HL Klemove

Vehicle model

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Introduction

- Vehicle model
  - 자동차를 처음 운전할 때 가장 먼저 해보는 것
    - Steering, brake, throttle을 작동시켰을 때 어떻게 차가 움직이는지 감을 익힌다
    - 머리속에 input 대비 output에 대한 모델을 만드는 것이 목적

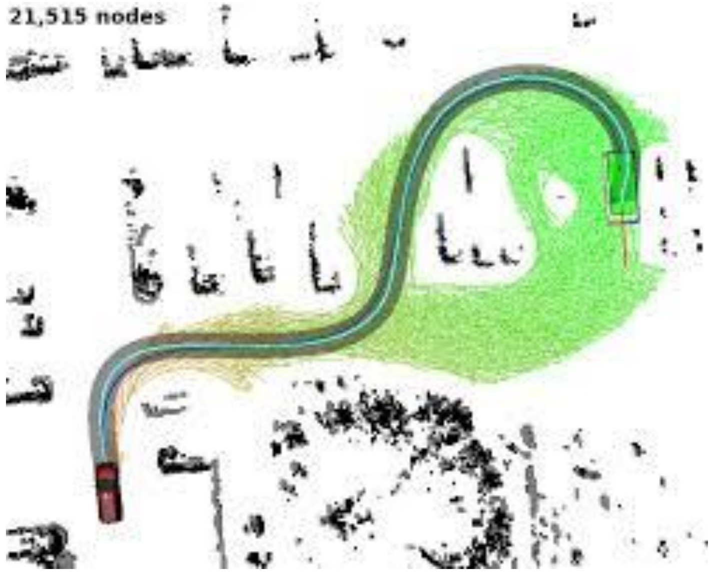
Introduction

- Motion planning and control for autonomous vehicle
  - 차량의 steering, brake, throttle 대비 차량이 움직이는 수학적 모델이 기초
    - From kinematic model to dynamic model
    - Kinematic model : force, torque의 영향을 무시하고 기구학적인 요소만 고려
    - Dynamic model : Kinematic model보다 높은 정확도 제공, 공기저항이나 road friction, slip 등이 더 고려됨

      https://youtu.be/WNIDcT0Zdj4?si=0stZ_GJ-VyQBzrS6

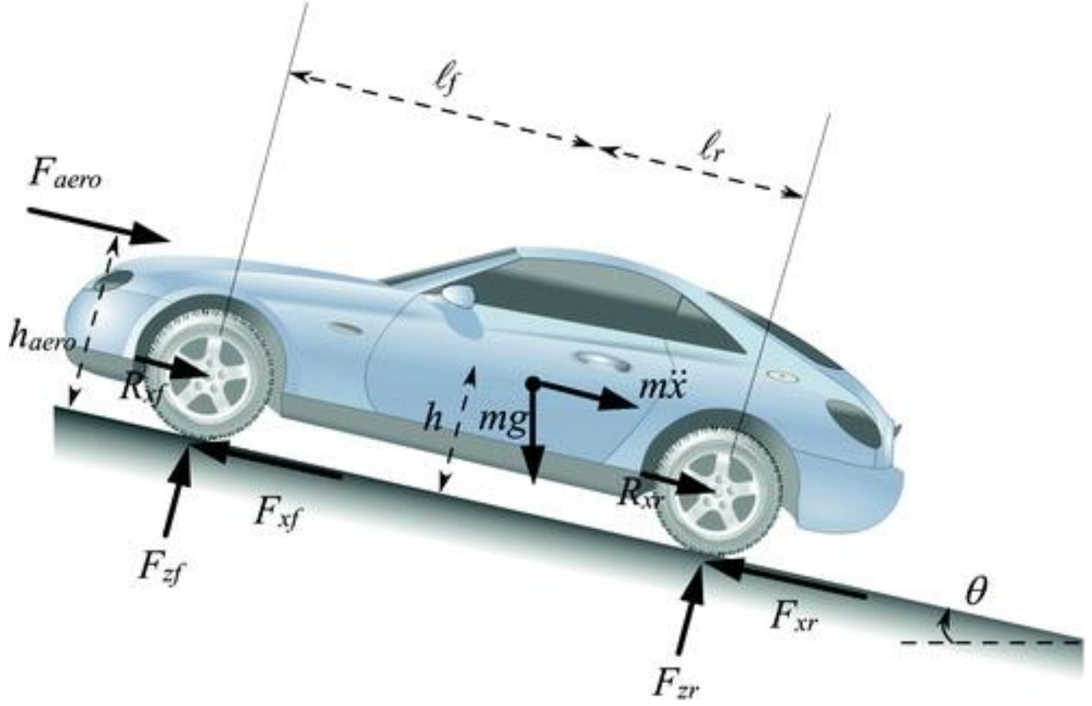
      https://youtu.be/3x3SqeSdrAE?si=2yyvn64GrfOXhBj0

      Paper: https://ddl.stanford.edu/sites/g/files/sbiybj9456/f/marty_avec2018_fullpaper.pdf



Longitudinal vehicle model

- Dynamic model
  - force balance on a vehicle

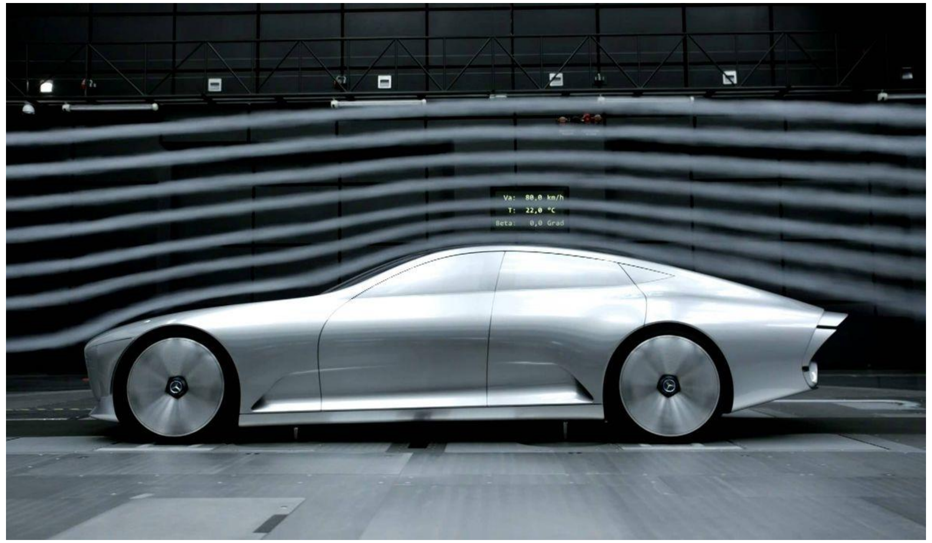


$$m\ddot{x} = \underbrace{F_{xf} + F_{xr}}_{\text{Tire forces}} - \underbrace{F_{aero}}_{\text{Aerodynamic forces}} - \underbrace{R_{xf} - R_{xr}}_{\text{Rolling resistance}} - \underbrace{mgsin\theta}_{\text{Gravity (road inclination)}}$$

Longitudinal vehicle model

- Aerodynamics
  - Exact model of air flow is too complicated
  - Semi-empirical drag force model is used

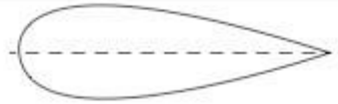
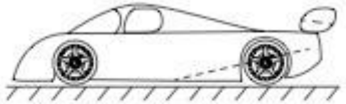
$$m\ddot{x} = F_{xf} + F_{xr} - \boxed{F_{aero}} - R_{xf} - R_{xr} - mgsin\theta$$



$$F_{aero} = \frac{1}{2}\rho \underline{V^2} C_D A$$

Dominant in high speed!

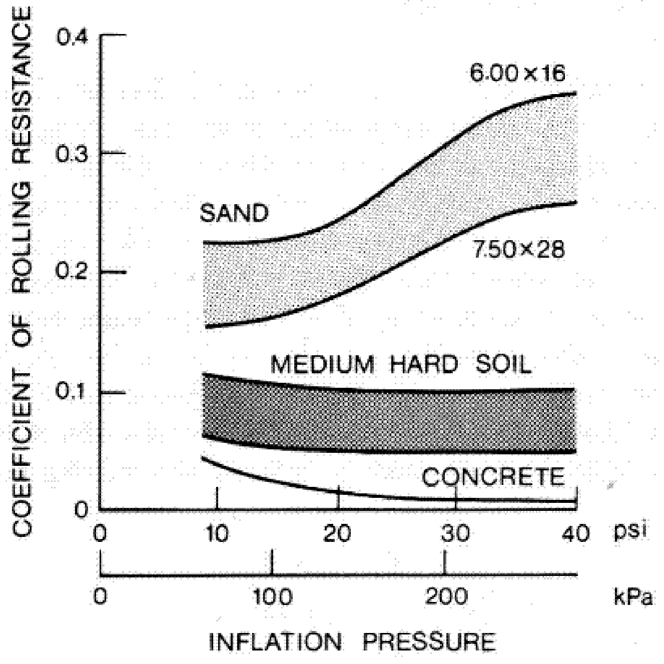
- ρ: air density
- C_D: drag coefficient
- V: relative velocity of the vehicle (with respect to the wind)
- A: Frontal aera of the vehicle

			C_L	C_D
1	Low drag body of revolution		0	0.04
2	Low drag vehicle near the ground		0.18	0.15
3	Generic automobile		0.28	0.35
4	Prototype race car		-3.00	0.75

Longitudinal vehicle model

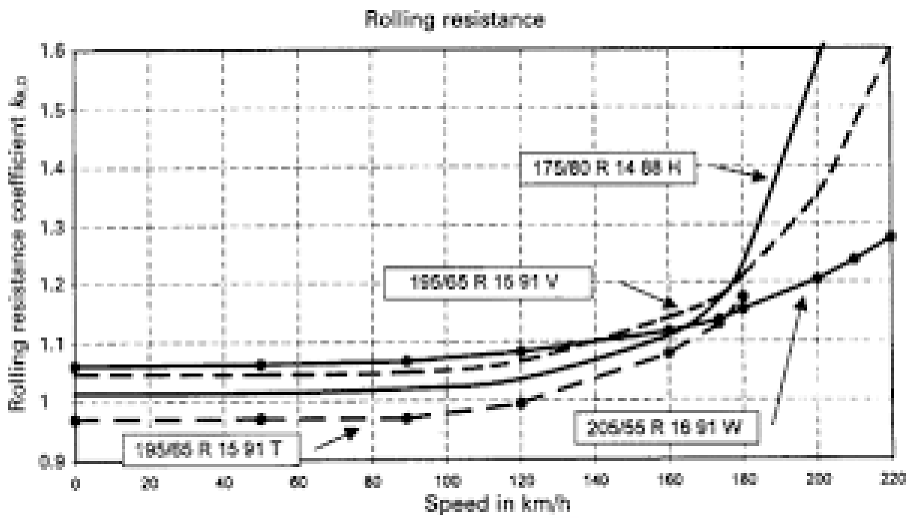
- Rolling resistance
  - The total rolling resistance is the sum of the resistances from all the wheels
  - Tire inflation pressure : depends on surface
  - Speed: the effect is small at low speed → assumed to be constant

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - \boxed{R_{xf} - R_{xr}} - mgsin\theta$$



$$R_x = R_{xf} + R_{xr} = f_r W$$

- f_r: rolling resistance coefficient
- W: wight of the vehicle

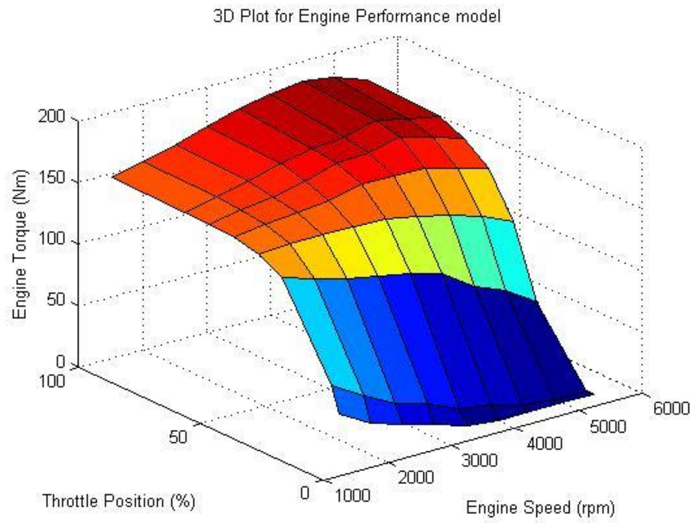


Longitudinal vehicle model

- Tire forces : traction & brake forces
  - Powertrain : throttle and brake system
  - https://youtu.be/aS615xkzmfs (참고)

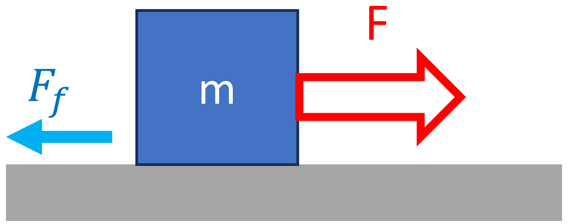
$$m\ddot{x} = \boxed{F_{xf} + F_{xr}} - F_{aero} - R_{xf} - R_{xr} - mgsin\theta$$

$$\tau = f(throttle, brake, \mathrm{rpm}, \cdots)$$

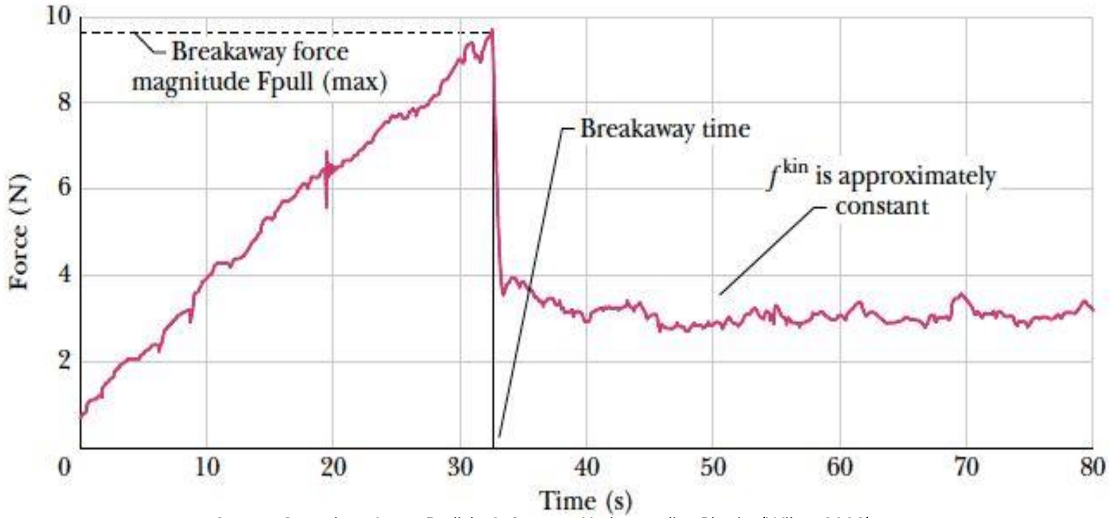


Longitudinal vehicle model

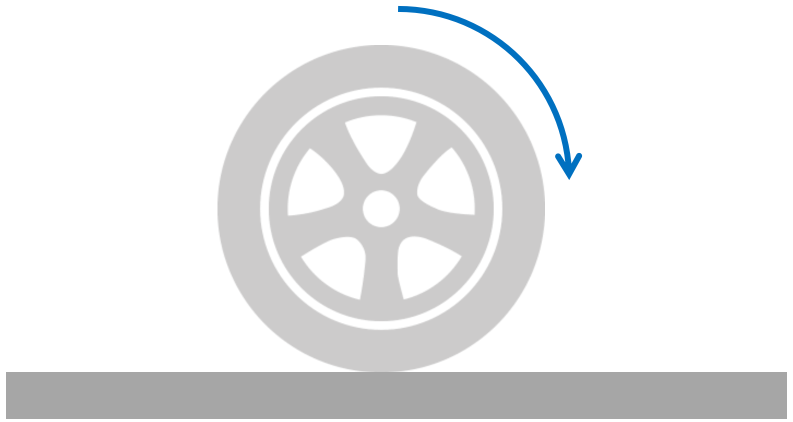
- Tire forces : traction & brake forces
  - Friction



$$F_{f,s} = \mu_s N$$
$$F_{f,m} = \mu_m N$$



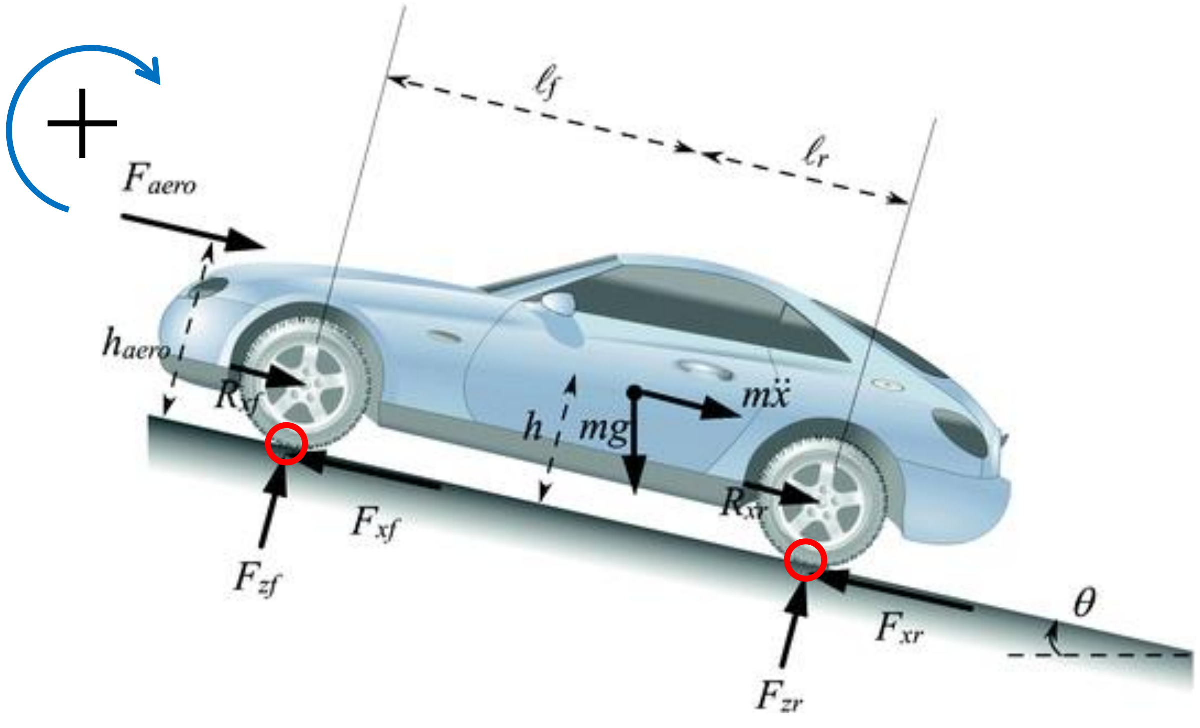
Source: Cummings, Laws, Redish, & Cooney, *Understanding Physics* (Wiley, 2003)



$$F_{f,s} = \mu_s N = \mu_s F_z$$

Longitudinal vehicle model

- Tire forces : traction & brake forces
  - Dynamic axle loads on a vehicle
    - Rotational system on each tire



Front wheel

$$h_{aero}F_{aero} + mgl_f\cos(\theta) + mgh\sin(\theta) + mh\ddot{x} - F_{zr}L = 0$$

Rear wheel

$$h_{aero}F_{aero} - mgl_r\cos(\theta) + mgh\sin(\theta) + mh\ddot{x} + F_{zf}L = 0$$

$$F_{zf} = \frac{mgl_r\cos(\theta) - mgh\sin(\theta) - h_{aero}F_{aero} - mh\ddot{x}}{L}$$

$$F_{zr} = \frac{mgl_f\cos(\theta) + mgh\sin(\theta) + h_{aero}F_{aero} + mh\ddot{x}}{L}$$

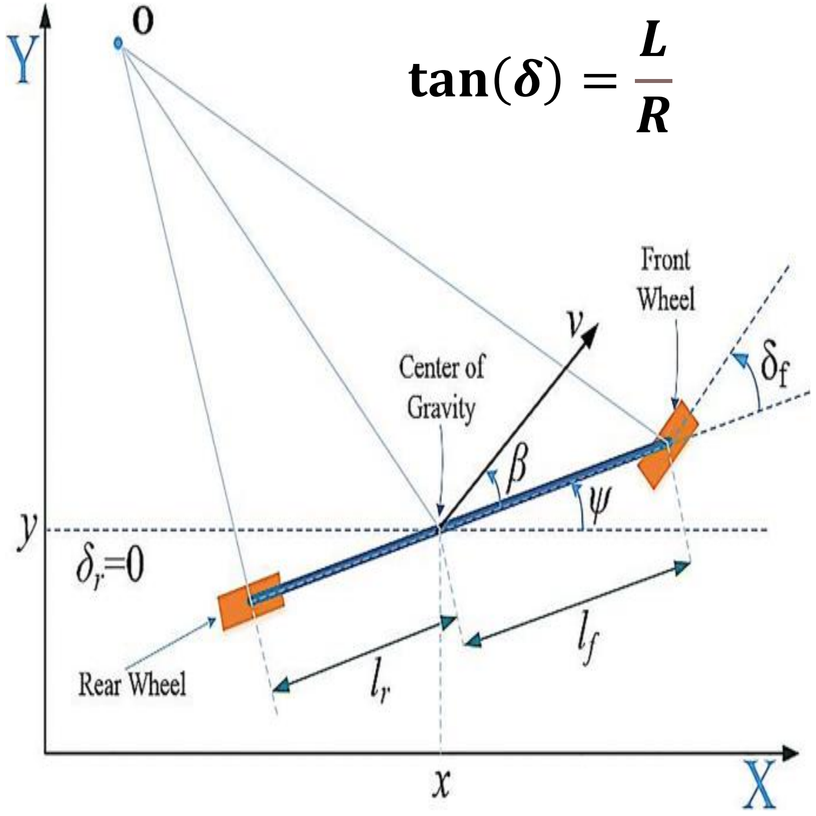
$$F_{zf} = \frac{mgl_r - h_{aero}F_{aero} - mh\ddot{x}}{L}$$

$$F_{zr} = \frac{mgl_f + h_{aero}F_{aero} + mh\ddot{x}}{L}$$

Lateral vehicle model

- Kinematic bicycle model
  - 구조적인 특성만을 이용한 모델
  - Force 를 사용하지 않음
  - 각 전륜 / 후륜을 하나의 바퀴로 근사(자전거)
  - No tire slip

$$\tan(\boldsymbol{\delta}) = \frac{\boldsymbol{L}}{\boldsymbol{R}}$$

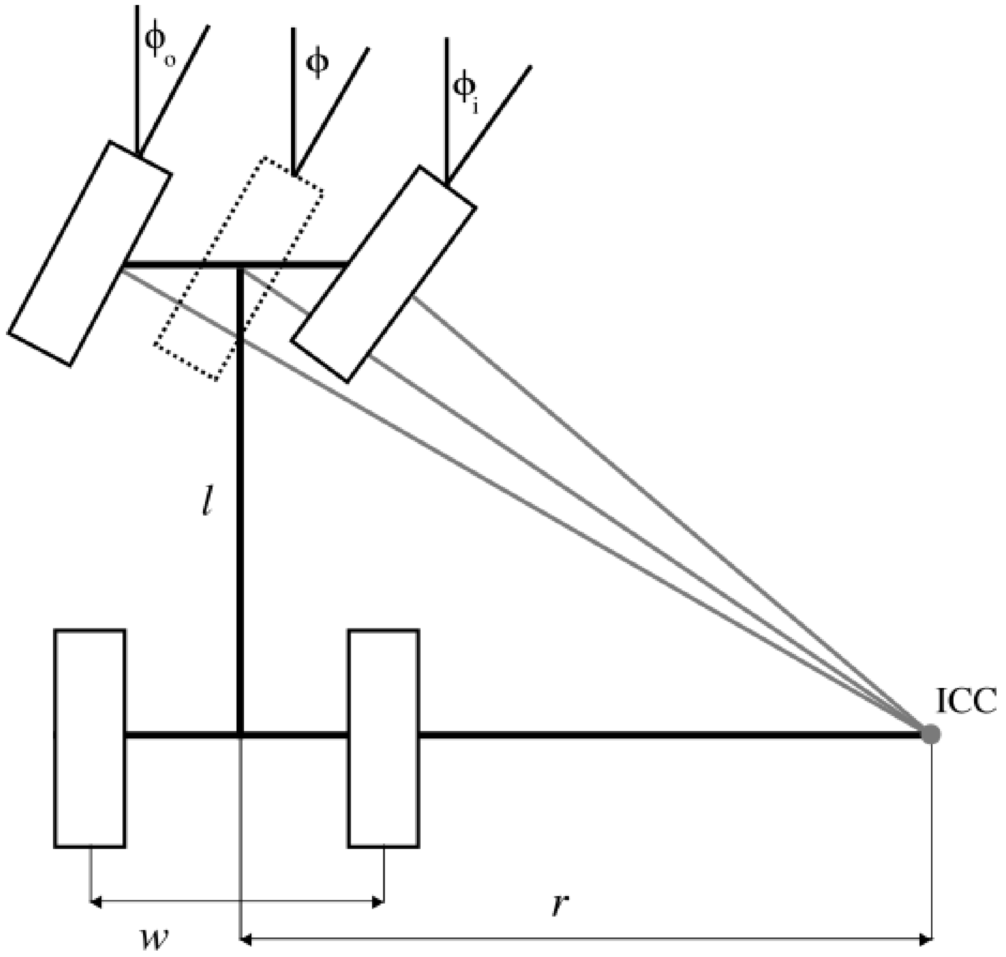


Rear wheel
$$\begin{cases} \dot{\psi} = \dfrac{V_x}{R} = V_x\left(\dfrac{\tan(\delta)}{L}\right) = \dfrac{V_x\tan(\delta)}{L} \\ \dot{X} = V_x\cos(\psi) \\ \dot{Y} = V_x\sin(\psi) \end{cases}$$

Center of mass
$$\begin{cases} \dot{\psi} = \dfrac{V_x\cos(\beta)}{R_c \approx R} = V_x\cos(\beta)\left(\dfrac{\tan(\delta)}{L}\right) = \dfrac{V_x cos(\beta)\tan(\delta)}{L} \\ \dot{X} = V_x\cos(\psi+\beta) \\ \dot{Y} = V_x\sin(\psi+\beta) \end{cases}$$

Lateral vehicle model

- Ackermann steering geometry

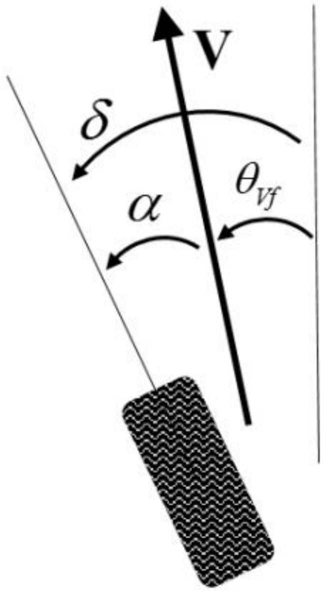


$$\tan(\delta_l) = \frac{l}{R + \frac{w}{2}}$$

$$\tan(\delta_r) = \frac{l}{R - \frac{w}{2}}$$

Lateral vehicle model

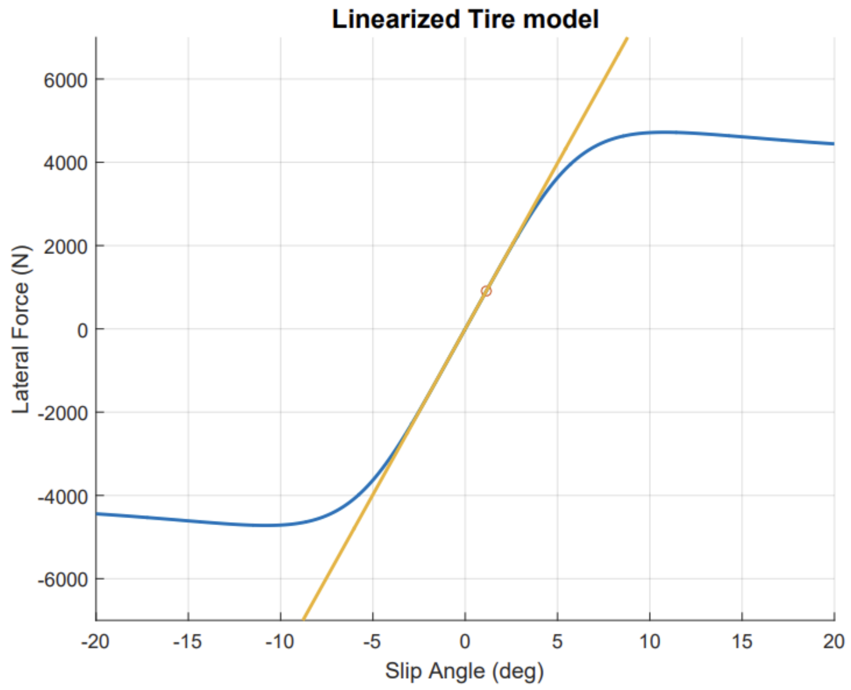
- Dynamic bicycle model
  - Tire forces : 횡방향(y) 힘의 근원지!
    - Tire slip : 타이어 방향과 실제 움직이는 방향(Heading angle)과의 차이
      (Heading angle → Tire direction)



$$\alpha = \delta - \theta_v$$

$$F_y = C_\alpha \alpha = C_\alpha(\delta - \theta_v)$$

C_α: Cornering stiffness



Lateral vehicle model

- Dynamic bicycle model
  - Bicycle model formulation
    - With small steering angle(δ)

$$ma_y = F_{yf} + F_{yr}$$

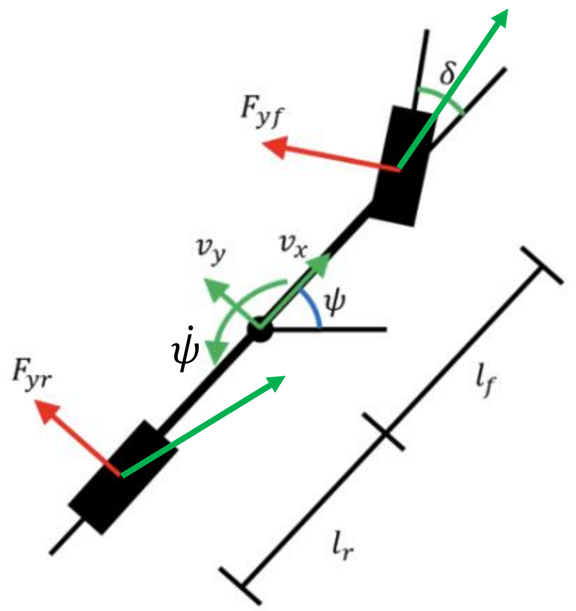
$$I\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

$$F_{yf} = 2C_{\alpha f}\alpha_f \cancel{\cos(\delta)}$$

$$F_{yr} = 2C_{\alpha r}\alpha_r$$

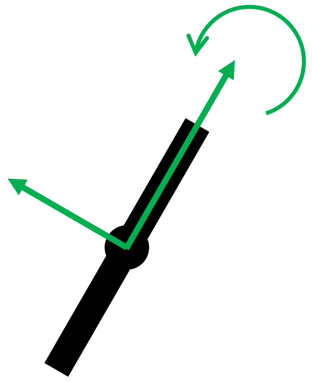
$$\delta - \theta_f = \alpha_f$$

$$-\theta_r = \alpha_r$$



$$\tan(\theta_f) = \frac{v_y + \dot{\psi} l_f}{v_x} \approx \theta_f$$

$$\tan(\theta_r) = \frac{v_y - \dot{\psi} l_r}{v_x} \approx \theta_r$$



Lateral vehicle model

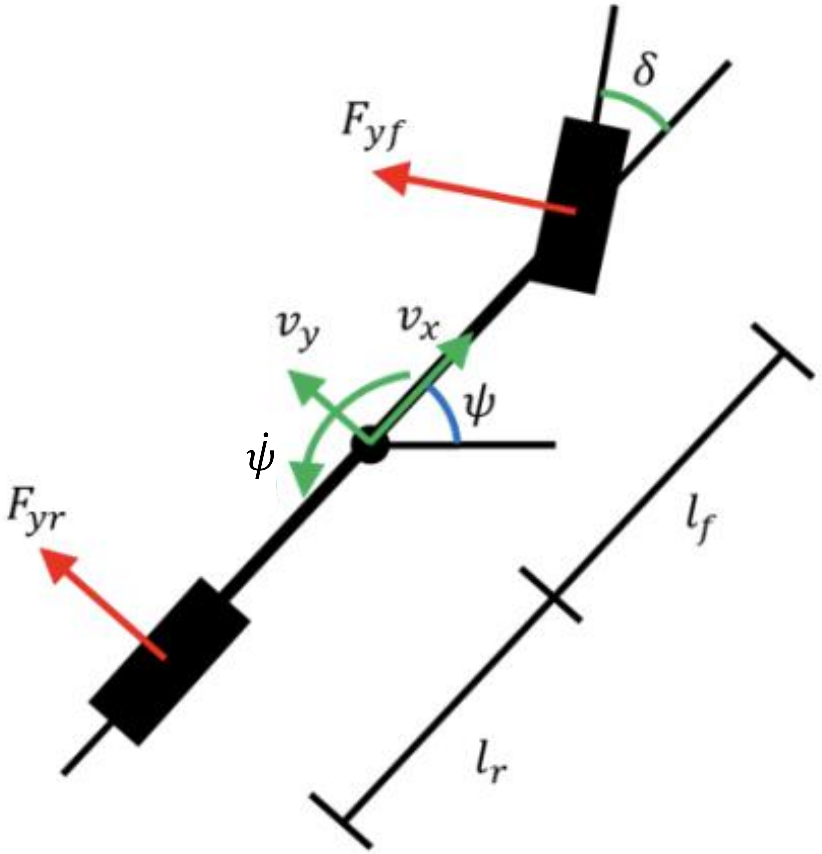
- Dynamic bicycle model
  - Acceleration of moving vehicle

$$a_y = \ddot{y} + \dot{\psi}^2 R$$

  - Dynamic equations

$$m\left(\ddot{y} + \dot{\psi}^2 R\right) = 2C_{\alpha f}\alpha_f + 2C_{\alpha r}\alpha_r$$

$$I\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$



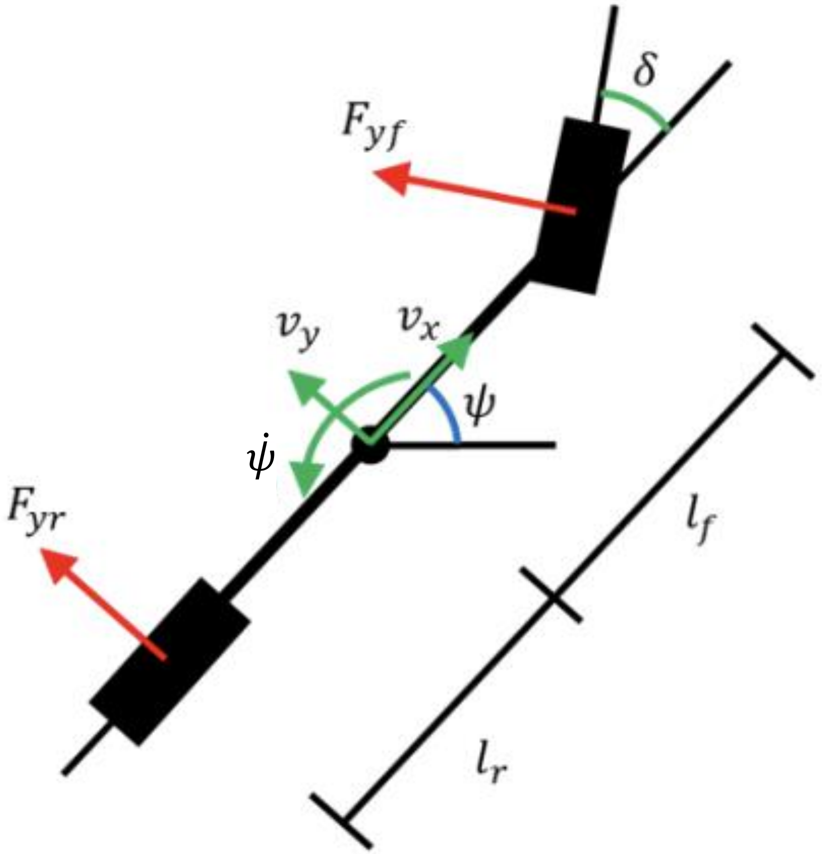
Lateral vehicle model

- Dynamic bicycle model

$$\ddot{y} = \left(-\frac{2C_{\alpha f} + 2C_{\alpha r}}{mv_x}\right)\dot{y} + \left(-v_x - \frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mv_x}\right)\dot{\psi} + \frac{2C_{\alpha f}}{m}\delta$$

$$\ddot{\psi} = \left(-\frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{Iv_x}\right)\dot{y} - \left(\frac{2C_{\alpha f}{l_f}^2 + 2C_{\alpha r}{l_r}^2}{Iv_x}\right)\dot{\psi} + \frac{2C_{\alpha f}l_f}{I}$$

$$\frac{d}{dt}\begin{pmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f} + 2C_{\alpha r}}{mv_x} & 0 & -v_x - \frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{Iv_x} & 0 & -\frac{2C_{\alpha f}{l_f}^2 + 2C_{\alpha r}{l_r}^2}{Iv_x} \end{pmatrix}\begin{pmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2C_{\alpha f}l_f}{I} \end{pmatrix}\delta$$



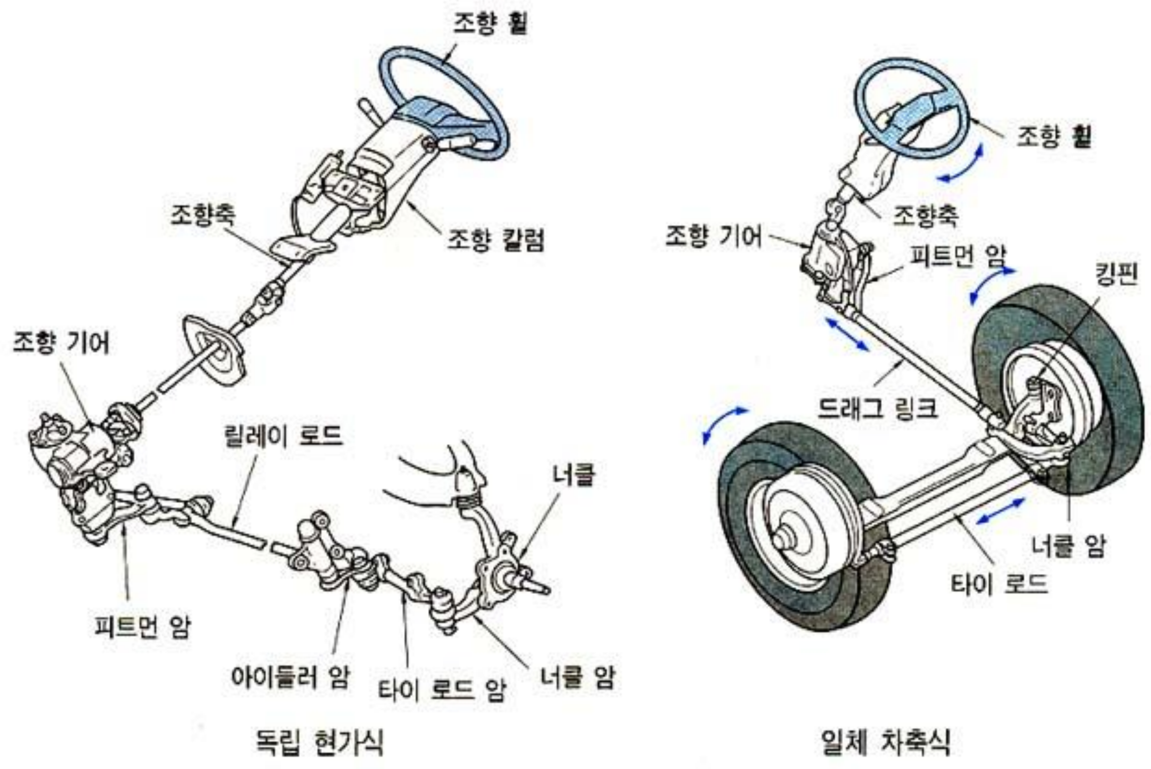
Lateral vehicle model

- Steering wheel angle, road wheel angle

δ_{SAS} : Steering wheel angle(from steering wheel sensor)

δ_R : Road wheel angle

$$\delta_{SAS} \Rightarrow \boxed{?} \Rightarrow \delta_R$$



http://jwkang7.wo.to/pds03/317.htm

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Thank You