

# SPHERICAL COORDINATE SYSTEM

In spherical coordinate system, a point  $P$  in space is represented by triplet  $(r, \theta, \phi)$  where,

- $r$  is distance between point and origin
- $\theta$  is clockwise angle from the  $X$  axis in  $XZ$  plane
- $\phi$  is angle between the  $Y$  axis and the point.

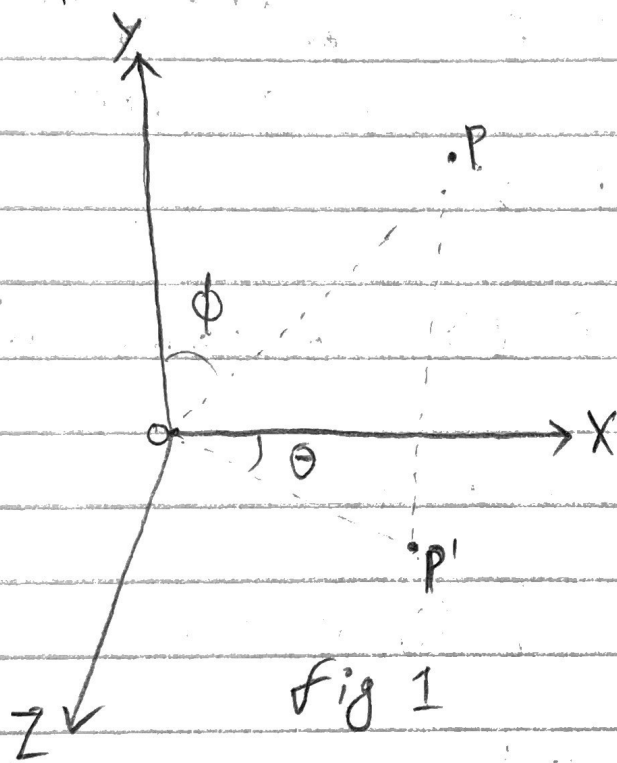


Fig 1

Note that in literature elsewhere, it is also common to see  $Z$  axis aligned in up direction in which case our figure would be like Fig 2 below. Fig 1 above is most apt for 3D graphics.

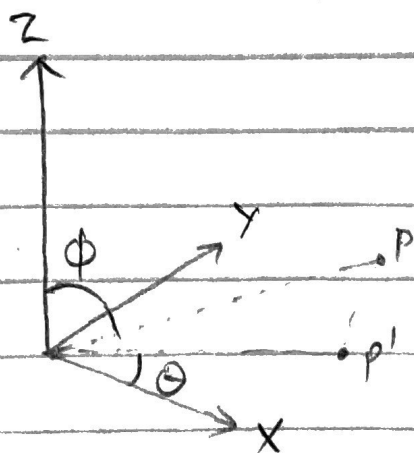


Fig 2  
(not referenced here)

Our motivation for spherical coord. system is mostly for expressing camera position in world space.

To express  $P(x, y, z)$  in terms of  $P(r, \theta, \phi)$ , from fig 1:

$$r^2 = x^2 + y^2 + z^2; \quad r = \sqrt{x^2 + y^2 + z^2}$$

$$\tan \theta = z/x; \quad \theta = \tan^{-1}(z/x)$$

$$\cos \phi = \frac{y}{\sqrt{x^2 + y^2 + z^2}}; \quad \phi = \cos^{-1}\left(\frac{y}{\sqrt{x^2 + y^2 + z^2}}\right)$$

To express  $P(r, \theta, \phi)$  in terms of  $P(x, y, z)$ , from fig 1:

$$y = r \cos \phi$$

$$OP' = r \sin \phi$$

$$x = OP' \cos \theta = r \sin \phi \sin \theta$$

$$y = OP' \sin \theta = r \sin \phi \sin \theta$$