Chapter 7. Statistical Intervals Based on a Single Sample

Peijia Zheng (郑培嘉)
School of Data & Computer Science
Sun Yat-Sen University
Email：zhpj@mail.sysu.edu.cn

Chapter 7: Statistical Intervals Based on A Single Sample

Introduction

- A point estimation provides no information about the **precision** and **reliability** of estimation.
- For example, using the statistic $\overline{X}$ to calculate a point estimate for the true average breaking strength (g) of paper towels of a certain brand, and suppose that $\overline{X} = 9322.7$. Because of sample variability, it is virtually never the case that $\overline{X} = \mu$. The point estimate says nothing about how close it might be to μ.
- An alternative to reporting a single sensible value for the parameter being estimated is to calculate and report an entire interval of plausible values—an interval estimate or confidence interval (CI)

7.1 Basic Properties of Confidence Intervals

- Considering a Simple Case

Suppose that the parameter of interest is a population mean μ and that

1. The population distribution is normal.
2. The value of the population standard deviation σ is known

- Normality of the population distribution is often a reasonable assumption.
- If the value of μ is unknown, it is implausible that the value of σ would be available.

In later sections, we will develop methods based on less restrictive assumptions.

7.1 Basic Properties of Confidence Intervals

- Example 7.1

  Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. A sample of n = 31 trained typists was selected , and the preferred keyboard height was determined for each typist. The resulting sample average preferred height was 80.0 cm. Assuming that preferred height is normally distributed with σ = 2.0 cm. Please obtain a CI for μ, the true average preferred height for the population of all experienced typists.

  Consider a random sample X_1, X_2, … X_n from the normal distribution with mean value μ and standard deviation σ . Then according to the proposition in Chapter 5, the sample mean is normally distribution with expected value μ and standard deviation $\delta / \sqrt{n}$

7.1 Basic Properties of Confidence Intervals

- Example 7.1 (Cont')

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$P(-z_{0.025} \le \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \le z_{0.025}) = 0.95 \quad \text{we have } z_{0.025} = 1.96$$

$$P(-1.96 \le \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \le 1.96) = 0.95$$

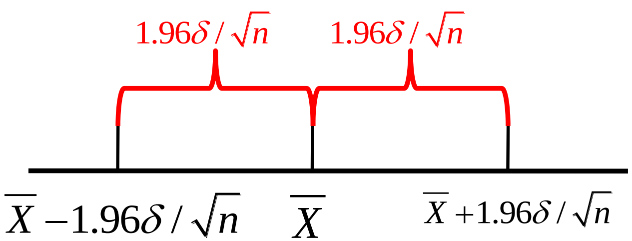
$$\bar{X} - 1.96 \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$

7.1 Basic Properties of Confidence Intervals

- Example 7.1 (Cont')

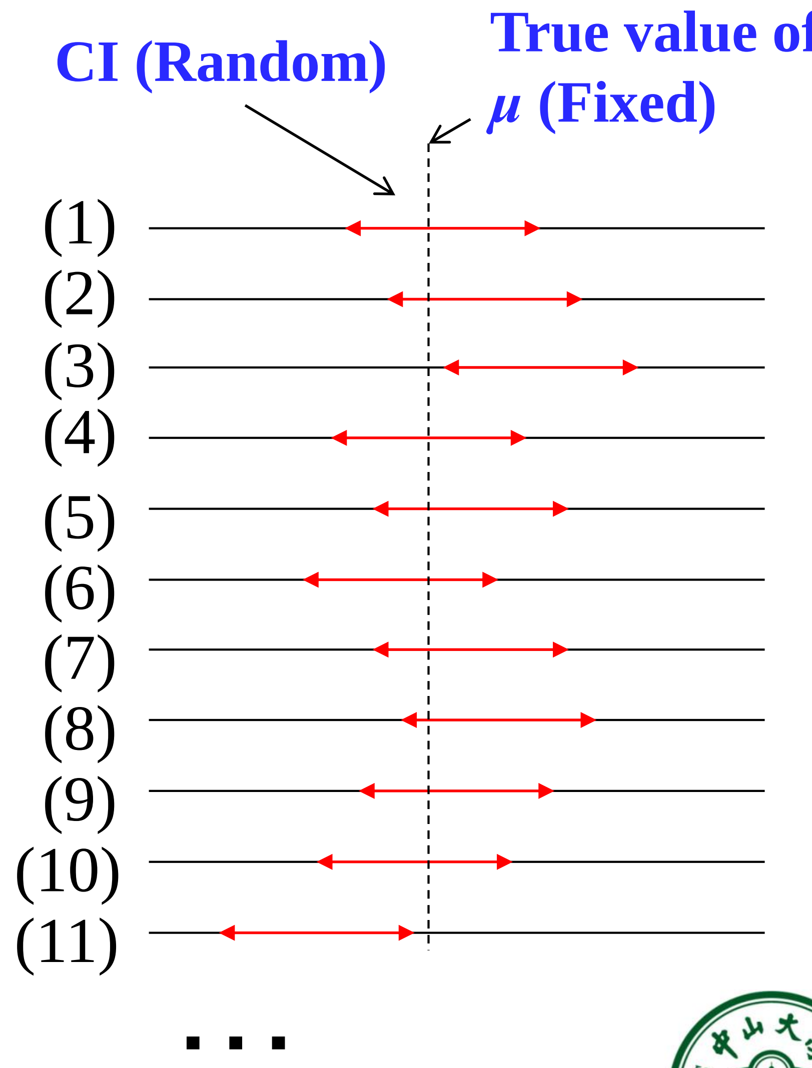
The CI of 95% is:

$$\bar{X} - 1.96 \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$



Interpreting a CI: It can be paraphrased as "the probability is 0.95 that the random interval includes or covers the true value of μ.

Interval number with different sample means



7.1 Basic Properties of Confidence Intervals

- Example 7.2 (Ex. 7.1 Cont')

  The quantities needed for computation of the 95% CI for average preferred height are δ=2, n=31and $\bar{x}=80$. The resulting interval is

$$\bar{x} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}} = 80.0 \pm (1.96)\frac{2.0}{\sqrt{31}} = 80.0 \pm .7 = (79.3, 80.7)$$

That is, we can be highly confident that 79.3 < μ < 80.7. This interval is relatively narrow, indicating that μ has been rather precisely estimated.

7.1 Basic Properties of Confidence Intervals

- Definition

If after observing $X_1=x_1$, $X_2=x_2$, … $X_n=x_n$, we compute the observed sample mean $\bar{x}$. The resulting fixed interval is called a 95% confidence interval for μ. This CI can be expressed either as

$$\left(\bar{x}-1.96\cdot\frac{\sigma}{\sqrt{n}},\bar{x}+1.96\cdot\frac{\sigma}{\sqrt{n}}\right)$$

is a 95% CI for μ

or as $$\bar{x}-1.96\cdot\frac{\sigma}{\sqrt{n}}<\mu<\bar{x}+1.96\cdot\frac{\sigma}{\sqrt{n}}$$ with a 95% confidence

Lower Limit

Upper Limit

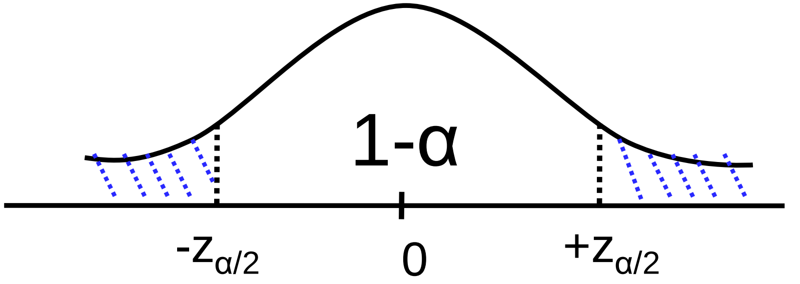
7.1 Basic Properties of Confidence Intervals

- Other Levels of Confidence

Why is Symmetry?

$P(a<z<b) = 1-\alpha$

Refer to the Definition Z_α



A 100(1- α)% confidence interval for the mean μ of a normal population when the value of σ is known is given by

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) \text{ or, } \quad \bar{x} \pm z_{\alpha/2} \cdot \sigma / \sqrt{n}$$

For instance, the 99% CI is $\bar{x} \pm 2.58 \cdot \sigma / \sqrt{n}$

7.1 Basic Properties of Confidence Intervals

- Example 7.3

Let's calculate a confidence interval for true average hole diameter using a confidence level of 90%.

This requires that 100(1-α) = 90, from which α = 0.1 and $z_{\alpha/2} = z_{0.05}$ = 1.645. The desired interval is then

$$5.426 \pm (1.645)\cdot\frac{0.100}{\sqrt{40}} = 5.426 \pm 0.26 = (5.400, 5.452)$$

7.1 Basic Properties of Confidence Intervals

- Confidence Level, Precision, and Choice of Sample Size

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

Then the width (Precision) of the CI

$$w = 2 \times z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Independent of the sample mean

Higher confidence level (larger $z_{\alpha/2}$) ➔ A wider interval

Reliability ⇔ **Precision**

Larger σ ➔ A wider interval

Smaller n ➔ A wider interval

Given a desired confidence level (α) and interval width (w), then we can determine the necessary sample size n, by

$$n = \left(2 z_{a/2} \cdot \frac{\sigma}{w}\right)^2$$

7.1 Basic Properties of Confidence Intervals

- Example 7.4

Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with standard deviation 25 millisec. A new operating system has been installed, and we wish to estimate the true average response time μ for the new environment. Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% CI has a width of no more than 10? The sample size n must satisfy

$$10 = 2 \cdot (1.96)\left(25/\sqrt{n}\right)$$

$$\Rightarrow \quad \sqrt{n} = 2 \cdot (1.96)(25)/10 = 9.80$$

$$\Rightarrow \quad n = (9.80)^2 = 96.04$$

Since n must be an integer, a sample size of 97 is required.

7.1 Basic Properties of Confidence Intervals

- Deriving a Confidence Interval

In the previous derivation of the CI for the unknown population mean $\theta = \mu$ of a normal distribution with known standard deviation σ, we have constructed the variable

$$h(X_1, X_2, \ldots, X_n; \mu) = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Two properties of the random variable

- depending functionally on the parameter to be estimated (i.e., μ)
- having the standard normal probability distribution, which does not depend on μ.

7.1 Basic Properties of Confidence Intervals

- The Generalized Case

  Let $X_1, X_2, \ldots, X_n$ denote a sample on which the CI for a parameter θ is to be based. Suppose a random variable $h(X_1, X_2, \ldots, X_n; \theta)$ satisfying the following two properties can be found:

1. The variable depends functionally on both $X_1, X_2, \ldots, X_n$ and θ.
2. The probability distribution of the variable does not depend on θ or on any other unknown parameters.

7.1 Basic Properties of Confidence Intervals

- In order to determine a 100(1-α)% CI of θ, we proceed as follows:

$$P(a < h(X_1, X_2, ..., X_n; \theta) < b) = 1 - \alpha$$

- Because of the second property, *a* and *b* do not depend on θ. In the normal example, we had a=-$Z_{\alpha/2}$ and b=$Z_{\alpha/2}$ Suppose we can isolate θ in the inequation:

$$P(l(X_1, X_2, ..., X_n) < \theta < u(X_1, X_2, ..., X_n)) = 1 - \alpha$$

So a 100(1-α)% CI is $[l(X_1, X_2, ..., X_n),\ u(X_1, X_2, ..., X_n)]$

- In general, the form of the *h* function is suggested by examining the distribution of an appropriate estimator $\hat{\theta}$.

- Example 7.5

A theoretical model suggest that the time to breakdown of an insulating fluid between electrodes at a particular voltage has an exponential distribution with parameter λ. A random sample of n = 10 breakdown times yields the following sample data :

$$x_1 = 41.53, x_2 = 18.73, \quad x_3 = 2.99, x_4 = 30.34, x_5 = 12.33,$$

$$x_6 = 117.52, x_7 = 73.02, x_8 = 223.63, \quad x_9 = 4.00, x_{10} = 26.78$$

A 95% CI for λ and for the true average breakdown time are desired.

$$h(X_1, X_2, \ldots, X_n; \lambda) = 2\lambda \sum X_i$$

It can be shown that this random variable has a probability distribution called a chi-squared distribution with 2*n* degrees of freedom. (Properties #2 & #1)

7.1 Basic Properties of Confidence Intervals

- Example 7.5 (Cont')

$$p\left(9.591 < 2\lambda \sum X_i < 34.170\right) = 0.95$$

$$p\left(9.591 / \left(2\sum X_i\right) < \lambda < 34.170 / \left(2\sum X_i\right)\right) = 0.95$$

For the given data, $\Sigma x_i = 550.87$, giving the interval (0.00871, 0.03101).

The 95% CI for the population mean of the breakdown time:

$$p\left(2\sum X_i / 34.170 < 1/\lambda < 2\sum X_i / 9.591\right) = 0.95$$

$$\left(2\sum x_i / 34.170, 2\sum x_i / 9.591\right) = (32.24, 114.87)$$

7.1 Basic Properties of Confidence Intervals

- Homework

  Ex. 1, Ex. 5, Ex. 10

7.2 Large-Sample Confidence Intervals for a Population Mean and Proportion

- The CI for μ given in the previous section assumed that the population distribution is normal and that the value of σ is known. We now present a **large-sample** CI whose validity does not require these assumptions.

- Let $X_1, X_2, \ldots X_n$ be a random sample from a population having a mean μ and standard deviation σ (any population, normal or un-normal).

  Provided that n is large (Large-Sample), the Central Limit Theorem (CLT) implies that $\bar{X}$ has approximately a normal distribution whatever the nature of the population distribution.

7.2 Large-Sample Confidence Intervals for a Population Mean and Proportion

Thus we have $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1) \Rightarrow P(-z_{\alpha/2} < \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} < z_{\alpha/2}) \approx 1 - \alpha$

Therefore, $\bar{x} \pm z_{\alpha/2} \cdot \sigma / \sqrt{n}$ is a large-sample CI for μ with a confidence level of **approximately** $100(1-\alpha)\%$.

That is , when n is large, the CI for μ given previously remains valid whatever the population distribution, provided that the qualifier "approximately" is inserted in front of the confidence level.

When σ is not known, which is generally the case, we may consider the following standardized variable

$$Z = \frac{\bar{X} - \mu}{S / \sqrt{n}} \qquad S \approx \delta$$

- Proposition

If n is sufficiently large (usually, n>40), the standardized variable

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

has approximately a standard normal distribution, meaning that

$$\bar{x} \pm z_{a/2} \cdot \frac{s}{\sqrt{n}}$$

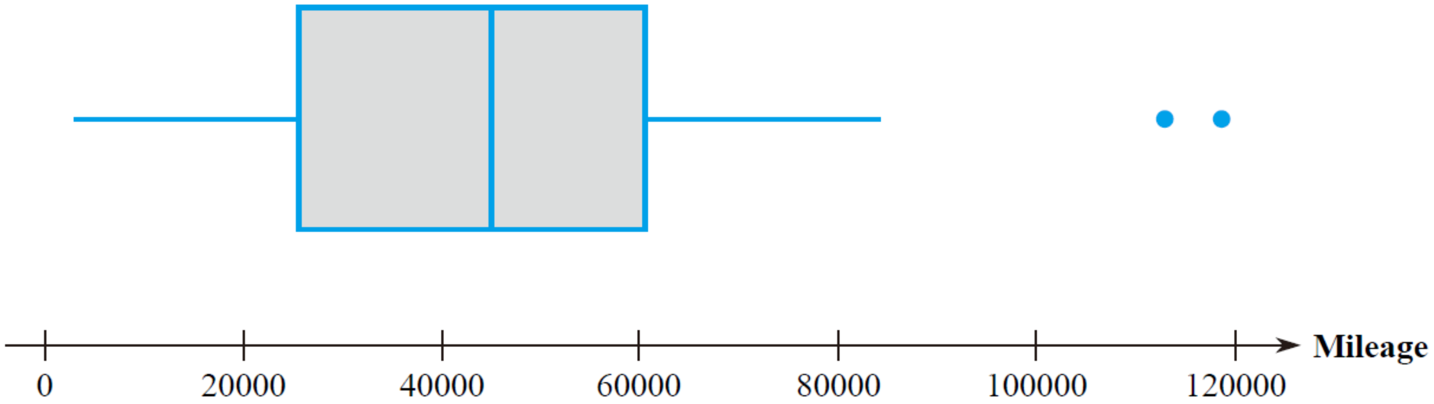
Compared with (7.5) in pp.272
$s \approx \delta$

is a large-sample confidence interval for μ with confidence level approximately 100(1-α)%.

Note: This formula is valid regardless of the shape of the population distribution.

- Example 7.6

2948	2996	7197	8338	8500	8759	12710	12925
15767	20000	23247	24863	26000	26210	30552	30600
35700	36466	40316	40596	41021	41234	43000	44607
45000	45027	45442	46963	47978	49518	52000	53334
54208	56062	57000	57365	60020	60265	60803	62851
64404	72140	74594	79308	79500	80000	80000	84000
113000	118634						



- Example 7.6 (Cont')

Summary quantities include $n = 50$, $\overline{x} = 45{,}679.4$, $\tilde{x} = 45{,}013.5$, $s = 26{,}641.675$, $f_s = 34{,}265$. The mean and median are reasonably close (if the two largest values were each reduced by 30,000, the mean would fall to 44,479.4, while the median would be unaffected). The boxplot and the magnitudes of s and f_s relative to the mean and median both indicate a substantial amount of variability. A confidence level of about 95% requires $z_{.025} = 1.96$, and the interval is

$$45{,}679.4 \pm (1.96)\left(\frac{26{,}641.675}{\sqrt{50}}\right) = 45{,}679.4 \pm 7384.7$$

$$= (38{,}294.7,\ 53{,}064.1)$$

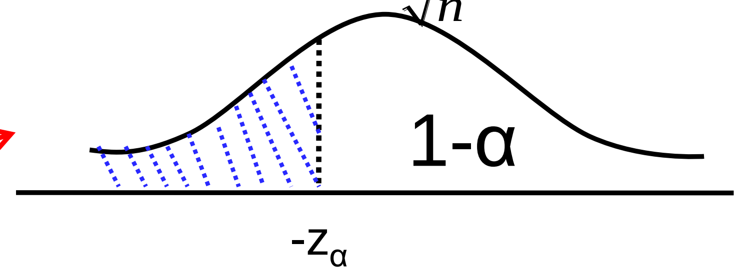
7.2 Large-Sample Confidence Intervals for a Population Mean and Proportion

- One-Sided Confidence Intervals **(Confidence Bounds)**

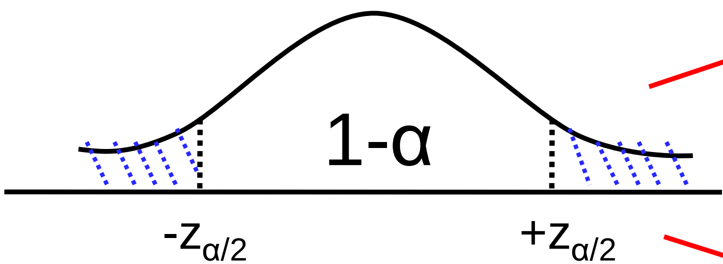
So far, the confidence intervals give both a lower confidence bound and an upper bound for the parameter being estimated.

In some cases, we will want only the upper confidence or the lower one.

$$\mu < \overline{x} + z_{\alpha} \cdot \frac{s}{\sqrt{n}} \ (upper \quad bound)$$

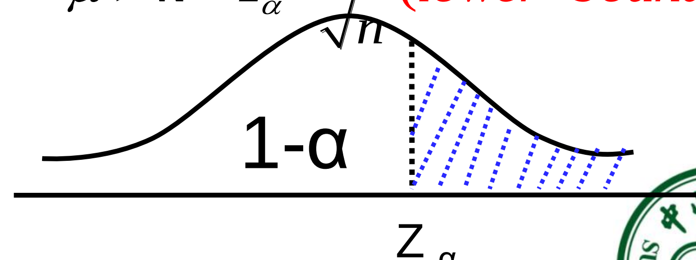


$$\mu \in (\overline{x} - z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}, \overline{x} + z_{\alpha/2} \cdot \frac{s}{\sqrt{n}})$$



Standard Normal Curve

$$\mu > \overline{x} - z_{\alpha} \cdot \frac{s}{\sqrt{n}} \ (lower \quad bound)$$



- Proposition

A large-sample upper confidence bound for μ is

$$\mu < \bar{x} + z_{\alpha} \cdot \frac{s}{\sqrt{n}}$$

and a large-sample lower confidence bound for μ is

$$\mu > \bar{x} - z_{\alpha} \cdot \frac{s}{\sqrt{n}}$$

Compared the formula (7.8) in pp.277

- Example 7.10

A sample of 48 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a sample standard deviation of 3.28 N/mm^2.

Then A lower confidence bound for true average shear strength μ with confidence level 95% is

$$17.17-(1.645)\frac{(3.28)}{\sqrt{48}}=17.17-0.78=16.39$$

Namely, with a confidence level of 95%, the value of μ lies in the interval $(16.39, \infty)$.

- Homework

  Ex. 12, Ex. 15, Ex. 16

7.3 Intervals Based on a Normal Population Distribution

- The CI for μ presented in the previous section is valid provided that n is large. The resulting interval can be used whatever the nature of the population distribution (with unknown μ and σ).
- If n is small, the CLT can not be invoked. In this case we should make a specific assumption.
- Assumption

  The population of interest is normal, X_1, X_2, … X_n constitutes a random sample from a normal distribution with both μ and δ unknown.

- Theorem

When $\overline{X}$ is the mean of a random sample of size n from a normal distribution with mean μ. Then the rv

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

has a probability distribution called a t distribution with n-1 degrees of freedom (df) .

only n-1 of these are "freely determined"

S is based on the **n** deviations $(X_1 - \overline{X}), (X_2 - \overline{X}), \ldots, (X_n - \overline{X})$

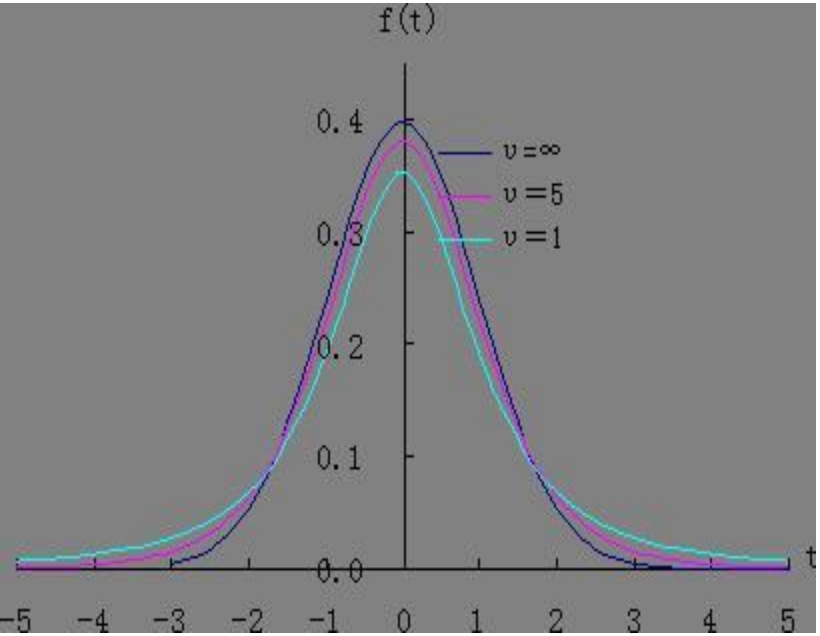
Notice that $\sum_{i=1}^{n}(X_i - \overline{X}) = 0$

- Properties of *t* Distributions

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

The only one parameter in T is the number of df: v=n-1

Let t_v be the density function curve for v df

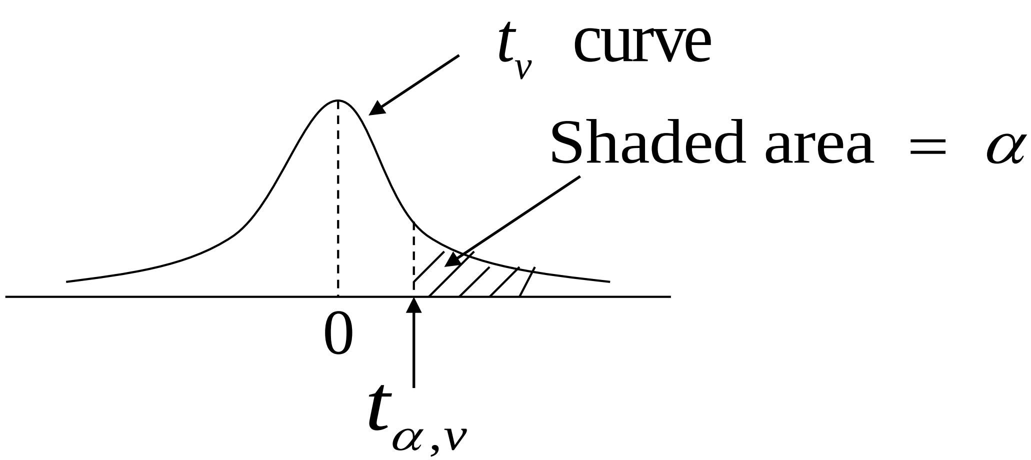


1. Each t_v curve is bell-shaped and centered at 0.
2. Each t_v curve is more spread out than the standard normal curve.
3. As v increases, the spread of the corresponding t_v curve decreases.
4. As v→ ∞, the sequence of t_v curves approaches the standard normal curve N(0,1) .

Rule: v ≥ 40 ~ N(0,1)

- Notation

Let $t_{\alpha,v}$ = the value on the measurement axis for which the area under the *t* curve with v df to the right of $t_{\alpha,v}$ is α; $t_{\alpha,v}$ is called a ***t* critical value**



Fixed α, v↑, $t_{\alpha,v}$↓

Fixed v, α↑, $t_{\alpha,v}$↓

Figure 7.7 A pictorial definition of $t_{\alpha,v}$

Refer to pp.164 for the similar definition of Z_α

The One-Sample t confidence Interval

The standardized variable *T* has a *t* distribution with *n*-1 df, and the area under the corresponding *t* density curve between $-t_{\alpha/2,n-1}$ and $t_{\alpha/2,n-1}$ is $1-\alpha$, so

$$P(-t_{\alpha/2,n-1} < T < t_{\alpha/2,n-1}) = 1-\alpha$$

Proposition: Let *x* and *s* be the sample mean and sample standard deviation computed from the results of a random sample from a normal population with mean μ. Then a 100(1-α)% confidence interval for μ is

$$\left(\bar{x} - t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}\right)$$

Or, compactly $\bar{x} \pm t_{\alpha/2,n-1} \cdot s/\sqrt{n}$

An upper confidence bound with 100(1-α)% confidence level for μ is $\bar{x} + t_{\alpha,n-1} \cdot s/\sqrt{n}$. Replacing + by – gives a lower confidence bound for μ.

Compared with the propositions in pp 286, 292 & 297

- Example 7.11

Consider the following observations

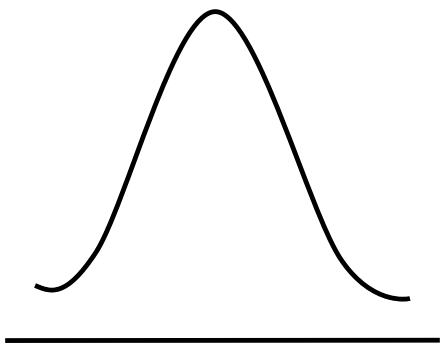
10490	**16620**	**17300**	**15480**	**12970**	**17260**	**13400**	**13900**
13630	**13260**	**14370**	**11700**	**15470**	**17840**	**14070**	**14760**

1. approximately normal by observing the probability plot.
2. n = 16 is small, and the population deviation σ is unknown, so we choose the statistic T with a t distribution of n – 1 = 15 df. The resulting 95% CI is

$$\bar{x} \pm t_{.025,15} \cdot \frac{s}{\sqrt{n}} = 14,532.5 \pm (2.131)\frac{2055.67}{\sqrt{16}} = (13,437.3,\ 15,627.7)$$

7.3 Intervals Based on a Normal Population Distribution

- A Prediction Interval for a Single Future Value



Population with unknown parameter

Given a random sample of size n from the population

$X_1, X_2, \ldots, X_n$

Estimation the population parameter, *e.g.* μ
(Point Estimation)

Deriving a Confidence Interval (CI) for the population parameter, *e.g.* μ
(Confidence Interval)

Deriving a Confidence Interval for a new arrival X_{n+1}
(Prediction Interval)

- Example 7.12

Consider the following sample of fat content (in percentage) of n = 10 randomly selected hot dogs

25.2 21.3 22.8 17.0 29.8 21.0 25.5 16.0 20.9 19.5

Assume that these were selected from a normal population distribution.

Please give a 95% **CI** for the population mean fat content.

$$\bar{x} \pm t_{.025,9} \cdot \frac{s}{\sqrt{n}} = 21.90 \pm 2.262 \cdot \frac{4.134}{\sqrt{10}} = 21.90 \pm 2.96$$

$$= (18.94, 24.86)$$

- Example 7.12 (Cont’)

  Suppose, however, we are only interested in predicting the fat content of the next hot dog in the previous example. How would we proceed?

  Point Estimation (point prediction):

$$\overline{X} = 21.90$$

  Can not give any information on reliability or precision.

- Prediction Interval (PI)

Let the fat content of the next hot dog be X_{n+1}. A sensible point predictor is $\bar{X}$. Let's investigate the prediction error $\bar{X} - X_{n+1}$.

$\bar{X} - X_{n+1}$ **is a normal rv** with **Why?**

$E(\bar{X} - X_{n+1}) = 0$ and

$$V(\bar{X} - X_{n+1}) = V(\bar{X}) + V(X_{n+1}) = \frac{\sigma^2}{n} + \sigma^2 = \sigma^2\left(1 + \frac{1}{n}\right)$$

7.3 Intervals Based on a Normal Population Distribution

- Example 7.12 (Cont')

$$E(\bar{X} - X_{n+1}) = 0$$

$$V(\bar{X} - X_{n+1}) = \sigma^2\left(1 + \frac{1}{n}\right)$$

$$\Rightarrow \quad Z = \frac{\bar{X} - X_{n+1}}{\sigma\sqrt{1 + \frac{1}{n}}} \sim N(0,1)$$

unknown

$$T = \frac{\bar{X} - X_{n+1}}{S\sqrt{1 + \frac{1}{n}}}$$ ~ t distribution with n-1 df

- Proposition

A prediction interval (PI) for a single observation to be selected from a normal population distribution is

$$\bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1+\frac{1}{n}}$$

The prediction level is 100(1-α)%

- Example 7.13 (Ex. 7.12 Cont')

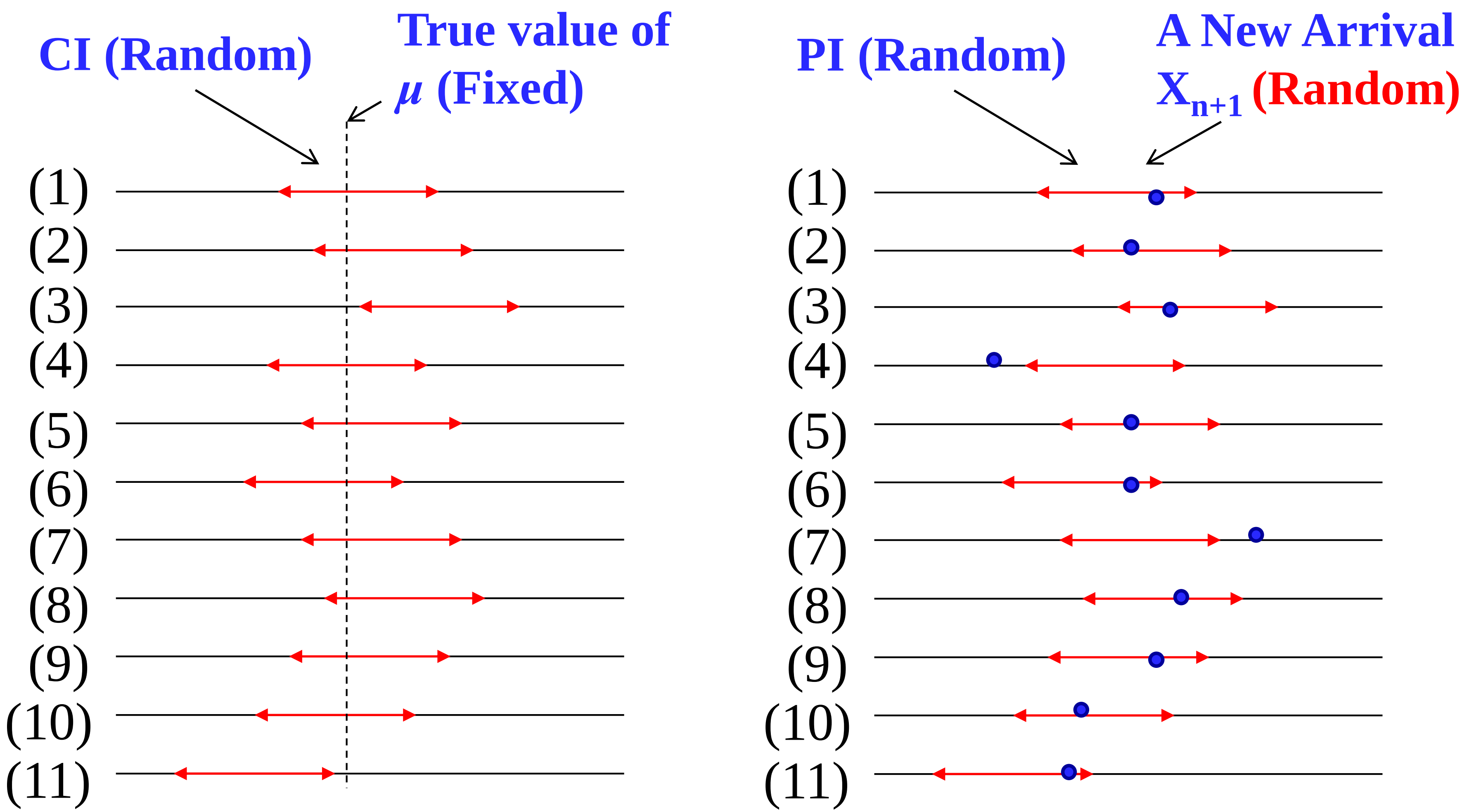
With n=10, sample mean is 21.90, and $t_{0.025,9}$=2.262, a 95% PI for the fat content of a single hot dog is

$$\bar{x} \pm t_{0.025,9} \cdot s\sqrt{1+1/n}$$

$$= 21.90 \pm 2.262 \cdot 4.134\sqrt{1+1/10} = 21.90 \pm 9.81$$

$$= (12.09, 31.71)$$

7.3 Intervals Based on a Normal Population Distribution



There is more variability in the PI than in CI due to X_{n+1}

7.3 Intervals Based on a Normal Population Distribution

- Homework

  Ex. 32, Ex.33

7.4 Confidence Intervals for the Variance and Standard Deviation of a Normal Population

- In order to obtain a CI for the variance σ^2 of a normal distribution, we start from its point estimator, S^2

- Theorem

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with parameter μ and σ^2 . Then the rv

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum (X_i - \overline{X})^2}{\sigma^2}$$

has a chi-squared (χ^2) probability distribution with ***n-1*** df.

Note: The two properties for deriving a CI in pp. 288 are satisfied.

- The Distributions of χ^2

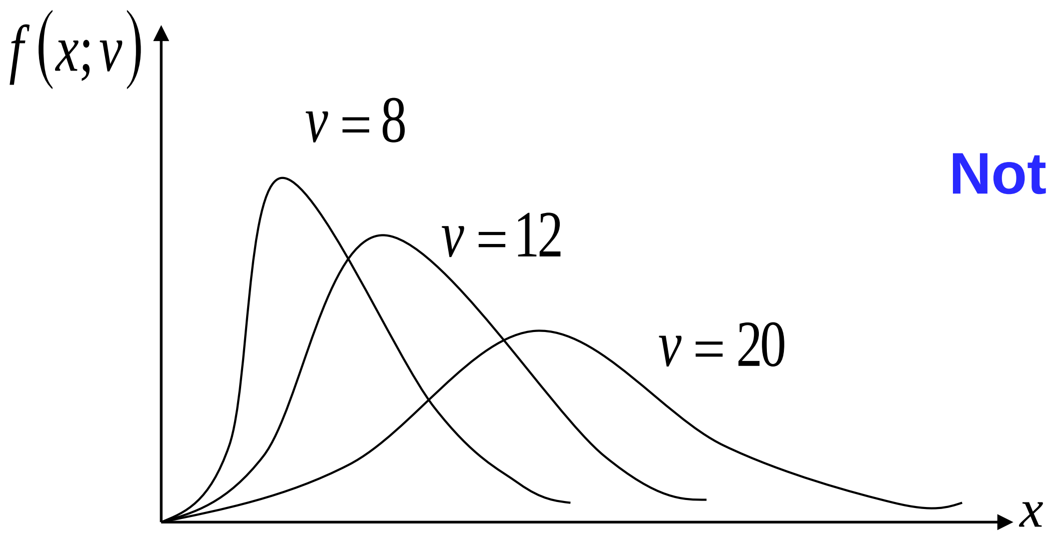


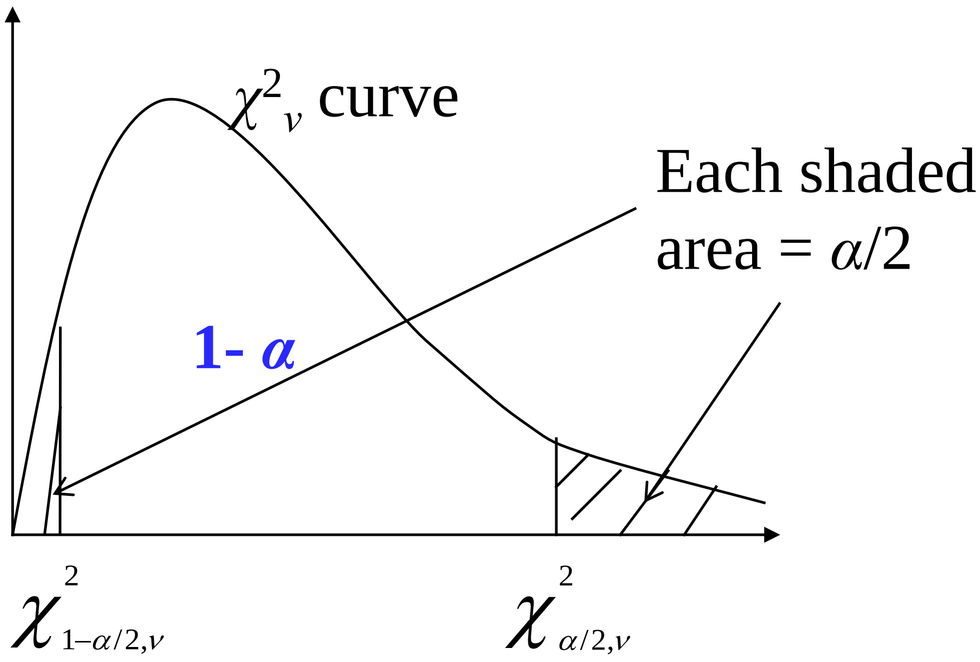
Figure 7.9 Graphs of chi-squared density functions

Not a Symmetric Shape

Refer to Table A.7 in 677

7.4 Confidence Intervals for the Variance and Standard Deviation of a Normal Population

- Chi-squared critical value $\chi^2_{\alpha,\nu}$



$$P\left(\chi^2_{1-\alpha/2,n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2,n-1}\right) = 1-\alpha \Rightarrow \frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

7.4 Confidence Intervals for the Variance and Standard Deviation of a Normal Population

- Proposition

A 100(1- α)% confidence interval for the variance σ^2 of a normal population is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

v=n-1

Lower Limit **Upper Limit**

A confidence interval for σ *is*

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}$$

- Example 7.15

The accompanying data on breakdown voltage of electrically stressed circuits was read from a normal probability plot. The straightness of the plot gave strong support to the assumption that breakdown voltage is approximately normally distributed .

1170	1510	1690	1740	1900	2000	2030	2100	2190
2200	2290	2380	2390	2480	2500	2580	2700	

Let σ^2 denote the variance of the breakdown voltage distribution and it is unknown. Determine the 95% confidence interval of σ^2.

- Example 7.15 (Cont')

The computed value of the sample variance is s^2 =137,324.3, the point estimate of σ^2. With df = n-1 =16, a 95% CI require $\chi^2_{0.975,16}$ = 6.908 and $\chi^2_{0.025,16}$ = 28.845. The interval is

$$\left(\frac{16(137,324.3)}{28.845},\ \frac{16(137,324.3)}{6.908}\right)=(76,172.3,\ 318,064.4)$$

Taking the square root of each endpoint yields (276.0,564.0) as the 95% CI for σ.

Summary of Chapter 7

- General method for deriving CIs (2 properties, p.288)

Case #1: (7.1)

CI for μ of a normal distribution with known σ;

Case #2: (7.2)

Large-sample CIs for μ of General distributions with unknown σ

Case #3: (7.3)

Small-sample CIs for μ of Gaussian distributions with unknown σ

- Both Sided Vs. One-sided CIs (p.297)
- PI (p.303) & CIs for σ^2 (7.4)

7.4 Confidence Intervals for the Variance and Standard Deviation of a Normal Population

- Homework

  Ex. 44