

HOMEWORK 2

ECE/CS 4720-7720 MACHINE LEARNING AND PATTERN RECOGNITION

Question 1

a) Write a generic Matlab function¹ to compute the Mahalanobis distance between two arbitrary samples x_1 and x_2 of a given Gaussian distribution with covariance Σ , mean μ , and dimension d .

b) Write another Matlab function¹ to call the function above and compute the discriminant function with the following generic form

$$g_i(x) = -\frac{1}{2}(x - \mu_i)^t \Sigma_i^{-1}(x - \mu_i) - \frac{d}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma_i| + \ln P(\omega_i)$$

also for any given d dimensional data, mean, covariance matrix and prior probabilities.

c) Use the dataset on page 79 of the textbook². The distributions are assumed to be Gaussian. Use the functions you wrote for Homework 1¹ to compute the mean and covariance for each class. Let $P(\omega_1) = 0.6$ and $P(\omega_2) = P(\omega_3) = 0.2$. Then use the Matlab functions that you wrote in parts a) and b) above to classify the following test points: $(1, 3, 2)$, $(4, 6, 1)$, $(7, -1, 0)$ and $(-2, 6, 5)$.

Question 2

A two-class dataset has gaussian likelihood functions and $P(\omega_1) = 4P(\omega_2)$. Let their statistics be $\mu_1 = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, $\mu_2 = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$ and $\Sigma_1 = \Sigma_2 = \begin{bmatrix} 4.1 & 0 \\ 0 & 2.8 \end{bmatrix}$

a) write a Matlab program that generates (say, 1000) samples from the two classes with the statistics above and plot them in 3D. (your plot should be similar to figure 2.10 (b) in the textbook). The classes above MUST be created from a Gaussian distribution with $N(\vec{0}, I)$.³

b) derive the decision boundary and plot this boundary on top of the generated samples.

c) plot the posterior probabilities.

d) redo part a), b) and c) for $\Sigma_1 = \Sigma_2 = \begin{bmatrix} 4.1 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$

e) redo part a), b) and c) for $\Sigma_1 = \begin{bmatrix} 2.1 & 1.5 \\ 1.5 & 3.8 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 4.1 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$

f) redo part a), b) and c) for $\Sigma_1 = \begin{bmatrix} 2.1 & 1.5 \\ 1.5 & 3.8 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 4.1 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$ and $P(\omega_1) = P(\omega_2)$

¹ you may use any computer language/package, but you may NOT use any function other than the basic operations: i.e. +, -, *, /

² the data is available from http://vigir.ee.missouri.edu/~gdesouza/ece7720/data_class3.mat

³ the following matlab functions may be useful for this part of the assignment: *mvrnd()*, *peaks()*, *meshgrid()*, *surf()*, and *mesh()*