



## Black-box modeling of ship maneuvering motion based on multi-output nu-support vector regression with random excitation signal

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### ABSTRACT

This paper proposes a novel method for offline black-box modeling of ship maneuvering by utilizing the training data from random maneuvers under medium rudder angle with random amplitude and duration. The identification algorithm adopted is a multi-output  $\nu$ ('nu')-Support Vector Regression, MO- $\nu$ -SVR, that has higher computational efficiency and better operability than a conventional  $\nu$ -SVR. The ONRT vessel is taken as the study object, and numerical simulations are conducted to provide the training, validation and testing datasets. The superiority of the proposed random maneuver over the standard zig-zag maneuver is demonstrated by a contrastive study where the excitation signals from the random maneuver and the 20°/20° zig-zag maneuver are used for training the model separately. To examine the robustness of the proposed modeling method and the identified model, three levels of white noise are added into the raw simulation data for training the model. To explore the effectiveness and generalization ability of the identified model on different motion patterns of ship maneuvering, course-keeping, course-changing, and turning motions are examined separately. The results demonstrate that the model trained by the excitation signals of the random maneuver has better generalization ability and robustness, verifying the feasibility and practicality of the proposed modeling method.

### 1. Introduction

Establishing an accurate model of ship maneuvering is an essential prerequisite for many of the applications, including but not limited to prediction of ship trajectory, path planning and motion control (Sutulo and Guedes Soares, 2011). Captive model tests, Computational Fluid Dynamics (CFD) method, empirical formulae or database method, and System Identification (SI) method are four applicable methods for modeling of ship maneuvering. Among them, SI method has the advantage of balancing the effectiveness and flexibility, requires little computational cost and is capable of adapting the changes in ship dynamic characteristics.

In the last decades, SI methods by utilizing the training data from physical or virtual free-running model tests have been extensively studied and applied in modeling of ship maneuvering. Most researches focused on offline modeling, which typically assumes that the system is time invariant with no dynamic changes under calm water condition

(Hwang, 1980; Luo and Zou, 2009; Sutulo and Guedes Soares, 2014; Moreno et al., 2019; Wang et al., 2019). Along with the rapid development of smart ship technologies and great concern arisen on them, online modeling for real-time prediction of ship maneuvering motion, which updates the dynamic model continuously by utilizing the incremental data in real time, has attracted increasingly attention as it is crucial to the realization of autonomous navigation. However, online modeling of ship maneuvering has not been effectively studied yet, and there still exists a large gap between the current offline modeling practice and the subsequent application in real-time prediction of ship maneuvering motion.

To realize real-time prediction of ship maneuvering motion, research on offline modeling method is necessary. Modeling of ship maneuvering online by using adaptive learning algorithm from scratch faces the problems of slow convergence and inferior performance caused by the biased initial samples (Agarwal et al., 2010). In most cases, obtaining an initial solution from an offline algorithm is the most efficient way to

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carry out online adapting modeling (Ma et al., 2003) and has been successfully applied in many fields (Nair and Clark, 2004; Agarwal et al., 2010; Hu et al., 2016; Liu and Zio, 2016). As regards the problem of online modeling of ship maneuvering, providing a robust and accurate pre-trained model as an initial model is of great significance, and previous studies on offline modeling of ship maneuvering may provide solutions.

On the one hand, an excitation signal that can be representative of ship maneuvering behavior is required to better excite the model. System identification is a process of obtaining a dynamic model from the observed input and output signals (Ljung, 1999). The performance of the identified model heavily depends on whether the input signal contains enough information of the system. Typically, previous studies collected data from the standard maneuvers such as 20°/20° and 10°/10° zig-zag maneuvers and 35° turning circle maneuver. These data are relatively easy to obtain, but another fact is that the standard maneuvers are proposed to evaluate ship maneuverability rather than to provide data for system identification. In some studies (Revestido Herrero and Velasco González, 2012; Perera et al., 2016; Wang et al., 2020a), it was pointed out that a single standard maneuver cannot provide sufficient information to effectively excite the dynamic features of ship maneuvering. To bridge the gap between the unsatisfactory excitation and the desirable model, Revestido Herrero and Velasco González (2012) reconstructed the model to adapt the excitation of zig-zag maneuvers. Wang and Zou (2018), Wang et al. (2019) combined several zig-zag maneuvers together to provide input signal, arguing that zig-zag maneuvers with varying rudder angles contain different dynamic features. Yoon and Rhee (2003), Rhee et al. (2004) managed to realize the optimal design of input signal based on pseudo-random binary sequence (PRBS). Further, Wang et al. (2020a) performed optimal design of input signal with multi-level pseudo-random signal to cover more nonlinear dynamic features. The mainstream idea is to use the excitation signal that only involves a few steering actions with manually selected or optimal designed amplitude and duration of rudder angles. Random excitation signal that involves frequent steering actions could be an option. Moreno et al. (2019) utilized the data of random maneuvers as training, validation and testing data, trying to establish a more robust model.

On the other hand, attention should be paid to the experimental cost and convenience in collecting the required data. To set up a maneuvering model that is not affected by the so-called scale effect for a real ship, sea trials need to be carried out, which is quite expensive. Generally, with respect to ship maneuverability with rudder as control device, the motion patterns of ship maneuvering can be categorized as course-keeping under small rudder angle, course-changing under medium rudder angle and turning under large rudder angle. The existing studies have made a lot of attempts to cover the control input space. The optimal design performed by Wang et al. (2020a) involves all ranges of rudder angles, including small, medium and large ones. However, steering a vessel with large rudder angle for a relatively long duration by following a rigid steering procedure requires an experimental site of large size. Besides, it is inconvenient to conduct in the sea trials, with the risk of capsizing.

Considering that a ship sailing at sea usually undergoes unsteady maneuvering motion under small or medium rudder angles, random maneuver with moderate rudder angle may be a feasible solution to facilitate the data collection, to reduce the cost and risk, and at the same time to reflect the dynamic features of ship maneuvering at sea. With random maneuver, the amplitude and frequency of the input signal can be arbitrary values, which can reach the goal of sufficiently exciting the model without any specific design of the input signal. Besides, the steering process does not need to follow a given rigid procedure, thus can be conducted conveniently on a daily voyage. The data collection can be performed repeatedly when needed, even when the vessel is on its daily voyage.

As for the identification algorithm, many techniques have been

successfully applied, such as maximum likelihood estimate (Källström and Åström, 1981), model reference approach (Van Amerongen, 1984), extended Kalman filter (Perera et al., 2011), genetic algorithm (Sutulo and Guedes Soares, 2014), least squares method (Wang et al., 2020a), Neural Network (NN) (Zhang and Zou, 2013) and Support Vector Machines (SVM) (Luo and Zou, 2009). Among them, SVM algorithm has a strict theoretical foundation following the principle of structural risk minimization. It can reach a global optimal solution and has better generalization ability. The studies carried out with different kinds of SVM algorithms have shown the effectiveness of this method in both white-box modeling and black-box modeling (Luo and Zou, 2009; Zhang and Zou, 2011; Moreno-Salinas et al., 2013; Luo et al., 2014; Wang et al., 2015; Zhu et al., 2017; Wang et al., 2019, 2020b; Xu and Guedes Soares, 2019).

White-box modeling with deterministic model structure based on prior knowledge is less susceptible to noise, since the model structure provides constraints for the system. On the other hand, however, selection of the model structure is a key problem that needs to reach a balance of model complexity and model capability. Parameter drift (Hwang, 1980) caused by multicollinearity (Belsley et al., 2005) is another critical issue that can be alleviated by carefully designed excitation signal but is nearly impossible to eliminate completely. Moreover, for a system affected by continuously varying external environmental factors and internal dynamic features, a fixed model structure is no longer applicable. To cope with the existing problems of white-box modeling, black-box modeling can be applied. In black-box modeling, no prior information is needed other than the input and output data of the system. Compared with white-box model, black-box model has better operability and flexibility, since there are no rigid constraints that need to be obeyed. It is more appropriate to choose black-box modeling approach when it comes to building an online adjustable model. However, black-box model is more likely to be sensitive to data noise for lack of prior knowledge. To boost the robustness of black-box model, the dynamic characteristics of the model must be fully excited by the rich dynamic information contained in the excitation signal of input data. From this aspect, design of the qualified excitation signal is highly desired.

In order to overcome the drawbacks of the existing studies on modeling of ship maneuvering and meet the actual application demands, the present study tries to develop an effective black-box modeling approach for offline modeling of a pre-trained model that can serve as an initial model for the subsequent online modeling of ship maneuvering. For the selection of excitation signal, the training dataset and validation dataset are constructed with the data obtained from a random maneuver that contains frequent steering actions with moderate rudder angle. For the evaluation of the model performance, the generalization ability of the model on different motion patterns of ship maneuvering and the robustness of the model to data noise with different levels are examined separately. In addition, improvement is made to the identification algorithm. A  $\nu$ ('nu')-Support Vector Regression ( $\nu$ -SVR) algorithm is adapted to providing multi-output ability to improve the identification efficiency and application operability.

The rest of the paper is organized as follows: Section 2 describes the principle and the whole process of the black-box modeling of ship maneuvering. Section 3 introduces the multi-output version of  $\nu$ -SVR (MO- $\nu$ -SVR) algorithm and specifies the hyperparameters that need to be tuned. A case study based on the simulation datasets is carried out in Section 4: Firstly, the study object and the data acquisition method are introduced. Then the design details of the proposed excitation signal and the evaluation scheme are elaborated. A black-box model is identified, and its robustness to white noise with different levels and its generalization ability on different motion patterns are examined. Meanwhile, a comparative study is conducted by using the excitation signals from the proposed random maneuvers and the standard zig-zag maneuvers. In the end, conclusions are summarized and future work is prospected in Section 5.

## 2. Problem formulation

### 2.1. Black-box model

The process of black-box modeling of a dynamic system is to obtain a nonlinear mapping from the input to the output of the system through training. For the dynamic modeling of ship motion considered here, the input includes the state variables and the control variables at the current time step, while the output includes the state variables at the next time step. This process can be described as

$$\dot{x}_s(t) = f(x_s(t), u(t)) \quad (1-a)$$

$$x_s(t+1) = h\dot{x}_s(t) + x_s(t) \quad (1-b)$$

where  $x_s(t)$  is the state vector and  $u(t)$  is the control vector at the current time step;  $f(\cdot)$  is the matrix form of the nonlinear mapping.  $x_s(t+1)$  is the state vector at the next time step, which is formulated by Euler method with the length of time step  $h$ .

Considering all the coupled motions, a ship can be regarded as a rigid body with motions of six degrees of freedom (DoF), including surge, sway, heave, roll, pitch, and yaw. In this study, focus is on the black-box modeling of the 3-DoF maneuvering motion (surge, sway, and yaw) in the horizontal plane. Referring to the well-known whole-ship model proposed by Abkowitz (1964) where the hydrodynamic forces and moments acting on a ship are expressed as functions of the kinematic variables and rudder angle, the speed components are taken as the state variables, and the rudder angle is taken as the control variable. From Eq.

(1-a) it follows the function of nonlinear mapping that serves as the black-box model to be identified:

$$u(t) = f_1(u(t), v(t), r(t), \delta(t)) \quad (2-a)$$

$$\dot{v}(t) = f_2(u(t), v(t), r(t), \delta(t)) \quad (2-b)$$

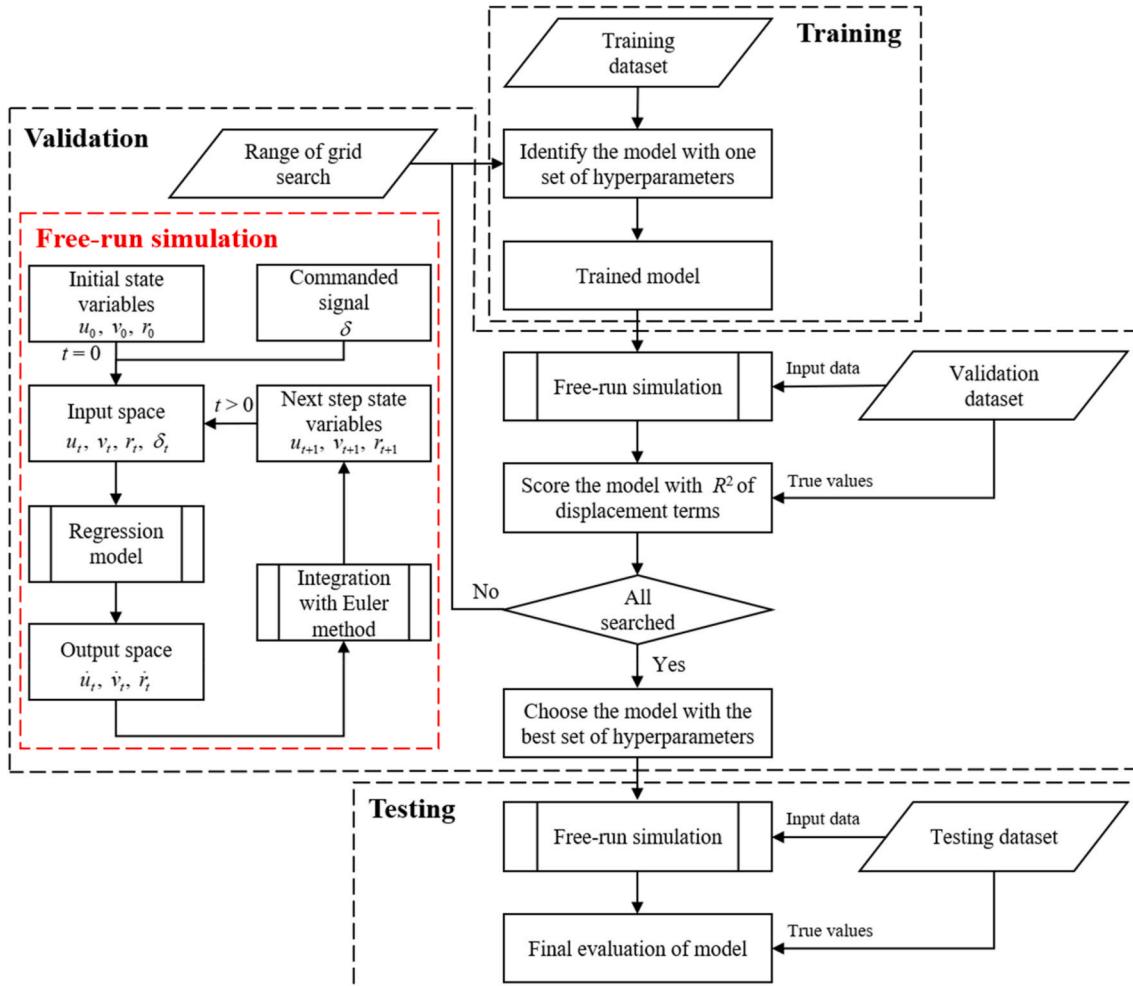
$$\dot{r}(t) = f_3(u(t), v(t), r(t), \delta(t)) \quad (2-c)$$

where  $u$ ,  $v$ ,  $r$  denote the surge speed, sway speed, and yaw rate, respectively;  $\delta$  denotes the rudder angle. They are the inputs of the black-box modeling.

### 2.2. Modeling process

Generally, establishing a reliable regression model requires three standard procedures, i.e., training, validation and testing with the training dataset, validation dataset and testing dataset, respectively. Firstly, the regression model is fitted on the training dataset. Then the hyperparameters in the model that control the complexity and accuracy of the fitted model are tuned on the validation dataset. Finally, the general performance of the tuned model is examined on the testing dataset. The flowchart of the identification modeling process is shown in Fig. 1.

Tuning of the hyperparameters is one of the decisive factors that affect the accuracy and robustness of any machine learning (ML) algorithms. In this study, grid search algorithm is applied to find the best hyperparameters. Grid search is an exhaustive searching through a



**Fig. 1.** Flowchart of the identification modeling process.

specified subset of the hyperparameter space, evaluating each case with a score. In this study, coefficient of determination, denoted as  $R^2$ , is chosen as the scoring strategy. Note that the scoring function is not directly applied to the original output data of  $u$ ,  $v$ , and  $r$ , but to the displacement of vessel motion in order to achieve a better model performance on trajectory prediction.  $R^2$  represents the proportion of target variance that is explained by the independent variables in the model, defined as

$$R^2(y, \hat{y}) = 1 - \frac{\sum_{i=1}^{n_s} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n_s} (y_i - \bar{y})^2} \quad (3)$$

where  $\hat{y}_i$  is the predicted value of the  $i$ -th sample;  $y_i$  is the corresponding true value;  $\bar{y}$  is the mean of all true values;  $n_s$  is the number of samples.  $R^2$  is a normalized term that would not change when the values are scaled.

Although the training procedure follows a point-to-point training method with the whole input space made up of true values, the other two procedures involve a different approach. Considering that the objective of the identified model is to predict ship maneuvering motion in multiple time steps, the validation and testing procedures are implemented with a free-run simulation. That is, the control variable  $\delta$ , as a command signal, is provided at each time step, while the state variables  $u$ ,  $v$ ,  $r$  are computed from the outputs of the previous time step as Eq. (1-b). Provided with the initial values of the state variables and a series of control variable, the regression process is implemented recursively.

### 3. Identification algorithm

#### 3.1. $\nu$ -SVR

Fundamentally, Support Vector Machines (SVM) method aims at mapping the input vector into a high-dimensional feature space to construct an optimal separating hyperplane. SVM was applied to regression and time series prediction successfully (Drucker et al., 1997; Müller et al., 1997) by regarding the hyperplane as a curve tube. This type of SVM algorithm is called Support Vector Regression (SVR). The standard SVR algorithm commonly used today is  $\epsilon$ -SVR proposed by Vapnik (1995). The procedure of  $\epsilon$ -SVR algorithm is as follows: At each data point  $x_i$ , an error within  $\epsilon$  is allowed within a so-called insensitive zone. Error above  $\epsilon$  is captured with slack variables  $\xi_i$  and  $\xi_i^*$ , and penalized along with the model complexity. The data points located precisely on the margin of the insensitive zone (the errors of these data points are equal to  $\epsilon$ ) are called support vectors (SVs).

Consider a set of data given as  $\{(x_1, y_1), \dots, (x_\ell, y_\ell)\} \in R^N \times R$ , where  $\ell$  is the number of the samples,  $N$  is the dimension of  $x_i$ . The regression model can be written in the following form:

$$f(x) = w^T \varphi(x) + b, \quad w \in R^M, \quad x \in R^N, \quad b \in R \quad (4)$$

where  $w$  is the weight vector;  $b$  is the bias;  $\varphi(\cdot)$  is a nonlinear function, mapping the original input to a high dimensional feature space;  $M$  is the dimension of the high dimensional feature space. For  $\epsilon$ -SVR algorithm, the model is trained by minimizing the structural risk as follow:

$$\min \tau(w, \xi^*) = \frac{1}{2} \|w\|^2 + C \frac{1}{\ell} \sum_{i=1}^{\ell} (\xi_i + \xi_i^*) \quad (5)$$

subject to

$$\begin{aligned} (w^T \varphi(x) + b) - y_i &\leq \epsilon + \xi_i \\ y_i - (w^T \varphi(x) + b) &\leq \epsilon + \xi_i^* \\ \xi_i^* &\geq 0 \end{aligned} \quad (6)$$

where  $\xi_i^*$  is the slack variables;  $C$  is a pre-defined constant that controls

the trade-off between model complexity and model precision. The first term of the right-hand side of Eq. (5) represents the model complexity, and the second term represents the  $\epsilon$ -insensitive error. Eq. (5) is a so-called “soft-margin” loss function.

The parameter  $\epsilon$  is a hyperparameter that determines the performance of the algorithm directly. It needs to be adjusted manually according to the level of noise contained in the data. To reduce the difficulty of choosing  $\epsilon$ , Schölkopf et al. (2000) proposed  $\nu$ -SVR which is a variant form of the standard  $\epsilon$ -SVR. In this algorithm, parameter  $\nu$  is used to directly control the number of support vectors and  $\epsilon$  is automatically determined based on the desirable number of SVs. Though the number of free parameters is not reduced, selection of  $\nu$  is much more explicit and easier to handle. This characteristic makes the  $\nu$ -SVR algorithm more suitable than the standard  $\epsilon$ -SVR algorithm.

In  $\nu$ -SVR algorithm,  $\epsilon$  still exists and serves as the same function as previous algorithm. However,  $\epsilon$  is no longer a prior-given hyperparameter and is added to the objective function as a parameter regularized by a new prior parameter,  $\nu$ . The width of the insensitive zone now is another minimization target. Hence, the optimization problem is modified as

$$\min \tau(w, \xi^*, \epsilon) = \frac{1}{2} \|w\|^2 + C \left( \nu \epsilon + \frac{1}{\ell} \sum_{i=1}^{\ell} (\xi_i + \xi_i^*) \right) \quad (7)$$

subject to

$$\begin{aligned} (w^T \varphi(x) + b) - y_i &\leq \epsilon + \xi_i \\ y_i - (w^T \varphi(x) + b) &\leq \epsilon + \xi_i^* \\ \xi_i^* &\geq 0, \quad \epsilon \geq 0 \end{aligned} \quad (8)$$

Using Lagrange multiplier method, Eq. (7) and Eq. (8) can be transformed into a dual optimization problem as

$$\begin{aligned} L(w, b, \alpha^*, \beta, \xi^*, \epsilon, \eta^*) &= \frac{1}{2} \|w\|^2 + C \nu \epsilon \\ &+ \frac{C}{\ell} \sum_{i=1}^{\ell} (\xi_i + \xi_i^*) - \beta \epsilon - \sum_{i=1}^{\ell} (\eta_i \xi_i + \eta_i^* \xi_i^*) \\ &- \sum_{i=1}^{\ell} \alpha_i (\xi_i + y_i - w^T \varphi(x) - b + \epsilon) \\ &- \sum_{i=1}^{\ell} \alpha_i^* (\xi_i^* - y_i + w^T \varphi(x) + b + \epsilon) \end{aligned} \quad (9)$$

where  $w$ ,  $b$ ,  $\xi_i^*$ , and  $\epsilon$  are the primal variables;  $\alpha_i^*$ ,  $\beta$ , and  $\eta_i^*$  are the Lagrange multipliers. To minimize Eq. (9), the optimal solution is at the saddle point of the Lagrangian, complying

$$\begin{aligned} \frac{\partial L}{\partial w} = 0 &\rightarrow w = \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) \varphi(x_i) \\ \frac{\partial L}{\partial b} = 0 &\rightarrow \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) = 0 \\ \frac{\partial L}{\partial \xi_i^*} = 0 &\rightarrow \frac{C}{\ell} - \alpha_i^* - \eta_i^* = 0 \\ \frac{\partial L}{\partial \epsilon} = 0 &\rightarrow C \nu - \sum_{i=1}^{\ell} (\alpha_i^* + \alpha_i) - \beta = 0 \end{aligned} \quad (10)$$

Substituting Eq. (10) into Eq. (9) and replacing the dot product of nonlinear function  $\varphi(\cdot)$  with a kernel function  $K(x_i, x_j) = \varphi(x_i)^T \varphi(x_j)$ , Eq. (9) can be further rewritten as

$$\max W(\alpha^*) = \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) y_i - \frac{1}{2} \sum_{i,j=1}^{\ell} (\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j) K(x_i, x_j) \quad (11)$$

subject to

$$\begin{aligned} \sum_{i=1}^{\ell} (\alpha_i - \alpha_i^*) &= 0 \\ \alpha_i^* &\in \left[0, \frac{C}{\ell}\right] \\ \sum_{i=1}^{\ell} (\alpha_i + \alpha_i^*) &\leq C\nu \end{aligned} \quad (12)$$

The global optimal solution can be reached under Karush-Kuhn-Tucker (KKT) condition and the weight vector  $\mathbf{w}$  is a linear combination of the SVs.  $\nu$  is an upper bound of the data points lying outside the insensitive zone and a lower bound of the SVs relative to the total number of input data. The value of  $\varepsilon$  is automatically decided accordingly.

In this study, Radial Basis Function (RBF) kernel is selected as the kernel function, which is expressed as

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\gamma \|\mathbf{x}_i - \mathbf{x}_j\|^2) \quad (13)$$

where  $\gamma$  is the width parameter of the RBF kernel.

### 3.2. MO- $\nu$ -SVR

Essentially, SVR is a single-output regression algorithm. Although the most suitable hyperparameters of the model may be found, one fitted model can only predict one feature. For modeling of ship maneuvering, the number of the features that need to be predicted is consistent with the degrees of freedom of motion to be modeled. Multiple models need to be built as in previous studies (Luo and Zou, 2009; Zhang and Zou, 2011; Luo et al., 2014; Wang et al., 2015, Wang et al., 2019, 2020b). However, using single-output algorithm for modeling of ship maneuvering will suffer from the following problems:

#### 1) Time-consuming

The training and validation procedures, including the hyperparameters tuning process that consumes a large amount of time, need to be implemented for each model. It does not seem to be a big problem for offline modeling of 3-DoF motions, but for the applications such as modeling of 6-DoF motions and online continuous modeling in real time, time consumption of single-output algorithms will be unfavorable.

#### 2) Poor operability for hyperparameters tuning

As mentioned in Section 2, free-run simulation is a more suitable way for tuning the hyperparameters. The state variables at the next time step are obtained by integration of the outputs. In the previous study that applied free-run simulation method with single-output regression model (Wang et al., 2020b), the hyperparameters of each model were tuned by using the true values of motions of other DoFs, since the single-output algorithm could not predict the motions of all DoFs simultaneously. This may cause accumulated errors.

Using a multi-output algorithm, the two disadvantages mentioned above can be overcome, since a multi-output regression model can predict all the features with one model:

$$\left. \begin{aligned} \dot{u}(t) &= f_1(u(t), v(t), r(t), \delta(t)) \\ \dot{v}(t) &= f_2(u(t), v(t), r(t), \delta(t)) \\ \dot{r}(t) &= f_3(u(t), v(t), r(t), \delta(t)) \end{aligned} \right\} \Rightarrow \mathbf{y}(t) = \begin{bmatrix} \dot{u}(t) \\ \dot{v}(t) \\ \dot{r}(t) \end{bmatrix} = \mathbf{f}(\mathbf{u}(t), \mathbf{v}(t), \mathbf{r}(t), \delta(t)) \quad (14)$$

where  $\mathbf{y}(t)$  is the output vector.

This paper proposes a multi-output  $\nu$ -SVR algorithm denoted as MO- $\nu$ -SVR. The basic idea is to extend the dimension of the model as

$$\begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{r} \end{bmatrix} = \hat{\mathbf{y}} = \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \mathbf{w}_3^T \end{bmatrix} \varphi(\mathbf{x}) + \mathbf{b}, \quad \mathbf{w} \in \mathbb{R}^M, \mathbf{x} \in \mathbb{R}^N, \hat{\mathbf{y}} \in \mathbb{R}^3, \mathbf{b} \in \mathbb{R}^3 \quad (15)$$

where the hat mark “ $\hat{\cdot}$ ” denotes the predicted values, to distinguish from the true values without the hat mark.

In MO- $\nu$ -SVR, the weight vector  $\mathbf{w}$  and the bias  $\mathbf{b}$  for each output variable are different and are calculated independently. The models of all DoFs are organized in a matrix form to fit and tune simultaneously. They share the same hyperparameters and can produce outputs simultaneously. The tuning process is based on the mean error of all three outputs. Using the same hyperparameters for all DoFs may reduce model accuracy to some extent, since the chosen hyperparameters may not be most suitable for all DoFs. Nevertheless, the results presented in the following section show that the influence is too small to cause noticeable deviations.

### 3.3. Hyperparameters

There is in total three hyperparameters to be tuned, i.e.,  $\gamma$  in RBF kernel function,  $C$  and  $\nu$  in MO- $\nu$ -SVR. The hyperparameter  $\gamma$  implicitly defines the distribution of the mapped input data in the feature space. Its value is chosen in the range of [0, 1]. A too large  $\gamma$  may result in a highly concentrated distribution that causes overfitting problem, decreasing the generalization ability and robustness of the model. On the other hand, if  $\gamma$  is too small, the distribution will be very flat and the mapping will behave almost linearly, making the model fail to learn the nonlinear features. The hyperparameter  $\nu$  gives an upper bound of the outliers and a lower bound of the SVs relative to the total number of the input data. Its value is in the range of [0, 1]. The limitation on the fractions of SVs and outliers directly decides the computational efficiency and model performance. Model with more SVs needs more computation time and tends to be more accurate, but is relatively poor in robustness. The hyperparameter  $C$ , common to all SVM algorithms, trades off the modeling error against the simplicity of the model. A smaller  $C$  tolerates larger modeling error with a simple model, while a large  $C$  aims to reach an accurate result with a complex model.

The tuning process is implemented by using grid search algorithm and free-run simulation. The hyperparameters are chosen according to the  $R^2$  score of each case in the grid. Considering the characteristics of the specific problem and each hyperparameter, the range of the grid is chosen as:  $\gamma \in [2^{-6}, 2^{-5}, 2^{-4}, 2^{-3}, 2^{-2}]$ ,  $\nu \in [0.2, 0.25, 0.3]$ ,  $C \in [2^{-2}, 2^{-1}, 2^0]$ .

## 4. Case study with a ONRT ship model

### 4.1. Study object

To evaluate the effectiveness of the proposed random excitation signal and MO- $\nu$ -SVR algorithm, a case study with a scaled model of the surface combatant ONR tumblehome (ONRT) is carried out. The main particulars of the ship model are given in Table 1.

**Table 1**  
Main particulars of the ONRT ship model.

Parameters	Value	Unit
Length $L$	3.147	m
Breadth $B$	0.384	m
Draft $d$	0.112	m
Displacement $\Delta$	72.6	kg
Transverse metacentric height $GM$	0.0423	m
Dimensionless radius of gyration for roll $k_{xx}/B$	0.344	–
Dimensionless radius of gyration for pitch, yaw $k_{yy}/L, k_{zz}/L$	0.246	–
Rudder area $A_R$	$0.012 \times 2$	$m^2$
Maximum rudder angle $\delta_{max}$	$\pm 35$	deg
Rudder rate $\dot{\delta}$	35	deg/s

#### 4.2. Data acquisition

Data used in this study are obtained from numerical simulations using a predetermined mathematical model of 3-DoF ship maneuvering motion. The effectiveness and accuracy of the simulation method have been proved in a previous study by Guo and Zou (2017). The mathematical model used for simulation is a 3-DoF MMG (Maneuvering Modeling Group) model adapted from the equations proposed by Umeda and Hashimoto (2002). It can be expressed in the following form:

$$\dot{u} = [-R(u_0) + X_{vv}(u)v^2 + X_{vr}(u)vr + X_{rr}(u)r^2 + X_P(u, n) + X_R(\delta, u, v, r) + (m + m_y)vr] / (m + m_x) \quad (16-a)$$

$$\dot{v} = [Y_v(u)v + Y_r(u)r + Y_{vv}(u)v^3 + Y_{vvr}(u)v^2r + Y_{vrr}(u)vr^2 + Y_{rrr}(u)r^3 + Y_P(u, v, r) + Y_R(\delta, u, v, r) - (m + m_x)ur] / (m + m_y) \quad (16-b)$$

$$\dot{r} = [N_v(u)v + N_r(u)r + N_{vvv}(u)v^3 + N_{vvr}(u)v^2r + N_{vrr}(u)vr^2 + N_{rrr}(u)r^3 + N_P(u, v, r) + N_R(\delta, u, v, r)] / (I_z + J_z) \quad (16-c)$$

where  $m$  is the mass of the ship;  $m_x, m_y$  are the added masses in surge and sway respectively;  $I_z$  is the moment of inertia in yaw;  $J_z$  is the added moment of inertia in yaw.  $R$  is the ship resistance at the design speed  $u_0$ ;  $n$  is the propeller revolutions.  $X, Y$ , and  $N$  with the motion variables  $u, v, r$  as subscripts denote the linear and nonlinear hydrodynamic derivatives of surge force, sway force and yaw moment. The subscripts ' $P$ ' and ' $R$ ' denote the force or moment components induced by propeller and rudder.

The data obtained from numerical simulation are initially pure, and do not contain the measurement errors that inevitably exist in real experimental data. Therefore, white noise with different levels is added into the raw data to imitate the measurement errors contained in real experimental data, following the method used in Sutulo and Guedes Soares (2014) and Wang et al. (2019):

$$\zeta_i = \zeta_{0i} + \zeta^{\max} k_0 k_\xi \xi_i \quad (17)$$

where  $\zeta_i$  is the polluted data;  $\zeta_{0i}$  is the original pure data.  $\zeta^{\max} k_0 k_\xi \xi_i$  is the disturbance term, where  $\zeta^{\max}$  is the maximum absolute value of the pure data;  $k_0$  is the general reduction factor corresponding to different noise level;  $k_\xi$  is the variable-specified reduction factor corresponding to different variables, which is set to 0.05 for  $\delta$ , 0.20 for  $u$ , and 1.00 for  $v$  and  $r$ , corresponding to the probable sensor accuracy of each input variable;  $\xi_i$  is the Gaussian white noise process with variance of 1.

Three different noise levels are selected as  $k_0 = 1\%, 5\%$ , and  $10\%$ , on account of data-collecting means. Together with the raw data, four datasets are constructed and referred to as Pure data (Pure), Noise Level 1 (NL1), Noise Level 2 (NL2) and Noise Level 3 (NL3), respectively. Noise Level 1 corresponds to the data from precise sensors or after pre-processed. Noise Level 2 and Noise Level 3 represent the data collected by a normal navigation unit or other conventional sensors (Wang et al., 2019).

#### 4.3. Design and pre-processing of datasets

Most previous studies collected data from the standard maneuvers such as zig-zag maneuvers and turning circle maneuvers either through physical experiments or numerical simulations. However, these standard maneuvers are proposed for evaluating ship maneuverability, not for system identification purpose. They might not effectively excite the dynamic characteristics of ships. Thus, training a model by using the data from these standard maneuvers may not obtain the model that can adequately reflect the dynamic characteristics of ship maneuvering, and only examining the identified model on these standard maneuvers cannot properly evaluate the model performance for all kinds of ship motion pattern.

In fact, most of ship maneuvers during sailing at sea are under small

or medium rudder angles and with unsteady motion features. On the other hand, while ship maneuver under small rudder angle may not fully stimulate the nonlinear dynamic characteristics of a ship, steering a ship with large rudder angle for a long duration may not only require an experimental site of larger size and longer experimental time, increasing the difficulty of experiment, but also increase the risk of capsizing. For these reasons, a random maneuver with frequent steering under moderate rudder angles is proposed to provide the excitation signal. It can properly represent most of the dynamic characteristics of ship maneuvering, and also can be conducted conveniently and safely, even during a daily voyage of the ship. Specifically, in this study, the random maneuver is performed under a random rudder angle within  $(-0.3 \text{ rad}, 0.3 \text{ rad})$ , approximately within  $(-17.2^\circ, 17.2^\circ)$ , with a sampling step of 0.2s, and the duration of each rudder angle signal is a random value between 5s and 10s. The data obtained from the random maneuver are divided into three datasets that are used for training, validation, and testing, respectively. The training dataset contains 1200 samples, and each of the other two datasets contains 400 samples. The black-box model is trained and tuned respectively using the datasets for training and validation that are intercepted from the data of the proposed random maneuver. In addition, some other maneuvers are performed to provide data for examining the model performance on different motion patterns, including course-keeping under small rudder angle, which is examined with the so-called  $0^\circ/5^\circ$  Very Small Zig-zag (VSZZ) maneuver (i.e., to keep the original course with  $5^\circ$  rudder angle) as recommended by IMO (2002); course-changing under medium rudder angle, which is examined with the proposed random maneuver and  $15^\circ/15^\circ$  zig-zag maneuver; turning under large rudder angle, which is examined with  $35^\circ$  turning circle maneuver. Table 2 provides an overview of the datasets for training, validation and testing.

White noise of different levels that is used to imitate the measurement error of different levels is added to the raw data (pure data) for training to examine the robustness of the proposed method. The pure and polluted training datasets are shown in Fig. 2. Note that the differences in the effects of noise on each input variable correspond to the maximum absolute value and probable sensor accuracy, as described in Eq. (17).

SVM algorithms are not scale invariant, and the features with large magnitude may dominate the objective function and make the estimator unable to learn from other features correctly. Thus, appropriate data pre-processing is desirable, which can largely improve the accuracy of the model. In this study, the data in the input space are standardized and centralized to have a mean of 0 and a variance of 1 by

$$y'_i = \frac{y_i - \bar{y}}{\sigma(y)} \quad (18)$$

where  $y_i$  is the original variable.  $y'_i$  is the scaled variable.  $\bar{y}$  and  $\sigma(y)$  are the mean and the variance computed from the samples in the training dataset, respectively. Note that the same scaling must be applied to the validation dataset and the testing dataset to obtain meaningful results.

#### 4.4. Identification results

After careful design and preparation of the datasets, the model is fitted and tuned with the proposed black-box modeling MO- $\nu$ -SVR algorithm. The training dataset and validation dataset from the proposed

**Table 2**

Datasets for training, validation and testing.

Datasets	Source of the datasets
Training	Random maneuver
Validation	Random maneuver
Testing	Course-keeping: VSZZ maneuver Course-changing: Random maneuver and $15^\circ/15^\circ$ zig-zag maneuver Turning: $35^\circ$ turning circle maneuver

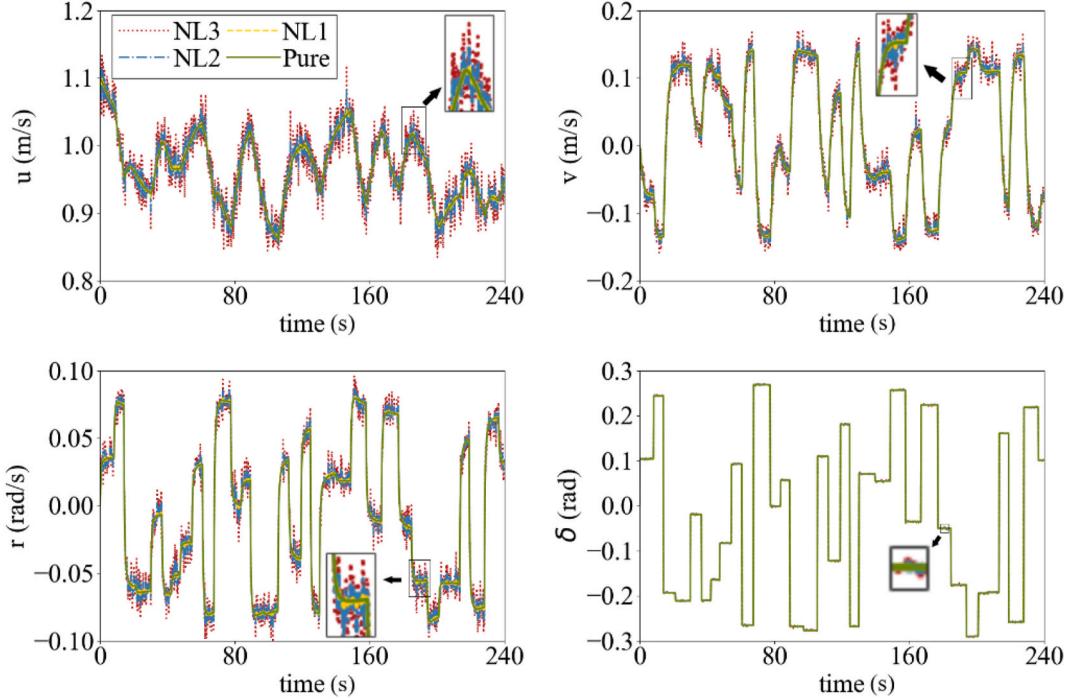


Fig. 2. Training datasets with white noise of different levels.

random maneuver contain the maneuvering information of course-changing motion under medium rudder angle. Identification results on the testing dataset with the same motion pattern under the interference of noise are presented and compared with those trained by the widely used  $20^\circ/20^\circ$  zig-zag maneuver.

Firstly, the dataset from the random maneuver that is intercepted for testing is examined. Fig. 3 presents the prediction results of the trained models. It can be seen that the identified model excited by the random maneuver is able to predict ship trajectory accurately under Noise Level 1 and Noise Level 2, while slight deviation occurs under Noise Level 3

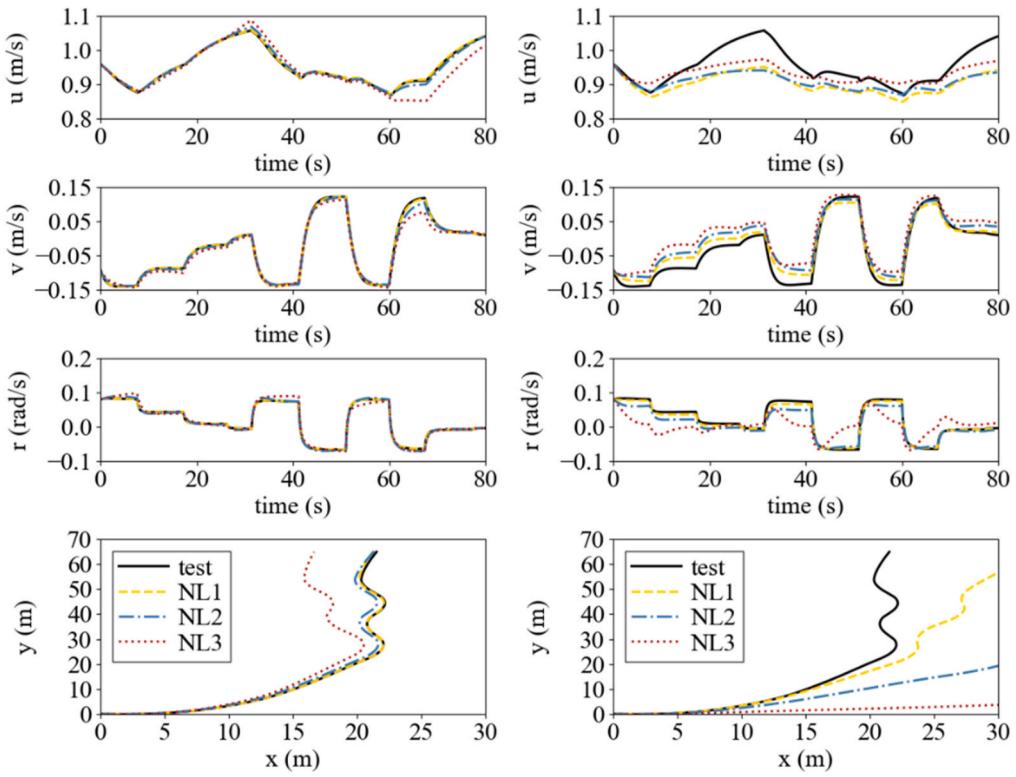


Fig. 3. Prediction results of random maneuver with the models affected by noise.

due to larger errors of  $u$  and  $v$  at the latter prediction steps. By contrast, the model excited by  $20^\circ/20^\circ$  zig-zag maneuver cannot achieve satisfactory prediction of ship trajectory even under the smallest noise. The errors of  $v$  and  $r$  are tolerable under slight noise disturbance, but the error of  $u$  is far too large.

However, only testing the trained models on a random maneuver may not be able to fully demonstrate the superiority of the proposed excitation signal. The better model performance that Fig. 3 exhibits may be due to overfitting, since similar type of random maneuver is utilized in both training and testing. To test the performance of the trained model comprehensively,  $15^\circ/15^\circ$  zig-zag maneuver is also examined. Fig. 4 shows the prediction results of the trained models on a  $15^\circ/15^\circ$  zig-zag maneuver as testing dataset, where  $\psi$  is the heading angle. The prediction results show that the model identified with the random maneuver still exhibits much better performance under all noise levels, while the model excited by  $20^\circ/20^\circ$  zig-zag maneuver fails to complete the prediction of  $15^\circ/15^\circ$  zig-zag maneuver after reversing the rudder angle for only one time under Noise Level 3.

Further, the prediction accuracy of the speed components in the testing dataset is evaluated by root mean square error (RMSE) defined as

$$RMSE = \sqrt{\frac{1}{n_s} \sum_{i=1}^{n_s} (y_i - \hat{y}_i)^2} \quad (19)$$

where  $\hat{y}_i$  is the predicted value of the  $i$ -th sample,  $y_i$  is the corresponding true value;  $n_s$  is the number of samples.

Comparisons between the trained models are shown in Fig. 5. It can be seen clearly that in general the accuracy of the model trained with the dataset of random maneuver under medium rudder angle is improved significantly with high anti-noise ability.

As can be seen in Fig. 5, the influence of noise on the model performance is noticeable; the prediction accuracy decreases gradually as the noise level increases. Overall, all the results predicted by the model identified by the proposed modeling method fit well with the true values. The prediction results on two different datasets from random maneuver and  $15^\circ/15^\circ$  zig-zag maneuver show that the identified models have good generalization ability for course-changing motion. The comparison of the prediction results between the models excited by random maneuver and by  $20^\circ/20^\circ$  zig-zag maneuver demonstrates that the rudder angle signal of random maneuver contains sufficient information for establishing the prediction model, while  $20^\circ/20^\circ$  zig-zag maneuver is not an ideal data source for this purpose. The performance differences between the two models can be attributed to the fact that the dataset of the proposed random maneuver is constructed using an input signal (rudder angle) that changes frequently with medium amplitude and random duration, thus the model can be fully excited; while in  $20^\circ/20^\circ$  zig-zag maneuver, the rudder angle changes only at a specific instant when the heading angle is equal to the executed rudder

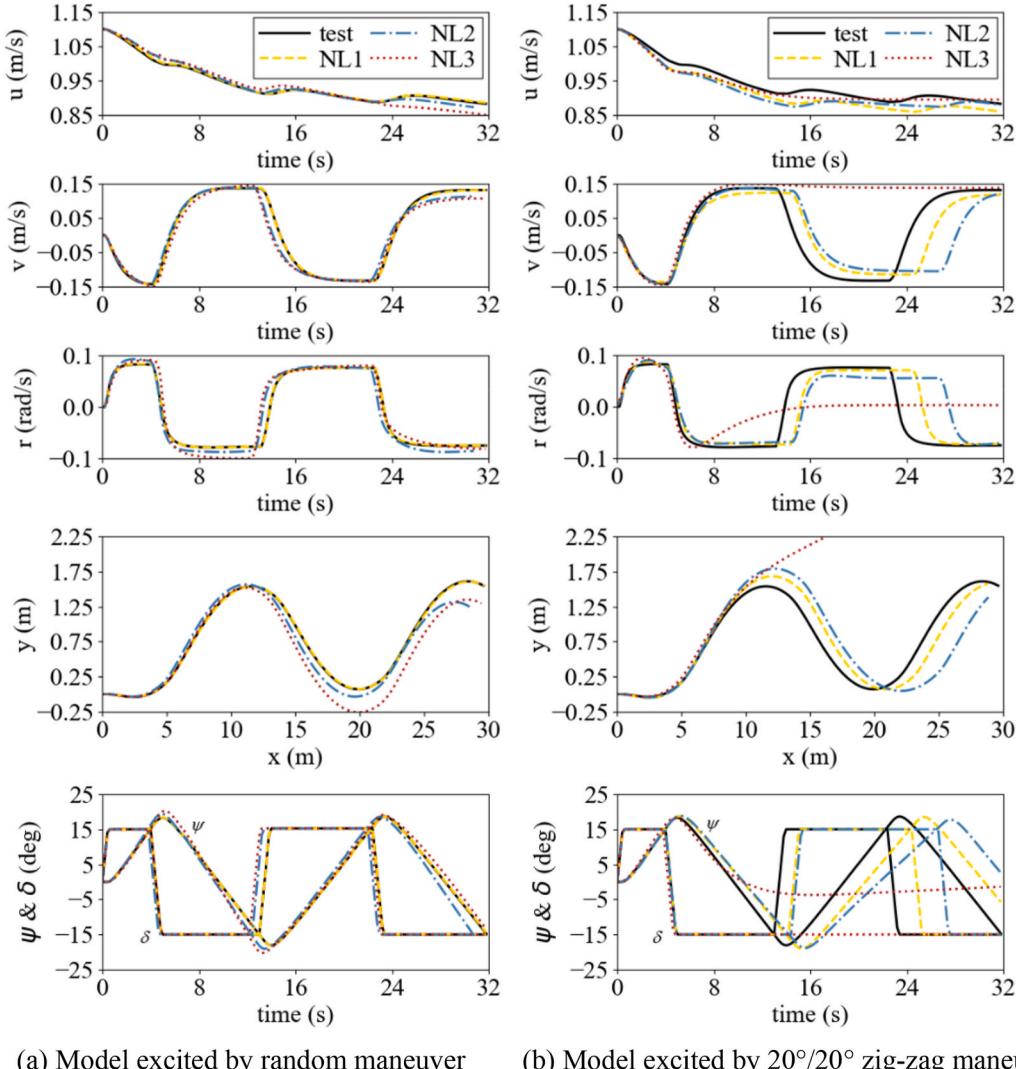


Fig. 4. Prediction results of  $15^\circ/15^\circ$  zig-zag maneuver with the models affected by noise.

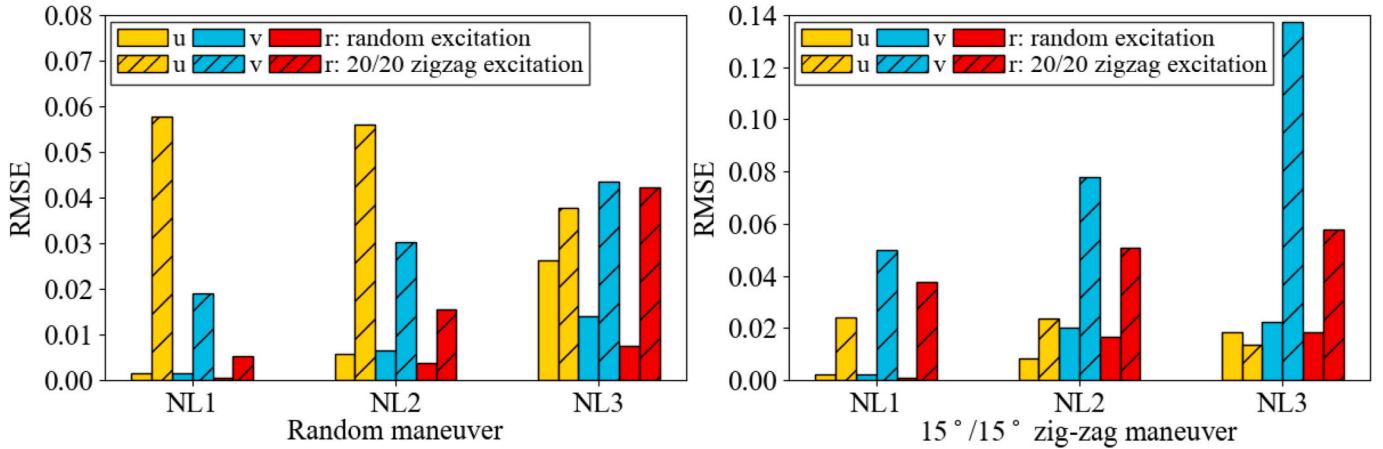


Fig. 5. Comparison of prediction accuracy evaluated by RMSE.

angle (i.e., 20°), and the change lasts only for a short period, thus the dataset does not contain sufficient dynamic information to fully excite the dynamic features of the model.

The designed random excitation signal does not include the information of course-keeping motion pattern under small rudder angle and turning motion pattern under large rudder angle. The generalization ability of the identified model is further examined on these two motion patterns with the datasets of Very Small Zig-zag (VSZZ) maneuver and 35° turning circle maneuver separately.

VSZZ is believed to be the most useful type of maneuver for representing the behavior of a ship steered to maintain a straight course. The difference between VSZZ and similar conventional zig-zag maneuvers such as 1°/5° zig-zag maneuver is that VSZZ must be initialized with a non-zero yaw rate  $r$  (IMO, 2002). Moreover, VSZZ should consist of a larger number of cycles than zig-zag maneuvers, since the concern is to investigate the course-keeping performance in a relatively long duration.

Fig. 6 presents the prediction results of the trained models with the dataset of a VSZZ maneuver as testing dataset. It shows that the VSZZ maneuver can be predicted accurately by the model trained with the random maneuver. By contrast, the model trained with 20°/20° zig-zag maneuver is not applicable at all.

The maneuvering behavior of a ship steered to turn under the maximum rudder angle can be demonstrated by 35° turning circle

maneuver. Fig. 7(a) illustrates the prediction results of 35° turning circle maneuver with the model excited by the proposed random signal (denoted as “original model”). As can be seen, the prediction fails since the surge speed approaches 0. The reason behind the failure is that the turning circle maneuver is implemented with the maximum rudder angle that is not covered in the training dataset. Nevertheless, it can be observed that despite the fact that the model is not able to predict ship trajectory over the steady turning phase, an acceptable prediction of trajectory can be achieved over a short period in the steering phase. To explore whether the identified model can adjust itself to fulfill the task of predicting the complete trajectory under full rudder, 50 samples of the 35° turning circle maneuver are appended to the end of the original training dataset to broaden the input space of the model. The added samples are intercepted from the dataset at the beginning of the 35° turning circle maneuver that includes the motion information in the steering phase and part of the transition phase. Then using the newly constructed datasets, the model is trained and tuned following the same approaches as used for identifying the original model. The obtained model is denoted as “adjusted model”. Fig. 7(b) shows the prediction results of 35° turning circle maneuver by the adjusted model. It can be seen that the adjusted model can predict completely and accurately the 35° turning circle maneuver, indicating that with slight adjustment to the training dataset, the model identified by the proposed black-box modeling method is able to predict the turning circle maneuver under

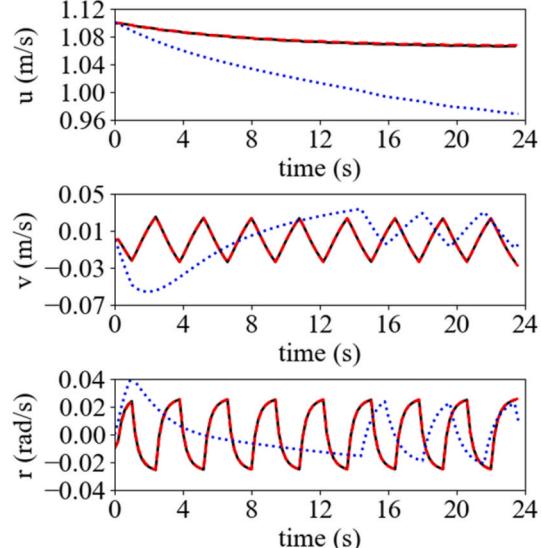
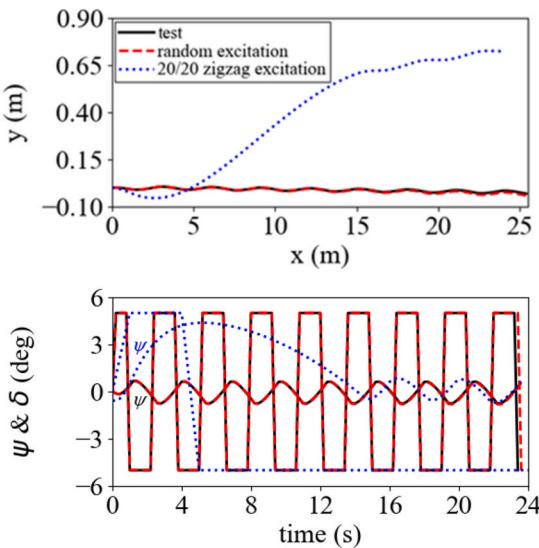
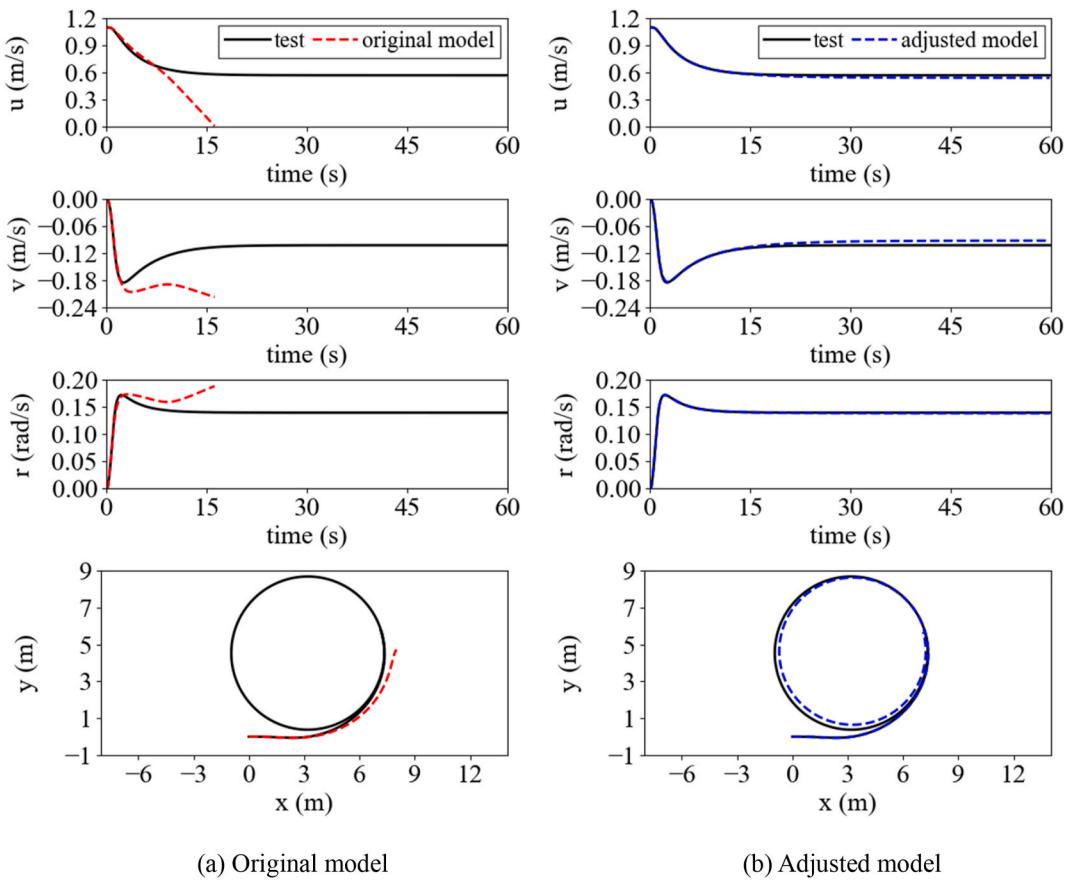


Fig. 6. Prediction results of VSZZ maneuver.



**Fig. 7.** Prediction results of 35° turning circle maneuver.

full rudder.

## 5. Conclusions

To obtain a pre-trained model for predicting ship maneuvering of different motion patterns that can be used as an initial model for the subsequent online modeling and real-time prediction of ship maneuvering motion, this paper proposes a novel scheme based on multi-output  $\nu$ -SVR (MO- $\nu$ -SVR) algorithm for establishing a black-box model of ship maneuvering, including design of random excitation signal, evaluation of the generalization ability of the identified model on various motion patterns of ship maneuvering and the robustness of the identified model to interference of noise with different levels.

Taking the ONRT ship model as the study object, a comprehensive case study is carried out. All the data used are obtained by numerical simulation with a 3-DoF MMG model of ship maneuvering motion. Firstly, the prediction accuracy and robustness of the MO- $\nu$ -SVR algorithm that uses a set of hyperparameters are proved. Secondly, a comparison between the models excited by the proposed random maneuver under moderate rudder angles and by the widely used 20°/20° zig-zag maneuver is conducted, which reveals that the excitation signal from the proposed random maneuver contains more information of the dynamic system than that from 20°/20° zig-zag maneuver. Thus, the model to be identified can be better excited by the proposed random maneuver, and has higher generalization ability and anti-noise ability; the instability of black-box modeling can be diminished to a certain extent. Thirdly, the generalization ability of the identified model obtained by the proposed method is demonstrated by testing the model on different motion patterns of ship maneuvering. The high prediction accuracy of course-keeping and course-changing motions under small and medium rudder angles indicates that the identified model performs well in the

prediction of the motion patterns of ship maneuvering that are typical for a ship sailing at sea. Besides, it is shown that continuous prediction of 35° turning circle maneuver under large rudder angle that is not covered in the training dataset can be achieved with slight adjustment by adding a few samples from the turning motion into the training dataset. The identified model can serve as a pre-trained model for the follow-up online black-box modeling of ship maneuvering, being applied effectively in the real-time prediction of various motion patterns of ship maneuvering.

The present study sets up a framework for offline black-box modeling of ship maneuvering with an easily tuned algorithm and conveniently collected training data that contain the excitation signal being able to fully stimulate the dynamic characteristics of a ship in maneuvering motion. A robust pre-trained black-box model that has high generalization ability on various motion patterns of ship maneuvering is established and validated. In the future, efforts will be made to relate the research to real scenarios in actual maritime activity and to apply the developed method in the practical navigation problems. The incremental algorithm with continuous learning ability will be applied for online modeling and real-time prediction of maneuvering motion for a ship sailing at sea; especially, the external disturbances from the environmental factors that may affect the ship dynamic characteristics will be taken into account.

## CRediT authorship contribution statement

**Yan-Yun Zhang:** Conceptualization, Methodology, Formal analysis, Simulation, Analysis, Writing – original draft. **Zi-Hao Wang:** Conceptualization, Methodology, Formal analysis, Analysis, Writing – review & editing. **Zao-Jian Zou:** Supervision, Funding acquisition, Project administration, Writing – review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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