Introduction to Control: Project
(2 credit points)

Prof. Boris Houska

Deadline for the report: Dec 23, 2019;
Project presentations will take place on Dec 16 and Dec 18, 2019.

1 Introduction

The EE160 Project is about the modeling, simulation, and control of a nonlinear system of your choice. Here, you have two options:

1. Pick at least one of the application examples (with given physcial models) from the appendix of this project sheet; or
2. propose a control problem of your own choice. If you want to go for this second option, please contact prof. Houska or one of the TAs. If you have a good idea, we will accept this as a project topic, too, as long as the project is related to the methods that are discussed in the lecture. The deadline for proposing such an individual project topic is December 4, 2019 (if you miss this deadline, you will have to work with one of the given models).

Independent of whether you go for one of the default project or come up with your own project, you will need to complete a report ($\geq$ 4 pages) and a presentation. Your presentation should consist of 4 slides and you will have to present either on Dec. 16 or on Dec. 18, 2019.

2 Main Requirements on the Project

The goal of this project is to design a closed-loop controller for at least one application that leads to a nonlinear control system with 2 or more differential states. The main tasks of the project are as follows:

1. Introduce your model and write in the form of a nonlinear control system in standard form
$$\dot{x}(t) = f(t, x(t), u(t))$$
Explain what x, u, and f are in your application.
2. Linearize your model at a suitable set point and write the linear approximation in the form
$$\dot{z}(t) = Az(t) + Bv(t) + b \ .$$
What are A, B, b, z, and v in your application?
3. Design a feedback control law of the form
$$u(t) = Kx(t) + u_{\mathrm{ref}}$$
by using at least one of the methods that has been introduced in the lecture. Alternatively, you may also use a time-varying reference, $u(t) = K(x(t) - x_{\mathrm{ref}}(t)) + u_{\mathrm{ref}}(t)$, if this is needed for your particular application.
4. Implement a Runge-Kutta integrator and simulate the nonlinear closed-loop dynamic for different initial values. Plot the closed-loop state trajectory and discuss your results.

3 Project Report

Write a short report (preferably in Latex) containing the following sections:

1. *Title and Authors* (find a good title + name of the author)
2. *Introduction* (describe the problem that you want to solve)
3. *Problem Formulation* (introduce a suitable mathematical notation to define the problem that you are trying to solve)
4. *Solution Method* (explain how you have designed your controller and how you have implemented your simulation code)
5. *Numerical Results* (plot/visualize and explain your numerical results)
6. *Conclusion* (summarize the highlights of your results)

4 Presentation

Every group / person will get a 3-5min time slot to present their project (please contact our TAs to schedule your talk). The presentation should consist of 4 slides, preferably in pdf format. Your three main tasks for the presentation are:

1. prepare 4 clean and efficient slides, namely,
   - one title slide containing the project title and the name of all authors
   - one slide introducing/explaining the problem that you are solving
   - one slide about the control design and your simulation results
   - one conclusion slide summarizing and assessing your results

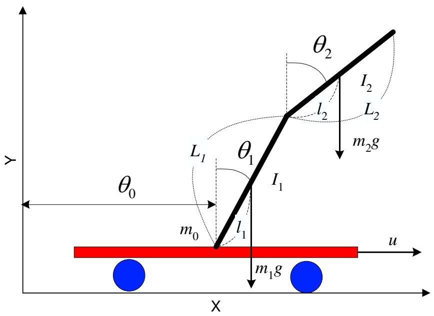
   Don't use small fonts; instead of the third slide, you may also show an animation or video (optional, not required).
2. present your work freely in English and by using your own words
3. answer questions by the professor / TAs, or other people from the audience.

Appendix

This appendix collects a few suggestions for possible project topics and some hints on how to derive a differential equation model.

A Double Pendulum on a Cart

Our first suggestion for a possible project is to consider a double pendulum on a trolley that can move on a horizontal plane as shown in the figure below.



A dynamic of this system can be written in the form,

$$D(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Hu$$

with $\theta = \begin{pmatrix}\theta_0 & \theta_1 & \theta_2\end{pmatrix}^T \in \mathbb{R}^3$ and matrices

$$D = \begin{pmatrix} d_1 & d_2\cos(\theta_1) & d_3\cos(\theta_2) \\ d_2\cos(\theta_1) & d_4 & d_5\cos(\theta_1-\theta_2) \\ d_3\cos(\theta_2) & d_5\cos(\theta_1-\theta_2) & d_6 \end{pmatrix}, \; G = \begin{pmatrix} 0 \\ -f_1\sin(\theta_1) \\ -f_2\sin(\theta_2) \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & -d_2\sin(\theta_1)\dot{\theta}_1 & -d_3\sin(\theta_2)\dot{\theta}_2 \\ 0 & 0 & d_5\sin(\theta_1-\theta_2)\dot{\theta}_2 \\ 0 & -d_5\sin(\theta_1-\theta_2)\dot{\theta}_1 & 0 \end{pmatrix}, \; H = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\begin{aligned}
d_1 &= m_0 + m_1 + m_2 \\
d_2 &= m_1 l_1 + m_2 L_1 = \left(\frac{1}{2}m_1 + m_2\right) L_1 \\
d_3 &= m_2 l_2 = \frac{1}{2} m_2 L_2 \\
d_4 &= m_1 l_1^2 + m_2 L_1^2 + I_1 = \left(\frac{1}{3}m_1 + m_2\right) L_1^2 \\
d_5 &= m_2 L_1 l_2 = \frac{1}{2} m_2 L_1 L_2 \\
d_6 &= \left(m_1 l_1 + m_2 L_1\right) g = \left(\frac{1}{2}m_1 + m_2\right) L_1 g \\
f_2 &= m_2 l_2 g = \frac{1}{2} m_2 L_2 g
\end{aligned}$$

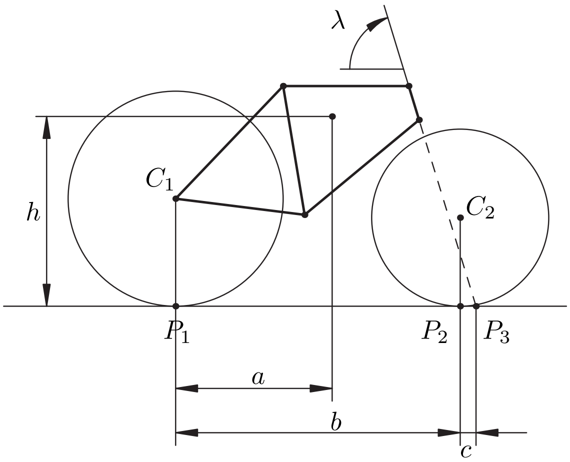
In order to setup a case study, you would still need to think about suitable values for the masses and the lengths of the pendulum arms.

Reference

1. Optimal control of a double inverted pendulum on a cart. `https://www.researchgate.net/publication/250107215_Optimal_Control_of_a_Double_Inverted_Pendulum_on_a_Cart`

B Bike

Another suggestion is to try to design a controller for a bicycle as shown in the figure below. [h]



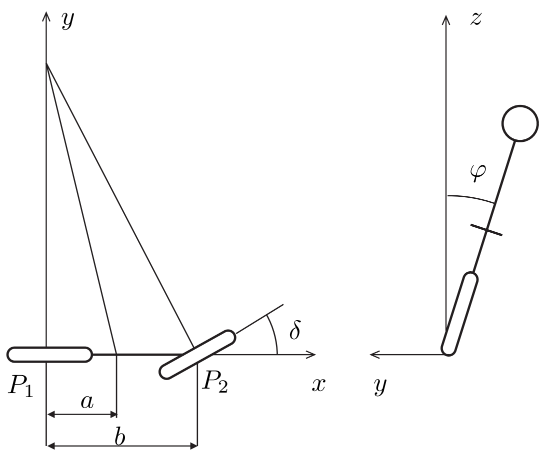
This project is very challenging, but if you would like to simplifty the model, you may assume $\lambda = \frac{\pi}{2}$ such that $P_2 = P_3$, $c = 0$. In this simple case, the dynamic equation is given by

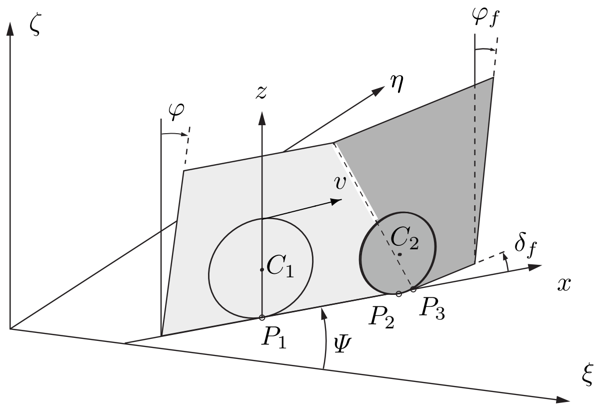
$$J\ddot{\varphi} - mgh\varphi = \frac{DV}{b}\dot{\delta} + \frac{mhv^2}{b}\delta$$

where $J \approx mh^2$ is the moment of inertia of the bicycle with respect to x-axis. The inertia with respect the the xz plane is $D = -J_{xz} \approx -mah$. In order to setup a complete case study, you would still need to figure out some details from the figures.

Reference

1. Bicycle dynamics and control: adapted bicycles for education and research. `https://ieeexplore.ieee.org/document/1499389`





ζ
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C Quadcopter

Another interesting case study could be the control of a quadcoptor as shown in the figure below.

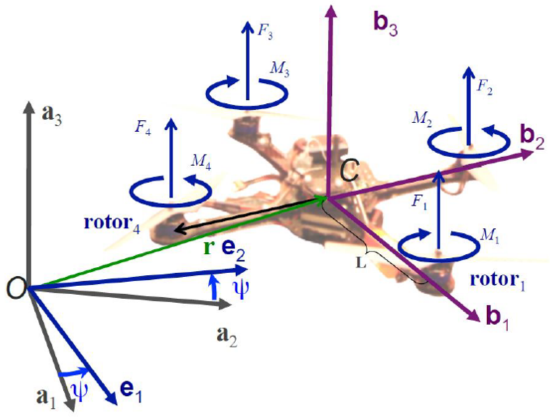


Figure 1: Quadcopter

We denote the position and velocity in the inertial frame by $p = [x\ y\ z]^\top$ and $\dot{p} = [\dot{x}\ \dot{y}\ \dot{z}]^\top$. Moreover, the roll, pitch, and yaw angles in the body frame are denoted by $r = [\phi\ \theta\ \psi]^\top$. The corresponding angular velocities are $\dot{r} = [\dot{\phi}\ \dot{\theta}\ \dot{\psi}]^\top$. In general, the dynamic model is complicated, but a simplified model for the motion is obtained by decoupling the rotational and lateral motions:

- **Newton's equation** A simplified differential equation model for p is given by

$$m\ddot{p} = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} + R \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{pmatrix} \tag{1}$$

  where m denotes the mass of the quadcopter and

$$R = \begin{pmatrix} \cos(\phi)\cos(\psi) - \cos(\theta)\sin(\phi)\sin(\psi) & -\cos(\psi)\sin(\phi) - \cos(\phi)\cos(\theta)\sin(\psi) & \sin(\theta)\sin(\psi) \\ \cos(\phi)\sin(\psi) + \cos(\psi)\cos(\psi)\sin(\phi) & \cos(\phi)\cos(\theta)\cos(\psi) - \sin(\psi)\sin(\phi) & -\sin(\theta)\cos(\psi) \\ \sin(\phi)\sin(\theta) & \cos(\phi)\sin(\theta) & \cos(\theta) \end{pmatrix}$$

- **Euler's equation** Next, the rotation of the quadcopter can be modelled as

$$I\dot{\omega} + \omega \times (I\omega) = \tau$$

  where I denotes the inertia

$$\omega = \begin{pmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{pmatrix} \dot{r}\ ,\ \tau = \begin{pmatrix} L(F_1 - F_3) \\ L(F_2 - F_4) \\ M_1 - M_2 + M_3 - M_4 \end{pmatrix}$$

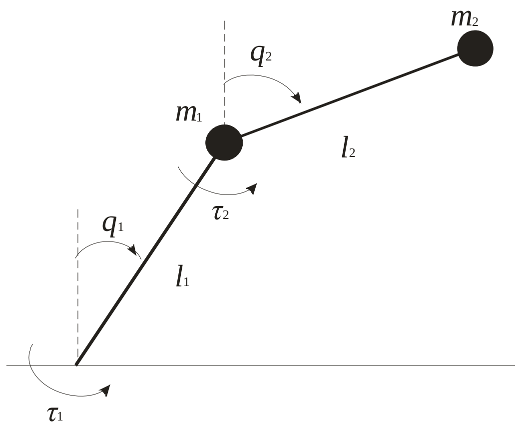
In order to setup a realistic case study, you would still need to figure out some detail and specify a trajectory that the quadcopter should follow.

Reference

1. Quadcopter Dynamics, Simulation, and Control: `http://andrew.gibiansky.com/downloads/pdf/Quadcopter%20Dynamics,%20Simulation,%20and%20Control.pdf`

D 2 DOF Robot Arm

The figure below shows a robot arm with two degrees of freedom. The torques q_1 and q_2 at the arms can be controlled by using a motor.



A differential equation model is given by

$$\begin{pmatrix} (m_1+m_2)l_1^2 & m_2 l_1 l_2 \cos(q_1-q_2) \\ m_2 l_1 l_2 \cos(q_1-q_2) & m_2 l_2 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} K_{q_1}\dot{q}_1 + m_2 l_1 l_2 \sin(q_1-q_2)\dot{q}_2 \\ K_{q_2}\dot{q}_2 + m_2 l_1 l_2 \sin(q_1-q_2)\dot{q}_1 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}.$$

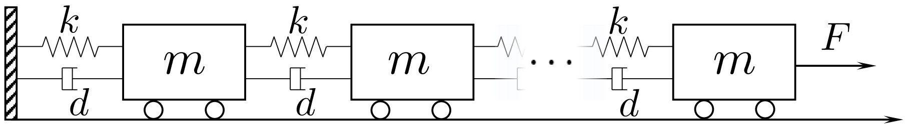
As for the other models, you would still need to figure out some details to setup a case study.

Reference

1. Nonlinear Control Design of Robot Arm.
`https://pdfs.semanticscholar.org/79a1/4ea6887dfa6c92a1c966cf45976333313a6f.pdf`

E Spring damper mass system

The figure below shows a spring mass damper system.



For the special case that there is only one mass, a dynamic equation model is given by

$$\begin{pmatrix} \dot{p}(t) \\ \dot{v}(t) \end{pmatrix} = \begin{pmatrix} \dot{v}(t) \\ -\frac{k(p(t))}{m} p(t) - \frac{d(v(t))}{m} v(t) + \frac{F(t)}{m} \end{pmatrix}$$

where the spring and damping coefficients may, in general, be functions of the position p and velocity v of the trolley, for example $k(p) = k_0 \exp(-p)$ and $d(v) = d_0 \exp(-v)$. If you like, you can try to generalize this example for more than one trolley.

Reference

1. Robust Model Predictive Control of continuous-time sampled-data nonlinear systems with Integral Sliding Mode. `https://ieeexplore.ieee.org/abstract/document/7074739`