## the whole problem:

(1)

$$\min_{F,\Omega,\theta,\lambda^+,\lambda^-} \quad F_{n+1} + \Omega$$

s.t.

$$\Omega \geq \sum_{\substack{a\in V^2 \\ a=(v,w)}} (F_v - F_w + t_w)\, x_a^k + \Gamma \theta^k + \sum_{v\in V} (\lambda_v^{k,+} + \lambda_v^{k,-}) \qquad \forall 1 \leq k \leq K$$

$$\theta^k + \lambda_v^{k,+} \geq \sum_{\substack{w\in V \\ (w,v)\in A}} u_v \cdot x^k_{(w,v)} \qquad \forall v\in V$$

$$\theta^k + \lambda_v^{k,+} \geq -\sum_{\substack{w\in V \\ (w,v)\in A}} u_v \cdot x^k_{(w,v)} \qquad \forall v\in V$$

$$z_{ij} = 1 \qquad \forall (i,j)\in E$$

$$z_{ij} + z_{ji} \leq 1 \qquad \forall i,j\in V,\ i<j$$

$$z_{ij} \geq z_{ip} + z_{pj} - 1 \qquad \forall i,j,p\in V,\ i\neq j\neq p$$

$$f_{ij}^k \leq \min(\tilde{r}_i^c, \tilde{r}_j^c)\, z_{ij} \qquad \forall i,j\in V,\ i\neq j,\ i\neq n+1,\ j\neq 0, \quad \forall k\in \{1,2,\dots$$

$$\sum_{j\in V\setminus\{i,0\}} f_{ij}^k = \tilde{r}_i^c \qquad \forall i\in V\setminus\{n+1\},\ \forall k\in K'$$

$$\sum_{i\in V\setminus\{j,n+1\}} f_{ij}^k = \tilde{r}_j^c \qquad \forall j\in V\setminus\{0\},\ \forall k\in K'$$

$$f_{ij}^k \geq 0 \qquad \forall i,j\in V,\ i\neq j,\ i\neq n+1,\ j\neq 0,\ \forall k\in K'$$

$$F_i \geq 0 \qquad \forall i\in V$$

$$x_a \leq M z_a \qquad \forall a\in E$$

$$z_{ij}\in\{0,1\} \qquad \forall i,j\in V,\ i\neq j$$

$$\tilde{r}_i^c = \begin{cases} R^c, & \text{falls } i=0,n+1, \\ r_i^c, & \text{otherwise} \end{cases} \qquad \forall k\in K'$$

(2)

## the subproblem:

$$M^{*t} = \max_{x,\xi^+,\xi^-} \sum_{a\in A} (F_v - F_w + t_w)\, x_a + q_a^+ u_w - q_a^+ u_w$$

s.t.

$$\sum_{(\cdot,v)\in\mathcal{E}^t} x_a - \sum_{(v,\cdot)\in\mathcal{E}^t} x_a \leq d_v^+ \qquad \forall v\in V\setminus\{v_0\}$$

$$-\sum_{(\cdot,v)\in\mathcal{E}^t} x_a + \sum_{(v,\cdot)\in\mathcal{E}^t} x_a \leq d_v^- \qquad \forall v\in V\setminus\{v_0\}$$

$$\sum_{v\in V} (\xi_v^+ + \xi_v^-) \leq \Gamma$$

$$q_a^+, q_a^- \leq x_a \qquad \forall a\in\mathcal{E}^t$$

$$x_a \leq M\cdot z_a \qquad \forall a\in\mathcal{E}^t$$

$$q^+_{(v,w)} \leq M\cdot\xi_w^+ \qquad \forall (v,w)\in\mathcal{E}^t$$

$$q^-_{(v,w)} \leq M\cdot\xi_w^-$$

$$q^+_{(v,w)} \geq M\cdot\xi_w^+ + x_{(v,w)} - M$$

$$q^-_{(v,w)} \geq M\cdot\xi_w^- + x_{(v,w)} - M$$

$$x_a, q_a^+, q_a^- \geq 0 \qquad \forall a\in\mathcal{E}^t$$

$$\xi_v^+, \xi_v^- \in\{0,1\} \qquad \forall v\in V$$

$\mathcal{E}^t = \{(i,j)\in V^2 \mid z_{ij}^{*t} > 0\}$
$Z^t$ is accordingly set

Cut: $\Omega \geq (M^{*t} - \lambda^t(1))\left(\sum_{(i,j)\in Z^t} z_{ij} - |Z^t|\right) + M^{*t}$ (a)

P.S. $N^t(1) = \{z \in \{0,1\}^{|Z^t|} \mid |Z^t| - \sum_{(i,j)\in Z^t} z_{ij} = 1\}$

$\lambda^t(1)$ : lower Bound of $\min_{N^t(1)} M^{*t}$

Starting with a (possibly empty) subset $\hat{X} \subseteq \widetilde{X}$ in each iteration, the LP (2) with only constraints corresponding to the nodes from $\hat{X}$ is solved optimally. For the optimal solution $F^*, \Omega^*$, a new vertex $x^*$ is computed by solving (SP2) If the objective is greater than $\Omega^*$, a violated constraint is found and we add $x^*$ to $\hat{X}$. Else, the solution $F^*, \Omega^*$ is optimal.

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**Algorithm 1** Exact algorithm for the Γ-robust two-stage project scheduling problem

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**Input:** Graph $G = (V, A)$, uncertainty parameter $\Gamma \in \mathbb{N}_{<|V|}$, execution times $(t_v)_{v\in V}$, uncertainties $(u_v)_{v\in V}$, initial vertex set $\hat{X} \subset \widetilde{X}$, sufficiently large constant $M$ $t=0$

**Output:** the finish time

```
while true do
    schedule_obj, F*, Ω* optimal solution of (1) with constraints only for X̂
    subproblem_obj, x* = optimal solution of (2)
    if ~~subproblem_obj~~ (a) > Ω then
        X̂ = X̂ ∪ {x} ,   t = t+1
    else
        break;
    end if
end while
return schedule_obj, (F_v)_{v∈V}
```

待解决问题（参照文献1）

1. 找到这个模型对应的 McCormick Heuristic 模型及算法
2. 模仿文献1 对算法进行评估

    2.1 分析本文件中算法1在不同大小下的收敛性

    2.2比较M=0.25, M=0.5, M=1时的 McCormick Heuristic 与 adaptive McCormick heuristic iterative recovery heuristic