EE160P 2019 Update
2-DOF Robot Arm

EE160P TAs

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1 Problem

We want to control the 2-DOF robot arm to some desired time-varying trajectory or fixed point, starting from some initial point near it (or even far from it) using state feedback with gain matrix K computed by LQR. The model is based on [1].

We use the following notations to denote physical quantities at 2 motors and 2 arms:

- State variables and their derivatives

  i motor electrical current

  q angle of arm, and therefore $\dot{q}$ the angular velocity and $\ddot{q}$ the angular acceleration

- Intermediate states

  τ motor output torque, $\tau = K_t i$,
  where K_t is called torque constant. In this project, $K_t = 1$.

- Control variable

  u motor command voltage, the input of the system

You need to work out the general form $\dot{x} = f(x, u)$ based on these two sets of equations:

$$Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + K_b\frac{\mathrm{d}q}{\mathrm{d}t} = u \tag{1}$$

where K_b has the same value of K_t under SI unit (Système International d'Unités).

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}K_t i_1\\ K_t i_2\end{bmatrix} = \begin{bmatrix}d_{11} & d_{12}s_{21}\\ d_{12}s_{21} & d_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}b_1\dot{q}_1 + d_{12}c_{21}\dot{q}_2^2\\ b_2\dot{q}_2 + d_{12}c_{21}\dot{q}_1^2\end{bmatrix} \tag{2}$$

where $s_{ij} = \sin(q_i - q_j)$ and $c_{ij} = \cos(q_i - q_j)$.

You can start by setting $x = \begin{bmatrix}i_1\, q_1\, \dot{q}_1\, i_2\, q_2\, \dot{q}_2\end{bmatrix}^\mathsf{T} \in \mathbb{R}^6$. Use Symbolic Toolbox if the calculation gets too complicated.

2 Parameters

motor inductance	L	$0.0036\,\mathrm{H}$
motor resistance	R	$0.5\,\Omega$
inertia para. 1	d_{11}	$1\,\mathrm{kg\,m^2}$
inertia para. 2	d_{12}	$-0.13\,\mathrm{kg\,m^2}$
inertia para. 3	d_{22}	$0.24\,\mathrm{kg\,m^2}$
damping coeff. 1	b_1	$3.6\,\mathrm{kg\,m^2\,s^{-1}}$
damping coeff. 2	b_2	$0.7\,\mathrm{kg\,m^2\,s^{-1}}$

References

[1] G. J. Liu and A. A. Goldenberg. "Robust control of robot manipulators incorporating motor dynamics". In: *Proceedings of 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS '93)*. Vol. 1. 1993, 68–75 vol.1. DOI: 10.1109/IROS.1993.583081.