Multi-Scale Deep Reinforcement Learning for Real-Time 3D-Landmark Detection in CT Scans

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December 9, 2018

Outline

Reinforcement Learning: Overview

Fundamentals

Reward Function

Value Function

Policy Function

Putting it all Together

Markov Decision Process (MDP)

Markov Property

Actions

How to perform Actions?

Ok I performed actions, now what?

Learning the Policy Function

Learning Problem

Conclusion

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In a Nutshell

► Theory and algorithms uniting multiple fields of machine learning, optimal control theory, psychology and neuroscience.

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- ► Each action influences the agents future state.
- Success is measured by a scalar reward signal.
- ► Goal: select actions to maximize future reward.

High level overview

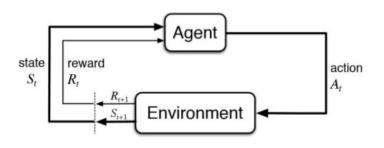


Figure: Agent-Environment interaction

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- ► Reward function provides the immediate, intrinsic desirability of environmental states.

Simple Visualization

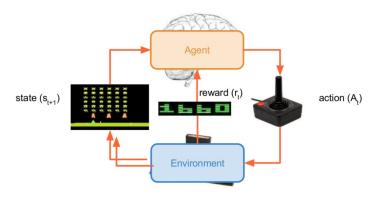


Figure: Atari Example for Reward Function

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- ► Value function estimates the worth of an environmental state in the long run.

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Figure: Mario trying to figure out life (like most grad students)

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- Mario "If I take the star, will it help me achieve a better reward by end of this level?"
- ▶ Value function $V(s_t, a_t)$ calculates this total reward quantity, given the current state and action agent takes.

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Figure: Mario (still) trying to figure out life

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► Mario - "Given that I'm under this brick ceiling, what action should I take?"

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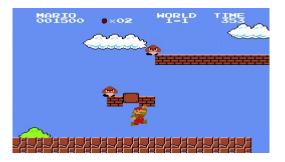


Figure: Mario (still) trying to figure out life

- ► Mario "Given that I'm under this brick ceiling, what action should I take?"
- ▶ Policy function π computes action a_t , given a current state s_t .

Fundamental Functions

Summary

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Simple Visualization



Figure: Super Mario Bros Atari emulator as an environment

Simple Visualization

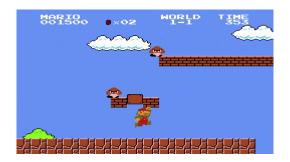


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Figure: Physics-based walking environment

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Markov Property

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- So what's a simple and intuitive way to do it?

Simple Algorithm

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- 7. Return $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, s_{H-1}, a_{H-1}, r_{H-1})$

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Trajectory and Total Reward

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$$R(\tau) = \sum_{i=0}^{H-1} \gamma^i * r_i$$

Trajectory and Total Reward

- ▶ Recall from the algorithm slide, that we returned a vector $\tau = (s_0, a_0, r_0, s_1, a_1, r_1,s_{H-1}, a_{H-1}, r_{H-1}).$
- ightharpoonup au is called the "trajectory" of the agent.
 - ► Trajectory is basically the path that the agent took in the current episode.
- Now, given a trajectory τ obtained after the agent performed actions for an episode of H time steps:
 - We need to find the total reward that we received after this whole episode.
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 - Larger the value of γ , greater the influence of later rewards on the return.
 - In other words, it larger γ favours long term rewards, and the agent becomes willing to forgo short term rewards.

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Fundamentals

Reward Function

Value Function

Policy Function

Putting it all Together

Markov Decision Process (MDP)

Markov Property

Actions

How to perform Actions?

Ok I performed actions, now what?

Learning the Policy Function

Learning Problem

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How to learn the policy function?

Objective Function and Aim

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$$J(\pi) = \mathbb{E}_{\pi} \left[\sum_{i=0}^{H-1} \gamma^i * r(s_t, a_t, s_{t+1}) | s_0 \right]$$

▶ In general: $\pi^* = argmax_{\pi}J(\pi)$

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- ► This was the fundamental formal definition of the reinforcement learning problem.
- ► The concepts introduced here are general terminologies, and should be a good starting point to explore state-of-art literature in (Deep) RL.