Problem Set 1.1

1.

(a)
$$\because \begin{bmatrix}3\\6\\9\end{bmatrix} = 3\begin{bmatrix}1\\2\\3\end{bmatrix}, \quad \begin{bmatrix}1\\2\\3\end{bmatrix} = 1\begin{bmatrix}1\\2\\3\end{bmatrix} \quad \therefore \begin{bmatrix}3\\6\\9\end{bmatrix} \text{ and } \begin{bmatrix}1\\2\\3\end{bmatrix} \text{ fills a line.}$$

(b) Let $v = \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad w = \begin{bmatrix}0\\2\\3\end{bmatrix}$, and we have all linear combinations $cv + dw$ where c and d are constants. This is the simplest form. Therefore v and w fill a plane.

(c) Let $u = \begin{bmatrix}2\\0\\0\end{bmatrix}, \quad v = \begin{bmatrix}0\\2\\2\end{bmatrix}, \quad w = \begin{bmatrix}2\\2\\3\end{bmatrix}$. u, v, w are 3 different vectors, so $cu+dv+ew$ is the simplest form where c, d, e are constants, and thus they fill all of R^3.