

SPHERICAL COORDINATE SYSTEM

In spherical coordinate system, a point P in space is represented by triplet (r, θ, ϕ) where,

- r is distance between point and origin
- θ is clockwise angle from the X axis in XZ plane
- ϕ is angle between the Y axis and the point.

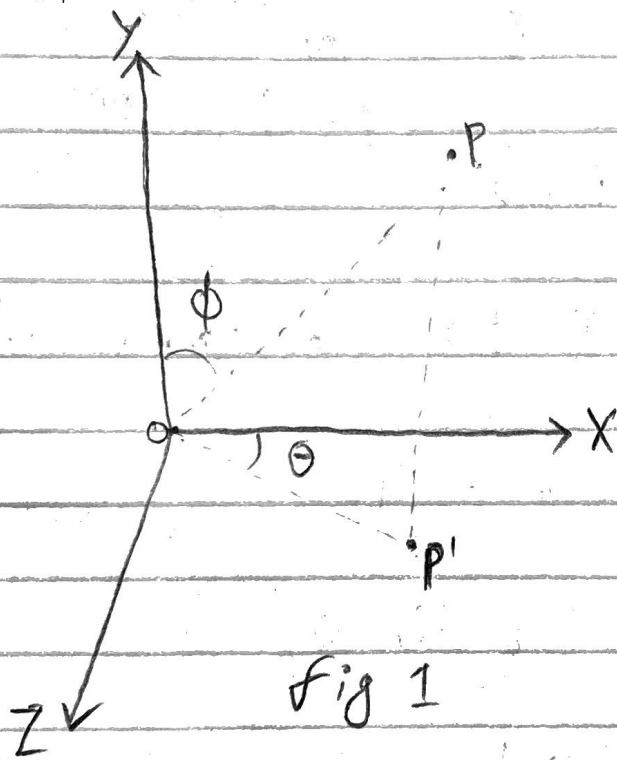


Fig 1

Note that in literature elsewhere, it is also common to see Z axis aligned in up direction in which case our figure would be like fig 2 below. Fig 1 above is most apt for 3D graphics.

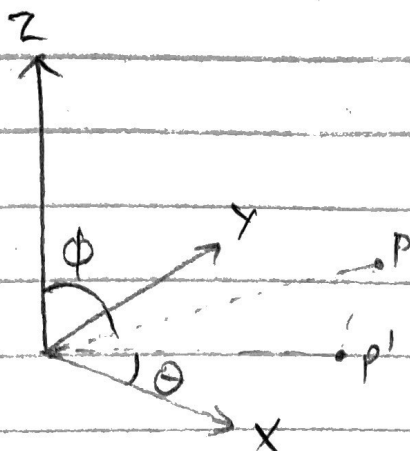


fig 2
(not referenced here)

Our motivation for spherical coord. system is mostly for expressing camera position in world space.

To express $P(x, y, z)$ in terms of $P(r, \theta, \phi)$, from fig 1:

$$r^2 = x^2 + y^2 + z^2; \quad r = \sqrt{x^2 + y^2 + z^2}$$

$$\tan \theta = z/x; \quad \theta = \tan^{-1}(z/x)$$

$$\cos \phi = \frac{y}{\sqrt{x^2 + y^2 + z^2}}; \quad \phi = \cos^{-1}\left(\frac{y}{\sqrt{x^2 + y^2 + z^2}}\right)$$

To express $P(r, \theta, \phi)$ in terms of $P(x, y, z)$, from Fig 1:

$$y = r \cos \phi$$

$$OP' = r \sin \phi$$

$$x = OP' \cos \theta = r \sin \phi \cos \theta$$

$$y = OP' \sin \theta = r \sin \phi \sin \theta$$