Chapter 01

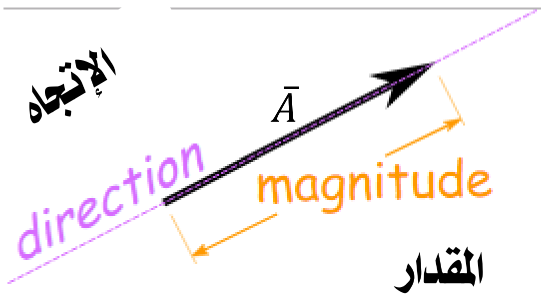
Vector Algebra

الكميات أو الـ Quantities بتنقسم اللي نوعين:

Scalars كيميات قياسية	Vectors كيميات متجهة
Scalar has "magnitude" only. الكيميات القياسية لها "مقدار" فقط.	Vector has "magnitude" and "Direction. المتجهات لها "مقدار" و"أتجاه".
Examples: Time, Temperature, Work, Length, mas, speed	Examples: Displacement, Wight, Velocity, Force, Acceleration

إزاي نعبر عن الـ Vector ؟

بيانياً "Graphically" ...عن طريق رسم متجه طوله هو المقدار وسهم ناحية الأتجاه.



عشان نميز بين الـVector والـScalar ... ح نضع فوق الكيميات الـVector.. "BAR"... "$\bar{A}$"

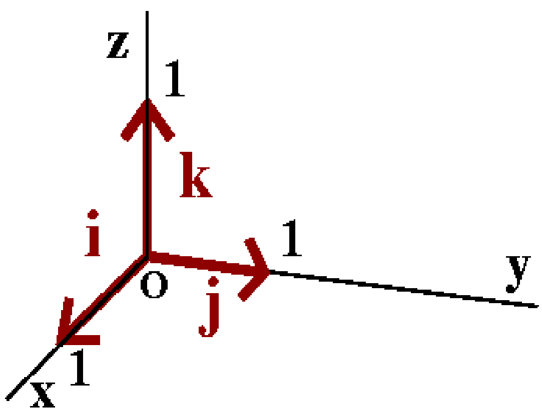
جبريا "Algebraically" ...وفيها ح نكتب الـ Vector بالشكل

$$\bar{A} = \langle a, b, c\rangle = a\underline{i} + b\underline{j} + c\underline{k}$$

1- $\underline{i}$ هو vector طوله بواحد في أتجاه x-axis.

2- $\underline{j}$ هو vector طوله بواحد في أتجاه y-axis.

3- $\underline{k}$ هو vector طوله بواحد في أتجاه z-axis.



Euclidian Spaces

الــ Euclidian Space هو فراغ فيه اكثر من ثلاثا أبعاد n-dimensional نوني الأبعاد

ويتكتب فيه المتجه علي أحدي الأشكال الأتية:

$$\bar{A} = \langle a_1, a_2, \cdots, a_n \rangle$$

$$\bar{A} = (a_1, a_2, \cdots, a_n)$$

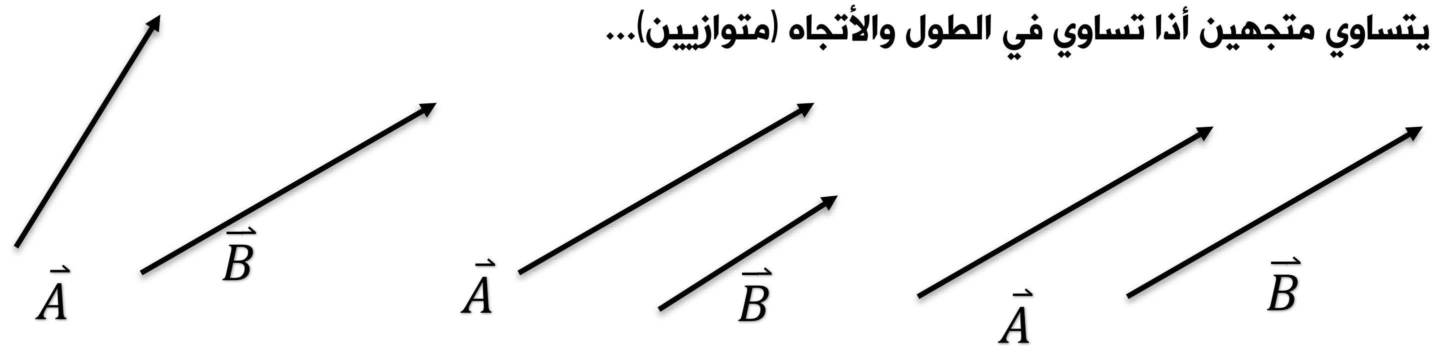
$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Operations of Vectors العمليات علي المتجهات

$\boxed{1}$ Equating of Vectors التساوي

Graphically:

يتساوي متجهين أذا تساوي في الطول والأتجاه (متوازيين)...



$\overline{\boldsymbol{A}} \neq \overline{\boldsymbol{B}}$ $\qquad$ $\overline{\boldsymbol{A}} \neq \overline{\boldsymbol{B}}$ $\qquad$ $\overline{\boldsymbol{A}} = \overline{\boldsymbol{B}}$

Algebraically:

بطريقة أخري .. لو كل المركبات المتناظرة تساوت

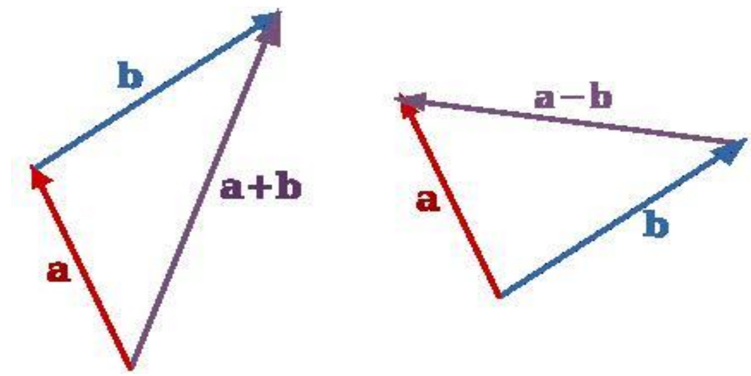
$$\overline{\mathrm{A}} = \langle a_1, a_2, \cdots, a_n \rangle$$

$$\overline{\mathrm{B}} = \langle b_1, b_2, \cdots, b_n \rangle$$

$$\boxed{\overline{\mathrm{A}} = \overline{\mathrm{B}}} \text{ if } \boxed{a_1 = b_1,\ \ a_2 = b_2\ , \cdots, a_n = b_n}$$

2 Addition and Subtraction الجمع والطرح

Graphically:



Algebraically:

$$\bar{\mathrm{A}} = \langle a_1, a_2, \cdots, a_n \rangle$$

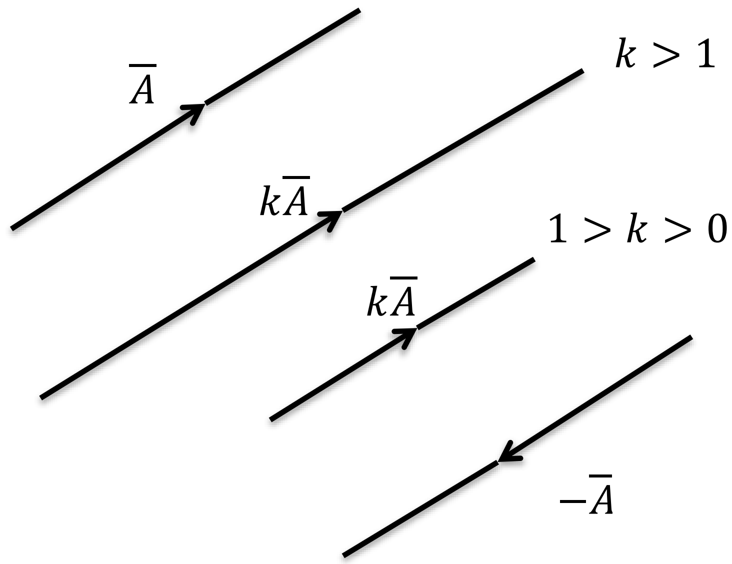
$$\bar{\mathrm{B}} = \langle b_1, b_2, \cdots, b_n \rangle$$

$$\boxed{\bar{\mathrm{A}} \pm \bar{\mathrm{B}} = \langle a_1 \pm b_1, a_2 \pm b_2, \cdots, a_n \pm b_n \rangle}$$

3 Multiplication by Scalar الضرب في عدد

ضرب المتجه في كمية Scalar يعني عدد لا يغير من أتجاه انما فقط يؤثر في طوله او مقداره

Graphically:



Algebraically:

$$\bar{\mathrm{A}} = \langle a_1, a_2, \cdots, a_n \rangle$$

$$\mathrm{k}\bar{\mathrm{A}} = \bar{\mathrm{A}} = \langle \mathrm{k}a_1, \mathrm{k}a_2, \cdots, \mathrm{k}a_n \rangle$$

Ex. 01 If $\bar{A} = 3\underline{i} + 2\underline{j} - \underline{k}$, $\bar{B} = 6\underline{i} + 3\underline{k}$. Find $4\bar{A}$ and $3\bar{A} - 2\bar{B}$.

Answer.

$$4A = \langle 12,8,-4\rangle$$

$$\begin{aligned} 3A - 2B &= \langle 9,6,-3\rangle - \langle 12,0,6\rangle \\ &= \langle -3,6,-9\rangle \end{aligned}$$

Ex. 02 If $\bar{A} = a\underline{i} + 2\underline{j} - \underline{k}$, $\bar{B} = 2\underline{i} + 3\underline{k}$, $\bar{C} = \underline{i} + 2b\underline{j} + c\underline{k}$, and $\bar{A} + \bar{B} = \bar{C}$. Find a, b, c.

Answer.

4 Norm (Magnitude, or Length) المعيار أو الطول

كل Vector وله مقدار او معيار وهو عبارة عن كمية Scalar

$$\bar{A} = \langle a_1, a_2, \cdots, a_n\rangle \xrightarrow{\text{Modulus}} \|\bar{A}\| = A = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$$

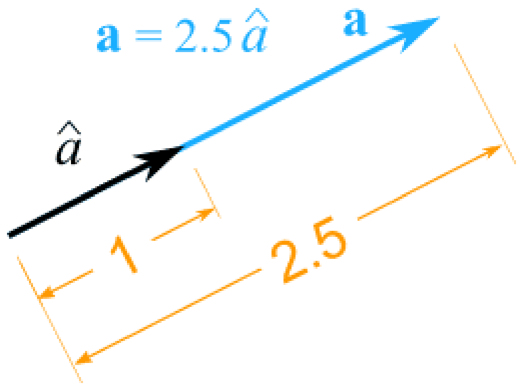
Ex. 03 If $\bar{A} = 3\underline{i} + 2\underline{j} - \underline{k}$, then evaluate $\|\bar{A}\|$.

Answer.

$$\|A\| = \sqrt{9+4+1} = \sqrt{14}$$

5 Unit Vector

لكل Vector نقدر نجيب Vector تاني في نفس أتجاهه لكن طوله بواحد



$$\overline{A} \xrightarrow{\text{Unit Vector}} \widehat{A} = \frac{\overline{A}}{|\overline{A}|}$$

Ex. 04 Find the unit vector of the resultant of two vectors

$$\overline{\boldsymbol{S}} = \langle \mathbf{3}, -\mathbf{5}, \mathbf{2} \rangle$$

and

$$\overline{\boldsymbol{T}} = \langle \mathbf{1}, \mathbf{2}, \mathbf{4} \rangle$$

Answer.

The resultant of $\overline{S}$ and $\overline{\boldsymbol{T}}$ is given by

$$\overline{R} = \overline{S} + \overline{T} = \langle 3, -5, 2 \rangle + \langle 1, 2, 4 \rangle = \langle 4, -3, 6 \rangle$$

Now,

$$|\overline{R}| = \sqrt{16 + 9 + 36} = \sqrt{61}$$

Then the unit vector is

$$\hat{R} = \frac{\overline{R}}{|\overline{R}|} = \frac{\langle 4, -3, 6 \rangle}{\sqrt{61}} = \langle \frac{4}{\sqrt{61}}, -\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}} \rangle$$

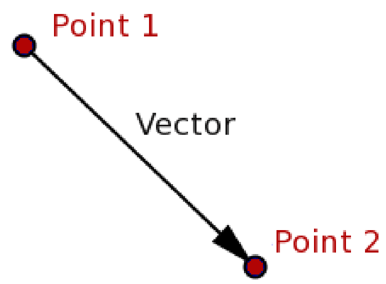
Ex. 05 Find a vector $\bar{b}$ of magnitude 8 and parallel to the vector $\bar{a} = 3\underline{i} + 2\underline{j} - 6\underline{k}$ but in the opposite direction.

Answer.

6 The vector joining between two points

The vector from the point (x_1, y_1, z_1) to the point (x_2, y_2, z_2) is given by:

$$(x_2 - x_1)\underline{i} + (y_2 - y_1)\underline{j} + (z_2 - z_1)\underline{k}$$



Ex. 06 Find the vector comes from the point $(1, 2, -3)$ going to $(2, 0, 4)$.

Answer.

The vector is

$$(2-1)\underline{i} + (0-2)\underline{j} + (4-(-3))\underline{k} = \underline{i} - 2\underline{j} + 7\underline{k}$$

7 Dot Product "Scalar Product"

Let

$$\bar{A} = \langle a_1, a_2, \cdots, a_n \rangle$$

$$\bar{B} = \langle b_1, b_2, \cdots, b_n \rangle$$

$$\boxed{\bar{A} \cdot \bar{B} = (a_1 b_1) + (a_2 b_2) + \cdots + (a_n b_n)}$$

Ex. 07 If $\bar{A} = 4\underline{i} + 2\underline{j} - \underline{k}$, $\bar{B} = 2\underline{i} - 5\underline{j} + 3\underline{k}$. Find $\bar{A} \cdot \bar{B}$.

Answer.

$$\bar{A} \cdot \bar{B} = (4)(2) + (2)(-5) + (-1)(3) = -5$$

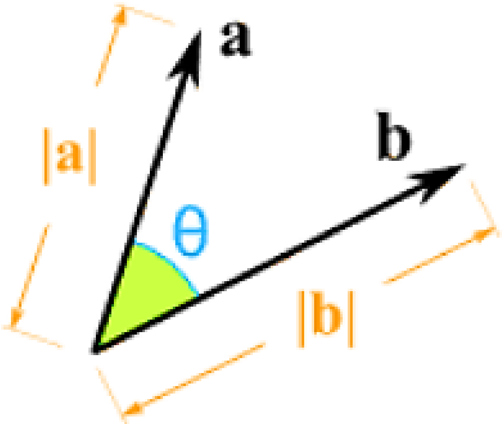
Note that: $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

Applications

I Angle الزاوية

لحساب زاوية θ بين المتجهين $\bar{A}$ و $\bar{B}$ نستخدم هذا القانون

$$\boxed{\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}||\bar{B}|}}$$



Ex. 08 Find the ACUT angle between the two vectors

$\bar{A} = 5\underline{i} + 2\underline{j} - 2\underline{k}$ and $\bar{B} = 3\underline{i} - 5\underline{j} + 6\underline{k}$

Answer.

$$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}||\bar{B}|}$$

$$\bar{A} \cdot \bar{B} = (5)(3) + (2)(-5) + (-2)(6) = -7$$

$$|\bar{A}| = \sqrt{5^2 + 2^2 + (-2)^2} = \sqrt{33}$$

$$|\bar{B}| = \sqrt{3^2 + (-5)^2 + 6^2} = \sqrt{70}$$

$$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}||\bar{B}|} = -\frac{7}{\sqrt{33}\sqrt{70}} \approx -0.456$$

$$\theta = 98.37^0 \equiv 81.63^o$$

Ex. 09 Find the angle between the vectors $\langle 12, -5\rangle$ and $\langle 3, 4\rangle$.

Answer.

Ex. 10 Find the angle between the vector $\bar{\mathbf{A}} = 3\underline{\mathbf{i}} - 5\mathbf{j} + 2\underline{\mathbf{k}}$ and y-axis.

Answer.

Let

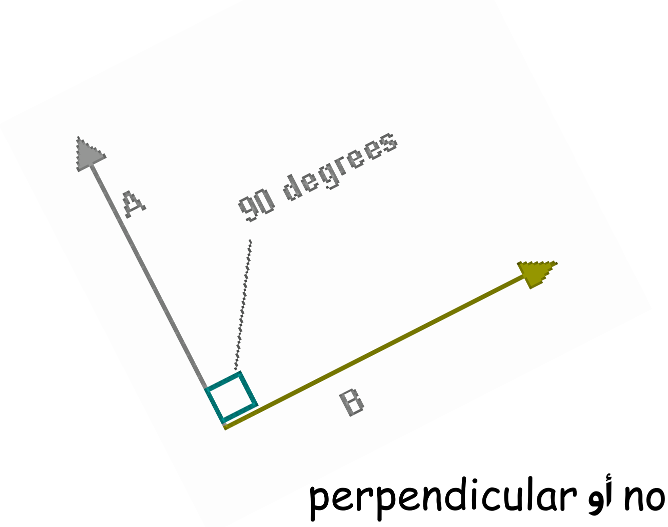
and $\bar{\mathrm{B}} = \underline{\mathrm{j}}\bar{\mathrm{A}} = 3\underline{\mathrm{i}} - 5\underline{\mathrm{j}} + 2\underline{\mathrm{k}}$

$$\bar{\mathrm{A}} \cdot \bar{\mathrm{B}} = -5$$

and $|\bar{\mathrm{B}}| = 1|\bar{\mathrm{A}}| = \sqrt{9+25+4} = \sqrt{38}$

$$\cos\theta = \frac{\bar{\mathrm{A}} \cdot \bar{\mathrm{B}}}{|\bar{\mathrm{A}}||\bar{\mathrm{B}}|} = -\frac{5}{\sqrt{38}} \quad \to \boxed{\theta = 144°12'}$$

$\boxed{\text{II}}$ Orthogonal Vectors تعامد المتجهات



شرط تعامد المتجهان $\bar{A}$ و $\bar{B}$ هو $\bar{A}\cdot\bar{B}=0$.

$$\boxed{\bar{\mathbf{A}} \perp \bar{\mathbf{B}} \Leftrightarrow \bar{\mathbf{A}}\cdot\bar{\mathbf{B}}}$$

متعامد يعني orthogonal أو orthonormal أو normal أو perpendicular

Ex. 11 Find the value of c making $\bar{\mathbf{A}} = \mathbf{c}\underline{\mathbf{i}} + \underline{\mathbf{j}} - 3\underline{\mathbf{k}}$ and $\bar{\mathbf{B}} = \mathbf{c}\underline{\mathbf{i}} - 2\mathbf{cj} + \underline{\mathbf{k}}$ be ORTHOGONAL.

Answer.

Since $\bar{A}$ and $\bar{B}$ are orthogonal. Hence $\bar{A}\cdot\bar{B}=0$

$$\left(\mathrm{c}\underline{\mathrm{i}} + \underline{\mathrm{j}} - 3\underline{\mathrm{k}}\right)\cdot\left(\mathrm{c}\underline{\mathrm{i}} - 2\mathrm{cj} + \underline{\mathrm{k}}\right) = 0$$

$$c^2 - 2c - 3 = 0$$

$$(c-3)(c+1) = 0$$

Factorization أعمل
تحليل وهات قيم c

Then

Or $\quad c = -1 c = 3$

Ex. 12 Find the value of c which makes the vectors $2\bar{\imath} - 2\bar{\jmath} - \bar{k}$ and be parallel.$-\bar{\imath} + \bar{\jmath} + c\bar{k}$

Answer.

$$\frac{2}{-1} = \frac{-2}{1} = \frac{-1}{c}$$

شرط توازي متجهين ان
تكون مركباتهم متناسبة

Hence, $c = \frac{1}{2}$.