf{y}_l=\begin{bmatrix}\mathbf{y}_{l,1}\\\mathbf{y}_{l,2}\end{bmatrix},\mathbf{R}=\begin{bmatrix}\mathbf{R}_{11}&\mathbf{R}_{12}\\\mathbf{R}_{21}&\mathbf{R}_{22}\end{bmatrix} \tag{25}$$

with $\mathbf{y}_{l,1}\in\mathbb{C}^{q\times1},\mathbf{y}_{l,2}\in\mathbb{C}^{(N-q)\times1},\mathbf{R}_{11}\in\mathbb{C}^{q\times q},\mathbf{R}_{12}\in\mathbb{C}^{q\times(N-q)},\mathbf{R}_{21}\in\mathbb{C}^{(N-q)\times q}$, and $\mathbf{R}_{22}\in\mathbb{C}^{(N-q)\times(N-q)}$. Also, $\hat{\tilde{\mathbf{x}}}_l^H\mathbf{P}_{\hat{\tilde{\mathbf{U}}}}\hat{\tilde{\mathbf{x}}}_l$ can be expressed as a similar form like (24). Hence, the distribution of the test statistic in (21) is independent of $\mathbf{M}$. Therefore, the proposed detector ensures the asymptotic CFAR property with respect to the covariance matrix $\mathbf{M}$.

IV. Performance Assessment

In this section, numerical examples are provided to assess the performances of the proposed detector (referred to as proposed GLRT). The proposed detector is compared with other related detectors, including the GLRT in compound-Gaussian noise with recursive maximum likelihood (ML) estimator [7], denoted as GLRT-ML, the GLRT in Gaussian noise plus subspace interference [17], denoted as GLRT-I, and the two-step procedure of GLRT in Gaussian noise and interference [17], denoted as GLRT-2S-I. In all simulations, we set $N=8$, $L=4$, $r=1$, $q=3$, $\alpha_l=2$, $\beta_l=5$, $\mathbf{H}=1/\sqrt{N}[1,\ldots,1]^T$ unless other stated. The $\mathbf{a}_l$'s have the same deterministic amplitude

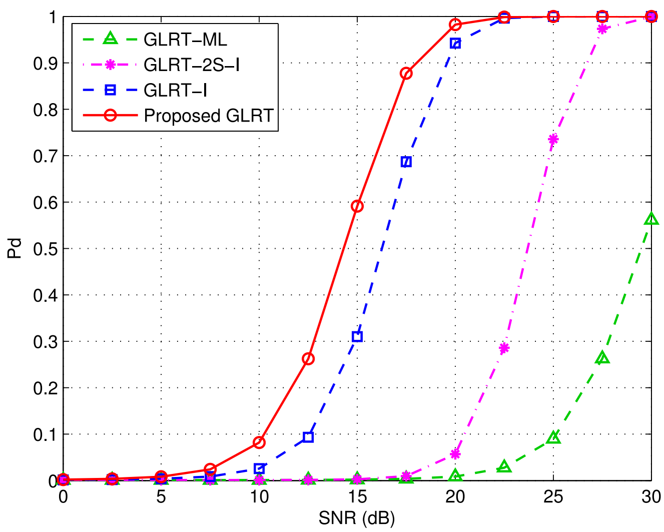

Fig. 2. Pd versus SNR: $N = 8$, $L = 4$, $\alpha = 2$, and $K = 12$.

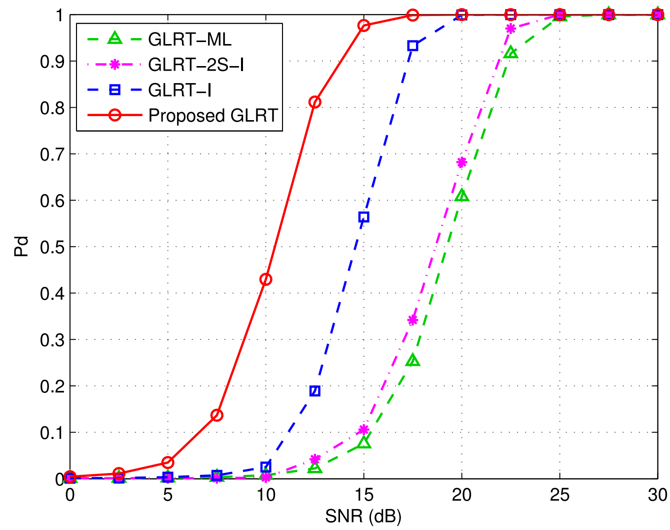

Fig. 3. Pd versus SNR: $N = 8$, $L = 4$, $\alpha = 2$, and $K = 48$.

and random phases that are independent variables with uniform distribution $[0, 2\pi]$. The mth column vector of the interference subspace matrix $\mathbf{J} = [\mathbf{j}_1, \ldots, \mathbf{j}_m, \ldots, \mathbf{j}_q]$ is given by

$$\mathbf{j}_m = \frac{1}{\sqrt{N}}[1, \exp(i2\pi r_m), \ldots, \exp(i2\pi r_m(N-1))]^T \quad (26)$$

where $r_m = 0.1m, m = 1, 2, \ldots, q$ denotes the normalized Doppler shift of the interference signal. As to $\mathbf{M}$, it is an exponentially correlated covariance matrix with one-lag correlation coefficient $\rho = 0.9$, namely, the (i, j)th element of $\mathbf{M}$ is $\sigma_n^2 \rho^{|i-j|}$, where σ_n^2 denotes the noise power. The interference coordinate $\mathbf{b}_l$ is independent and identically distributed complex Gaussian vector with zero mean and covariance matrix $\sigma_J^2 \mathbf{I}_q$, where σ_J^2 denotes the interference power. The interference-to-noise ratio (INR) is defined by σ_J^2/σ_n^2. In the following, we set INR = 15 dB. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{\text{tr}(\mathbf{A}^H\mathbf{H}^H\mathbf{M}^{-1}\mathbf{H}\mathbf{A})}{N}. \quad (27)$$

To decrease the computation load, the probability of false alarm (Pfa) is set to 10^{-3}. For the Monte Carlo simulation, the detection threshold and the probability of detection (Pd) are obtained from 100/Pfa and 10^5 Monte Carlo runs, respectively.

The detection performances for different size of secondary data are illustrated in Figs. 2 and 3, where K=12 and 48, respectively. As shown in Fig. 2, the proposed detector is better than other detectors in nonhomogeneous environments. It is also observed that the performance of the GLRT-ML is the worst. This is because the GLRT-ML does not consider the presence of the interference. Fig. 3 compares the detection performance of the proposed detector with the existing ones when the training data are sufficient. In this case, the proposed detector still maintains highest probabilities of detection. Moreover, the detection performance of the GLRT-ML improves, as the number of the training data increases.

The detection performance of the proposed detector for different values of the shape parameter α are given in Fig. 4, where $N = 8$, $L = 4$, and $K = 24$. The results show that the detection ability of the proposed detector is enhanced when the noise is

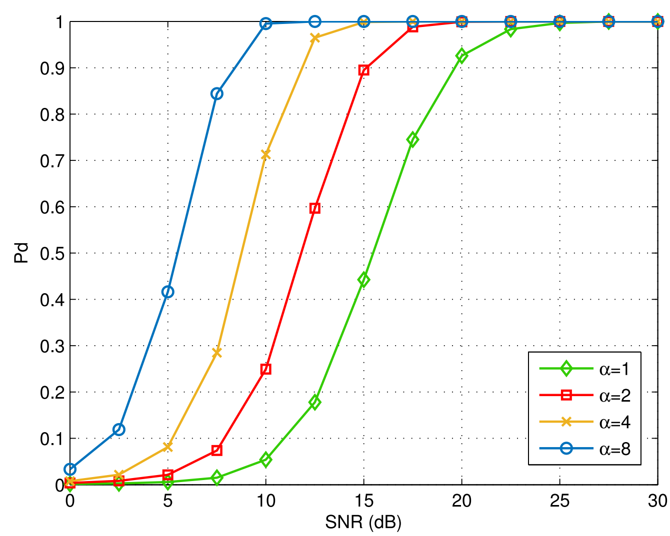

Fig. 4. Pd versus SNR for different α: $N = 8$, $L = 4$, and $K = 24$.

more speckle (i.e., for larger α). This phenomenon is consistent with the results in [21] and [22].

V. Conclusion

The letter dealt with the problem of detection of distributed targets in compound-Gaussian clutter in the presence of subspace structured interference. We adopted the two-step GLRT rule to design an effective detector. The proposed detector was shown to be asymptotic CFAR with respect to the covariance matrix structure. The performance assessment showed that the proposed detector exhibits a considerable performance improvement over the conventional detectors in nonhomogeneous environments. However, a limitation of the detector is that it must know the scale parameter in advance. Future work will concern the unknown scale parameter in real environments as well as fluctuation models for target returns.

REFERENCES

[1] E. Conte, M. Di Bisceglie, C. Galdi, and G. Ricci, "A procedure for measuring the coherence length of the sea texture," *IEEE Trans. Instrum. Meas.*, vol. 46, no. 4, pp. 836–841, Aug. 1997.

[2] A. Farina, F. Gini, M. V. Greco, and L. Verrazzani., "High resolution sea clutter data: Statistical analysis of recorded live data," *IEE Proc.-Radar, Sonar Navig.*, vol. 144, no. 3, pp. 121–130, Jun. 1997.

[3] K. J. Sangston and K. R. Gerlach, "Coherent detection of radar targets in a non-Gaussian background," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 30, no. 2, pp. 330–340, Apr. 1994.

[4] E. Conte, A. De Maio, and G. Ricci, "CFAR detection of distributed targets in non-Gaussian disturbance," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 38, no. 2, pp. 612–621, Apr. 2002.

[5] E. Conte and A. De Maio, "Distributed target detection in compound-Gaussian noise with Rao and Wald tests," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 2, pp. 568–582, Apr. 2003.

[6] Y. He, T. Jian, F. Su, C. Qu, and X. Gu, "Novel range-spread target detectors in non-Gaussian clutter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 46, no. 3, pp. 1312–1328, Jul. 2010.

[7] X. F. Shuai, L. J. Kong, and J. Y. Yang, "Performance analysis of GLRT-based adaptive detector for distributed targets in compound-Gaussian clutter," *Signal Process.*, vol. 90, no. 1, pp. 16–23, 2010.

[8] L. L. Scharf and B. Friedlander, "Matched subspace detectors," *IEEE Trans. Signal Process.*, vol. 42, no. 8, pp. 2146–2157, Aug. 1994.

[9] J. Liu, Z. Zhang, Y. Cao, and M. Yang, "Distributed target detection in subspace interference plus Gaussian noise," *Signal Process.*, vol. 95, no. 4, pp. 88–100, Feb. 2014.

[10] O. Besson, "Detection in the presence of surprise or under nulled interference," *IEEE Signal Process. Lett.*, vol. 14, no. 5, pp. 352–354, May. 2007.

[11] D. Ciuonzo, A. De Maio, and D. Orlando, "A unifying framework for adaptive radar detection in homogeneous plus structured interference-part I: On the maximal invariant statistic," *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2894–2906, Jun. 2016.

[12] D. Ciuonzo, A. De Maio, and D. Orlando, "A unifying framework for adaptive radar detection in homogeneous plus structured interference-part II: Detectors design," *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2907–2919, Jun. 2016.

[13] A. De Maio and D. Orlando, "Adaptive radar detection of a subspace signal embedded in subspace structured plus Gaussian interference via invariance," *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 2156–2167, Apr. 2016.

[14] A. Aubry, A. De Maio, D. Orlando, and M. Piezzo, "Adaptive detection of point-like targets in the presence of homogeneous clutter and subspace interference," *IEEE Signal Process. Lett.*, vol. 21, no. 7, pp. 848–852, Jul. 2014.

[15] F. Bandiera, O. Besson, D. Orlando, G. Ricci, and L. L. Scharf, "GLRT-based direction detectors in homogeneous noise and subspace interference," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2386–2394, Jun. 2007.

[16] W. Liu, J. Liu, L. Huang, D. Zou, and Y. Wang, "Rao tests for distributed target detection in interference and noise," *Signal Process.*, vol. 117, pp. 333–342, May 2015.

[17] F. Bandiera, A. De Maio, A. S. Greco, and G. Ricci, "Adaptive radar detection of distributed targets in homogeneous and partially homogeneous noise plus subspace interference," *IEEE Trans. Signal Process.*, vol. 55, no. 4, pp. 1223–1237, Apr. 2007.

[18] E. Conte, A. De Maio, and G. Ricci, "Recursive estimation of the covariance matrix of a compound-Gaussian process and its application to adaptive CFAR detection," *IEEE Trans. Signal Process.*, vol. 50, no. 8, pp. 1908–1915, Aug. 2002.

[19] F. Pascal, Y. Chitour, J. Ovarlez, P. Forster, and P. Larzabal, "Covariance structure maximum likelihood estimates in compound Gaussian noise: Existence and algorithm analysis," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 34–38, Jan. 2008.

[20] P. Stoica and R. Moses, *Spectral Analysis of Signals*. Upper Saddle River, NJ, USA: Prentice-Hall, 2005.

[21] F. Bandiera, O. Besson, and G. Ricci, "Knowledge-aided covariance matrix estimation and adaptive detection in compound-Gaussian noise," *IEEE Trans. Signal Process.*, vol. 58, no. 10, pp. 5391–5396, Oct. 2010.

[22] Y. Gao, G. Liao, S. Zhu, and D. Yang, "A persymmetric GLRT for adaptive detection in compound-Gaussian clutter with random texture," *IEEE Signal Process. Lett.*, vol. 20, no. 6, pp. 615–618, Jun. 2013.