Masterproblem

$$
\begin{aligned}
\min_{F,\Omega,\theta,\lambda^+,\lambda^-} \quad & \sum_{v\in V} c_v F_v + \Omega && (9)\\
\text{s.t.} \quad & \Omega \ge \sum_{\substack{a\in A\\ a=(v,w)}} (F_v - F_w + t_w) x_a^k + \Gamma\theta^k \\
& \quad + \sum_{v\in V} (\lambda_v^{k,+} + \lambda_v^{k,-}) && \forall 1 \le k \le K\\
& \theta^k + \lambda_v^{k,+} \ge \sum_{w\in V\,:\,(w,v)\in A} u_v \cdot x^k_{(w,v)} && \forall v\in V,\ 1\le k\le K\\
& \theta^k + \lambda_v^{k,-} \ge - \sum_{w\in V\,:\,(w,v)\in A} u_v \cdot x^k_{(w,v)} && \forall v\in V,\ 1\le k\le K\\
& F, \theta^k, \lambda^{k,+}, \lambda^{k,-} \ge 0 && \forall 1\le k\le K
\end{aligned}
$$

Subproblem

$$
\begin{aligned}
\max_{x,\xi^+,\xi^-} \quad & \sum_{a=(v,w)\in A} (F_v - F_w + t_w) x_a + q_a^+ \cdot u_w - q_a^- \cdot u_w \\
\text{s.t.} \quad & \sum_{a=(\cdot,v)\in A} x_a - \sum_{a=(v,\cdot)\in A} x_a \le d_v^+ && \forall v\in V\backslash\{v_0\}\\
& - \sum_{a=(\cdot,v)\in A} x_a + \sum_{a=(v,\cdot)\in A} x_a \le d_v^- && \forall v\in V\backslash\{v_0\}\\
& x_a \ge 0 && \forall a\in A\\
& \sum_{v\in V} \xi_v^+ + \xi_v^- \le \Gamma\\
& q_a^+, q_a^- \le x_a && \forall a\in A\\
& q^+_{(v,w)} \le M\cdot \xi_w^+ && \forall (v,w)\in A\\
& q^-_{(v,w)} \le M\cdot \xi_w^- && \forall (v,w)\in A\\
& q^+_{(v,w)} \ge M\cdot \xi_w^+ + x_{(v,w)} - M && \forall (v,w)\in A\\
& q^-_{(v,w)} \ge M\cdot \xi_w^- + x_{(v,w)} - M && \forall (v,w)\in A\\
& x_a, q_a^+, q_a^- \ge 0 && \forall a\in A\\
& \xi_v^+, \xi_v^- \in \{0,1\} && \forall v\in V \quad (10)
\end{aligned}
$$

Algorithm 1 Exact algorithm for the Γ-robust two-stage project scheduling problem

Input: Graph $G=(V,A)$, uncertainty parameter $\Gamma\in\mathbb{N}_{\le|V|}$, execution times $(t_v)_{v\in V}$, uncertainties $(u_v)_{v\in V}$, costs per time unit $(c_v)_{v\in V}$, costs per time unit for finishing earlier than scheduled $(d_v^-)_{v\in V}$, costs per time unit for finishing later than scheduled $(d_v^+)_{v\in V}$, initial vertex set $\hat{X}\subset\tilde{X}$, sufficiently large constant M

Output: the costs for the Γ-robust two-stage problem and the finish times for all tasks $(F_v)_{v\in V}$

```
while true do
    schedule_obj, F*, Ω* optimal solution of (9) with constraints only for X̂
    subproblem_obj, x* = optimal solution of (10)
    if subproblem_obj > Ω then
        X̃ = X̃ ∪ {x}
    else
        break;
    end if
end while
return schedule_obj, (F_v)_{v∈V}
```

McCormick Heuristic

$$
\begin{aligned}
\min_{\substack{F,\mu^\pm,\theta,\alpha^\pm\\ \beta^\pm,\lambda^\pm,\phi^\pm}} \quad & \sum_{v\in V\backslash\{v_0\}} \left(d_v^-\mu_v^- + d_v^+\mu_v^+\right) + \sum_{v\in V}\left(c_v\cdot F_v + \phi_v^+ + \phi_v^-\right)\\
& + \Gamma\theta + M\cdot \sum_{a\in A}\left(\lambda_a^+ + \lambda_a^-\right)\\
\text{s.t.} \quad & \mu_w^+ - \mu_w^- - \alpha_a^+ - \alpha_a^- + \lambda_a^+ + \lambda_a^- \ge -F_w + t_w && \forall a=(v_0,w)\in A\\
& \mu_w^+ - \mu_v^+ - \mu_w^- + \mu_v^- - \alpha_a^+ - \alpha_a^- + \lambda_a^+ + \lambda_a^- && \forall a=(v,w)\in A\\
& \quad \ge F_v - F_w + t_w && v\ne v_0\\
& \theta - M\cdot \sum_{a=(\cdot,v)\in A} (\beta_a^+ - \lambda_a^+) + \phi_v^+ \ge 0 && \forall v\in V\\
& \theta - M\cdot \sum_{a=(\cdot,v)\in A} (\beta_a^- - \lambda_a^-) + \phi_v^- \ge 0 && \forall v\in V\\
& \alpha_a^+ + \beta_a^+ - \lambda_a^+ \ge u_w && \forall a=(v,w)\in A\\
& \alpha_a^- + \beta_a^- - \lambda_a^- \ge -u_w && \forall a=(v,w)\in A\\
& F,\mu^+,\mu^-,\theta,\alpha^+,\alpha^-,\beta^+,\beta^-,\lambda^+,\lambda^-,\phi^+,\phi^- \ge 0 && (11)
\end{aligned}
$$

Algorithm 2 Adaptive McCormick heuristic for the Γ-robust two-stage project scheduling problem

Input: Graph $G=(V,A)$, uncertainty parameter $\Gamma\in\mathbb{N}_{\le|V|}$, execution times $(t_v)_{v\in V}$, uncertainties $(u_v)_{v\in V}$, costs per time unit $(c_v)_{v\in V}$, costs per time unit for finishing earlier than scheduled $(d_v^-)_{v\in V}$, costs per time unit for finishing later than scheduled $(d_v^+)_{v\in V}$

Output: the costs for the Γ-robust two-stage problem and the finish times for all tasks $(F_v)_{v\in V}$

```
step = -1/2
M_opt = M
cost_opt, F_opt optimal solution of McCormick heuristic with M = M_opt
while step ≥ 0.01 and number iterations ≤ 30 do
    M_temp = M_opt · (1 + step)
    cost_temp, F_temp optimal solution of McCormick heuristic with M = M_temp
    if cost_temp < cost_opt then
        M_opt, cost_opt, F_opt = M_temp, cost_temp, F_temp
    else
        step = -1/2 · step
    end if
end while
return cost_opt, F_opt
```

Iterative Recovery Heuristic

Algorithm 3 Iterative Recovery Heuristic

Input: Graph $G = (V, A)$, uncertainty parameter $\Gamma \in \mathbb{N}_{\leq |V|}$, execution times $\left(t_v\right)_{v \in V}$, uncertainties $\left(u_v\right)_{v \in V}$, costs per time unit $\left(c_v\right)_{v \in V}$, costs per time unit for finishing earlier than scheduled $\left(d_v^-\right)_{v \in V}$, costs per time unit for finishing later than scheduled $\left(d_v^+\right)_{v \in V}$

Output: the costs for the Γ-robust two-stage problem and the finish times for all tasks $(F_v)_{v \in V}$

F solution of the deterministic project scheduling problem G, t

ξ, y optimal solution of the second stage problem for schedule F with cost obj

while number iterations ≤ 30 **do**

  $\tilde{F}$ with $\tilde{F}_v = F_v + 1$ if $y_v > 0$ and $\tilde{F}_v = F_v - 1$ if $y_v < 0$

  $\tilde{\xi}, \tilde{y}$ optimal solution of the second stage problem for schedule $\tilde{F}$ with cost $\tilde{\text{obj}}$

  if $\tilde{\text{obj}}/\text{obj} > 1 - 10^{-5}$ **then**

    $F, \xi, y, \text{obj} = \tilde{F}, \tilde{\xi}, \tilde{y}, \tilde{\text{obj}}$

  else

    break

  end if

end while

return obj, F