IGME 309 EX 9

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1 Introduction

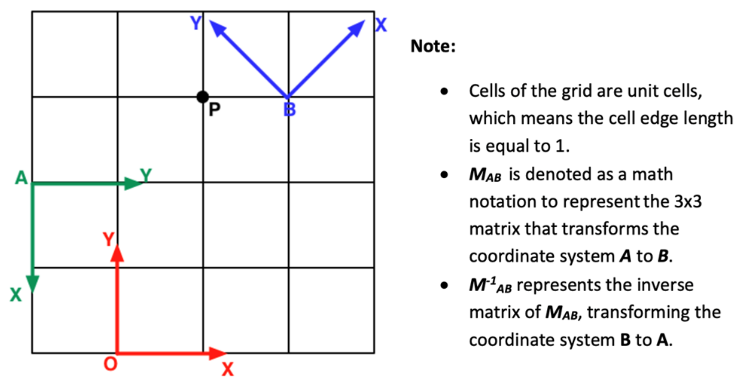

Figure 1: Image of transformations

1.1 Q1

What are the coordinates of P in the coordinate system O?

The origin of coordinate system O is translated one unit to the right, so in the standard basis the origin would be $(1, 0)$. In this case, we can count the units:

$$P_O = (1, 3)$$

1.2 Q2

What are the coordinates of P in the coordinate system A?

This time, the coordinate system is rotated 90 degrees clockwise and translated 2 units up from the origin. so that positive-x is now facing downward. We can just count the spaces from point A. We count two spaces right, and one space up. That means:

$$P_A = (-1, 2)$$

1.3 Q3

What are the coordinates of P in the coordinate system B?

This coordinate system is rotated 45 degrees counterclockwise, with an origin at $(3, 3)$. We can still do this visually, but we must imagine diagonal x-axis extending further as to cut the bottom-right square from P in half. The distance in the x-direction to the point P is then one half of the diagonal of a square, which is $\frac{\sqrt{2}}{2}$. Likewise in y, so we get that

$$P_B = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

1.4 Q4

Derive and calculate the values of M_{AB}.

We can think of this transformation, M_{AB}, as a rotation 135 degrees counterclockwise, and then moving three units right and one unit up. The transformations are not commutative in this case. We should apply the rotation first about the origin A, then apply the translation. We will denote the translation as $M_{AB_{translate}}$, and the rotation as $M_{AB_{rotate}}$

The composition of matrix operations forms the transformation

$$M_{AB} = M_{AB_{translate}} M_{AB_{rotate}}$$

We will need to convert any such vector $\mathbf{v_2} \in \mathbb{R}^2 = [v_x, v_y]^T$ into an augmented vector $\mathbf{v_3} \in \mathbb{R}^3 = [v_x, v_y, 1]^T$ to apply these transformations as a matrix operation.

$$M_{AB_{translate}} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Where t_x and t_y represent the translation vector $\mathbf{t} = [3, 1]$. We obtain

$$M_{AB_{translate}} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Intuitively, this matrix leaves x and y alone, but because $\mathbf{v}_{3_z} = 1$, the linear combination of columns can be represented as

$$
\begin{aligned}
M_{AB_{translate}}\mathbf{v_3} &= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \\
&= v_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + v_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} v_x + t_x \\ v_y + t_y \\ 1 \end{bmatrix}
\end{aligned}
$$

Which is the $\mathbb{R}^3$ version of the translation. Finally, we apply the rotation. We want a point (1, 0) in the $\mathcal{A}$-basis, which is (0, -1) in the standard basis to rotate 135 degrees counterclockwise. So, (1, 0) should be transformed to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, which is just $(\cos(135), \sin(135))$. (0, 1) should be transformed to $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$, which by standard rotation matrix convention is $(-\sin(135), \cos(135))$. We leave z alone in this case, so we get the following matrix:

$$
\begin{aligned}
M_{AB_{rotation}} &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(135) & -\sin(135) & 0 \\ \sin(135) & \cos(135) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

We can now obtain the final transformation matrix:

$$
\begin{aligned}
M_{AB} &= M_{AB_{translate}} M_{AB_{rotate}} \\
&= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

1.5 Q5

Use M_{AO} and M_{BA} to represent M_{OB}. There is no need to calculate matrix values. Please use the provided math notations (M_{AO} and M_{BA}), their inverse forms, and the multiplication sign as necessary to express your answer.

M_{OB} is defined as the transformation from coordinate system O to coordinate system B.

We have $M_{AO}^{-1} = M_{OA}$ and $M_{BA}^{-1} = M_{AB}$, so we can calculate the transformations as the product of these matrices, applied from right-to-left:

$$
\begin{aligned}
M_{OB} &= M_{AB} M_{OA} && (1) \\
&= M_{BA}^{-1} M_{AO}^{-1} && (2)
\end{aligned}
$$