2021 — 2022 学年《编译原理》第 2-3 次编程作业
AST and derivatives of the extended regular expressions

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Contents

2019 级弘毅班《编译原理》第 2-3 次编程作业
正则表达式的抽象语法树与求导

本次作业 (02) 将使用递下降分析法对正则表达式进行语法分析，并输出抽象语法树. 在此基础上对正则表达式进行求导 (derivative)，其对应的线性形式即可看成是 NFA 或 DFA 的状态转移，从而完成正则表达式到 DFA 的直接转换 (assignment 03) (see partial_derivative.pdf).

1 扩展正则表达式 (EREGEX)

增加了 3 个二元运算:

1. 差: `e1 - e2`, `L(e1 - e2) = L(e1) - L(e2)`.
2. 交错乘积: `e1 ^ e2`, `L(e1 ^ e2) = {` $s_1t_1 \cdots s_nt_n \mid s_1 \cdots s_n \in L(e_1) \wedge t_1 \cdots t_n \in L(e_2)$ `}`

   ```
   a ^ b = a b | b a
   ab ^ ba = (a ^ b) (a ^ b)
   a ^ b ^ c = a b c | a c b | b a c | b c a | c a b | c b a
   ```

3. 交: `e1 & e2`, `L(e1 & e2) = L(e1)` $\cap$ `L(e2)`.

优先级由低到高排列: `|`, `-`, `^`, `&`, `concat`, `*`.

1.1 EREGEX 的文法

消除左递归后的文法:

```
reg -> term_or reg'
reg' -> '|' term_or reg' | epsilon

term_or -> term_diff term_or'
term_or' -> '-' term_alt term_or' | epsilon

term_alt -> term_and term_alt'
term_alt' -> '^' term_and term_alt'

term_and -> term term_and'
term_and' -> '&' term term_and' | epsilon

term_concat -> kleene term_concat'
term_concat' -> kleene term_concat' | epsilon

kleene -> fac kleene'
kleene' -> * kleene' | epsilon

fac -> ALPHA | '(' reg ')'
```

设非终结符 `term_xxx` 和 `term_xxx'`，其中 `xxx` is `or`, `alt`, `and`, or `concat`. 则文法可统一为:

```
expr(op1) -> expr(op2) expr1(op1)
expr1(op1) -> op1 expr(op2) expr1(op1)
            | epsilon

where op1 = |, op2 = -;
      op1 = -, op2 = ^;
      op1 = ^, op2 = &;
      op1 = &, op2 = Seq;
```

这样其对应的递归调用函数 (see `parser.c`):

```
AST_PTR expr(Kind op)
{
  AST_PTR left;
  switch (op) {
  case Or: left = expr(Diff);
    return expr1(Or, left);
  case Diff: left = expr(Alt);
```

```
    return expr1(Diff, left);
  case Alt: left = expr(And);
    return expr1(Alt, left);
  default:  left = term();
    return expr1(And, left);
  }
}

AST_PTR expr1(Kind op, AST_PTR left)
{
  AST_PTR right, tmp;
  char op_ch;
  switch (op) {
  case Or: op_ch = '|'; break;
  case Diff: op_ch = '-'; break;
  case Alt: op_ch = '^'; break;
  default: op_ch = '&';
  }
  if (*current == op_ch ) {
    next_token ();
    right = expr(op);
    tmp = arrangeOpNode(op, left, right);
    return expr1(op, tmp);
  } else
    return left;
}
```

1.2 抽象语法树 (AST)

详见 ast.h:

```
typedef enum { Or = 1, Diff = 2, Alt = 3, And = 4, Seq = 5,
               Star = 6, Alpha = 7, Epsilon = 8, Empty = 9} Kind;
/* in order of increased precedence */

typedef struct ast {
  Kind op;
  struct ast *lchild, *rchild;
  int hash;
  int nullable;  /* = 1, if E is nullable, it will use to determine
                    if NFA is final */
  char *exp_string;  /* mostly simplified exp of E, use to
                        determine if 2 regex are equals */
  int state;   /* for  assignment 03!!!
                  state number. trap state is 0,
                  the original exp is 1 */
  LF_PTR lf; /* for assignment 03!!!
                linear form of NFA */
}  AST;
```

1.3 符号表

语法分析或后续求导过程中出现的子正则表达式都进符号表 (ast.h):

```
typedef struct exptab {
  struct exptab *next; /* for collision */
  AST_PTR exp;
} *EXPTAB; /* for hash table of expressions */
```

```
#define HASHSIZE  8011

/* defined in ast.c */
EXPTAB exptab[HASHSIZE] = {NULL}; /* symbol table */
```

the key is the mostly simplified expression string stored in struct `ast.exp_string`.

the hash function is (`ast.c`):

```
int hash(char *s)
{
  unsigned int hv = 7, len = strlen(s);
  for (int i = 0; i < len; i++) {
    hv = hv*31 + s[i];
  }
  return (int) (hv % HASHSIZE) ;
}
```

接口函数：

```
AST_PTR lookup(char *exp_string)
{
  int hv = hash(exp_string);

  EXPTAB t = exptab[hv];

  if (t == NULL) return NULL;

  while (t != NULL) {
    if (strcmp(exp_string, t -> exp -> exp_string) == 0) {
      break;
    }
    t = t -> next;
  }
  if (t == NULL) return NULL;
  return t -> exp;
}
```

if `lookup()` returns `NULL`, it will be stored in `exptab` by

```
AST_PTR insert(AST_PTR exp)
{
  int hv = exp->hash;

  EXPTAB new = (EXPTAB) safe_allocate(sizeof(*new));
  new -> next = exptab[hv];
  new -> exp = exp;
  exptab[hv] = new;

  return exp;
}
```

1.4 Algebraic laws of EREGEX

求导法直接求 FA 是把正则表达式最为 FA 的状态，其求解过程需要判断两个状态是否相等，即两个正则表达式是否等价 (`e1 = e2` iff `L(e1) = L(e2)`). 但严格按数学定义来判断相等，只能用其最小状态 DFA 同构，即本问题求 DFA，因此行不通. 在此我们用正则表达式的代数变换，对所分析的正则表达式，利用代数定律求其最简代数形式。若最简代数形式所对应的字符串 `exp_string` (see above `lookup()`) 相等，则相等. 正则表达式的代数定律如下：

1. 空的化简:

   x $\emptyset$ = $\emptyset$ x = x & $\emptyset$ = $\emptyset$ & x = x ^ $\emptyset$ = $\emptyset$ ^ x = $\emptyset$.
   $\emptyset$ - x = $\emptyset$, x - $\emptyset$ = x.

2. 空串的吸收:

   x ε = ε x = x ^ ε = ε ^ x = x.
   x | $\emptyset$ = $\emptyset$ | x = x.

3. 交换律:

   x | y = y | x
   连续的并运算 | 必须调整为左结合且对应的运算子表达式严格按字典序排列. e.g.

   (c | b) | (e | (f | a)) = (((((a | b) | c) | d) | e)) | f

   这时 `exp_string` 是"a|b|c|d|e|f".

4. 连接运算的结合律:

   (xy)z = x(yz).
   因 (xy)z 和 x(yz) 的 `exp_string` 是相同的, 即 "xyz", 因此不必对 x(yz) 调整为最左结合. 运算 & 和 ^ 也不需调整.

5. 幂等率 (| 和 &):

   x | x = x, x & x = x.

6. Kleene 闭包: x** = x*

7. 分配率:

   (x | y)z = xz | yz, x(y | z) = xy | xz.
   为了加速求导法的收敛, 必须用分配率：
   (x | y)z = (xz | yz), x(y | z) = xy | xz.

1.5 Nullability

a regex x is nullable iff ε in L(x). so

x	N(x)
x	0
ε	1
$\emptyset$	0
x | y	N(x) || N(y)
xy	N(x) && N(y)
x*	1
x - y	N(x) && !N(y)
x ^ y	N(x) && N(y)
x & y	N(x) && N(y)

1.6 AST

以下 AST 构造函数没有使用交换律和结合率进行化简 (in `ast.c`): .

```
AST_PTR mkEpsilon (void)
{
  AST_PTR tree_tmp;

  tree_tmp = lookup ("");

  if (tree_tmp != NULL) return tree_tmp;
```

```
  tree_tmp = (AST_PTR) safe_allocate(sizeof(*tree_tmp));
  tree_tmp->op = Epsilon;
  tree_tmp->exp_string = strdup("");
  tree_tmp->hash = hash(tree_tmp->exp_string);
  tree_tmp->nullable = 1;
  tree_tmp->state = -1; /* no associates to any state */
  tree_tmp->lf = NULL;
  tree_tmp->lchild = NULL;
  tree_tmp->rchild = NULL;
  return insert(tree_tmp);
}

AST_PTR mkEmpty (void)
{
  AST_PTR tree_tmp;

  tree_tmp = lookup ("");

  if (tree_tmp != NULL) return tree_tmp;

  tree_tmp = (AST_PTR ) safe_allocate(sizeof(*tree_tmp));
  tree_tmp->op = Empty;
  tree_tmp->exp_string = strdup("");
  tree_tmp->hash = hash(tree_tmp->exp_string);
  tree_tmp->nullable = 0;
  tree_tmp->lf = NULL;
  tree_tmp->state = -1;
  tree_tmp->lchild = NULL;
  tree_tmp->rchild = NULL;
  return insert(tree_tmp);
}



AST_PTR mkOpNode(Kind op, AST_PTR tree1, AST_PTR tree2)
{
  char *exp_string = (char *)safe_allocate(strlen(tree1->exp_string) +
                                           strlen(tree2->exp_string) + 6);
  char *lp1="", *rp1="", *lp2="", *rp2="";
  char *op_string;
  AST_PTR tree_tmp;

  switch (op) {
  case Alt: op_string = "^"; break;
  case Diff: op_string = "-"; break;
  case And: op_string = "&"; break;
  case Or: op_string = "|"; break;
  default: op_string = "";
  }

  if (op == Seq || op == Alt) {
    if (tree1->op == Epsilon) return tree2;
    if (tree1->op == Empty) return tree1;
    if (tree2->op == Epsilon) return tree1;
    if (tree2->op == Empty) return tree2;
  }

  if (op == And) {
```

```
    if (tree1->op == Epsilon) return tree1;
    if (tree1->op == Empty) return tree1;
    if (tree2->op == Epsilon) return tree2;
    if (tree2->op == Empty) return tree2;
  }
  if (op == Diff) {
    if (tree1->op == Empty) return tree1;
    if (tree2->op == Empty) return tree1;
  }

  if (tree1 == tree2) {
    if (op == Or || op == And) return tree1;
    if (op == Diff) return mkEmpty();
  }

  if (op == Or)
    sprintf(exp_string,"%s%s%s", tree1->exp_string,
            op_string, tree2->exp_string);
  else {
    if (op == Diff && tree2->op == Diff) {
      lp2 ="("; rp2 = ")";
    } else {
    if (tree1->op < op) {
      lp1 ="("; rp1=")";
    }
    if (tree2->op < op) {
      lp2 ="("; rp2 = ")";
    }
    }
    sprintf(exp_string,"%s%s%s%s%s%s%s", lp1, tree1->exp_string, rp1,
            op_string, lp2, tree2->exp_string, rp2);
  }
  tree_tmp = lookup (exp_string);

  if (tree_tmp != NULL)  {
    free(exp_string);
    return tree_tmp;
  }

  tree_tmp = (AST_PTR ) safe_allocate(sizeof *tree_tmp);
  tree_tmp->hash = hash(exp_string);
  tree_tmp->op = op;
  tree_tmp->exp_string = exp_string;
  tree_tmp->nullable = (op == Or?tree1->nullable || tree2->nullable:
                       (op == Diff? tree1->nullable*(tree1->nullable - tree2->nullable)
                        : tree1->nullable && tree2->nullable));
  tree_tmp->lf = NULL;
  tree_tmp->state = -1;
  tree_tmp->lchild = tree1;
  tree_tmp->rchild = tree2;

  return insert(tree_tmp);
}


AST_PTR mkStarNode(AST_PTR tree)
{
  char *exp_string = (char *) safe_allocate(strlen(tree->exp_string) + 4);
```

```
  char *lp = "", *rp = "";
  AST_PTR tree_tmp;

  if (tree->op == Star || tree->op == Epsilon ||
      tree->op == Empty) return tree;

  if (tree->op == Or && tree->lchild->op == Epsilon) return mkStarNode(tree->rchild);

  if (tree->op != Alpha) {
    lp = "("; rp = ")";
  }

  sprintf(exp_string, "%s%s%s%c", lp, tree->exp_string, rp, '*');

  tree_tmp = lookup (exp_string);

  if (tree_tmp != NULL)  {
    free(exp_string);
    return tree_tmp;
  }

  tree_tmp = (AST_PTR) safe_allocate(sizeof(*tree_tmp));

  tree_tmp->hash = hash(exp_string);
  tree_tmp->op = Star;
  tree_tmp->exp_string = exp_string;
  tree_tmp->nullable = 1;
  tree_tmp->state = -1;
  tree_tmp->lf = NULL;
  tree_tmp->lchild = tree;
  tree_tmp->rchild = NULL;
  return insert(tree_tmp);
}
```

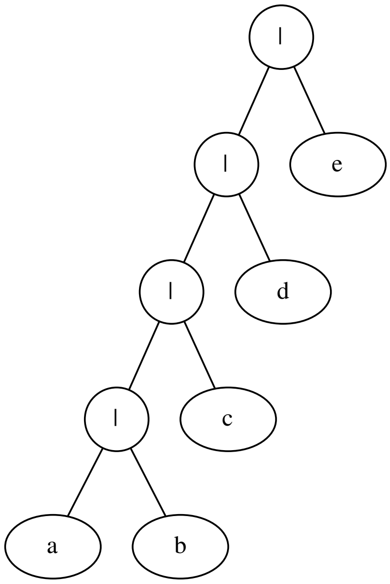
1.7 交换律和结合律

请实现交换律化简正则表达式函数 `AST_PTR arrangeOpNode(Kind op, AST_PTR tree1, AST_PTR tree2)`, 其中 `op` 为 `|`, `^` 或 `&`. 要求对连续的 `|` 均转换为最左结合的 `|` 运算.
实现函数 `AST_PTR arrangeSeqNode(AST_PTR tree1, AST_PTR tree2)`, 用结合率化简正则表达式. 如输入"`(a|b)c`", 化简后输出"`ac|bc`"; 输入"`a(b|c)`", 输出"`ab|ac`".

1.8 测试

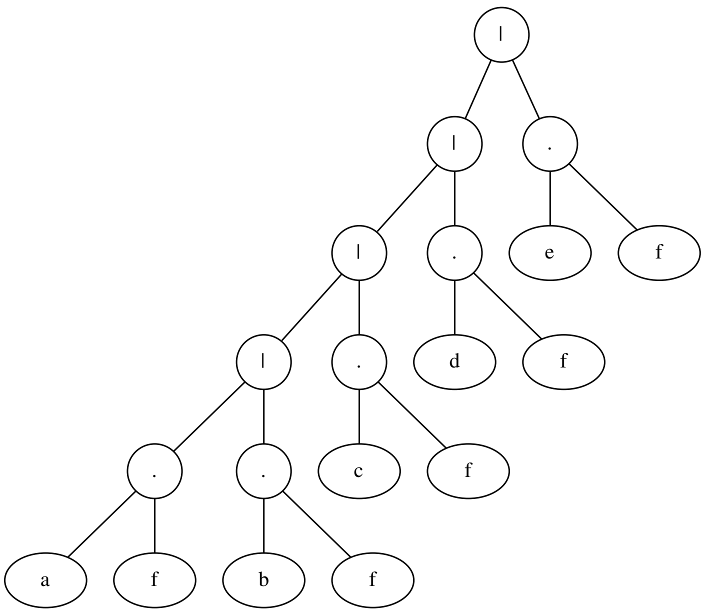
借助于 graphviz 可视化 AST, 函数 `int graphviz_ast_aux(AST_PTR)` 和 `void graphviz_ast(AST_PTR)` 可把 AST 转换成 graphviz 输入文件格式 (`.gv`). 大家可用样本程序 (`REG2FDFA.EXE` for DOS, `reg2dfa` for Linux) 输入正则表达式，则程序输出 graphviz 文件 `ast.gv`. "`dot -Tpdf -o ast.pdf ast.gv`" (Linux) 转换为可视化的 AST 树结构 (pdf 文件).
以下是测试用例：

1. `a|((b|d)|(c|e))`
   the simplified exp is `a|b|c|d|e`



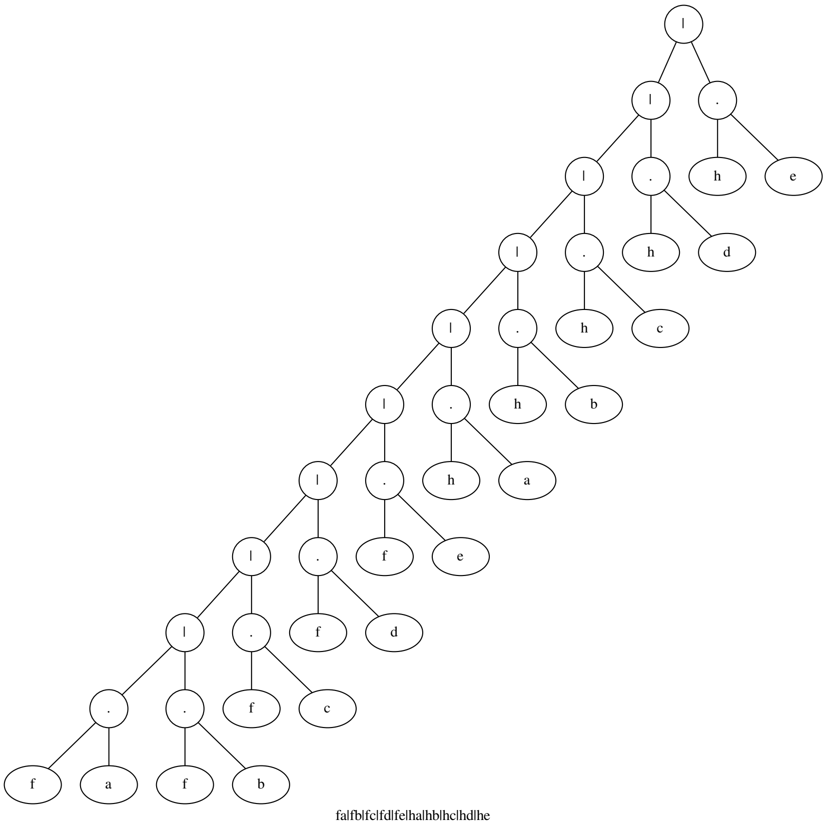
a|b|c|d|e

2. `(a|((b|d)|(c|e)))f`
   the simplified exp is `af|bf|cf|df|ef`

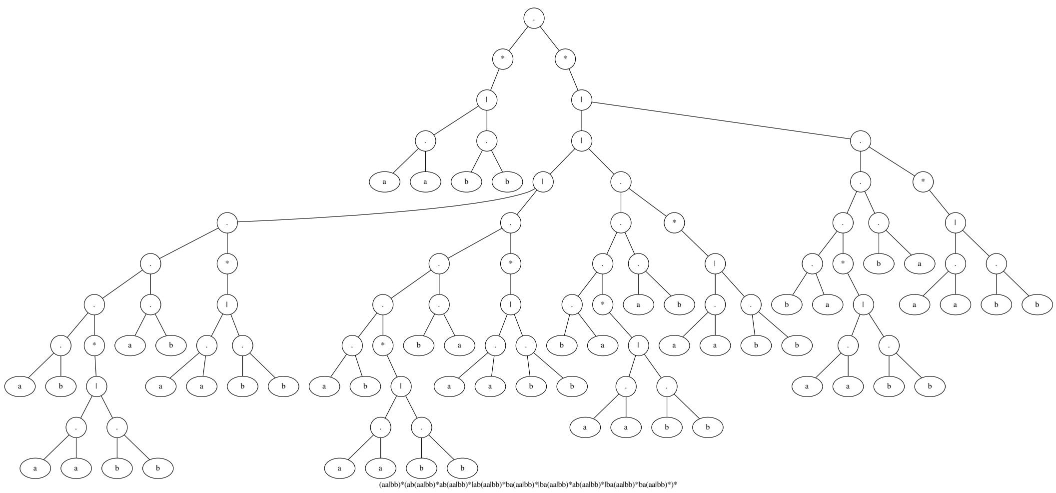


af|bf|cf|df|ef

3. `(f|h)(a|((b|d)|(c|e)))`
   the simplified exp is `fa|fb|fc|fd|fe|ha|hb|hc|hd|he`



4. `(aa|bb)*((ab|ba)(aa|bb)*(ab|ba)(aa|bb)*)*`
   the simplified exp is `(aa|bb)*(ab(aa|bb)*ab(aa|bb)*|ab(aa|bb)*ba(aa|bb)*`
   `|ba(aa|bb)*ab(aa|bb)*|ba(aa|bb)*ba(aa|bb)*)*`



1.9 TODO

implement `AST_PTR arrangeOpNode(Kind op, AST_PTR tree1, AST_PTR tree2)` and `AST_PTR arrangeSeqNode(AST_PTR tree1, AST_PTR tree2)`, so the exe will output the same `ast.gv` as the sample program.
please send your `ast.c` as attached file to `mailto:595180978@qq.com?subject=ID(02)` where the `ID` is your student id number.

2 偏导数 (assignment 03)

用 Thompson 算法转换正则表达式得到 NFA 有 `O(n)` 个状态, 其中 `n` 是正则表达式的长度 (用到的字母的个数). 该 NFA 用到了很多 ε 转换边. 而偏导数法得到的 NFA 无 ε 转换边，且状态数 $\leq$ `n + 1`. 且由此得到的 DFA ，其状态数比子集构造法的要少，即更接近最小状态 DFA.

2.1 线性形式

设 `r` 是正则表达式：

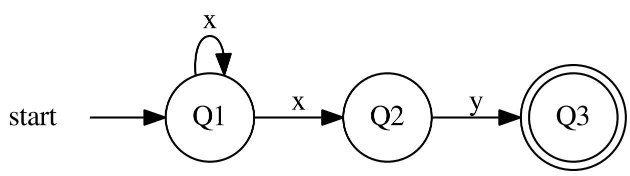
1. *linear form* of `r`:
   `lf(r) = N(r) || a1 r1 | a2 r2 | ...  | an rn`,
   其中 `ai` 是字母，`ri` 是正则表达式.
   `N(r) = 1` iff $\varepsilon \in$ `L(r)`, 否则 `N(r) = 0`.
2. 如: `lf(x*xy) = 0 || x x*xy | x y`,
   `N(x*xy) = 0`, `a1 = a2 = x`, `r1 = x*xy`, `r2 = y`.
   `ai` 和 `aj` 可以是相同的字母，即 *undeterministic*.

   若所有的 `ai` 都是两两不同的, 即 *deterministic* linear form.
3. 我们可利用分配率在求解过程中，把相同的 `ai` 所对应的 `ri` 合并即可得到 deterministic 形式, 如：
   `lf(x*xy) = 0 || x (x*xy|y)`.
   其中 `(x*xy|y)` 相对于 `x` 的偏导数.

若将 `r` 和 `ri` 看成 NFA 的状态 `Q` 和 `Qi`, 则有 `trans(Q, ai) = Qi`. 若 `N(Qi) = 1` , 状态 `Qi` 为接受状态. 对求 `Q` 的 `lf` 过程中新产生的状态 `Qi` 递归求解其 `lf` 直到没有新的状态产生，即可得到 NFA (可证明 NFA 最多有 `n + 1` 个状态, 即算法收敛, see detail in partial_derivative.pdf).

如：设 `Q1 = x*xy`, 则对应的 NFA:

```
Q1 = 0 || x Q1 | x Q2 | Q1 = x*xy
Q2 = 0 || y Q3 | Q2 = y
Q3 = 1 || Q3 = ε
```



NFA of x*xy

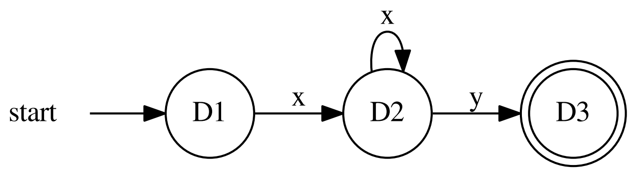
若 `lf` 重组，则
`Q1 = 0 || x (Q1 | Q2)`.

设 `D1 = Q1`, `D2 = Q1 | Q2 = x*xy | y`. 对 `D2` 计算 `lf`:
`D2 = 0 || x xx*y | x y | y` ε.

重组 `D2` 为：

`D2 = 0 || x (xx*y | y) | y` ε `= 0 || x D1 | y D3`
其中 `D3 =` ε. 这样对应的 DFA 为:

```
D1 = 0 || x D2 | D1 = x*xy
D2 = 0 || x D2 | y D3 | D2 = x*xy|y
D3 = 1 || D3 = ε
```



DFA of x*xy

2.2 线性形式的计算

`lf` 可用以下规则通过对正则表达式的抽象语法树的后序遍历计算出来 (即综合属性):

regex	lf
ε	`1 || ∅`
$\emptyset$	`0 || ∅`
`x`	`0 || x ε`
`A | B`	`N(A)∨N(B) || (lf(A) | lf (B))` if `lf(A) = a1 A1 | ... | an An, lf(B) = b1 B1 | ... | bm Bm` then `lf(A) | lf(B) = a1 A1 | ... | an An | b1 B1 | ... | bm Bm`
`A B`	`0 || (lf(A))B` if `N(A) = 0` if `lf(A) = a1 A1 | ... | an An`, then `(lf(A))B = a1 (A1 B) | ... | an (An B)`
`A B`	`N(B) || (lf(A))B | lf(B)` if `N(A) = 1`
`A*`	`1 || (lf(A))A*`

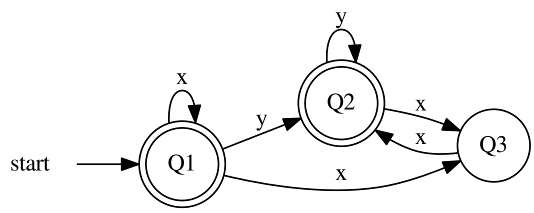
2.2.1 Example 1. NFA of `x*(y|xx)*`

```
  lf(x*(y|xx)*)
= 0((y|xx)*) || (lf(x*))(y|xx)* | lf((y|xx)*)
= 1 || ((lf(x))x*)(y|xx)* | lf((y|xx)*)
= 1 || x (x*(y|xx)*) | lf((y|xx)*)
= 1 || x (x*(y|xx)*) | (lf(y|xx))(y|xx)*
= 1 || x (x*(y|xx)*) | (lf(y) | lf(xx))(y|xx)*
= 1 || x (x*(y|xx)*) | (y ε | (lf(x))x)(y|xx)*
= 1 || x (x*(y|xx)*) | (y ε | (x ε)x) (y|xx)*
= 1 || x (x*(y|xx)*) | (y ε | x x )(y|xx)*
= 1 || x (x*(y|xx)*) | y (y|xx)* | x x(y|xx)*
= 1 || x Q1 | y Q2 | x Q3
(where Q1 = x*(y|xx)*, Q2 = (y|xx)*, Q3 = x(y|xx)*)

  lf(Q2) = lf((y|xx)*)
= 1 || (lf(y|xx))(y|xx)*
= 1 || (lf(y) | lf(xx))(y|xx)*
= 1 || (y ε | lf(x)x)(y|xx)*
= 1 || y (y|xx)* | x x(y|xx)*
= 1 || y Q2 | x Q3

  lf(Q3) = lf(x(y|xx)*)
= 0 || (lf(x))(y|xx)*
= 0 || x (y|xx)*
= 0 || x Q2
```

so, we have NFA:

```
Q1 = 1 || x Q1 | y Q2 | x Q3 | Q1 = x*(y|xx)*
```

```
Q2 = 1 || y Q2 | x Q3 | Q2 = (y|xx)*
Q3 = 0 || x Q2 | Q3 = x(y|xx)*
```



NFA of x*(y|xx)*

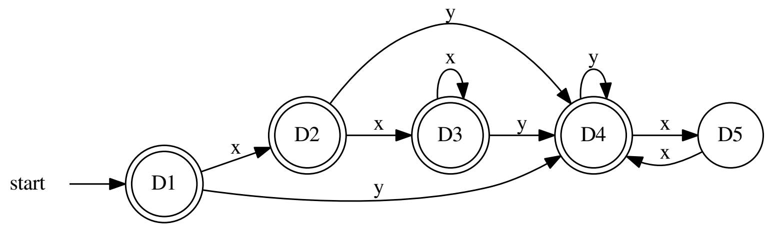
2.2.2 Example 2. DFA of `x*(y|xx)*`

对上例求 `lf` 过程中相同字母对应的右边因子用 `|` 运算重组便可直接得到 DFA!

```
  lf(D1) = lf(x*(y|xx)*)
= O((y|xx)*) || (lf(x*))(y|xx)* | lf((y|xx)*)
= 1 || x (x*(y|xx)*) | y (y|xx)* | x x(y|xx)*
= 1 || x (x*(y|xx)*|x(y|xx)*) | y (y|xx)* = 1 || x D2 | y D4
(where D1 = x*(y|xx)*, D2 = x*(y|xx)*|x(y|xx)*, D4 = (y|xx)*)

  lf(D2) = lf(x*(y|xx)*|x(y|xx)*)
= 1 || x (x*(y|xx)*|x(y|xx)*|(y|xx)*) | y (y|xx)*
= 1 || x D3 | y D4
(where D3 = x*(y|xx)*|x(y|xx)*|(y|xx)*)

  lf(D3) = lf(x*(y|xx)*|x(y|xx)*|(y|xx)*)
= 1 || x (x*(y|xx)*|x(y|xx)*|(y|xx)*) | y (y|xx)*
= 1 || x D3 | y D4

  lf(D4) = lf((y|xx)*)
= 1 || y (y|xx)* | x (x(y|xx)*)
= 1 || y D4 | x D5
(where D5 = x(y|xx)*)

  lf(D5) = lf(x(y|xx)*)
= 0 || y (y|xx)* | x (x(y|xx)*)
= 0 || x (y|xx)* D4
= 0 || x D4
```

so, we have DFA:

```
D1 = 1 || x D2 | y D4 | D1 = x*(y|xx)*
D2 = 1 || x D3 | y D4 | D2 = x*(y|xx)*|x(y|xx)*
D3 = 1 || x D3 | y D4 | D3 = x*(y|xx)*|x(y|xx)*|(y|xx)*
D4 = 1 || y D4 | x D5 | D4 = (y|xx)*
D5 = 0 || x D4 | D5 = x(y|xx)*
```



DFA of x*(y|xx)*

2.3 算法

lf 的数据结构见 ast.h:

```
typedef struct lf {
  struct lf *next;
  char symbol;
  AST_PTR exp;
} LF;

typedef LF *LF_PTR;
```

lf 的计算可抽象为下述两函数 (见 nfa.c):

```
static LF_PTR (*union_method)();
/* the union of 2 lf */

static LF_PTR (*seq_method)();
/* concatenation of a linear form with a regex */
```

对 NFA, union_method() 为:

```
LF_PTR lf_union(LF_PTR lf1, LF_PTR lf2)
{
  LF_PTR tmp;
  tmp = lf1 = lf_clone(lf1);
  lf2 = lf_clone(lf2);

  if (lf1 == NULL) return lf2;
  while (tmp != NULL) {
    if (tmp->next == NULL) {
      tmp->next = lf2;
      break;
    }
    tmp = tmp -> next;
  }
  return lf1;
}
/* if
     lf1 = a1 A1 | ... | an An,
     lf2 = b1 B1 | ... | bm Bm
   then
     lf_union(lf1, lf2) = a1 A1 | ... | an An | b1 B1 | ... | bm Bm
*/
```

seq_method() 为:

```
LF_PTR lf_concate(LF_PTR lf, AST_PTR exp)
{
  LF_PTR tmp;
  tmp = lf = lf_clone(lf);
  if (tmp == NULL) return NULL;
  while (tmp != NULL) {
    tmp->exp = mkOpNode(Seq, tmp->exp, exp);
    tmp = tmp->next;
  }
  return lf;
}
/* if
     lf = a1 A1 | ... | an An,  exp = B
   then
     lf_concate(lf, lf2) = a1 (A1 B) | ... | an (An B)
*/
```

对 DFA, `union_method()` 为:

```
LF_PTR lf_union_plus(LF_PTR lf1, LF_PTR lf2)
{
  LF_PTR head, tmp, new, lf;
  head = lf = lf_clone(lf1);

  if (lf1 == NULL) return lf2;

  while (lf2 != NULL) {
    tmp = lf;
    while (tmp != NULL) {
      if (tmp->symbol == lf2->symbol) {
        tmp->exp = arrangeOpNode(Or, tmp->exp, lf2->exp);
        goto NEXT;
      }
      tmp = tmp->next;
    }
    new = mk_lf(lf2->symbol, lf2->exp);
    new->next = head;
    head = new;
  NEXT:
    lf2 = lf2 -> next;
  }
  return head;
}

/* if
     lf1 = a1 A1 | ... | an An | c1C1 | ... | ciCi,
     lf2 = a1 B1 | ... | an Bn | d1D1 | ....| djDj
   then
     lf_union_plus(lf1, lf2) = a1 (A1 | B1) | ... | an (An | Bn) |
       c1C1 | ... | ciCi | d1D1 | ....| djDj
*/
```

`seq_method()` 为:

```
LF_PTR lf_concate_plus(LF_PTR lf, AST_PTR exp)
{
  LF_PTR tmp;
  tmp = lf = lf_clone(lf);
  if (tmp == NULL) return NULL;
  while (tmp != NULL) {
```

```
    tmp->exp = arrangeSeqNode(Seq, tmp->exp, exp);
      /* apply distributive law */
    tmp = tmp->next;
  }
  return lf;
}
/* if
     lf = a1 (A1 | B1) | ... | an (An | Bn),  exp = C
   then
     lf_concate_plus(lf, lf2) = a1 (A1 C |B1 C) | ... | an (An C | Bn C)
*/
/* use distributive law to factorize regex, so more opptunity to
   test if two regex are equals */
```

则递归计算 `lf` 的函数如下（TODO）:

```
void linear_form(AST_PTR exp, int stated)
/* lf calculation of exp,
   if stated is 1, and add all states (Ai) (see addstate(AST_PTR)) and
   recursively call linear_form(Ai, 1) where exp->lf = a1 A1 | ... | an An

   for lf of AB and nullabe(A) = 1, then

   lf(AB) = (lf(A))B + lf(B)

   so lf(A) and lf(B) are not states for AB. so they don't needed
   attribute the state number, hence the recursive call lf are
   linear_form(A, 0) and linear_form(B, 0).

   the same case for lf(A*).

*/
{
  /* TODO */
}
```

3 测试

to run the test, just `./reg2dfa < exN`, where `N` is number of the test example. `dot -Tpdf -o mdfa.pdf mdfa.gv` to see the MDFA graph.

3.1 ex1: `(a|b)*(babab(a|b)*bab|bba(a|b)*bab)(a|b)*`

NFA states: 13, DFA states 21, MDFA states 10.
Thompson & Subset (JFLAP): NFA 68, DFA 62, MDFA 10.

3.2 ex2: `((a*b*a*b*)*(a*b*a*b*)*(a*b*a*b*)*(a*b*a*b*)*)*`

NFA states: 17, DFA states 3, MDFA states 1.
Thompson & Subset (JFLAP): NFA 84, DFA 3, MDFA 1.

3.3 ex3: `(a*b*a|b*a*b)*`

NFA states: 5, DFA states 3, MDFA states 1.
Thompson & Subset (JFLAP): NFA 3, DFA 3, MDFA 1.

3.4 ex4: `(ba*b*|ab*a*)*`

NFA states: 5, DFA states 8, MDFA states 1.
Thompson & Subset (JFLAP): NFA 2, DFA 9, MDFA 1.

3.5 ex5: `((ab|ba)*aa|(ab|ba)*bb)*(ab|ba)*`

NFA states: 12, DFA states 4, MDFA states 2.
Thompson & Subset (JFLAP): NFA 66, DFA 8, MDFA 2.

3.6 ex6: `(aa|bb)*((ab|ba)(aa|bb)*(ab|ba)(aa|bb)*)*`

NFA states: 9, DFA states 4, MDFA states 4.
Thompson & Subset (JFLAP): NFA 82, DFA 17, MDFA 4.

3.7 ex7:

```
((aa|ab(bb)*ba)*(b|ab(bb)*a)(a(bb)*a)*(b|a(bb)*ba))*(aa|ab(bb)*ba)*(b|ab(bb)*a)(a(bb)*a)*
```

NFA states: 64, DFA states 7, MDFA states 4.

3.8 ex8:

```
(a(aa)*|aa(aaa)*|aaa(aaaaa)*|aaaaa(aaaaaaa)*|aaa(aaaaaaaaaaa)*|aaa(aaaaaaaaaaaa)*|
aaa(aaaaaaaaaaaaa)*)
```

NFA states: 54, DFA states 60061, MDFA states 30030.

3.9 ex9:

```
(a(aa)*|aa(aaa)*|aaa(aaaaa)*|aaaaa(aaaaaaa)*|aaa(aaaaaaaaaaa)*|aaa(aaaaaaaaaaaa)*|
aaa(aaaaaaaaaaaaa)*)*
```

NFA states: 54, DFA states 4, MDFA states 1.

if we disacitve distributive law in `lf_concate_plus()`, `rege2dfa` will cause memory exhausted!

4 Discussions

分配率 (`lf_concate_plus()`) 可加速求 DFA 时的收敛速度, 即非常显著地减少 DFA 的状态数，如:

`(a(aa)*|aa(aaa)*|aaa(aaaaa)*|aaaaa(aaaaaaa)*)*` 的 DFA 状态数为

1.

但若用没有分配率的 `lf_concate()` 作为 `seq_method()`, 则生成的 DFA 有 211 个状态.

但也有例外的情况. 如由 `0 - 3` 组成的且没有数字重复的正则表达式 (see `rep0_3.txt`) :

```
(1|!)(01)*(0|!)(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!)(3(2((0(10)*(1|!)|1(01)*(0|!))2)*(1|!)
(01)*(0|!)|(0(10)*(1|!)|1(01)*(0|!))(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!)))*(3|!)
```

使用分配率: DFA 12 states, MDFA 5 states. 不使用: DFA 13 states.

而 `0 - 4` 的无重复的正则表达式 (see `rep0_4.txt`):

```
(1|!)(01)*(0|!)(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!)(3(2((0(10)*(1|!)|1(01)*(0|!))2)*(1|!)
(01)*(0|!)|(0(10)*(1|!)|1(01)*(0|!))(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!)))*(3|!)
(4(3((2((0(10)*(1|!)|1(01)*(0|!))2)*(1|!)(01)*(0|!)|(0(10)*(1|!)|1(01)*(0|!))
(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!))3)*(1|!)(01)*(0|!)(2(0(10)*(1|!)|1(01)*(0|!)))*
(2|!)|(2((0(10)*(1|!)|1(01)*(0|!))2)*(1|!)(01)*(0|!)|(0(10)*(1|!)|1(01)*(0|!))(2(0(10)*
(1|!)|1(01)*(0|!)))*(2|!))(3(2((0(10)*(1|!)|1(01)*(0|!))2)*(1|!)(01)*(0|!)|(0(10)*
(1|!)|1(01)*(0|!))(2(0(10)*(1|!)|1(01)*(0|!)))*(2|!)))*(3|!)))*(4|!)
```

使用分配率: DFA 的状态数激增导致 memory exausted! 不使用：DFA 31 states, MDFA 6 states.

5 TODO

5.1 Implement `linear_form()` in `nfa.c`

Implement `lf` for the ordinary regex (union, concatenation, and star).

5.2 EREGEX(Bonus)

the `lf` of EREGEX is recursively defined as

<table>
<tr><th>regex</th><th>lf</th></tr>
<tr><td>A ^ B</td><td>N(A)∧N(B) || ((lf(A) ^ B) | (A ^ lf (B)))
where if lf(A) = a1 A1 | ... | an An,
then lf(A) ^ B = a1 (A1 ^ B) | ... | an (An ^ B)</td></tr>
<tr><td>A & B</td><td>N(A)∧N(B) || (lf(A)) & (lf(B))
if lf(A) = a1 A1 | ... | an An, lf(B) = a1 B1 | ... | an Bn then
(lf(A)) & (lf(B)) = a1 (A1 & B1) | ... | an (An & Bn)</td></tr>
<tr><td>A - B</td><td>N(B)∧¬N(B) || (lf(A)) - (lf(B))
if lf(A) = a1 A1 | ... | an An, lf(B) = a1 B1 | ... | an Bn then
(lf(A)) - (lf(B)) = a1 (A1 - B1) | ... | an (An - Bn)</td></tr>
</table>

Implement `linear_form()` for extended regex operations. as test example (`rep0_9B.txt`):

```
(0|1|2|3|4|5|6|7|8|9)*-(0|1|2|3|4|5|6|7|8|9)*(00|11|22|33|44|55|66|77|88|99)(0|1|2|3|4|5|6|7|8|9)*
```

will generate 12 state MDFA where one is trap state.

because the different op `-` will diverged for NFA (e.g. `(a|b)*-(a|b)*ab(a|b)*`). we should disacitve `lf` for NFA if `-` is presented in regex (see `is_minus(exp)` in `nfa.c`).

5.3 LR parser for EREGEX

We can also use YACC to generate LR parser of EREGEX. to add the macro definition in the regex definition, we can add `Eq` of AST type `Kind`:

```
typedef enum { Eq = 0, Or = 1, Diff = 2, Alt = 3, And = 4, Seq = 5,
               Star = 6, Alpha = 7, Epsilon = 8, Empty = 9} Kind;
/* in order of increasing precdence */
```

and YACC grammar:

```
%{
#include <ctype.h>
#include <stdlib.h>
#include "ast.h"

#define YYSTYPE AST_PTR
#define MAX_BUFFER 1024
static char input_buffer[MAX_BUFFER] = "\0";
static char * current = input_buffer;
%}

%token ALPHA
%right '='
%left '|'
%left '-'
```

```
%left '^'
%left '&'
%left ALPHA '(' '!'
%left CONCAT
%nonassoc '?'
%nonassoc '*'
%nonassoc '+'

%%

root : root  line
|
;
line : reg ';' {
  if ($1->op != Eq) {
    printf("the simplified exp is %s\n", $1->exp_string);
    print_tree($1);
    printf("\n");
    reg2nfa($1);
  }
;
reg : ALPHA { $$ = mkLeaf(*current); }
| '!'  { $$ = mkEpsilon(); }
| '(' reg ')' { $$ = $2;  }
| reg '=' reg { $$ = mkEqNode($1, $3); }
| reg '|' reg { $$ = arrangeOpNode(Or, $1, $3); }
| reg '-' reg { $$ = arrangeOpNode(Diff, $1, $3); }
| reg '^' reg { $$ = arrangeOpNode(Alt, $1, $3); }
| reg '&' reg { $$ = arrangeOpNode(And, $1, $3); }
| reg reg %prec CONCAT { $$ = arrangeSeqNode($1, $3); }
| reg '*' { $$ = mkStarNode($1); }
| reg '+' { $$ = arrangeSeqNode($1, mkStarNode($1));
             /* e+ = e e* */ }
| reg '?' { $$ = arrangeOpNode(Or, $1, mkEpsilon());
            /* e? = e | epsilon */ }
;
```

so the the regex of no repeatation of digits can be recursive defined as (rep0_9A.txt):

```
A = 1? (0 1)* 0?;
B = 1 (0 1)* 0? | 0 (1 0)* 1?;
C = A(2 B)* 2?;
D = 2 (B 2)* A | B (2 B)* 2?;
E = C(3 D)* 3?;
F = 3(D 3)* C | D (3 D)* 3?;
G = E (4 F)* 4?;
H = 4(F 4)* E | F (4 F)* 4?;
I = G (5 H)* 5?;
J = 5(H 5)* G | H (5 H)* 5?;
K = I (6 J)* 6?;
L = 6(J 6)* I | J (6 J)* 6?;
M = K (7 L)* 7?;
N = 7(L 7)* K | L (7 L)* 7?;
O = M (8 N)* 8?;
P = 8(N 8)* M | N (8 N)* 8?;
Q = O (9 P)* 9?;
Q;
```

if disacitve distributive law, `reg2dfa` will generate 59 state NFA, 1892 state DFA, and 11 state MDFA.

please send your `nfa.c` as attached file to `mailto:595180978@qq.com?subject=ID(03)` where the `ID` is your student id number.

–hfwang

February 28, 2022