
:::tip[About the tutor]

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:::

- E&M
- Optics
- Modern Physics

Intro

All of simple physics in 5 equations

$$
\begin{aligned}
&1. F = q(E + \nu \times B)\\
&2. \iiint E \cdot dA = \frac{Q_{inside}}{\varepsilon_0}\\
&3. \iiint B \cdot dA = 0\\
&4. \iint E \cdot dl = -\frac{d\Phi_B}{dt}\\
&5. \iint B \cdot dl = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\end{aligned}
$$

Gravity , Electric and Magnetic Force has magical similarity!

Chap. 25 Charge and Coulomb's law

:::important[Crucial constant nums]

$e = 1.602 \times 10^{-19} C$

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N \cdot m^2 \cdot C^{-2}$

$1C = 6 \times 10^{18} e$

$m_e = 9.1 \times 10^{-31} kg$

:::

25-2

$1n, 1p = 3$ quarks

$1p(+e) = 2 \times (\frac{2}{3}e) + (-\frac{1}{3}e)$

$1n(0) = 2 \times (-\frac{1}{3}e) + \frac{2}{3}e$

25-3

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\hat{r}$$

Electrostatics in matter

Atom force **H**

Ionic Crystal force **NaCl**

Covalent Bond force **H-H**

Metal force **Au**

Coulomb's laws an exact result for stationary

charges and **not an approximation** form some higher

law.

- Gravitational vs Electric Force

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}{G\frac{m_1 m_2}{r^2}} = 4.17 \times 10^{42}$$

:::note[e.g 25-3]

for a man ,he push his arms apart with a force of 450N,how many charge can he hold outstretched?

$$F = 450N Q = r\sqrt{\frac{F}{k}} = 4.47 \times 10^{-4}$$

:::

:::note[e.g 25-3-2]

How many electrons in a person by **Feynman** treat people like water

$$\lambda_p = \frac{NA}{M_r(H_2O)} \times e(H_2O) = 3.3 \times 10^{23} e/g$$

then ,assume he is 80kg

$$n_e = \lambda_p m = 2.6 \times 10^{28} e$$

1% of a person

$$1\% \times 2.6 \times 10^{28} \times 1.6 \times 10^{-19} = 4.2 \times 10^7 C$$

then for two people 0.75m apart:

$$F = 9 \times 10^9 \times \left(\frac{4.2 \times 10^7 C}{0.75m}\right)^2 = 2.8 \times 10^{25} \approx W_{earth} = 6 \times 10^{24} kg \times 9.8 m/s^2$$

:::

you should distinguish the difference between the vec and scalars

25-4 Conductors and Insulators

- **Insulators** : $\leqslant 1e_c$ per cm^3,*Glass,Plastics,Dry wood*
- **Conductors** : $\approx 10^{23} e_c$ per cm^3, *Aluminum,Copper,Silver...*
- **Semiconductor** : $10^{10} \to 10^{12}$ per cm^3, *Silicon,Germanium*
- **Superconductor** : $R = 0, B = 0$

(the following picture is from OCR,so some fault occurred)

Hg (T_c=4.2K, 1911)
NhSn (T_c=23K, 1969) ,
$\mathrm{YBa_2Cu_3O}$, (T_c=90K, 1987)
HgBaCaCuO (T_c=156K, 1988)
$\mathrm{ReO_1, F_2FeeA_3}$ ($\mathrm{T_c}$=55K, 2008)
$\mathrm{(BaK)Fe_2As_2}$, (T_c=39K, 2008)
$\mathrm{H_3S}$ (T_c=210K, High Pressure, 2016)
$\mathrm{LaH_{10}}$(T_c=250K, High Pressure, 2019)
Pe(Te, So) (T_c=144K, 2008)
(Ti, K, Rb, CS))Fe_s(Se_s, (T_c=30K, 2010).........
$\mathrm{ThNi_2Se_2, ThNi_2S_2}$($T_c$=3.7$K$, 2013)

:::important[usage of $B = 0$]

- NMR(**Nuclear Magnetic Resonance**)
- Brain research
- **Magnetic Levitation** (Maglev)

:::

25-5 Continuous Charge Distribution

The charge density

- $\lambda = \frac{\mathrm{d}q}{\mathrm{d}x}$
- $\sigma = \frac{\mathrm{d}q}{\mathrm{d}A}$
- $\rho = \frac{\mathrm{d}q}{\mathrm{d}V}$

to calculate $q_0 \to$ (distribution)ρ

$$\mathrm{d}\vec{F_{q_0}} = \frac{1}{4\pi\varepsilon_0}\frac{q_0(\rho\mathrm{d}V)}{|\vec{r}-\vec{r'}|^3}(\vec{r}-\vec{r'})$$

For a uniform ring of charge :

$$\begin{aligned}
&\lambda = \frac{q}{2\pi R} \\
&dF = \frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda R d\phi}{(z^2+R^2)} \\
&F_z = \int dF_z = \int dF\cos\theta \\
&= \int \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda R d\phi}{(z^2+R^2)}\frac{z}{\sqrt{z^2+R^2}} \\
&= \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda R z}{(z^2+R^2)^{3/2}}\int_0^{2\pi} d\phi \\
&= \frac{1}{4\pi\varepsilon_0}\frac{q_0 q z}{(z^2+R^2)^{3/2}}
\end{aligned}$$

for approximation

$$z \to \infty, F_z \to \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{z^2}$$

For a uniform disk of charge :

$$
\begin{aligned}
\sigma &= \frac{q}{\pi R^2} \\
\mathrm{d}q &= \sigma \mathrm{d}A = 2\pi\sigma\omega\mathrm{d}\omega \\
\mathrm{d}F_z &= \frac{1}{4\pi\varepsilon_0}\frac{q_0(2\pi\sigma\omega\mathrm{d}\omega)}{(z^2+\omega^2)^{\frac{3}{2}}} \\
F_z &= \frac{1}{4\pi\varepsilon_0} q_0 2\pi\sigma z \int_0^R \frac{\omega\mathrm{d}\omega}{(z^2+\omega^2)^{\frac{3}{2}}} \\
&= \frac{1}{4\pi\varepsilon_0}\frac{2q_0 q}{R^2}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)
\end{aligned}
$$

for approximation

$$z \to \infty, F_z \to \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{z^2}$$

25-6 Charge conservation

$$e^+ + e^- \to 2\gamma n \to p + e^- + \nu_e$$

Chap. 26 Electric Fields

26-1 Field

- scalar field
- Vector field

Mass-field-mass: not action at a distance

26-2 E field

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

26-3 The Electric field of point charge

we just deal with **electrostatics** instead of **electrodynamics**

for a single charge q

$$\vec{E} = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$

for the Electric Dipole ($p = 2Qa = Ql$)

remember $\vec{p} = q \cdot \vec{l}$,the direction is from $-q$ to $+q$

Figure 1: image-20241013135307542

$$E_x(x,0)=0E_y(x,0)=-2\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\sin\theta\sin\theta=\frac{a}{r},r=\sqrt{x^2+a^2}E_y(x,0)=-2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{(x^2+a^2)^{\frac{3}{2}}}\to-2k\frac{Qa}{r^3}$$

then for y-axis

$$E_x\left(0,r\right)=0E_y(0,r)=\frac{Q}{4\pi\varepsilon_0}\frac{4ar}{r^4\left(1-\frac{a^2}{r^2}\right)^2}\to E_y(0,r)\approx+4\frac{1}{4\pi\varepsilon_0}\frac{Qa}{r^3}=4k\frac{Qa}{r^3}$$

:::note[NaCl e.g.]

$2a=0.236nm,p_t=2ea=1.6\times3.78\times10^{-29}$

$p_e=3\times10^{-29}$(measured)

it indicates that the electron is not entirely removed from Na to Cl

:::

for $x>>a$

we can approximate E more precisely

$$E=\frac{pk}{x^3}[1+(-\frac{3}{2})(\frac{a}{x})^2+\cdots]\propto\frac{1}{r^3}$$

26-4 The Electric field of Continuous Charge Distribution

once again we turn to distribution densities:d$q=\lambda dx=\sigma dA=\rho dV$

Figure 2: image-20241013141019977

$$dE_x=-\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\sin\theta dE_y=+\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\cos\theta$$

then

$$E_x=0,E_y=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}k\frac{\lambda\mathrm{d}\theta}{r}\cos\theta=\frac{2k\lambda}{r}=\frac{\lambda}{2\pi\varepsilon_0r}\propto\frac{1}{r}$$

Figure 3: image-20241013141723256

$$
\begin{aligned}
dE &= \frac{\lambda ds}{4\pi\varepsilon_0 r^2} = \frac{\lambda ds}{4\pi\varepsilon_0(z^2+R^2)} \\
dE_z &= dE\cos\theta \\
&= \frac{\lambda ds}{4\pi\varepsilon_0(z^2+R^2)} \cdot \frac{z}{(z^2+R^2)^{1/2}} \\
&= \frac{z\lambda ds}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \\
E_z &= \frac{z}{4\pi\varepsilon_0(z^2+R^2)^{\frac{3}{2}}} \int_0^{2\pi R} \lambda \mathrm{d}s \to \frac{q}{4\pi\varepsilon_0 z^2}
\end{aligned}
$$

Figure 4: image-20241013142455784

$$
\begin{aligned}
&dq = 2\pi\omega \cdot d\omega \cdot \sigma \\
&dE = \frac{zdq}{4\pi\varepsilon_0(z^2+\omega^2)^{3/2}} = \frac{z2\pi\sigma\omega d\omega}{4\pi\varepsilon_0(z^2+\omega^2)^{3/2}} \\
&E = \int dE = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{\omega d\omega}{\left(z^2+\omega^2\right)^{3/2}} \\
&= \frac{\sigma z}{4\varepsilon_0}\int_0^R \frac{d(z^2+\omega^2)}{(z^2+\omega^2)^{3/2}} \\
&= \frac{\sigma}{2\varepsilon_0}(1 - \frac{1}{\sqrt{1+\frac{R^2}{z^2}}}) \\
&R >> z : E \to \frac{\sigma}{2\varepsilon_0} \\
&z >> R : E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}\left(\frac{R}{z}\right)^2 - \frac{3}{8}\left(\frac{R}{z}\right)^4 + \cdots\right) \to \frac{q}{4\pi\varepsilon_0 z^2}
\end{aligned}
$$

Summary

when $r >> a/R$

- Dipole $p = 2Qa = Ql$
  - in line : $E = \frac{4Qa}{4\pi\varepsilon_0 r^3} = \frac{2p}{4\pi\varepsilon_0 r^3}$
  - off line : $E = \frac{2Qa}{4\pi\varepsilon_0 r^3} = \frac{p}{4\pi\varepsilon_0 r^3}$
- Point $E = \frac{q}{4\pi\varepsilon_0 r^2}$

- Line $E = \frac{2\lambda}{4\pi\epsilon_0 r}$
- Ring $E = \frac{q}{4\pi\varepsilon_0 z^2}$
- Disk $E = \frac{q}{4\pi\varepsilon_0 z^2}$

26-5 E-Field Lines

$$\begin{cases} E_x = E_r \sin\theta \cos\phi \\ E_y = E_r \sin\theta \sin\phi \\ E_z = E_r \cos\theta \end{cases}$$

26-6 Point Charge in E-field

:::note[deflecting electrode system of an ink-jet printer]

$$\begin{aligned} \text{An ink drop} : m &= 1.3 \times 10^{-10} kg \quad L = 1.6cm \\ q &= -1.5 \times l0^{-13} C, \quad E = 1.4 \times 10^6 N/C \\ v &= 18m/s \end{aligned}$$

$$y = \frac{1}{2}\frac{qE}{m}\frac{L^2}{v^2} \approx 0.64mm$$

one letter -> about 100 drops

100000 drops/s $\rightarrow$ 1000 letters/s

:::

:::important[bonus:measure the elementary charge $e = 1.602 \times 10^{-19} C$]

skip

:::

then in an ununiformed field,for example

$$\begin{aligned} &F = qE \neq const \quad F(z) \\ &\frac{d^2z}{dt^2} = \frac{q}{m}E(z) \\ &E = \frac{\text{Q}z}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \\ &\frac{d^2z}{dt^2} = \frac{q}{m}\frac{Qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \end{aligned}$$

Figure 5: image-20241013144629047

26-7 Dipole in E-field

- Torque:$\vec{\tau} = pE\sin\theta = \vec{p} \times \vec{E}$
- $W = \int_{\theta_0}^{\theta} = pE(\cos\theta - \cos\theta_0) = -\Delta U, U(\theta) = -\vec{p} \cdot \vec{E}$

we can divide molecules to

- Dipole-molecule H_2O
- non-dipole molecule C_2O

:::note[the max torque on H2O]

$$\tau = qE\sin\theta = 9.3{\times}10^{-26} N{\cdot}m\theta : \pi \to 0 : W = 2pE = 1.9{\times}10^{-25} J\epsilon_{int} = \frac{3kT}{2} {-}6.3{\times}10^{-21} >> \epsilon_{elect}$$

:::

26-8 Atom Nuclear Model

- Thomson model plum pudding

Rutherford:

$$\begin{aligned} &E_{\max} = \frac{Q}{4\pi\varepsilon_0 R^2} = 1.2 \times 10^{13} N/C \\ &R = 1.0 \times 10^{-10} m \\ &Q = 79e \\ &U = 6\text{Mev} = 9.6 \times 10^{-13} J \\ &\nu = \sqrt{\frac{2U}{m}} = 1.7 \times 10^7 m/s \end{aligned}$$

$$\begin{aligned} &F = qE_{\max} = ma \\ &a = \frac{q}{m} E_{\max} \\ &\Delta\nu = a\Delta t = \frac{q}{m} E_{\max} \frac{2R}{\nu} = 6.6 \times 10^3 m/s \\ &\therefore \theta = tg^{-1} \frac{\Delta\nu}{\nu} = tg^{-1} (\frac{6.6 \times 10^3}{1.7 \times 10^7}) \approx 0.02 \end{aligned}$$

then , the result disobeys the experimental result!

Chap. 27 Gauss Law

27-2(3) Flux

We need to at first define flux and the vector of surface of a closed region.

$$
\begin{aligned}
&\Phi = \oint \vec{\nu} \bullet d\vec{A} \\
&d\Phi = \vec{\nu} \bullet d\vec{A} \\
&d\vec{A} = dydz\vec{i} + dzdx\vec{j} + dxdy\vec{k} \\
&\vec{\nu} = \nu_x\vec{i} + \nu_y\vec{j} + \nu_z\vec{k} \\
&\vec{\nu}\acute{\text{L}}d\vec{A} = \nu_x dydz + \nu_y dzdx + \nu_z dxdy
\end{aligned}
$$

if no source or sink of fluid

$$\Phi = \oint \vec{\nu} \cdot \mathrm{d}\vec{A} = 0$$

for electrostatic case

$$\Phi = \oint \vec{E} \bullet \mathrm{d}\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Geometry and Surface Integrals

we can illustrate that Gauss's Law -> Coulomb's Law

for a +Q sphere having radius R

$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$

27-5(6) Application Gauss's Law

:::note[Uniform charged sphere]

$$q = \frac{4}{3}\pi a^3 \rho \Rightarrow E = \frac{\rho a^3}{3\varepsilon_0 r^2}(r > a) E = \frac{\rho r}{3\varepsilon_0}(r < a)$$

:::

Conductors——Spherical Symmetry

since E=0 inside a conductor $Q_{inside} = 0$, charges are only on the surface

For a conductor,we obtain a cylinder gauss plane:

$$\varepsilon_0 E\Delta A + 0 + 0 = \sigma\Delta A E = \frac{\sigma}{\varepsilon_0}$$

Line——Cylindrical Symmetry

$$E \cdot 2\pi r h \cdot \varepsilon_0 = \lambda h \rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Aline charge λ (C/m) is placed along the axis of an uncharged conducting cylinder of inner radius $r_i = a$, and outer radius $r_o = b$ as shown.

we need to get σ_o

$$E_o = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma_0 b}{\varepsilon_0 r} \Rightarrow \sigma_o = \frac{\lambda}{2\pi b}$$

Sheets——Planar Symmetry

$$\varepsilon_0(2EA) = \sigma A \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

for two paralleled sheets

$$E_{in} = \frac{\sigma}{\varepsilon_0}$$

Figure 6: image-20241013155219398

$$E_A = \frac{-\sigma_1}{2\varepsilon_0} E_B = \frac{+\sigma_1}{2\varepsilon_0} E_C = 0 E_D = \frac{+\sigma_1}{2\varepsilon_0}$$

always based on gauss plane

Figure 7: image-20241013160823665

$$Q_2 = -3Q_1$$

$$\sigma_i = -\frac{Q_1}{4\pi R_2^2} \sigma_o = -\frac{2Q_1}{4\pi R_2^2} E = \begin{cases} 0 & r < R_1 \\ \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2} & R_1 < r < R_2 \\ -\frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{r^2} & r > R_2 \end{cases}$$

Once we connect the two spheres with a wire:,it becomes a whole conductor with $-2Q_1$

Figure 8: image-20241013162838562

:::note[eg2 Cylinders]

$$\sigma_i = -\frac{\lambda}{2\pi R}, \sigma_o = \sigma_t + \frac{\lambda}{2\pi R}$$

$$E_r = \begin{cases} \frac{\lambda}{2\pi\varepsilon_0 r} & r < R \\ \frac{\lambda}{2\pi\varepsilon_0, r} + \frac{\sigma R}{\varepsilon_0 r} & r > R \end{cases}$$

27-7 Experimental Tests of Gauss' Law and Coulomb's Law

just skip

Chap. 28 Electric Potential U & V

28-1 Potential Energy

Figure 9: image-20241013164129339

The union energy of two points of charge :

$$U_b - U_a = -W_{a,b} = -\int_a^b \vec{F} \cdot \mathrm{d}\vec{l}$$

The circuit Law of electrostatics field

$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = 0 \nabla \times \vec{E} = 0$$

The gauss' law(the same as **Math Analysis**):

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{q}{\varepsilon_0} = \iiint \frac{\rho}{\varepsilon_0} \mathrm{d}V$$

28-2 Electric Potential

$$V_p = \frac{U_p}{q_0}$$

$$V_B - V_A \equiv \frac{W_{AB}}{q_0} = -\int_A^B \vec{E} \cdot \mathrm{d}\vec{l}$$

28-3 Calculate Potential from E

$$W_{ab} = -\int_a^b \vec{E}\cdot \mathrm{d}\vec{l} V_p = W_{\infty p} = -\int_\infty^p \vec{E}\cdot \mathrm{d}\vec{l}$$

for a point charge:

$$V_b - V_a = -\int_{r_a}^{r_b} \frac{q}{4\pi\varepsilon_0 r^2}\mathrm{d}r = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Figure 10: image-20241013170530297

$$r >> a V_r = \frac{1}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2} r_2 - r_1 \approx 2a\cos\theta, r_1 r_2 = r^2 V_r = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$$

Figure 11: image-20241013171208250

$$\begin{aligned} V(r) &= \sum_i V_i(r_i) \\ &= \frac{1}{4\pi\varepsilon_0}(\frac{q}{r-d} + \frac{-2q}{r} + \frac{q}{r+d}) \\ &= \frac{1}{4\pi\varepsilon_0}\frac{2qd^2}{r(r^2-d^2)} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{2qd^2}{r^3(1-d^2/r^2)} \to \frac{Q}{4\pi\varepsilon_0 r^3}(d << r) \end{aligned}$$

- Point charge $\propto \frac{1}{r}$
- Dipole $\propto \frac{1}{r^2}$
- Quadrupole $\propto \frac{1}{r^3}$

we can trace back to the distribution reflected by n-dipoles.

:::[egs for potential calculation]

for a charged sphere shell

$$E = [r \geqslant R]\cdot \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V(P) = \frac{q}{4\pi\varepsilon_0 \max\{r_P, R\}}$$

$$U=\frac{1}{2}\int V\mathrm{d}q=\frac{q^2}{8\pi\varepsilon_0 R}$$

$$W=mc^2=\frac{e^2}{8\pi\varepsilon_0 R}\Rightarrow R_e\approx 1.4\times 10^{-15}m$$

for a ring

$$V=\oint\frac{1}{4\pi\varepsilon_0}\frac{\lambda\mathrm{d}s}{\sqrt{z^2+R^2}}=\frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

for a disk

$$\begin{aligned}
&dq=2\pi\omega\cdot d\omega\cdot\sigma\\
&dV=\frac{2\pi\omega\cdot d\omega\cdot\sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}}\\
&V=\int_0^R\frac{2\pi\omega\cdot d\omega\cdot\sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}}=\frac{\sigma}{2\varepsilon_0}(\sqrt{z^2+R^2}-z)\to\frac{q}{4\pi\varepsilon_0 z}
\end{aligned}$$

:::

Sparks

High electric fields can ionize nonconducting materials(**dielectrics**)

(Insulator->Conductor)

:::note[ball breakdown]

we have two ball shell with same potential V

Ball 2 is as twice large as Ball 1

as V goes up, the Ball 1 will breakdown first

$$E_s=\frac{Q}{4\pi\varepsilon_0 r^2},V=\frac{Q}{4\pi\varepsilon_0 r}\Rightarrow E=\frac{V}{r}$$

then $r_1<r_2\Rightarrow E_1>E_2$

:::

- $\Delta V>0$ means we go uphill
- $\Delta V<0$ means we go downhill

/pic/phy2/Potential_2.png

Figure 12: image-20241015101813495

$$V(r) = V_\infty - \int_\infty^c \vec{E}_l \cdot \vec{l} - \int_c^b \vec{E}_l \cdot \vec{l} - \int_b^a \vec{E}_l \cdot \vec{l} - \int_a^r \vec{E}_l \cdot \vec{l} = V_\infty - \left(\int_\infty^c + \int_b^a \right) \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \mathrm{d}r - \int_a^r \frac{1}{4\pi\varepsilon_0} \frac{Qr}{a^3} \mathrm{d}r = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \right.$$

you can split into two patterns

28-4 Equipotentials

when in the equipotential surface we can conclude

$$\vec{E} \cdot \mathrm{d}\vec{l} \equiv V$$

28-5 Potential of a charged conductors

Claim: The surface of a conductor is an equipotential surfacewhen two sphere conductors are attatched to each others.

$$\frac{Q_A}{Q_B} = \frac{r_A}{r_B}$$

for example , when a point of charge is placed off-center inside a sphere conductor,the inside surface will be inuniform and the outside surface will be uniform

28-6 Calculate E from Potential

$$V_P = \int_P^\infty \vec{E} \cdot \mathrm{d}\vec{l}$$

- Graphically the E-field line is the fastest-descending line of equipotential surfaces
- Math:

  from $\mathrm{d}W = -q_0 \mathrm{d}V$

$$\mathrm{d}W = \vec{F} \cdot \mathrm{d}\vec{l} = q_0 \vec{F} \cdot \mathrm{d}\vec{l} = q_0 E \mathrm{d}l \cos\theta E \cos\theta = -\frac{\mathrm{d}V}{\mathrm{d}l}$$

$$E_l = -\frac{\mathrm{d}V}{\mathrm{d}l} \Rightarrow \vec{E} = -\nabla V$$

– Cartesian coordinates:

$$\nabla V = \frac{\partial V}{\partial x}\bar{x} + \frac{\partial V}{\partial y}\bar{y} + \frac{\partial V}{\partial z}\bar{z}$$

– Spherical coordinates:

$$\nabla V = \frac{\partial V}{\partial r}\bar{x} + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{y} + \frac{1}{r\sin\varphi}\frac{\partial V}{\partial \varphi}\bar{\varphi}$$

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}$$

$$r = \sqrt{x^2+y^2+z^2}\theta = \arccos\frac{z}{\sqrt{x^2+y^2+z^2}} = \arccos\frac{z}{r} = \begin{cases} \arctan\frac{\sqrt{x^2+y^2}}{z} & \text{if } z > 0 \\ \pi + \arctan\frac{\sqrt{x^2+y^2}}{z} & \text{if } z < 0 \\ +\frac{\pi}{2} & \text{if } z = 0 \text{ and } \sqrt{x^2 \\ \text{undefined} & \text{if } x = y = z = 0 \end{cases}$$

$$\begin{aligned} V &= 3x^2 + 2xy - z^2 \\ \vec{E} &= (-6x - 2y, -2x, 2z)^T \\ \vec{E} &= \frac{2aq}{4\pi\varepsilon_0 r^3}(2\cos\theta, \sin\theta, 0)^T_{sp} \end{aligned}$$

then we have two eg

/pic/P_dipole.png

Figure 13: image-20241015114126009

$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2} \to \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}(r >> a)$$

then

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2} E_r = -\frac{\partial V}{\partial r} E_\theta = -\frac{\partial V}{\partial \theta} \Rightarrow \vec{E} = \frac{2aq}{4\pi\varepsilon_0 r^3}((2\cos\theta)\hat{r} + \sin\theta\hat{\theta})$$

/pic/kk.png

Figure 14: image-20241015114612342

it's easy to see that $\arg\max_\theta ||\vec{E}|| = \frac{\pi}{2}$

:::note[eg. disk]

/pic/phy2/P_disk.png

Figure 15: image-20241015114925818

$$\begin{aligned}
&dq = 2\pi\omega \cdot d\omega \cdot \sigma \\
&dV = \frac{dq}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} = \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} \\
&V = \int_0^R \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right) \\
&E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{R^2+z^2}} - 1\right) \\
&= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)
\end{aligned}$$

:::important[a e.g for getting v with ΔU]

$$\frac{1}{2}mv^2 = K = -\Delta U = -q\Delta V \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

we can measure \$$\alpha$\$$partical's large velocity by measuring eletrostatic information like difference of potential$

:::

Appendix method of images

/pic/phy2/imaging.png

Figure 16: image-20241015120204563

we can see

$$E(\vec{r}_s) = \frac{\sigma(\vec{r}_s)}{\varepsilon_0}$$

we can see that the induced charge distribution generated with a point of charge and a conductor sheet is the same as which is made by two charges. It can be proved with symmetry theorem.

Chap. 30 Capacitance and Dielectrics

30-1 Capacitors

Classic Capacitors

- flashbulb:capacitor draw energy from battery (s) then release it through bulb(ms)
- Laser pulse
- thermonuclear fusion $10^{14}W, 10^{-9}s, 10^{8}K$

Definition of Capacitance : two spatially seperated conductors($+q/-q$) $C = \frac{q}{\Delta V}$

one single conductor is capacitor!

:::note[eg:Parallel Plate capacitor]

$$q = \sigma A \Delta V = -\int_A^B \vec{E} \cdot \mathrm{d}\vec{l}$$

:::

:::note[eg:Parallel Plate capacitor]

$$q = \sigma A \Delta V = -\int_A^B \vec{E} \cdot \mathrm{d}\vec{l}$$

since

$$EA\varepsilon_0 = \sigma A \Rightarrow E = \frac{\sigma}{\varepsilon_0} \Rightarrow \Delta V = \frac{q}{A\varepsilon_0} d \Rightarrow C = \frac{\varepsilon_0 A}{d}$$

:::

- condenser:$C \propto \frac{1}{d} \xrightarrow{\text{fixed } \Delta V} Q \propto \frac{1}{d}, I = \frac{\mathrm{d}Q}{\mathrm{d}t}$:the vibration -> different I

:::note[eg:Cylindrical Capacitor]

Figure 17: image-20241018170156213

$+Q, -Q$ on surface,ΔV

$$2\pi r \cdot L \cdot E \cdot \varepsilon_0 = Q \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 L r}$$

$$\Delta V = \int_b^a \vec{E} \cdot \mathrm{d}\vec{l} = \frac{Q}{2\pi\varepsilon L \ln\frac{b}{a}}, C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

:::note[ex:TV signals tansmit/coaxial cable]

$a = r_i = 0.15, b = r_o = 2.1$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln(2.1/0.15)} = 21 \times 10^{-12} F/m = 21 pF/m$$

:::

:::note[eg:spherical capacitor]

Figure 18: image-20241018170915119

$$\vec{E} = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2} \Rightarrow \Delta V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \Rightarrow C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

for earth $R = 6.37 \times 10^6 m \Rightarrow C = 7.1 \times 10^{-4}\mathrm{F} = 710\mu\mathrm{F}$

:::

Summary

- Parallel Plate:$C=\frac{\varepsilon_0 A}{d}$
- Cylindrical Capacitor:$C=\frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$

- Spherical $C=4\pi\varepsilon\_0\frac{ab}{b-a}$

Parallel & Series

Figure 19: image-20241018171312744

Figure 20: image-20241018171326385

for capacitors in parallel

$$V=\frac{Q_1}{C_1}=\frac{Q_2}{C_2}\Rightarrow Q_2=Q_1\frac{C_2}{C_1}C=C_1+C_2$$

for capacitors in series

$$V_{ab}=\frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}\Rightarrow C=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}$$

Figure 21: image-20241018171611735

$$\frac{1}{C}=\frac{1}{C_3}+\frac{1}{C_1+C_2}$$

$$C=\frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}\ln\frac{d}{c}}$$

30-2 Energy storage in E-field

for paralleled capacitor

$$\mathrm{d}W=V(q)\mathrm{d}q=\frac{q\mathrm{d}q}{C}W=\int_0^Q\frac{q\mathrm{d}q}{C}=\frac{1}{2}\frac{Q^2}{C}=\frac{CV^2}{2}\Rightarrow=\frac{1}{2}CV^2$$

in some questions ,we need to verify the fact whether Q is const or V is const.

$$U=\frac{1}{2}\frac{Q}{C^2}=\frac{1}{2}\frac{Q^2}{A\varepsilon_0/d}\Rightarrow E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 A}\Rightarrow U=\frac{1}{2}E^2\varepsilon_0 Ad$$

Figure 22: image-20241018171919786

the energy density $u = \frac{W}{Ad} = \frac{1}{2}\varepsilon_0 E^2 (J \cdot m^{-3})$

you can calculate with u with Cylindrical Capacitor

$$U = \int_a^b \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\int \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)^2 L2\pi r \mathrm{d}r = \frac{1}{2}\frac{Q^2}{2\pi\varepsilon_0 L}\ln\frac{b}{a} = \frac{1}{2}\frac{Q^2}{C}$$

:::note[ACT2 cylindrical capacitors]

with two cylindrical capacitors ((a, b) vs $(2a, 2b)$)

$$C = \frac{2\pi\varepsilon_o L}{\ln\left(\frac{r_{outer}}{r_{inner}}\right)}$$

so $C_1 = C_2$

::: note[P687 30-7]

Problem 30- 7 (page 687) . An isolated conducting sphere whose radius R is 6.85cm carries a charge $q = 1.25nC$.(a) How much energy is stored in the electric field of this charged conductor? (b) What is the energy density () at the surface of the sphere? (c)What is the radius R_ϑ of the imaginary spherical surface such that one-half of the stored potential energy lies within it?

R=6.85cm, q=1.25nC

(a)**U=?**

(b)**u=? (at the surface of the sphere)**

(c)R_0 =? At $R < R_0, U' = \frac{1}{2}U$

(a)

$$C = 4\pi\boldsymbol{\varepsilon}_0 R$$
$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\boldsymbol{\varepsilon}_0 R} = 1.03 \times 10^{-7}J = 103nJ$$

(b)

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$
$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{q^2}{16\pi^2\varepsilon_0^2 R^4} = \frac{q^2}{32\pi^2\varepsilon_0^2 R^4} = 25.4nJ/cm^3$$

(c)

$$\begin{aligned}
&\int_R^{R_0} \frac{1}{2}\varepsilon_0 E^2 d\nu = \int_{R_0}^{\infty} \frac{1}{2}\varepsilon_0 E^2 d\nu \\
&\int_R^{R_0} \frac{1}{2}\varepsilon_0 \frac{q^2}{16\pi^2\varepsilon_0^2 r^4} 4\pi r^2 dr = \int_{R_0}^{\infty} \frac{1}{2}\varepsilon_0 \frac{q^2}{16\pi^2\varepsilon_0^2 r^4} 4\pi r^2 dr \\
&\int_R^{R_0} \frac{dr}{r^2} = \int_{R_0}^{\infty} \frac{dr}{r^2} \\
&\frac{1}{R} - \frac{1}{R_0} = \frac{1}{R_0} \\
&R_0 = 2R = 13.7cm
\end{aligned}$$

:::

30-3 Dielectrics

Capacitor with dielectrics

- **Empirical observation**: Inserting a non-conducting material between the plates of a capacitor changes the VALUE of the capacitance.
- dielectric constant $C = \kappa_e C_0$,$\kappa_e > 1$(glass=5.6,water=78)
- C_0 means the capacitance with vacuum (air)

with dielectric constant κ_e and const Q

$$V = \frac{Q}{C} = \frac{V_0}{\kappa_e} \Rightarrow E = \frac{E_0}{\kappa_e}$$

with const V

$Q' = \kappa_e C_0 V$

- parallel-plate :$C = \frac{\kappa_e \varepsilon_0 A}{d}$
- cylindrical $C = \frac{\kappa_e 2\pi\varepsilon_0 L}{\ln \frac{b}{a}}$
- spherical $C = 4\pi\varepsilon_0\kappa_e \frac{ab}{b-a}$

for point charge $E = \dfrac{Q}{4\pi\varepsilon_0\kappa_e r^2}$

:::note[the increasing C]

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d_1 + d_2} > \frac{\varepsilon_0 A}{d}(d > d_1 + d_2))$$

Polarization effect :

$$V = Ed = (E_0 - E')d < E_0 d C = \frac{q}{(E_0 - E')d} > C_0$$

:::

microscopic mechanism of polarization

- Non-polar dielectrics $\vec{p} = q\vec{d} = 0$
- Polar dielectrics $\vec{p} = q\vec{d} \neq 0$

for non-polar dielectrics

we have

Induced electric dipole moment

Electric displacement polarization

Figure 23: image-20241020222558545

Polar dielectrics

$$\sum \vec{p} \neq 0$$

Alignment polarization

In high frequency field , Electric displacement polarization plays an important role

Polarization

Polarization intensity $\vec{P}$

$$\vec{P} = \frac{\sum \vec{p}_m}{\Delta V}(C \cdot m^{-2}) = nq\vec{l}$$

Figure 24: image-20241020222921535

$$
\begin{aligned}
&dN = ndV = nldA\cos\theta \\
&dq' = qdN = nqldA\cos\theta \\
&= PdA\cos\theta \\
&= \vec{P} \bullet d\vec{A} \\
&\oint \vec{P} \bullet d\vec{A} = \sum_{out} q' = -\sum_{in} q' \\
&dq' = \vec{P} \bullet d\vec{A} = P\cos\theta \cdot dA \\
&\sigma' = \frac{dq'}{dA} = P\cos\theta = \vec{P} \bullet \vec{n} = P_n
\end{aligned}
$$

Depolarization Field

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

Figure 25: image-20241020223121085

$$
\begin{aligned}
&\sigma_e' = P_n = P\cos\theta \\
&dE' = \frac{dq'}{4\pi\varepsilon_0 R^2} = \frac{\sigma_e' dA}{4\pi\varepsilon_0 R^2} = \frac{P\cos\theta dA}{4\pi\varepsilon_0 R^2} \\
&dA = Rd\theta \cdot R\sin\theta d\varphi \\
&= R^2 \sin\theta d\theta d\varphi \\
&dE' = \frac{P}{4\pi\varepsilon_0}\cos\theta\sin\theta d\theta d\varphi \\
&dE_z' = dE'\cos(\pi-\theta) = -dE'\cos\theta \\
&= -\frac{P}{4\pi\varepsilon_0}\cos^2\theta\sin\theta d\theta d\varphi \\
&E_z' = \oint_z dE_z' = -\frac{P}{4\pi\varepsilon_0}\int_0^{\pi}\cos^2\theta\sin\theta d\theta\int_0^{2\pi} d\varphi = -\frac{P}{3\varepsilon_0}
\end{aligned}
$$

:::note[eg2 parallel plate]

$$\sigma_e' = P\cos\theta = PE' = \frac{\sigma_e'}{\varepsilon_0}$$

:::

Polarization law

$$\vec{P} \Rightarrow \sigma_e' \Rightarrow \vec{E}' \Rightarrow \vec{E}[\vec{P}(\vec{E})]$$

for general isotropic materials

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} (\kappa_e = 1 + \chi_e)$$

χ_e: Polarization coefficient

for crystal materials:

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \chi_{xx}\chi_{xy}\chi_{xz} \\ \chi_{yx}\chi_{yy}\chi_{yz} \\ \chi_{zx}\chi_{zy}\chi_{zz} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_0 E_x \\ \boldsymbol{\varepsilon}_0 E_y \\ \boldsymbol{\varepsilon}_0 E_z \end{pmatrix}$$

:::note[eg:ferroelectric material]

Electric hysteresis effect Similar to the magnetic hysteresis effect.

:::

Electric Displacement vector$\vec{D}$

$$\vec{E}_0 \to \vec{P} \to \boldsymbol{\sigma'}_e \to \vec{E'} \to \vec{E} = \vec{E}_0 + \vec{E'}$$

$ $ means:

- Electric displacement vec
- electric induction

Figure 26: image-20241020224048323

let q_0 be inside charge,q' be induced charge

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \sum_{In} (q_0 + q')$$

$$\oint \vec{P} \bullet d\vec{A} = -\sum_{In} q'$$

$$\iint \varepsilon_0 \vec{E} \bullet d\vec{A} = \sum_{In} q_0 - \oint \int \vec{P} \bullet d\vec{A}$$

$$\oint (\varepsilon_0 \vec{E} + \vec{P}) \bullet d\vec{A} = \sum_{In} q_0$$

$$\oint \vec{D} \bullet d\vec{A} = \sum_{In}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = (1 + \chi_e)\varepsilon_0 \vec{E} = \kappa_e \varepsilon_0 \vec{E}$$

$$\boxed{\oint \vec{D} \bullet d\vec{A} = \sum_{In} q_0}$$

:::note[eg paralleled plate]

$$\oint \vec{D} \bullet dA = \sum q_0$$

$$D_1 \Delta A + D_2 \Delta A = \sigma_{e0} \Delta A$$

$$\vec{E}_1 = 0, D_1 = \kappa_{e1} \varepsilon_0 E_1 = 0, \therefore D_1 = 0$$

$$\therefore D = D_2 = \sigma_{e0} = \varepsilon_0 E_0$$

$$\therefore E = \frac{D}{\kappa_e \varepsilon_0} = \frac{\varepsilon_0 E_0}{\kappa_e \varepsilon_0} = \frac{E_0}{\kappa_e}$$

$$\vec{D} = \kappa_e \varepsilon_0 \vec{E}$$

:::

:::note[eg charge in a hole]

$$\oint \vec{D} \bullet d\vec{A} = \sum q_0$$

$$4\pi r^2 D = q_0$$

$$D = \frac{q_0}{4\pi r^2}$$

$$E = \frac{D}{\kappa_e \varepsilon_0} = \frac{q_0}{4\pi \varepsilon_0 \kappa_e r^2} = \frac{E_0}{\kappa_e}$$

$\oint \vec{D} \cdot \mathrm{d}\vec{l} \neq 0$

:::

Pressure Electric Effect

Figure 27: image-20241020224455711

Chap 31 The Steady Current*(Optional)

31-1 The Steady Current and Conduction Law

$$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{\mathrm{d}q}{\mathrm{d}t}$$

the current density vector $\vec{j}$

$$\begin{aligned} di &= \vec{j} \bullet d\vec{A}, \\ i &= \iint_A \vec{j} \bullet d\vec{A} \\ &= \iint_A j \cos\theta dA \end{aligned}$$

the steady current condition

$$\iint_A \vec{j} \bullet d\vec{A} = 0 \Leftrightarrow j_1 \Delta A_1 = j_2 \Delta A_2$$

Ohm Law

- **linear devices** :**Metal, liquid containing acid, alkali, salt**
- **nonlinear devices** :**Evacuated tube, transistor**

Figure 28: image-20241020225112215

Conductance $G = \frac{1}{R} = \frac{\mathrm{d}I}{\mathrm{d}V} (Unit : S)$

Resistivity, & conductivity :

$$R = \int \rho \frac{\mathrm{d}l}{A}, \sigma = \frac{1}{\rho}$$

or differential form:$j = \frac{E}{\rho} = \sigma E$

Figure 29: image-20241020225354688

$$R = \int \rho \frac{dl}{A} = \int_a^\infty \rho \frac{dr}{2\pi r^2}$$
$$= \frac{\rho}{2\pi}[-\frac{1}{r}]_a^\infty = \frac{\rho}{2\pi a}$$

:::note[$\rho - T$ relativity]

for metal

$$\rho(T) = \rho_0 + \alpha T$$

:::

Electric power and Joule Law

$$P = \frac{W}{\Delta t} = iV = i^2 R = \frac{V^2}{R}$$

Microscopic explanation of Ohm Law

/pic/phy2/Ohm_micro.png

Figure 30: image-20241021092841895

:::note[drift speed of electric charge]

$j = 2.4A/mm^2 b = 8.4{\times}10^{28} m^{-3} j = \frac{\Delta i}{\Delta A} = neuu = \frac{j}{ne} = 1.8{\times}10^{-4} m/s << v_t \approx 10^5 m/s$

:::

31-2 Source and Electromotive Force(emf)

Chap 32/33 The Steady Magnetic Field

32-1 Basic phenomena

first hard disk 1957:50 platters

Basic Phenomena of Magnetism

- attract small bits of metal
- have two poles
- like poles repel, and unlike poles attract
- oersted experiment: $B, i \to F$
- Solenoid is similar to a bar magnet
- Interaction between electric currents

/pic/phy2/bar_b.png

Figure 31: image-20241021084128203

the magnetic field line distribution is like electric field of a electdipole pair.

Magnetic Monopoles

since no monopole has ever been found $\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0$

Magnetic field

(Ampere) molecular current: he bind solenoid and a magnet

electric charge in motion is the source of Magnetic Fields

Ampere's Law(1820.12.4)

:::note[Ampere's Law]

/pic/phy2/Ampere.png

Figure 32: image-20241021085424748

- Current element $i\mathrm{d}\vec{s}$

$$\mathrm{d}F_{12} \propto \frac{i_1 i_2 \mathrm{d}s_1 \mathrm{d}s_2}{r_{12}^2} = \frac{\mu_0}{4\pi} \cdot \frac{i_2 \mathrm{d}\vec{s}_2 \times (i_1 \mathrm{d}\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}$$

$\mu_0 = 4\pi \times 10^{-7}(N \cdot A^{-2})$:Permeability constant

:::

the consider two examples

/pic/phy2/B_eg1.png

$$
\begin{aligned}
d\vec{F}_{12} &= \frac{\mu_0}{4\pi}\frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2} \\
d\vec{s}_1 &\perp \hat{r}_{12} \\
\therefore dF_{12} &= \frac{\mu_0}{4\pi}\frac{i_1 i_2 ds_1 ds_2}{r_{12}^2} \\
d\vec{F}_{21} &= \frac{\mu_0}{4\pi}\frac{i_1 d\vec{s}_1 \times (i_2 d\vec{s}_2 \times \hat{r}_{21})}{r_{21}^2} \\
d\vec{s}_2 &\perp \hat{r}_{21} \\
\therefore dF_{21} &= \frac{\mu_0}{4\pi}\frac{i_1 i_2 ds_1 ds_2}{r_{12}^2} \\
\mathrm{d}F_{12} &= \mathrm{d}F_{21}
\end{aligned}
$$

/pic/B_eg2.png

$$
\mathrm{d}F_{12} = 0 \mathrm{d}F_{21} = \frac{\mu_0}{4\pi}\frac{i_1 i_2 \mathrm{d}s_1 \mathrm{d}s_2}{r_{12}^2}
$$

The Magnetic Induction Strength

- Coulomb's Law

$$
\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \vec{F}_{12} = q_2\vec{E}_1, \vec{E}_1 = \frac{\vec{F}_{12}}{q_2} \therefore \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}^2}\hat{r}_{12}
$$

- Ampere's Law

$$
d\vec{F}_{12} = \frac{\mu_0}{4\pi}\frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}
$$

let us call $i_2 \mathrm{d}\vec{s}_2$ the element of a test electric current

$$
\begin{aligned}
&d\vec{F}_2 = i_2 d\vec{s}_2 \times \frac{\mu_0}{4\pi} \oint_{\mathrm{L}_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2} \\
&\text{Define:} \quad \vec{B}_1 = \frac{\mu_0}{4\pi} \oint_{\mathrm{L}_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2} \\
&\therefore \quad d\vec{F}_2 = i_2 d\vec{s}_2 \times \vec{B}_1
\end{aligned}
$$

$$
\begin{aligned}
&d\vec{F}_2 = i_2 d\vec{s}_2 \times \vec{B}_1 \\
&dF_2 = i_2 ds_2 B_1 \sin\theta \\
&\theta = 0, dF_2 = 0 \\
&\theta = \frac{\pi}{2}, dF_2 \text{maximun} \\
&\boxed{\textbf{Define: } B_1 = \frac{(dF_2)_{\max}}{i_2 ds_2}} \\
&\Rightarrow \vec{B}_1 = \frac{\mu_0}{4\pi} \oint_{L_1} \frac{i_1 \mathrm{d}\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2}
\end{aligned}
$$

the Unit T(Tesla=$1N/(m \cdot A) = 10^4$ Gauss)

32-2 The magneticn field of a Current-Carrying loop

:::note[Biot-Savart Law]

/pic/phy2/Biot-Savart.png

$$
\vec{B} = \frac{\mu_0}{4\pi} \oint_L \frac{i d\vec{s} \times \hat{r}}{r^2}
$$

/pic/B_line.png

$$B = \int_{A_1}^{A_2} dB = \frac{\mu_0}{4\pi} \int_{A_1}^{A_2} \frac{i \sin\theta dx}{r^2}$$

$$r_0 = r\sin(\pi - \theta) = r\sin\theta, r = \frac{r_0}{\sin\theta}$$

$$x = -r_0 ctg\theta, \quad dx = \frac{r_0 d\theta}{\sin^2\theta}$$

$$B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 i}{4\pi} \frac{\sin\theta \cdot \frac{r_0 d\theta}{\sin^2\theta}}{r_0^2} = \frac{\mu_0 i}{4\pi r_0} \int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

$$= \frac{\mu_0 i}{4\pi r_0}(\cos\theta_1 - \cos\theta_2)$$

$$= \frac{\mu_0 i}{2\pi r_0} \propto \frac{1}{r_0}$$

/pic/phy2/B_circular.png

$$\left|d\vec{B}\right| = \left|d\vec{B}'\right|$$
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2}$$
$$dB_x = dB\cdot\cos\alpha$$
$$dB = \frac{\mu_0}{4\pi}\frac{ids}{r^2}\sin\theta$$
$$\theta = \frac{\pi}{2}, \sin\theta = 1, r = r_0/\sin\alpha$$
$$B_x = \int dB\cos\alpha$$
$$B = \frac{\mu_0 i}{4\pi}\oint\frac{\sin^2\alpha}{r_0^2}\cos\alpha ds$$
$$= \frac{\mu_0 i}{4\pi r_0^2}\sin^2\alpha\cos\alpha\cdot 2\pi R$$
$$B = \frac{\mu_0}{2}\frac{iR^2}{(R^2+r_0^2)^{\frac{3}{2}}} \xrightarrow{r_0=0} B = \frac{\mu_0 i}{2R}$$
$$\xrightarrow{r_0>>R} B = \frac{\mu_0 iR^2}{2r_0^3}$$

we can define magnetic dipole moment μ (just like $\vec{p} = q\vec{l}$)

$$B = \frac{\mu_0 iR^2}{2r_0^3} = \frac{\mu_0 i\pi R^2}{2\pi r_0^3} = \frac{\mu_0 iA}{2\pi r_0^3}$$

Define: $\mu = iA = i\pi R^2(\vec{\mu} = i\vec{A})$

$$B = \frac{\mu_0}{2}\frac{iR^2}{r_0^3} = \frac{\mu_0}{2\pi}\frac{i\pi R^2}{r_0^3} = \frac{\mu_0}{2\pi}\frac{\mu}{r_0^3}$$

:::

/pic/phy2/B_circ.png

Figure 33: image-20241023110346439

:::note[p757 sample33-5]

/pic/eg757.png

$$\mathrm{d}B = \frac{\mu_0 di}{2\pi d} = \frac{\mu_0 \frac{i}{a} dx}{2\pi d} d = \frac{R}{\cos\theta} \Rightarrow B_x = \int dB\cos\theta = \frac{\mu_0 i}{2\pi a R}\int \cos^2\theta dx$$

then

$dx = \mathrm{d}(R\tan\theta) = R\frac{\mathrm{d}}{\cos^2\theta}$

$$B_x = \frac{\mu_0 i}{2\pi a}\int_{-\theta}^{\theta} d\theta = \frac{\mu_0 i}{\pi a}\alpha = \frac{\mu_0 i}{\pi a}\arctan\frac{a}{2R}$$

$R >> a : (\alpha \to \tan\alpha) B = \frac{\mu_0 i}{2\pi R}$

$R \to 0, B = \frac{\mu_0 i}{2a}$

:::note[**Bohr model of the hydrogen atom**]

$$\begin{aligned}
&a_0 = 0.529A = 5.29\times 10^{-11} m\\
&\nu = 6.63\times 10^{15} Hz\\
&i = e\nu = 1.60\times 10^{-19}\times 6.63\times 10^{15} = 1.63\times 10^{-3} A\\
&B = \frac{\mu_0 i}{2R} = \frac{4\pi\times 10^{-7}\times 1.06\times 10^{-3}}{2\times 5.29\times 10^{-11}} = 12.6T\\
&\mu_{\mathrm{B}} = iA = 1.63\times 10^{-3}\times\pi\times(5.29\times 10^{-11})^2\\
&\ = 0.923\times 10^{-23} A\cdot m^2
\end{aligned}$$

called as Bohr Magnon

:::

B of Solenoid

A constant magnetic field can (in principle) be produced by an ∞ sheet of current. In practice, however, a constant magnetic field is often produced by a solenoid.

Parameters:i, n, R, L,we assume $R << L$

for a circular loop

$$B = \frac{\mu_0}{2}\frac{iR^2}{(R^2 + r_0^2)^{3/2}}$$

`/pic/phy2/B_solenoid.png`

Figure 34: image-20241023112023529

$$
\begin{aligned}
dB &= \frac{\mu_0}{2}\frac{R^2 indl}{[R^2+(x-l)^2]^{3/2}} \\
B &= \frac{\mu_0}{2}\int_{-L/2}^{L/2}\frac{R^2 indl}{\left[R^2+(x-l)^2\right]^{3/2}} \\
r &= \sqrt{R^2+(x-l)^2} = \frac{R}{\sin\beta} \\
\frac{x-l}{R} &= ctg\beta \Rightarrow dl = \frac{R}{\sin^2\beta}d\beta
\end{aligned}
$$

then

$$
\begin{aligned}
B &= \frac{\mu_0}{2}\int_{\beta_1}^{\beta_2}\frac{R^2 ni\frac{R}{\sin^2\beta}d\beta}{(\frac{R^2}{\sin^2\beta})^{3/2}} \\
&= \frac{\mu_0}{2}\cdot ni\int_{\beta_1}^{\beta_2}\sin\beta d\beta \\
&= \frac{1}{2}\mu_0 ni(\cos\beta_1-\cos\beta_2)
\end{aligned}
$$

use

$$
\cos\beta_1 = \frac{x+L/2}{\sqrt{R^2+(x+L/2)^2}}\cos\beta_2 = \frac{x-L/2}{\sqrt{R^2+(x-L/2)^2}}
$$

with

- $L\to\infty, \beta_1 = 0, \beta_2 = \pi$

  $B = \frac{1}{2}\mu_0 ni(1+1) = \mu_0 ni$
- $\beta_1 = 0, \beta_2 = \frac{\pi}{2}$

  $B = \frac{1}{2}\mu_0 ni(1-0) = \frac{1}{2}\mu_0 ni$

The field outside the ideal solenoid is zero.

:::note[eg:multi-layers solenoid]

/pic/B_outside_solenoid.png

Figure 35: image-20241023112523551

/pic/phy2/B_multilayers.png

$$B = \frac{1}{2}\mu_0 ni(\cos\beta_1 - \cos\beta_2)$$

With this

$$\begin{aligned} ni = \frac{Ni}{L} \Rightarrow \frac{jLdr}{L} = jdr = \frac{Ni}{2l(R_2 - R_1)}dr \\ \cos\beta_2 = -\cos\beta_1, \cos\beta_1 = \frac{l}{\sqrt{l^2 + r^2}} \end{aligned}$$

$$\begin{aligned} dB &= \frac{1}{2}\mu_0 \frac{Ni}{2l(R_2 - R_1)} \cdot \frac{2l}{\sqrt{l^2 + r^2}}dr \\ B &= \mu_0 jl \int_{R_1}^{R_2} \frac{dr}{\sqrt{l^2 + r^2}} \\ &= \mu_0 jl \ln \frac{R_2 + \sqrt{R_2^2 + l^2}}{R_1 + \sqrt{R_1^2 + l^2}} \end{aligned}$$

in practice

we define $\gamma = \frac{l}{R_1}, \alpha = \frac{R_2}{R_1}$

$$B_0 = \mu_0 h R_1 \gamma \ln \frac{\alpha + \sqrt{\alpha^2 + \gamma^2}}{1 + \sqrt{1 + \gamma^2}}$$

:::

32-3 Gauss Law and Ampere's Loop Law for B

let's first review

for $\vec{E}$

- Gauss law:

$$\oint \vec{E} \bullet d\vec{A} = \frac{1}{\varepsilon_0} \sum q, \nabla \bullet \vec{E} = \frac{\rho_e}{\varepsilon_0}$$

- Loop law

$$\oint \vec{E} \bullet d\vec{l} = 0, \nabla \times \vec{E} = 0(\vec{E} = -\nabla V)$$

$$\Phi_B = \iint \vec{B} \bullet d\vec{A} = \iint B\cos\theta dA(Unit : T \cdot m^2 = Wb)$$

for $\vec{B}$

- Gauss law:

$$\oint \vec{B} \bullet d\vec{A} = 0, \nabla \bullet \vec{B} = 0$$

- Loop law

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum i, \nabla \times \vec{B} = \mu_0 \vec{J}$$

-

$$\oint \vec{B} \mathrm{d}\vec{l} = \mu_0 \sum i + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{E} \mathrm{d}\vec{S}$$

The differential form is

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

`/pic/phy2/B-Ampere.png`

Figure 36: image-20241030100813902

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0(i_1 + i_3 - 2i_2)$$

:::note[egs]

[e.g.1]

inf long wire R,i uniform distribution

$$B \cdot 2\pi r = \mu_0 i \cdot \frac{r^2}{\pi R^2} \Rightarrow B = \frac{\mu_0 i r}{2\pi R^2}(r < R) \propto r$$

$$B \cdot 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}(r > R) \propto \frac{1}{r}$$

[e.g.2]

Consider an ∞ sheet of current described by n wires/length each carrying current i **into** the screen as shown. Calculate the B field.

for symmetry:vertical direction on the screen

we catch a w*w square

$$\oint \vec{B} \bullet \mathrm{d}\vec{l} = 2Bw = \mu_0 nwi \Rightarrow B = \frac{1}{2}\mu_0 ni$$

[e.g.3]

calculate B for \$$\infty$\$*solenoid*

view it as two sheets

$$\oint \vec{B} \bullet \mathrm{d}\vec{l} = Bw = \mu_0 nwi \Rightarrow B = \mu_0 ni$$

[e.g.4]

the B of Toroid (N total turns with current i)

/pic/Toroid.png

$$\oint \vec{B} \bullet \mathrm{d}\vec{l} = B2\pi r = \mu_0 Ni \rightarrow B = \mu_0 ni(n = \frac{N}{2\pi r})$$

:::

- Power door locks
- Magnetic cranes
- Electronic Switch “relay

32-4 The magnetic force on a carrying-current wire

remember

$$\mathrm{d}\vec{F} = i\mathrm{d}\vec{s} \times \vec{B}(\mathrm{d}\vec{F}_2 = i_2\mathrm{d}\vec{s}_2 \times \left(\oint_{L_1} \frac{i_1\mathrm{d}\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2}\right))$$

:::note[EG32-5,P738]

/pic/eg738.png

$$F_2 = F_\perp = \int_0^\pi iBR\mathrm{d}\theta \sin\theta = 2iBR$$

$$F = F_1 + F_2 + F_3 = iB(2L + 2R)$$

:::

for two parallel conductors

/pic/phy2/B-two-par.png

$$f = \frac{\mu_0 i_1 i_2}{2\pi d} i_1 = i_2 = ii \sqrt{\frac{fd}{\frac{\mu_0}{2\pi}}}$$

we define $i = 1A$ to be the current that make two $1m$ apart parallel conductors have force density $2 \times 10^{-7} N/m$

for convenience sake, we define $\hat{n}$ the unit normal vector of current loop

$$\vec{\mu} = iA\hat{n}\vec{\tau} = \vec{\mu} \times \vec{B}$$

this holds for arbitrary shape loop

/pic/phy2/torque-shape.png

$$\begin{aligned}
dF_1 &= ids_1 B \sin\theta_1 \\
dF_2 &= ids_2 B \sin\theta_2 \\
dF_1 &= dF_2 = iBdh \\
d\tau &= dF_1 x_1 + dF_2 x_2 = iBdA
\end{aligned}$$

for magnetic dipole

we define $\vec{\tau} = iA(\vec{n} \times \vec{B})$

then

$$U_p = -\int_{\infty}^{p} \tau \cdot d\vec{\theta} = \int \mu B \sin\theta d\theta = \mu B \cos\theta = \vec{\mu} \bullet \vec{B} \Rightarrow U = -\mu \bullet \vec{B}$$

remind that

$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \bullet \vec{E}$$

we have

$$\mu = A\vec{i}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \bullet \vec{B}$$

/pic/phy2/potential-energy-of-dipole.png

Figure 37: image-20241030105437447

System	(J/T)
Nucleus of N atom	2.0x10^-28
Proton	1.4x10^-26
Electron	9.3x10^-24
N atom	2.8x10^-23
Typical small coil	5.4x10^-6
Small bar magnet	5.0
Superconducting coil	400
The Earth	8.0x10^22

:::important[MRI (Magnetic Resonance Imaging)]

Proton Spin and Magnetic Moment:

- A single proton possesses a positive charge $ +|e| $ and an intrinsic angular momentum, known as “spin.”

- Naively imagining the charge circulating in a loop gives rise to a magnetic dipole moment $ $\mu$$.

  Behavior in an External Magnetic Field (B):

  - **Classically:** Torques will be present unless the magnetic moment $ $ is aligned with or against the magnetic field (B).
  - **Quantum Mechanics (QM):** The spin is always aligned either with or against the magnetic field (B).

  – **Anti-aligned State:** Energy $ U_2 = $\mu B$$**Aligned State:**$Energy$$$U_1 = -\mu B$$

- **Energy Difference:** $ $\Delta U = U_2 - U_1 = 2\mu B$$

$$\mu_{proton} = 1.36 \times 10^{-26} \text{ A m}^2 \quad B = 1 \text{ Tesla } (= 10^4 \text{ Gauss})$$
$$\Delta U = 2\mu B = 2.7 \times 10^{-26} \text{J}$$

$$h\nu = \Delta U \Rightarrow \nu = \frac{2.7\times10^{-26}}{6.6\times10^{-34}Js} = 41\text{MHz}$$

:::

more applications

:::note[app:Galvanometers]

/pic/galvanometers.png

Figure 38: image-20241030110554772

1. **Magnetic Force on a Current Loop:**
   - When a loop of wire carrying an electric current is placed in a magnetic field, the field exerts a torque on the loop, attempting to align the loop’s magnetic dipole moment with the field.
2. **Structure of a Galvanometer:**
   - Inside a galvanometer, there is a rotating coil attached to a pivot.
   - To return the pointer to its equilibrium position, a spring is included in the galvanometer, which creates a torque in the opposite direction to the rotation of the coil.

:::

motor is almost the same ,we just skip it

- fixed voltage:DC motors
- fixed current:AC motors