Interim Report of Course Project

COMP 409

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1 Encoding of Einstein's Puzzle

My approach of encoding from Einstein's Puzzle to CNF formulas is straightforward. Basically, I used a proposition for each possible combination of nationality and another property in house location, pets, house color, beverge and cigar.

For example, "The Dane raise a cat" if and only if $a[Dane][Pet][Cat]$ is true. More specifically, the following form showes how we encode a proposition:

		1	2	3	4	5
	Nationality	Brit	Swede	Dane	Norwegian	German
1	House Location	Left	Left-mid	Mid	Right-mid	Right
2	Pets	Dog	Bird	Cat	Horse	Fish
3	House Color	Red	White	Blue	Yellow	Green
4	Beverage	Tea	Milk	Beer	Water	Coffee
5	Cigar	Pall Mall	Dunhill	Blends	Blue masters	Prince

For convenience of implementation, I used index which took value from $\{1, 2, 3, 4, 5\}$ to encode each proposition. For example, $a[Dane][Pet][Cat]$ is encoded as $a[3][2][3]$ in the table. The first index determines nationality. The second one decides the other property and the third one indicates the value of that property. It's easy to see that I need $5 \cdot 5 \cdot 5 = 125$ variables.

Transforming variables from $a[1][1][1]$ to $a[5][5][5]$ into a_1 to a_{125} just needs some simple algebra. For example $a[3][2][3]$ became a_{63} because $63 = (3-1) \cdot 25 + (2-1) \cdot 5 + 3$. In final implementation, I used a_1 to a_{125} as variables in input, which is a requirement of DIMACS format.

Before we encode every condition of Einstein's Puzzle into CNF clause, note that there are some constrains in this encoding:

- For every person and every property, at least one variable should be true, which means for example every person must raise an animal, formally:

$$\forall x, y \in \{1, 2, 3, 4, 5\}.a[x][y][1] \vee a[x][y][2] \vee \cdots a[x][y][5]$$

- For every person and every property, the person can only have one value on that property:

$$\forall x, y, i \neq j \in \{1, 2, 3, 4, 5\}.\neg(a[x][y][i] \wedge a[x][y][j])$$

- Two different persons can not have the same value on the same property, which means, for instance, the Swede and the Dane can not both raise a cat:

$$\forall x_1, x_2, y, z \in \{1, 2, 3, 4, 5\}.\neg(a[x_1][y][z] \wedge a[x_2][y][z])$$

Next we need to transform every condition in Einstein's Puzzle to one or a group of formulas. I will show three representative examples here and others will follow similarly:

- The Brit lives in the red house: $a[1][3][1]$
- The green house's owner drinks coffee: $\forall x\{1,2,3,4,5\}.a[x][3][5] \rightarrow a[x][4][5]$
- The man who smokes Blends lives next to the one who keeps cats:

  $\forall x, y \in \{1,2,3,4,5\}, z \in \{2,3,4\}.((a[x][5][3] \wedge a[y][2][3]) \rightarrow (a[x][1][z] \rightarrow (a[y][1][z-1] \vee a[y][1][z+1]))$

Finally I transform Einstein's Puzzle to a CNF case with 125 variables and 1581 clauses.

2 Solution of Einstein's Puzzle

The sat solver gave one solution which is actually the only solution to the formula. The solution is:

s cnf 1 125 1581

v -1 -2 3 -4 -5 -6 7 -8 -9 -10 11 -12 -13 -14 -15 -16 17 -18 -19 -20 21 -22 -23 -24 -25 -26 -27 -28 -29 30 31 -32 -33 -34 -35 -36 37 -38 -39 -40 -41 -42 43 -44 -45 -46 -47 -48 49 -50 -51 52 -53 -54 -55 -56 -57 -58 59 -60 -61 -62 63 -64 -65 66 -67 -68 -69 -70 -71 -72 73 -74 -75 76 -77 -78 -79 -80 -81 -82 83 -84 -85 -86 -87 -88 89 -90 -91 -92 -93 94 -95 -96 97 -98 -99 -100 -101 -102 -103 104 -105 -106 -107 -108 -109 110 -111 -112 -113 -114 115 -116 -117 -118 -119 120 -121 -122 -123 -124 125

Since $a_{110} = a[5][2][5] = a[German][Pet][Fish] = 1$, the German owns the fish.

The encoding can be found in my website:

- PDF file:`www.cs.rice.edu/~zz59/Zhi-Wei-Zhang/509_project/Einstein's_Puzzle_in_CNF.pdf`
- TXT file: `www.cs.rice.edu/~zz59/Zhi-Wei-Zhang/509_project/Einstein's_Puzzle_in_CNF.txt`