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# A Novel Type of Skeleton for Polygons

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**Abstract** A new internal structure for simple polygons, the straight skeleton, is introduced and discussed. It is composed of pieces of angular bisectores which partition the interior of a given $n$-gon $P$ in a tree-like fashion into $n$ monotone polygons. Its straight-line structure and its lower combinatorial complexity may make the straight skeleton preferable to the widely used medial axis of a polygon. As a seemingly unrelated application, the straight skeleton provides a canonical way of constructing a polygonal roof above a general layout of ground walls.

**Keywords**: Simple polygon, angular bisectors, internal skeleton, roof construction

## 1 Introduction and basic properties

The purpose of this paper is to introduce and discuss a new and interesting internal structure for simple polygons in the plane. The new structure, called the *straight skeleton*, is solely made up of straight line segments which are pieces of angular bisectors of polygon edges. It uniquely partitions the interior of a given $n$-gon $P$ into $n$ monotone polygons, one for each edge of $P$.

The straight skeleton, in general, differs from the well-known *medial axis* of $P$ which consists of all interior points whose closest point on $P$'s boundary is not unique; see e.g. Lee [L]. If $P$ is convex then both structures are identical. Otherwise, the medial axis contains parabolically curved segments in the neighborhood of reflex vertices of $P$ which are avoided by the straight skeleton. If $P$ is rectilinear then the straight skeleton is the medial axis of $P$ for the $L_\infty$-metric. Skeletons have numerous applications inside and outside computer science as is documented e.g. in Kirkpatrick [K].

While the medial axis is a Voronoi-diagram-like concept, the straight skeleton is not defined using a distance function but rather by an appropriate *shrinking process* for $P$. Imagine that the boundary of $P$ is contracted towards $P$'s interior, in a self-parallel manner and at the same speed for all edges. Lengths of edges might decrease or increase in this process. Each vertex of $P$ moves along the angular bisector of its incident edges. This situation continues as long as the boundary does not change topologically. There are two possible types of changes:

(1) *Edge event*: An edge shrinks to zero, making its neighboring edges adjacent now.

(2) *Split event*: An edge is split, i.e., a reflex vertex runs into this edge, thus splitting the whole polygon. New adjacencies occur between the split edge and each of the two edges incident to the reflex vertex.

After either type of event, we are left with a new, or two new, polygons which are shrunk recursively if they have non-zero area. Note that certain events will occur simultaneously even if $P$ is in general position, namely three edge events letting a triangle collapse to a point. The shrinking process gives a hierarchy of nested polygons; see Figure 1(a).

The straight skeleton, $S(P)$, is defined as the union of the pieces of angular bisectors traced out by polygon vertices during the shrinking process. $S(P)$ is a unique structure defining a polygonal partition of $P$. Each edge $e$ of $P$ sweeps out a certain area which we call the *face* of $e$. Bisector pieces are called *arcs*, and their endpoints which are not vertices of $P$ are called *nodes*, of $S(P)$. See Figure 1(b).

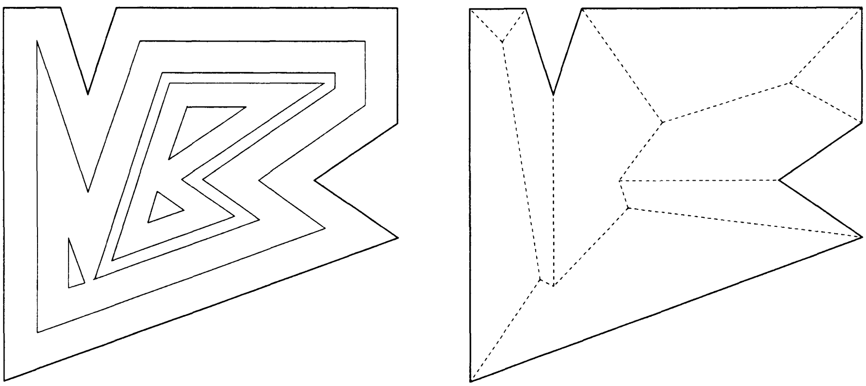

**Figure 1:** (a) Polygon hierarchy and (b) straight skeleton

As far as it is known to the authors, no attention has been paid to the straight skeleton in the literature. We show that $S(P)$ has several useful properties. For example, its tree structure implies that, if $P$ is non-convex, $S(P)$ is of smaller combinatorial size than the medial axis of $P$. The latter, though also being a tree, has to distinguish between curved and straight parts of arcs. To be precise, if $P$ is an $n$-gon with $r$ reflex vertices then $S(P)$ realizes $2n-3$ arcs whereas the medial axis of $P$ realizes $2n+r-3$ arcs, $r$ of which are parabolically curved. As a particularly nice property, $S(P)$ partitions $P$ into monotone polygons.

A three-dimensional interpretation of $S(P)$, the roof model, is discussed in Section 2 and Section 3. This leads us to the interesting and practically relevant question of constructing a roof of fixed slope above a given layout $P$ of ground walls. The roof model allows us to gain more insight into the structure of straight skeletons and, in particular, gives a way to define $S(P)$ non-procedurally. On the