BNC: Homework 1

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1 Problem 1: Hodgkin-Huxley Model

1.1 Background

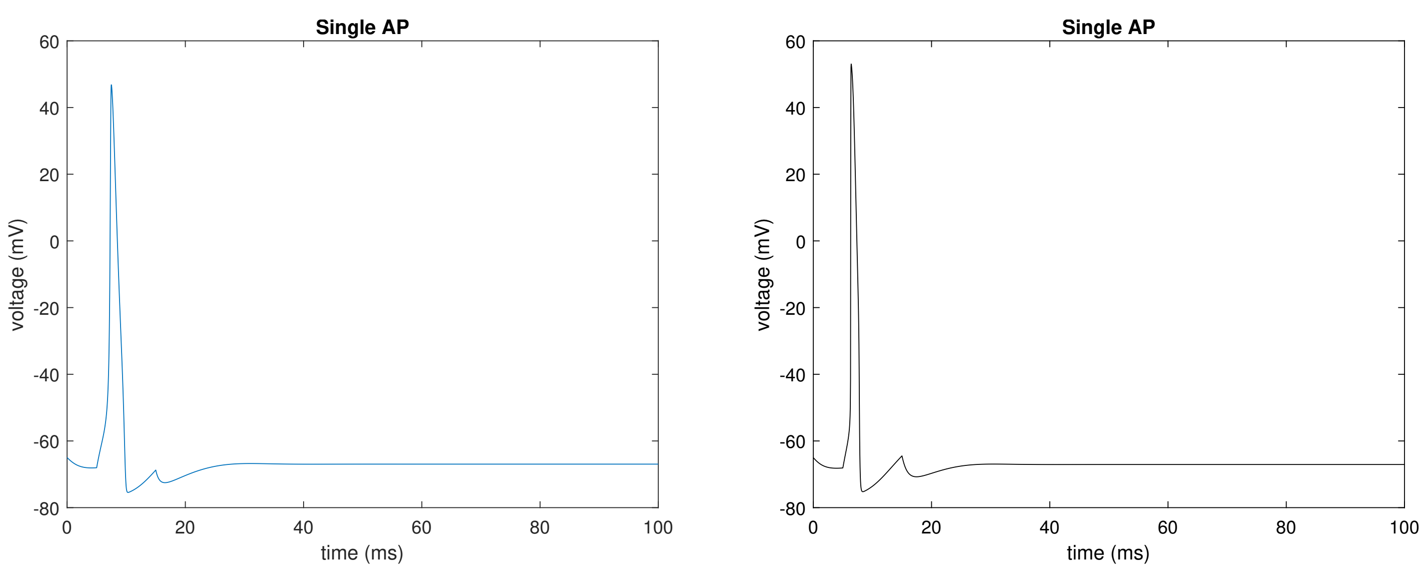
The Hodgkin-Huxley model is a classical model that is used to demonstrate the shape of an action potential over a time course. This problem deals with investigating the inner workings of the Hodgkin Huxley (HH) model. The figures below exemplify what happens when a threshold is met to produce an action potential (AP).

1.2 Method

Used the following equations to model the HH-model.
Original: $C\frac{dV}{dt} = I - g_k n^4(V - E_k) - g_{Na}m^3h(V - E_{Na}) - g_L(V - E_L)$
Reduced: $C\frac{dV}{dt} = I - g_k n^4(V - E_k) - g_{Na}m^3_{\infty}(V)(0.89 - 1.1n)(V - E_{Na}) - g_L(V - E_L)$
Then, adjusted the current to produce varying results shown below.

1.3 Results

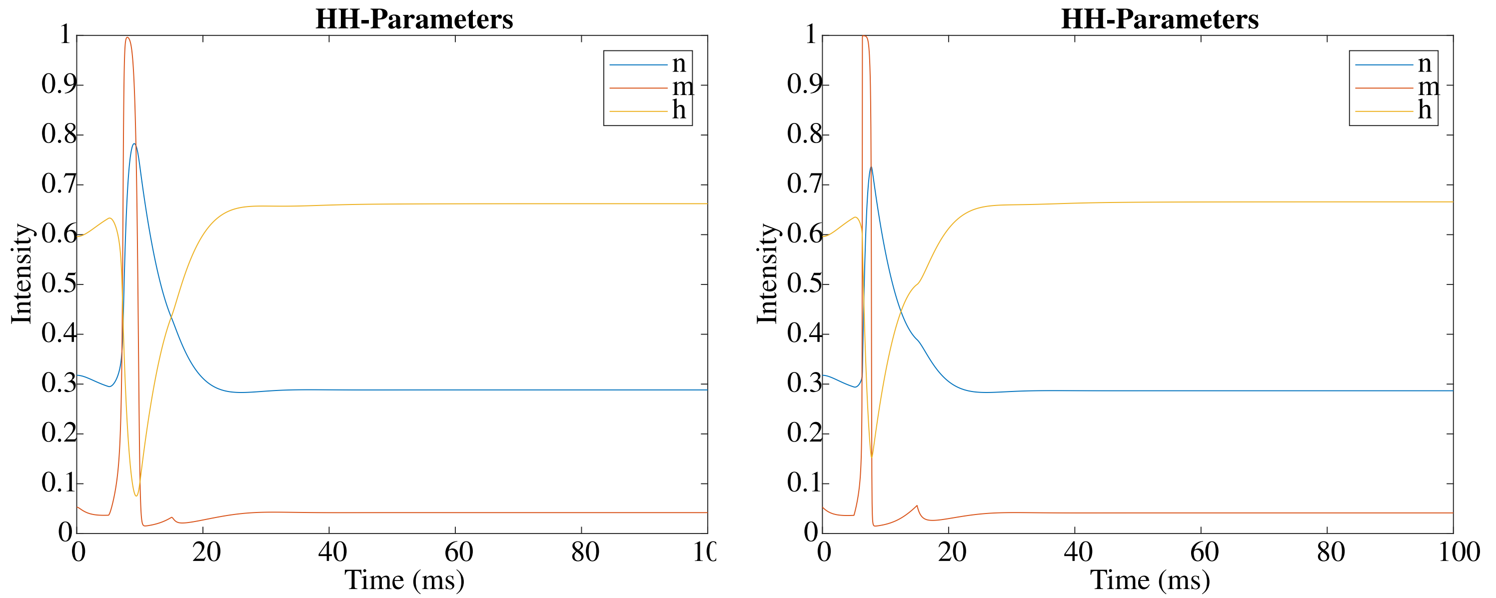
Original: $C\frac{dV}{dt} = I - g_k n^4(V - E_k) - g_{Na}m^3h(V - E_{Na}) - g_L(V - E_L)$



(a) Original HH model

(b) Reduced HH Model

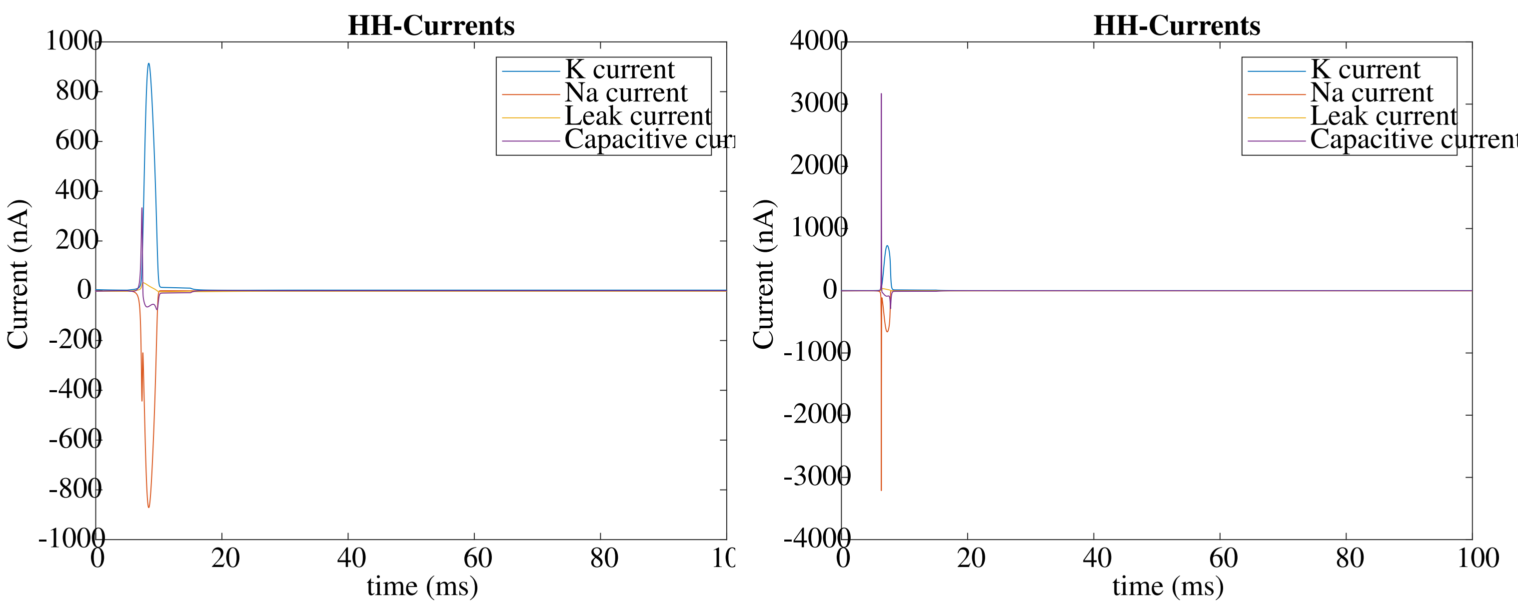
Figure 1: a) A singular AP of both HH models. The plot details depolarization and hyper-polarization phases as well as a spike that is characteristic of an all-or-nothing mechanism for producing an AP.



(a) Original HH model

(b) Reduced HH Model

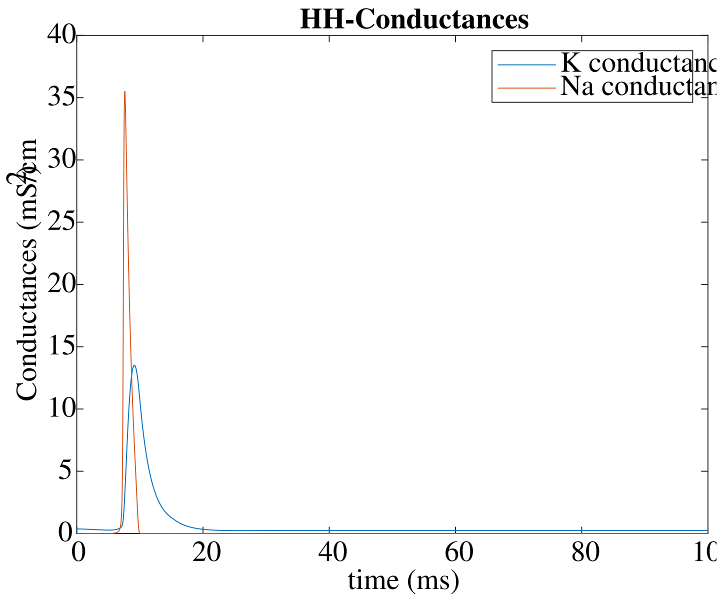
Figure 2: The evolution of the m,n,and h parameters of the HH model. m is representative of the sodium activation, h is representative of the sodium inactivation, and n is representative of potassium activation. The parameters are voltage dependent as they are changing over the course of the AP.



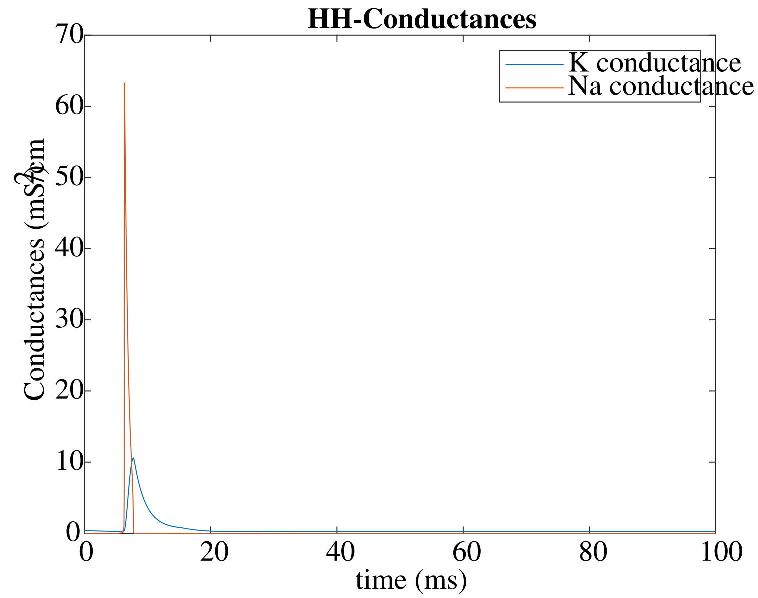
(a) Original HH model

(b) Reduced HH Model

Figure 3: This plot details the currents of the HH model: the sodium current is negative (inward current), the potassium current is positive (outward current), leak current (sourced from chlorine ions and other ions), and capacitive current (sourced from the potential difference of the membrane).



(a) Original HH model



(b) Reduced HH Model

Figure 4: The HH model has two pertinent conductances that dictate the system: potassium and sodium conductances. This figure demonstrates the expected quick, but large magnitude by which sodium channels increases conductivity while the the slow and smaller magnitude of the increase in conductivity of potassium channels.

1.3.1 Original Model

Original: $C\frac{dV}{dt} = I - g_k n^4(V - E_k) - g_{Na}m^3h(V - E_{Na}) - g_L(V - E_L)$

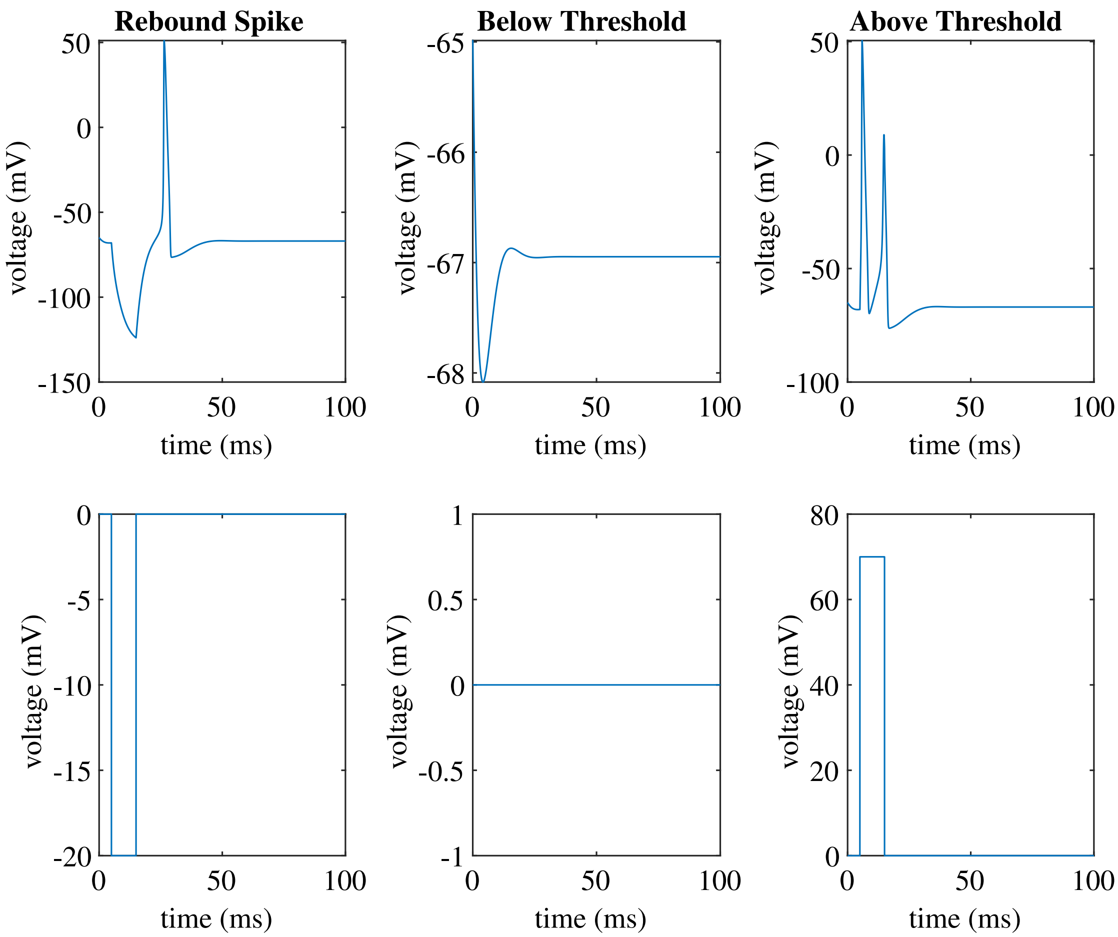


Figure 5: Behavior of HH-model at a) hyperpolarizing current, b) subthreshold current, and c) suprathreshold current.

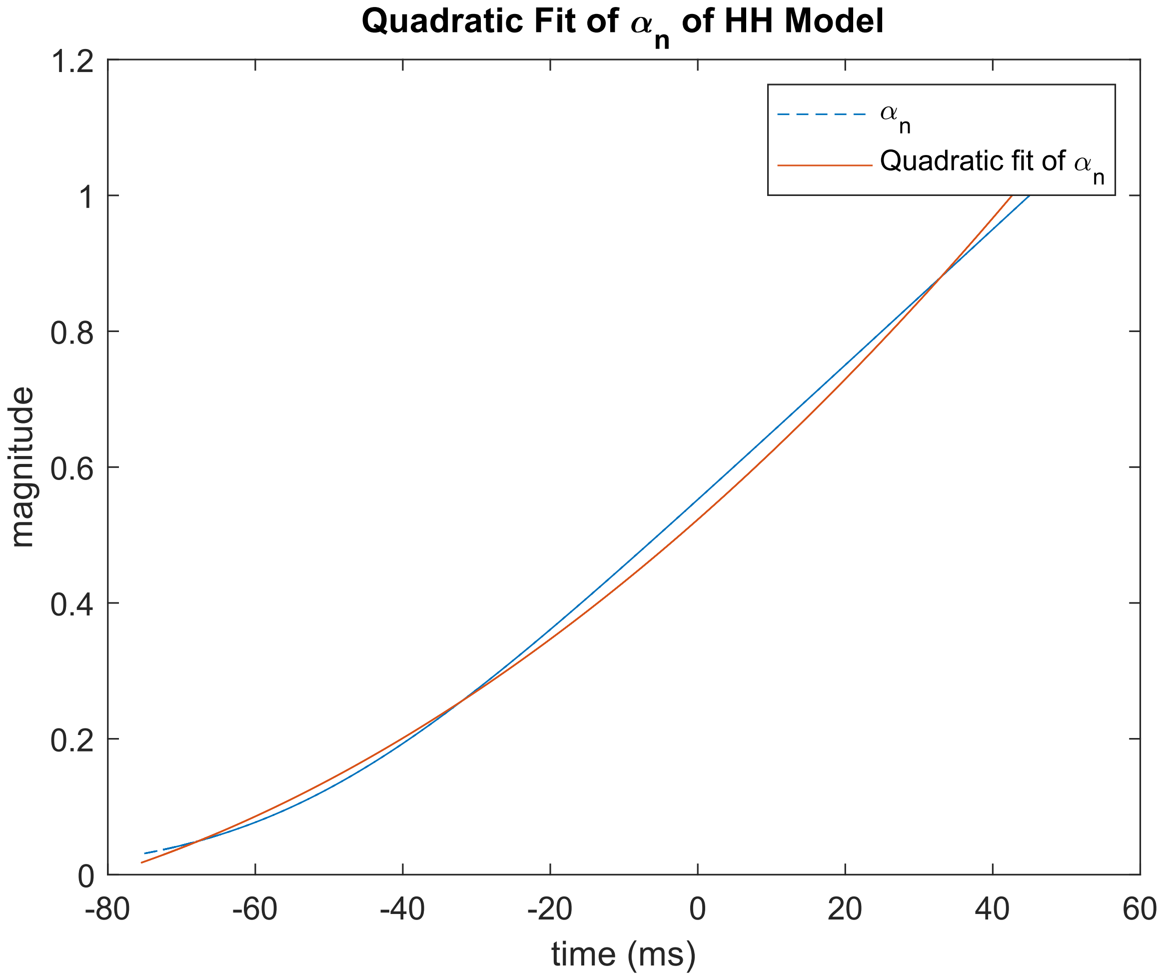


Figure 6: Determining the quadratic fit of α_n parameter.

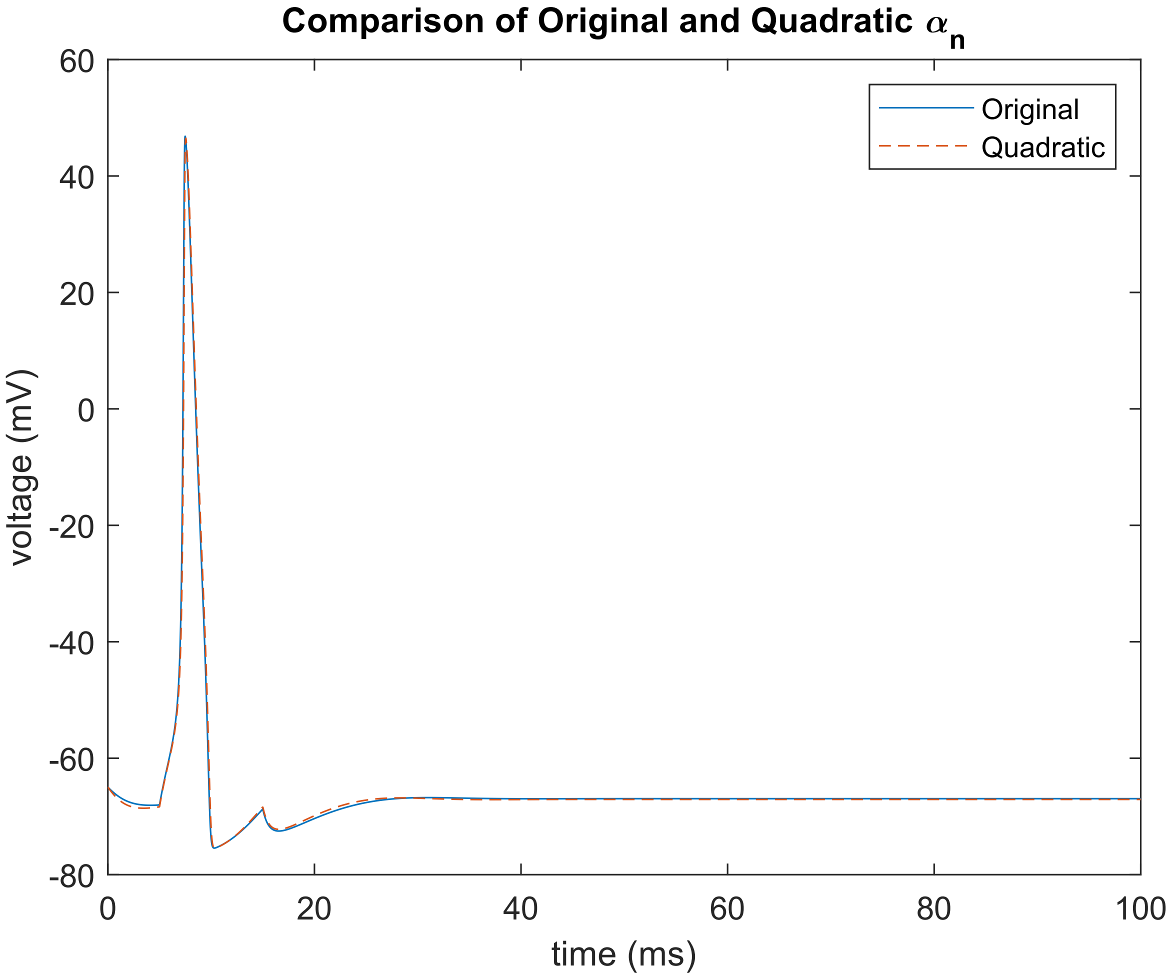


Figure 7: Comparison of the quadratic fit and the original α_n parameter.

1.3.2 Reduced Model

Reduced: $C\frac{dV}{dt} = I - g_k n^4(V - E_k) - g_{Na} m_\infty^3(V)(0.89 - 1.1n)(V - E_{Na}) - g_L(V - E_L)$

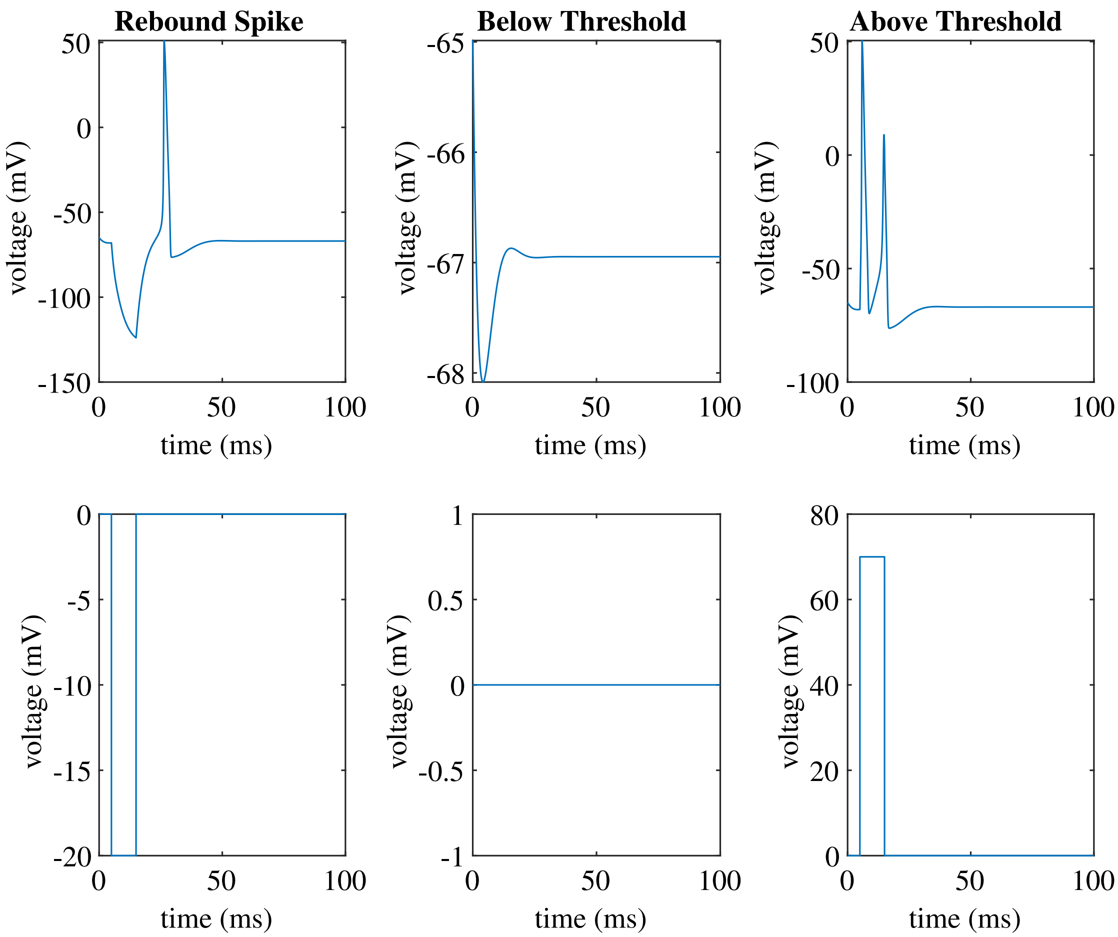


Figure 8: Behavior of HH-model at a) hyperpolarizing current, b) subthreshold current, and c) suprathreshold current.

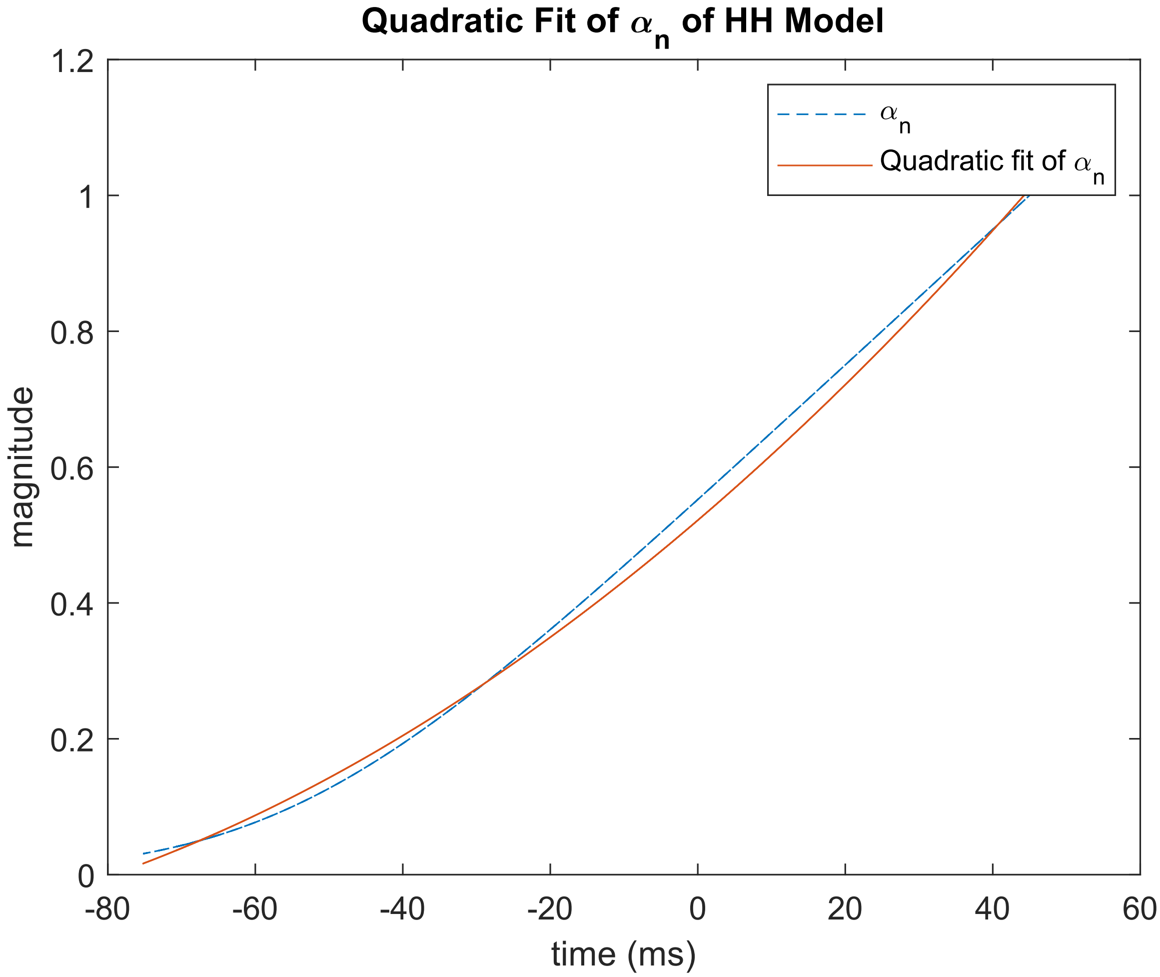


Figure 9: Determining the quadratic fit of α_n parameter.

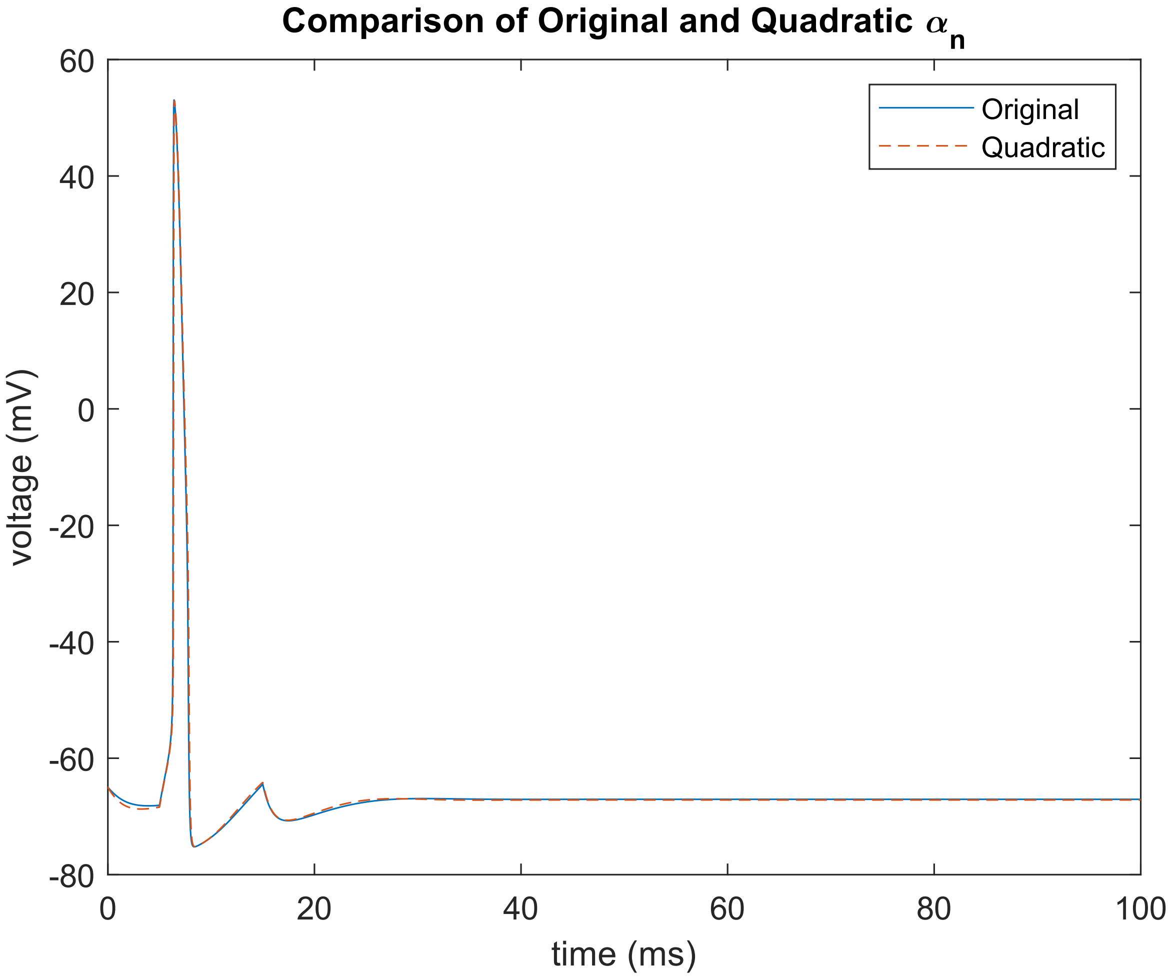


Figure 10: Comparison of the quadratic fit and the original α_n parameter.

1.4 Discussions

The simplified model takes into account that the m varies so fast in comparison to n and h, that its approximate behavior can be reflected by $m_\infty = \frac{\alpha_m}{\alpha_m+\beta_m}$. Furthermore, when looking the summation of the n and h parameters, it is almost, allows for the h to simplified to $h = 0.89 - 1.1n$. The above plots show that the reduced and original versions of the Hodgkin-Huxley model are very similar despite the modifications.

2 Problem 2

2.1 Background

Further exploring the HH-model particularly by implementing changes in the frequency of stimulus.

2.2 Method

Built both models in Matlab and then proceeded to change the frequencies to produce different effects. Also, constructed the two-neuron model in Matlab.

2.3 Results

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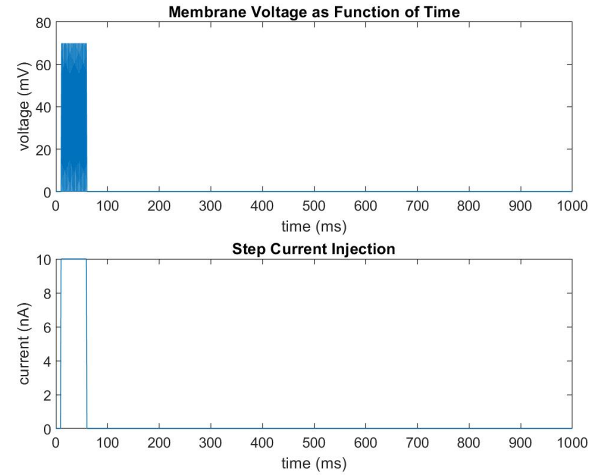
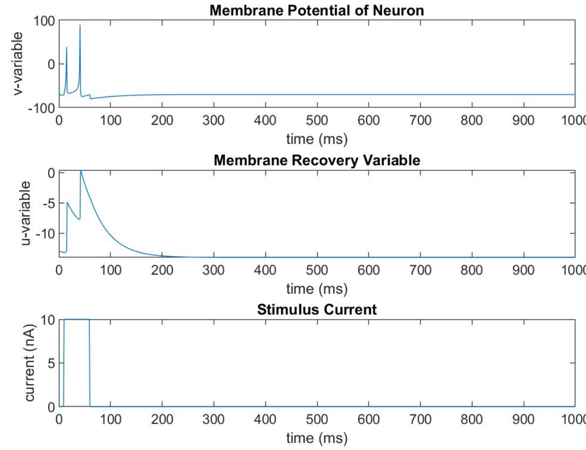
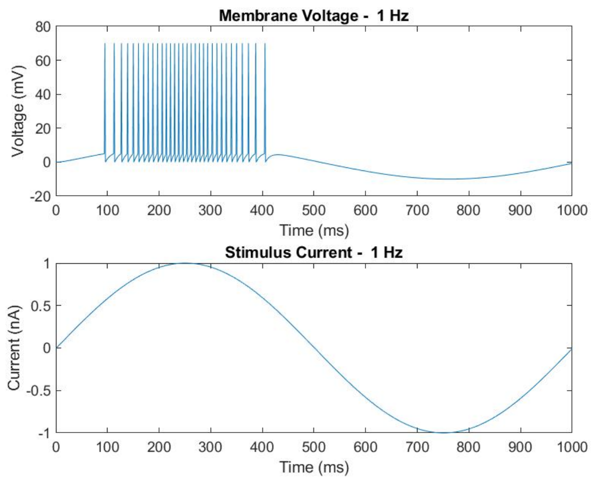


(a) IAF model of membrane potential.

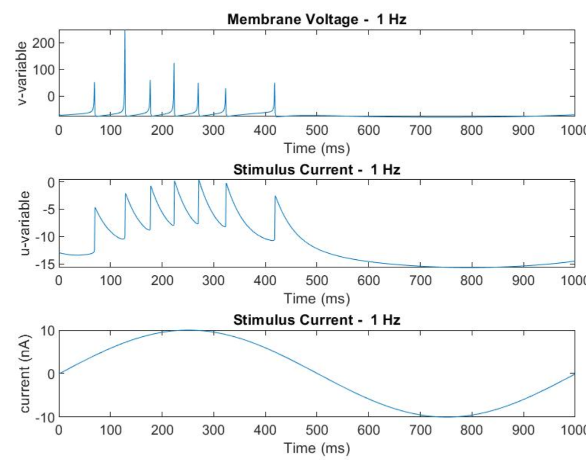


(b) Izhikivech model of membrane potential.

Figure 11: Comparison of models with simple injection current at 10 nA.

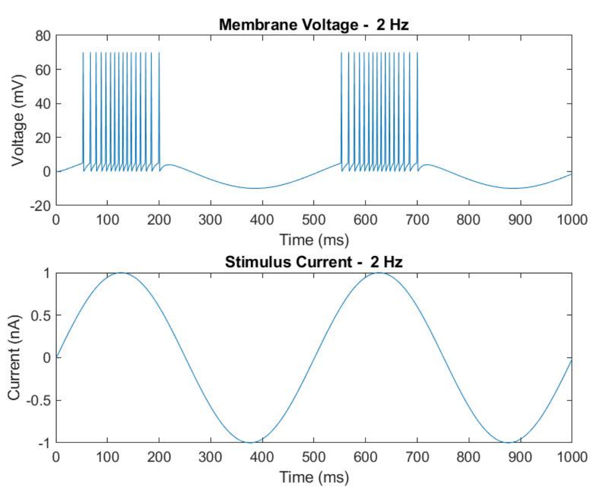


(a) IAF model

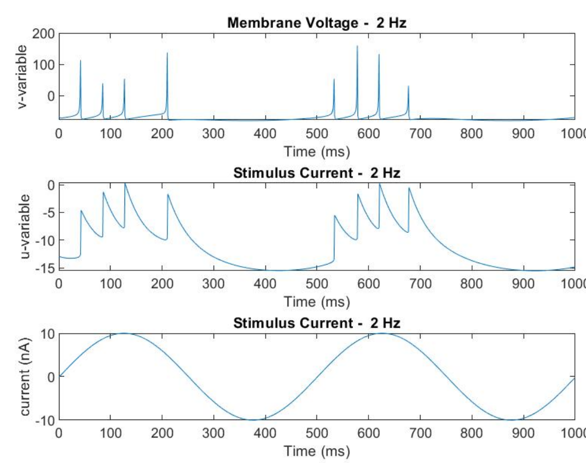


(b) Izhikivech model

Figure 12: Comparison of models at 1 Hz

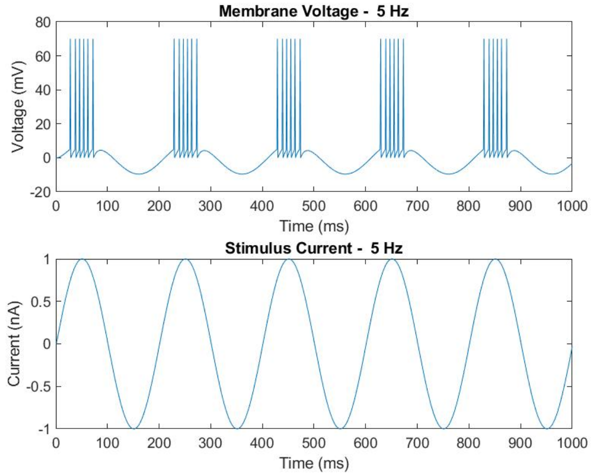


(a) IAF model

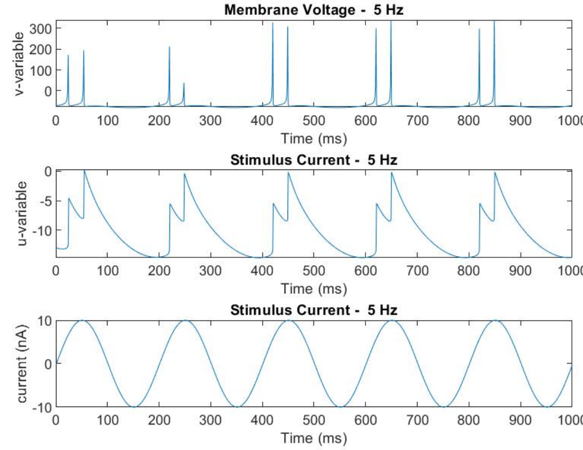


(b) Izhikivech model

Figure 13: Comparison of models at 2 Hz

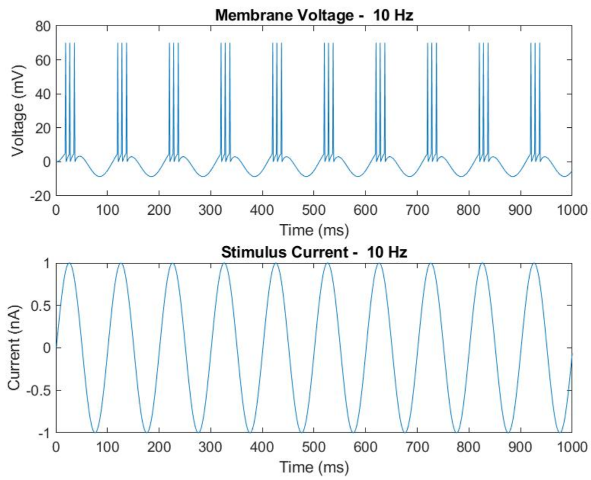


(a) IAF model

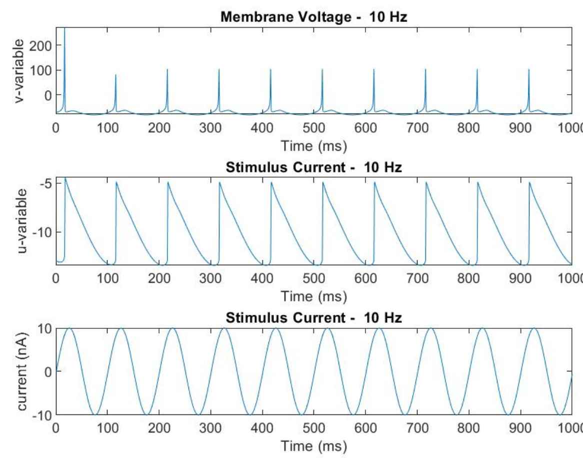


(b) Izhikivech model

Figure 14: Comparison of models at 5 Hz

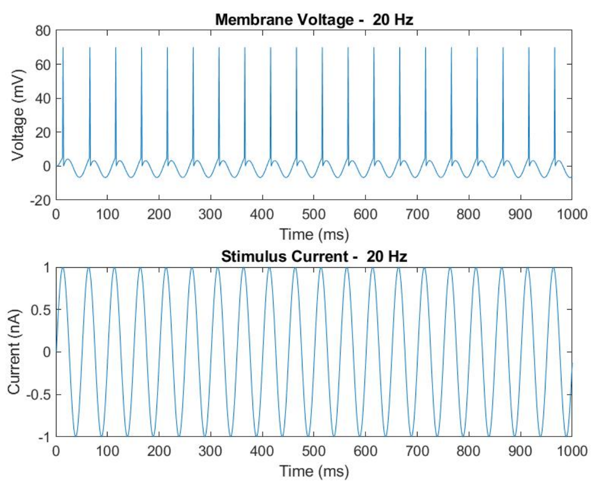


(a) IAF model

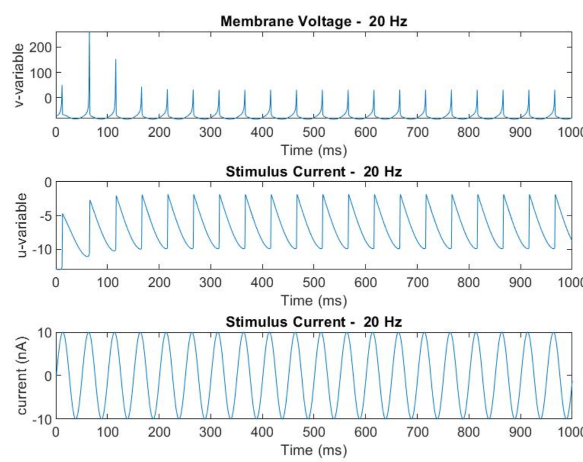


(b) Izhikivech model

Figure 15: Comparison of models at 10 Hz

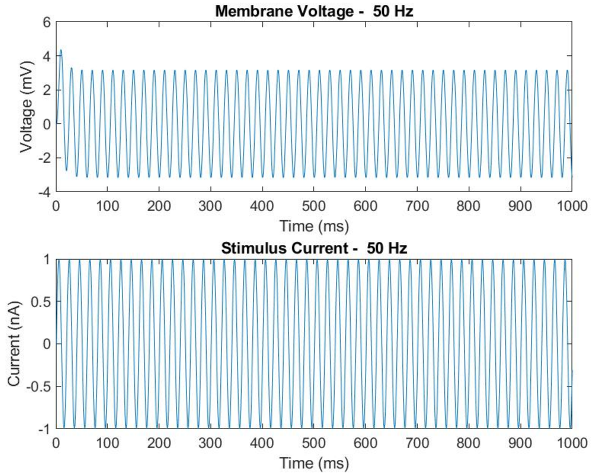


(a) IAF model

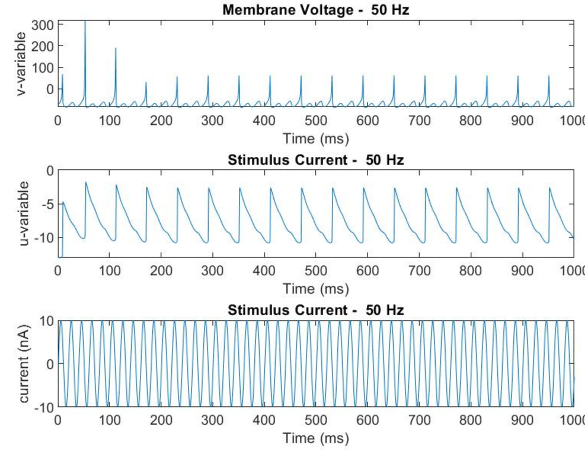


(b) Izhikivech model

Figure 16: Comparison of models at 20 Hz

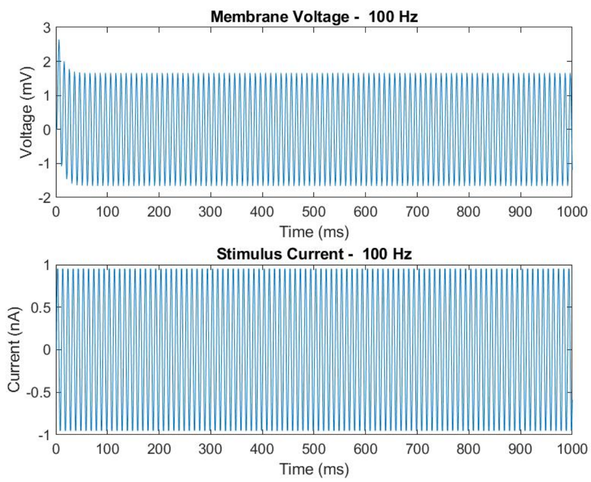


(a) IAF model

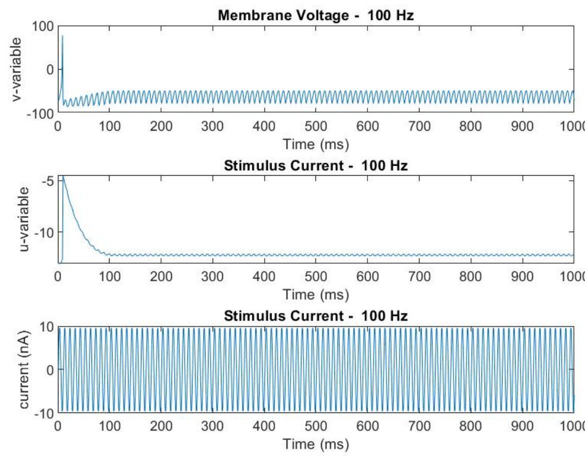


(b) Izhikivech model

Figure 17: Comparison of models at 50 Hz



(a) IAF model



(b) Izhikivech model

Figure 18: Comparison of models at 100 Hz

2.3.1 Two-Neuron Oscillator

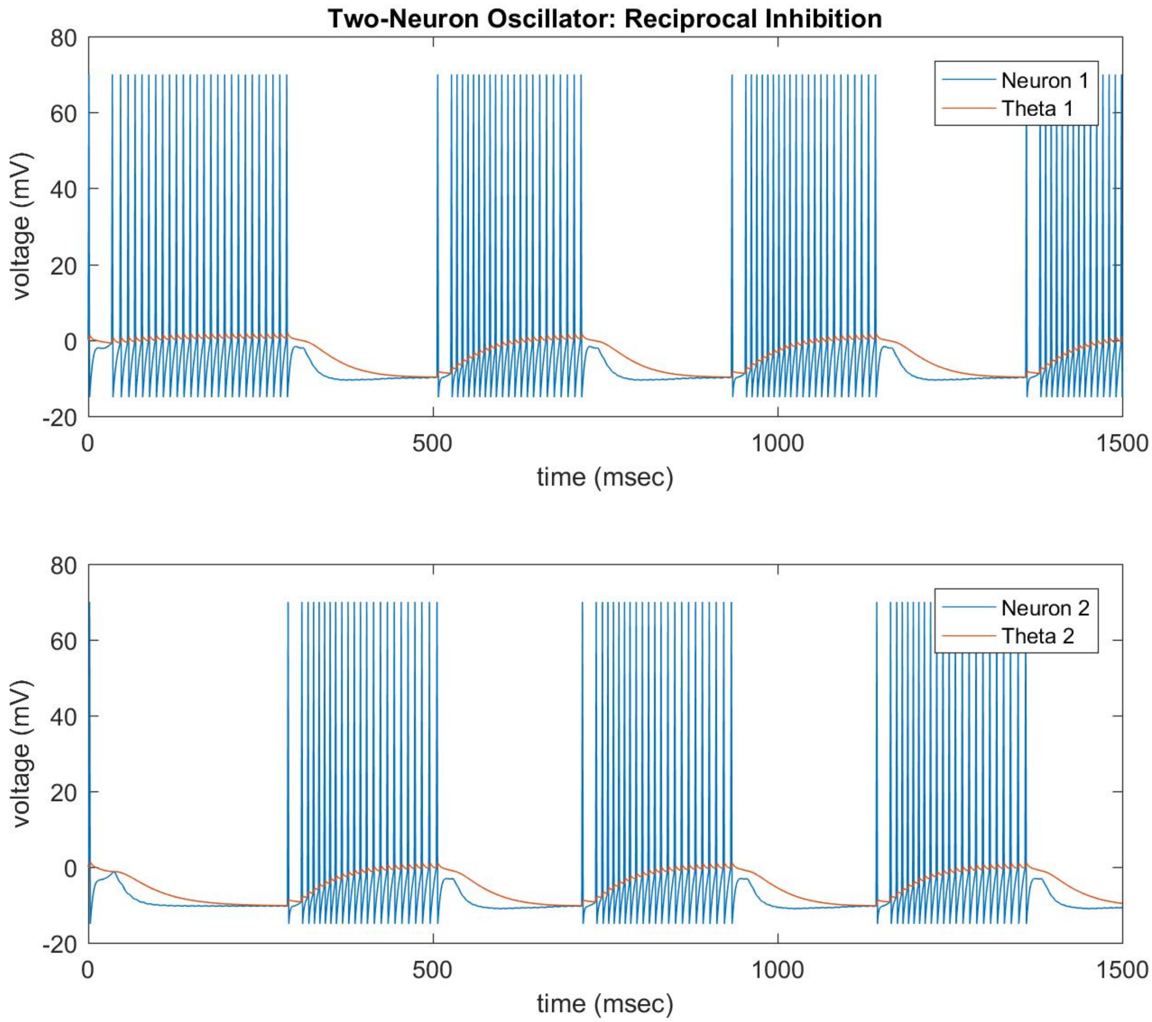


Figure 19: This figure demonstrates the oscillatory and adaptive behavior of a two neuron system by which the firing of one inhibits the other as can be seen in the figure.

2.4 Discussions

This was an exploration into how frequency can change the action potentials produced. In both models they produced the same effect by creating more action potentials as the frequency increased. The two-neuron oscillator problem was a focus of understanding the inhibitory and excitatory relationship between neurons that affect firing.

3 Problem 3: Linear-Non-Linear Filter

3.1 Background

Constructing a Linear-Non-Linear Filter that follows the method used in "Neural encoding of rapidly fluctuating odors"[**?**]. Provided a system with a stimulus over the course of five trials to lead varying responses of 20 second durations.

3.2 Method

To simulate the linear-non-linear model in **??**, the method applied relied on least-square estimate for the $filter, W$, on a $stimulus, S$, to output a $firingrateresponse, R$, that was mathematically represented as $SW = R$. Hence, the filter was optimized to minimize error $(R - SW)^2$. A moving window of 100ms for 2s at a time was used to group the Stimulus,S, into 100 ms time bins during the previous 2 seconds while R, is the response also grouped into 100 ms time bins.

Originally, the responses were simply just indicated by whether firing occurred or not. In order to translate to response rates, the number of spikes were counted over the course of 100 ms for each trial. Furthermore, the trials 1-4 response rates were averaged to lead to the average response rate as seen in Fig. 3.

Then, used the Matlab function **pinv** to compute the filter for the averaged response and trial 5 as shown in Fig. 4. The appropriate filters were then applied to determine if it were possible to recreate trial 5 with the linear-non-linear filter.

3.3 Results

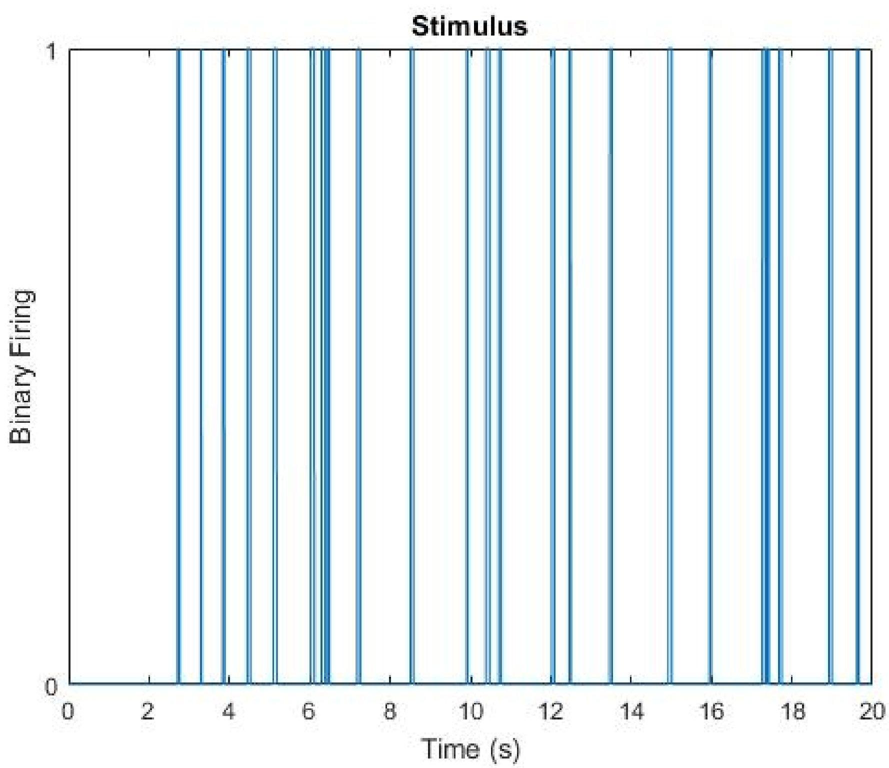


Figure 20: The stimulus presented over the course of 20 seconds for all 5 trials.

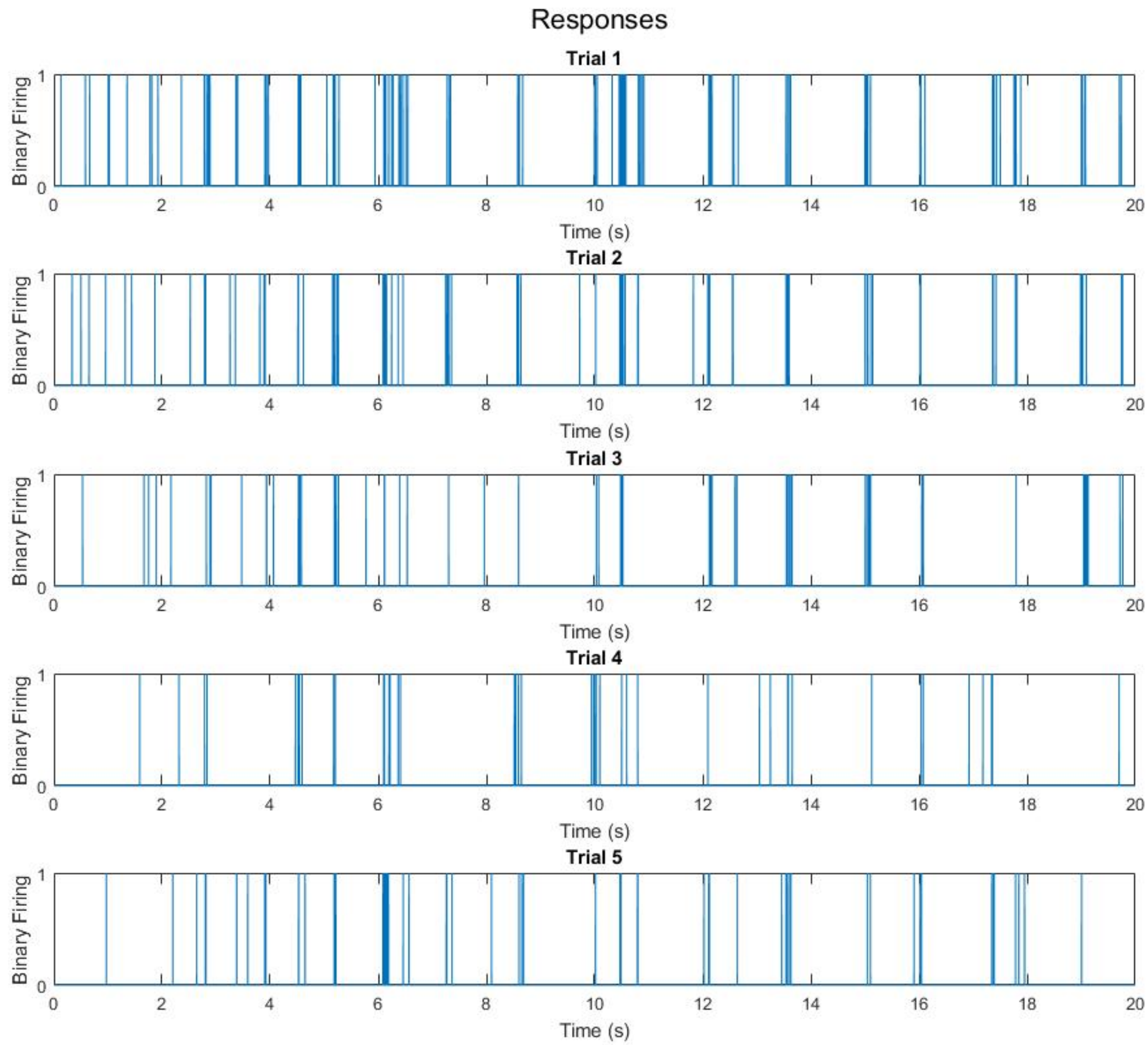
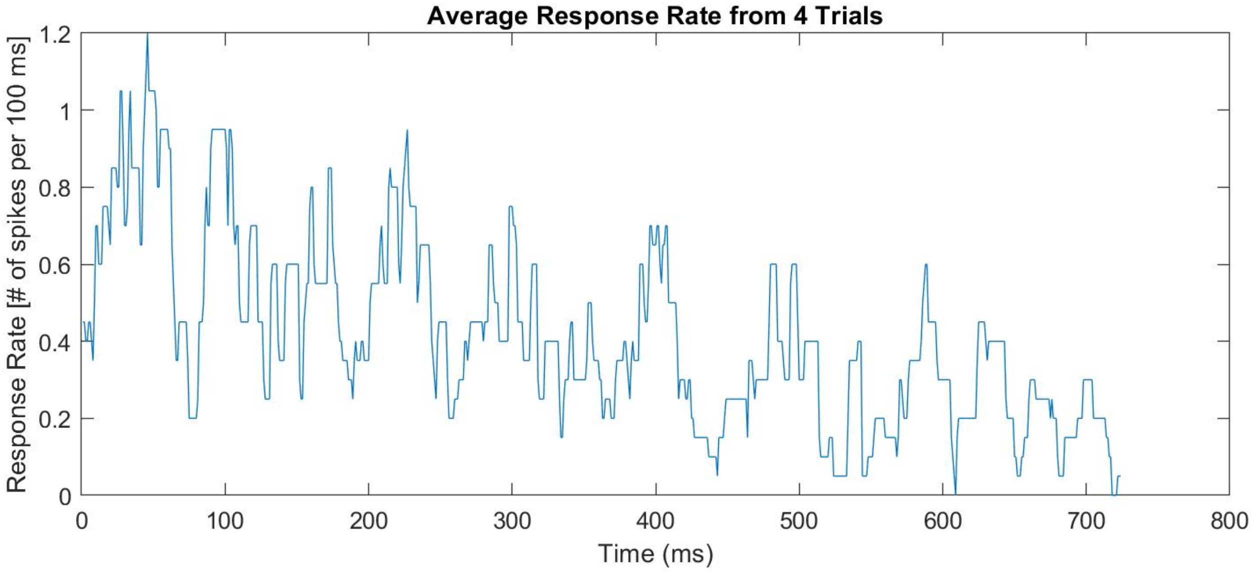


Figure 21: The corresponding binary firing response for each trial.



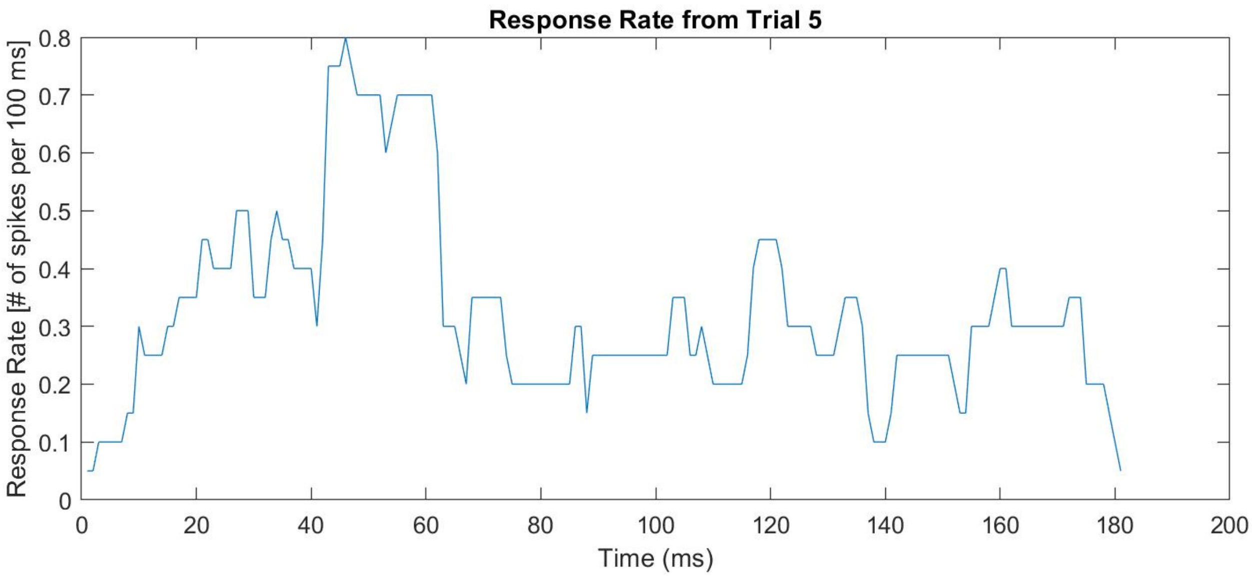


Figure 22: Averaged response rates from trials 1-4 & response rates for trial 5.

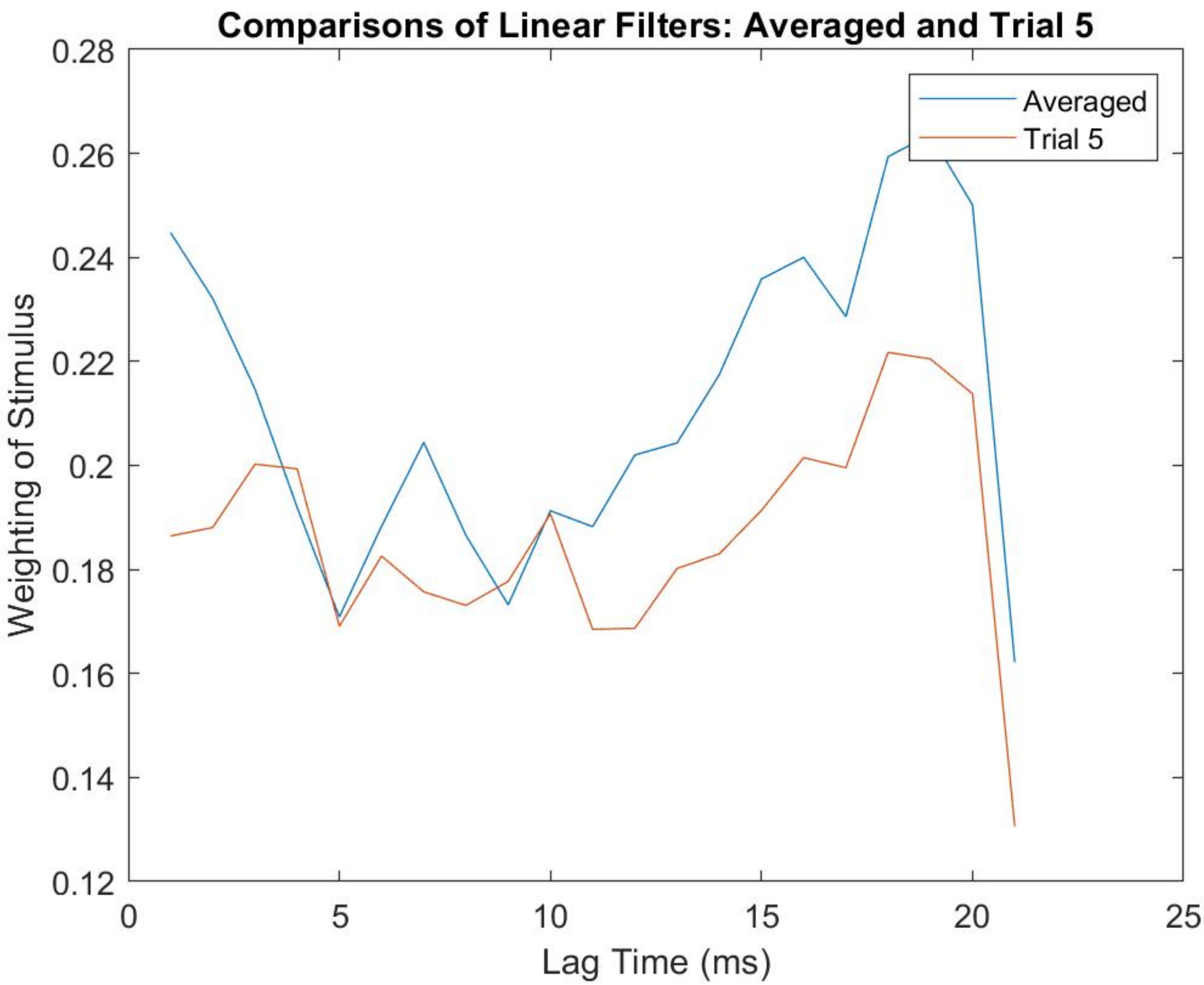


Figure 23: Averaged linear filters from trials 1-4 & linear filter for only trial 5.

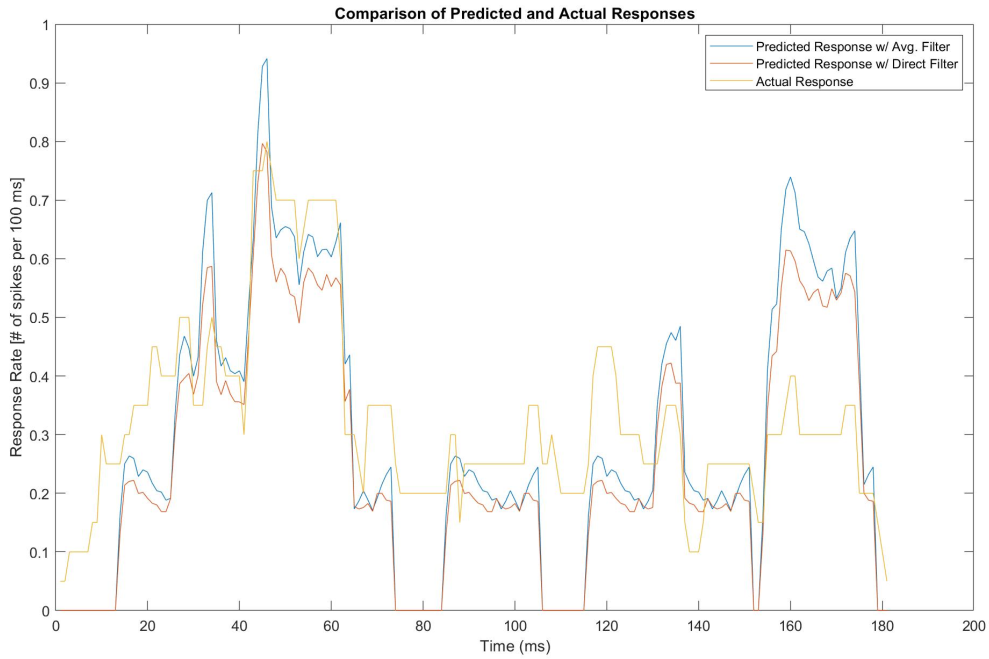


Figure 24: Application of filters on stimulus to reproduced the 5th trial responses.

3.4 Discussions

The predicted response that was constructed from using the linear-non-linear filter was able to produce a response that was similar to that of the actual response as seen in Fig. 5 However, the sampling rate could be downsampled to lead to a much smoother figure.

4 Problem 4: Poisson Spike Train

4.1 Background

Spike trains can be predicted by using known response rates. This problem served as a continuation of the last problem to demonstrate this process.

4.2 Method

The spiking information from trial 5 of the previous problem were used to build a spike spike train that corresponded to the a randomly generated vector of values between 0 and 1 that had would only lead to a spike if it the generated value was below the normalized response rate ($\frac{\lambda(t)}{\lambda_{max}}$).

Additionally, information from the spike train such as the interspike interval (ISI), the time between a spike and the succeeding spike, was determined to demonstrate the properties of the randomly generated spike train: Fano factor,the coefficient of variation, and the ISI distribution.These properties are indicative of essentially the spread of the ISIs produced from the spike train.

$$\text{Coefficient of Variation} = \frac{\sigma}{\mu} \tag{1}$$

$$\text{Fano factor} = \frac{\sigma^2}{\mu} \tag{2}$$

http://www.math.hcmus.edu.vn/ ntlanh/IntroML/Slides/3.

4.3 Results

Coefficient of variation = 1.4645
Fano factor = 6.5304

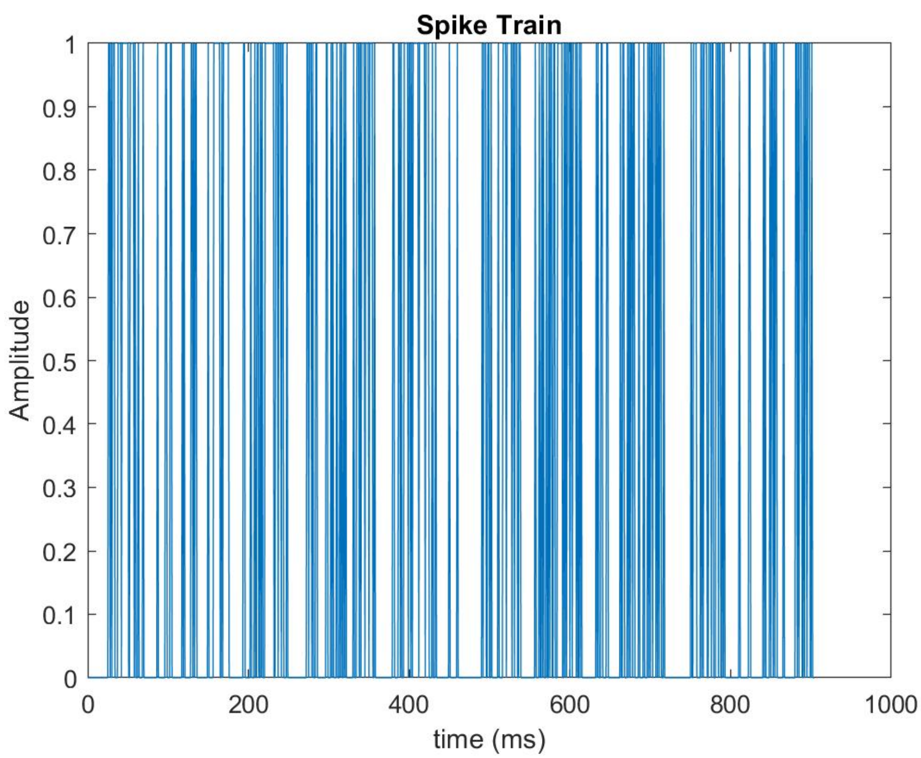


Figure 25: Generated spike train from previous problem.

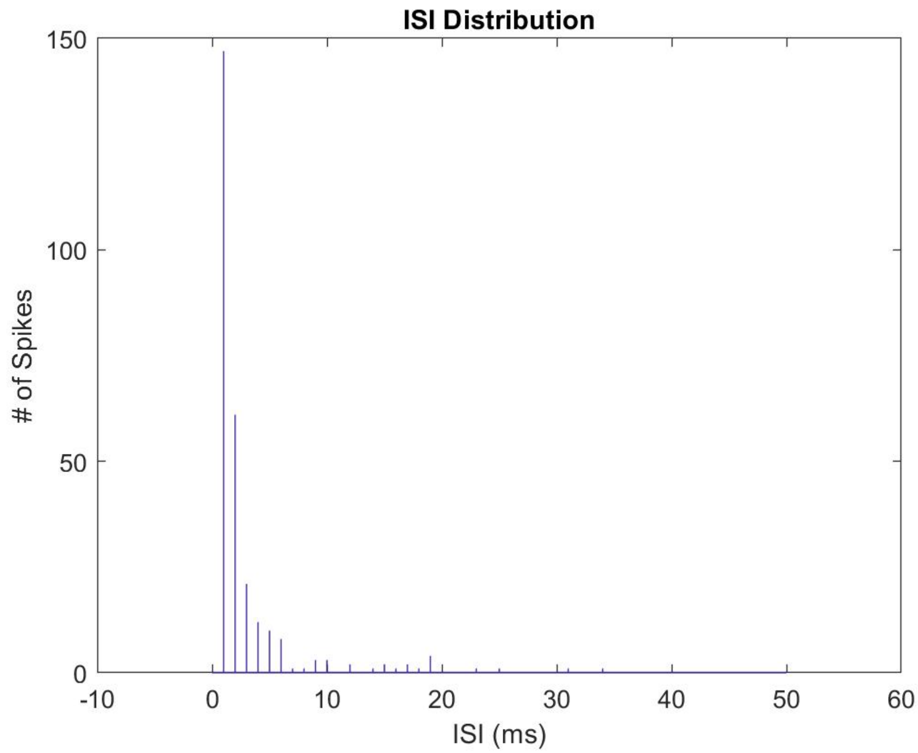


Figure 26: The ISI distribution for the generated spike train.

The results were reasonable as the generated spike train fired more than not, so the duration of ISI would be inversely proportional the number of times the spike train experienced that period time before spiking again. In fact, the histogram produced of the ISI distributions is reminiscent of an exponential decay. The fano factor and the coefficient of variation also supported the idea that spike train spikes irregular, but with a consistency to spike more often than not, so shorter ISI durations in a specified time window.