1 the whole problem

$$
\begin{aligned}
\min_{F,\Omega,\theta,\lambda^+,\lambda^-} \quad & F_{n+1} + \Omega && (1)\\
s.t. \quad & \Omega \geq \sum_{(v,w)=a\in V^2} (F_v - F_w + t_w)x_a^k + \Gamma\theta^k + \sum_{v\in V}(\lambda_v^{k,+} + \lambda_v^{k,-}) \quad \forall 1 \leq k \leq K,\\
& \theta^k + \lambda_v^{k,+} \geq \sum_{w\in V,(w,v)\in A} u_v \cdot x_{(w,v)}^k \quad \forall v \in V,\\
& \theta^k + \lambda_v^{k,-} \geq -\sum_{w\in V,(w,v)\in A} u_v \cdot x_{(w,v)}^k \quad \forall v \in V,\\
& z_{ij} = 1 \quad \forall (i,j) \in E,\\
& z_{ij} + z_{ji} \leq 1 \quad \forall i,j \in V, i < j,\\
& z_{ij} \geq z_{ip} + z_{pj} - 1 \quad \forall i,j,p \in V, i \neq j \neq p,\\
& f_{ij}^k \leq min(\widetilde{r}_i^c, \widetilde{r}_j^c) z_{ij} \quad \forall i,j \in V, i \neq j, i \neq n+1, j \neq 0, \forall k \in \{1,2,...,K\}, \forall c \in C,\\
& \sum_{j\in V\setminus\{i,0\}} f_{ij}^k = \widetilde{r}_i^c \quad \forall i \in V \setminus \{n+1\}, \forall k \in \{1,2,...,K\}, \forall c \in C,\\
& \sum_{i\in V\setminus\{j,n+1\}} f_{ij}^k = \widetilde{r}_j^c \quad \forall j \in V \setminus \{0\}, \forall k \in \{1,2,...,K\}, \forall c \in C,\\
& f_{ij}^k \geq 0 \quad \forall i,j \in V, i \neq j, i \neq n+1, j \neq 0, \forall k \in \{1,2,...,K\},\\
& F_i \geq 0 \quad \forall i \in V\\
& x_a \leq M z_a \quad \forall a \in E,\\
& z_{ij} \in \{0,1\} \quad \forall i,j \in V, i \neq j,\\
& \widetilde{r}_i^c = R^c, falls \;\; i = 0, n+1, \forall k \in \{1,2,...,K\}, \forall c \in C,\\
& \widetilde{r}_i^c = r_i^c, otherwise, \forall k \in \{1,2,...,K\}, \forall c \in C.
\end{aligned}
$$

2 subproblem

$\varepsilon^t := \{(i,j) \in V^2 | z^*_{ij} > 0\}$, Z^t is accordingly set.

$$
\begin{aligned}
M^{*t} = \max_{x,\xi^+,\xi^-} & \sum_{a\in A} (F_v - F_w + t_w)x_a + q_a^+ u_w - q_a^- u_w \qquad (2)\\
s.t. \quad & \sum_{(\cdot,v)\in\varepsilon^t} x_a - \sum_{(v,\cdot)\in\varepsilon^t} x_a \le d_v^+ \quad \forall v \in V \setminus \{v_0\},\\
& -\sum_{(\cdot,v)\in\varepsilon^t} x_a + \sum_{(v,\cdot)\in\varepsilon^t} x_a \le d_v^+ \quad \forall v \in V \setminus \{v_0\},\\
& \sum_{v\in V} (\xi_v^+ + \xi_v^-) \le \Gamma,\\
& q_a^+, q_a^- \le x_a \quad \forall a \in \varepsilon^t,\\
& x_a \le M z_a \quad \forall a \in \varepsilon^t,\\
& q_{(v,w)}^+ \le M \cdot \xi_w^+ \quad \forall (v,w) \in \varepsilon^t,\\
& q_{(v,w)}^- \le M \cdot \xi_w^- \quad \forall (v,w) \in \varepsilon^t,\\
& q_{(v,w)}^+ \ge M \cdot \xi_w^+ + x_{(v,w)} - M \quad \forall (v,w) \in \varepsilon^t,\\
& q_{(v,w)}^- \ge M \cdot \xi_w^- + x_{(v,w)} - M \quad \forall (v,w) \in \varepsilon^t,\\
& x_a, q_a^+, q_a^- \ge 0 \quad \forall a \in \varepsilon^t,\\
& \xi_v^+, \xi_v^- \in \{0,1\} \quad \forall v \in V
\end{aligned}
$$

3 cut and algorithm

Cut:

$$\Omega \ge (M^{*t} - \lambda^t(1))(\sum_{(i,j)\in Z^t} z_{ij} - |Z^t|) + M^{*t}$$

$N^t(1) = \{z \in \{0,1\}^{|Z^t|} | |Z^t| - \sum_{(i,j)\in Z^t} z_{ij} = 1\}$, $\lambda^t(1)$ is the lower bound of $\min_{N^t(1)} M^{*t}$.

Starting with a (possibly empty) subset $\hat{X} \subset \widetilde{X}$ in each iteration, the LP (1) with only constraints corresponding to the nodes from $\hat{X}$ is solved optimally. For the optimal solution F^*, Ω^*, a new vertex x^* is computed by solving (2). If the objective is greater than Ω^*, a violated constraints is found and we add x^* to $\hat{X}$. Else, the solution F^*, Ω^* is optimal.

Algorithm 1 Exact Alogorithm

Input: Graph $G = (V, A)$, uncertainty parameter $\Gamma \in \mathbb{N}_{<|V|}$, execution times $(t_v)_{v \in V}$, uncertainties $(u_v)_{v \in V}$, costs per time unit for finishing earlier than scheduled $(d_v^-)_{v \in V}$, costs per time unit for finishing later than scheduled $(d_v^+)_{v \in V}$, flowvalue $(f_{ij}^k)_{(i,j) \in V^2}$, resource requirment $(\widetilde{r}_i^c)_{i \in V}$, initial vertex set $\hat{X} \subset \widetilde{X}$, sufficiently large constant M, $t = 0$

Output: the finish time

while true **do**

$schedule_obj$, F^*, Ω^* optimal solution of (1) with constraints only for $\hat{X}$

$subproblem_obj$, $x^* =$ optimal solution of (2)

if $(M^{*t} - \lambda^t(1))(\sum_{(i,j) \in Z^t} z_{ij} - |Z^t|) + M^{*t} > \Omega$ **then**

$\hat{X} = \hat{X} \cup \{x\}$, $t = t + 1$

else

break;

end if

end while

return $schedule_obj$, $(F_v)_{v \in V}$