〈7장 적분 문제〉

20231609 정희선

#11 $\int \cos^5 x \cdot \sin^5 x \, dx$

$= \int \sin x \cdot \cos x \cdot \cos^4 x \cdot \sin^4 x \, dx$

$= \int \sin x \cdot \cos x \cdot (1-\sin^2 x)^2 \cdot \sin^4 x \, dx$

$\sin x = t$, $\cos x \, dx = dt$

$= \int t^5 \cdot (1-t^2)^2 \, dt$

$t^5 (1-2t^2+t^4)$

$t^9 - 2t^7 + t^5$

$= \frac{1}{10} t^{10} - \frac{2}{8} t^8 + \frac{1}{6} t^6 + C$

$= \frac{1}{10} \cdot \sin^{10} x - \frac{1}{4} \cdot \sin^8 x + \frac{1}{6} \cdot \sin^6 x + C$

#12 $\int \tan^4 x \, dx$

$= \int (\sec^2 x - 1)^2 \, dx$

$= \int \sec^4 x - 2\sec^2 x + 1 \, dx$

$= \int \sec^2 x \cdot \sec^2 x \, dx - 2\int \sec^2 x \, dx + \int 1 \, dx$

$\tan x = t$

$\sec^2 x \cdot dx = dt$

$\int (\tan^2 x + 1) \sec^2 x \, dx$

$= \int t^2 + 1 \, dt - 2 \tan x + x + C$

$\frac{1}{3} t^3 + t$

$= \frac{1}{3} \tan^3 x - \tan x + x + C$

#13 $\int e^x \cdot \sin x \, dx = \sin x \cdot e^x - \int \cos x \cdot e^x \, dx$

$= \cos x \cdot e^x + \int \sin x \cdot e^x \, dx$

$\int e^x \sin x \, dx = \sin x \cdot e^x - \cos x \cdot e^x - \int \sin x \cdot e^x \, dx$

$2 \cdot \int e^x \cdot \sin x \, dx = e^x (\sin x - \cos x)$

$\int e^x \cdot \sin x \, dx = \frac{1}{2} \cdot e^x (\sin x - \cos x)$

#14 $\int \sin\left(x + \frac{\pi}{6}\right) \cos x \, dx$

$= \sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2} \int \sin x \cos x \, dx + \frac{1}{2} \int \cos^2 x \, dx$

$\frac{1}{2} \sin 2x$ $\qquad$ $\frac{1+\cos 2x}{2}$

$= \frac{\sqrt{3}}{4} \int \sin 2x \, dx + \frac{1}{4} \int 1 + \cos 2x \, dx$

$= -\frac{\sqrt{3}}{8} \cos 2x + \frac{1}{4} x + \frac{1}{8} \sin 2x + C$