Problem & project. Video Game Design

Professor Cloud has been consulting in the design of the most anticipated game of the year: ***Takehome Fantasy XII***. One of the levels in the game is a maze that players must navigate through multiple rooms from an entrance to an exit. Each room can be empty, contain a monster, or contain a life potion. As the player wanders through the maze, points are added or subtracted from her ***life points*** L. Drinking a life potion increases L, but battling a monster decreases L. If L drops to 0 or below, the player dies.

As shown in Figure 1, the maze can be represented as a digraph $G = (V, E)$, where vertices correspond to rooms and edges correspond to (one-way) corridors running from room to room. A vertex-weight function $f : V \to Z$ represents the room contents:

If $f(v) = 0$, the room is empty.

If $f(v) > 0$, the room contains a life potion. Every time the player enters the room, her life points L increase by $f(v)$.

If $f(v) < 0$, the room contains a monster. Every time the player enters the room, her life points L drop by $|f(v)|$, killing her if L becomes nonpositive.

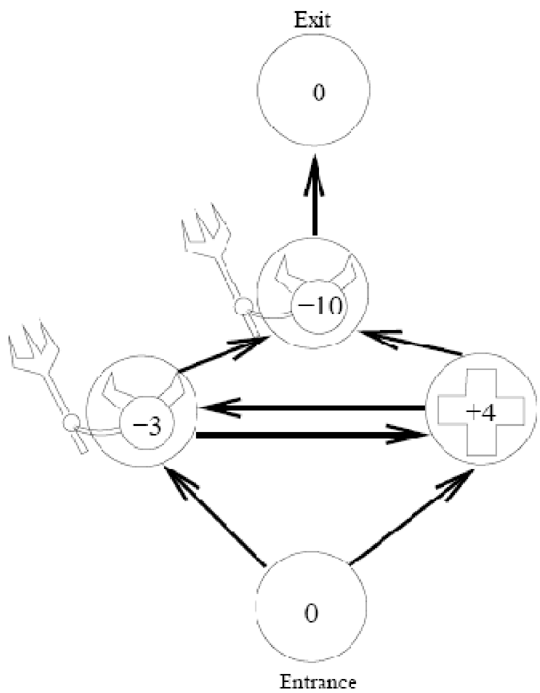

Figure 1 An example of a 1-admissible maze

The ***entrance*** to the maze is a designated room $s \in V$, and the ***exit*** is another room $t \in V$. Assume that a path exists from s to every vertex $v \in V$, and that a path exists from every vertex $v \in V$ to t. The player starts at the entrance s with $L = L_0 > 0$ life points. For simplicity, assume that the entrance is empty: $f(s) = 0$.

Professor Cloud has designed a program to put monsters and life potions randomly into the maze, but some mazes may be impossible to safely navigate from entrance to exit unless the player enters with a sufficient number $L_0 > 0$ of life points. A path from

s to t is ***safe*** if the player stays alive along the way, i.e., her life points never become nonpositive. Define a maze to be ***r-admissible*** if a safe path through the maze exists when the player begins with $L_0 = r$ life points.

Help the professor by designing an efficient algorithm to determine the minimum value r for which a given maze is r-admissible, or determine that no such r exists.

(a) Find a safe path in the maze in Figure 1.

(b) Formulate the problem as an equivalent problem where the weights are on the edges,and prove equivalence.

(c) Assume for this problem part that there are no cycles whose traversal gives a net increase in life points. Given r, how would you check whether the maze is r-admissible?

(d) Now assume that there can be cycles in the graph whose traversal gives a net increasein life points. Given r, how would you check whether the maze is r-admissible?

(e) How can you find the minimum r for which the maze is r-admissible?

(f) Write a program to implement your algorithms.