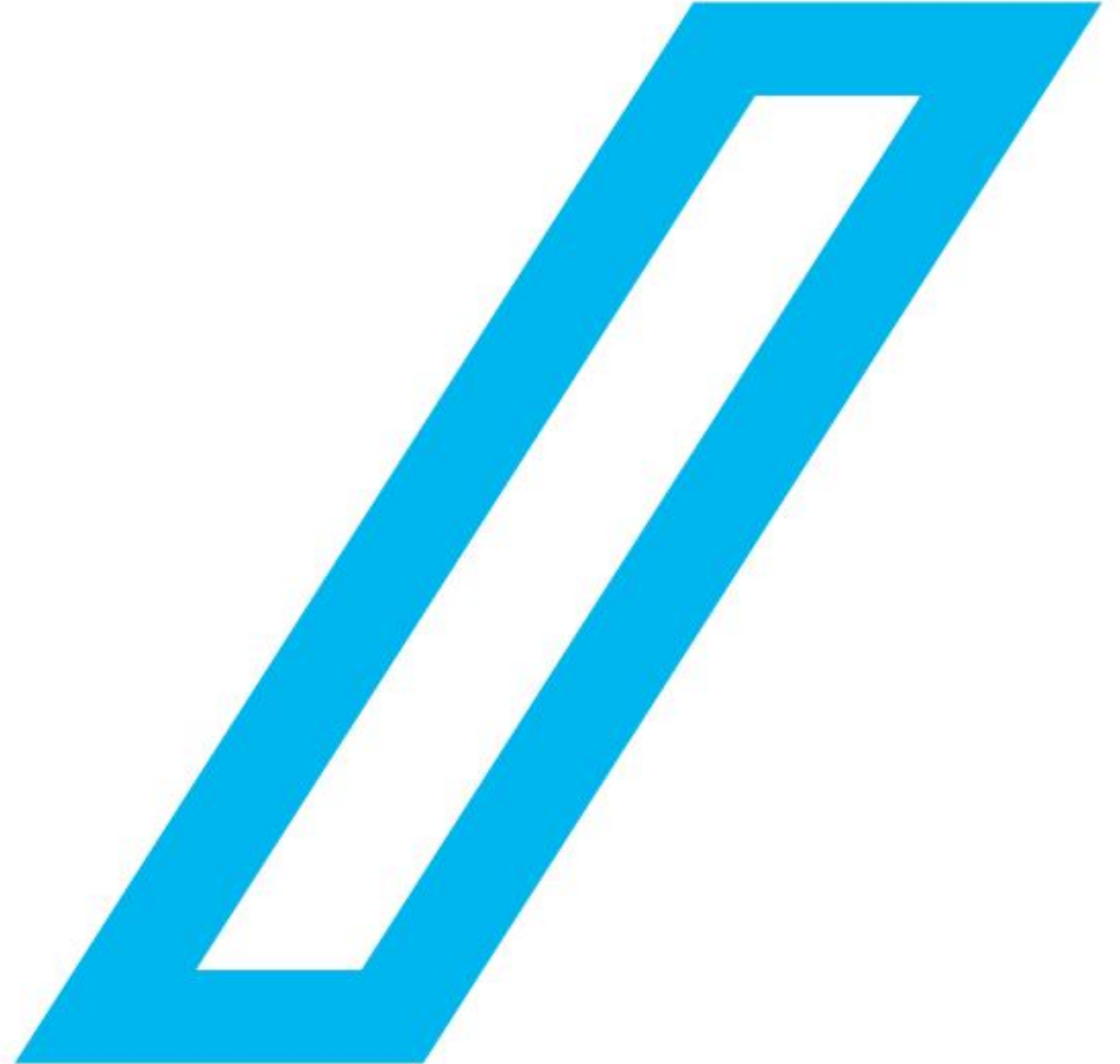


Path planning

Lecturer : Seungmok Song



Contents

1. Introduction
2. Path planning
3. Searching based path planning



Introduction

- Path planning with driving lane



Introduction

- Path planning



Path planning

- Path planning with lane

- Road model

- Clothoid: 곡률의 변화가 연속적!

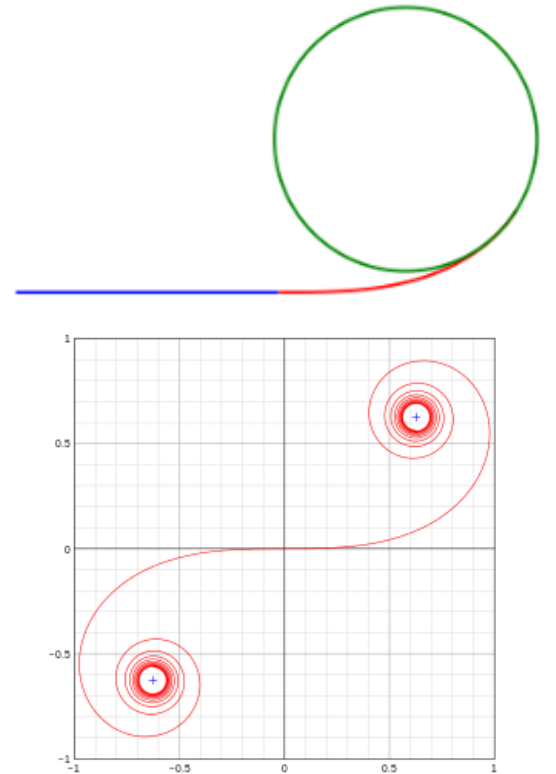
$$\text{Curvature } \kappa = \frac{1}{R} \approx l$$

$$x(t) = aC(t) = a \int_0^t \cos\left(\frac{\pi}{2}u^2\right) du$$

$$y(t) = aS(t) = a \int_0^t \sin\left(\frac{\pi}{2}u^2\right) du$$

$$x = \ell - \frac{\ell^5}{40A^4} + \frac{\ell^9}{3456A^8} - \frac{\ell^{13}}{599040A^{12}} \dots$$

$$y = \frac{\ell^3}{6A^2} - \frac{\ell^7}{336A^6} + \frac{\ell^{11}}{42240A^{10}} - \frac{\ell^{15}}{9676800A^{14}} \dots$$

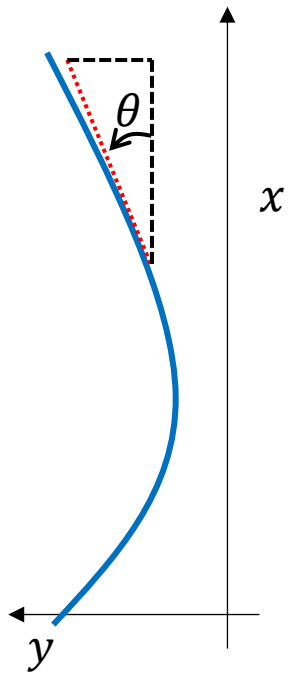


Path planning

- Path planning with lane

- Road model

- 3rd order polynomial : local coordinate 에서 clothoid 를 표현하기에 적합

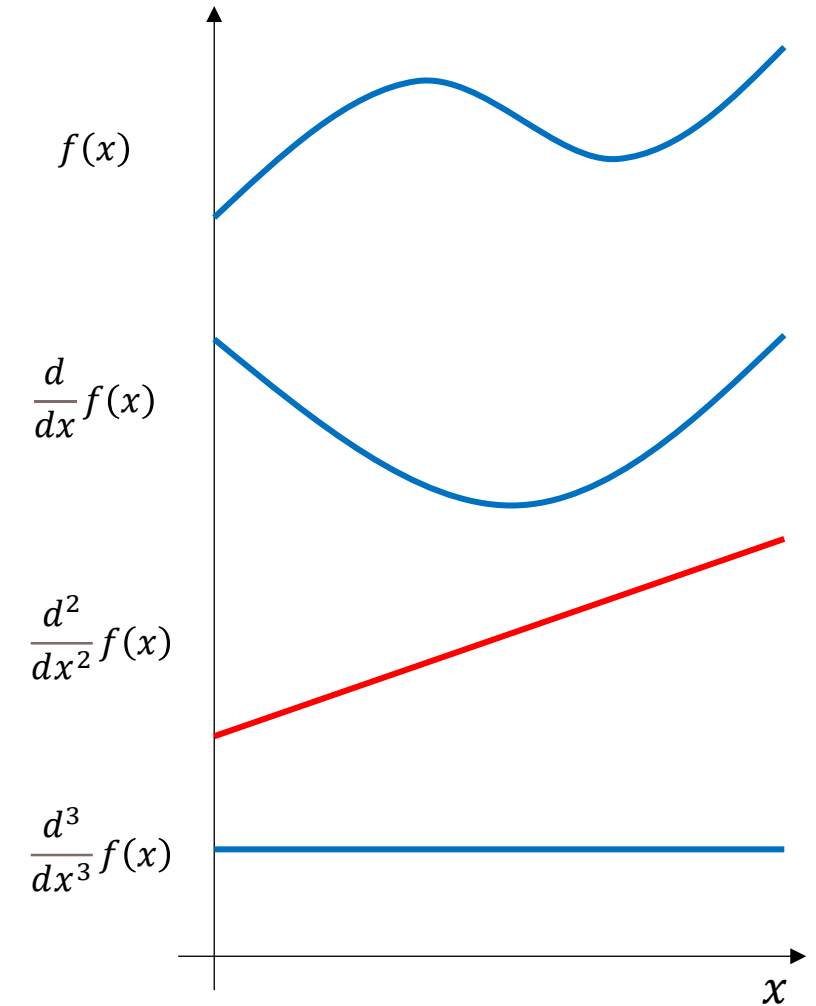
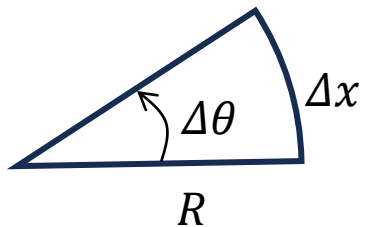


$$y = f(x) = c_3x^3 + c_2x^2 + c_1x + c_0$$

$$\theta \approx \tan(\theta) = \frac{dy}{dx} = \frac{d}{dx}f(x)$$

$$\frac{d\theta}{dx} = \kappa = \frac{1}{R} = \frac{d^2}{dx^2}f(x)$$

$$\frac{d\kappa}{dx} = \frac{d^3}{dx^3}f(x)$$



Path planning

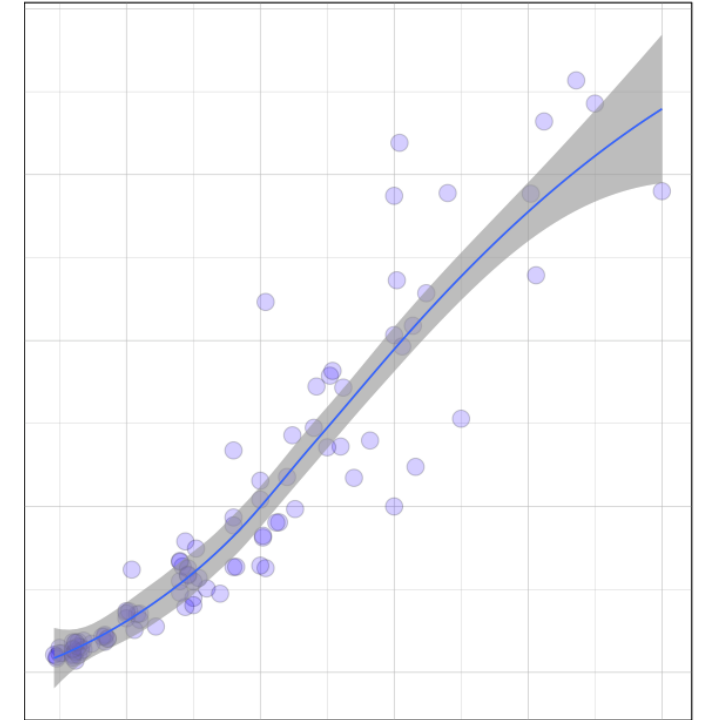
- Path planning with lane
 - Road model
 - Fitting to 3rd order polynomial : least square

$$\begin{bmatrix} (x_1, y_1) \\ (x_2, y_2) \\ \vdots \\ (x_m, y_m) \end{bmatrix} \Rightarrow y = f(x) = c_3x^3 + c_2x^2 + c_1x + c_0$$

$$\begin{bmatrix} x_1^3 & x_1^2 & x_1^1 & x_1^0 \\ x_2^3 & x_2^2 & x_2^1 & x_2^0 \\ \vdots & \vdots & \vdots & \vdots \\ x_m^3 & x_m^2 & x_m^1 & x_m^0 \end{bmatrix} \begin{bmatrix} c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix} \approx \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

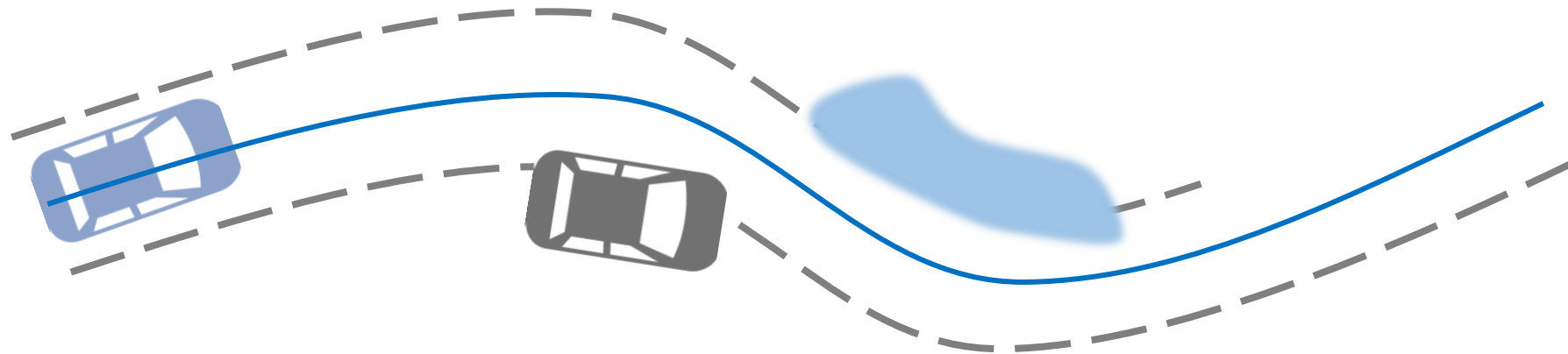
$$XC \approx Y$$

$$C = \text{inv}(X^T X) X^T Y$$



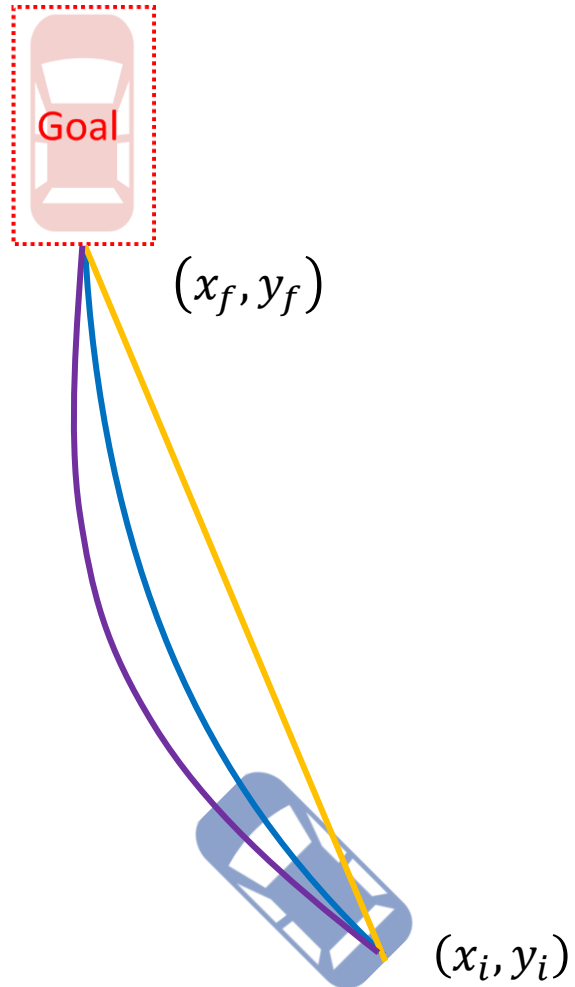
Path planning

- Path planning with lane
 - Both lane detected
 - Use mean value of each lane
 - Either lane detected
 - Make virtual lane
 - Lane width estimation



Path planning

- Path planning without lane
 - With goal destination



$$y = f(x) = c_1x + c_0 : 2 \text{ known information}$$

$$y = f(x) = c_2x^2 + c_1x + c_0 : 3 \text{ known information}$$

$$y = f(x) = c_3x^3 + c_2x^2 + c_1x + c_0 : 4 \text{ known information}$$

\vdots

Path planning

- Path planning without lane
 - With leading vehicle

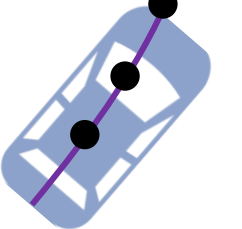


$$y = f(x) = c_1x + c_0 : 2 \text{ known information}$$

$$y = f(x) = c_2x^2 + c_1x + c_0 : 3 \text{ known information}$$

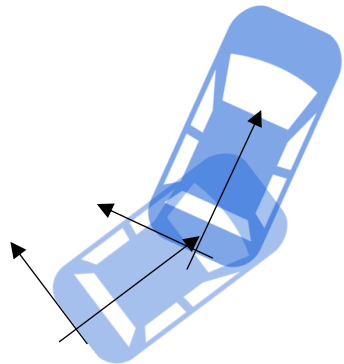
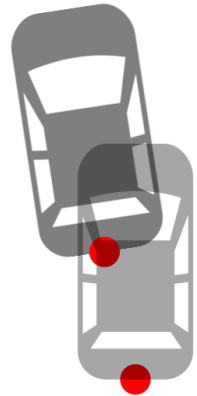
$$y = f(x) = c_3x^3 + c_2x^2 + c_1x + c_0 : 4 \text{ known information}$$

⋮



Path planning

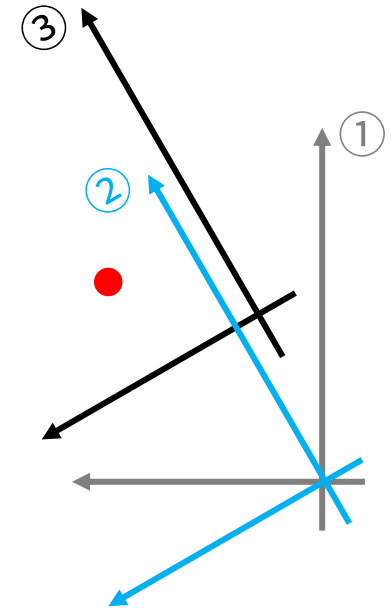
- Path planning without lane
 - With leading vehicle
 - CTRV(등속 선회운동)



$$(x_1, y_1) \rightarrow (x_2, y_2) \rightarrow (x_3, y_3)$$

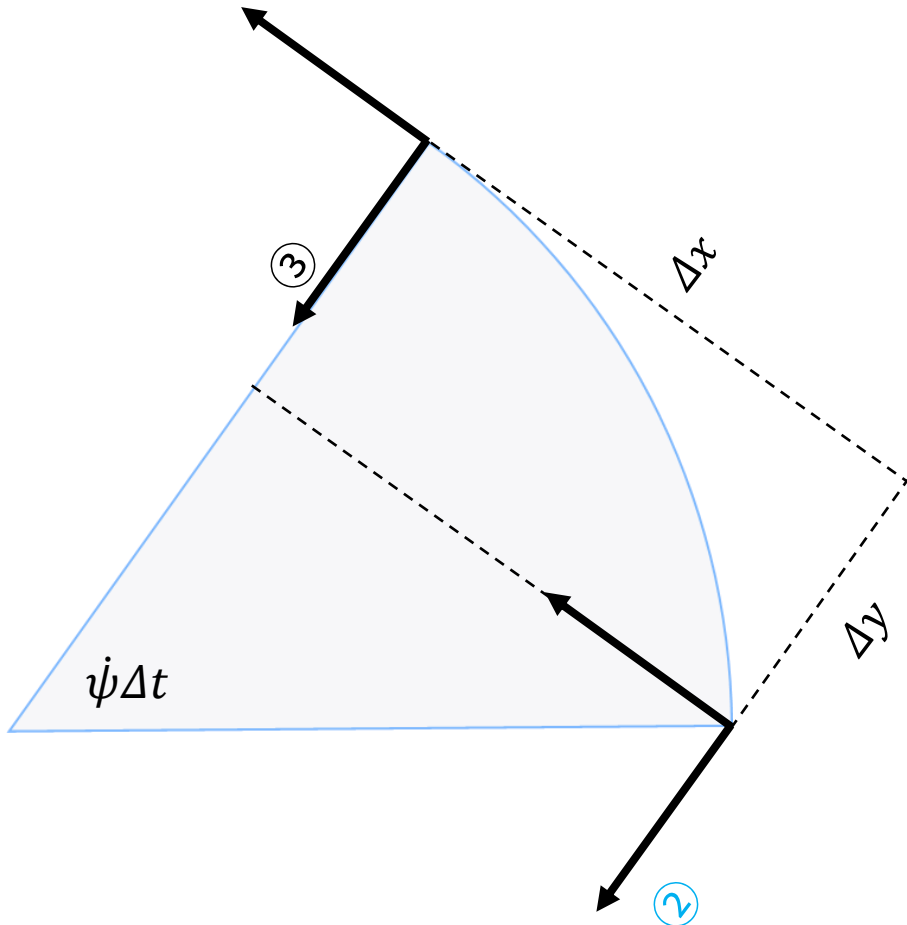
Step 1. ①→②좌표계 회전 : $-\dot{\psi}\Delta t$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \quad \theta = -\dot{\psi}\Delta t$$



Path planning

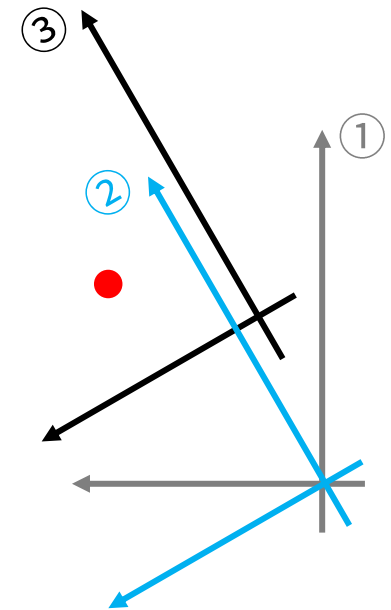
- Path planning without lane
 - With leading vehicle
 - CTRV(등속 선회운동)



Step 2. ② → ③ 좌표계 회전 : $-\dot{\psi}\Delta t$

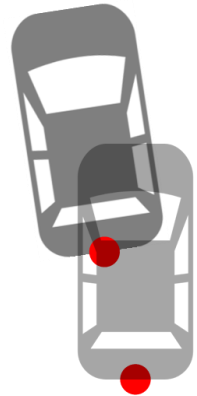
$$\Delta x = R \sin(\dot{\psi}\Delta t), \quad \Delta y = R (1 - \cos(\dot{\psi}\Delta t))$$

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -R \sin(\dot{\psi}\Delta t) \\ R (1 - \cos(\dot{\psi}\Delta t)) \end{pmatrix}$$

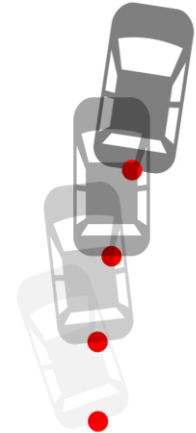


Path planning

- Path planning without lane
 - With leading vehicle
 - CTRV(등속 선회운동)



$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} \cos(\dot{\psi}\Delta t) & \sin(\dot{\psi}\Delta t) \\ -\sin(\dot{\psi}\Delta t) & \cos(\dot{\psi}\Delta t) \end{pmatrix} \begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} -R\sin(\dot{\psi}\Delta t) \\ R(1 - \cos(\dot{\psi}\Delta t)) \end{pmatrix}$$



$$P_{k+1}^T = \begin{pmatrix} \cos(\dot{\psi}\Delta t) & \sin(\dot{\psi}\Delta t) \\ -\sin(\dot{\psi}\Delta t) & \cos(\dot{\psi}\Delta t) \end{pmatrix} P_k^T + \begin{pmatrix} -R\sin(\dot{\psi}\Delta t) \\ R(1 - \cos(\dot{\psi}\Delta t)) \end{pmatrix}, P_k = \begin{pmatrix} (x_1, y_1) \\ (x_2, y_2) \\ \vdots \\ (x_n, y_n) \end{pmatrix}$$

