Lec 01/02

2019年2月25日 星期一 上午9:52

Measure Theory vs Probability Theory

Independence

$P(A\cap B) = P(A)P(B)$

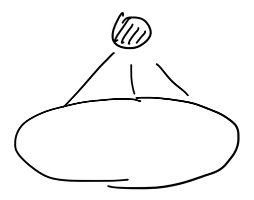
Chapter 0

Branch Process

$Z_0 = 0$

$Z_1 =$

$Z_2 =$



$P(X=k) = p_k$

$EX = \mu$

母函数. $f(\theta) = E(\theta^X) = P(X=0) + \sum_{k\in\mathbb{Z}^+} \theta^k (X=k)$

($f(\theta)$ 与 $p_1, p_2 \cdots p_n$ 关联起来) X

$f(0) = P(X=0)$. $f(1) = 1$. $f'(1) = EX$

$Z_n \to f_n(\theta) = E(\theta^{Z_n}) = P(Z_n = 0) + \sum_{k\in\mathbb{Z}^+} \theta^k P(Z_n = k)$

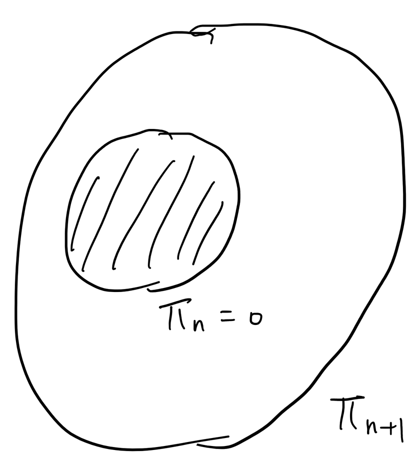
$Z_n = X_1^n + X_2^n + \cdots + X_{Z_{n-1}}^n$

$$\begin{aligned} f_n(\theta) = E(\theta^{Z_n}) &= EE(\theta^{Z_n} \mid Z_{n-1}) \\ &= EE(\theta^{X_1^n}\cdot\theta^{X_2^n}\cdots X^{X^n_{Z_{n-1}}} \mid Z_{n-1}) \\ &= E\left(\left(E(\theta^X)\right)^{Z_{n-1}}\right) \\ &= E(f(\theta)^{Z_{n-1}}) \\ &= f_{n-1}(f(\theta)) \end{aligned}$$

$\Rightarrow f_n(\theta) = f^n(\theta) = f\circ f\circ\cdots f(\theta)$

$\pi_n = P(Z_n = 0) = f_n(0)$ $\pi_n \uparrow$ $\pi_n = f_n(0) = f(\pi_{n-1})$

$\pi = \lim \pi_n$. $\longrightarrow$ 直觉：消亡概率



$\Leftrightarrow \mu(\lim Z_n = 0) = \lim \mu(Z_n = 0)$ → μ是测度. 具有可列可加性. 关键在赋予概率空间的结构

$\pi_n = 0$

$\pi_{n+1} = 0$

$E(Z_n) = \mu^n$, $M_n = Z_n/\mu^n$

$E(M_{n+1} \mid Z_1, \cdots Z_n) = E(M_{n+1} \mid Z_n) = E\left(\frac{Z_{n+1}}{\mu^{n+1}} \Big| Z_n\right) = \frac{\mu Z_n}{\mu^{n+1}} = M_n$

$\Rightarrow$ Markov process

M_n是鞅

Probability theory

In pure and applied probability, the Laplace transform is defined as an expected value. If X is a random variable with probability density function f, then the Laplace transform of f is given by the expectation

$\mathcal{L}\{f\}(s) = \mathrm{E}[e^{-sX}]$. 随机变量X的Laplace反演，本质上是pdf的反演

By convention, this is referred to as the Laplace transform of the random variable X itself. Replacing s by $-t$ gives the moment generating function of X. The Laplace transform has applications throughout probability theory, including first passage times of stochastic processes such as Markov chains, and renewal theory.

Of particular use is the ability to recover the cumulative distribution function of a continuous random variable X by means of the Laplace transform as follows[16]

$$F_X(x) = \mathcal{L}^{-1}\left\{\frac{1}{s}\mathrm{E}\left[e^{-sX}\right]\right\}(x) = \mathcal{L}^{-1}\left\{\frac{1}{s}\mathcal{L}\{f\}(s)\right\}(x).$$

鞅收敛定理 $M_n \to M_\infty$ (a.s.)

$e^{-\lambda M_n} \to e^{-\lambda M_\infty}$ (a.s)

有界收敛定理 $Ee^{-\lambda M_n} \to Ee^{-\lambda M_\infty} = \lim Ee^{-\lambda M_n}$

$\mathcal{L}(\lambda) = Ee^{-\lambda M_\infty} = \lim Ee^{-\frac{\lambda Z_n}{\mu^n}} = \lim f_n(e^{-\frac{\lambda}{\mu^n}})$

M_∞的Laplace变换

$\mathcal{L}(\lambda\mu) = \lim f_n(e^{-\frac{\lambda}{\mu^{n-1}}}) = \lim f(f_{n-1}(e^{-\frac{\lambda}{\mu^{n-1}}})) = \lim f(Ee^{-\frac{\lambda Z_{n-1}}{\mu^{n-1}}}) = f(\mathcal{L}(\lambda))$

$\boxed{\text{E1.1}}$ $C \subseteq N$

$\rho(C) = \lim_{n\to\infty} \frac{|C\cap\{1,2\cdots n\}|}{n}$ 存在. $C \in CES$

Césaro 测度 CES不是一个代数

Pf. ① $C \subseteq N$. $f_n(c)$: $\frac{1}{2}$, 1

$A = \{3c : c\in C\} \cup \{3a-1 : a\in N\setminus C\}$

$B = \{3c : c\in C\} \cup \{3b-2 : b\in N\setminus C\}$

$A\cap B = C$. C不可测.

但 $\rho(A) = \rho(B) = \frac{1}{3}$

EX1 E1.1

EX2 $L(\lambda\mu) = f(L(\lambda))$