INVERSE TRANSPOSE OF NORMAL VECTORS

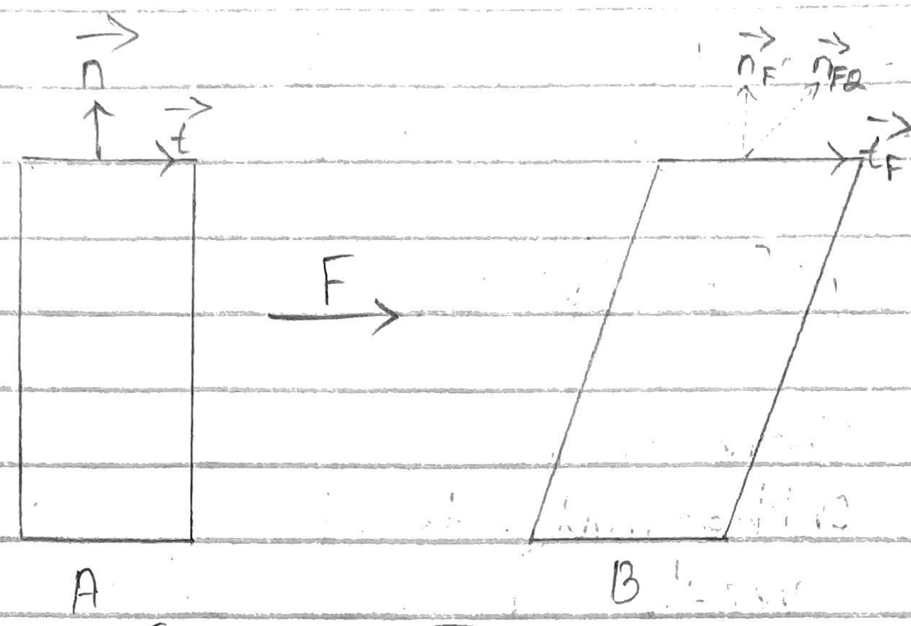

Fig: sheer Transformation

Consider a transformation matrix F (sheer transforma-tion) that transforms a quad A to B. $\vec{n}$ and $\vec{t}$ are the original normal and tangent vectors.

It is a given that tangents transform correctly, so $\vec{t}$ correctly transforms to $\vec{t_F}$. However, if we apply F to the normal $\vec{n}$ then it is transformed to $\vec{n_{F2}}$, and that is not perpendicular to $\vec{t_F}$ anymore. Thus, we need a different matrix F' that transforms $\vec{n}$ to $\vec{n_F}$ correctly.

As, $\vec{n}$ and $\vec{t}$ are original normal and tangent vectors, we know:

$$\vec{n}^T . \vec{t} = 0$$

This is same as,

$$\vec{n} \, I \, \vec{t} = 0 \text{; which is same as,}$$

$$\vec{n} \, F^{-1} F \vec{t} = 0 .$$

$F\vec{t}$ term in equation above is the transformed tangent vector. I'll denote that as $\vec{t}_F$

$$\vec{n}^T F^{-1} \vec{t}_F = 0.$$

Because their scalar product is 0, $\vec{n}^T F^{-1}$ must be orthogonal to it. So, the correctly transformed vector $\vec{n}_F^T$ must be

$$\vec{n}_F^T = \vec{n}^T F^{-1}$$

transposing both sides to express $\vec{n}_F^T$ as column vector:

$$\vec{n}_F = (\vec{n}^T F^{-1})^T$$

Since $(AB)^T = B^T A^T$,

$$\vec{n}_F = (F^{-1})^T \vec{n}$$

Thus F' matrix that transforms normal vectors correctly is $(F^{-1})^T$