
:::tip[About the tutor]

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- E&M
- Optics
- Modern Physics

Intro

All of simple physics in 5 equations

$$1. F = q(E + v \times B)$$

$$2. \iiint E \cdot dA = \frac{Q_{inside}}{\epsilon_0}$$

$$3. \iiint B \cdot dA = 0$$

$$4. \iint E \cdot dl = -\frac{d\Phi_B}{dt}$$

$$5. \iint B \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Gravity , Electric and Magnetic Force has magical similarity!

Chap. 25 Charge and Coulomb's law

:::important[Crucial constant nums]

$$e = 1.602 \times 10^{-19} C$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 N \cdot m^2 \cdot C^{-2}$$

$$1C = 6 \times 10^{18} e$$

$$m_e = 9.1 \times 10^{-31} kg$$

...

25-2

$1n, 1p = 3$ quarks

$$1p(+e) = 2 \times (\frac{2}{3}e) + (-\frac{1}{3}e)$$

$$1n(0) = 2 \times (-\frac{1}{3}e) + \frac{2}{3}e$$

25-3

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}$$

Electrostatics in matter

Atom force **H**

Ionic Crystal force **NaCl**

Covalent Bond force **H-H**

Metal force **Au**

Coulomb's laws an exact result for stationary

charges and **not an approximation** form some higher law.

- Gravitational vs Electric Force

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}}{G \frac{m_1 m_2}{r^2}} = 4.17 \times 10^{42}$$

...note[e.g 25-3]

for a man ,he push his arms apart with a force of 450N,how many charge can he hold outstretched?

$$F = 450NQ = r\sqrt{\frac{F}{k}} = 4.47 \times 10^{-4}$$

...

...note[e.g 25-3-2]

How many electrons in a person by **Feynman**

treat people like water

$$\lambda_p = \frac{NA}{M_r(H_2O)} \times e(H_2O) = 3.3 \times 10^{23} e/g$$

then ,assume he is 80kg

$$n_e = \lambda_p m = 2.6 \times 10^{28} e$$

1% of a person

$$1\% \times 2.6 \times 10^{28} \times 1.6 \times 10^{-19} = 4.2 \times 10^7 C$$

then for two people 0.75m apart:

$$F = 9 \times 10^9 \times \left(\frac{4.2 \times 10^7 C}{0.75m} \right)^2 = 2.8 \times 10^{25} \approx W_{earth} = 6 \times 10^{24} kg \times 9.8m/s^2$$

...

you should distinguish the difference between the vec and scalars

25-4 Conductors and Insulators

- **Insulators** : $\leq 1e_c$ per cm^3 , *Glass,Plastics,Dry wood*
- **Conductors** : $\approx 10^{23}e_c$ per cm^3 , *Aluminum,Copper,Silver...*
- **Semiconductor** : $10^{10} \rightarrow 10^{12}$ per cm^3 , *Silicon,Germanium*
- **Superconductor** : $R = 0, B = 0$

(the following picture is from OCR,so some fault occurred)

Hg ($T_c=4.2K$, 1911)
 NbSn ($T_c=23K$, 1969) ,
 YBa₂Cu₃O, ($T_c=90K$, 1987)
 HgBaCaCuO ($T_c=156K$, 1988)
 ReO₁, F₂FeAs₃ ($T_c=55K$, 2008)
 (BaK)Fe₂As₂, ($T_c=39K$, 2008)
 H₃S ($T_c=210K$, High Pressure, 2016)
 LaH₁₀($T_c=250K$, High Pressure, 2019)
 Fe(Te, Se) ($T_c=144K$, 2008)
 (Ti, K, Rb, Cs)Fe_s(Se_s, ($T_c=30K$, 2010).....
 ThNi₂Se₂, ThNi₂S₂($T_c=3.7K$, 2013)

:::important[usage of $B = 0$]

- NMR(**N**uclear **M**agnetic **R**esonance)
- Brain research
- **Magnetic Levitation** (Maglev)

...

25-5 Continuous Charge Distribution

The charge density

- $\lambda = \frac{dq}{dx}$
- $\sigma = \frac{dq}{dA}$
- $\rho = \frac{dq}{dV}$

to calculate $q_0 \rightarrow (\text{distribution})\rho$

$$d\vec{F}_{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_0(\rho dV)}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

For a uniform ring of charge :

$$\begin{aligned}\lambda &= \frac{q}{2\pi R} \\ dF &= \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R d\phi}{(z^2 + R^2)} \\ F_z &= \int dF_z = \int dF \cos \theta \\ &= \int \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R d\phi}{(z^2 + R^2)} \frac{z}{\sqrt{z^2 + R^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R z}{(z^2 + R^2)^{3/2}} \int_0^{2\pi} d\phi \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_0 q z}{(z^2 + R^2)^{3/2}}\end{aligned}$$

for approximation

$$z \rightarrow \infty, F_z \rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{z^2}$$

For a uniform disk of charge :

$$\begin{aligned}
\sigma &= \frac{q}{\pi R^2} \\
dq &= \sigma dA = 2\pi\sigma\omega d\omega \\
dF_z &= \frac{1}{4\pi\epsilon_0} \frac{q_0(2\pi\sigma\omega d\omega)}{(z^2 + \omega^2)^{\frac{3}{2}}} \\
F_z &= \frac{1}{4\pi\epsilon_0} q_0 2\pi\sigma z \int_0^R \frac{\omega d\omega}{(z^2 + \omega^2)^{\frac{3}{2}}} \\
&= \frac{1}{4\pi\epsilon_0} \frac{2q_0q}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)
\end{aligned}$$

for approximation

$$z \rightarrow \infty, F_z \rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_0q}{z^2}$$

25-6 Charge conservation

$$e^+ + e^- \rightarrow 2\gamma n \rightarrow p + e^- + \nu_e$$

Chap. 26 Electric Fields

26-1 Field

- scalar field
- Vector field

Mass-field-mass: not action at a distance

26-2 E field

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

26-3 The Electric field of point charge

we just deal with **electrostatics** instead of **electrodynamics**

for a single charge q

$$\vec{E} = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \hat{r}$$

for the Electric Dipole ($p = 2Qa = Ql$)

remember $\vec{p} = q \cdot \vec{l}$, the direction is from $-q$ to $+q$

Figure 1: image-20241013135307542

$$E_x(x, 0) = 0, E_y(x, 0) = -2 \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \sin \theta \sin \theta = \frac{a}{r}, r = \sqrt{x^2 + a^2} E_y(x, 0) = -2 \frac{1}{4\pi\epsilon_0} \frac{Qa}{(x^2 + a^2)^{\frac{3}{2}}} \rightarrow -2k \frac{Qa}{r^3}$$

then for y-axis

$$E_x(0, r) = 0, E_y(0, r) = \frac{Q}{4\pi\epsilon_0} \frac{4ar}{r^4 \left(1 - \frac{a^2}{r^2}\right)^2} \rightarrow E_y(0, r) \approx +4 \frac{1}{4\pi\epsilon_0} \frac{Qa}{r^3} = 4k \frac{Qa}{r^3}$$

:::note[NaCl e.g.]

$$2a = 0.236nm, p_t = 2ea = 1.6 \times 3.78 \times 10^{-29}$$

$$p_e = 3 \times 10^{-29}(\text{measured})$$

it indicates that the electron is not entirely removed from Na to Cl

:::

for $x \gg a$

we can approximate E more precisely

$$E = \frac{pk}{x^3} \left[1 + \left(-\frac{3}{2}\right) \left(\frac{a}{x}\right)^2 + \dots \right] \propto \frac{1}{r^3}$$

26-4 The Electric field of Continuous Charge Distribution

once again we turn to distribution densities: $dq = \lambda dx = \sigma dA = \rho dV$

Figure 2: image-20241013141019977

$$dE_x = -\frac{1}{4\pi\epsilon_0} \frac{\lambda d\theta}{r} \sin \theta, dE_y = +\frac{1}{4\pi\epsilon_0} \frac{\lambda d\theta}{r} \cos \theta$$

then

$$E_x = 0, E_y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k \frac{\lambda d\theta}{r} \cos \theta = \frac{2k\lambda}{r} = \frac{\lambda}{2\pi\epsilon_0 r} \propto \frac{1}{r}$$

Figure 3: image-20241013141723256

$$\begin{aligned}
 dE &= \frac{\lambda ds}{4\pi\epsilon_0 r^2} = \frac{\lambda ds}{4\pi\epsilon_0(z^2 + R^2)} \\
 dE_z &= dE \cos \theta \\
 &= \frac{\lambda ds}{4\pi\epsilon_0(z^2 + R^2)} \cdot \frac{z}{(z^2 + R^2)^{1/2}} \\
 &= \frac{z \lambda ds}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \\
 E_z &= \frac{z}{4\pi\epsilon_0(z^2 + R^2)^{\frac{3}{2}}} \int_0^{2\pi R} \lambda ds \rightarrow \frac{q}{4\pi\epsilon_0 z^2}
 \end{aligned}$$

Figure 4: image-20241013142455784

$$\begin{aligned}
 dq &= 2\pi\omega \cdot d\omega \cdot \sigma \\
 dE &= \frac{z dq}{4\pi\epsilon_0(z^2 + \omega^2)^{3/2}} = \frac{z 2\pi\sigma\omega d\omega}{4\pi\epsilon_0(z^2 + \omega^2)^{3/2}} \\
 E &= \int dE = \frac{\sigma z}{2\epsilon_0} \int_0^R \frac{\omega d\omega}{(z^2 + \omega^2)^{3/2}} \\
 &= \frac{\sigma z}{4\epsilon_0} \int_0^R \frac{d(z^2 + \omega^2)}{(z^2 + \omega^2)^{3/2}} \\
 &= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{z^2}}}\right) \\
 R \gg z : E &\rightarrow \frac{\sigma}{2\epsilon_0} \\
 z \gg R : E &= \frac{\sigma}{2\epsilon_0} \left(\frac{1}{2} \left(\frac{R}{z}\right)^2 - \frac{3}{8} \left(\frac{R}{z}\right)^4 + \dots \right) \rightarrow \frac{q}{4\pi\epsilon_0 z^2}
 \end{aligned}$$

Summary

when $r \gg a/R$

- Dipole $p = 2Qa = Ql$
 - in line : $E = \frac{4Qa}{4\pi\epsilon_0 r^3} = \frac{2p}{4\pi\epsilon_0 r^3}$
 - off line : $E = \frac{2Qa}{4\pi\epsilon_0 r^3} = \frac{p}{4\pi\epsilon_0 r^3}$
- Point $E = \frac{q}{4\pi\epsilon_0 r^2}$

- Line $E = \frac{2\lambda}{4\pi\epsilon_0 r}$
- Ring $E = \frac{q}{4\pi\epsilon_0 z^2}$
- Disk $E = \frac{q}{4\pi\epsilon_0 z^2}$

26-5 E-Field Lines

$$\begin{cases} E_x = E_r \sin \theta \cos \phi \\ E_y = E_r \sin \theta \sin \phi \\ E_z = E_r \cos \theta \end{cases}$$

26-6 Point Charge in E-field

:::note[deflecting electrode system of an ink-jet printer]

$$\begin{aligned} \text{An ink drop : } m &= 1.3 \times 10^{-10} kg \quad L = 1.6 cm \\ q &= -1.5 \times 10^{-13} C, \quad E = 1.4 \times 10^6 N/C \\ v &= 18 m/s \end{aligned}$$

$$y = \frac{1}{2} \frac{qE}{m} \frac{L^2}{v^2} \approx 0.64 mm$$

one letter -> about 100 drops

100000 drops/s \rightarrow 1000 letters/s

:::

:::important[bonus:measure the elementary charge $e = 1.602 \times 10^{-19} C$]

skip

:::

then in an ununiformed field, for example

$$\begin{aligned} F &= qE \neq const \quad F(z) \\ \frac{d^2 z}{dt^2} &= \frac{q}{m} E(z) \\ E &= \frac{Qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \\ \frac{d^2 z}{dt^2} &= \frac{q}{m} \frac{Qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \end{aligned}$$

Figure 5: image-20241013144629047

26-7 Dipole in E-field

- Torque: $\vec{\tau} = pE \sin \theta = \vec{p} \times \vec{E}$
- $W = \int_{\theta_0}^{\theta} pE(\cos \theta - \cos \theta_0) = -\Delta U, U(\theta) = -\vec{p} \cdot \vec{E}$

we can divide molecules to

- Dipole-molecule H_2O
- non-dipole molecule C_2O

:::note[the max torque on H_2O]

$$\tau = qE \sin \theta = 9.3 \times 10^{-26} N \cdot m \theta : \pi \rightarrow 0 : W = 2pE = 1.9 \times 10^{-25} J \epsilon_{int} = \frac{3kT}{2} - 6.3 \times 10^{-21} >> \epsilon_{elect}$$

:::

26-8 Atom Nuclear Model

- Thomson model plum pudding

Rutherford:

$$E_{\max} = \frac{Q}{4\pi\epsilon_0 R^2} = 1.2 \times 10^{13} N/C$$

$$R = 1.0 \times 10^{-10} m$$

$$Q = 79e$$

$$U = 6 \text{ Mev} = 9.6 \times 10^{-13} J$$

$$\nu = \sqrt{\frac{2U}{m}} = 1.7 \times 10^7 m/s$$

$$F = qE_{\max} = ma$$

$$a = \frac{q}{m} E_{\max}$$

$$\Delta\nu = a\Delta t = \frac{q}{m} E_{\max} \frac{2R}{\nu} = 6.6 \times 10^3 m/s$$

$$\therefore \theta = tg^{-1} \frac{\Delta\nu}{\nu} = tg^{-1} \left(\frac{6.6 \times 10^3}{1.7 \times 10^7} \right) \approx 0.02$$

then , the result disobeys the experimental result!

Chap. 27 Gauss Law

27-2(3) Flux

We need to at first define flux and the vector of surface of a closed region.

$$\begin{aligned}\Phi &= \oint \vec{v} \bullet d\vec{A} \\ d\Phi &= \vec{v} \bullet d\vec{A} \\ d\vec{A} &= dydz\vec{i} + dzdx\vec{j} + dxdy\vec{k} \\ \vec{v} &= v_x\vec{i} + v_y\vec{j} + v_z\vec{k} \\ \vec{v} \bullet d\vec{A} &= v_x dydz + v_y dzdx + v_z dxdy\end{aligned}$$

if no source or sink of fluid

$$\Phi = \oint \vec{v} \cdot d\vec{A} = 0$$

for electrostatic case

$$\Phi = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

Geometry and Surface Integrals

we can illustrate that Gauss's Law \rightarrow Coulomb's Law

for a +Q sphere having radius R

$$E \cdot 4\pi R^2 = \frac{Q}{\epsilon_0}$$

27-5(6) Application Gauss's Law

...note[Uniform charged sphere]

$$q = \frac{4}{3}\pi a^3 \rho \Rightarrow E = \frac{\rho a^3}{3\epsilon_0 r^2} (r > a) E = \frac{\rho r}{3\epsilon_0} (r < a)$$

...

Conductors——Spherical Symmetry

since $E=0$ inside a conductor $Q_{inside} = 0$, charges are only on the surface

For a conductor, we obtain a cylinder gauss plane:

$$\varepsilon_0 E \Delta A + 0 + 0 = \sigma \Delta A E = \frac{\sigma}{\varepsilon_0}$$

Line——Cylindrical Symmetry

$$E \cdot 2\pi r h \cdot \varepsilon_0 = \lambda h \rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

A line charge λ (C/m) is placed along the axis of an uncharged conducting cylinder of inner radius $r_i = a$, and outer radius $r_o = b$ as shown.

we need to get σ_o

$$E_o = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma_o b}{\varepsilon_0 r} \Rightarrow \sigma_o = \frac{\lambda}{2\pi b}$$

Sheets——Planar Symmetry

$$\varepsilon_0 (2EA) = \sigma A \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

for two parallel sheets

$$E_{in} = \frac{\sigma}{\varepsilon_0}$$

Figure 6: image-20241013155219398

$$E_A = \frac{-\sigma_1}{2\varepsilon_0} E_B = \frac{+\sigma_1}{2\varepsilon_0} E_C = 0 E_D = \frac{+\sigma_1}{2\varepsilon_0}$$

always based on gauss plane

Figure 7: image-20241013160823665

$$Q_2 = -3Q_1$$

$$\sigma_i = -\frac{Q_1}{4\pi R_2^2} \sigma_o = -\frac{2Q_1}{4\pi R_2^2} E = \begin{cases} 0 & r < R_1 \\ \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} & R_1 < r < R_2 \\ -\frac{1}{4\pi\varepsilon_0} \frac{2Q_1}{r^2} & r > R_2 \end{cases}$$

Once we connect the two spheres with a wire, it becomes a whole conductor with $-2Q_1$

Figure 8: image-20241013162838562

:::note[eg2 Cylinders]

$$\sigma_i = -\frac{\lambda}{2\pi R}, \sigma_o = \sigma_t + \frac{\lambda}{2\pi R}$$

$$E_r = \begin{cases} \frac{\lambda}{2\pi\epsilon_0 r} & r < R \\ \frac{\lambda}{2\pi\epsilon_0 r} + \frac{\sigma R}{\epsilon_0 r} & r > R \end{cases}$$

27-7 Experimental Tests of Gauss' Law and Coulomb's Law

just skip

Chap. 28 Electric Potential U & V

28-1 Potential Energy

Figure 9: image-20241013164129339

The union energy of two points of charge :

$$U_b - U_a = -W_{a,b} = - \int_a^b \vec{F} \cdot d\vec{l}$$

The circuit Law of electrostatics field

$$\oint \vec{E} \cdot d\vec{l} = 0 \nabla \times \vec{E} = 0$$

The gauss' law(the same as **Math Analysis**):

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} = \iiint \frac{\rho}{\epsilon_0} dV$$

28-2 Electric Potential

$$V_p = \frac{U_p}{q_0}$$

$$V_B - V_A \equiv \frac{W_{AB}}{q_0} = - \int_A^B \vec{E} \cdot d\vec{l}$$

28-3 Calculate Potential from E

$$W_{ab} = - \int_a^b \vec{E} \cdot d\vec{l} V_p = W_{\infty p} = - \int_{\infty}^p \vec{E} \cdot d\vec{l}$$

for a point charge:

$$V_b - V_a = - \int_{r_a}^{r_b} \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

Figure 10: image-20241013170530297

$$r \gg a V_r = \frac{1}{4\pi\epsilon_0} \frac{r_2 - r_1}{r_1 r_2} r_2 - r_1 \approx 2a \cos \theta, r_1 r_2 = r^2 V_r = \frac{1}{4\pi\epsilon_0} \frac{2aq \cos \theta}{r^2} = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2}$$

Figure 11: image-20241013171208250

$$\begin{aligned} V(r) &= \sum_i V_i(r_i) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r-d} + \frac{-2q}{r} + \frac{q}{r+d} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{2qd^2}{r(r^2 - d^2)} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2qd^2}{r^3(1 - d^2/r^2)} \rightarrow \frac{Q}{4\pi\epsilon_0 r^3} (d \ll r) \end{aligned}$$

- Point charge $\propto \frac{1}{r}$
- Dipole $\propto \frac{1}{r^2}$
- Quadrupole $\propto \frac{1}{r^3}$

we can trace back to the distribution reflected by n-dipoles.

:::[egs for potential calculation]

for a charged sphere shell

$$E = [r \geq R] \cdot \frac{q}{4\pi\epsilon_0 r^2}$$

$$V(P) = \frac{q}{4\pi\epsilon_0 \max\{r_P, R\}}$$

$$U = \frac{1}{2} \int V dq = \frac{q^2}{8\pi\epsilon_0 R}$$

$$W = mc^2 = \frac{e^2}{8\pi\epsilon_0 R} \Rightarrow R_e \approx 1.4 \times 10^{-15} m$$

for a ring

$$V = \oint \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{\sqrt{z^2 + R^2}} = \frac{q}{4\pi\epsilon_0 \sqrt{z^2 + R^2}}$$

for a disk

$$\begin{aligned} dq &= 2\pi\omega \cdot d\omega \cdot \sigma \\ dV &= \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\epsilon_0 \sqrt{z^2 + \omega^2}} \\ V &= \int_0^R \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\epsilon_0 \sqrt{z^2 + \omega^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z) \rightarrow \frac{q}{4\pi\epsilon_0 z} \end{aligned}$$

...

Sparks

High electric fields can ionize nonconducting materials(**dielectrics**)

(Insulator->Conductor)

:::note[ball breakdown]

we have two ball shell with same potential V

Ball 2 is as twice large as Ball 1

as V goes up, the Ball 1 will breakdown first

$$E_s = \frac{Q}{4\pi\epsilon_0 r^2}, V = \frac{Q}{4\pi\epsilon_0 r} \Rightarrow E = \frac{V}{r}$$

then $r_1 < r_2 \Rightarrow E_1 > E_2$

...

- $\Delta V > 0$ means we go uphill
- $\Delta V < 0$ means we go downhill

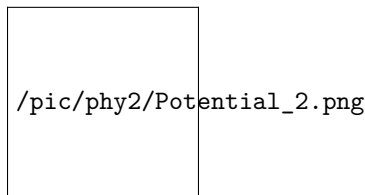


Figure 12: image-20241015101813495

$$V(r) = V_{\infty} - \int_{\infty}^c \vec{E}_l \cdot \vec{l} - \int_c^b \vec{E}_l \cdot \vec{l} - \int_b^a \vec{E}_l \cdot \vec{l} - \int_a^r \vec{E}_l \cdot \vec{l} = V_{\infty} - \left(\int_{\infty}^c + \int_b^a \right) \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr - \int_a^r \frac{1}{4\pi\epsilon_0} \frac{Qr}{a^3} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{r} \right)$$

you can split into two patterns

28-4 Equipotentials

when in the equipotential surface we can conclude

$$\vec{E} \cdot d\vec{l} \equiv V$$

28-5 Potential of a charged conductors

Claim: The surface of a conductor is an equipotential surface when two sphere conductors are attached to each other.

$$\frac{Q_A}{Q_B} = \frac{r_A}{r_B}$$

for example, when a point of charge is placed off-center inside a sphere conductor, the inside surface will be non-uniform and the outside surface will be uniform.

28-6 Calculate E from Potential

$$V_P = \int_P^{\infty} \vec{E} \cdot d\vec{l}$$

- Graphically the E-field line is the fastest-descending line of equipotential surfaces
- Math:

from $dW = -q_0 dV$

$$dW = \vec{F} \cdot d\vec{l} = q_0 \vec{F} \cdot d\vec{l} = q_0 E dl \cos \theta E \cos \theta = -\frac{dV}{dl}$$

$$E_l = -\frac{dV}{dl} \Rightarrow \vec{E} = -\nabla V$$

– Cartesian coordinates:

$$\nabla V = \frac{\partial V}{\partial x} \bar{x} + \frac{\partial V}{\partial y} \bar{y} + \frac{\partial V}{\partial z} \bar{z}$$

– Spherical coordinates:

$$\nabla V = \frac{\partial V}{\partial r} \bar{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \bar{\theta} + \frac{1}{r \sin \varphi} \frac{\partial V}{\partial \varphi} \bar{\varphi}$$

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2} \theta = \arccos \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \arccos \frac{z}{r} = \begin{cases} \arctan \frac{\sqrt{x^2 + y^2}}{z} & \text{if } z > 0 \\ \pi + \arctan \frac{\sqrt{x^2 + y^2}}{z} & \text{if } z < 0 \\ +\frac{\pi}{2} & \text{if } z = 0 \text{ and } \sqrt{x^2 + y^2} > 0 \\ \text{undefined} & \text{if } x = y = z = 0 \end{cases}$$

$$V = 3x^2 + 2xy - z^2$$

$$\vec{E} = (-6x - 2y, -2x, 2z)^T$$

$$\vec{E} = \frac{2aq}{4\pi\epsilon_0 r^3} (2 \cos \theta, \sin \theta, 0)_{sp}^T$$

then we have two eg

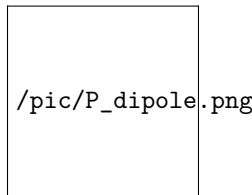


Figure 13: image-20241015114126009

$$V(r) = \frac{q}{4\pi\epsilon_0} \frac{r_2 - r_1}{r_1 r_2} \rightarrow \frac{1}{4\pi\epsilon_0} \frac{2aq \cos \theta}{r^2} (r \gg a)$$

then

$$V(r, \theta) = \frac{1}{4\pi\epsilon_0} \frac{2aq \cos \theta}{r^2} E_r = -\frac{\partial V}{\partial r} E_\theta = -\frac{\partial V}{\partial \theta} \Rightarrow \vec{E} = \frac{2aq}{4\pi\epsilon_0 r^3} ((2 \cos \theta) \hat{r} + \sin \theta \hat{\theta})$$

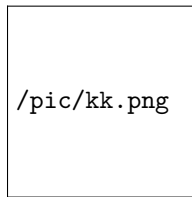


Figure 14: image-20241015114612342

it's easy to see that $\arg \max_{\theta} ||\vec{E}|| = \frac{\pi}{2}$

:::note[eg. disk]

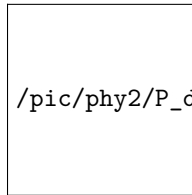


Figure 15: image-20241015114925818

$$\begin{aligned} dq &= 2\pi\omega \cdot d\omega \cdot \sigma \\ dV &= \frac{dq}{4\pi\epsilon_0 \sqrt{z^2 + \omega^2}} = \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\epsilon_0 \sqrt{z^2 + \omega^2}} \\ V &= \int_0^R \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\epsilon_0 \sqrt{z^2 + \omega^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z) \\ E_z &= -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\epsilon_0} \left(\frac{2z}{2\sqrt{R^2 + z^2}} - 1 \right) \\ &= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{1 + (R/z)^2}} \right) \end{aligned}$$

:::important[a e.g for getting v with ΔU]

$$\frac{1}{2}mv^2 = K = -\Delta U = -q\Delta V \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

we can measure α *partical' slargevelocitybymeasuringelectrostaticinformationlikedifferenceofpotential*
 \dots

Appendix method of images

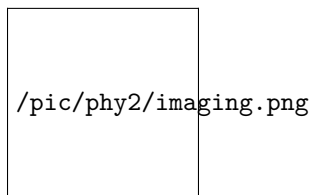


Figure 16: image-20241015120204563

we can see

$$E(\vec{r}_s) = \frac{\sigma(\vec{r}_s)}{\varepsilon_0}$$

we can see that the induced charge distribution generated with a point of charge and a conductor sheet is the same as which is made by two charges. It can be proved with symmetry theorem.

Chap. 30 Capacitance and Dielectrics

30-1 Capacitors

Classic Capacitors

- flashbulb:capacitor draw energy from battery (s) then release it through bulb(ms)
- Laser pulse
- thermonuclear fusion $10^{14}W, 10^{-9}s, 10^8K$

Definition of Capacitance : two spatially seperated conductors(+q/-q) $C = \frac{q}{\Delta V}$

one single conductor is capacitor!

:::note[eg:Parallel Plate capacitor]

$$q = \sigma A \Delta V = - \int_A^B \vec{E} \cdot d\vec{l}$$

...

:::note[eg:Parallel Plate capacitor]

$$q = \sigma A \Delta V = - \int_A^B \vec{E} \cdot d\vec{l}$$

since

$$EA\epsilon_0 = \sigma A \Rightarrow E = \frac{\sigma}{\epsilon_0} \Rightarrow \Delta V = \frac{q}{A\epsilon_0} d \Rightarrow C = \frac{\epsilon_0 A}{d}$$

...

- condenser: $C \propto \frac{1}{d} \xrightarrow{\text{fixed } \Delta V} Q \propto \frac{1}{d}, I = \frac{dQ}{dt}$: the vibration \rightarrow different I

:::note[eg:Cylindrical Capacitor]

Figure 17: image-20241018170156213

$+Q, -Q$ on surface, ΔV

$$2\pi r \cdot L \cdot E \cdot \epsilon_0 = Q \Rightarrow E = \frac{Q}{2\pi\epsilon_0 L r}$$

$$\Delta V = \int_b^a \vec{E} \cdot d\vec{l} = \frac{Q}{2\pi\epsilon_0 L \ln \frac{b}{a}}, C = \frac{2\pi\epsilon_0 L}{\ln \frac{b}{a}}$$

:::note[ex:TV signals transmit/coaxial cable]

$$a = r_i = 0.15, b = r_o = 2.1$$

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln(2.1/0.15)} = 21 \times 10^{-12} F/m = 21 pF/m$$

...

:::note[eg:spherical capacitor]

Figure 18: image-20241018170915119

$$\vec{E} = \frac{q\hat{r}}{4\pi\epsilon_0 r^2} \Rightarrow \Delta V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \Rightarrow C = \frac{4\pi\epsilon_0 ab}{b-a}$$

for earth $R = 6.37 \times 10^6 m \Rightarrow C = 7.1 \times 10^{-4} F = 710 \mu F$

...

Summary

- Parallel Plate: $C = \frac{\epsilon_0 A}{d}$
- Cylindrical Capacitor: $C = \frac{2\pi\epsilon_0 L}{\ln \frac{b}{a}}$
- Spherical $C = 4\pi\epsilon_0 \frac{ab}{b-a}$

Parallel & Series

Figure 19: image-20241018171312744

Figure 20: image-20241018171326385

for capacitors in parallel

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_2 = Q_1 \frac{C_2}{C_1} \Rightarrow C = C_1 + C_2$$

for capacitors in series

$$V_{ab} = \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow C = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

Figure 21: image-20241018171611735

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2}$$

$$C = \frac{2\pi\epsilon_0 L}{\ln \frac{b}{a} \ln \frac{d}{c}}$$

30-2 Energy storage in E-field

for paralleled capacitor

$$dW = V(q) dq = \frac{q dq}{C} \Rightarrow W = \int_0^Q \frac{q dq}{C} = \frac{1}{2} \frac{Q^2}{C} = \frac{CV^2}{2} \Rightarrow \frac{1}{2} CV^2$$

in some questions, we need to verify the fact whether Q is const or V is const.

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{A\epsilon_0/d} \Rightarrow E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \Rightarrow U = \frac{1}{2} E^2 \epsilon_0 A d$$

Figure 22: image-20241018171919786

the energy density $u = \frac{W}{Ad} = \frac{1}{2}\epsilon_0 E^2 (J \cdot m^{-3})$

you can calculate with u with Cylindrical Capacitor

$$U = \int_a^b \frac{1}{2} \epsilon_0 E^2 = \frac{\epsilon_0}{2} \int \left(\frac{\lambda}{2\pi\epsilon_0 r} \right)^2 L 2\pi r dr = \frac{1}{2} \frac{Q^2}{2\pi\epsilon_0 L} \ln \frac{b}{a} = \frac{1}{2} \frac{Q^2}{C}$$

:::note[ACT2 cylindrical capacitors]

with two cylindrical capacitors $((a, b)$ vs $(2a, 2b)$)

$$C = \frac{2\pi\epsilon_0 L}{\ln \left(\frac{r_{outer}}{r_{inner}} \right)}$$

so $C_1 = C_2$

::: note[P687 30-7]

Problem 30- 7 (page 687) . An isolated conducting sphere whose radius R is 6.85cm carries a charge $q = 1.25nC$. (a) How much energy is stored in the electric field of this charged conductor? (b) What is the energy density (u) at the surface of the sphere? (c) What is the radius R_0 of the imaginary spherical surface such that one-half of the stored potential energy lies within it?

R=6.85cm, q=1.25nC

(a) $U=?$

(b) $u=?$ (at the surface of the sphere)

(c) $R_0=?$ At $R < R_0, U' = \frac{1}{2}U$

(a)

$$C = 4\pi\epsilon_0 R$$

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\epsilon_0 R} = 1.03 \times 10^{-7} J = 103nJ$$

(b)

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \frac{q^2}{16\pi^2 \epsilon_0^2 R^4} = \frac{q^2}{32\pi^2 \epsilon_0 R^4} = 25.4nJ/cm^3$$

(c)

$$\begin{aligned}
\int_R^{R_0} \frac{1}{2} \varepsilon_0 E^2 d\nu &= \int_{R_0}^{\infty} \frac{1}{2} \varepsilon_0 E^2 d\nu \\
\int_R^{R_0} \frac{1}{2} \varepsilon_0 \frac{q^2}{16\pi^2 \varepsilon_0^2 r^4} 4\pi r^2 dr &= \int_{R_0}^{\infty} \frac{1}{2} \varepsilon_0 \frac{q^2}{16\pi^2 \varepsilon_0^2 r^4} 4\pi r^2 dr \\
\int_R^{R_0} \frac{dr}{r^2} &= \int_{R_0}^{\infty} \frac{dr}{r^2} \\
\frac{1}{R} - \frac{1}{R_0} &= \frac{1}{R_0} \\
R_0 &= 2R = 13.7 \text{ cm}
\end{aligned}$$

...

30-3 Dielectrics

Capacitor with dielectrics

- **Empirical observation:** Inserting a non-conducting material between the plates of a capacitor changes the VALUE of the capacitance.
- dielectric constant $C = \kappa_e C_0, \kappa_e > 1$ (glass=5.6, water=78)
- C_0 means the capacitance with vacuum (air)

with dielectric constant κ_e and const Q

$$V = \frac{Q}{C} = \frac{V_0}{\kappa_e} \Rightarrow E = \frac{E_0}{\kappa_e}$$

with const V

$$Q' = \kappa_e C_0 V$$

- parallel-plate : $C = \frac{\kappa_e \varepsilon_0 A}{d}$
- cylindrical $C = \frac{\kappa_e 2\pi \varepsilon_0 L}{\ln \frac{b}{a}}$
- spherical $C = 4\pi \varepsilon_0 \kappa_e \frac{ab}{b-a}$

$$\text{for point charge } E = \frac{Q}{4\pi \varepsilon_0 \kappa_e r^2}$$

:::note[the increasing C]

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d_1 + d_2} > \frac{\varepsilon_0 A}{d} (d > d_1 + d_2)$$

Polarization effect :

$$V = Ed = (E_0 - E')d < E_0dC = \frac{q}{(E_0 - E')d} > C_0$$

:::

microscopic mechanism of polarization

- Non-polar dielectrics $\vec{p} = q\vec{d} = 0$
- Polar dielectrics $\vec{p} = q\vec{d} \neq 0$

for non-polar dielectrics

we have

Induced electric dipole moment

Electric displacement polarization

Figure 23: image-20241020222558545

Polar dielectrics

$$\sum \vec{p} \neq 0$$

Alignment polarization

In high frequency field , Electric displacement polarization plays an important role

Polarization

Polarization intensity \vec{P}

$$\vec{P} = \frac{\sum \vec{p}_m}{\Delta V} (C \cdot m^{-2}) = nq\vec{l}$$

Figure 24: image-20241020222921535

$$\begin{aligned}
dN &= ndV = nldA \cos \theta \\
dq' &= qdN = nql dA \cos \theta \\
&= PdA \cos \theta \\
&= \vec{P} \bullet d\vec{A} \\
\oint \vec{P} \bullet d\vec{A} &= \sum_{out} q' = - \sum_{in} q' \\
dq' &= \vec{P} \bullet d\vec{A} = P \cos \theta \cdot dA \\
\sigma' &= \frac{dq'}{dA} = P \cos \theta = \vec{P} \bullet \vec{n} = P_n
\end{aligned}$$

Depolarization Field

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

Figure 25: image-20241020223121085

$$\begin{aligned}
\sigma'_e &= P_n = P \cos \theta \\
dE' &= \frac{dq'}{4\pi\epsilon_0 R^2} = \frac{\sigma'_e dA}{4\pi\epsilon_0 R^2} = \frac{P \cos \theta dA}{4\pi\epsilon_0 R^2} \\
dA &= Rd\theta \cdot R \sin \theta d\varphi \\
&= R^2 \sin \theta d\theta d\varphi \\
dE' &= \frac{P}{4\pi\epsilon_0} \cos \theta \sin \theta d\theta d\varphi \\
dE'_z &= dE' \cos(\pi - \theta) = -dE' \cos \theta \\
&= -\frac{P}{4\pi\epsilon_0} \cos^2 \theta \sin \theta d\theta d\varphi \\
E'_z &= \oint_z dE'_z = -\frac{P}{4\pi\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta \int_0^{2\pi} d\varphi = -\frac{P}{3\epsilon_0}
\end{aligned}$$

:::note[eg2 parallel plate]

$$\sigma'_e = P \cos \theta = PE' = \frac{\sigma'_e}{\epsilon_0}$$

:::

Polarization law

$$\vec{P} \Rightarrow \sigma'_e \Rightarrow \vec{E}' \Rightarrow \vec{E}[\vec{P}(\vec{E})]$$

for general isotropic materials

$$\vec{P} = \chi_e \epsilon_0 \vec{E} (\kappa_e = 1 + \chi_e)$$

χ_e : Polarization coefficient

for crystal materials:

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \chi_{xx} \chi_{xy} \chi_{xz} \\ \chi_{yx} \chi_{yy} \chi_{yz} \\ \chi_{zx} \chi_{zy} \chi_{zz} \end{pmatrix} \begin{pmatrix} \epsilon_0 E_x \\ \epsilon_0 E_y \\ \epsilon_0 E_z \end{pmatrix}$$

:::note[eg:ferroelectric material]

Electric hysteresis effect Similar to the magnetic hysteresis effect.

:::

Electric Displacement vector \vec{D}

$$\vec{E}_0 \rightarrow \vec{P} \rightarrow \boldsymbol{\sigma}'_e \rightarrow \vec{E}' \rightarrow \vec{E} = \vec{E}_0 + \vec{E}'$$

\$ \$ means:

- Electric displacement vec
- electric induction

Figure 26: image-20241020224048323

let q_0 be inside charge, q' be induced charge

$$\begin{aligned}
\varepsilon_0 \oint \vec{E} \bullet d\vec{A} &= \sum_{In} (q_0 + q') \\
\oint \vec{P} \bullet d\vec{A} &= - \sum_{In} q' \\
\iint \varepsilon_0 \vec{E} \bullet d\vec{A} &= \sum_{In} q_0 - \oint \int \vec{P} \bullet d\vec{A} \\
\oint (\varepsilon_0 \vec{E} + \vec{P}) \bullet d\vec{A} &= \sum_{In} q_0 \\
\oint \vec{D} \bullet d\vec{A} &= \sum_{In} \\
\vec{D} = \varepsilon_0 \vec{E} + \vec{P} &= (1 + \chi_e) \varepsilon_0 \vec{E} = \kappa_e \varepsilon_0 \vec{E} \\
\boxed{\oint \vec{D} \bullet d\vec{A} = \sum_{In} q_0}
\end{aligned}$$

:::note[eg paralleled plate]

$$\begin{aligned}
\oint \vec{D} \bullet dA &= \sum q_0 \\
D_1 \Delta A + D_2 \Delta A &= \sigma_{e0} \Delta A \\
\vec{E}_1 = 0, D_1 = \kappa_{e1} \varepsilon_0 E_1 = 0, \therefore D_1 &= 0 \\
\therefore D = D_2 = \sigma_{e0} = \varepsilon_0 E_0 \\
\therefore E = \frac{D}{\kappa_e \varepsilon_0} = \frac{\varepsilon_0 E_0}{\kappa_e \varepsilon_0} = \frac{E_0}{\kappa_e} \\
\vec{D} = \kappa_e \varepsilon_0 \vec{E}
\end{aligned}$$

...

:::note[eg charge in a hole]

$$\begin{aligned}
\oint \vec{D} \bullet d\vec{A} &= \sum q_0 \\
4\pi r^2 D &= q_0 \\
D &= \frac{q_0}{4\pi r^2} \\
E &= \frac{D}{\kappa_e \varepsilon_0} = \frac{q_0}{4\pi \varepsilon_0 \kappa_e r^2} = \frac{E_0}{\kappa_e}
\end{aligned}$$

$$\oint \vec{D} \cdot d\vec{l} \neq 0$$

...

Pressure Electric Effect

Figure 27: image-20241020224455711

Chap 31 The Steady Current*(Optional)

31-1 The Steady Current and Conduction Law

$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

the current density vector \vec{j}

$$\begin{aligned} di &= \vec{j} \bullet d\vec{A}, \\ i &= \iint_A \vec{j} \bullet d\vec{A} \\ &= \iint_A j \cos \theta dA \end{aligned}$$

the steady current condition

$$\iint_A \vec{j} \bullet d\vec{A} = 0 \Leftrightarrow j_1 \Delta A_1 = j_2 \Delta A_2$$

Ohm Law

- linear devices :Metal, liquid containing acid, alkali, salt
- nonlinear devices :Evacuated tube, transistor

Figure 28: image-20241020225112215

Conductance $G = \frac{1}{R} = \frac{dI}{dV} (Unit : S)$

Resistivity, & conductivity :

$$R = \int \rho \frac{dl}{A}, \sigma = \frac{1}{\rho}$$

or differential form: $j = \frac{E}{\rho} = \sigma E$

Figure 29: image-20241020225354688

$$R = \int \rho \frac{dl}{A} = \int_a^\infty \rho \frac{dr}{2\pi r^2}$$

$$= \frac{\rho}{2\pi} \left[-\frac{1}{r} \right]_a^\infty = \frac{\rho}{2\pi a}$$

note[$\rho - T$ relativity]

for metal

$$\rho(T) = \rho_0 + \alpha T$$

...

Electric power and Joule Law

$$P = \frac{W}{\Delta t} = iV = i^2 R = \frac{V^2}{R}$$

Microscopic explanation of Ohm Law

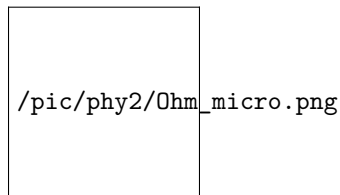


Figure 30: image-20241021092841895

note[drift speed of electric charge]

$$j = 2.4 A/mm^2 b = 8.4 \times 10^{28} m^{-3} j = \frac{\Delta i}{\Delta A} = n e u = \frac{j}{ne} = 1.8 \times 10^{-4} m/s \ll v_t \approx 10^5 m/s$$

...

31-2 Source and Electromotive Force(emf)

Chap 32/33 The Steady Magnetic Field

32-1 Basic phenomena

first hard disk 1957:50 platters

Basic Phenomena of Magnetism

- attract small bits of metal
- have two poles
- like poles repel, and unlike poles attract
- oersted experiment: $B, i \rightarrow F$
- Solenoid is similar to a bar magnet
- Interaction between electric currents

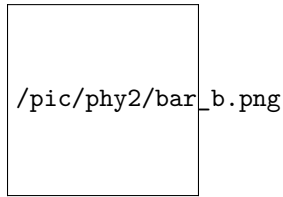


Figure 31: image-20241021084128203

the magnetic field line distribution is like electric field of a electdipole pair.

Magnetic Monopoles

since no monopole has ever been found $\oint \vec{B} \cdot d\vec{A} = 0$

Magnetic field

(Ampere) molecular current: he bind solenoid and a magnet

electric charge in motion is the source of Magnetic Fields

Ampere's Law(1820.12.4)

:::note[Ampere's Law]

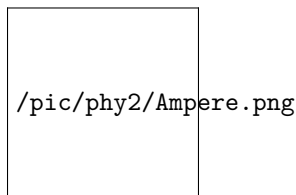


Figure 32: image-20241021085424748

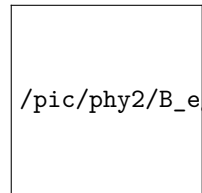
- Current element $id\vec{s}$

$$dF_{12} \propto \frac{i_1 i_2 ds_1 ds_2}{r_{12}^2} = \frac{\mu_0}{4\pi} \cdot \frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}$$

$\mu_0 = 4\pi \times 10^{-7} (N \cdot A^{-2})$: Permeability constant

...

the consider two examples



/pic/phy2/B_eg1.png

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi} \frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}$$

$$d\vec{s}_1 \perp \hat{r}_{12}$$

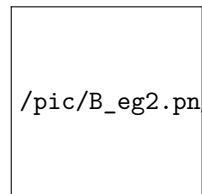
$$\therefore dF_{12} = \frac{\mu_0}{4\pi} \frac{i_1 i_2 ds_1 ds_2}{r_{12}^2}$$

$$d\vec{F}_{21} = \frac{\mu_0}{4\pi} \frac{i_1 d\vec{s}_1 \times (i_2 d\vec{s}_2 \times \hat{r}_{21})}{r_{21}^2}$$

$$d\vec{s}_2 \perp \hat{r}_{21}$$

$$\therefore dF_{21} = \frac{\mu_0}{4\pi} \frac{i_1 i_2 ds_1 ds_2}{r_{12}^2}$$

$$dF_{12} = dF_{21}$$



/pic/B_eg2.png

$$dF_{12} = 0, dF_{21} = \frac{\mu_0}{4\pi} \frac{i_1 i_2 ds_1 ds_2}{r_{12}^2}$$

The Magnetic Induction Strength

- Coulomb's Law

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}, \vec{F}_{12} = q_2 \vec{E}_1, \vec{E}_1 = \frac{\vec{F}_{12}}{q_2} \therefore \vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}^2} \hat{r}_{12}$$

- Ampere's Law

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi} \frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}$$

let us call $i_2 d\vec{s}_2$ the element of a test electric current

$$d\vec{F}_2 = i_2 d\vec{s}_2 \times \frac{\mu_0}{4\pi} \oint_{L_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2}$$

Define: $\vec{B}_1 = \frac{\mu_0}{4\pi} \oint_{L_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2}$

$$\therefore d\vec{F}_2 = i_2 d\vec{s}_2 \times \vec{B}_1$$

$$d\vec{F}_2 = i_2 d\vec{s}_2 \times \vec{B}_1$$

$$dF_2 = i_2 ds_2 B_1 \sin \theta$$

$$\theta = 0, dF_2 = 0$$

$$\theta = \frac{\pi}{2}, dF_2 \text{ maximum}$$

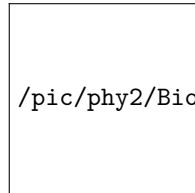
Define: $B_1 = \frac{(dF_2)_{\max}}{i_2 ds_2}$

$$\Rightarrow \vec{B}_1 = \frac{\mu_0}{4\pi} \oint_{L_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2}$$

the Unit T (Tesla = $1N/(m \cdot A) = 10^4$ Gauss)

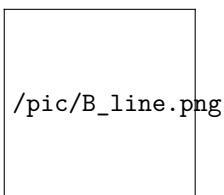
32-2 The magnetic field of a Current-Carrying loop

:::note[Biot-Savart Law]

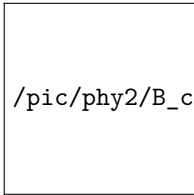


/pic/phy2/Biot-Savart.png

$$\vec{B} = \frac{\mu_0}{4\pi} \oint_L \frac{id\vec{s} \times \hat{r}}{r^2}$$



$$\begin{aligned}
 B &= \int_{A_1}^{A_2} dB = \frac{\mu_0}{4\pi} \int_{A_1}^{A_2} \frac{i \sin \theta dx}{r^2} \\
 r_0 &= r \sin(\pi - \theta) = r \sin \theta, r = \frac{r_0}{\sin \theta} \\
 x &= -r_0 \cot \theta, \quad dx = \frac{r_0 d\theta}{\sin^2 \theta} \\
 B &= \int_{\theta_1}^{\theta_2} \frac{\mu_0 i \sin \theta \cdot \frac{r_0 d\theta}{\sin^2 \theta}}{r_0^2} = \frac{\mu_0 i}{4\pi r_0} \int_{\theta_1}^{\theta_2} \sin \theta d\theta \\
 &= \frac{\mu_0 i}{4\pi r_0} (\cos \theta_1 - \cos \theta_2) \\
 &= \frac{\mu_0 i}{2\pi r_0} \propto \frac{1}{r_0}
 \end{aligned}$$



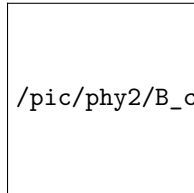
/pic/phy2/B_circular.png

$$\begin{aligned}
 |d\vec{B}| &= |d\vec{B}'| \\
 d\vec{B} &= \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \hat{r}}{r^2} \\
 dB_x &= dB \cdot \cos \alpha \\
 dB &= \frac{\mu_0}{4\pi} \frac{ids}{r^2} \sin \theta \\
 \theta &= \frac{\pi}{2}, \sin \theta = 1, r = r_0 / \sin \alpha \\
 B_x &= \int dB \cos \alpha \\
 B &= \frac{\mu_0 i}{4\pi} \oint \frac{\sin^2 \alpha}{r_0^2} \cos \alpha ds \\
 &= \frac{\mu_0 i}{4\pi r_0^2} \sin^2 \alpha \cos \alpha \cdot 2\pi R \\
 B &= \frac{\mu_0}{2} \frac{iR^2}{(R^2 + r_0^2)^{\frac{3}{2}}} \xrightarrow{r_0=0} B = \frac{\mu_0 i}{2R} \\
 &\xrightarrow{r_0 \gg R} B = \frac{\mu_0 i R^2}{2r_0^3}
 \end{aligned}$$

we can define magnetic dipole moment μ (just like $\vec{p} = q\vec{l}$)

$$B = \frac{\mu_0 i R^2}{2r_0^3} = \frac{\mu_0 i \pi R^2}{2\pi r_0^3} = \frac{\mu_0 i A}{2\pi r_0^3} \quad \text{Define: } \mu = iA = i\pi R^2 (\vec{\mu} = i\vec{A})$$

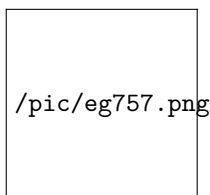
...



/pic/phy2/B_circ.png

Figure 33: image-20241023110346439

:::note[p757 sample33-5]



$$dB = \frac{\mu_0 di}{2\pi d} = \frac{\mu_0 \frac{i}{a} dx}{2\pi d} = \frac{R}{\cos \theta} \Rightarrow B_x = \int dB \cos \theta = \frac{\mu_0 i}{2\pi a R} \int \cos^2 \theta dx$$

then

$$dx = d(R \tan \theta) = R \frac{d}{\cos^2 \theta}$$

$$B_x = \frac{\mu_0 i}{2\pi a} \int_{-\theta}^{\theta} d\theta = \frac{\mu_0 i}{\pi a} \alpha = \frac{\mu_0 i}{\pi a} \arctan \frac{a}{2R}$$

$$R \gg a : (\alpha \rightarrow \tan \alpha) B = \frac{\mu_0 i}{2\pi R}$$

$$R \rightarrow 0, B = \frac{\mu_0 i}{2a}$$

note[**Bohr model of the hydrogen atom**]

$$a_0 = 0.529 \text{ \AA} = 5.29 \times 10^{-11} \text{ m}$$

$$\nu = 6.63 \times 10^{15} \text{ Hz}$$

$$i = e\nu = 1.60 \times 10^{-19} \times 6.63 \times 10^{15} = 1.63 \times 10^{-3} \text{ A}$$

$$B = \frac{\mu_0 i}{2R} = \frac{4\pi \times 10^{-7} \times 1.06 \times 10^{-3}}{2 \times 5.29 \times 10^{-11}} = 12.6 \text{ T}$$

$$\begin{aligned} \mu_B = iA &= 1.63 \times 10^{-3} \times \pi \times (5.29 \times 10^{-11})^2 \\ &= 0.923 \times 10^{-23} \text{ A} \cdot \text{m}^2 \end{aligned}$$

called as Bohr Magnon

...

B of Solenoid

A constant magnetic field can (in principle) be produced by an ∞ sheet of current. In practice, however, a constant magnetic field is often produced by a solenoid.

Parameters: i, n, R, L , we assume $R \ll L$

for a circular loop

$$B = \frac{\mu_0}{2} \frac{iR^2}{(R^2 + r_0^2)^{3/2}}$$

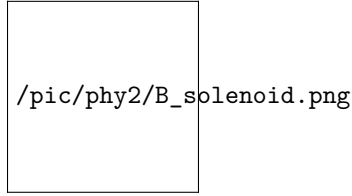


Figure 34: image-20241023112023529

$$dB = \frac{\mu_0}{2} \frac{R^2 i n dl}{[R^2 + (x-l)^2]^{3/2}}$$

$$B = \frac{\mu_0}{2} \int_{-L/2}^{L/2} \frac{R^2 i n dl}{[R^2 + (x-l)^2]^{3/2}}$$

$$r = \sqrt{R^2 + (x-l)^2} = \frac{R}{\sin \beta}$$

$$\frac{x-l}{R} = \text{ctg} \beta \Rightarrow dl = \frac{R}{\sin^2 \beta} d\beta$$

then

$$B = \frac{\mu_0}{2} \int_{\beta_1}^{\beta_2} \frac{R^2 n i \frac{R}{\sin^2 \beta} d\beta}{(\frac{R^2}{\sin^2 \beta})^{3/2}}$$

$$= \frac{\mu_0}{2} \cdot n i \int_{\beta_1}^{\beta_2} \sin \beta d\beta$$

$$= \frac{1}{2} \mu_0 n i (\cos \beta_1 - \cos \beta_2)$$

use

$$\cos \beta_1 = \frac{x + L/2}{\sqrt{R^2 + (x + L/2)^2}} \quad \cos \beta_2 = \frac{x - L/2}{\sqrt{R^2 + (x - L/2)^2}}$$

with

- $L \rightarrow \infty, \beta_1 = 0, \beta_2 = \pi$
 $B = \frac{1}{2} \mu_0 n i (1 + 1) = \mu_0 n i$
- $\beta_1 = 0, \beta_2 = \frac{\pi}{2}$
 $B = \frac{1}{2} \mu_0 n i (1 - 0) = \frac{1}{2} \mu_0 n i$

The field outside the ideal solenoid is zero.

:::note[eg:multi-layers solenoid]

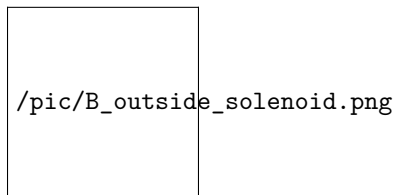
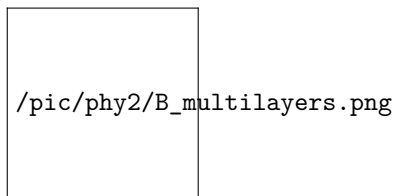


Figure 35: image-20241023112523551



$$B = \frac{1}{2} \mu_0 n i (\cos \beta_1 - \cos \beta_2)$$

With this

$$n i = \frac{N i}{L} \Rightarrow \frac{j L d r}{L} = j d r = \frac{N i}{2 l (R_2 - R_1)} d r$$

$$\cos \beta_2 = -\cos \beta_1, \cos \beta_1 = \frac{l}{\sqrt{l^2 + r^2}}$$

$$d B = \frac{1}{2} \mu_0 \frac{N i}{2 l (R_2 - R_1)} \cdot \frac{2 l}{\sqrt{l^2 + r^2}} d r$$

$$\begin{aligned} B &= \mu_0 j l \int_{R_1}^{R_2} \frac{d r}{\sqrt{l^2 + r^2}} \\ &= \mu_0 j l \ln \frac{R_2 + \sqrt{R_2^2 + l^2}}{R_1 + \sqrt{R_1^2 + l^2}} \end{aligned}$$

in practice

we define $\gamma = \frac{l}{R_1}, \alpha = \frac{R_2}{R_1}$

$$B_0 = \mu_0 h R_1 \gamma \ln \frac{\alpha + \sqrt{\alpha^2 + \gamma^2}}{1 + \sqrt{1 + \gamma^2}}$$

...

32-3 Gauss Law and Ampere's Loop Law for B

let's first review

for \vec{E}

- Gauss law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \sum q, \nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}$$

- Loop law

$$\oint \vec{E} \cdot d\vec{l} = 0, \nabla \times \vec{E} = 0 (\vec{E} = -\nabla V)$$

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \iint B \cos \theta dA (\text{Unit} : T \cdot m^2 = Wb)$$

for \vec{B}

- Gauss law:

$$\oint \vec{B} \cdot d\vec{A} = 0, \nabla \cdot \vec{B} = 0$$

- Loop law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum i, \nabla \times \vec{B} = \mu_0 \vec{J}$$

•

$$\oint \vec{B} d\vec{l} = \mu_0 \sum i + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} d\vec{S}$$

The differential form is

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

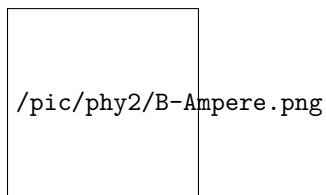


Figure 36: image-20241030100813902

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_1 + i_3 - 2i_2)$$

:::note[egs]

[e.g.1]

inf long wire R, i uniform distribution

$$B \cdot 2\pi r = \mu_0 i \cdot \frac{r^2}{\pi R^2} \Rightarrow B = \frac{\mu_0 i r}{2\pi R^2} (r < R) \propto r$$

$$B \cdot 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r} (r > R) \propto \frac{1}{r}$$

[e.g.2]

Consider an ∞ sheet of current described by n wires/length each carrying current i **into** the screen as shown. Calculate the B field.

for symmetry:vertical direction on the screen

we catch a $w \times w$ square

$$\oint \vec{B} \cdot d\vec{l} = 2Bw = \mu_0 nwi \Rightarrow B = \frac{1}{2}\mu_0 ni$$

[e.g.3]

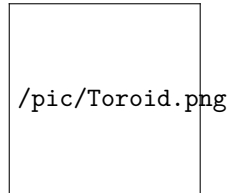
calculate B for ∞ solenoid

view it as two sheets

$$\oint \vec{B} \cdot d\vec{l} = Bw = \mu_0 nwi \Rightarrow B = \mu_0 ni$$

[e.g.4]

the B of Toroid (N total turns with current i)



$$\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 Ni \rightarrow B = \mu_0 ni \left(n = \frac{N}{2\pi r}\right)$$

...

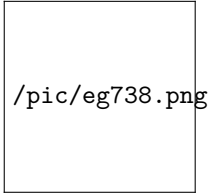
- Power door locks
- Magnetic cranes
- Electronic Switch “relay”

32-4 The magnetic force on a carrying-current wire

remember

$$d\vec{F} = i d\vec{s} \times \vec{B} \quad (d\vec{F}_2 = i_2 d\vec{s}_2 \times \left(\oint_{L_1} \frac{i_1 d\vec{s}_1 \times \hat{r}_{12}}{r_{12}^2} \right))$$

...note[EG32-5,P738]

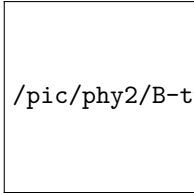


$$F_2 = F_{\perp} = \int_0^{\pi} iBR d\theta \sin \theta = 2iBR$$

$$F = F_1 + F_2 + F_3 = iB(2L + 2R)$$

...

for two parallel conductors



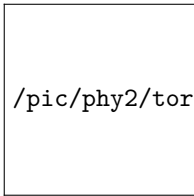
$$f = \frac{\mu_0 i_1 i_2}{2\pi d} i_1 = i_2 = ii \sqrt{\frac{fd}{\frac{\mu_0}{2\pi}}}$$

we define $i = 1A$ to be the current that make two $1m$ apart parallel conductors have force density $2 \times 10^{-7} N/m$

for convenience sake, we define \hat{n} the unit normal vector of current loop

$$\vec{\mu} = iA\hat{n}\vec{\tau} = \vec{\mu} \times \vec{B}$$

this holds for arbitrary shape loop



$$dF_1 = ids_1 B \sin \theta_1$$

$$dF_2 = ids_2 B \sin \theta_2$$

$$dF_1 = dF_2 = iBdh$$

$$d\tau = dF_1 x_1 + dF_2 x_2 = iBdA$$

for magnetic dipole

we define $\vec{\tau} = iA(\vec{n} \times \vec{B})$

then

$$U_p = - \int_{\infty}^p \vec{\tau} \cdot d\vec{\theta} = \int \mu B \sin \theta d\theta = \mu B \cos \theta = \vec{\mu} \bullet \vec{B} \Rightarrow U = -\vec{\mu} \bullet \vec{B}$$

remind that

$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \bullet \vec{E}$$

we have

$$\mu = A\vec{i}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \bullet \vec{B}$$

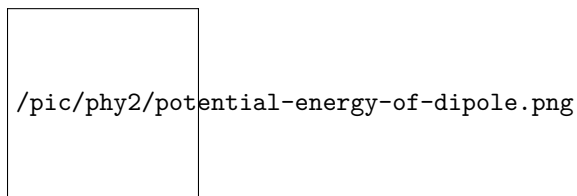


Figure 37: image-20241030105437447

System	(J/T)
Nucleus of N atom	2.0×10^{-28}
Proton	1.4×10^{-26}
Electron	9.3×10^{-24}
N atom	2.8×10^{-23}
Typical small coil	5.4×10^{-6}
Small bar magnet	5.0
Superconducting coil	400
The Earth	8.0×10^{22}

::important[MRI (Magnetic Resonance Imaging)]

Proton Spin and Magnetic Moment:

- A single proton possesses a positive charge $+|e|$ and an intrinsic angular momentum, known as “spin.”
- Naively imagining the charge circulating in a loop gives rise to a magnetic dipole moment μ .

Behavior in an External Magnetic Field (B):

- **Classically:** Torques will be present unless the magnetic moment μ is aligned with or against the magnetic field (B).
- **Quantum Mechanics (QM):** The spin is always aligned either with or against the magnetic field (B).
- **Anti-aligned State:** Energy $U_2 = \mu B$ **Aligned State:** Energy $U_1 = -\mu B$
- **Energy Difference:** $\Delta U = U_2 - U_1 = 2\mu B$

$$\mu_{proton} = 1.36 \times 10^{-26} \text{ A m}^2 \quad B = 1 \text{ Tesla } (= 10^4 \text{ Gauss})$$

$$\Delta U = 2\mu B = 2.7 \times 10^{-26} \text{ J}$$

$$h\nu = \Delta U \Rightarrow \nu = \frac{2.7 \times 10^{-26}}{6.6 \times 10^{-34} \text{ Js}} = 41 \text{ MHz}$$

:::

more applications

:::note[app:Galvanometers]

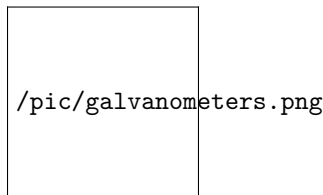


Figure 38: image-20241030110554772

1. Magnetic Force on a Current Loop:

- When a loop of wire carrying an electric current is placed in a magnetic field, the field exerts a torque on the loop, attempting to align the loop’s magnetic dipole moment with the field.

2. Structure of a Galvanometer:

- Inside a galvanometer, there is a rotating coil attached to a pivot.
- To return the pointer to its equilibrium position, a spring is included in the galvanometer, which creates a torque in the opposite direction to the rotation of the coil.

:::

motor is almost the same ,we just skip it

- fixed voltage:DC motors
- fixed current:AC motors