Practical SAT Solving

Lecture 5

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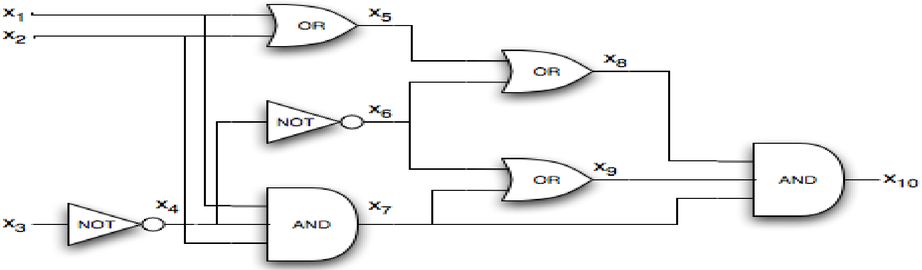
INSTITUTE FOR THEORETICAL COMPUTER SCIENCE



Lecture Outline: Today

- Repetition
- More Details on implementing DPLL
    - Literal Selection Heuristics
    - Efficient Unit Propagation

"Modern" DPLL Algorithm with "Trail"

boolean `mDPLL(`**ClauseSet** S, **PartialAssignment** α)
{
  while ((S, α) contains a unit clause $\{L\}$) **{**
    add $\{L = 1\}$ to α
  }
  if (a literal is assigned both 0 and 1 in α) **return** false;
  if (all literals assigned) **return** true;
  choose a literal L not assigned in α occurring in S;
  if (mDPLL(S, $\alpha \cup \{L = 1\}$) **return** true;
  else if (mDPLL(S, $\alpha \cup \{L = 0\}$) **return** true;
  else return false;
}

(S, α): clause set S as "seen" under partial assignment α

DPLL: Implementation Issues

- How can we implement unit propagation efficiently?
- (How can we implement pure literal elimination efficiently?)
- Which literal L to use for case splitting?
- How can we efficiently implement the case splitting step?

Properties of a good decision heuristic

Properties of a good decision heuristic

- Fast to compute
- Yields efficient sub-problems
    - More short clauses?
    - Less variables?
    - Partitioned problem?

Bohm's Heuristic

- Best heuristic in 1992 for random SAT (in the SAT competition)
- Select the variable x with the maximal vector $(H_1(x), H_2(x), \dots)$

$$H_i(x) = \alpha \max(h_i(x), h_i(\overline{x})) + \beta \min(h_i(x), h_i(\overline{x}))$$

- where $h_i(x)$ is the number of not yet satisfied clauses with i literals that contain the literal x.
- α and β are chosen heuristically ($\alpha = 1$ and $\beta = 2$).
- Goal: satisfy or reduce size of many preferably short clauses

MOMS Heuristic

- Maximum Occurrences in clauses of Minimum Size
- Popular in the mid 90s
- Choose the variable x with a maximum $S(x)$.

$$S(x) = (f^*(x) + f^*(\overline{x})) \times 2^k + (f^*(x) \times f^*(\overline{x}))$$

- where $f^*(x)$ is the number of occurrences of x in the smallest not yet satisfied clauses, k is a parameter
- Goal: assign variables with high occurrence in short clauses

Jeroslow-Wang Heuristic

- Considers all the clauses, shorter clauses are more important
- Choose the literal x with a maximum $J(x)$.

$$J(x) = \sum_{x \in c, c \in F} 2^{-|c|}$$

- Two-sided variant: choose variable x with maximum $J(x) + J(\overline{x})$
- Goal: assign variables with high occurrence in short clauses
- Much better experimental results than Bohm and MOMS
- One-sided version works better

(R)DLCS and (R)DLIS Heuristics

- (Randomized) Dynamic Largest (Combined | Individual) Sum
- Dynamic = Takes the current partial assignment in account
- Let C_P (C_N) be the number of positive (negative) occurrences
- DLCS selects the variable with maximal $C_P + C_N$
- DLIS selects the variable with maximal $\max(C_P, C_N)$
- RDLCS and RDLIS does a random selection among the best
  - Decrease greediness by randomization
- Used in the famous SAT solver GRASP in 2000

LEFV Heuristic

- Last Encountered Free Variable
- During unit propagation save the last unassigned variable you see, if the variable is still unassigned at decision time use it otherwise choose a random
- Very fast computation: constant memory and time overhead
  - Requires 1 int variable (to store the last seen unassigned variable)
- Maintains search locality
- Works well for pigeon hole and similar formulas

How to Implement Unit Propagation

The Task

Given a partial truth assignment ϕ and a set of clauses F identify all the unit clauses, extend the partial truth assignment, repeat until fix-point.

Simple Solution

- Check all the clauses
- Too slow
- Unit propagation cannot be efficiently parallelized (is P-complete)

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In the context of DPLL the task is actually a bit different

- The partial truth assignment is created incrementally by adding (decision) and removing (backtracking) variable value pairs
- Using this information we will avoid looking at all the clauses

How to Implement Unit Propagation

The Real Task

We need a data structure for storing the clauses and a partial assignment ϕ that can efficiently support the following operations

- detect new unit clauses when ϕ is extended by $x_i = v$
- update itself by adding $x_i = v$ to ϕ
- update itself by removing $x_i = v$ from ϕ
- support restarts, i.e., un-assign all variables at once

Observation

- We only need to check clauses containing x_i

Occurrences List and Literals Counting

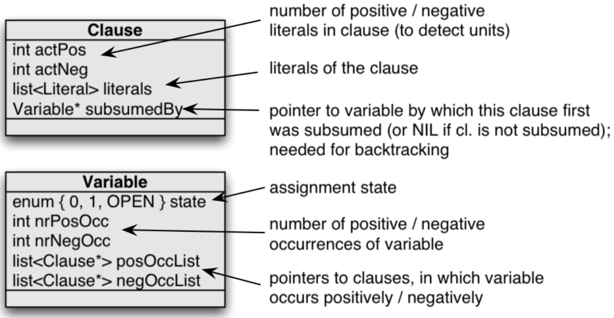
The Data Structure

- For each clause remember the number unassigned literals in it
- For each literal remember all the clauses that contain it

Operations

- If $x_i = T$ is the new assignment look at all the clauses in the occurrence of of $\overline{x_i}$. We found a unit if the clause is not SAT and counter=2
- When $x_i = v$ is added or removed from ϕ update the counters

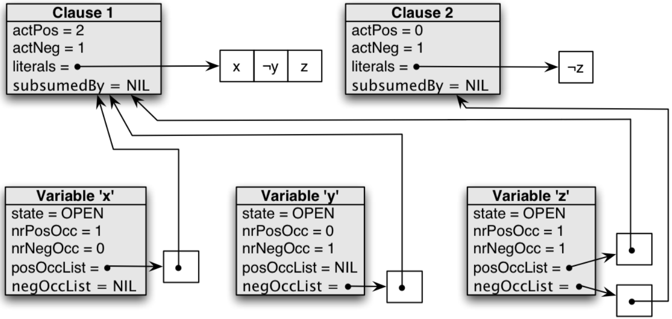
“Traditional” Approach



Crawford, Auton (1993)

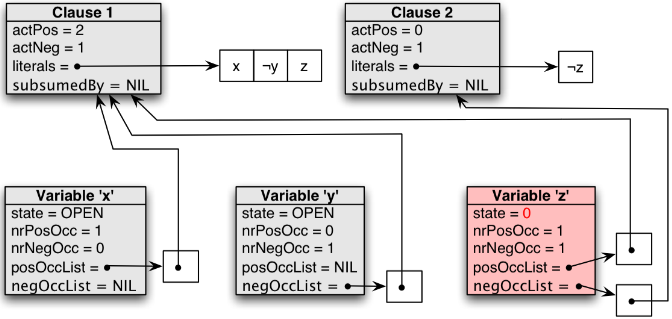
Traditional Approach: Example

$F = \{\{x, \neg y, z\}, \{\neg z\}\}$



Traditional Approach: Example

$F = \{\{x, \neg y, z\}, \{\neg z\}\}$ unit propagation: set $z = 0$



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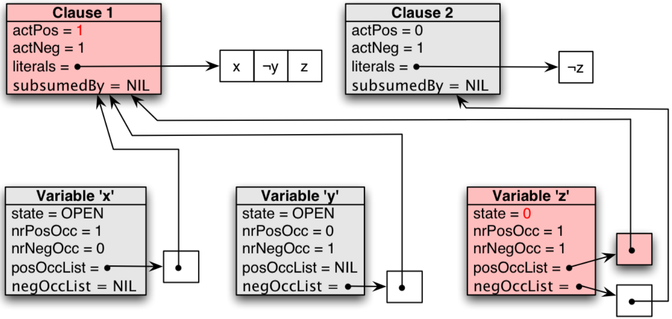

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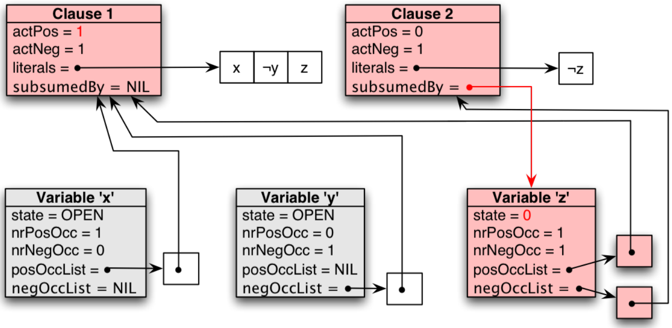
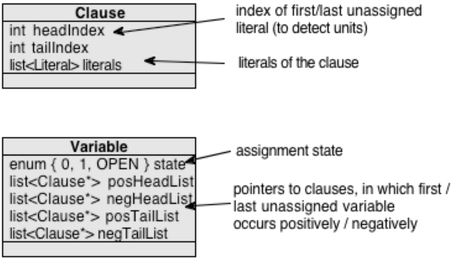


Head/Tail Lists



Zhang, Stickel (1996)

Head/Tail Lists: Example

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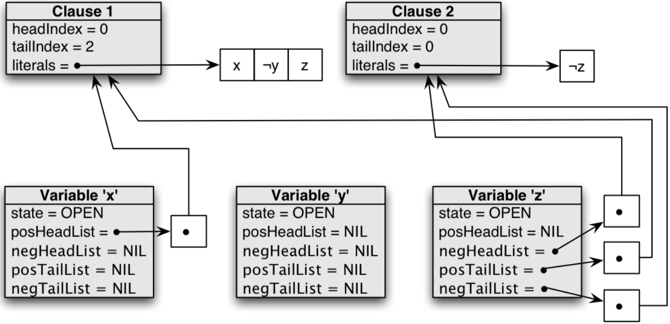
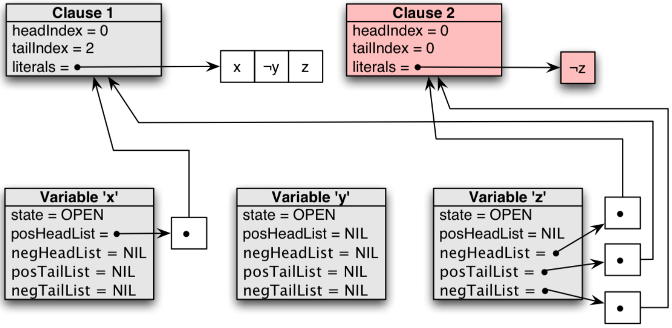


Head/Tail Lists: Example

$F = \{\{x, \neg y, z\}, \{\neg z\}\}$ detected unit clause: $\{\neg z\}$



Head/Tail Lists: Example

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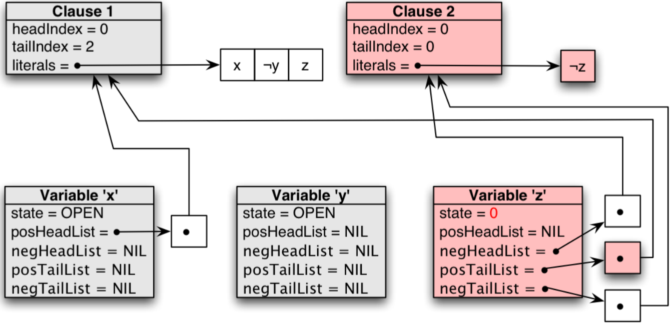

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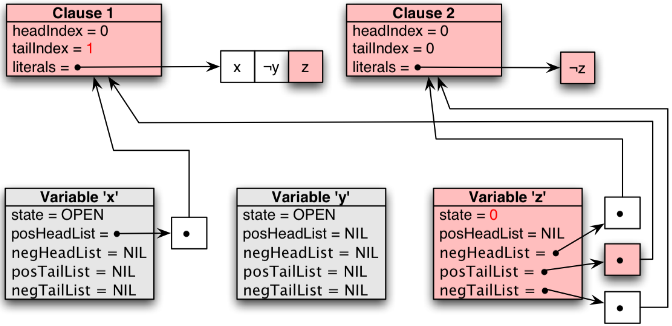

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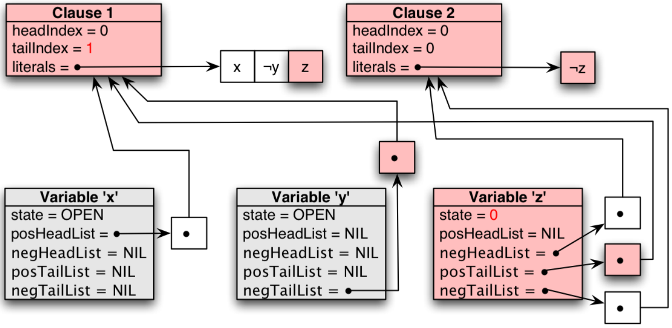


2 watched literals

The Data Structure

- In each non-satisfied clause "watch" two non-false literals
- For each literal remember all the clauses where it is watched

Maintain the invariant: two watched non-false literals in non-sat clauses

- If a literal becomes false find another one to watch
- If that is not possible the clause is unit

Advantages

2 watched literals

The Data Structure

- In each non-satisfied clause "watch" two non-false literals
- For each literal remember all the clauses where it is watched

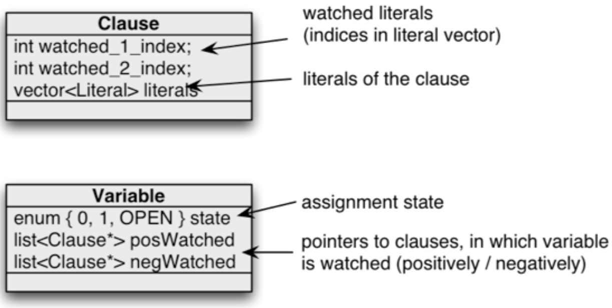
Maintain the invariant: two watched non-false literals in non-sat clauses

- If a literal becomes false find another one to watch
- If that is not possible the clause is unit

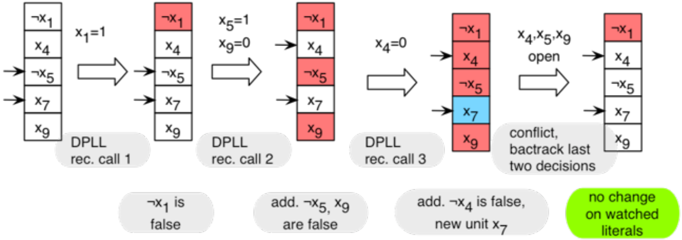
Advantages

- visit fewer clauses: when $x_i = T$ is added only visit clauses where $\overline{x_i}$ is watched
- no need to do anything at backtracking and restarts
  - watched literals cannot become false

2 Watched Literals: Data Structures



2 Watched Literals: Example



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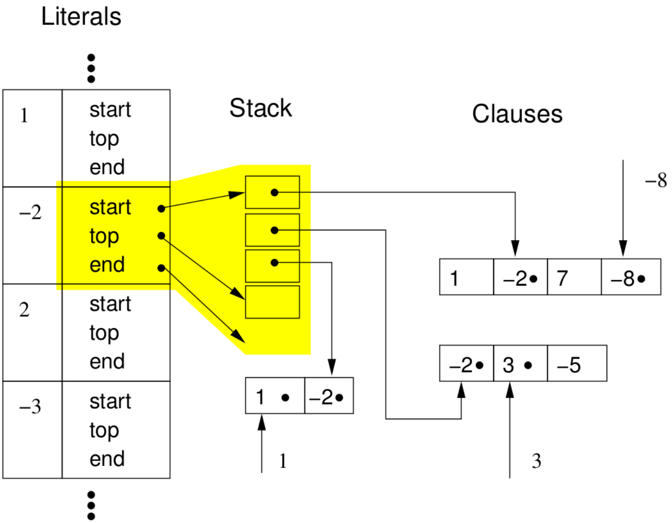
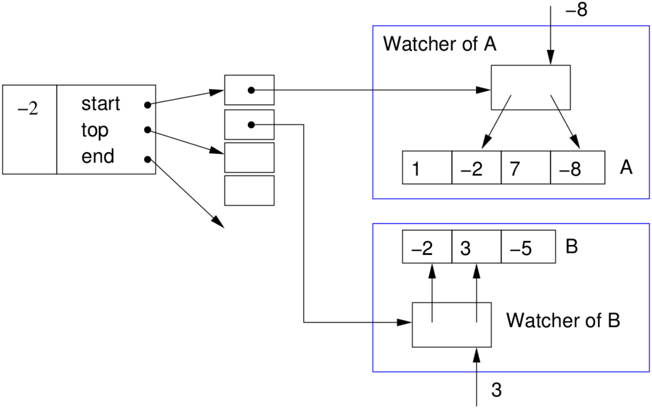


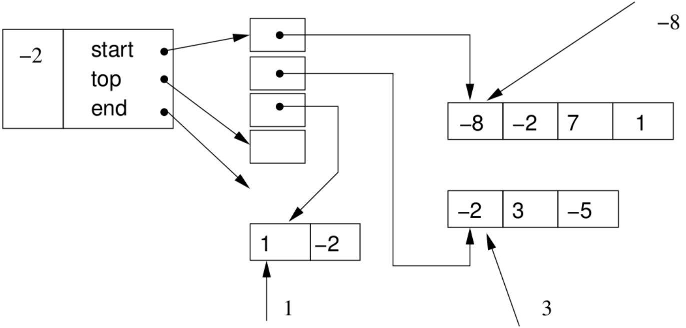
Literals
1
start
top
end
–2
start
top
end
2
start
top
end
–3
start
top
end
Stack
Clauses
–8
1
–2
7
–8
–2
3
–5
1
–2
1
3

Limmat



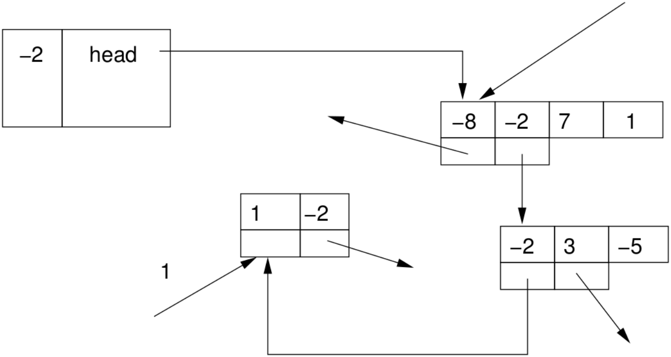
- Good for parallel SAT solvers with shared clause database

MiniSat



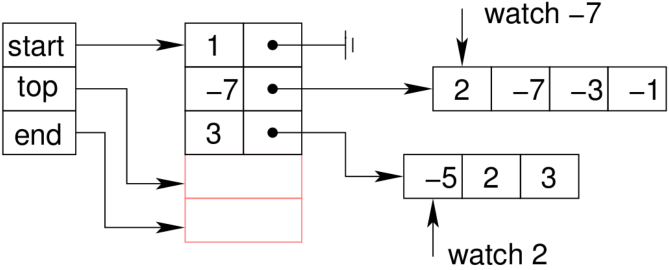
invariant: first two literals are watched

PicoSat



invariant: first two literals are watched

Lingeling



- often the other watched literal satisfies the clause
- for binary clauses no need to store the clause

MiniSAT `propagate()`-Function

```
CRef Solver::propagate()
{
  CRef   confl    = CRef_Undef;
  int    num_props = 0;

  while (qhead < trail.size()){
  Lit p   = trail[qhead++]; // propagate 'p'.
  vec<Watcher>&  ws   = watches.lookup(p);
  Watcher   *i, *j, *end;
  num_props++;

  for (i = j = (Watcher*)ws, end = i + ws.size();
    i != end;){
    // Try to avoid inspecting the clause:
    Lit blocker = i->blocker;
    if (value(blocker) == l_True){
    *j++ = *i++; continue; }

    // Make sure the false literal is data[1]:
    CRef    cr    = i->cref;
    Clause&   c    = ca[cr];
    Lit   false_lit = ~p;
    if (c[0] == false_lit)
    c[0] = c[1], c[1] = false_lit;
    assert(c[1] == false_lit);
    i++;

    // If 0th watch is true, clause is satisfied.
    Lit    first = c[0];
    Watcher w    = Watcher(cr, first);
    if (first != blocker && value(first) == l_True){
    *j++ = w; continue; }

    // Look for new watch:
    for (int k = 2; k < c.size(); k++)
      if (value(c[k]) != l_False){
        c[1] = c[k]; c[k] = false_lit;
        watches[~c[1]].push(w);
        goto NextClause; }

    // Did not find watch -- clause is unit
    *j++ = w;
    if (value(first) == l_False){
      confl = cr;
      qhead = trail.size();
      // Copy the remaining watches:
      while (i < end)
        *j++ = *i++;
    }else
      uncheckedEnqueue(first, cr);

  NextClause:;
  }
  ws.shrink(i - j);
 }
 propagations += num_props;
 simpDB_props -= num_props;

 return confl;
}
```