2.3. minimax criterion

zero-one loss function $\lambda_{11}=\lambda_{22}=0$ $\lambda_{12}=\lambda_{21}=1$

(a)

prove $\int_{R_2} p(x|w_1)dx = \int_{R_1} p(x|w_2)dx$

Answer: priors: $P(w_1)$, $P(w_2)=1-P(w_1)$

$$R(P(w_1)) = P(w_1)\int_{R_2} p(x|w_1)dx + (1-P(w_2))\int_{R_1} p(x|w_2)dx$$

Obtain the prior with minimum risk:

$$\frac{d}{dP(w_1)} R(P(w_1)) = \int_{R_2} p(x|w_1)dx - \int_{R_1} p(x|w_2)dx = 0$$

$$\Rightarrow \int_{R_2} p(x|w_1)dx = \int_{R_1} p(x|w_2)dx$$

(b) The solution is not unique

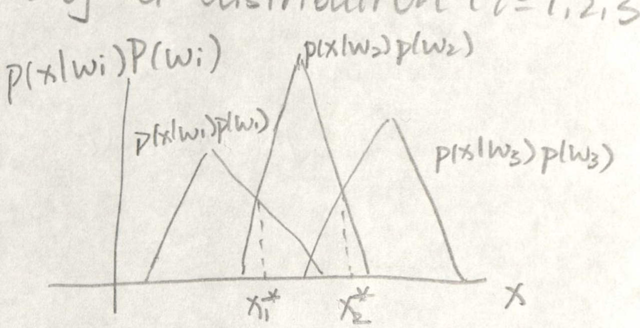
i.e. Quadratic in x_0

2.5. Generalize the minimax decision rule

$$p(x|w_i) = T(\mu_i, \delta_i) \equiv \begin{cases} (\delta_i - |x-\mu_i|)/\delta_i^2 & \text{for } |x-\mu_i|<\delta_i \\ 0 & \text{otherwise} \end{cases}$$

where $\delta_i>0$ is the half-width of a distribution ($i=1,2,3$)

Assume that $\mu_1<\mu_2<\mu_3$



(a) $p(w_1)p(x|w_1) = p(w_2)p(x|w_2)$

$p(w_2)p(x|w_2) = p(w_3)p(x|w_3)$

$$\Rightarrow \begin{cases} x_1^* = \\ x_2^* = \end{cases} \quad \text{O} \vee \text{O}$$

(b) $R = \int_{R_2} p(w_1)p(x|w_1)dx + \int_{R_1} p(w_2)p(x|w_2)dx$

$+ \int_{R_3} p(w_2)p(x|w_2)dx + \int_{R_4} p(w_3)p(x|w_2)dx$

$= P(w_1)\frac{1}{2\delta_1^2}(\mu_1+\delta_1-x_1^*)^2 + P(w_2)\frac{1}{2\delta_2^2}[(\delta_2-\mu_2+x_1^*)^2 +$

$(\mu_2+\delta_2-x_2^*)^2] + [1-P(w_1)-P(w_2)]\frac{1}{2\delta_3^2}(\delta_3-\mu_3+x_2^*)^2$

$$\frac{\partial E}{\partial P(w_1)} = \frac{\partial E}{\partial P(w_2)} = 0 \Rightarrow x_i^* = \frac{b_i+\sqrt{c_i}}{a_i} \quad i=1,2$$

(c) (d) ↑

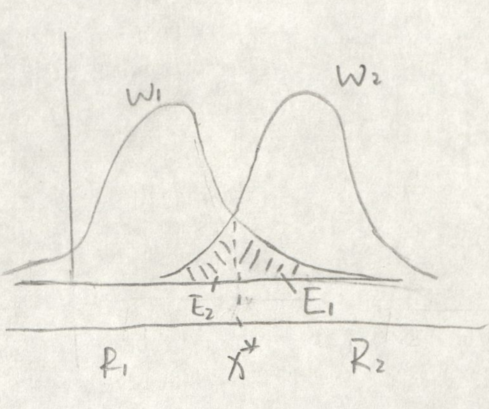
2.7. $$p(x|w_i) = \frac{1}{\pi b}\cdot\frac{1}{1+(\frac{x-a_i}{b})^2} \quad i=1,2$$

Assume zero-one error loss, $a_2 > a_1$, same "width" b, and equal priors.

(a) maximum acceptable error rate for classifying a pattern that is actually in w_1 as if it were in w_2 is E_1.

$$E_1 = \int_{R_2} p(x|w_1)p(w_1)\,dx$$

$$= \frac{1}{2}\int_{x^*}^{+\infty}\frac{1}{\pi b}\frac{1}{1+(\frac{x-a_1}{b})^2}dx$$



(b) $E_2 = \int_{R_1} p(x|w_2)p(w_2)\,dx$

(c) $E = E_1 + E_2$

(d) ↑

(e) For the Bayes case, the decision point is midway between the peaks of the two distribution