A Least Square Parameter Identification Method for Nonlinear Motion Model of Unmanned Surface Vehicle Based on Euler Discrete Difference

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Abstract: Parameter identification method for the nonlinear motion model of unmanned surface vehicle (USV) is addressed in this paper. A least square (LS) based parameter identification algorithm is proposed combining with Euler discrete difference, which greatly reduces the difficulty of identification system designing and improves the accuracy and running speed of the algorithm. A kind of more accurate nonlinear USV model is studied in this paper compared with traditional simple linear model, and model transformation of the USV model is carefully designed in order to facilitate the design process of the identification system. Some computer simulation experiments are carried out, and simulation data are analyzed. Compared with the original USV model parameters, parameters calculated by the proposed algorithm are basically the same as the original data, which shows the validity and reliability of the least square parameter identification method designed based on Euler discrete difference in this paper.

Key Words: Unmanned Surface Vehicle, Nonlinear Motion Model, Parameter Identification, Least Square, Euler Discrete Difference

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1 INTRODUCTION

Unmanned surface vehicle(USV) is a kind of highly intelligent autonomous surface sailing vessel, it has great commercial value and wide application prospect in military application, scientific investigation, and civil field et al[1-4]. In recent years, the USV has been widely concerned and studied by researchers all over the world[5-8]. Motion mathematical modeling is an important technology in the development of USV. It is an important prerequisite for USV to complete ship maneuverability prediction and controller design and analysis. Compared with the traditional constrained model test and CFD (computational fluid dynamics) hydrodynamic calculation, the system parameter identification technology has obvious advantages in the mathematical modeling of ship motion, the experiment cost is lower and the calculation time is shorter. Many researchers from different countries have carried out a lot of research work on the parameter identification technology of ship motion system and achieved a lot of research results.

A kind of system identification method based on Support Vector Regression(SVR) is proposed by Hou for identifying the nonlinear roll motion equation of a vessel in regular waves[9], and then a parameter identification method based on a combination of random decrement technique and SVR is proposed to identify the coefficients in the roll motion equation of a floating structure by using the measured roll response in irregular waves[10]. System identify of vessel steering associated with unstructured uncertainties is considered in literature [11], and support vector machines (SVM) optimized by the artificial bee colony algorithm was designed for system identification of large container ship in paper [12]. By taking into account of some aspects of, for example, selection of the sampling period, smoothing of the data acquired in the tests, et al, parameter estimation accuracy of a nonlinear manoeuvring model of a torpedo shaped unmanned underwater vehicle is improved in [13]. A novel hybrid identification method based on the least support vector regression algorithm (LS-SVR) and the artificial bee colony algorithm is presented by Zhu to deal with the identification of the simplified ship dynamic model while the outliers exist in the measurements[14], while an optimal truncated least square support vector machine (LS-SVM) is proposed for the parameter estimation of a nonlinear manoeuvring model in shallow water by Xu[15]. A multi-objective strategy to identify the parameters of pitch and heave coupled motions is proposed in [16], and a practical and robust system identification modeling method for ship manoeuvring motion is presented in [17] to alleviate the impact of noise-induced problems. An optimized support vector regression algorithm is presented, in order to solve the problem of how to derive a simplistic model feasible for describing dynamics of different types of ships for maneuvering simulation employed to study maritime traffic and further more to provide ship models for simulation-based engineering test-beds[18]. A novel Nonlinear Least Squares Support Vector Machine (NLS-SVM) is introduced in [19] to accurately identify the ship's roll model parameters in shallow water, and solve the problems of difficult estimating nonlinear damping coefficients by traditional methods. Aiming at the inconvenience of obtaining model parameters under the traditional experimental method, parameter identification of unmanned marine vehicle's manoeuvring model based on extended Kalman filter and support vector machine is studied in [20]. A novel identification approach to nonlinear

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ship manoeuvring models based on Bayes' rule is presented in [21], while the combination of LS-SVM and cuckoo search (CS) algorithm is proposed to identify the dynamic models of USV in [22]. A novel nonlinear kernel-based LS-SVM is proposed by Xu to approximate the nonlinear manoeuvring model, which is a robust method for regression modelling with a low computational cost by reducing the dimensionality of the kernel matrix[23]. An optimal design scheme of excitation signals is presented to determine the training data that provides the maximum dynamic information to improve the stability and accuracy of the identification of ship manoeuvring models in [24], where a multi-level pseudo-random sequence is selected as the optimized object for covering the maximum nonlinear dynamic characteristics. Multi-innovation least squares and improved multi-innovation extended Kalman filtering are proposed by Xie[25], and the LS-SVM is used to estimate the dynamic parameters of a nonlinear marine vessel steering model in real-time by Xu[26].

Different from the above literatures, a least square parameter identification method for nonlinear motion model of USV based on Euler discrete difference is proposed in this paper, which greatly reduces the difficulty of identification system designing and improves the accuracy and running speed of the algorithm. Nonlinear USV model is studied in this paper, and model transformation of the USV model is carefully designed in order to facilitate the design process of the identification system. An improved least square parameter identification algorithm is designed by combing model transformation and Euler discrete difference, and then some computer simulation experiments are carried out to verify its validity and reliability.

2 NONLINEAR MODEL OF USV

According to[27], the three degree of freedom (DOF) horizontal nonlinear model of USV can be described as follows:

$$\begin{bmatrix} m-X_{\dot{u}} & 0 & 0 \\ 0 & m-Y_{\dot{v}} & mx_G-Y_{\dot{r}} \\ 0 & mx_G-N_{\dot{v}} & I_{zz}-N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix} \quad (1)$$

where

$$\begin{aligned} \Delta f_1 &= X_u\Delta u + X_{uu}\Delta u^2 + X_{uuu}\Delta u^3 + X_{vv}v^2 + X_{rr}r^2 + X_{\delta\delta}\times \\ &\delta^2 + X_{\delta\delta u}\delta^2\Delta u + X_{vr}vr + X_{v\delta}v\delta + X_{v\delta u}v\delta\Delta u + X_{uvv}\Delta uv^2 \\ &+X_{urr}\Delta ur^2 + X_{uvr}\Delta uvr + X_{r\delta}r\delta + X_{ur\delta}\Delta ur\delta + X_0 \end{aligned} ,$$

$$\begin{aligned} \Delta f_2 &= Y_u\Delta u + Y_{uu}\Delta u^2 + Y_v v + Y_r r + Y_\delta\delta + Y_{vvv}v^3 + Y_{\delta\delta\delta}\delta^3 + \\ &Y_{vvr}v^2 r + Y_{vv\delta}v^2\delta + Y_{v\delta\delta}v\delta^2 + Y_{\delta u}\delta\Delta u + Y_{vu}v\Delta u + Y_{ru}r\Delta u + \\ &Y_{\delta uu}\delta\Delta u^2 + Y_{rrr}r^3 + Y_{vrr}vr^2 + Y_{vuu}v\Delta u^2 + Y_{ruu}r\Delta u^2 + Y_{r\delta\delta}\times \\ &r\delta^2 + Y_{rr\delta}r^2\delta + Y_{rv\delta}rv\delta + Y_0 \end{aligned} ,$$

$$\begin{aligned} \Delta f_3 &= N_u\Delta u + N_{uu}\Delta u^2 + N_v v + N_r r + N_\delta\delta + N_{vvv}v^3 + N_{\delta\delta\delta} \\ &\delta^3 + N_{vvr}v^2 r + N_{vv\delta}v^2\delta + N_{v\delta\delta}v\delta^2 + N_{\delta u}\delta\Delta u + N_{vu}v\Delta u + \\ &N_{ru}r\Delta u + N_{\delta uu}\delta\Delta u^2 + N_{rrr}r^3 + N_{vrr}vr^2 + N_{vuu}v\Delta u^2 + \\ &N_{ruu}r\Delta u^2 + N_{r\delta\delta}r\delta^2 + N_{rr\delta}r^2\delta + N_{rv\delta}rv\delta + N_0 \end{aligned} .$$

and m represents mass of USV, and x_G means x coordinate of the center of gravity. u is longitudinal velocity, and v is lateral velocity, while r is angular velocity of USV. I_{zz} is moment of inertia of the USV and δ is rudder angle. Others such as $X_{\dot{u}}$ 、 $Y_{\dot{v}}$ 、 $Y_{\dot{r}}$ et al are all hydrodynamic parameters of USV, and all details can refer to reference [27].

3 IDENTIFICATION ALGORITHM DESIGN

3.1 USV Model Transformation Design Based on Euler Discrete Difference

Firstly, dimensionless transformation design is needed for USV motion model expressed in equation (1), and dimensional variables and dimensionless variables can be transformed as follows[27]:

$m' = m/(0.5\rho L^3)$, $u' = u/V$, $v' = v/V$, $r' = r/(V/L)$,

where m' 、 u' 、 v' et al are all dimensionless variables, and represents the density of water, is the speed of USV, while means the length of USV. Then equation (1) could be rewritten in the following form as in equation (2):

$$\begin{bmatrix} m'-X_{\dot{u}}' & 0 & 0 \\ 0 & m'-Y_{\dot{v}}' & m'x_G'-Y_{\dot{r}}' \\ 0 & m'x_G'-N_{\dot{v}}' & I_{zz}'-N_{\dot{r}}' \end{bmatrix} \begin{bmatrix} \dot{u}' \\ \dot{v}' \\ \dot{r}' \end{bmatrix} = \begin{bmatrix} \Delta f_1' \\ \Delta f_2' \\ \Delta f_3' \end{bmatrix} \quad (2)$$

where

$$\begin{aligned} \Delta f_1' &= X_u'\Delta u' + X_{uu}'\Delta u'^2 + X_{uuu}'\Delta u'^3 + X_{vv}'v'^2 + X_{rr}'r'^2 \\ &+X_{\delta\delta}'\delta'^2 + X_{\delta\delta u}'\delta'^2\Delta u' + X_{vr}'v'r' + X_{v\delta}'v'\delta' + X_{v\delta u}'\times \\ &v'\delta'\Delta u' + X_{uvv}'\Delta u'v'^2 + X_{urr}'\Delta u'r'^2 + X_{uvr}'\Delta u'v'r' + \\ &X_{r\delta}'r'\delta' + X_{ur\delta}'\Delta u'r'\delta' + X_0' \end{aligned} ,$$

$$\begin{aligned} \Delta f_2' &= Y_u'\Delta u' + Y_{uu}'\Delta u'^2 + Y_v'v' + Y_r'r' + Y_\delta'\delta' + Y_{vvv}'v'^3 + \\ &Y_{\delta\delta\delta}'\delta'^3 + Y_{vvr}'v'^2 r' + Y_{vv\delta}'v'^2\delta' + Y_{v\delta\delta}'v'\delta'^2 + Y_{\delta u}'\delta'\Delta u' \\ &+Y_{vu}'v'\Delta u' + Y_{ru}'r'\Delta u' + Y_{\delta uu}'\delta'\Delta u'^2 + Y_{rrr}'r'^3 + Y_{vrr}'v'r'^2 \\ &+Y_{vuu}'v'\Delta u'^2 + Y_{ruu}'r'\Delta u'^2 + Y_{r\delta\delta}'r'\delta'^2 + Y_{rr\delta}'r'^2\delta' + \delta'\times \\ &Y_{rv\delta}'r'v' + Y_0' \end{aligned} ,$$

$$\begin{aligned} \Delta f_3' &= N_u'\Delta u' + N_{uu}'\Delta u'^2 + N_v'v' + N_r'r' + N_\delta'\delta' + N_{vvv}'\times \\ &v'^3 + N_{\delta\delta\delta}'\delta'^3 + N_{vvr}'v'^2 r' + N_{vv\delta}'v'^2\delta' + N_{v\delta\delta}'v'\delta'^2 + \delta'\times \\ &N_{\delta u}'\Delta u' + N_{vu}'v'\Delta u' + N_{ru}'r'\Delta u' + N_{\delta uu}'\delta'\Delta u'^2 + N_{rrr}'\times \\ &r'^3 + N_{vrr}'v'r'^2 + N_{vuu}'v'\Delta u'^2 + N_{ruu}'r'\Delta u'^2 + N_{r\delta\delta}'r'\delta'^2 \\ &+N_{rr\delta}'r'^2\delta' + N_{rv\delta}'r'v'\delta' + N_0' \end{aligned} .$$

In order to have the mathematical operation of the USV identification system more effective, acceleration variables of USV are discretized by Euler discrete difference, and then we could obtain that:

$$\dot{u}_t = (u_{t+1} - u_t)/h,\ \dot{v}_t = (v_{t+1} - v_t)/h,\ \dot{r}_t = (r_{t+1} - r_t)/h \quad (3)$$

where h means the sampling time. And relationships between dimensional and dimensionless variables of accelerations could be expressed as follows:

$$\dot{u} = \dot{u}'\times L/V^2,\ \dot{v} = \dot{v}'\times L/V^2,\ \dot{r} = \dot{r}'\times L/V^2 \quad (4)$$

Then we can derive that:

$$\begin{aligned} u_{t+1} = {} & u_t + h/[L(m'-X_{\dot{u}}')]\times[X_u'\Delta uV + X_{uu}'\Delta u^2 + \\ & X_{uuu}'\Delta u^3/V + X_{vv}'v^2 + X_{rr}'r^2L^2 + X_{\delta\delta}'\delta^2V^2 + X_{\delta\delta u}'\times \\ & \delta^2\Delta uV + X_{vr}'vrL + X_{v\delta}'v\delta V + X_{v\delta u}'v\delta\Delta u + X_{uvv}'\times \\ & \Delta uv^2/V + X_{urr}'\Delta ur^2L^2/V + X_{uvr}'\Delta uvrL/V + X_{r\delta}'r\times \\ & \delta LV + X_{ur\delta}'\Delta ur\delta L + X_0'V^2] \end{aligned} \tag{5}$$

$$\begin{aligned} v_{t+1} = {} & v_t + h(I_z'-N_{\dot{r}}')/(SL)\times[Y_u'\Delta uV + Y_{uu}'\Delta u^2 + v\times \\ & Y_v'V + Y_r'rLV + Y_\delta'\delta V^2 + Y_{vvv}'v^3/V + Y_{\delta\delta\delta}'\delta^3V^2 + v^2\times \\ & Y_{vvr}'rL/V + Y_{vv\delta}'v^2\delta + Y_{v\delta\delta}'v\delta^2V + Y_{\delta u}'\delta\Delta uV + Y_{vu}'v\times \\ & \Delta u + Y_{ru}'r\Delta uL + Y_{\delta uu}'\delta\Delta u^2 + Y_{rrr}'r^3L^3/V + Y_{ruu}'r\Delta u^2L/ \\ & V + Y_{vrr}'vr^2L^2/V + Y_{vuu}'v\Delta u^2/V + Y_{r\delta\delta}'r\delta^2LV + Y_{rr\delta}'\delta\times \\ & r^2L^2 + Y_{rv\delta}'rv\delta L + Y_0'V^2] - h(m'x_G'-Y_{\dot{r}}')\times[N_u'\Delta u\times \\ & V + N_{uu}'\Delta u^2 + N_v'vV + N_r'rLV + N_\delta'\delta V^2 + N_{vvv}'v^3/ \\ & V + N_{\delta\delta\delta}'\delta^3V^2 + N_{vvr}'v^2rL/V + N_{vv\delta}'v^2\delta + N_{v\delta\delta}'v\delta^2\times \\ & V + N_{\delta u}'\delta\Delta uV + N_{vu}'v\Delta u + N_{ru}'r\Delta uL + N_{\delta uu}'\delta\Delta u^2 + \\ & N_{rrr}'r^3L^3/V + N_{vrr}'vr^2L^2/V + N_{vuu}'v\Delta u^2/V + N_{ruu}'\times \\ & r\Delta u^2L/V + N_{r\delta\delta}'r\delta^2LV + N_{rr\delta}'r^2\delta L^2 + N_{rv\delta}'rv\delta L + \\ & N_0'V^2]/(SL) \end{aligned} \tag{6}$$

$$\begin{aligned} r_{t+1} = {} & r_t + h(m'-Y_{\dot{v}}')/(SL^2)\times[N_u'\Delta uV + N_{uu}'\Delta u^2 + v\times \\ & N_v'V + N_r'rLV + N_\delta'\delta V^2 + N_{vvv}'v^3/V + N_{\delta\delta\delta}'\delta^3V^2 + \\ & N_{vvr}'v^2rL/V + N_{vv\delta}'v^2\delta + N_{v\delta\delta}'v\delta^2V + N_{\delta u}'\delta\Delta uV + v\times \\ & N_{vu}'\Delta u + N_{ru}'r\Delta uL + N_{\delta uu}'\delta\Delta u^2 + N_{rrr}'r^3L^3/V + vr^2\times \\ & N_{vrr}'L^2/V + N_{vuu}'v\Delta u^2/V + N_{ruu}'r\Delta u^2L/V + N_{r\delta\delta}'r\times \\ & \delta^2LV + N_{rr\delta}'r^2\delta L^2 + N_{rv\delta}'rv\delta L + N_0'V^2] - h(m'x_G'- \\ & N_{\dot{v}}')/(SL^2)\times[Y_u'\Delta uV + Y_{uu}'\Delta u^2 + Y_v'vV + Y_r'rLV + \\ & Y_\delta'\delta V^2 + Y_{vvv}'v^3/V + Y_{\delta\delta\delta}'\delta^3V^2 + Y_{vvr}'v^2rL/V + Y_{vv\delta}'\times \\ & v^2\delta + Y_{v\delta\delta}'v\delta^2V + Y_{\delta u}'\delta\Delta uV + Y_{vu}'v\Delta u + Y_{ru}'r\Delta uL + \\ & Y_{\delta uu}'\delta\Delta u^2 + Y_{rrr}'r^3L^3/V + Y_{vrr}'vr^2L^2/V + Y_{vuu}'v\Delta u^2/ \\ & V + Y_{ruu}'r\Delta u^2L/V + Y_{r\delta\delta}'r\delta^2LV + Y_{rr\delta}'r^2\delta L^2 + Y_{rv\delta}'r\times \\ & v\delta L + Y_0'V^2] \end{aligned} \tag{7}$$

where $S=(m'-Y_{\dot{v}}')(I_z'-N_{\dot{r}}')-(m'x_G'-Y_{\dot{r}}')(m'x_G'-N_{\dot{v}}')$.

3.2 Parameter Identification Algorithm Design Based on Least Square

In order to better design the identification algorithm based on least square, we can design the following matrix variables:

$$\boldsymbol{Y}_{ut} = [u_{t+1}],\ \boldsymbol{Y}_{vt} = [v_{t+1}],\ \boldsymbol{Y}_{rt} = [r_{t+1}] \tag{8}$$

$$\begin{aligned} \boldsymbol{H}_{ut} = {} & [u_t, \Delta u_tV_t, \Delta u_t^2, \Delta u_t^3/V_t, v_t^2, r_t^2L^2, \delta_t^2V_t^2, \delta_t^2V_t\times \\ & \Delta u_t,, v_tr_tL, v_t\delta_tV_t, v_t\delta_t\Delta u_t, u_tv_t^2/V_t, \Delta u_tr_t^2L^2/V_t, \Delta u_t\times \\ & v_tr_tL/V_t, r_t\delta_tLV_t, \Delta u_tr_t\delta_tL, V_t^2] \end{aligned} \tag{9}$$

$$\begin{aligned} \boldsymbol{H}_{vt} = {} & [v_t, \Delta u_tV_t, \Delta u_t^2, v_tV_t, r_tLV_t, \delta_tV_t^2, v_t^3/V_t, \delta_t^3V_t^2, \\ & v_t^2r_tL/V_t, v_t^2\Delta\delta_t, v_t\delta_t^2V_t, \delta_t\Delta u_tV_t, v_t\Delta u_t, r_t\Delta u_tL, \delta_t\times \\ & \Delta u_t^2, r_t^3L^3/V_t, v_tr_t^2L^2/V_t, v_tu_t^2/V_t, r_t\Delta u_t^2L/V_t, r_t\times \\ & \delta_t^2LV_t, r_t^2\delta_tL^2, r_tv_t\delta_tL, V_t^2] \end{aligned} \tag{10}$$

$$\begin{aligned} \boldsymbol{H}_{rt} = {} & [r_t, \Delta u_tV_t, \Delta u_t^2, v_tV_t, r_tLV_t, \delta_tV_t^2, v_t^3/V_t, \delta_t^3V_t^2, \\ & v_t^2r_tL/V_t, v_t^2\Delta\delta_t, v_t\delta_t^2V_t, \delta_t\Delta u_tV_t, v_t\Delta u_t, r_t\Delta u_tL, \delta_t\times \\ & \Delta u_t^2, r_t^3L^3/V_t, v_tr_t^2L^2/V_t, v_tu_t^2/V_t, r_t\Delta u_t^2L/V_t, r_t\times \\ & \delta_t^2LV_t, r_t^2\delta_tL^2, r_tv_t\delta_tL, V_t^2] \end{aligned} \tag{11}$$

$$\boldsymbol{\theta}_{ut} = [1, a_1, a_2, \cdots, a_{16}]^{\mathrm{T}} \tag{12}$$

$$\boldsymbol{\theta}_{vt} = [1, b_1, b_2, \cdots, b_{22}]^{\mathrm{T}} \tag{13}$$

$$\boldsymbol{\theta}_{rt} = [1, c_1, c_2, \cdots, c_{22}]^{\mathrm{T}} \tag{14}$$

where $a_i\,(\mathrm{i}=1,...,16)$ 、$b_j\,(\mathrm{j}=1,...,22)$ 、$c_k\,(k=1,...,22)$ are initial parameters given before identification algorithm runs.

Supposing that the desired parameters identification results are recorded as:

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{ut} & \hat{\boldsymbol{\theta}}_{vt} & \hat{\boldsymbol{\theta}}_{rt} \end{bmatrix} \tag{15}$$

Then the identification error function can be designed as:

$$\boldsymbol{e} = \frac{1}{2}\left(\boldsymbol{Y} - \boldsymbol{H}\hat{\boldsymbol{\theta}}\right)^2 \tag{16}$$

where $\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_{ut} & \boldsymbol{Y}_{vt} & \boldsymbol{Y}_{rt} \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{ut} & \boldsymbol{H}_{vt} & \boldsymbol{H}_{rt} \end{bmatrix}^{\mathrm{T}}$.

In order to get the best results of all the identification parameters, we can take the derivative of $\hat{\boldsymbol{\theta}}$ as follows:

$$\frac{\partial \boldsymbol{e}}{\partial \hat{\boldsymbol{\theta}}} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{Y} - \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\hat{\boldsymbol{\theta}} \tag{17}$$

It is easy to know that, the optimal value of identification parameters $\hat{\boldsymbol{\theta}}$ can be obtained while the above derivation is equal to 0. So we can get that:

$$\boldsymbol{H}^{\mathrm{T}}\boldsymbol{Y} - \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\hat{\boldsymbol{\theta}} = 0 \tag{18}$$

and then

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{Y} \tag{19}$$

3.3 Structural Improvement of The Parameter Identification Algorithm

In accordance with what addressed in literature [20], parameter drift still exists in the parameter identification system which is designed based on the traditional least square method. A novel kind of structure transformation is presented here to compensate the influence of parameter drift, and the design process is described in detail below.

Step1, design the following new variables $\Delta\boldsymbol{Y}_{ut}$, $\Delta\boldsymbol{Y}_{vt}$ and $\Delta\boldsymbol{Y}_{rt}$ based on $\boldsymbol{Y}_{ut}$, $\boldsymbol{Y}_{vt}$ and $\boldsymbol{Y}_{rt}$ as follows:

$$\Delta\boldsymbol{Y}_{ut} = \left[(m'-X_{\dot{u}}')\left(\boldsymbol{Y}_{ut} - \boldsymbol{Y}_{u(t-1)}\right)L/hV^2\right] \tag{20}$$

$$\begin{aligned} \Delta\boldsymbol{Y}_{vt} = {} & [(m'-Y_{\dot{v}}')(\boldsymbol{Y}_{vt} - \boldsymbol{Y}_{v(t-1)})L/hV^2 + (m'x_G'-Y_{\dot{r}}')\times \\ & (\boldsymbol{Y}_{rt} - \boldsymbol{Y}_{r(t-1)})L^2/hV^2] \end{aligned} \tag{21}$$

$$\begin{aligned} \Delta\boldsymbol{Y}_{rt} = {} & [(m'x_G'-N_{\dot{v}}')(\boldsymbol{Y}_{vt} - \boldsymbol{Y}_{v(t-1)})L/hV^2 + (I_z'- \\ & N_{\dot{r}}')(\boldsymbol{Y}_{rt} - \boldsymbol{Y}_{r(t-1)})L^2/hV^2] \end{aligned} \tag{22}$$

Step2, subtracting all the first column elements of matrix $\boldsymbol{H}$ and divide the others by V_t^2, we can get new state vectors as follows:

$$\begin{aligned}\Delta \boldsymbol{H}_{ut} = [&\Delta u_t / V_t, \Delta u_t^2 / V_t^2, \Delta u_t^3 / V_t^3, v_t^2 / V_t^2, r_t^2 L^2 / \\ &V_t^2, \delta_t^2, \delta_t^2 \Delta u_t / V_t, v_t r_t L / V_t^2, v_t \delta_t / V_t, v_t \delta_t \Delta u_t / V_t^2, \\ &\Delta u_t v_t^2 / V_t^3, \Delta u_t r_t^2 L^2 / V_t^3, \Delta u_t v_t r_t L / V_t^3, r_t \delta_t L / V_t, \\ &\Delta u_t r_t \delta_t L / V_t^2, 1]\end{aligned} \tag{23}$$

$$\begin{aligned}\Delta \boldsymbol{H}_{yt} = [&\Delta u_t / V_t, \Delta u_t^2 / V_t^2, v_t / V_t, r_t L / V_t, \delta_t, v_t^3 / V_t^3, \\ &\delta_t^3, v_t^2 r_t L / V_t^3, v_t^2 \delta_t / V_t^2, v_t \delta_t^2 / V_t, \ \delta_t \Delta u_t / V_t, v_t \Delta u_t / \\ &V_t^2, r_t \Delta u_t L / V_t^2, \delta_t \Delta u_t^2 / V_t^2, r_t^3 L^3 / V_t^3, v_t r_t^2 L^2 / V_t^3, \\ &v_t \Delta u_t^2 / V_t^3, r_t \Delta u_t^2 L / V_t^3, \ r_t \delta_t^2 L / V_t, r_t^2 \delta_t L^2 / V_t^2, \\ &r_t v_t \delta_t L / V_t^2, 1]\end{aligned} \tag{24}$$

$$\begin{aligned}\Delta \boldsymbol{H}_{rt} = [&\Delta u_t / V_t, \Delta u_t^2 / V_t^2, v_t / V_t, r_t L / V_t, \delta_t, v_t^3 / V_t^3, \\ &\delta_t^3, v_t^2 r_t L / V_t^3, v_t^2 \delta_t / V_t^2, v_t \delta_t^2 / V_t, \ \delta_t \Delta u_t / V_t, v_t \Delta u_t / \\ &V_t^2, r_t \Delta u_t L / V_t^2, \delta_t \Delta u_t^2 / V_t^2, r_t^3 L^3 / V_t^3, v_t r_t^2 L^2 / V_t^3, \\ &v_t \Delta u_t^2 / V_t^3, \ r_t \Delta u_t^2 L / V_t^3, \ r_t \delta_t^2 L / V_t, r_t^2 \delta_t L^2 / V_t^2, r_t \times \\ &v_t \delta_t L / V_t^2, 1]\end{aligned} \tag{25}$$

Step3, referring to the above parameter identification algorithm presented in (19), the structure of the improved algorithm can be formalized as follows:

$$\Delta \hat{\boldsymbol{\theta}} = \left(\Delta \boldsymbol{H}^{\mathrm{T}} \Delta \boldsymbol{H} \right)^{-1} \Delta \boldsymbol{H}^{\mathrm{T}} \Delta \boldsymbol{Y} \tag{26}$$

where $\Delta\boldsymbol{Y} = [\Delta\boldsymbol{Y}_{ut}, \Delta\boldsymbol{Y}_{vt}, \Delta\boldsymbol{Y}_{rt}]^{\mathrm{T}}$, $\Delta\boldsymbol{H} = [\Delta\boldsymbol{H}_{ut}, \Delta\boldsymbol{H}_{vt}, \Delta\boldsymbol{H}_{rt}]^{\mathrm{T}}$, $\Delta\boldsymbol{\theta} = [\Delta\boldsymbol{\theta}_{ut}, \Delta\boldsymbol{\theta}_{vt}, \Delta\boldsymbol{\theta}_{rt}]^{\mathrm{T}}$, and $\Delta\hat{\boldsymbol{\theta}}$ represents the parameter identification value of $\Delta\boldsymbol{\theta}$, while $\Delta\boldsymbol{\theta}$ is the USV's nonlinear model parameter matrix that need to be identified, as follows:

$$\begin{aligned}\Delta \boldsymbol{\theta}_{ut} = [&X_u', X_{uu}', X_{uuu}', X_{vv}', X_{rr}', X_{\delta\delta}', X_{\delta\delta u}', \\ &X_{vr}', X_{v\delta}', X_{v\delta u}', X_{uvv}', X_{urr}', X_{uvr}', X_{r\delta}', X_{ur\delta}', \\ &X_0']^{\mathrm{T}}\end{aligned} \tag{27}$$

$$\begin{aligned}\Delta \boldsymbol{\theta}_{vt} = [&Y_u', Y_{uu}', Y_v', Y_r', Y_\delta', Y_{vvv}', Y_{\delta\delta\delta}', Y_{vvr}', Y_{vv\delta}', \\ &Y_{v\delta\delta}', Y_{\delta u}', Y_{vu}', Y_{ru}', Y_{\delta uu}', Y_{rrr}', Y_{vrr}', Y_{vuu}', Y_{ruu}', Y_{r\delta\delta}', \\ &Y_{rr\delta}', Y_{rv\delta}', Y_0']^{\mathrm{T}}\end{aligned} \tag{28}$$

$$\begin{aligned}\Delta \boldsymbol{\theta}_{rt} = [&N_u', N_{uu}', N_v', N_r', N_\delta', N_{vvv}', N_{\delta\delta\delta}', N_{vvr}', \\ &N_{vv\delta}', N_{v\delta\delta}', N_{\delta u}', N_{vu}', N_{ru}', N_{\delta uu}', N_{rrr}', N_{vrr}', \\ &N_{vuu}', N_{ruu}', N_{r\delta\delta}', N_{rr\delta}', N_{rv\delta}', N_0']^{\mathrm{T}}\end{aligned} \tag{29}$$

Substituting the actual voyage data or simulation experiment data of USV into the above identification algorithms and equations, and then we can get the final parameters identification results through continuous cyclic calculation and analysis. It should be noted that the more voyage data provided, the more accurate the parameters obtained by the parameter identification algorithm would be. However, considering that sometime the basic parameters of ships are incomplete, and usually information such as the longitudinal and lateral velocities of ships and their accelerations may usually not recorded in detail in actual voyage data. As in most literatures, computer simulation experiment data are used to test and verify the parameter identification algorithm proposed in the paper.

4 SIMULATION EXPERIMENT

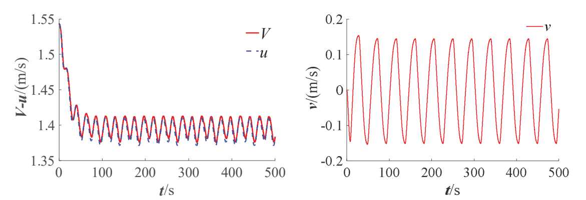
Some simulation experiments are carried out in this section to test and verify the algorithm designed in this paper. Some voyage data of computer simulation experiments are provided at first, and then the parameters are obtained by the parameter identification algorithm presented. Finally some detailed and in-depth analysis of those results is done, which is very helpful for identification system design of USV.

4.1 Simulation Data Collection

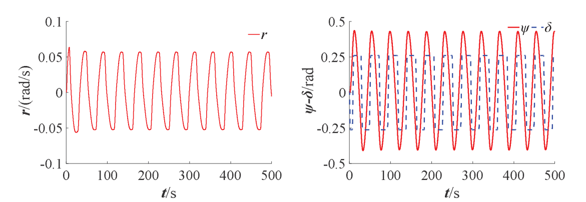
According to the model test data of "Mariner" shows in [28], main scale parameters and hydrodynamic parameters of the ship are shown in Table I and Table II respectively, 15° zigzag maneuvering motion simulation tests are carried out to collect the data required for identification, on which the model parameters can be identified, and the simulation data is shown as follows in Figure 1.

Table1. Main scale parameters of Mariner

Parameter	Value	Parameter	Value
L/m	6.437	B/m	0.927
d/m	0.329	∇ / m^3	1.187
$X_{\dot{u}}'$	-4.2×10^{-4}	m'	7.98×10^{-3}
x_G'	-2.3×10^{-2}	$Y_{\dot{v}}'$	-7.48×10^{-3}
$Y_{\dot{r}}'$	-9.354×10^{-5}	I_{zz}'	3.92×10^{-4}
$N_{\dot{v}}'$	4.646×10^{-5}	$N_{\dot{r}}'$	-4.38×10^{-4}



(a) Longitudinal velocity and actual speed (b) Data of lateral velocity



(c) Data of heading angular velocity (d) Data of heading and rudder angle

Figure 1. 15° zigzag maneuvering motion simulation data.

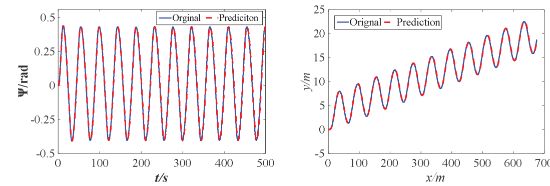
4.2 Analysis of Parameter Identification Results

Based on the above 15° zigzag maneuvering motion simulation data collected, the improved least square algorithm(A1) and the traditional unimproved algorithm(A2) is respectively tested to conduct parameter identification processing, and results are shown below as in Table II, where A1 represents the improved least square identification algorithm proposed in this paper while A2 represents the traditional unimproved algorithm.

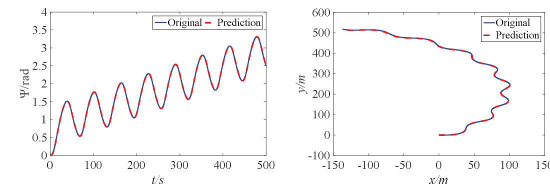
Table2. Results of parameter identification algorithms

Parameter	X_u'	X_{uu}'	X_{uuu}'	X_{vv}'
Original	-1.840×10^{-3}	-1.110×10^{-3}	-2.150×10^{-3}	-8.990×10^{-3}
A1	-1.843×10^{-3}	-1.137×10^{-3}	-2.090×10^{-3}	-8.610×10^{-3}
A2	9.961×10^{-2}	8.917×10^{-1}	-1.888×10^{-1}	6.684×10^{-1}
Parameter	X_{rr}'	$X_{\delta\delta}'$	$X_{\delta\delta u}'$	X_{vr}'
Original	1.800×10^{-4}	-9.500×10^{-4}	-1.900×10^{-3}	7.980×10^{-3}
A1	2.199×10^{-4}	-9.413×10^{-4}	-1.857×10^{-3}	8.137×10^{-3}
A2	1.592×10^{-1}	1.249×10^{-2}	1.251×10^{-1}	6.670×10^{-1}
Parameter	$X_{v\delta}'$	$X_{v\delta u}'$	X_{uvv}'	X_{urr}'
Original	9.300×10^{-4}	9.300×10^{-4}	0	0
A1	9.544×10^{-4}	7.672×10^{-4}	3.131×10^{-5}	1.018×10^{-4}
A2	1.942×10^{-1}	1.851	4.838	-1.080×10^{-3}
Parameter	X_{uvr}'	$X_{r\delta}'$	$X_{ur\delta}'$	X_0'
Original	0	0	0	0
A1	1.752×10^{-4}	4.920×10^{-5}	3.201×10^{-5}	3.526×10^{-8}
A2	-3.418×10^{-2}	-4.589×10^{-4}	6.158×10^{-5}	5.400×10^{-8}
Parameter	Y_u'	Y_{uu}'	Y_v'	Y_r'
Original	-8.000×10^{-5}	-4.000×10^{-5}	-1.160×10^{-2}	-4.990×10^{-3}
A1	-9.525×10^{-5}	-7.755×10^{-5}	-1.172×10^{-2}	-4.964×10^{-3}
A2	4.433	7.729	1.097×10^{2}	4.555
Parameter	Y_δ'	Y_{vvv}'	$Y_{\delta\delta\delta}'$	Y_{vvr}'
Original	2.780×10^{-3}	-8.078×10^{-2}	-9.000×10^{-4}	1.536×10^{-1}
A1	3.092×10^{-3}	-8.210×10^{-2}	-1.071×10^{-3}	1.644×10^{-1}
A2	1.479	-1.453×10^{4}	-6.459×10^{1}	-2.396×10^{4}
Parameter	$Y_{vv\delta}'$	$Y_{v\delta\delta}'$	$Y_{\delta u}'$	Y_{vu}'
Original	1.190×10^{-2}	-4.000×10^{-5}	5.560×10^{-3}	-1.160×10^{-2}
A1	1.026×10^{-2}	-7.403×10^{-4}	6.454×10^{-3}	-1.109×10^{-2}
A2	-7.111×10^{3}	-1.164×10^{3}	-1.213×10^{1}	-1.830×10^{3}
Parameter	Y_{ru}'	$Y_{\delta uu}'$	Y_{rrr}'	Y_{vrr}'
Original	-4.990×10^{-3}	2.780×10^{-3}	0	0
A1	-4.383×10^{-3}	4.915×10^{-3}	-5.180×10^{-4}	-2.951×10^{-3}
A2	-2.083×10^{1}	-1.025	2.841	9.926
Parameter	Y_{vuu}'	Y_{ruu}'	$Y_{r\delta\delta}'$	$Y_{rr\delta}'$
Original	0	0	0	0
A1	3.262×10^{-3}	3.197×10^{-3}	-3.901×10^{-4}	-1.116×10^{-3}
A2	5.962	3.011	-2.688×10^{-1}	-4.826×10^{-1}
Parameter	$Y_{rv\delta}'$	Y_0'	N_u'	N_{uu}'
Original	0	-4.000×10^{-5}	6.000×10^{-5}	3.000×10^{-5}
A1	-3.159×10^{-3}	-4.587×10^{-5}	7.675×10^{-5}	4.166×10^{-5}
A2	-1.158×10^{-2}	-4.584×10^{-5}	1.917	8.070
Parameter	N_v'	N_r'	N_δ'	N_{vvv}'
Original	-2.640×10^{-3}	-1.660×10^{-3}	-1.390×10^{-3}	1.636×10^{-2}
A1	-3.439×10^{-3}	-2.150×10^{-3}	-1.764×10^{-3}	2.019×10^{-2}
A2	2.862×10^{1}	6.284	3.455×10^{-1}	-2.856×10^{3}
Parameter	$N_{\delta\delta\delta}'$	N_{vvr}'	$N_{vv\delta}'$	$N_{v\delta\delta}'$
Original	4.500×10^{-4}	-5.483×10^{-2}	-4.890×10^{-3}	1.300×10^{-4}
A1	6.035×10^{-4}	-6.632×10^{-2}	-4.232×10^{-3}	5.562×10^{-4}
A2	-1.408×10^{1}	-4.604×10^{3}	-1.449×10^{3}	-2.515×10^{2}
Parameter	$N_{\delta u}'$	N_{vu}'	N_{ru}'	$N_{\delta uu}'$
Original	-2.780×10^{-3}	-2.640×10^{-3}	-1.660×10^{-3}	-1.390×10^{-3}
A1	-3.581×10^{-3}	-3.682×10^{-3}	-2.335×10^{-3}	-2.343×10^{-3}
A2	-3.265	4.129	1.192×10^{2}	-1.246×10^{-1}
Parameter	N_{rrr}'	N_{vrr}'	N_{vuu}'	N_{ruu}'
Original	0	0	0	0
A1	3.445×10^{-4}	2.054×10^{-3}	-2.349×10^{-3}	-1.672×10^{-3}
A2	5.244×10^{-1}	2.175	1.270	6.240×10^{-1}
Parameter	$N_{r\delta\delta}'$	$N_{rr\delta}'$	$N_{rv\delta}'$	N_0'
Original	0	0	0	3.000×10^{-5}
A1	2.130×10^{-4}	6.538×10^{-4}	2.132×10^{-3}	3.824×10^{-5}
A2	-2.614×10^{-2}	-3.768×10^{-2}	2.708×10^{-3}	3.822×10^{-5}

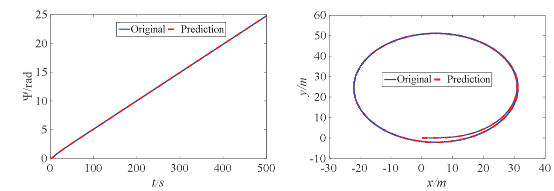
Above parameter identification results show that the improved least square identification algorithm proposed in this paper is better in accuracy and reliability than the traditional unimproved algorithm. Furthermore, some maneuvering prediction tests are carried out to verify the effectiveness and stability of the identification results, while the heading angle and trajectory of USV are shown below:



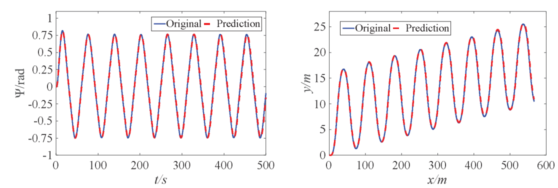
(a) heading angle predicion result (b) trajectory prediction result
Figure 2. 15° zigzag maneuvering prediction results



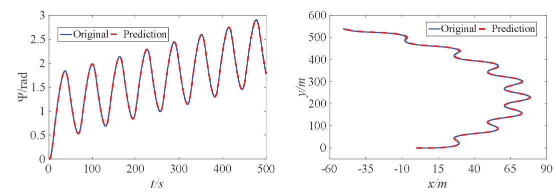
(a) heading angle predicion result (b) trajectory prediction result
Figure 3. 15° sine steering angle maneuvering prediction results



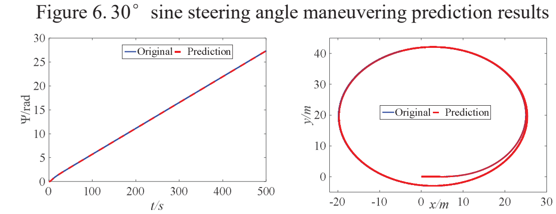
(a) heading angle predicion result (b) trajectory prediction result
Figure 4. 15° steady turning maneuvering prediction results



(a) heading angle predicion result (b) trajectory prediction result
Figure 5. 30° zigzag maneuvering prediction results



(a) heading angle predicion result (b) trajectory prediction result
Figure 6. 30° sine steering angle maneuvering prediction results



(a) heading angle predicion result (b) trajectory prediction result
Figure 7. 30° steady turning maneuvering prediction results

Model parameter identification results show in Table II and maneuvering prediction results show in Figures 2-7 all show that, the improved least square parameter identification algorithm designed is stable, reliable and effective, and it could be used in the identification system design of USV.

5 CONCLUSION

An improved least square parameter identification method is proposed in this paper for nonlinear motion model of an unmanned surface vehicle based on Euler discrete difference. Model transformation of the USV model is carefully designed in order to facilitate the design process of the identification system, and a novel kind of parameter identification structure transformation is presented here to compensate the influence of parameter drift. Some voyage data of simulation experiments are collected and then the parameters are obtained by the parameter identification algorithm presented. Several maneuvering prediction tests are carried out and the simulation results show the reliability, effectiveness and stability of the improved algorithm. Further, the algorithm will be applied to the actual USV's parameter identification system and some sea trials will be carried out.

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