Zhiqian Zhou

Question 1

a)

Using the library function in matlab.

mean

| | |
|---|---|
| mean1 | [9.9317,5.0477] |
| mean2 | [6.8948,-3.0668] |
| mean3 | [-1.9662,-2.0525] |
| mean4 | [-1.8583,2.9649] |

covariance

| | |
|---|---|
| cov1 | [2.9288,1.3406;1.3406,4.1460] |
| cov2 | [1.9065,1.2952;1.2952,2.0355] |
| cov3 | [6.6951,1.0105;1.0105,1.1735] |
| cov4 | [1.9036,-0.5367;-0.5367,0.6648] |

Calculating covariance by hand.

Using the formula Cov(X,Y) = E((X-μ)(Y-v)) directly.

| | |
|---|---|
| c1 | [2.9288,1.3406;1.3406,4.1460] |
| c2 | [1.9065,1.2952;1.2952,2.0355] |
| c3 | [6.6951,1.0105;1.0105,1.1735] |
| c4 | [1.9036,-0.5367;-0.5367,0.6648] |

Using the formula E((X-μ)(Y-v)) = E(XY)-μv to simplify the process.

| | |
|---|---|
| co1 | [2.9215,1.3373;1.3373,4.1357] |
| co2 | [1.9017,1.2919;1.2919,2.0304] |
| co3 | [6.6840,1.0088;1.0088,1.1716] |
| co4 | [1.8989,-0.5354;-0.5354,0.6632] |

The result is different.

In the first case, the formula uses unbiased estimator. Its denominator is n-1 instead of n. In the second case, since it's calculate the mean to gain the covariance, its denominator is n.

I believe that if I implement the process in another language, the result can be different as different precision might be reserved in different language.

b)

using function [value,vector] = eigen(class) at line 116 in cov.m

eigenvalue

| | |
|---|---|
| eigenValue1 | [5.0097;2.0651] |
| eigenValue2 | [3.2678;0.6742] |
| eigenValue3 | [6.8743;0.9944] |
| eigenValue4 | [2.1038;0.4646] |

eigenvector

| | |
|---|---|
| vector1 | [0.5416,0.8407;-0.8407,0.5416] |
| vector2 | [0.6893,0.7245;-0.7245,0.6893] |
| vector3 | [0.9847,0.1745;-0.1745,0.9847] |
| vector4 | [-0.9369,0.3495;0.3495,0.9369] |

c)

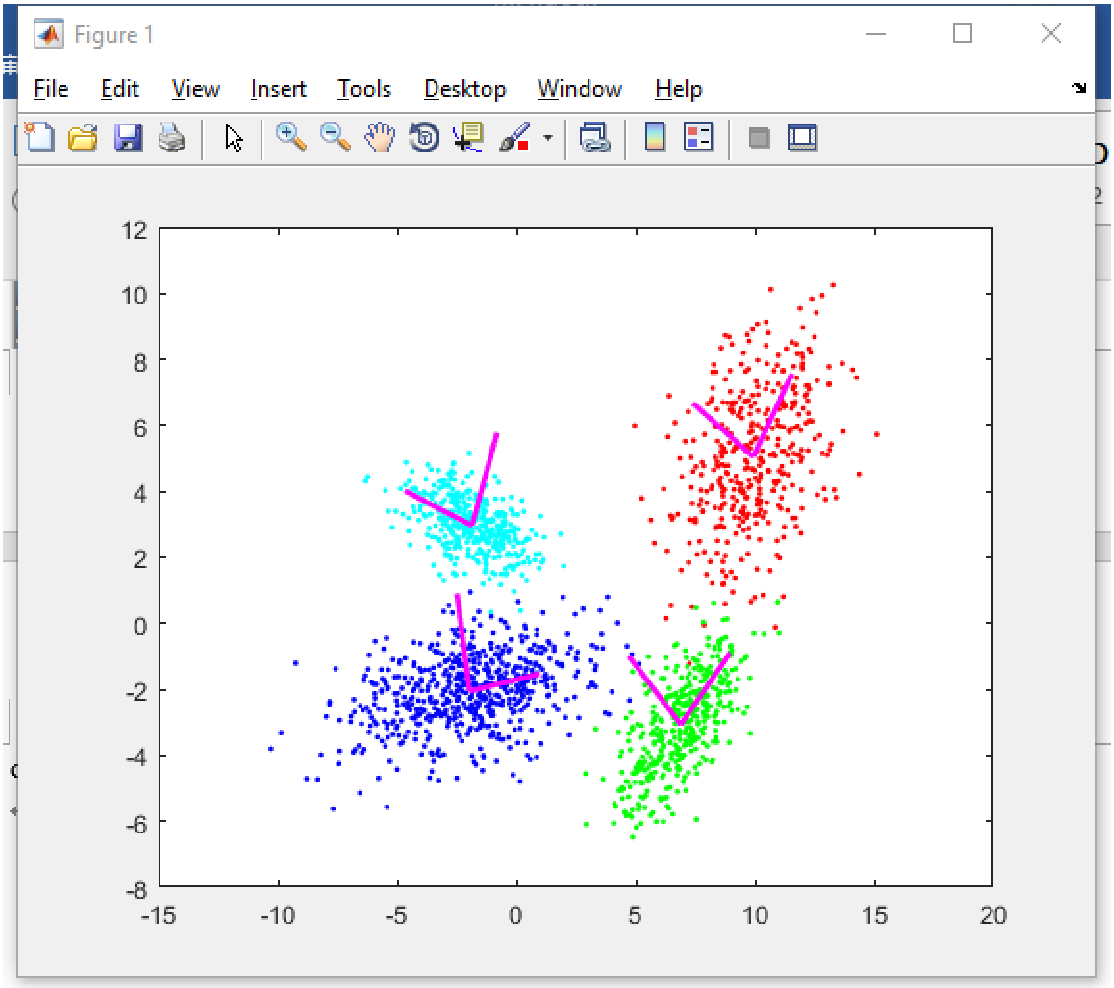

Figure 1
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Assignment 1. Question 2.

2.2 a) overall risk:

$$R = \int_{R_1} [\lambda_{11} P(\omega_1) p(x|\omega_1) + \lambda_{12} P(\omega_2) p(x|\omega_2)] dx + \int_{R_2} [\lambda_{12} P(\omega_2) p(x|\omega_1) + \lambda_{22} P(\omega_2) p(x|\omega_2)] dx.$$

$\because P(\omega_2) = 1 - P(\omega_1)$, $\int_{R_1} p(x|\omega_1) dx = 1 - \int_{R_2} p(x|\omega_1) dx$.

$\therefore R_{mm} = \lambda_{22} + (\lambda_{12} - \lambda_{22}) \int_{R_1} p(x|\omega_2) dx$

$= \lambda_{11} + (\lambda_{21} - \lambda_{11}) \int_{R_2} p(x|\omega_1) dx$

$\because \lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = 1$.

$\therefore R_{mm} = \int_{R_1} p(x|\omega_2) dx = \int_{R_2} p(x|\omega_1) dx$.

b) No, it isn't. i.e. the quadratic

2.5. assume that $\mu_1 + \delta_1 < \mu_3 - \delta_3$

a) For decision point $x_1^*$, $x_2^*$.

$p(x_1^*|\omega_1) = p(x_1^*|\omega_2)$.

$$\frac{\delta_1 - |x_1^* - \mu_1|}{\delta_1^2} = \frac{\delta_2 - |x_1^* - \mu_2|}{\delta_2^2}$$

$(|x_1^* - \mu_1| < \delta_1$ and $|x_1^* - \mu_2| < \delta_2)$.

~~Likewise, $p(x_2^*|\omega_1) = p(x_2^*|\omega_2)$.~~

assume that $\mu_1 < \mu_2 - \delta_2 < \mu_1 + \delta_1 < \mu_2$

$$\delta_1 \delta_2^2 - \delta_1^2 \delta_2 = \delta_2^2 (x_1^* - \mu_1) + \delta_1^2 (x_1^* - \mu_2).$$

$$(\delta_2^2 + \delta_1^2) x_1^* = \delta_2^2 (\delta_1 + \mu_1) - \delta_1^2 (\delta_2 - \mu_2)$$

$$x_1^* = \frac{\delta_2^2 (\delta_1 + \mu_1) + \delta_1^2 (\mu_2 - \delta_2)}{\delta_1^2 + \delta_2^2}$$

likewise, $p(x_2^*|\omega_1) = p(x_2^*|\omega_2)$

$$x_2^* = \frac{\delta_3^2 (\delta_2 + \mu_2) + \delta_2^2 (\mu_3 - \delta_3)}{\delta_2^2 + \delta_3^2}$$

b) minimax decision rule:

$$\alpha_i = \arg\min \{ R(\alpha_i | \vec{x}) \}.$$

for $x_1$: $\alpha_i = \arg\min \begin{cases} \lambda_{11} P(w_1|\vec{x}_1) + \lambda_{12} P(w_2|\vec{x}_2) \\ \lambda_{21} P(w_1|\vec{x}_1) + \lambda_{22} P(w_2|\vec{x}_2). \end{cases}$

$$= \begin{cases} P(w_2|\vec{x}_1) & \text{if } P(w_2|\vec{x}_1) < P(w_1|\vec{x}_1) \\ P(w_1|\vec{x}_1) & \text{otherwise.} \end{cases}$$

Likewise, for $x_2$:

$$\alpha_i = \begin{cases} P(w_3|\vec{x}_2) & \text{if } P(w_3|\vec{x}_2) < P(w_2|\vec{x}_2) \\ P(w_2|\vec{x}_2) & \text{otherwise.} \end{cases}$$

c) $p(x|w_1) = \begin{cases} 1-|x|, & |x| < 1 \\ 0 & \text{otherwise} \end{cases}$

$p(x|w_2) = \begin{cases} 2-4|x-0.5| & |x-0.5| < 0.5 \\ 0 & \text{otherwise} \end{cases}$

$p(x|w_3) = \begin{cases} 1-|x-1| & |x-1| < 1 \\ 0 & \text{otherwise.} \end{cases}$

$P(x_1^*|w_1) = P(x_1^*|w_2)$

$1-|x_1^*| = 2-4|x_1^*-0.5| \quad x \in (0,1).$

$4|x_1^*-0.5| - |x_1^*| = 1$

① $0.5 \leq x_1^* < 1$, $x_1^* = 1$ (x)

② $0 \leq x_1^* < 0.5$, $x_1^* = 0.2$

③ Likewise, $x_2^* = 0.8$

d) ~~$R_{mm} = \int_{R_1} p(x|w_2)dx = \int_{R_2} p(x|w_1)dx$~~

For $x_1^*$ $R_{mm} = \int_{R_1} p(x|w_2)dx = \int_0^{0.2} 4x\,dx = 2x^2 \Big|_0^{0.2} = 0.08$

For $x_2^*$ $R_{mm} = \int_{R_{21}} p(x|w_2)dx = \int_{0.8}^{1} -4x+4\,dx = -2x^2+4x \Big|_{0.8}^{1} = 0.08$

2.7 a) $P(x\in R_2, \omega_1) = P(x\in R_2|\omega_1)P(\omega_1)$

$= \int_{R_2} p(x|\omega_1)P(\omega_1)dx.$

∵ $p(\omega_1)=p(\omega_2)=\frac{1}{2}$

∴ $E_1 = \frac{1}{2}\int_{R_2}\frac{1}{\pi b}\cdot\frac{1}{1+(\frac{x-a_1}{b})^2}dx.$

b) $E_2 = \frac{1}{2\pi b}\int_{R_1}\frac{1}{1+(\frac{x-a_2}{b})^2}dx.$

c) $E = E_1+E_2 = \frac{1}{2\pi b}[\int_{R_2}\frac{1}{1+(\frac{x-a_1}{b})^2}dx + \int_{R_1}\frac{1}{1+(\frac{x-a_2}{b})^2}dx.]$

d) $E = \frac{1}{2\pi}[\int_{R_2}\frac{1}{1+(x+1)^2}dx + \int_{R_1}\frac{1}{1+(x-1)^2}dx$

$= \frac{1}{2\pi}[\arctan(x+1)\Big|_{x^*}^{+\infty} + \arctan(x-1)\Big|_{-\infty}^{x^*}]$

∵ $E_1 = \frac{1}{2\pi}\arctan(x+1)\Big|_{x^*}^{+\infty} = 0.1$

∴ $\frac{\pi}{2} - \arctan(x^*+1) = \frac{\pi}{5}$

$\tan\frac{3}{10}\pi = x^*+1$

∴ $x^* = \tan\frac{3}{10}\pi - 1$

∴ $E = \frac{1}{2\pi}[0.1 + \arctan(x^*-1) - (-\frac{\pi}{2})]$

$= \frac{1}{2\pi}[0.1+\frac{\pi}{2}+\arctan(\tan\frac{3\pi}{10}-2)]$

e) Bayes cases:

$p(x^*|\omega_1) = p(x^*|\omega_2)$

∴ $\cancel{\text{arct}\ \frac{\pi}{2} - \arctan(x^*+1) = \arctan(x^*-1)+\frac{\pi}{2}}$

$\frac{1}{\pi}\cdot\frac{1}{1+(x^*+1)^2} = \frac{1}{\pi}\cdot\frac{1}{1+(x^*-1)^2}$

$(x^*+1)^2 = (x^*-1)^2$

∴ $x^* = 0$

∴ $E_B = \frac{1}{2\pi}[\arctan(x+1)\Big|_0^{+\infty} + \arctan(x-1)\Big|_{-\infty}^0] = \frac{1}{2\pi}[\frac{\pi}{2}-\frac{\pi}{4}+(-\frac{\pi}{4}) - (-\frac{\pi}{2})]$

$= \frac{1}{2\pi}\cdot\frac{\pi}{2} = \frac{1}{4}$