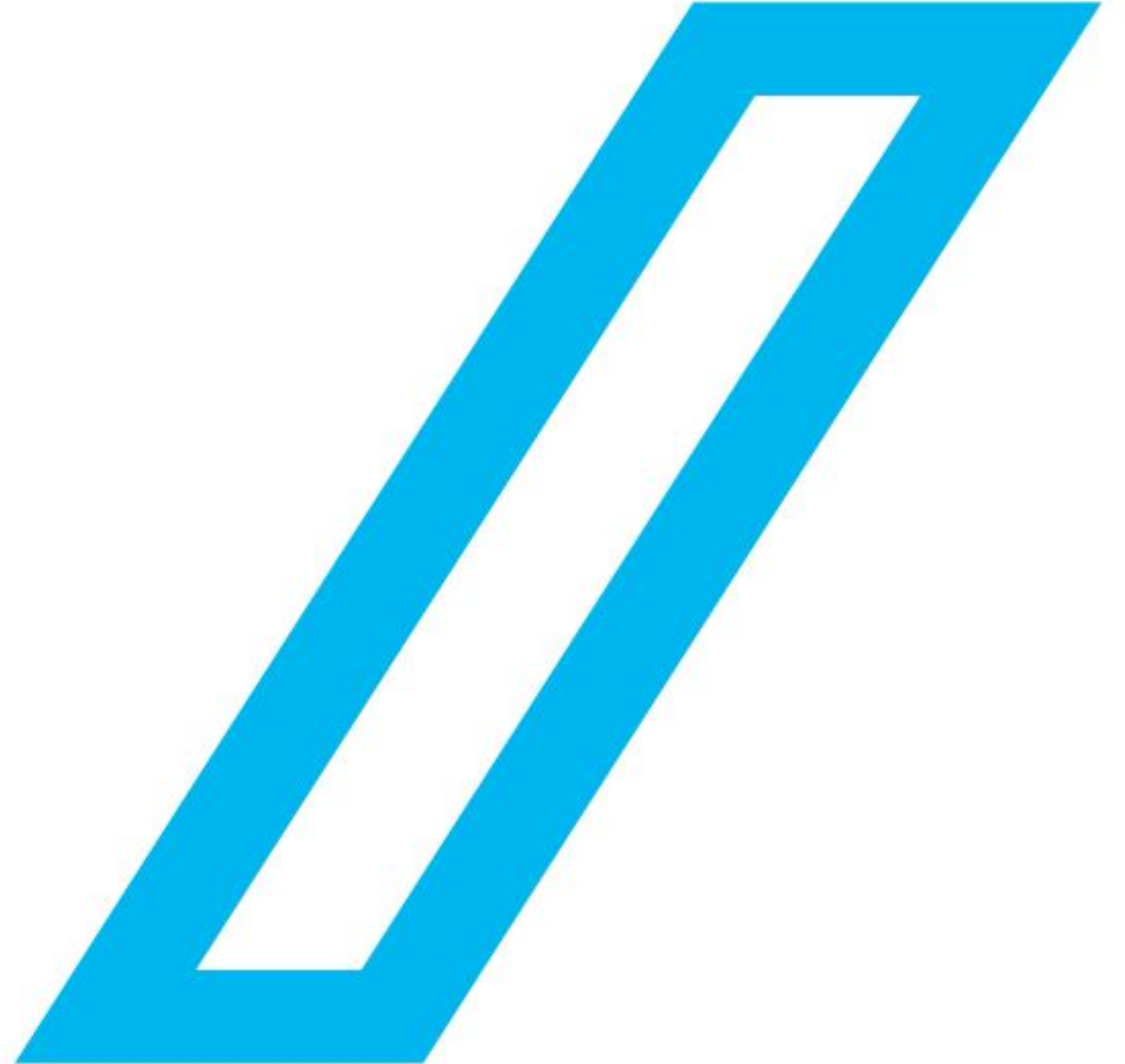


# Prediction

Lecturer : Seungmok Song



# Contents

1. Introduction
2. Physics based prediction
3. Maneuver based prediction



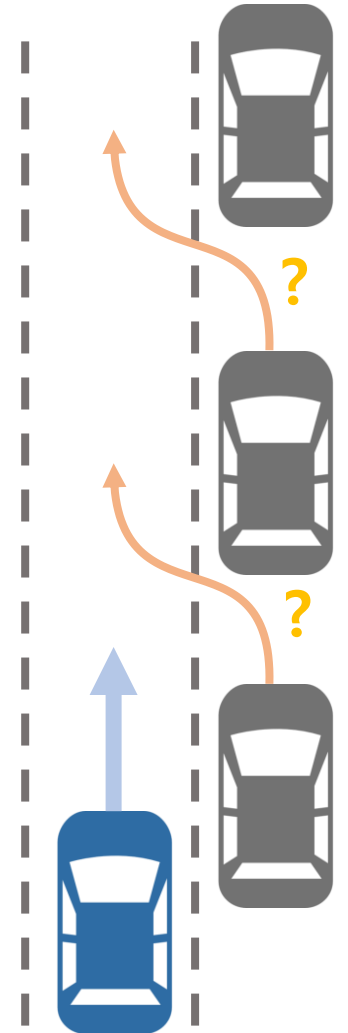
# Introduction

- Prediction
  - 주변 Object의 Future trajectory나 Intention 을 예측



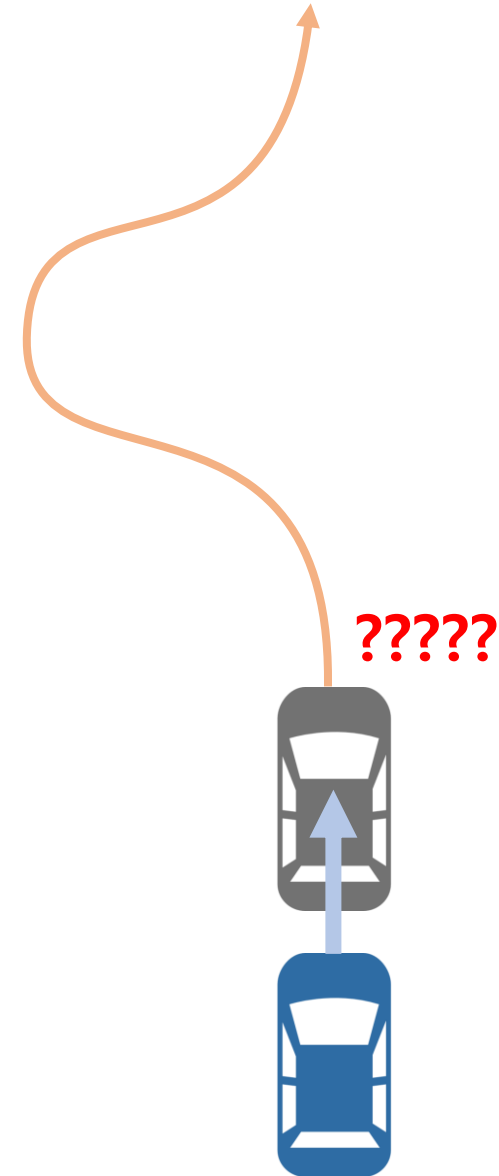
# Introduction

- Prediction
  - 초보 운전 vs 숙련된 운전자



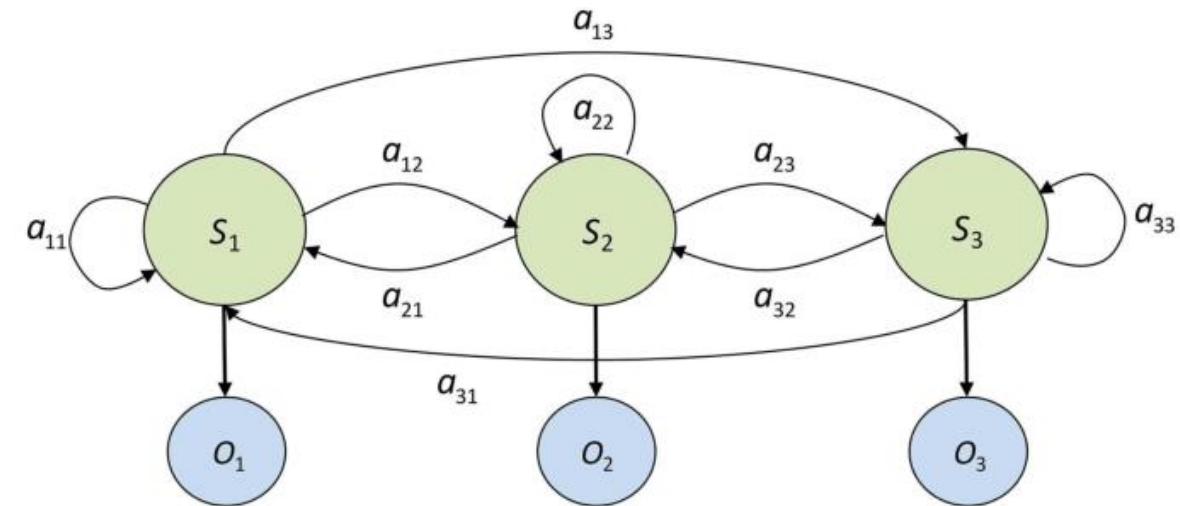
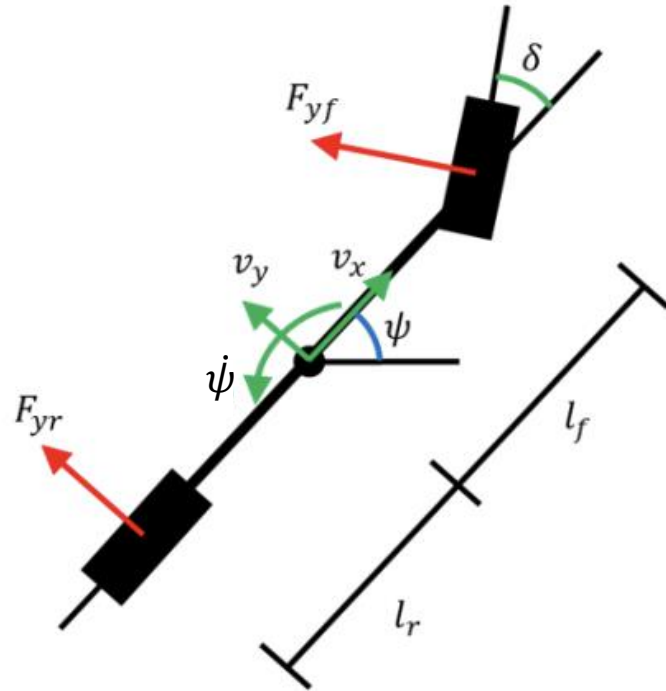
# Introduction

- Prediction
  - 초보 운전 vs 숙련된 운전자



# Introduction

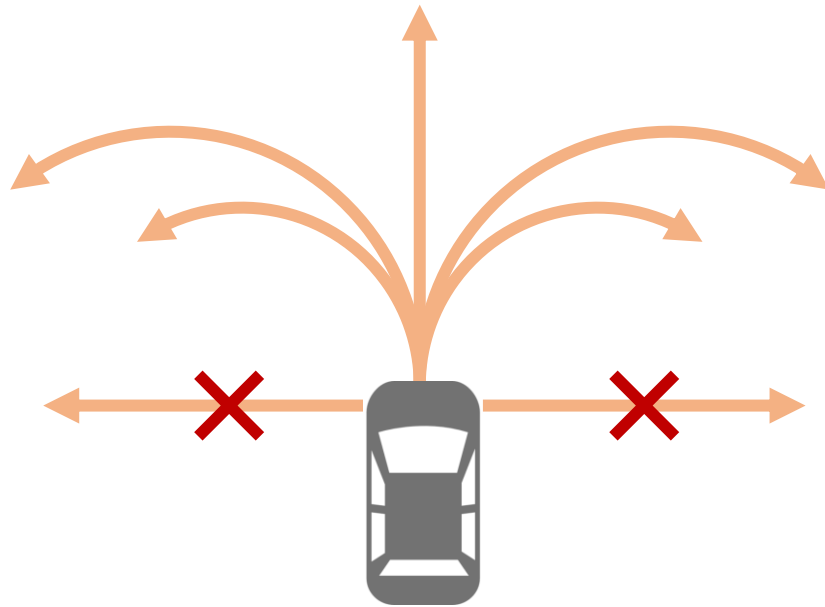
- Model based prediction
  - Physics based prediction
  - Maneuver based prediction



# Physics based prediction

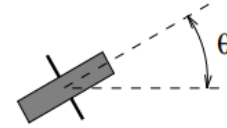
- Introduction

- 자동차는 정해진 물리 법칙 상에서 움직임
- 짧은 시간 예측에 유리
- 주변 환경을 고려하지 않음
- CV / CA / CTRV / CTRA model



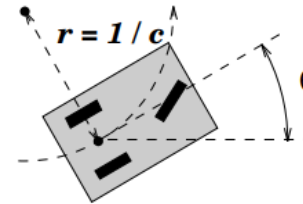
## Nonholonomic Systems

Unicycle



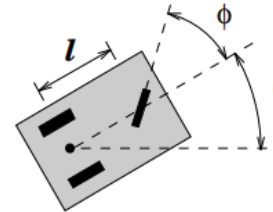
$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ v_\theta \end{pmatrix}$$

Car with fast steering



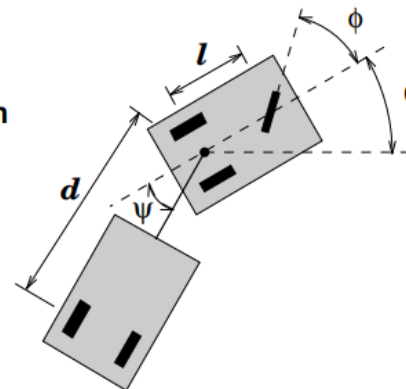
$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ v c \end{pmatrix}$$

Car with slow steering



$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ \tan(\phi)/l & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ v_\phi \end{pmatrix}$$

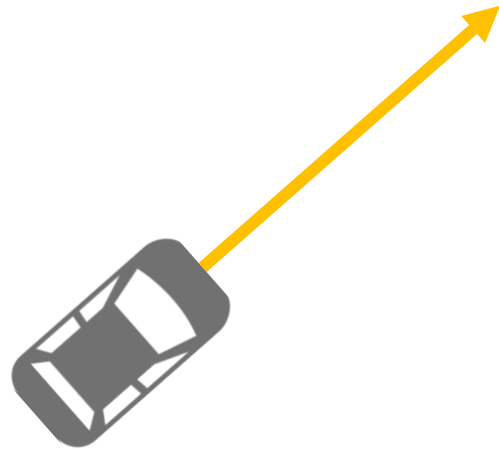
Car with trailer



$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ \theta \\ \psi \\ \phi \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ \tan(\phi)/l & 0 \\ \sin(\psi)/d & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ v_\phi \end{pmatrix}$$

# Physics based prediction

- Constant velocity model (CV model)
  - 현재의 주행 속도를 유지한다는 가정
  - 가장 간단한 모델



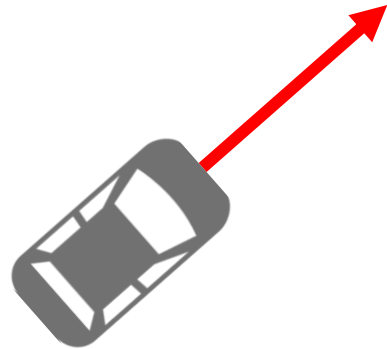
$$\text{State : } X = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} \quad \left( \begin{array}{l} x_{k+1} = x_k + v_{x,k} \Delta t \\ y_{k+1} = y_k + v_{y,k} \Delta t \\ v_{x,k+1} = v_{x,k} \\ v_{y,k+1} = v_{y,k} \end{array} \right)$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ v_{x,k+1} \\ v_{y,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ v_{x,k} \\ v_{y,k} \end{bmatrix}$$



# Physics based prediction

- Constant acceleration model (CA model)
  - 현재의 가속도를 유지한다는 가정
  - 주의해야 할 점..?



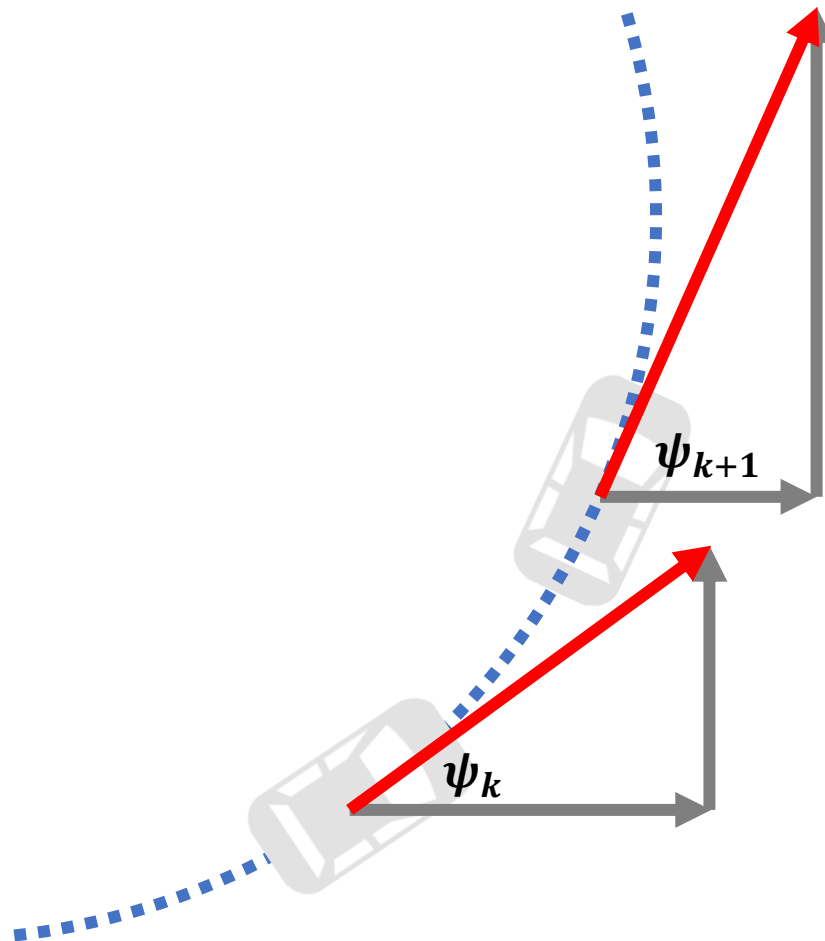
$$\text{State : } X = \begin{bmatrix} x \\ v \\ a \end{bmatrix} \quad \left( \begin{array}{l} x_{k+1} = x_k + v_k \Delta t + \frac{1}{2} a_k \Delta t^2 \\ v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \end{array} \right)$$

$$\begin{bmatrix} x_{k+1} \\ v_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v_k \\ a_k \end{bmatrix}$$

# Physics based prediction

- Constant turn rate & velocity model (CTRV model)

- 현재의 yaw rate(turn rate) 와 속도를 유지한다는 가정
- If ( yaw rate  $\ll 1$  ) ?



State :  $X = \begin{bmatrix} x \\ y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$

$$\begin{cases} x_{k+1} = x_k + \int_0^{\Delta t} v_{k,\tau} \cos(\psi_{k,\tau}) d\tau \\ y_{k+1} = y_k + \int_0^{\Delta t} v_{k,\tau} \sin(\psi_{k,\tau}) d\tau \\ v_{k,\tau} = v_k(\text{const}) \\ \psi_{k,\tau} = \psi_k + \dot{\psi}_k \tau \end{cases} \Rightarrow \begin{cases} x_{k+1} = x_k + \int_0^{\Delta t} v_k \cos(\psi_k + \dot{\psi}_k \tau) d\tau \\ y_{k+1} = y_k + \int_0^{\Delta t} v_k \sin(\psi_k + \dot{\psi}_k \tau) d\tau \end{cases}$$

$$\begin{aligned} x_{k+1} &= x_k + \frac{v_k}{\dot{\psi}_k} \left( \sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k) \right) \\ y_{k+1} &= y_k + \frac{v_k}{\dot{\psi}_k} \left( -\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k) \right) \\ v_{k+1} &= v_k \\ \psi_{k+1} &= \psi_k + \dot{\psi}_k \Delta t \\ \dot{\psi}_{k+1} &= \dot{\psi}_k \end{aligned}$$

# Physics based prediction

- Constant turn rate & acceleration model (CTRA model)

- 현재의 yaw rate(turn rate) 와 가속도를 유지한다는 가정

$$X = \begin{bmatrix} x \\ y \\ v \\ a \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\begin{cases} v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \\ \psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t \\ \dot{\psi}_{k+1} = \dot{\psi}_k \\ x_{k+1} = x_k + \int_0^{\Delta t} v_{k,\tau} \cos(\psi_{k,\tau}) d\tau \\ y_{k+1} = y_k + \int_0^{\Delta t} v_{k,\tau} \sin(\psi_{k,\tau}) d\tau \end{cases}$$

$$\begin{cases} x_{k+1} = x_k + \frac{v_k}{\dot{\psi}_k} \left( \sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k) \right) + \frac{a_k}{\dot{\psi}_k^2} \left( \cos(\psi_k + \dot{\psi}_k \Delta t) + \dot{\psi}_k \Delta t \sin(\psi_k + \dot{\psi}_k \Delta t) - \cos(\psi_k) \right) \\ y_{k+1} = y_k - \frac{v_k}{\dot{\psi}_k} \left( \cos(\psi_k + \dot{\psi}_k \Delta t) - \cos(\psi_k) \right) + \frac{a_k}{\dot{\psi}_k^2} \left( \sin(\psi_k + \dot{\psi}_k \Delta t) - \dot{\psi}_k \Delta t \cos(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k) \right) \\ v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \\ \psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t \\ \dot{\psi}_{k+1} = \dot{\psi}_k \end{cases}$$

# Physics based prediction

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- Process noise
  - 그럼 모델대로 움직이는가?

# Physics based prediction

- Basic Kalman filter

- ① Prediction

$$\tilde{x}_k = A\hat{x}_{k-1} + Bu_k$$

$$\tilde{y}_k = C\tilde{x}_k$$

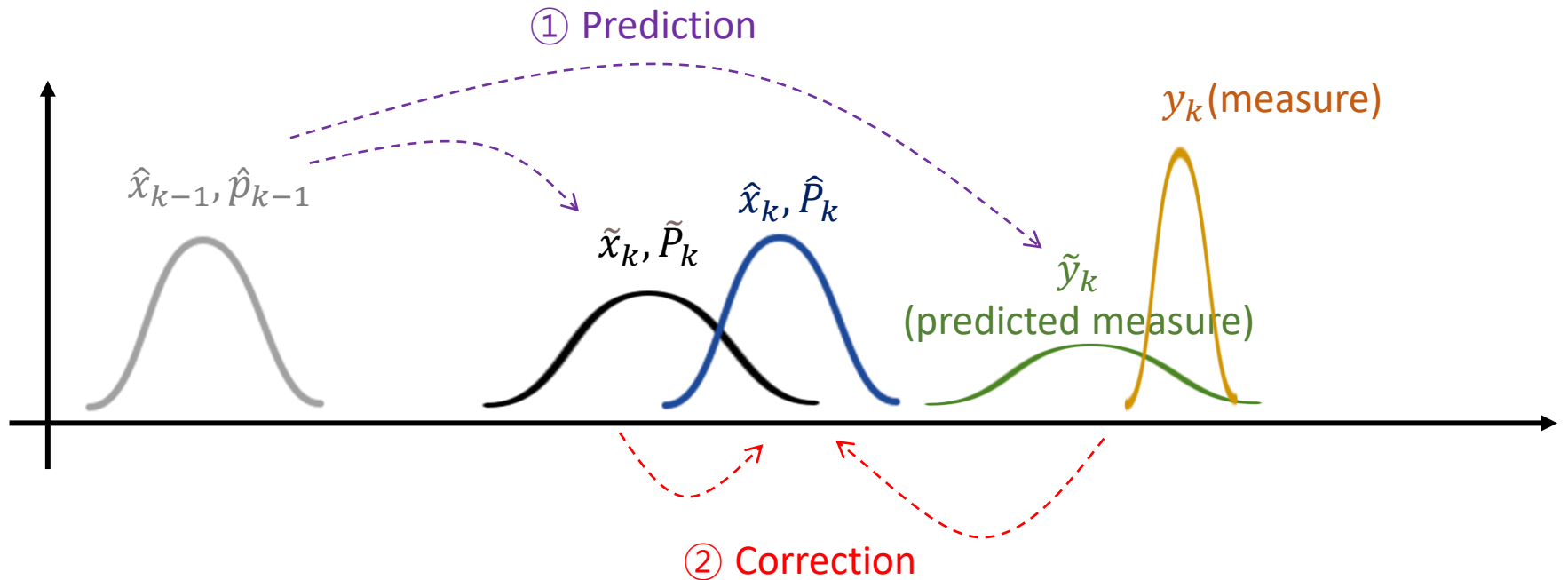
$$\tilde{P}_k = A P_{k-1} A^T + \underline{Q_k}$$

- ② Correction

$$\hat{x}_k = \tilde{x}_k + \underline{K_k}(y_k - \tilde{y}_k)$$

$$K_k = \frac{\tilde{P}_k C^T}{C \tilde{P}_k C^T + \underline{R_k}}$$

$$P_k = (I - K_k C) \tilde{P}_k$$



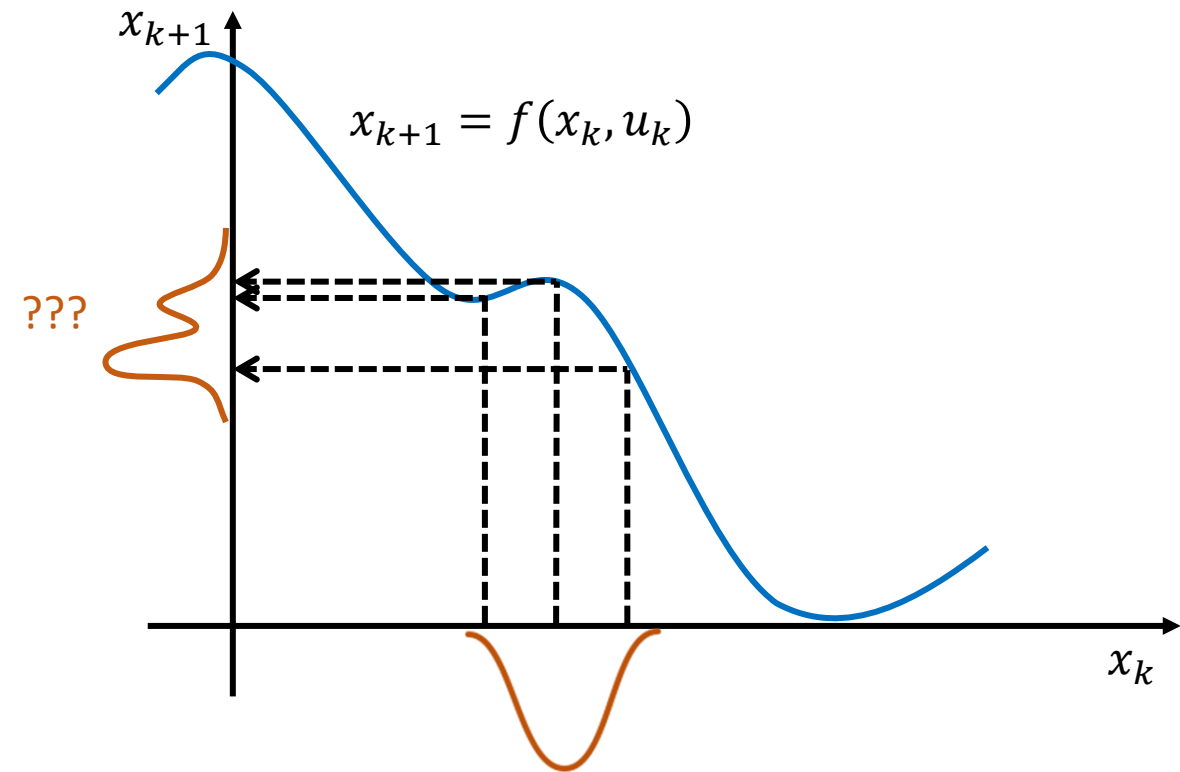
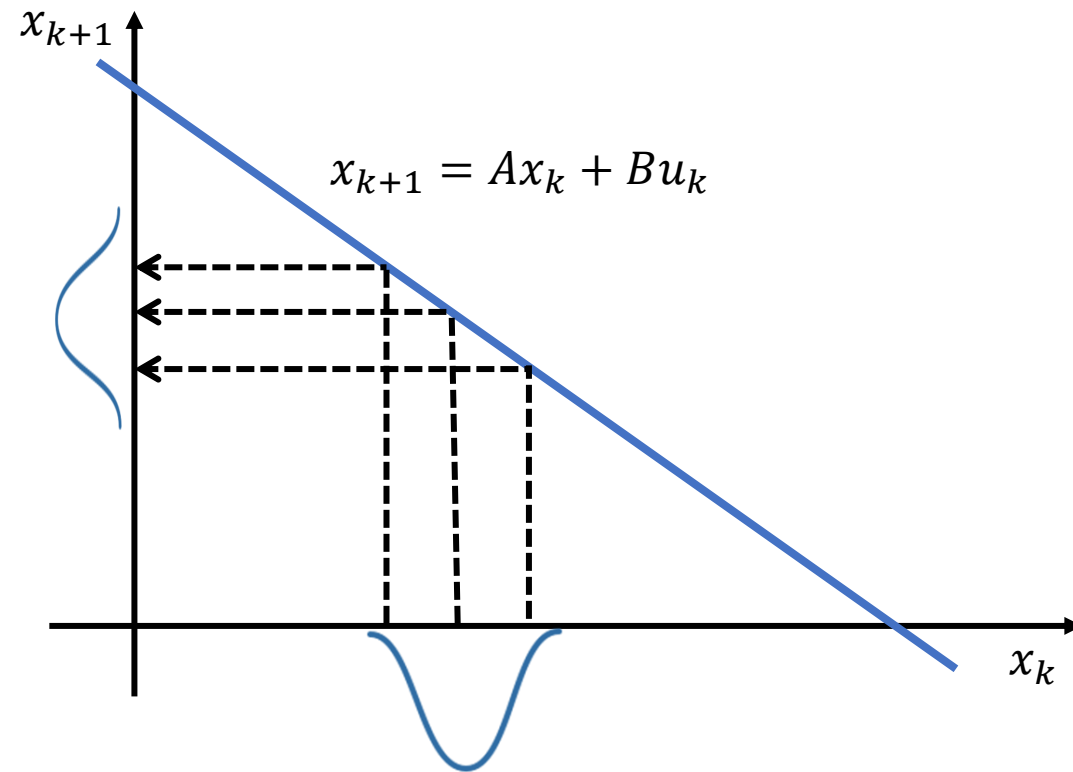
$K_k$  (Kalman gain) : 예측된 값에 측정값과의 차이를 얼마나 반영할 것인지에 대한 값

$Q_k$  (Model noise) : Kalman gain 을 크게 하는 요소 (Tuning parameter)

$R_k$  (Sensor noise) : Kalman gain 을 작게 하는 요소 (Tuning parameter)

# Physics based prediction

- Nonlinear Kalman filter
  - Linear vs Nonlinear model



# Physics based prediction

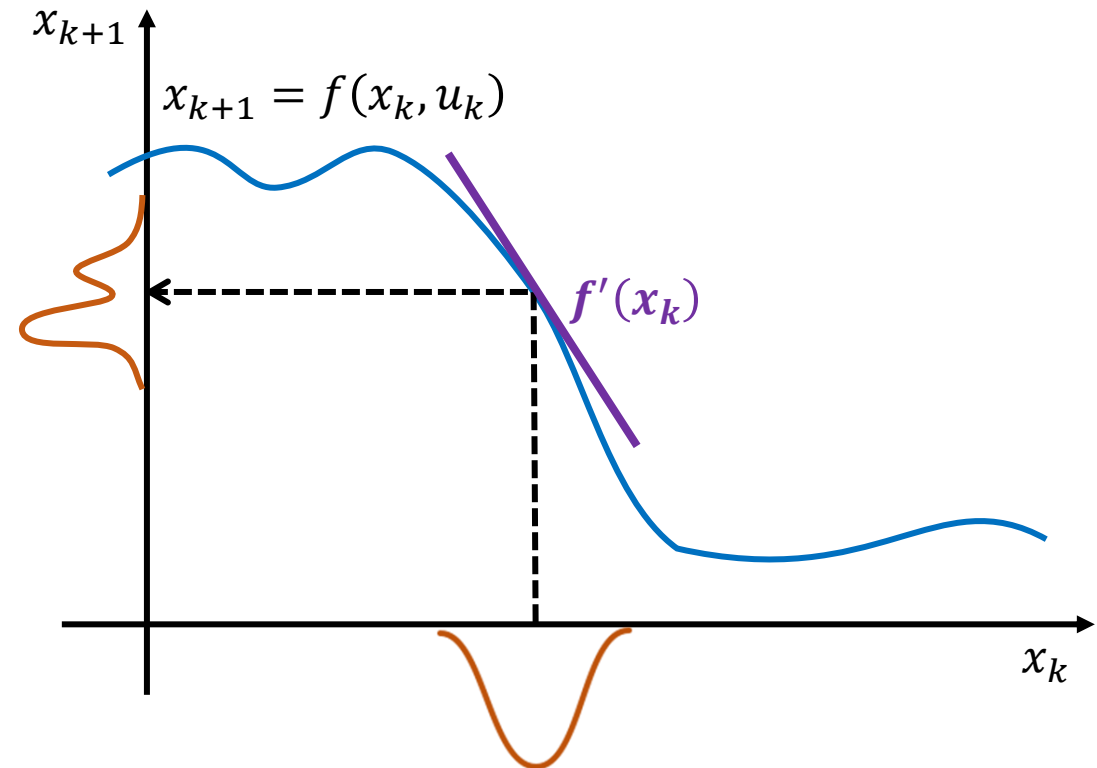
- Nonlinear Kalman filter

- Idea 1 : Taylor series

- 특정 함수는 한 점에서의 도함수 값들과 다항식의 무한 합으로 나타낼 수 있음

$$f(x) = \underline{f(a) + f'(a)(x - a)} + \frac{1}{2}f''(a)(x - a)^2 + \dots$$

$$\begin{aligned} f(x, y, z) = & f(x_i, y_i, z_i) + (x - x_i) \left. \frac{\partial f}{\partial x} \right|_{x_i, y_i, z_i} \\ & + (y - y_i) \left. \frac{\partial f}{\partial y} \right|_{x_i, y_i, z_i} \\ & + (z - z_i) \left. \frac{\partial f}{\partial z} \right|_{x_i, y_i, z_i} \end{aligned}$$



# Physics based prediction

- Nonlinear Kalman filter
  - Example : CTRV Model

$$\begin{aligned}
 x_{k+1} &= x_k + \frac{v_k}{\dot{\psi}_k} \left( \sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k) \right) = \mathbf{f}_1(\mathbf{x}_k, \mathbf{y}_k, \dots) \\
 y_{k+1} &= y_k + \frac{v_k}{\dot{\psi}_k} \left( -\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k) \right) = \mathbf{f}_2(\mathbf{x}_k, \mathbf{y}_k, \dots) \\
 v_{k+1} &= v_k = \mathbf{f}_3(\mathbf{x}_k, \mathbf{y}_k, \dots) \\
 \psi_{k+1} &= \psi_k + \dot{\psi}_k \Delta t = \mathbf{f}_4(\mathbf{x}_k, \mathbf{y}_k, \dots) \\
 \dot{\psi}_{k+1} &= \dot{\psi}_k = \mathbf{f}_5(\mathbf{x}_k, \mathbf{y}_k, \dots)
 \end{aligned}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ v_{k+1} \\ \psi_{k+1} \\ \dot{\psi}_{k+1} \end{bmatrix} = \begin{bmatrix} \frac{\delta f_1}{\delta x} & \frac{\delta f_1}{\delta y} & \frac{\delta f_1}{\delta v} & \frac{\delta f_1}{\delta \psi} & \frac{\delta f_1}{\delta \dot{\psi}} \\ \frac{\delta f_2}{\delta x} & \frac{\delta f_2}{\delta y} & \frac{\delta f_2}{\delta v} & \frac{\delta f_2}{\delta \psi} & \frac{\delta f_2}{\delta \dot{\psi}} \\ \frac{\delta f_3}{\delta x} & \frac{\delta f_3}{\delta y} & \frac{\delta f_3}{\delta v} & \frac{\delta f_3}{\delta \psi} & \frac{\delta f_3}{\delta \dot{\psi}} \\ \frac{\delta f_4}{\delta x} & \frac{\delta f_4}{\delta y} & \frac{\delta f_4}{\delta v} & \frac{\delta f_4}{\delta \psi} & \frac{\delta f_4}{\delta \dot{\psi}} \\ \frac{\delta f_5}{\delta x} & \frac{\delta f_5}{\delta y} & \frac{\delta f_5}{\delta v} & \frac{\delta f_5}{\delta \psi} & \frac{\delta f_5}{\delta \dot{\psi}} \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ v_k \\ \psi_k \\ \dot{\psi}_k \end{bmatrix} + \xi$$

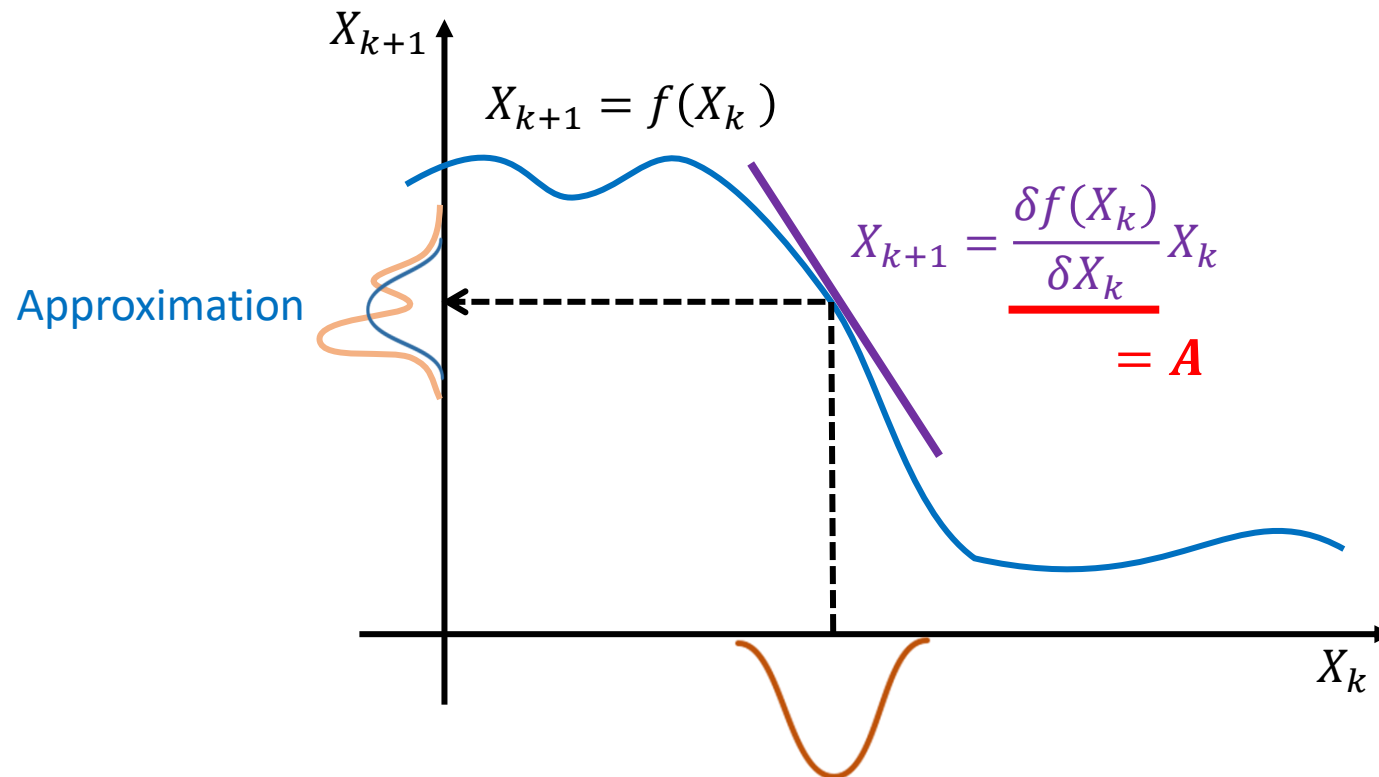
Jacobian matrix

$$= \frac{\delta f(X_k)}{\delta X_k}$$



# Physics based prediction

- Nonlinear Kalman filter
  - Extended Kalman filter



## ① Prediction

$$\tilde{x}_k = A\hat{x}_{k-1} + Bu_k$$

$$\tilde{y}_k = C\tilde{x}_k$$

$$\tilde{P}_k = AP_{k-1}A^T + Q_k$$

## ② Correction

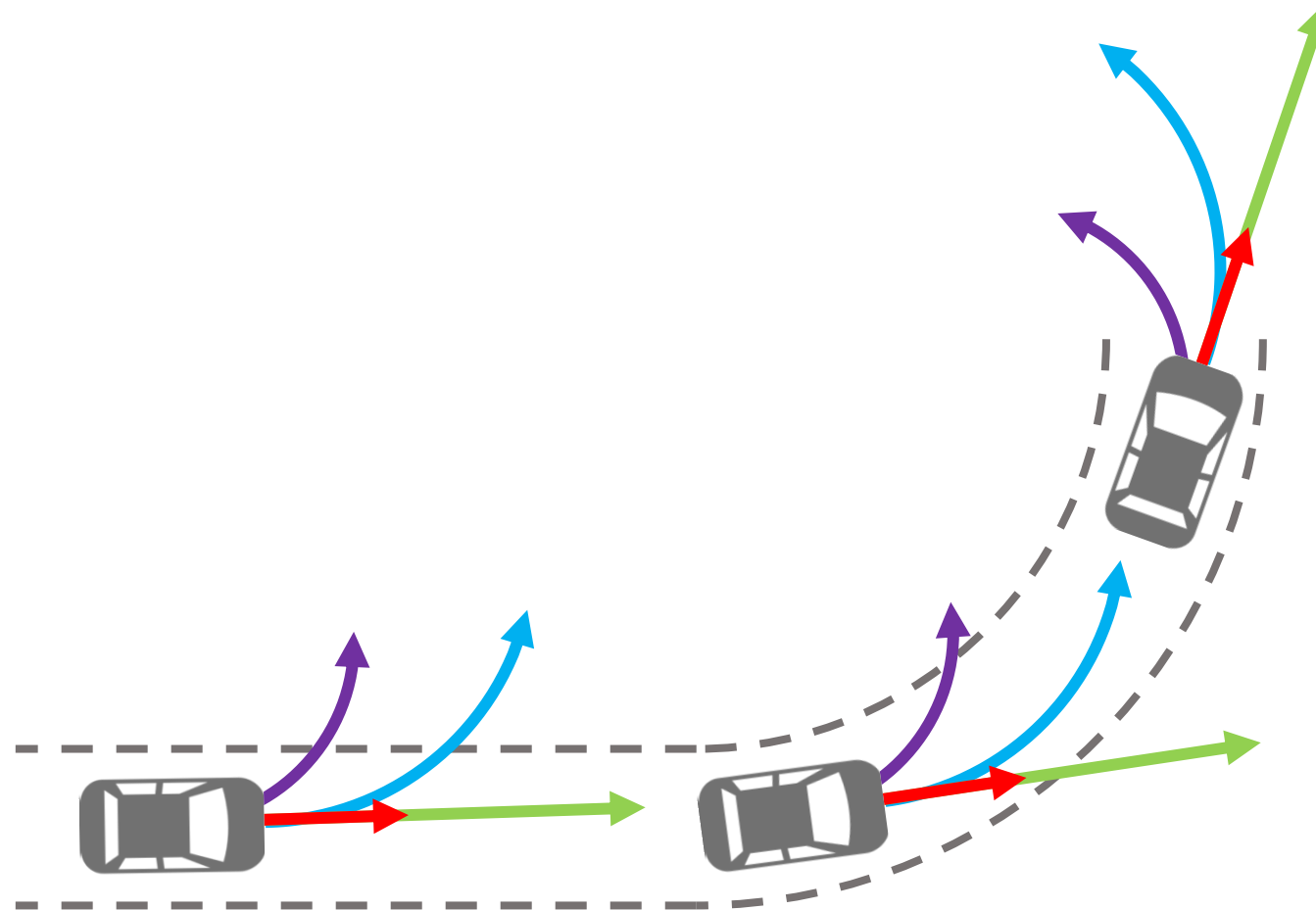
$$\hat{x}_k = \tilde{x}_k + K_k(y_k - \tilde{y}_k)$$

$$K_k = \frac{\tilde{P}_k C^T}{C\tilde{P}_k C^T + R_k}$$

$$P_k = (I - K_k C)\tilde{P}_k$$

# Physics based prediction

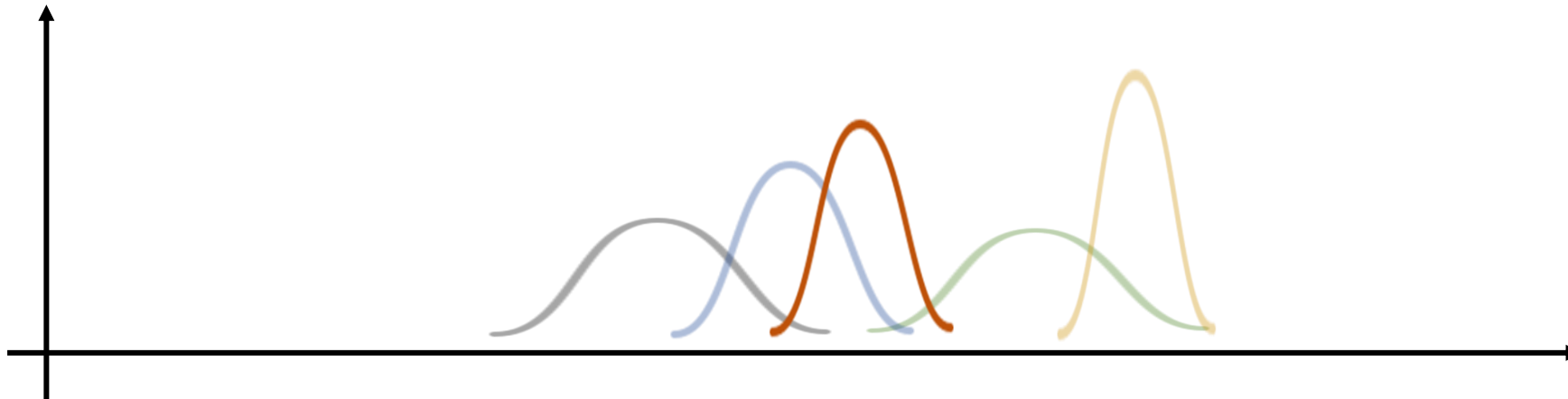
- Which model is the best?
  - CV, CA, CTRV, CTRA



# Physics based prediction

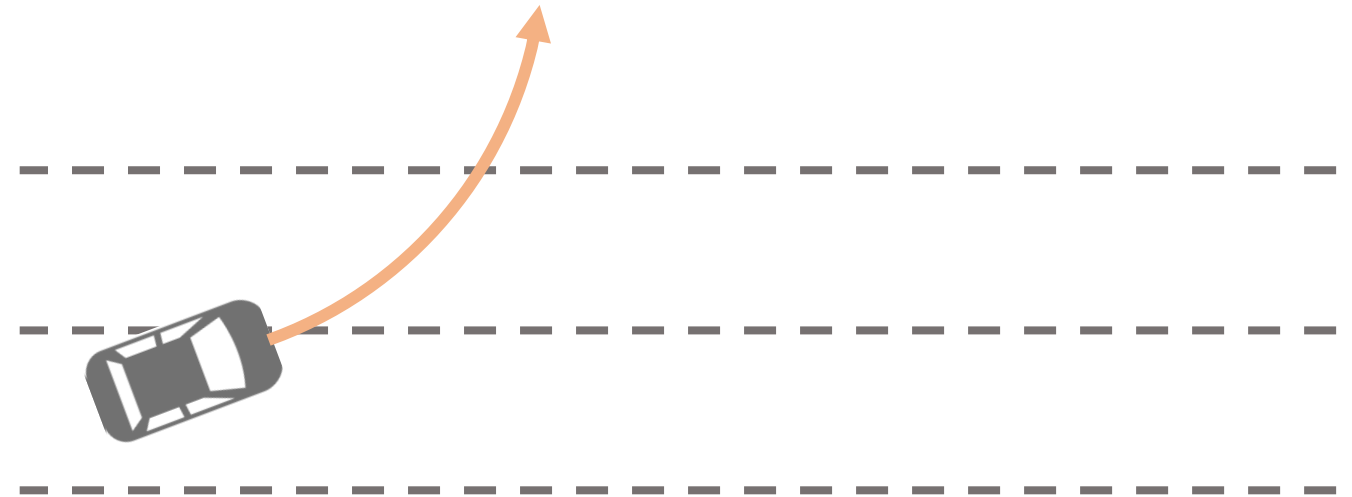
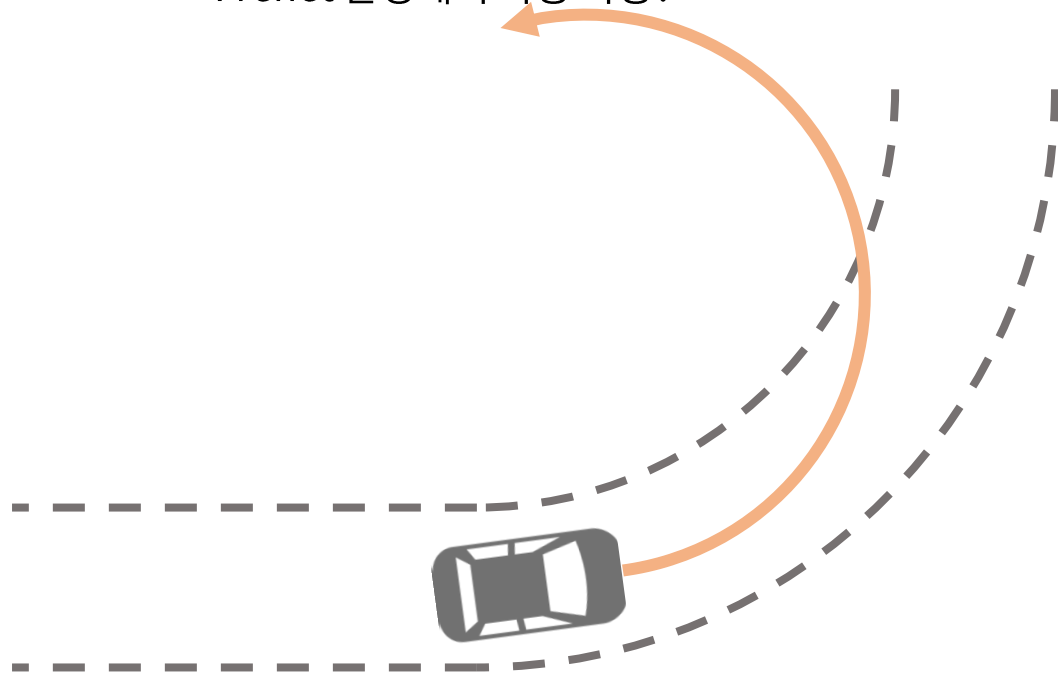
- Interacting multiple model (IMM)
  - Measure가 나올 확률이 높은 모델 사용

$$P(x_k|y_k) = \sum_i^{Model} P_i(x_k|y_k) \cdot \mu_{i,k}$$
$$, \mu_{i,k} = \frac{P(y_{(1:k)}|M_i)P(M_i)}{P(y_{(1:k)})}$$



# Physics based prediction

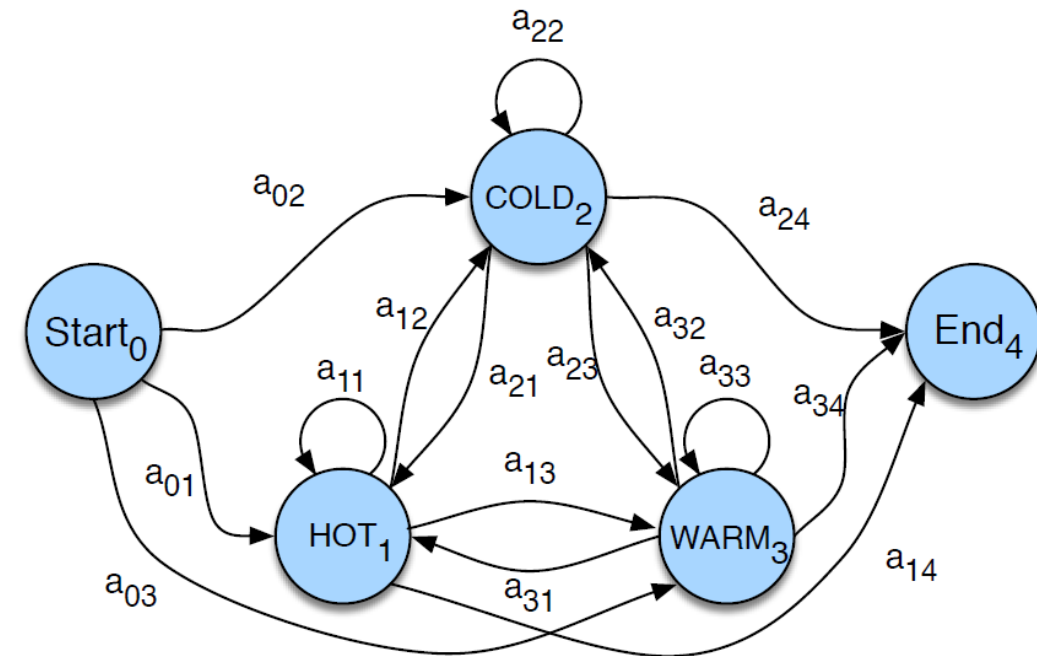
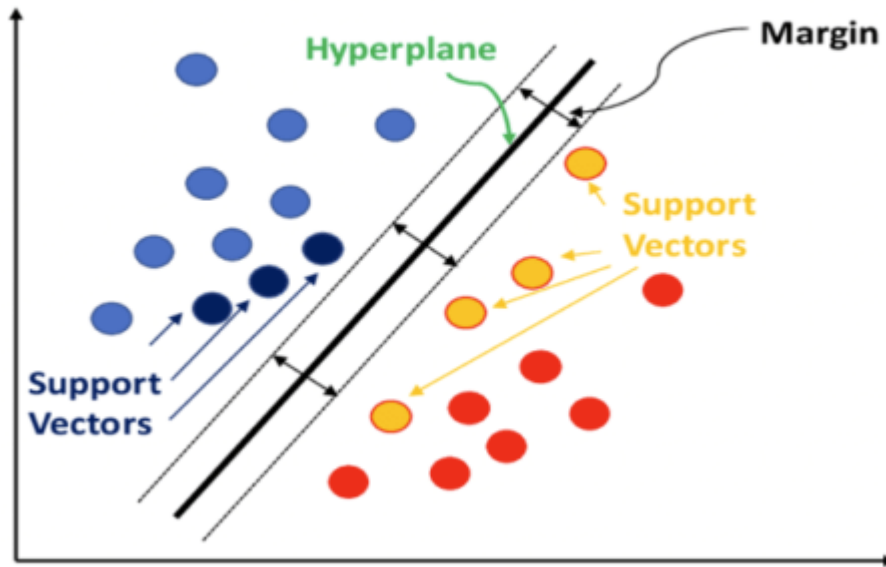
- Limits of physics-based prediction
  - Long-term prediction에는 적합하지 않음
  - 도로 환경(차선 등)을 고려할 수 없음
  - Frenet 환경에서 사용 가능?



# Maneuver based prediction

- Introduction

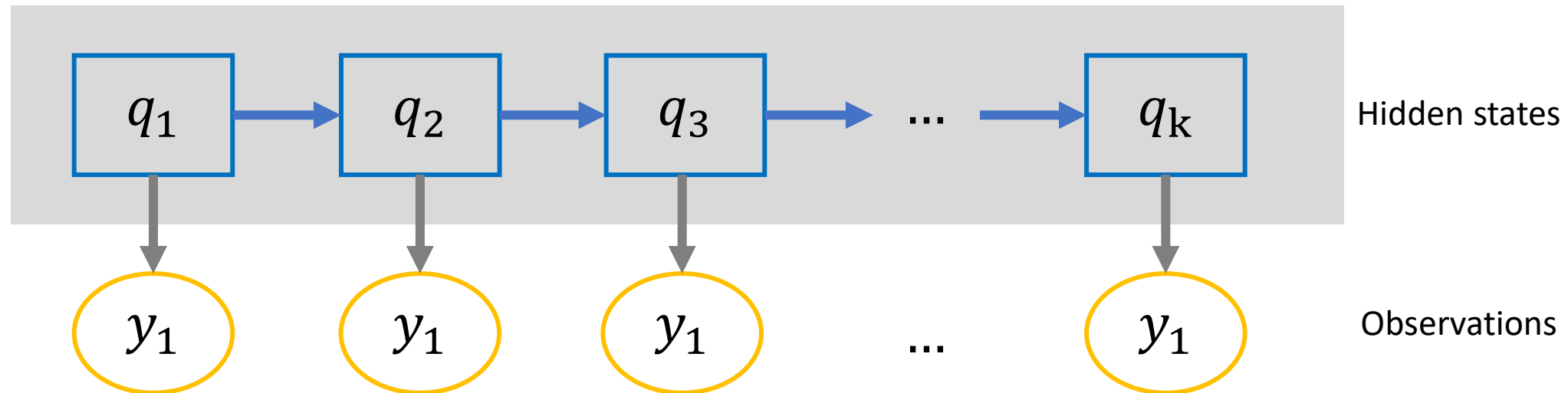
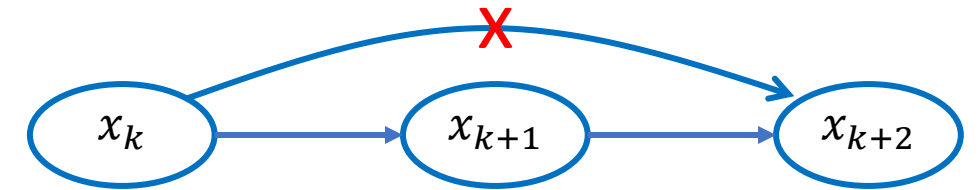
- 다소 긴 시간 예측에 유리
- 매우 긴 시간에 대한 예측은 불가
- 주변 환경을 고려하도록 설계
- Bayesian, SVM, HMM, ...



# Maneuver based prediction

- Hidden Markov model

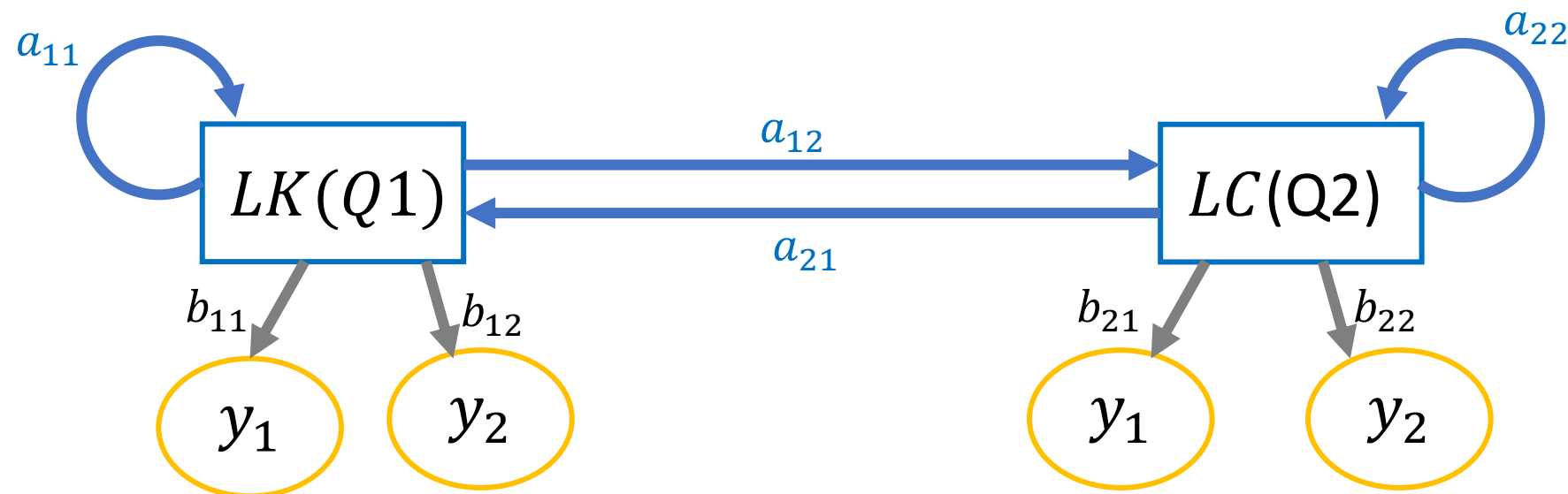
- Main idea : 관측된 데이터는 숨겨진 State ( Hidden state) 에서 비롯됨
- Hidden state (Q) : State which we cannot observe. (Intension, Tendency, ...)
- Observation (Y) : Value which we are able to observe. ( Distance from centerline, acceleration, velocity, ...)
- Assumption : Markov chain(model)



# Maneuver based prediction

- Hidden Markov model

- Transition matrix (A)
- Emission matrix (B)
- Initial probability ( $\pi$ )

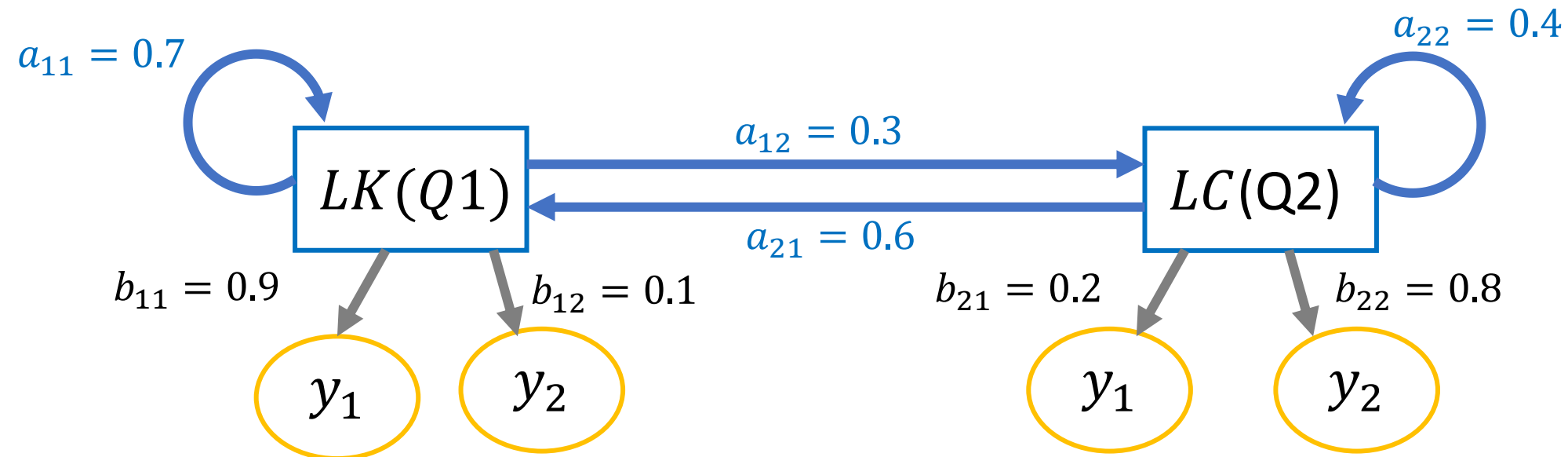


$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

# Maneuver based prediction

- Hidden Markov model
  - Example :  $P(y = \{y_1, y_1, y_2\})$



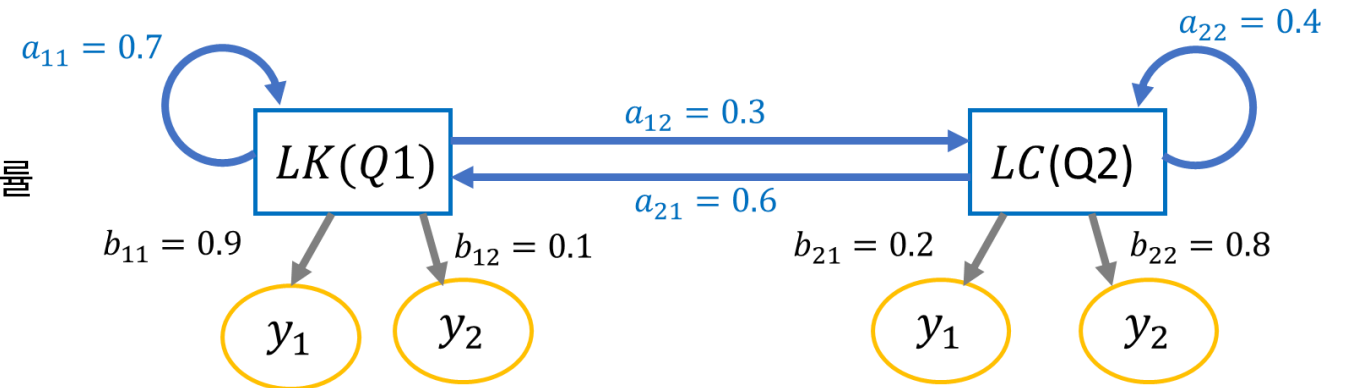


# Maneuver based prediction

- Hidden Markov model

- DP Problem : 반복되는 계산 값을 저장해서 사용
- Forward probability ( $\alpha$ ) : 관측값이 주어졌을 때(1:k) 어떤 State(k)에 있을 확률

$$\alpha_{k,j} = \sum_q \alpha_{k-1} a_{qj} b_{jy}$$



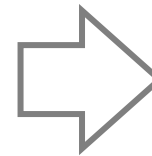
LK(Q1)

$$\alpha_1 = \pi_1 b_{11} = 0.9 \times 0.9 = 0.81$$

LC(Q2)

$$\alpha_2 = \pi_2 b_{21} = 0.1 \times 0.1 = 0.01$$

y<sub>1</sub>



LK(Q1)

$$\alpha_1 = 0.81 \times 0.7 \times 0.1 + 0.01 \times 0.6 \times 0.1$$

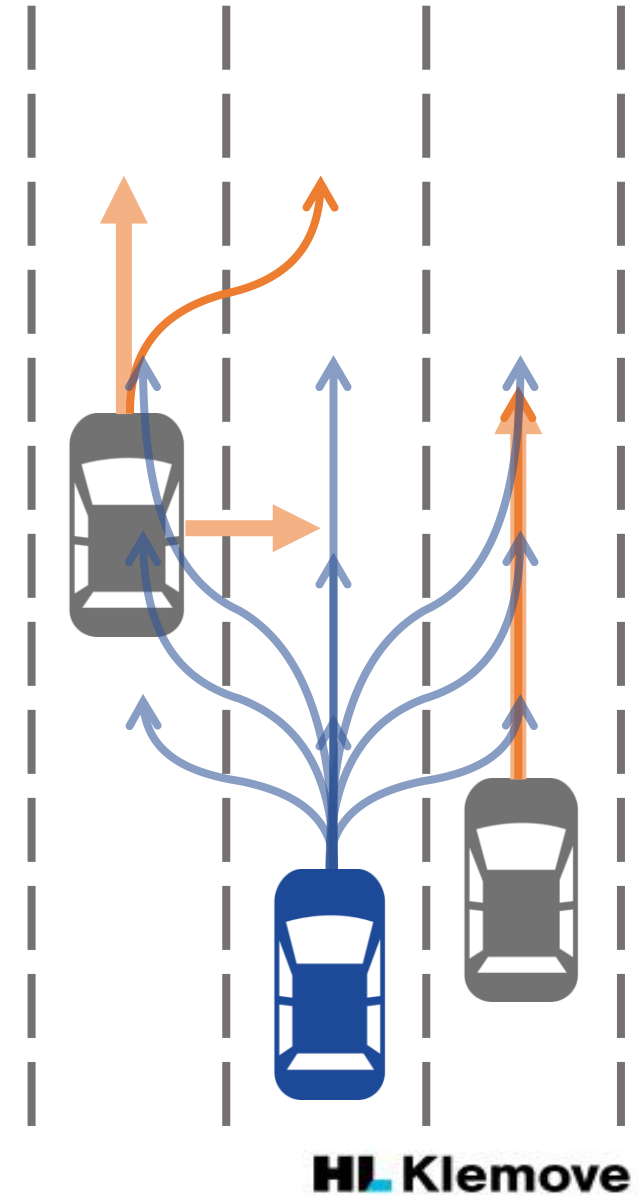
LC(Q2)

$$\alpha_2 = 0.81 \times 0.3 \times 0.8 + 0.01 \times 0.4 \times 0.8$$

y<sub>2</sub>

# Prediction

- Prediction on frenet frame
  - 선회 모델이 불필요
  - 모든 차들은 기본적으로 차선을 따라 주행한다!
  - 종방향 : Physical prediction
    - CV / CA 모델
  - 횡방향 : Maneuver prediction or physical prediction
    - Lane keeping / Lane change / CV / CA
  - 자차의 각 Trajectory candidate 별 Cost 계산 가능!



Thank You

