25

Reciprocating Air Compressor

Introduction

An *air compressor* is a machine which takes in atmospheric air, compresses it with the help of some mechanical energy and delivers it at higher pressure. It is also called *air pump*. An air compressor increases the pressure of air by decreasing its specific volume using mechanical means. Thus compressed air carries an immense potential of energy. The controlled expansion of compressed air provides motive force in air motors, pneumatic hammers, air drills, sand-blasting machines and paint sprayers, etc.

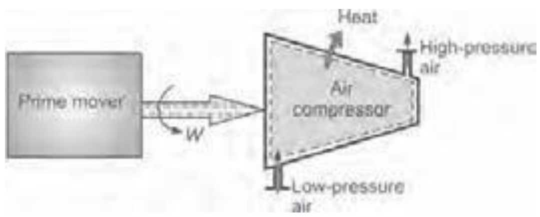


Fig. 25.1 *Schematic of an air compressor*

The schematic of an air compressor is shown in Fig. 25.1. The compressor recieves energy input from a prime mover (an engine or electric motor). Some part of this energy input is used to overcome the frictional effects, some part is lost in the form of heat and the remaining part is used to compress air to a high pressure.

25.1 USES OF COMPRESSED AIR

Compressed air has wide applications in industries as well as in commercial equipment. It is used in

1. Air refrigeration and cooling of large buildings,
2. Driving pneumatic tools in shops like drills, rivetters, screw drivers, etc.
3. Driving air motors in mines, where electric motors and IC engines cannot be used because of fire risks due to the presence of inflammable gases, etc.
4. Cleaning purposes,
5. Blast furnaces,
6. Spray painting and spraying fuel in Diesel engines,
7. Hard excavation work, tunneling, boring, mining, etc.
8. Starting of heavy-duty diesel engines,
9. Operating air brakes in buses, trucks and trains etc.
10. Inflating automobile and aircraft tyres,
11. Supercharging internal combustion engines,
12. Conveying solid and powder materials in pipelines,

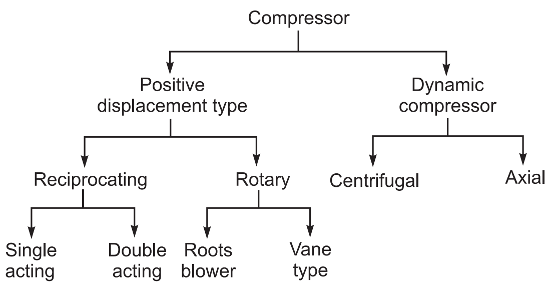
13. Process industries,
14. Operating lifts, hoists, crains and to operate pumps etc.
15. Pump sets for oil and gas transmission line,
16. Automobile suspension system.

25.2 CLASSIFICATION

The compressors are mainly classified as

(i) Reciprocating compressors, and
(ii) Rotary compressors.

The air compressors can broadly be classified as



A *reciprocating compressor* is used to produce high-pressure gas. It uses the displacement of piston in the cylinder for compression. It handles a low mass of gas and a high pressure ratio.

The *rotary compressors* are used for low and medium pressures. They usually consist of a bladed wheel or impeller that spins inside a circular housing. They handle a large mass of gas.

These compressors may be single stage or multistage to increase the pressure ratio.

25.3 RECIPROCATING COMPRESSOR TERMINOLOGY

In connection to reciprocating compressors, the following terms are defined:

1. Single-acting compressor is a compressor in which suction, compression and delivery of a gas take place only on one side of the piston during a cycle of one revolution of the crank shaft.

2. Double-acting compressor is a compressor in which suction, compression and delivery of gas take place on both sides of the piston and two cycles take place during one revolution of the crank shaft.

3. Single-stage compressor is a compressor in which the compression of gas to final delivery pressure is carried out in one cylinder only.

4. Multistage compressor is a compressor in which the compression of gas to the final pressure is carried out in more than one cylinder in series.

5. Pressure ratio is defined as the ratio of absolute discharge pressure to absolute suction pressure.

6. Free air is the air that exists under atmospheric condition.

7. Compressor displacement volume is the volume created when the piston travels a stroke. It is given as

$$V = \frac{\pi}{4} d^2 L \qquad ...(25.1)$$

where d is the bore of the cylinder and L the is stroke of the piston.

8. Induction-volume rate or volume-flow rate into the compressor is expressed in m^3/s and is given as

$\dot{V}$ = Volume inducted per cycle × No. of inductions per revolution × Number of revolutions per second

For the *single-acting reciprocating compressor*, only one cycle (thus, one induction) takes place for each revolution of the crank. Thus, for a compressor without clearance

$$\dot{V} = \frac{\pi}{4} d^2 L \frac{N}{60} \qquad ...(25.2)$$

For the *double-acting reciprocating compressor*, the induction takes place on both sides of the piston for each revolution. Thus,

$$\dot{V} = \frac{\pi}{4} d^2 L \left(\frac{2N}{60}\right) \qquad ...(25.3)$$

9. Capacity of a compressor is the actual quantity of air delivered per unit time at atmospheric conditions.

***10. Free Air delivery* (*FAD*)** It is the discharge volume of the compressor corresponding to ambient conditions.

11. Piston speed is the linear speed of the piston measured in m/min. It is expressed as

$$\mathcal{V}_{piston} = 2LN \qquad ...(25.4)$$

25.4 COMPRESSED AIR SYSTEMS

Compressed air systems consist: intake air filters, inter-stage coolers, after coolers, air dryers, moisture drain traps, receivers, piping network, control valves and lubricators.

1. Intake Air Filters They prevent dust from entering the compressor. Dust causes sticking of valves, scoured cylinders, excessive wear, etc.

2. Inter-stage Coolers These are placed between consecutive stages of multistage compressor. They reduce the temperature of compressed air, before it enters the next stage of compression.

3. After Coolers They remove heat of compression and moisture in the air by reducing the temperature in a water-cooled heat exchanger, after compression is completed.

4. Air-dryers The remaining traces of moisture, after an after-cooler are removed by using air dryers, for using compressed air in instruments and pneumatic equipment. The moisture is removed by using adsorbents like silica gel or activated carbon, or refrigerant dryers, or heat of compression dryers.

5. Moisture Drain Traps Moisture drain traps are used for removal of moisture in the compressed air. These traps are manual drain cocks, timer based/automatic drain valves, etc.

6. Air Receivers are cylindrical tanks into which the compressed air is discharged after final stage of compression from the air compressor. Receiver acts as storage tank and it helps to reduce pulsations and pressure variations from the compressed in the discharge line.

25.5 RECIPROCATING AIR COMPRESSOR

A machine which takes in air or gas during suction stroke at low pressure and then compresses it to high pressure in a piston–cylinder arrangement is known as a reciprocating compressor. External work must be supplied to the compressor to achieve required compression. This work is used to run the compressor. A part of the work supplied to the compressor is lost to overcome the frictional resistance between rubbing surfaces of the piston and cylinder. The cylinder of air compressor is cooled to minimise the work input.

The air compressed by a reciprocating compressor cannot directly be used for an application. The reciprocating motion of the piston gives rise to pulsating flow through the discharge valve of the compressor. Thus, the compressed air is discharged from the air compressor to an air receiver.

25.5.1 Construction

Figure 25.2 shows the sectional view of a single-stage air compressor. It consists of a piston, cylinder with cooling arrangement, connecting rod, crank, inlet and delivery valves. The piston fitted with piston rings, reciprocates in the cylinder. The prime mover (an engine or electric motor) drives the crank shaft, the crank rotates and converts rotary motion into reciprocating motion of piston

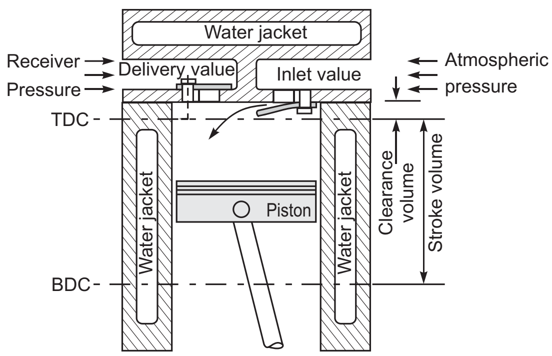


Fig. 25.2 *Sectional view of single-stage reciprocating air compressor*

with the help of a connecting rod. The cylinder head consists of spring-loaded inlet and delivery valves, which are operated by a small pressure difference across them. The light spring pressure gives a rapid closing action. The piston rings seal the gap between the piston and cylinder wall. The cylinder is surrounded by a water jacket or metallic fins for proper cooling of air during compression.

The double-acting air compressor is shown in Fig. 25.3. Its construction is very similar to that of a single-acting air compressor, except for two inlet and two delivery valves on two ends of the cylinder in order to allow air entry and delivery on two sides of the piston. When the piston compresses the air on its one side, it creates suction on the other side. Thus, the suction and compression of air take place on two sides of the piston simultaneously.

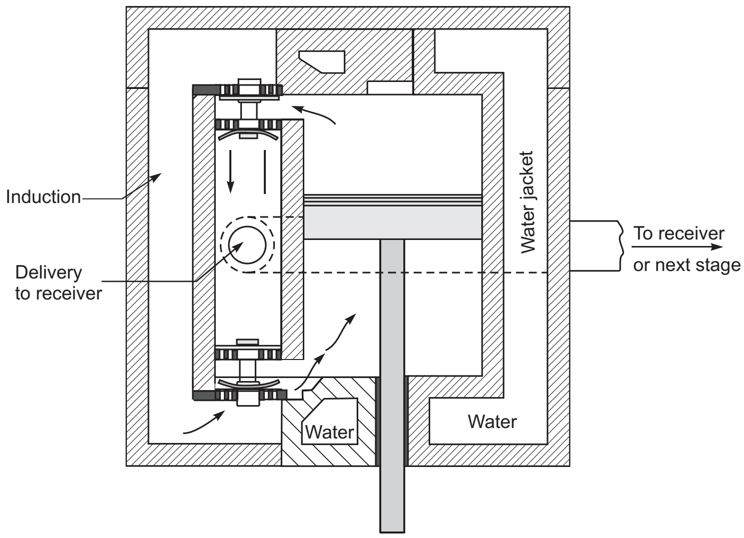


Fig. 25.3 *Double-acting reciprocating air compressor*

25.5.2 Working of a Single-Acting Air Compressor

As the piston moves in a downward stroke (from TDC to BDC), any residual compressed air left in the cylinder from the previous cycle expands first. On further movement of the piston, the pressure in the cylinder falls below the atmospheric pressure. The atmospheric air pushes the inlet valve to open and fresh air enters the cylinder as shown in Fig. 25.4. The line c-1 represents the induction stroke. During this stroke, the compressed air in the storage tank acts on the delivery valve, thus it remains closed. As the piston begins its return stroke from BDC to TDC, the pressure in the cylinder increases, and closes the inlet valve. The air in the cylinder is compressed by piston as shown by the curve 1-2.

During the compression stroke, as air pressure reaches a value, which is slightly more than the pressure of compressed air acting outside the delivery valve, the delivery valve opens and the compressed air is discharged from the cylinder to storage tank. At the end of the compression stroke, the piston once again moves downward, the pressure in the cylinder falls below the atmospheric pressure, the delivery valve closes and inlet valve opens for next cycle. The suction, compression and delivery of air take place with two strokes of the piston which is one revolution of the crank.

Figure 25.4 shows the p–V diagram for a reciprocating compressor without clearance. The processes are summarized below:

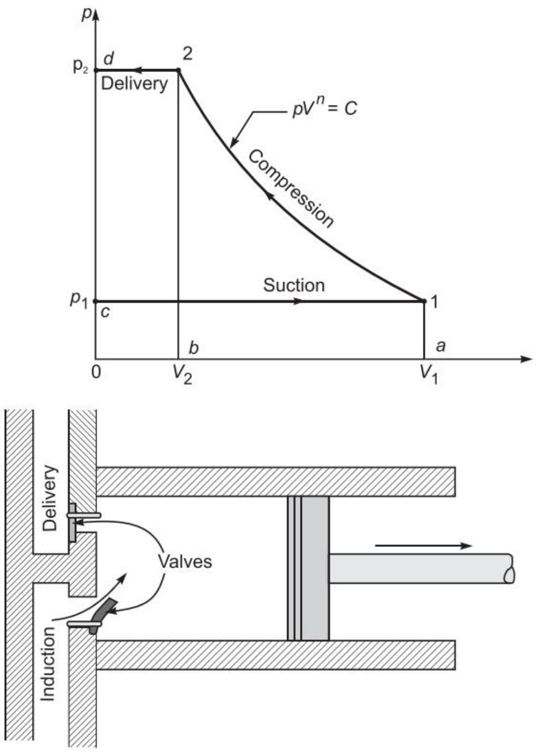


Fig. 25.4 *p–V diagram for a reciprocating compressor without clearance*

Process c–1 Suction stroke—inlet valve opens and air enters the compressor at constant pressure p_1

Process 1–2 Polytropic compression of air from pressure p_1 to pressure p_2

Process 2–d Discharge of compressed air through delivery valve at const. pressure p_2

Process d–c No air in the cylinder and return of piston for suction stroke

25.5.3 Indicated Work for a Single-acting Compressor without Clearance

The theoretical p–V diagram for single-stage, single-acting reciprocating air compressor without clearance is shown in Fig. 25.4. The net work done in the cycle is equal to the area behind the curve on p–V diagram and it is the work done on air.

Indicated work done on the air per cycle

= Area behind the curve, i.e., area c–1–2–d–c

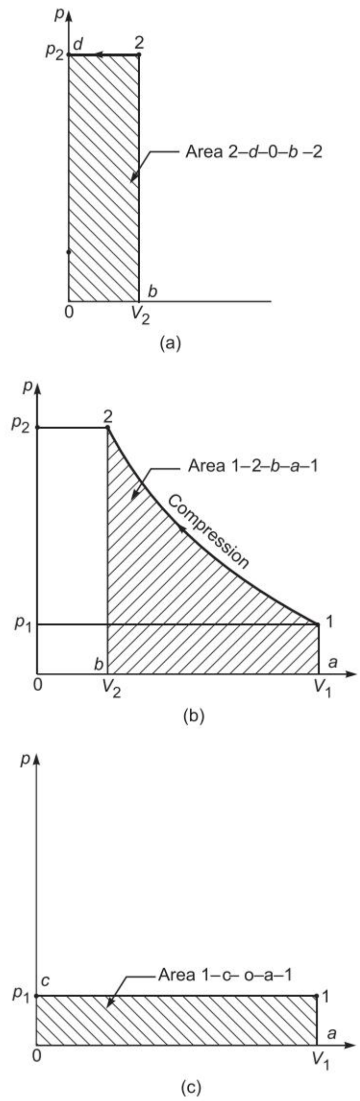


Fig. 25.5 *Three process areas on p–V diagram*

= Area 2–d–0–b–2 + Area 1–2–b–a–1 – Area 1–c–0–a–1

These three areas are shown in Fig. 25.5 as (a), (b) and (c), respectively.

Area 2–d–0–b–2 = p_2V_2 (Flow work during discharge at constant pressure p_2)

Area 1–2–b–a–1 = $-\int pdV$ (Piston dispalcement work from p_1 to p_2, –

sign is taken for compression)

Area $1\text{–}c-0-a-1 = p_1V_1$ (Flow work during suction at constant pressure p_1)

During compression process 1–2; the pressure and volume are related as

$$pV^n = \text{C (constant)}$$

Thus we get $-\int p\,dV = \dfrac{p_2V_2 - p_1V_1}{n-1}$

Therefore, the total indicated work input to compressor is

$$W_{in} = p_2V_2 + \frac{p_2V_2 - p_1V_1}{n-1} - p_1V_1 \qquad \text{...(25.5)}$$

$$= (p_2V_2 - p_1V_1)\left[\frac{1}{n-1} - 1\right]$$

$$W_{in} = \frac{n}{n-1}(p_2V_2 - p_1V_1) \text{ (kJ/cycle)} \qquad \text{...(25.6)}$$

Using charecteristic gas equation as

$$pV = m_a RT$$

Equation (25.6) can be modified as

$$W_{in} = \frac{n}{n-1} m_a R(T_2 - T_1) \text{ (kJ/cycle)} \qquad \text{...(25.7)}$$

Other expression for indicated work can be derived by arranging Eq. (25.7) as

$$W_{in} = \frac{n}{n-1} m_a RT_1\left[\frac{T_2}{T_1} - 1\right]$$

It is convenient to express the temperature T_2 in terms of delivery and intake pressure ratio.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Then $$W_{in} = \frac{n}{n-1} m_a RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{ (kJ/cycle)} \qquad \text{...(25.8)}$$

or $$W_{in} = \frac{n}{n-1} p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{(kJ/cycle)} \qquad \text{...(25.9)}$$

where V_1 is the volume inducted per cycle.

25.5.4 Mean Effective Pressure (p_m)

It is a hypothetical average pressure, which if acted on the piston during the entire compression stroke will require the same power input as required during the actual cycle.

Net work input in a cycle,

$$W_{in} = p_m \times \text{(Swept volume)}$$

$$= p_m \times V_s$$

Thus $$p_m = \frac{W_{net}}{V_s} = \frac{\text{Work output}}{\text{Swept volume}} \qquad \text{...(25.10)}$$

From a given indicator diagram the indicated mean effective pressure can be obtained as

$$p_m = \frac{\text{Area of indicator diagram (mm}^2)}{\text{Length of the indicator diagram (mm)}} \times \text{Spring constant (kPa/mm)} \qquad \text{...(25.11)}$$

25.5.5 Power and Mechanical Efficiency

***1. Indicated Power* (*IP*)** The work done on air per unit time is called *indicated power input to the compressor.* The power required by an air compressor, running at N rpm is given as

Indicated power IP

= Work input per cycle × No. of cycles per unit time

or $$IP = \frac{W_{in}\,N\,k}{60} \text{ (kW)} \qquad \text{...(25.12)}$$

From an indicated diagram, It is calculated as

IP = Indicated mean effective pressure × Swept volume rate

$$= \frac{p_{mi}\,L\,A\,N\,k}{60} \text{ (kW)} \qquad \text{...(25.13)}$$

where for Eq. (25.12) and Eq. (25.13);

W_{in} = Indicated work input per cycle

p_{mi} = Indicated mean effective pressure, (kPa or kN/m^2)

L = Stroke length, (m)

A = $(\pi/4)d^2$, cross-sectional area of cylinder of bore, d, (m)

N = number of rotation per minute

k = number of suction per revolution of crank shaft

= 1 for single-acting reciprocating compressor

= 2 for double-acting reciprocating compressor

***2. Brake Power* (*BP*)** The actual power (brake power or shaft power) input to the compressor is more than the indicated power because some work is required to overcome the irreversibilities and mechanical frictional effects.

Brake power;

$$BP = \text{Indicated power + Frictional power} \quad ...(25.14)$$

3. Mechanical Efficiency η_{mech} The mechanical efficiency of the compressor is given by

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Brake power}} \quad ...(25.15)$$

The brake power is derived from a driving motor or engine. The input of a driving motor can be expressed as

$$\text{Motor power} = \frac{\text{Shaft power (or brake power)}}{\text{Mechanical efficiency of motor and drive}} \quad ...(25.16)$$

Example 25.1 *A single-stage reciprocating air compressor takes in 1.4 kg of air per minute at 1 bar and 17°C and delivers it at 6 bar. Assuming compression process follows the law $pV^{1.35}$ = constant, calculate indicated power input to compressor.*

Solution

Given A single-stage reciprocating air compressor

$\dot{m}_a$ = 1.4 kg/min p_1 = 1 bar

T_1 = 17°C = 290 K p_2 = 6 bar

n = 1.35 bar

Law $pV^{1.35}$ = C

To find Indicator power input to compressor.

Assumptions

(i) Negligible clearance volume in the compressor.

(ii) No throttling effects on valve opening and closing.

(iii) Air as an ideal gas with R = 0.287 kJ/kg·K.

Analysis The delivery temperature of air

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (290\text{ K})\times\left(\frac{6}{1}\right)^{\frac{1.35-1}{1.35}}$$

$$= 461.46\text{ K}$$

The rate of work input to compressor, Eq. (25.7)

$$W_{in} = \frac{n}{n-1}\,\dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.35}{1.35-1}\times(1.4\text{ kg/min})\times(0.287\text{ kJ/kg}\cdot\text{K})\times(461.46 - 290)\text{ (K)}$$

$$= 265.72\text{ kJ/min}$$

Indicated power input;

$$IP = \frac{W_{in}}{60} = \frac{(265.72\text{ kJ/min})}{(60\text{ s/min})} = \mathbf{4.43\text{ kW}}$$

Note: The indicated work input to compressor can also be calculated by using Eq. (25.8).

Example 25.2 *A single-acting, single-cylinder reciprocating air compressor has a cylinder diameter of 200 mm and a stroke of 300 mm. Air enters the cylinder at 1 bar; 27°C. It is then compressed polytropically to 8 bar according to the law $pV^{1.3}$= constant. If the speed of the compressor is 250 rpm, calculate the mass of air compressed per minute, and the power required in kW for driving the compressor.*

Solution

Given A single-acting, single-cylinder reciprocating air compressor

d = 200 mm = 0.2 m L = 300 mm = 0.3 m

p_1 = 1 bar = 100 kPa p_2 = 8 bar

N = 250 rpm T_1 = 27°C = 300 K

n = 1.3

To find

(i) The mass of air compressed in kg/min,

(ii) Power input to compressor in kW.

Assumptions

(i) Negligible clearance volume in the cylinder.

(ii) Air as an ideal gas with R = 0.287 kJ/kg·K.

Analysis The swept volume of the cylinder per cycle

$$V_s = V_1 = \left(\frac{\pi}{4}\right)d^2 L$$

$$= \left(\frac{\pi}{4}\right)\times(0.2\text{ m})^2\times(0.3\text{ m})$$

$$= 9.424\times10^{-3}\text{ m}^3$$

The mass of air, using perfect gas equation

$$m_a = \frac{p_1V_1}{RT_1} = \frac{(100\text{ kPa})\times(9.424\times10^{-3}\text{ m}^3)}{(0.287\text{ kJ/kg}\cdot\text{K})\times(300\text{ K})}$$

$$= 0.0109\text{ kg/cycle}$$

The mass flow rate of air;

$\dot{m}_a$ = mass of air × number of suction/min = $m_a N$

$= 0.0109 \times 250 =$ **2.74 kg/min**

Temperature of air after compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= (300\,\text{K}) \times \left(\frac{8}{1}\right)^{\frac{1.3-1}{1.3}} = 484.75\ \text{K}$$

The work input to compressor, Eq. (25.7)

$$W_{in} = \frac{n}{n-1} m_a R (T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times (2.74\ \text{kg/min}) \times (0.287\ \text{kJ/kg}\cdot\text{K}) \times (484.75 - 300)\ (\text{K})$$

$= 629.56$ kJ/min or **10.49 kW**

Example 25.3 *A single-acting, single-cylinder reciprocating air compressor is compressing 20 kg/min. of air from 110 kPa, 30°C to 600 kPa and delivers it to a receiver. Law of compression is $pV^{1.25}$ = constant. Mechanical efficiency is 80%. Find the power input to compressor, neglecting losses due to clearance, leakages and cooling.*

Solution

Given A single-stage reciprocating air compressor

$\dot{m}_a = 20$ kg/min $p_1 = 110$ kPa

$T_1 = 30°\text{C} = 303$ K $p_2 = 600$ kPa

Law $pV^{1.25} = C$

$\eta_{mech} = 0.8$

To find Power input to compressor.

Assumptions

(i) Negligible clearance volume in the compressor.

(ii) No throttling effects on valve opening and closing.

(iii) Air as an ideal gas with $R = 0.287$ kJ/kg · K.

Analysis The delivery temperature of air

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (303\ \text{K}) \times \left(\frac{600}{110}\right)^{\frac{1.25-1}{1.25}}$$

$= 425.4$ K

The indicated power input to compressor,

$$IP = \frac{n}{n-1}\dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.25}{1.25-1} \times \left(\frac{20}{60}\ \text{kg/s}\right) \times (0.287\ \text{kJ/kg}\cdot\text{K}) \times (425.4 - 303)\ (\text{K})$$

$= 58.55$ kW

The motor (brake) power

$$BP = \frac{IP}{\eta_{mech}} = \frac{58.55\ \text{kW}}{0.8} = \mathbf{73.18\ kW}$$

Example 25.4 *A single-cylinder, double-acting, reciprocating air compressor receives air at 1 bar; 17°C, compresses it to 6 bar according to the law $pV^{1.25}$= constant. The cylinder diameter is 300 mm. The average piston speed is 150 m/min at 100 rpm. Calculate the power required in kW for driving the compressor. Neglect clearance.*

Solution

Given A double-acting, single-cylinder reciprocating air compressor

$d = 300$ mm $= 0.3$ m $p_1 = 1$ bar $= 100$ kPa

$p_2 = 6$ bar $N = 100$ rpm

$T_1 = 17°\text{C} = 290$ K $n = 1.25$

$k = 2$ $\mathcal{V}_{piston} = 150$ m/min

To find Power input to compressor in kW.

Analysis The piston speed is given as

$\mathcal{V}_{piston} = 2LN$

Stroke of piston;

$$L = \frac{\mathcal{V}_{piston}}{2N} = \frac{150\ \text{m/min}}{2 \times (100\ \text{rotation/min})}$$

$= 0.75$ m

The swept volume of the cylinder per cycle

$$V_s = V_1 = \left(\frac{\pi}{4}\right) d^2 L = \frac{\pi}{4} \times (0.3\ \text{m})^2 \times (0.7\ 5\text{m})$$

$= 0.053\ \text{m}^3$

The indicated work input to compressor by Eq. (25.9)

$$W_{in} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.25}{1.25-1} \times 100 \times 0.053 \times \left[\left(\frac{6}{1}\right)^{\frac{1.25-1}{1.25}} - 1\right]$$

$= 11.42$ kJ/cycle

For a double-acting reciprocating compressor, the indicated power

$$IP = \frac{W_{in} N k}{60} \text{ (kW)}$$

$$= \frac{11.42 \times 100 \times (2 \text{ for double acting})}{60}$$

$$= \mathbf{38.1 \text{ kW}}$$

Example 25.5 *A single-stage, single-acting, reciprocating air compressor takes in 1 m³ air per minute at 1 bar and 17°C and delivers it at 7 bar. The compressor runs at 300 rpm and follows the law $pV^{1.35}$= constant. Calculate the cylinder bore and stroke required, assuming stroke-to-bore ratio of 1.5. Calculate the power of the motor required to drive the compressor, if the mechanical efficiency of the compressor is 85% and that of motor transmissions is 90 %. Neglect clearance volume and take R = 0.287 kJ/kg·K for air.*

Solution

Given A single-stage, single-acting, reciprocating air compressor

$\dot{V}$ = 1 m³/min $\quad p_1$ = 1 bar = 100 kPa

T_1 = 17°C = 290 K $\quad p_2$ = 7 bar

N = 300 rpm $\quad n$ = 1.35

$\eta_{transmission}$ = 0.9 $\quad \eta_{mech}$ = 0.85

L/d = 1.5 $\quad R$ = 0.287 kJ/kg·K

To find

(i) Cylinder bore, and strokes,

(ii) Motor power.

Analysis Volume sucked in per cycle

$$V_s = \frac{\dot{V}}{N k} = \frac{1 \text{ m}^3/\text{min}}{(300 \text{ rpm}) \times 1} = \frac{1}{300} \text{m}^3$$

The cylinder (swept) volume also given as,

$$V_s = \frac{\pi}{4} d^2 L = \frac{\pi}{4} d^2 (1.5d) = 1.5 \times \left(\frac{\pi}{4}\right) d^3$$

Equating two equations

$$1.5 \times \left(\frac{\pi}{4}\right) d^3 = \frac{1}{300}$$

We get

cylinder bore,

$$d = 0.1414 \text{ m} = \mathbf{141.4 \text{ mm}}$$

and stroke;

$$L = 1.5\, d = \mathbf{212.10 \text{ mm}}$$

The mass flow rate of air per minute,

$$\dot{m}_a = \frac{p_1 \dot{V}}{R T_1} = \frac{(100 \text{ kPa}) \times (1 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (290 \text{ K})}$$

$$= 1.2 \text{ kg/min}$$

The temperature of air after compression

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (290 \text{ K}) \times \left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}}$$

$$= 480.28 \text{ K}$$

The rate of work input to compressor, Eq. (25.7)

$$\dot{W} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.35}{1.35 - 1} \times 1.2 \times 0.287 \times 480.28 - 290$$

$$= 252.77 \text{ kJ/min}$$

Indicated power required;

$$IP = \frac{(252.77 \text{ kJ/min})}{(60 \text{ s/min})} = \mathbf{4.21 \text{ kW}}$$

The brake power input to compressor;

$$\text{Brake power} = \frac{IP}{\eta_{mech}} = \frac{(4.21 \text{ kW})}{0.85} = 4.956 \text{ kW}$$

The motor power required;

$$\text{Motor power} = \frac{\text{Brake power}}{\eta_{transmission}} = \frac{(4.956 \text{ kW})}{0.9}$$

$$= \mathbf{5.5 \text{ kW}}$$

25.6 MINIMIZING COMPRESSION WORK

The work done on the gas for compression can be minimized when the compression process is executed in an internally reversible manner, i.e., by minimizing the irreversibilities. The other way of reducing the compression work is to keep the specific volume of gas as small as possible during compression process. It is achieved by keeping the gas temperature as low as possible during the compression. Since specific volume of gas is proportional to temperature, therefore, the cooling arrangement is provided on the compressor to cool the gas during the compression.

For better understanding of the effect of cooling during compression process, we consider three types of compression processes executed between same pressure levels (p_1 and p_2); an isentropic compression 1–2″ (involves no cooling), a polytropic compression 1–2 (involves partial cooling) and an isothermal compression 1–2′ (involves perfect cooling) as shown in Fig. 25.6.

(a) The indicated compression work per cycle for a polytropic compression process 1–2 is given by Eq. (25.9)

$$W_{Poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

(b) Isentropic compression process 1–2″

An equation for indicated work input can be obtained as Eq. (25.9) by replacing n by γ. That is,

$$W_{isentropic} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad ...(25.17)$$

(c) Isothermal compression process 1–2′: With perfect cooling ($T_2 = T_1$);

Indicated work input for isothermal compression is given by area c–1–2′–d–c.

Area c–1–2′–d–c = Area a–1–2′–b + Area b–2′–d–0–Area a–1–c–0

$$W_{iso} = -\int_{V_1}^{V_2} pdV + p_2 V_2 - p_1 V_1$$

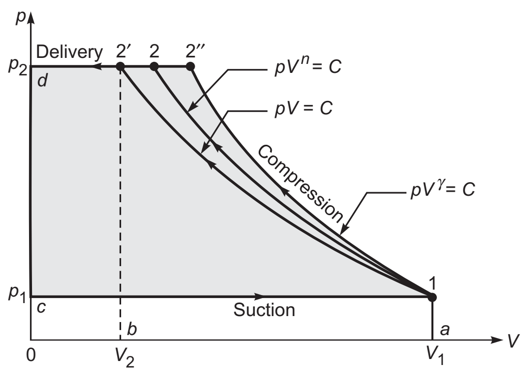


Fig. 25.6 *Three types of compression processes on a p–V diagram*

For an isothermal process; using

$$p = \frac{C}{V} \quad (\text{Since } pV = C)$$

$$\text{We get} - \int_{V_1}^{V_2} pdV = p_1 V_1 \ln\left(\frac{V_1}{V_2}\right)$$

For isothermal process;

$$p_2 V_2 = p_1 V_1 \text{ and } \frac{V_1}{V_2} = \frac{p_2}{p_1}$$

$$\therefore \qquad W_{iso} = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) \qquad ...(25.18)$$

where V_1 is the volume of the air inducted per cycle.

The three processes are plotted on a p–V diagram in Fig. 25.6 for same inlet state and exit pressure. The area of the indicator diagram is the measure of compression work. The only type of compression can influence the magnitude of area of indicator diagram and length of the line 2–d.

It is interesting to observe from this diagram that among the three processes considered, the area with isentropic compression is *maximum.* Thus it requires maximum work input and with isothermal compression, the area of indicator diagram is *minimum.* Thus, the *compressor with isothermal compression will require minimum* work input.

25.6.1 Adiabatic Efficiency

This term is seldom used in practice for reciprocating compressors. The adiabatic efficiency of an air compressor is defined as the ratio of isentropic work input to actual work input.

$$\eta_{adiabatic} = \frac{\text{Isentropic work input}}{\text{Actual work input}} \qquad ...(25.19)$$

25.6.2 Compressor Efficiency

It compares the indicated work input to isothermal work input to the compressor and it is defined as ratio of isothermal work input to indicated work input

$$\eta_{comp} = \frac{\text{Isothermal work input}}{\text{Indicated work input}} \qquad ...(25.20)$$

25.6.3 Isothermal Efficiency

It compares the actual work done on the gas with isothermal compression work, and is defined as the ratio of isothermal work input to actual work input during compression, i.e.,

$$\eta_{iso} = \frac{\text{Isothermal work input}}{\text{Actual work input}} \quad ...(25.21)$$

25.6.4 Methods for Improving Isothermal Efficiency

As illustrated with the help of Fig. 25.6, the compression of gas in isothermal manner requires minimum work input. With isothermal compression, the temperature remains constant throughout the compression process.

$$T_2 = T_1$$

Isothermal compression is only possible when all the heat generated during compression is dissipated to cooling medium around the cylinder wall. It is possible, when the compressor runs very slowly.

In actual practice, the compression process should approach isothermal compression even with high-speed compressors. The various methods are adopted to reduce the temperature of gas during the compression and keep it more closely to isothermal compression.

1. Water Spray It was an old method in which water was injected into the cylinder during compression of air to keep the temperature of air constant. But this method has certain disvantages, and thus became obsolete.

2. Water Jacketing It is commonly and successfully used practice for all types of reciprocating compressors. The water is circulated around the cylinder through the water jacket which helps to cool the air during compression.

3. External Fins For a small-capacity compressor, the effective cooling can be achieved by attaching fins of conducting material around the cylinder. The fins increase surface area of the cylinder for heat transfer.

4. Inter-cooler If the very high pressure ratio is required then air is compressed in stages. The intercooler is used between two stages of compression for cooling of compressed air after one stage, before entering the next stage. The water jackets are also used around the cylinder of compressor of each stage.

5. By Suitable Cylinder Proportions If the compressor has large surface to volume ratio, the greater surface area will be available for heat transfer and cooling will be more effective.

It is possible by choosing a cylinder with large bore and short piston stroke. The large cylinder head dissipates heat in a much more effective way, which contains hottest compressed air all the time.

Example 25.6 *A single-acting, single-stage reciprocating air compressor of 250–mm bore and 350–mm stroke runs at 200 rpm The suction and delivery pressures are 1 bar and 6 bar, respectively. Calculate the theoretical power required to run the compressor under each of the following conditions of compression:*

(a) isothermal,
(b) polytropic n = 1.3, and
(c) isentropic, γ = 1.4.

Neglect the effect of clearance and also calculate isothermal efficiency in each of the above cases.

Solution

Given A single-acting, single-stage reciprocating air compressor:

p_1 = 1 bar = 100 kPa p_2 = 6 bar
d = 250 mm = 0.25 m L = 350 mm
N = 200 rpm

and three types of compression with n = 1, 1.3 and 1.4

To find

(i) Theoretical power required to run the compressor for
  (a) Isothermal compression,
  (b) Polytropic compression,
  (c) Isentropic compression.
(ii) Isothermal efficiency in each case.

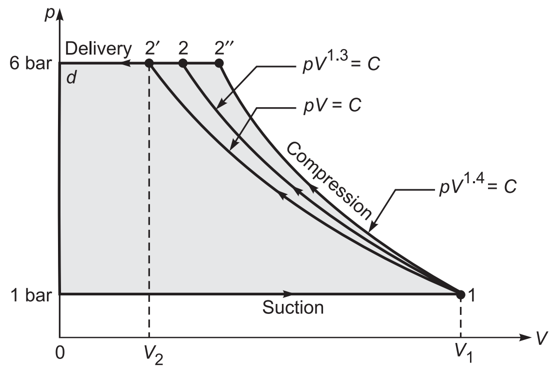


Fig. 25.7

Analysis The cylinder volume (without clearance)

$$V_1 = V_s = \left(\frac{\pi}{4}\right)d^2L = \left(\frac{\pi}{4}\right) \times (0.25 \text{ m})^2 \times (0.35 \text{ m})$$

$$= 0.0171 \text{ m}^3$$

The volume flow rate of air

$$\dot{V}_1 = \text{Cylinder volume} \times \text{No of suctions per second}$$

$$= 0.0171 \times \frac{200}{60} = 0.0572 \text{ m}^3/\text{s}$$

(i) *Theoretical power required*

(a) For isothermal compression,

$$\text{Power } \dot{W}_{iso} = p_1\dot{V}_1 \ln\left(\frac{p_2}{p_1}\right)$$

$$= 100 \times 0.0572 \times \ln\left(\frac{6}{1}\right)$$

$$= 10.261 \text{ kJ/s or } \mathbf{10.261\ kW}$$

(b) Polytropic compression,

$$\dot{W}_{Poly} = \frac{n}{n-1}p_1\dot{V}_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{1.3}{1.3-1}\right) \times 100 \times 0.0572 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{12.7\ kW}$$

(c) Isentropic compression,

$$\dot{W}_{isentropic} = \frac{\gamma}{\gamma-1}p_1\dot{V}_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= \left(\frac{1.4}{1.4-1}\right) \times 100 \times 0.0572 \times \left[\left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= \mathbf{13.38\ kW}$$

(ii) *Isothermal efficiency*

$$\eta_{iso} = \frac{\text{Isothermal work}}{\text{Actual work}} \times 100$$

(a) For isothermal compression,

$$\eta_{iso} = \mathbf{100\%}$$

(b) For polytropic compression,

$$\eta_{iso} = \frac{10.261}{12.7} \times 100 = \mathbf{80.8\%}$$

(c) For isentropic compression,

$$\eta_{iso} = \frac{10.261}{13.38} \times 100 = \mathbf{76.67\%}$$

25.7 CLEARANCE VOLUME IN A COMPRESSOR

The *clearance volume* is the space left in the cylinder when the piston reaches its topmost position, i.e., TDC. It is provided

(i) to avoid the piston striking the cylinder head, and
(ii) to accommodate the valve's actuation inside the cylinder, because suction and delivery valves are located in the clearance volume.

A compressor should have the smallest possible clearance volume, because the compressed air left in the clearance volume, first re-expands in the cylinder during suction, thus reducing suction capacity.

The ratio of clearance volume to swept volume is defined as the *clearance ratio* or *percentage clearance*. The value of clearance ratio may vary from 2 to 10%.

25.7.1 Effects of Clearance Volume

1. The volume of air taken in per stroke is less than the swept volume, thus the volumetric efficiency decreases.
2. More power input is required to drive the compressor for same pressure ratio, due to increase in volume to be handled.
3. The maximum compression pressure is controlled by the clearance volume.

25.7.2 Indicated Compression Work with Clearance

The clearance volume is generally kept very small. The work done on the air in the clearance space during compression stroke is approximately equal to the work done by the air when it re-expands during suction stroke. Therefore, the work of compression is not affected by clearance space in the compressor. But the mass of air inducted is reduced, and thus the volumetric efficiency of the compressor will be less.

Figure 25.8 shows an indicator diagram for a reciprocating air compressor with clearance. After delivery of compressed air, the air remaining in the clearance volume at pressure p_2 expands, when the piston proceeds for the next suction stroke. As soon as the pressure p_1 reaches at the state 4, the induction of fresh charge starts and continues to the end of the stroke at state 1.

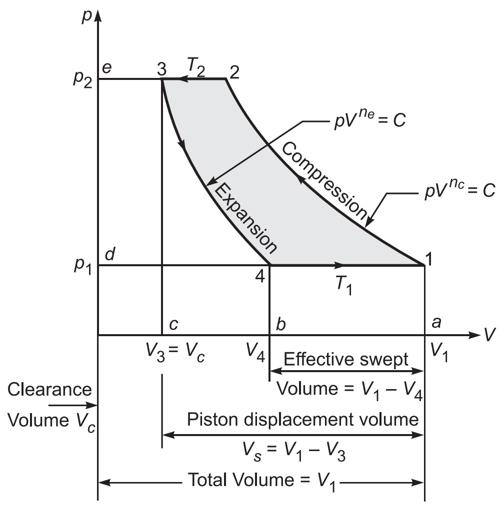


Fig. 25.8 *Indicator diagram for reciprocating air compressor with clearance*

The indicated work done is given by the area 1–2–3–4–1 on a p–V diagram.

Indicated work

= Area 1–2–3–4–1

= Area 1–2–e–d – Area 3–e–d–4

Similar to derivation of Eq. (25.9), the compression work equivalent to area 1–2–e–d, can be obtained with index of compression n_c.

Area 1–2–e–d;

$$W_{Comp} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] \quad \text{...(25.22a)}$$

The work done by gas during expansion (index n_e) can be also obtained as above

Area 3–e–d–4;

$$W_{Expan} = \frac{n_e}{n_e - 1} p_4 V_4 \left[\left(\frac{p_3}{p_4} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

$$= \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] \quad \text{...(25.22b)}$$

Since $p_4 = p_1$ and $p_3 = p_2$

Net work of compression;

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] \quad \text{...(25.23)}$$

If indices of compression and expansion are same, i.e., $n_c = n_e = n$, then

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{...(25.24)}$$

Example 25.7 *An ideal single-stage, single-acting reciprocating air compressor has a displacement volume of 14 litre and a clearance volume of 0.7 litre. It receives the air at a pressure of 1 bar and delivers it at a pressure of 7 bar. The compression is polytropic with an index of 1.3 and re-expansion is isentropic with an index of 1.4. Calculate the net indicated work of a cycle.*

Solution

Given A single-stage, single-acting reciprocating air compressor.

$p_1 = 1$ bar = 100 kPa $\quad p_2 = 7$ bar
$V_s = 14$ litre $\quad V_c = 0.7$ litre
$n_c = 1.3 \quad n_e = 1.4$

To find Indicated work input per cycle.

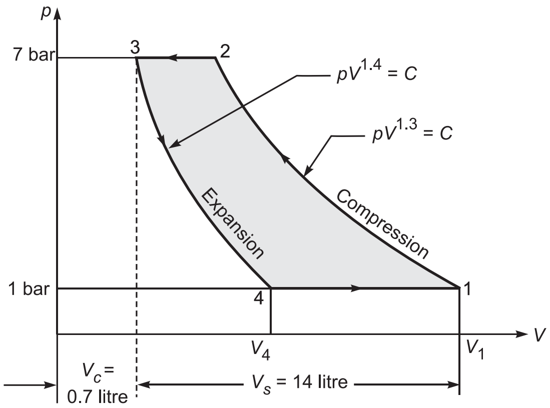


Fig. 25.9

Analysis The total volume of cylinder;

$$V_1 = V_s + V_c$$
$$= 14 + 0.7 = 14.7 \text{ litre} \quad \text{or} \quad 0.0147 \text{ m}^3$$

The volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} = 0.7 \times \left(\frac{7}{1}\right)^{\frac{1}{1.4}}$$
$$= 2.81 \text{ litre} \quad \text{or} = 0.00281 \text{ m}^3$$

Indicated work input per cycle, Eq. (25.23)

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_c-1}{n_c}} - 1\right] - \frac{n_e}{1 - n_e} p_1 V_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_e-1}{n_e}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 100 \times 0.0147 \times \left[\left(\frac{7}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] - \frac{1.4}{1.4-1} \times 100 \times 0.00281 \times \left[\left(\frac{7}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= 3.61 - 0.731 = \mathbf{2.88\ kJ/cycle}$$

Example 25.8 *A single-stage, double-acting reciprocating air compressor takes in 14 m³ of air per minute measured at 1.013 bar and 15°C. The delivery pressure is 7 bar and the compressor speed is 300 rpm. The compressor has a clearance volume of 5% of swept volume with a compression and re-expansion index of n = 1.3. Calculate the swept volume of the cylinder, the delivery temperature and the indicated power.*

Solution

Given A single-stage, double-acting reciprocating air compressor, with

$\dot{V}_1 = 14$ m³/min $\quad T_1 = 15°\text{C} = 288$ K
$N = 300$ rpm $\quad n = 1.3$
$p_1 = 1.03$ bar = 101.3 kPa $\quad p_2 = 7$ bar
$V_c = 0.05\,V_s$

To find

(i) The swept volume of cylinder,
(ii) The delivery temperature, and
(iii) Indicated power.

Assumptions

(i) No throttling effect on valve opening and closing.
(ii) Effect of piston rod on underside of cylinder is negligible.
(iii) Air as an ideal gas with specific gas constant R = 0.287 kJ/kg·K.

Analysis The p–V diagram for given data of compressor is shown in Fig. 25.10.

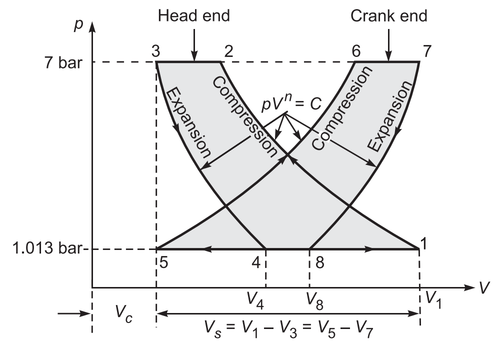


Fig. 25.10 *p–V diagram for double-acting reciprocating air compressor*

(i) *Swept volume of cylinder*

The total volume of cylinder,

$$V_1 = V_s + V_c = V_s + 0.05\,V_s = 1.05\,V_s$$

The volume inducted per cycle

$$V_1 - V_4 = \frac{\text{Volume induction per minute}}{\text{No. of suction per revolution} \times \text{No. of revolution per minute}}$$

$$= \frac{14\ \text{m}^3/\text{min}}{2\ \text{suction per rev.} \times 300\ \text{rev./min}}$$

$$= 0.0233\ \text{m}^3/\text{cycle}$$

The volume V_4 after re-expansion of compressed air.

$$V_4 = V_3\left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = V_c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 0.05\,V_s \times \left(\frac{7}{1.013}\right)^{\frac{1}{1.3}}$$

$$= 0.221\,V_s$$

Then $V_1 - V_4$

$$= 1.05\,V_s - 0.221\,V_s = 0.829\,V_s$$

or $0.829\,V_s = 0.0233\ \text{m}^3/\text{cycle}$

$\therefore$ swept volume,

$$V_s = \frac{0.0233}{0.829} = \mathbf{0.0281\ m^3/cycle}$$

The swept volume of the cylinder is 0.0281 m^3.

(ii) *The delivery temperature of air*

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= 288 \times \left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}}$$

$$= 450\ \text{K} = \mathbf{177°C}$$

(iii) *Indicated power*

$$IP = \frac{n}{n-1}\,p_1\dot{V}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times (101.3\ \text{kPa}) \times \left(\frac{14}{60}\ \text{kg/s}\right)\left[\left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{57.58\ kW}$$

25.8 ACTUAL INDICATOR DIAGRAM

The actual indicator diagram on a p–V plane for a single-stage reciprocating air compressor is shown in Fig. 25.11. It is similar to theoretical one (Fig. 25.8) except for induction and delivery processes. The variation during these processes is due to valve action effects.

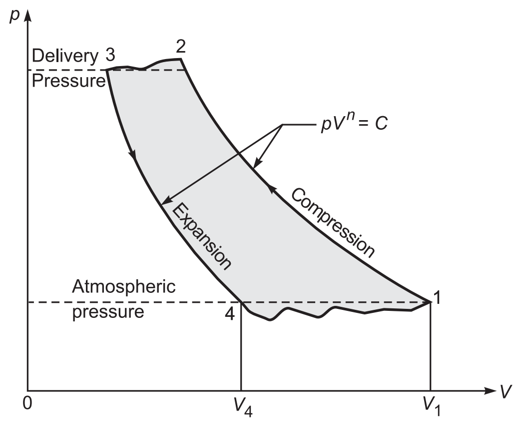


Fig. 25.11 *Actual indicator diagram for reciprocating compressor*

There must be some pressure difference across the valves to operate them. During the suction process 4–1, the pressure drops in the cylinder until the inlet valve is forced by atmospheric air to open. During the suction stroke, the piston creates *vacuum* in the cylinder. Thus pressure reduces and atmospheric air enters the cylinder.

Similarly, during delivery process 2–3, some more pressure is required to open the delivery valve and to displace the compressed air through narrow valve passage. Thus, *gas throttling* takes place during delivery, reducing the pressure gradually to the state 3.

The waviness of lines during these processes is due to valve bounce and wire drawing effect through the valves.

25.9 VOLUMETRIC EFFICIENCY

Actual volume sucked into the cylinder during the suction stroke is always less than the swept volume.

It is due to

(i) resistance offered by inlet valve to incoming air,
(ii) temperature of incoming air, and
(iii) back pressure of residual gas left in the clearance volume.

The *volumetric efficiency,* η_{vol} of the air compressor is defined as the ratio of actual volume of air sucked into the compressor, measured at atmospheric pressure and temperature to the piston displacement volume.

In terms of mass ratio, the volumetric efficiency is defined as the ratio of actual mass of air sucked per stroke to the mass of air corresponding to piston displacement volume at atmospheric conditions.

$$\eta_{vol} = \frac{\text{Actual mass sucked}}{\text{Mass corresponding to swept volume at Atmospheric pressure and temperature}} \quad ...(25.25)$$

$$= \frac{\text{Effective swept volume}}{\text{Piston displacement volume}} \quad ...(25.26)$$

Figure 25.12 shows an indicator diagram for a reciprocating air compressor showing effective swept volume, and piston displacement volume.

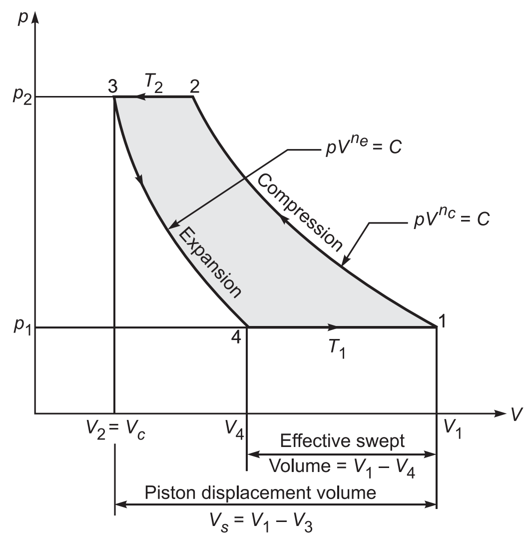


Fig. 25.12 *Indicator diagram for a reciprocating compressor showing effective swept volume, and piston displacement volume*

The volumetric efficiency can be expressed in terms of effective volume and piston displacement volume as;

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_c} = \frac{V_s + V_c - V_4}{V_s + V_c - V_c} = \frac{V_s + V_c - V_4}{V_s}$$

$$= 1 + \frac{V_c}{V_s} - \frac{V_4}{V_s} \times \frac{V_c}{V_c} \quad ...(25.27)$$

Introducing $c = \dfrac{V_c}{V_s}$ as clearance ratio, and using $V_c = V_3$, then

$$\eta_{vol} = 1 + c - c\left(\frac{V_4}{V_3}\right) \quad ...(25.28)$$

For expansion of gas in clearance volume

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n_e}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}}$$

Then $$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} \quad ...(25.29)$$

If index of expansion and index of compression are same, then

$$\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_c}} = \frac{V_1}{V_2}$$

and $$\eta_{vol} = 1 + c - c\left(\frac{V_1}{V_2}\right) \quad ...(25.30)$$

The volumetric efficiency decreases with pressure ratio $\left(\dfrac{p_2}{p_1}\right)$ in the compressor, its variation is shown in Fig. 25.13. The factors which lower volumetric efficiency are the following:

1. Too Large Clearance Volume Re-expansion of residual compressed air in the clearance space will reduce effective suction stroke $(V_1 - V_4)$ and therefore, the mass of fresh air entering into the cylinder reduces and volumetric efficiency decreases.

2. Obstruction at Intet Valve Obstruction due to narrow valve passage causes throttling of air in the cylinder. Throttling reduces the pressure in the

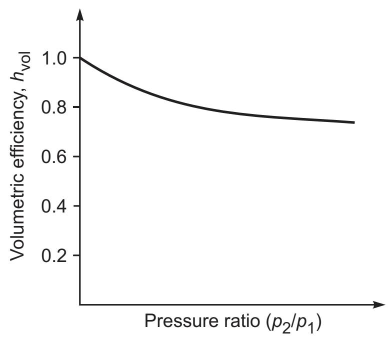


Fig. 25.13 *Effect of pressure ratio on volumetric efficiency of a reciprocating compressor*

cylinder during intake stroke and discharge pressure during delivery stroke, and thus the pressure ratio in the cylinder decreases. It leads to reduced FAD and volumetric efficiency.

3. High speed of Compressor With high speed of the compressor, the pressure drop across the inlet valve and delivery valve increases. Further, the temperature of compressed air increases due to less available time for cooling. Both these factors reduce volumetric efficiency of the compressor.

4. Heated Cylinder Walls The heated cylinder walls increase the temperature of the intake air. Thus, the specific volume of air increases which will reduce FAD and volumetric efficiency of the compressor.

5. Leakage Past the Piston Leakages across the piston will reduce the vacuum during suction and mass of compressed air above the piston. Both these effects will increase compression work input and decrease in volumetric efficiency.

25.10 FREE AIR DELIVERY (FAD)

The volume of compressed air corresponding to atmospheric conditions is known as *free air delivery* (FAD). FAD is the volume of compressed air measured in m^3/min, reduced to atmospheric pressure and temperature.

The free air delivered volume is less than the compressor displacement volume due to the following reasons:

1. *Obstruction at inlet valve* It offers the resistance to air flow through the narrow passage of valve.
2. *Re-expansion of high pressure air in clearance volume* It reduces effective suction stroke.
3. *Presence of hot cylinder walls of compressor* Air gets heated as it enters the cylinder. Thus, it expands and reducing the mass of air sucked into the cylinder.

In the actual indicator diagram as shown in Fig. 25.11, the air is sucked at a pressure and temperature which are lower than that of free (atmospheric) air. Using the property relation for an ideal gas as

$$\frac{p_f V_f}{T_f} = \frac{p_1 (V_1 - V_4)}{T_1}$$

Then free air delivery (FAD)

$$V_f = \frac{p_1 T_f}{p_f T_1} (V_1 - V_4) \qquad \text{...(25.31)}$$

where the suffix f denotes free (ambient) conditions while the suffix 1 indicates actual suction conditions. Then volumetric efficiency with respect to free air delivery;

$$\eta_{vol,\ overall} = \frac{V_f}{V_1 - V_c} = \frac{p_1 T_f}{p_f T_1}\left(\frac{V_1 - V_4}{V_1 - V_c}\right)$$

$$= \frac{p_1 T_f}{p_f T_1}\left[1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}}\right] \qquad \text{...(25.32)}$$

Example 25.9 *A single-stage, single-acting reciprocating air compressor receives air at 1.013 bar, 27°C and delivers it at 9.5 bar. The compressor has a bore = 250 mm, and stroke = 300 mm and it runs at 200 rpm. The mass-flow rate of air is 200 kg/h. Calculate the volumetric efficiency of the compressor.*

Solution

Given A single-stage, single-acting reciprocating air compressor

$p_1 = 1.03$ bar $= 101.3$ kPa $T_1 = 27°C = 300$ K
$N = 200$ rpm $p_2 = 9.5$ bar
$d = 250$ mm $= 0.25$ m $L = 300$ mm $= 0.3$ m
$\dot{m}_{act} = 200$ kg/h

To find The volumetric efficiency of the compressor.

Assumptions

(i) Neglecting clearance volume.
(ii) Specific gas constant of air, $R = 0.287$ kJ/kg·K.

Analysis The volume swept per cycle

$$V_1 = \left(\frac{\pi}{4}\right) d^2 L$$
$$= \left(\frac{\pi}{4}\right) \times (0.25\ \text{m})^2 \times (0.3\ \text{m})$$
$$= 0.0147\ \text{m}^3$$

The mass of air inducted per cycle

$$m_a = \frac{p_1 V_1}{R T_1} = \frac{(101.3\ \text{kPa}) \times (0.0147\ \text{m}^3)}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (300\ \text{K})}$$
$$= 0.0173\ \text{kg/cycle}$$

The mass-flow rate per hour

$$\dot{m}_a = \text{mass per cycle} \times \text{No. of suctions/revolution} \times \text{No. of revolutions/h}$$
$$= (0.0173\ \text{kg/cycle}) \times (1\ \text{suction/rev}) \times (200 \times 60\ \text{rev/h})$$
$$= 207.6\ \text{kg/h}$$

The mass of air actually sucked, $\dot{m}_{act} = 200$ kg/h

Thus
$$\eta_{vol} = \frac{\text{Actual mass sucked}}{\text{Mass corresponds to swept volume at atmospheric pressure and temperature}}$$
$$= \frac{200\ \text{kg}}{207.6\ \text{k}} = 0.963 \quad \text{or} \quad \mathbf{96.3\%}$$

Example 25.10 *A single-stage, double-acting reciprocating air compressor has a FAD of 14 m^3/min measured at 1.013 bar and 27°C. The pressure and temperature of the cylinder during induction are 0.95 bar and 45°C. The delivery pressure is 7 bar and the index of compression and expansion is 1.3. Calculate the indicated power required and volumetric efficiency. The clearance volume is 5% of the swept volume.*

Solution

Given A single-stage, double-acting reciprocating air compressor

$p_f = 1.03$ bar $= 101.3$ kPa
$T_f = 27°C = 300$ K
$p_2 = 7$ bar
FAD, $V_f = 14$ m^3/min
$p_1 = 0.95$ bar $= 95$ kPa
$T_1 = 45°C = 318$ K
$n_c = n_e = n = 1.3$
$V_c = 0.05\, V_s$

To find

(i) Indicated power, and
(ii) Volumetric efficiency.

Assumptions

(i) The compression and expansion are reversible.
(ii) Air as an ideal gas with $R = 0.287$ kJ/kg·K.

Analysis The p–V diagram for given data is shown in Fig. 25.14.

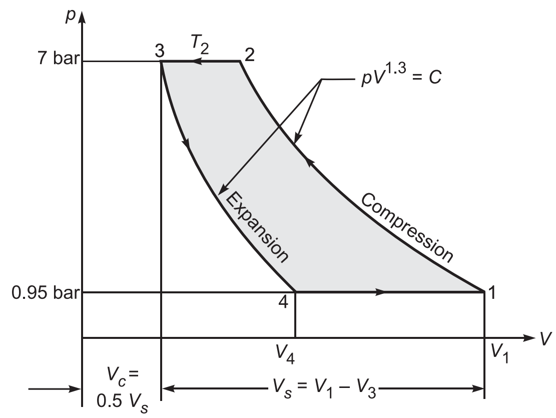


Fig. 25.14

The mass-flow rate corresponding to FAD at atmospheric conditions, p_f and T_f;

$$\dot{m}_a = \frac{p_f V_f}{R T_f}$$
$$= \frac{101.3 \times 14}{0.287 \times 300} = 16.47\ \text{kg/min}$$

The temperature T_2 after compression,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$
$$= (318\ \text{K}) \times \left(\frac{7}{0.95}\right)^{\frac{1.3-1}{1.3}} = 504.18\ \text{K}$$

(i) The indicated power,

$$IP = \frac{n}{n-1}\,\dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times \left(\frac{16.47}{60}\text{kg/s}\right) \times (0.287\text{ kJ/kg}\cdot\text{K}) \times (504.18\text{ K} - 318\text{ K})$$

$$= \mathbf{63.56\ kW}$$

(ii) The volumetric efficiency can be obtained by using Eq. (25.32)

$$\eta_{vol,\,overall} = \frac{p_1 T_f}{p_f T_1}\left[1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$$

$$= \frac{0.95 \times 300}{1.013 \times 318} \times \left[1 + 0.05 - 0.05 \times \left(\frac{7}{0.95}\right)^{\frac{1}{1.3}}\right]$$

$$= 0.723 \text{ or } \mathbf{72.3\%}$$

Example 25.11 *A single-stage, single-acting reciprocating air compressor has a bore of 20 cm and a stroke of 30 cm. The compressor runs at 600 rpm. The clearance volume is 4% of the swept volume and index of expansion and compression is 1.3. The suction conditions are at 0.97 bar and 27°C and delivery pressure is 5.6 bar. The atmospheric conditions are at 1.01 bar and 17°C. Determine,*

(a) *The free air delivered in m^3/min*

(b) *The volumetric efficiency referred to the free air conditions*

(c) *The indicated power*

Solution

Given Single-stage, single-acting reciprocating air compressor

d = 20 cm = 0.2 m	L = 30 cm = 0.3 m
N = 600 rpm	n = 1.3
p_1 = 0.97 bar = 97 kPa	T_1 = 27°C = 300 K
p_f = 1.01 bar = 101 kPa	T_f = 17°C = 290 K
V_c = 0.04 V_s	p_2 = 5.6 bar

To find

(i) Free air delivery, m^3/min,

(ii) Volumetric efficiency at ambient conditions; $\eta_{vol,\,overall}$, and

(iii) Indicated power.

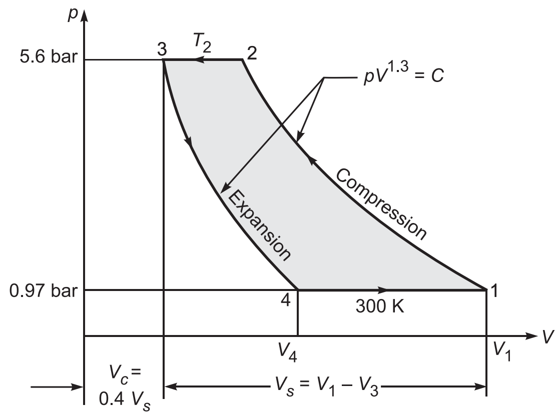


Fig. 25.15

Analysis

(i) *Free air delivery* (FAD)

The swept volume;

$$V_s = V_1 - V_3 = \left(\frac{\pi}{4}\right) d^2 L$$

$$= \left(\frac{\pi}{4}\right) \times (0.2\text{ m})^2 \times (0.3\text{ m})$$

$$= 0.009425\text{ m}^3$$

Clearance volume

$$V_c = 0.04\ V_s = 0.04 \times 0.009425$$

$$= 3.77 \times 10^{-4}\text{ m}^3$$

Total volume

$$V_1 = V_s + V_c$$

$$= 0.009425 + 3.77 \times 10^{-4}$$

$$= 0.0098\text{ m}^3$$

The volume V_4, after re-expansion of compressed air in clearance space,

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$V_4 = 3.77 \times 10^{-4} \times \left(\frac{5.6}{0.97}\right)^{\frac{1}{1.3}}$$

$$= 0.00145\text{ m}^3$$

Effective swept volume,

$$V_1 - V_4 = 0.0098 - 0.00145 = 0.00835\text{ m}^3$$

The free air delivery per cycle,

$$V_f = \frac{p_1 T_f}{p_f T_1}(V_1 - V_4)$$

$$= \frac{0.97 \times 290}{1.01 \times 300} \times 0.00835$$

$= 0.00775$ m^3/cycle

Free air delivered per minute

$= V_f \times$ No. of cycle per minute

$= 0.00775 \times 600 =$ **4.65 m^3/min**

(ii) The *volumetric efficiency referred to free air conditions*

$$\eta_{vol,\ overall} = \frac{V_f}{V_s} = \frac{0.00775}{0.009425}$$

$= 0.822$ or **82.2%**

(iii) *Indicated power*

Indicated work input using Eq. (25.24)

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 97 \times 0.00835 \times \left[\left(\frac{5.6}{0.97} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$= 1.75$ kJ/cycle

Since N cycles take place within a minute, the indicated power

$$IP = W_{in} \left(\frac{N}{60} \right)$$

$$= (1.75 \text{ kJ/cycle}) \times \left(\frac{600}{60} \text{ cycle/s} \right)$$

= 17.5 kW

Example 25.12 *A single-stage, double-acting air compressor delivers air at 7 bar. The pressure and temperature at the end of the suction stroke are 1 bar and 27°C. It delivers 2 m^3 of free air per minute when the compressor is running at 300 rpm. The clearance volume is 5% of the stroke volume. The pressure and temperature of the ambient air are 1.03 bar and 20°C. The index of compression is 1.3, and index of expansion is 1.35. Calculate*

(a) Volumetric efficiency of the compressor,

(b) Indicated power of the compressor,

(c) Brake Power, if mechanical efficiency is 80%.

Solution

Given A single-stage, double-acting reciprocating air compressor

$p_1 = 1$ bar $= 100$ kPa	$p_2 = 7$ bar
$N = 300$ rpm	$T_1 = 27°C = 300$ K
$\dot{V}_f = 2$ m^3/min	$p_f = 1.03$ bar $= 103$ kPa
$V_c = 0.05\ V_s$	$T_f = 20°C = 293$ K
$\eta_{mech} = 0.8$	$n_c = 1.3$, and
$n_e = 1.35$	

To find

(i) Volumetric efficiency,

(ii) Indicated power, and

(iii) Brake Power.

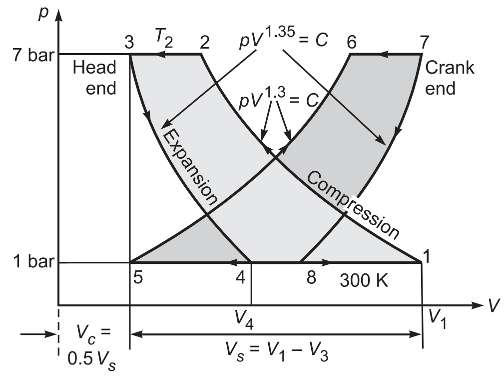


Fig. 25.16 *p-V dagram for double-acting compressor*

Analysis

(i) *Volumetric efficiency*

Clearance volume; $V_c = V_3 = 0.05\ V_s$

The volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}}$$

$$= 0.05\ V_s \times \left(\frac{7}{1} \right)^{\frac{1}{1.35}} = 0.2113 V_s$$

The total volume of cylinder;

$$V_1 = V_s + V_c = 1.05\ V_s$$

Effective swept volume

$$V_1 - V_4 = 1.05\ V_s - 0.2113\ V_s = 0.8386\ V_s$$

The volumetric efficiency

$$\eta_{vol} = \frac{V_1 - V_4}{V_s} = \frac{0.8386 V_s}{V_s}$$

$= 0.8386$ or **83.86%**

Free air delivered per cycle is given as

$$V_f = \frac{p_1 T_f}{p_f T_1}\ (V_1 - V_4)$$

Thus volume of air inducted per min at suction condition

$$\dot{V}_1 - \dot{V}_4 = \dot{V}_f \frac{p_f T_1}{p_1 T_f}$$

$$= (2\ \text{m}^3/\text{min}) \times \frac{(1.03\ \text{bar}) \times (300\ \text{K})}{(1\ \text{bar}) \times (293\ \text{K})}$$

$$= 2.109\ \text{m}^3/\text{min}$$

It is the volume sucked per minute, which can also be expressed as

$$\dot{V}_1 - \dot{V}_4 = \eta_{vol} V_s \times N \times \text{No. of suction per revolution}$$

$2.109\ \text{m}^3/\text{min} = 0.8386\ V_s \times (300\ \text{rpm}) \times 2$

or $\quad V_s = 0.00419\ \text{m}^3$

Now $\quad V_1 = 1.05 \times 0.00419 = 0.0044\ \text{m}^3$

and $\quad V_4 = 0.2113 \times 0.00419 = 8.856 \times 10^{-4}\ \text{m}^3$

(ii) *Indicated power*

Indicated work input per cycle, Eq. (25.23)

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_c - 1}{n_c}} - 1\right] - \frac{n_e}{1 - n_e} p_1 V_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_e - 1}{n_e}} - 1\right]$$

$$= \frac{1.3}{1.3 - 1} \times 100 \times 0.0044 \times \left[\left(\frac{7}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] - \frac{1.35}{1.35 - 1} \times 100 \times 8.856 \times 10^{-4} \times \left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}} - 1\right]$$

$$= 1.080 - 0.224 = 0.856\ \text{kJ/cycle}$$

The indicated power input to compressor

$$IP = W_{in}\left(\frac{N}{60}\right) \times 2\ \text{(for double acting)}$$

$$= (0.856\ \text{kJ/cycle}) \times \left(\frac{300}{60}\ \text{cycle/s}\right) \times 2$$

$$= \mathbf{8.558\ kW}$$

(iii) The brake (shaft) power

$$\text{Brake power} = \frac{\text{Indicated power}}{\eta_{mech}} = \frac{8.558}{0.8}$$

$$= \mathbf{10.7\ kW}$$

Example 25.13 *A single-stage, double-acting reciprocating air compressor works between 1 bar and 10 bar. The compression follows the law $pV^{1.35}$ = constant. The piston speed is 200 m/min and the compressor speed is 120 rpm. The compressor consumes an indicated power of 62.5 kW with volumetric efficiency of 90%. Calculate*

(a) diameter and stroke of the cylinder

(b) Clearance volume as percentage of stroke volume

Solution

Given A single-stage, single-acting reciprocating air compressor

p_1 = 1 bar = 100 kPa $\qquad p_2$ = 10 bar

N = 120 rpm $\qquad \mathcal{V}_{piston}$ = 200 m/min

η_{vol} = 0.9 $\qquad IP$ = 62.5 kW

and $\quad pV^{1.35} = \text{C}$

To find

(i) Cylinder bore and piston stroke,

(ii) Clearance ratio.

Analysis

(i) *Cylinder bore* and *piston stroke*

The indicated work input per cycle for double acting compressor can be obtained as

$$W_{in} = IP\left(\frac{60}{2N}\ \text{s/cycle}\right)$$

$$= (62.5\ \text{kJ/s}) \times \left(\frac{60}{2 \times 120}\ \text{s/cycle}\right)$$

$$= 15.625\ \text{kJ/cycle}$$

Further,

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$15.625 = \frac{1.35}{1.35-1} \times (100 \text{ kPa}) \times (V_1 - V_4) \times \left[\left(\frac{10}{1}\right)^{\frac{1.35-1}{1.35}} - 1\right]$$

or $(V_1 - V_4) = 0.0496 \text{ m}^3$

The volumetric efficiency is given as

$$\eta_{vol} = \frac{V_1 - V_4}{V_s}$$

Stroke volume

$$\text{or} \quad V_s = \frac{0.0496 \text{ m}^3}{0.9} = 0.0551 \text{ m}^3$$

The piston speed is given by

$$v_{piston} = 2LN$$

Stroke length;

$$L = \frac{200}{2 \times 120} = 0.833 \text{ m or } \mathbf{833 \text{ mm}}$$

Further, the stroke volume can be expressed as

$$V_s = \left(\frac{\pi}{4}\right) d^2 L$$

$$\text{Bore} \quad d = \sqrt{\frac{0.0551}{(\pi/4) \times 0.833}} = 0.290 \text{ m} \quad \text{or} \quad \mathbf{290 \text{ mm}}$$

(ii) *Clearance ratio*

The volumetric efficiency is given by

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\text{or} \quad 0.9 = 1 + c - c \times \left(\frac{10}{1}\right)^{\frac{1}{1.35}}$$

Clearance ratio;

$$c = 0.222 \quad \text{or} \quad \mathbf{2.22\%}$$

Example 25.14 *A single-stage, single-acting reciprocating air compressor delivers 0.6 kg/min of air at 6 bar. The temperature and pressure at the suction stroke are 30°C and 1 bar, respectively. The bore and stroke are 100 mm and 150 mm respectively. The clearance volume is 3% of the swept volume and index of expansion and compression is 1.3. Determine,*

(a) the volumetric efficiency of compressor,

(b) the power required, if mechanical efficiency is 85%,

(c) speed of the compressor.

Solution

Given A single-stage, single-acting reciprocating air compressor

$p_1 = 1$ bar	$p_2 = 6$ bar
$\dot{m}_a = 0.6$ kg/min	$T_1 = 30°C = 303$ K
$L = 150$ cm	$d = 100$ mm
$c = 0.03$	$n = 1.3$
$\eta_{mech} = 0.85$	

To find

(i) Volumetric efficiency,

(ii) Power required by compressor, and

(iii) Speed of the compressor.

Assumptions

(i) Suction takes place at free air conditions.

(ii) The specific gas constant for air $R = 0.287$ kJ/kg·K.

Analysis

(i) The *volumetric efficiency* is given by

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\text{or} \quad = 1 + 0.03 - 0.03 \times \left(\frac{6}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.910 \quad \text{or} \quad \mathbf{91.0\%}$$

(ii) *Power* required by compressor

Indicated power input

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times \left(\frac{0.6 \text{ kg}}{60 \text{ s}}\right) \times 0.287 \times 303 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= 1.929 \text{ kW}$$

Brake power required;

$$BP = \frac{IP}{\eta_{mech}} = \frac{1.929}{0.85} = \mathbf{2.27\ kW}$$

(iii) *Speed of the compressor*

The volume-flow rate of air at suction conditions

$$\dot{V}_1 - \dot{V}_4 = \frac{\dot{m}_a RT_1}{p_1}$$

$$= \frac{0.6 \times 0.287 \times 303}{100}$$

$$= 0.5217\ \text{m}^3/\text{min}$$

Piston-displacement volume rate

$$\dot{V}_s = \frac{\dot{V}_1 - \dot{V}_4}{\eta_{vol}} = \frac{0.5217\ \text{m}^3/\text{min}}{0.91}$$

$$= 0.5733\ \text{m}^3/\text{min}$$

Further, $\dot{V}_s = \left(\frac{\pi}{4}\right) d^2 L N$

$\therefore$ 0.5733

$$= \left(\frac{\pi}{4}\right) \times (0.1\ \text{m})^2 \times (0.1\ 5\text{m}) \times N$$

Speed of compressor;

N = **487 rpm**

Example 25.15 *A single-stage, double-acting reciprocating air compressor is driven by an electric motor consuming 40 kW, compresses 5.5 m^3/min air according to the law $pV^{1.3}$ = constant. It receives atmospheric air at 20°C and 745 mm of Hg barometer and delivers at a gauge pressure of 700 kPa. Calculate isothermal, volumetric, mechanical and overall efficiencies for the following data from an indiactor diagram for head and tail end.*

Length of the indicator diagram = 6.75 cm

Area of the head end = 7.6 cm^2

Area of the tail end = 7.8 cm^2

Spring scale = 200 kPa/cm

The diameter of the piston and piston rod are 25 and 2.5 cm, respectively. The stroke length is 30 cm. The compress or runs at 300 rpm.

Solution

Given A single-stage, double-acting reciprocating air compressor.

p_1 = 745 mm of Hg	p_{g_2} = 700 kPa
N = 300 rpm	$\dot{V}_f$ = 5.5 m^3/min
BP = 40 kW	L = 30 cm
d = 25 cm	d_{rod} = 2.5 cm, n = 1.3
$T_1 = T_f$ = 20°C = 293 K	n = 1.3 cm

For indicator diagram,

a_1 = 7.6 cm^2	a_2 = 7.8 cm^2
l = 6.75 cm	k = 200 kPa/cm

To find

(i) Overall efficiency,
(ii) Isothermal efficiency,
(iii) Mechanical efficiency, and
(iv) Volumetric efficiency.

Assumptions

(i) Suction takes place at free air conditions.
(ii) Negligible clearance volume on head and tail side of the cylinder.
(iii) Specific gas constant for air R = 0.287 kJ/kg·K.

Analysis The suction pressure corresponds to 745 mm of Hg;

$$p_1 = \frac{745}{760} \times (101.3\ \text{kPa}) = 99.3\ \text{kPa}$$

Absolute delivery pressure,

$$p_2 = p_1 + p_{g_2} = 99.3 + 700 = 799.3\ \text{kPa}$$

Mass of air inducted per second

$$\dot{m}_a = \frac{p_1 \dot{V}_f}{RT_f} = \frac{(9\ 93\ \text{kPa}) \times (5.5\ \text{m}^3/\text{min})}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (293\ \text{K})}$$

$$= 6.49\ \text{kg/min or } 0.1082\ \text{kg/s}$$

Temperature after polytropic compression

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= (293\ \text{K}) \times \left(\frac{793.3}{93.3}\right)^{\frac{1.3-1}{1.3}}$$

$$= 474.12\ \text{K}$$

Actual power input to air in the compressor;

$$\dot{W}_{in} = \frac{n}{n-1} \dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times 0.1082 \times 0.287 \times (474.12 - 293)$$

$$= 24.37\ \text{kW}$$

(i) *Overall efficiency*

$$\eta_{Overall} = \frac{\dot{W}_{in}}{BP} = \frac{24.37\ \text{kW}}{40\ \text{kW}}$$

$$= 0.6093 \quad \text{or} \quad \mathbf{60.93\%}$$

Isothermal power input to the compressor;

$$\dot{W}_{iso} = p_1 \dot{V}_f \ln\left(\frac{p_2}{p_1}\right)$$

$$= (99.3 \text{ kPa}) \times \left(\frac{5.5 \text{ m}^3}{60 \text{ s}}\right) \times \ln\left(\frac{793.3}{93.3}\right)$$

$$= 19.48 \text{ kW}$$

(ii) *Isothermal efficiency*

$$\eta_{iso} = \frac{\dot{W}_{iso}}{BP} = \frac{19.48}{40} = 0.487 \quad \text{or} \quad \mathbf{48.7\%}$$

For head end of the cylinder

Cross-sectional area,

$$A_1 = \left(\frac{\pi}{4}\right) d^2 = \left(\frac{\pi}{4}\right) \times (0.25 \text{ m})^2$$

$$= 0.049 \text{ m}^2$$

Indicated mean effective pressure;

$$p_{m_1} = \frac{a_1}{l} k = \frac{7.6 \text{ cm}^2}{6.75 \text{ cm}} \times (200 \text{ kPa/cm})$$

$$= 225.18 \text{ kPa}$$

For tail end of the cylinder

Cross-sectional area,

$$A_2 = \left(\frac{\pi}{4}\right)\left[d^2 - d_p^2\right]$$

$$= \left(\frac{\pi}{4}\right) \times \left[(0.25 \text{ m})^2 - (0.025 \text{ m})^2\right]$$

$$= 0.0486 \text{ m}^2$$

Indicated mean effective pressure;

$$p_{m_2} = \frac{a_2}{l} k = \frac{7.8 \text{ cm}^2}{6.75 \text{ cm}} \times (200 \text{ kPa/cm})$$

$$= 231.11 \text{ kPa}$$

Total indicated power input to head and tail sides of the compressor

$$IP = \frac{p_{m_1} L A_1 N}{60} + \frac{p_{m_2} L A_2 N}{60}$$

$$= \frac{225.18 \times 0.3 \times 0.049 \times 300}{60} + \frac{231.11 \times 0.3 \times 0.0486 \times 300}{60}$$

or $\quad IP = 4.965 + 5.054 = 33.4$ kW

(iii) *Mechanical efficiency*

$$\eta_{mech} = \frac{IP}{BP} = \frac{33.4 \text{ kW}}{40 \text{ kW}}$$

$$= 0.835 \quad \text{or} \quad \mathbf{83.5\%}$$

Total displacement volume per minute from head and tail ends of the cylinder;

$$\dot{V}_{total} = (A_1 + A_2)\, L N$$

$$= (0.049 + 0.0486) \times 0.3 \times 300$$

$$= 8.78 \text{ m}^3/\text{min}$$

(iv) *Volumetric efficiency*

$$\eta_{vol} = \frac{\dot{V}_f}{\dot{V}_{total}} = \frac{5.5 \text{ m}^3/\text{min}}{8.78 \text{ m}^3/\text{min}}$$

$$= 0.626 \quad \text{or} \quad \mathbf{62.6\%}$$

Example 25.16 *During the overhauling of an old compressor, a distance piece of 9 mm thickness is inserted accidentally between the cylinder head and cylinder. Before overhaul, the clearance volume was 3 per cent of the swept volume. The compressor receives atmospheric air at 1 bar and it is designed to deliver air at a gauge pressure of 7 bar with a stroke of 75 cm. If the compression and re-expansion follow the law $pV^{1.3}$ = constant, determine the percentage change in*

(a) volume of free air delivered,

(b) power necessary to drive the compressor.

Solution

Given A reciprocating air compressor before and after overhaul

$p_1 = 1$ bar	$p_{g_2} = 7$ bar
$t = 9$ mm	$V_c = 0.03\, V_s$
$L = 75$ cm	$n = 1.3$

To find

(i) Percentage change in volume of free air delivered,

(ii) Percentage change in power necessary to drive the compressor.

Analysis The absolute pressure of delivered air

$$p_2 = p_1 + p_{g_2} = 1 \text{ bar} + 7 \text{ bar} = 8 \text{ bar}$$

The clearance space

Before overhaul,

$$L_{c_1} = 0.03L = 0.03 \times 75 = 2.25 \text{ cm}$$

After overhaul,

$$L_{c_2} = L_{c_1} + t = 2.25 \text{ cm} + 0.9 \text{ cm}$$

$$= 3.15 \text{ cm}$$

The clearance volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = V_3 \times \left(\frac{8}{1}\right)^{\frac{1}{1.3}} = 4.95\, V_c$$

Using cylinder cross-section area A, before overhaul;

$$V_4 = 4.95 \times 2.25\ A = 11.14\ A\ \text{cm}^3$$

After overhaul,

$$V_{4a} = 4.95 \times 3.15\ A = 15.59\ A\ \text{cm}^3$$

Total volume of compressor

Before overhaul;

$$V_1 = V_s + AL_{c_1} = 75\ A + 2.25A = 77.25\ A\ \text{cm}^3$$

After overhaul;

$$V_{1a} = V_s + AL_{c_2} = 75\ A + 3.15A = 78.15\ A\ \text{cm}^3$$

The effective suction volume

Before overhaul;

$$V_1 - V_4 = 77.25\ A\ \text{cm}^3 - 11.14\ A\ \text{cm}^3 = 66.11\ A\ \text{cm}^3$$

After overhaul;

$$V_{1a} - V_{4a} = 78.15\ A\ \text{cm}^3 - 15.59\ A\ \text{cm}^3 = 62.56\ A\ \text{cm}^3$$

(i) Percentage change in FAD

$$= \frac{66.11A - 62.56\ A}{66.11A} \times 100 = \mathbf{5.37\%}$$

(ii) Indicator work input

$$W_1 = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

In calculation of indicated work input, all terms except $(V_1 - V_4)$ will remain constant before and after overhaul. Therefore, percentage change in indicated work input will be equal to percentage change in FAD, i.e. % change in work input = **5.37%**

Example 25.17 *A 4-cylinder, single-stage, double acting air compressor delivers air at 7 bar. The pressure and temperature at the end of the suction stroke are 1 bar and 27°C. It delivers 30 m³ of free air per minute when the compressor is running at 300 rpm. The pressure and temperature of the ambient air are 1 bar and 17°C. The clearance volume is 4% of the stroke volume. The stroke-to-bore ratio is 1.2. The index of compression and expansion is 1.32. Calculate*

(a) Volumetric efficiency of the compressor,
(b) Indicated power of the compressor,
(c) Size of motor, if mechanical efficiency is 85%,
(d) Cylinder dimensions.

Solution

Given A 4-cylinder, single-stage, double-acting reciprocating air compressor

No. of cylinders = 4	$k = 2$
$p_1 = 1$ bar = 100 kPa	$T_1 = 27°\text{C} = 300$ K
$p_2 = 7$ bar	$\dot{V}_f = 30\ \text{m}^3/\text{min}$
$N = 300$ rpm	$p_f = 1$ bar
$c = 0.04$	$T_f = 17°\text{C} = 290$ K
$\eta_{mech} = 0.85$	$n = 1.32$
$L/d = 1.2$	

To find

(i) Volumetric efficiency,
(ii) Indicated power,
(iii) Size of motor (brake power), and
(iv) Bore of cylinder and piston stroke.

Analysis

(i) *Volumetric efficiency*

Clearance ratio;

$$c = 0.04$$

The volumetric efficiency;

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = 1 + 0.04 - 0.04 \times \left(\frac{7}{1}\right)^{\frac{1}{1.32}} = 0.8653 \quad \text{or} \quad \mathbf{86.53\%}$$

(ii) *Indicated power*

The effective swept volume;

$$\dot{V}_1 - \dot{V}_4 = \frac{p_f \dot{V}_f}{T_f} \times \frac{T_1}{p_1} = \frac{(1\ \text{bar}) \times (30\ \text{m}^3/\text{min})}{(290\ \text{K})} \times \left(\frac{300\ \text{K}}{1\ \text{bar}}\right) = 31.03\ \text{m}^3/\text{min} \quad \text{or} \quad 0.517\ \text{m}^3/\text{s}$$

Indicated power;

$$IP = \frac{n}{n-1} p_1 (\dot{V}_1 - \dot{V}_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.32}{1.32 - 1} \times (100\ \text{kPa}) \times (0.517\ \text{m}^3/\text{s}) \times \left[\left(\frac{7}{1}\right)^{\frac{1.32-1}{1.32}} - 1\right]$$

$$= \mathbf{128.60\ kW}$$

(iii) The motor (brake) power

$$BP = \frac{IP}{\eta_{mech}} = \frac{128.6}{0.85} = 151.3 \text{ kW}$$

(iv) *Cylinder dimensions*

The piston displacement volume of one cylinder can be obtained by using volumetric efficiency as

$$V_s = \frac{\dot{V}_1 - \dot{V}_4}{\text{No. of cylinders} \times \eta_{vol}}$$

$$= \frac{31.03}{4 \times 0.8653} = 8.965 \text{ m}^3/\text{min}$$

The piston displacement volume rate with L = 1.2 d; for double-acting cylinder can be expressed as

$$V_s = \left(\frac{\pi}{4}\right) d^2 L N k$$

$$= \left(\frac{\pi}{4}\right) d^2 \times (1.2 d) \times 300 \times 2$$

Bore $\quad d = \sqrt[3]{\dfrac{8.965}{(\pi/4) \times 1.2 \times 300 \times 2}}$

$= 0.251$ m or **251 mm**

Stroke; $\quad L = 1.2\, d =$ **301.4 mm**

Example 25.18 *A twin-cylinder single-stage, single-acting reciprocating air compressor running at 300 rpm has pressure ratio of 8. The clearance is 3 per cent of the swept volume. It compresses 30 m³/min free air at 101.3 kPa and 20°C according to $pV^{1.3}$ = constant. The temperature rise during suction stroke is 25°C . The loss of pressure through intake and discharge valve is 8 and 150 kPa, respectively. Determine*

(a) Indicated power required by the compressor,
(b) Brake power, assuming compression efficiency of 85% and mechanical efficiency of 95% ,
(c) Cylinder dimesions for same bore and stroke size.

Solution

Given A single-stage, single-acting reciprocating air compressor

No. of cylinders	= 2	c	= 0.03
N	= 300 rpm	n	= 1.3
$\dot{V}_f$	= 30 m³/min	p_f	= 101.3 kPa
T_f	= 20°C = 293 K	$p_f - p_1$	= 8 kPa
$\frac{p_2}{p_1}$	= 8	L	= d

T_1 = 20°C + 25°C = 45°C or 318 K

$p_2 - p_d$ = 150 kPa

η_{comp} = 0.85

η_{mech} = 0.95

To find

(i) Indicated power necessary to drive the compressor,
(ii) Brake power, and
(iii) Cylinder dimesions for same bore and stroke size.

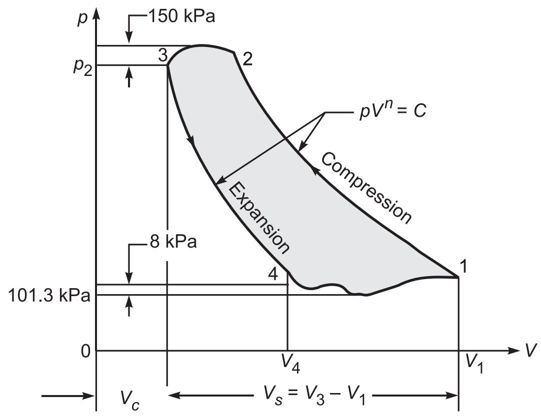


Fig. 25.17

Analysis The suction pressure

$$p_1 = p_f - 8 \text{ kPa} = 101.3 - 8 = 93.3 \text{ kPa}$$

Using the property relation for an ideal gas as

$$\frac{p_f \dot{V}_f}{T_f} = \frac{p_1 (\dot{V}_1 - \dot{V}_4)}{T_1}$$

It gives

$$\dot{V}_1 - \dot{V}_4 = \frac{p_f \dot{V}_f}{T_f} \times \frac{T_1}{p_1} = \frac{101.3 \times 30}{293} \times \frac{318}{93.3}$$

$= 35.35$ m³/min

(i) *The indicated power input to the compressor*

$$IP = \frac{n}{n-1} p_1 (\dot{V}_1 - \dot{V}_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3 - 1} \times 93.3 \times \left(\frac{35.5}{60}\right) \times \left[(8)^{\frac{1.3-1}{1.3}} - 1 \right]$$

= **146.7 kW**

(ii) *The brake power input to the compressor*

Brake power;

$$BP = \frac{\text{Indicated power}}{\eta_{comp} \times \eta_{mech}} = \frac{146.7}{0.85 \times 0.95}$$

$$= \mathbf{181.67\ kW}$$

(iii) *Cylinder dimesions for same bore and stroke size:*

The volumetric efficiency is given as

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.03 - 0.03 \times (8)^{1/1.3}$$

$$= 0.8814$$

Further, the volumetric efficiency is also given as

$$\eta_{vol} = \frac{(\dot{V}_1 - \dot{V}_4)}{\dot{V}_s}$$

∴ total displacement volume for two cylinders;

$$\dot{V}_s = \frac{(\dot{V}_1 - \dot{V}_4)}{\eta_{vol}} = \frac{35.35}{0.8814}$$

$$= 40.1\ m^3/min$$

For a two-cylinder, single-acting reciprocating air compressor, the displacement volume per minute is also expressed as

$$\dot{V}_s = 2 \times \left(\frac{\pi}{4}\right) d^2 L N$$

For $d = L$;

$$40.1 = 2 \times \left(\frac{\pi}{4}\right) d^3 \times 300$$

It gives bore and stroke sizes as

$$d = L = 0.44\ m \quad \text{or} \quad \mathbf{440\ mm}$$

25.11 LIMITATIONS OF SINGLE-STAGE COMPRESSION

Usually, the pressure ratio for a single-stage reciprocating air compressor is limited to 7. Increase in pressure ratio in a single-stage reciprocating air compressor causes the following undesirable effects:

1. Greater expansion of clearance air in the cylinder and as a consequence, it decreases effective suction volume ($V_1 - V_4$) and therefore, there is a decrease in fresh air induction.
2. With high delivery pressure, the delivery temperature increases. It increases specific volume of air in the cylinder, thus more compression work is required.
3. Further, for high pressure ratio, the cylinder size would have to be large, strong and heavy working parts of the compressor will be needed. It will increase balancing problem and high torque fluctuation will require a heavier flywheel installation.

All the above problems can be reduced to minimum level with multistage compression.

25.12 MULTISTAGE COMPRESSION

As discussed in preceding sections, the compressor requires minimum work input with isothermal compression. But the delivery temperature T_2 increases with pressure ratio and the volumetric efficiency decreases as pressure ratio increases.

All the above problems can be reduced to minimum level by compressing the air in more than one cylinders with intercooling between stages, for the same pressure ratio. The compression of air in two or more cylinders in series is called *multistage compression.* Air cooling between stages provides the means to achieving an appreciable reduction in the compression work and maintaining the air temperature within safe operating limits.

25.12.1 Advantages of Multistage Compression

1. The gas can be compressed to a sufficiently high pressure.
2. Cooling of air is more efficient with intercoolers and cylinder wall surface.
3. By cooling the air between the stages of compression, the compression can be brought to isothermal and power input to the compressor can be reduced considerably.

4. By multistaging, the pressure ratio of each stage is lowered. Thus, the air leakage past the piston in the cylinder is also reduced.
5. The low pressure ratio in a cylinder improves volumetric efficiency.
6. Due to phasing of operation in stages, in a multistage compressor, the negative and positive forces are balanced to a large extent. Thus, more uniform torque and better mechanical balance can be achieved.
7. Due to low pressure ratio in stages, the compressor speed could be higher for same isothermal efficiency.
8. Low working temperature in each stage helps to sustain better lubrication.
9. The low-pressure cylinder is made lighter and high-pressure cylinders are made smaller for reduced pressure ratio in each stage.

25.12.2 Work Done in Multistage Compressor with Intercooler

Figure 25.18 shows a schematic for two-stage compression. The air at p_1 and T_1 is first drawn into the first stage or low pressure (LP) cylinder. It is partially compressed to some intermediate pressure, p_2 and temperature T_2 and is then discharged to an intercooler which ideally cools the air to its initial temperature T_1. The cooled air then enters the second stage or high pressure (HP) cylinder and is compressed to a delivery pressure p_3 and temperature T_3. The corresponding indicator diagram is shown on a p–V plane in Fig. 25.19.

The cycle 1–2–3–4–1 represents first-stage compression cycle. The air is discharged from LP

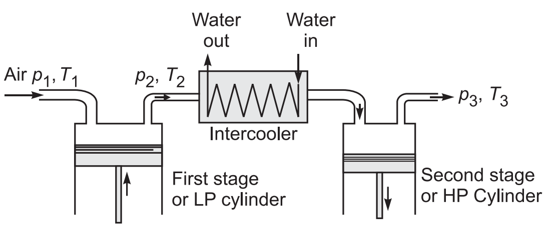


Fig. 25.18 *Two stage compression with intercooler*

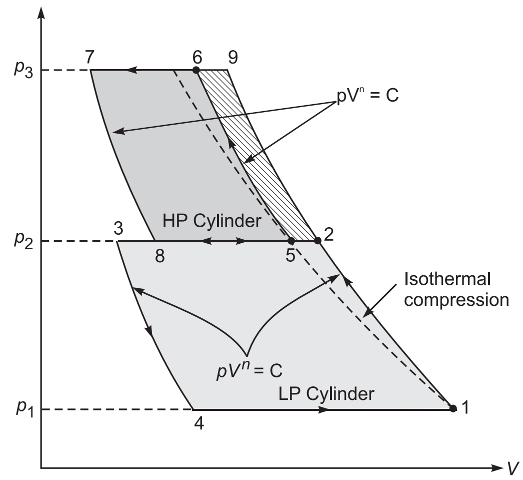


Fig. 25.19 *Indicator diagram for two-stage compressor*

cylinder at intermediate pressure p_2 and temperature T_2, related with inlet pressure p_1 and temperature T_1 as

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

The air is then cooled in an intercooler, if intercooling is complete (perfect), the air will enter the HP cylinder at the same temperature at which it enters the LP cylinder. The second-stage compression cycle in an HP cylinder is shown by cycle 5–6–7–8–5. The line 1–2–9 represents the single-stage compression from initial pressure p_1 to delivery pressure p_3. The shadded area 2–9–6–5–2 represents the saving in compression work obtained by intercooling.

The total indicator work

$$W_{in} = W_{LP} + W_{HP}$$

$$= \frac{n}{n-1} p_1 (V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$+ \frac{n}{n-1} p_1 (V_5 - V_8)\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

In terms of mass of air inducted per cycle;

$$W_{in} = \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{in} = \frac{n}{n-1} \times m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \text{(kJ/cycle)} \quad ...(25.33)$$

where, $m_a = \dfrac{p_f V_f}{R T_f} = \dfrac{p_1 (V_1 - V_4)}{R T_1} = \dfrac{p_2 (V_5 - V_8)}{R T_1}$

Since same mass of air is handled by both cylinders, the suffix f represents free air conditions.

For given mass-flow rate $\dot{m}_a$ (kg/s), the indicated power

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \text{(kW)} \quad ...(25.34)$$

25.12.3 Heat Rejected per Stage of Compression

In a two-stage air compressor, air rejects heat (i) during compression process, and (ii) after compression in intercooler.

$$Q_{stage} = Q_{comp} + Q_{cooling}$$

Heat rejected during polytropic compression process

$$Q_{comp} = m_a C_n (T_2 - T_1) \text{ (kJ)}$$

where $C_n = \dfrac{C_p - n C_v}{n-1}$, the polytropic specific heat, measured in kJ/kg·K.

For perfect intercooling, the temperature of air after cooling should be reduced to initial temperature T_1. Therefore, the heat rejected in intercooler

$$Q_{cooling} = m_a C_p (T_2 - T_1) \text{ (kJ)} \quad ...(25.35)$$

Using $C_p = \gamma C_v$, we get

$$C_n = \frac{\gamma C_v - n C_v}{n-1} = \frac{(\gamma - n)}{n-1} C_v$$

Then total heat rejected;

$$Q_{stage} = m_a \frac{(\gamma - n)}{n-1} C_v (T_2 - T_1) + m_a C_p (T_2 - T_1)$$

$$= m_a \left[\frac{(\gamma - n)}{n-1} C_v + C_p \right] (T_2 - T_1) \text{ (kJ)} \quad ...(25.36)$$

For heat rejected per kg of air

$$q_{stage} = \left[\frac{(\gamma - n)}{n-1} C_v + C_p \right] (T_2 - T_1) \text{ (kJ/kg)} \quad ...(25.37)$$

25.12.4 Actual Indicator Diagram for a Two-Stage Compressor

The actual indicator diagram on a p–V plane for a two-stage reciprocating air compressor is shown in Fig. 25.20. The variation during suction and delivery processes is due to valve action effects.

The indicator diagrams for low-pressure and high-pressure cylinders overlap due to pressure drop taking place in intercooler and clearance effect.

During the suction process, pressure drops in the cylinder until the inlet valve is forced to open by air. Similarly, during delivery process , some more pressure is required to open the delivery valve and to displace the compressed air through a narrow valve passage. Thus, *gas throttling* takes place during delivery, which reduces the pressure gradually.

25.12.5 Condition for Minimum Compression Work: Optimum Intermediate Pressure

The intermediate pressure p_2 influences the work to be done on the gas and its distribution between

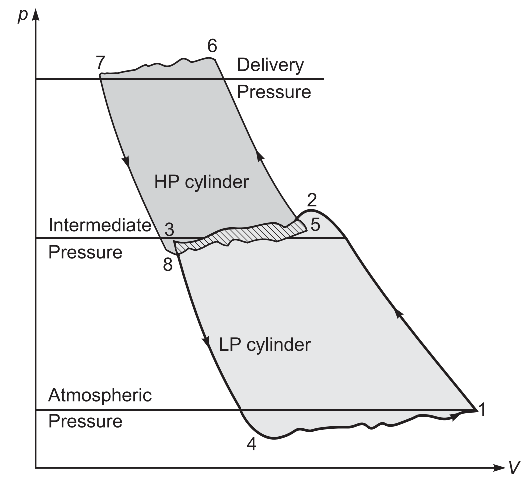


Fig. 25.20 *Actual indicator diagram for two-stage reciprocating air compressor*

stages. The intermediate pressure, which makes work input minimum, is always important.

The total power input to a two-stage reciprocating air compressor with complete intercooling is given by Eq. (25.34);

$$IP = \frac{n}{n-1}\dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

If p_1, T_1 and p_3 are fixed then the optimum value of intermediate pressure p_2 for minimum work input can be obtained by applying condition of minima, i.e.,

$$\frac{d(IP)}{dp_2} = 0$$

$$\frac{n}{n-1}\dot{m}_a R T_1 \frac{d}{dp_2}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] = 0$$

$$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} \frac{d}{dp_2}(p_2)^{\frac{n-1}{n}} + (p_3)^{\frac{n-1}{n}} \frac{d}{dp_2}(p_2)^{\frac{1-n}{n}} = 0$$

$$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} \left(\frac{n-1}{n}\right)(p_2)^{\frac{n-1}{n}-1} + (p_3)^{\frac{n-1}{n}} \left(\frac{1-n}{n}\right)(p_2)^{\frac{1-n}{n}-1} = 0$$

or $$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} (p_2)^{-\frac{1}{n}} - (p_3)^{\frac{n-1}{n}} (p_2)^{\frac{1-2n}{n}} = 0$$

or $$(p_1)^{\frac{1-n}{n}} (p_2)^{-\frac{1}{n}} = (p_3)^{\frac{n-1}{n}} (p_2)^{\frac{1-2n}{n}}$$

or $$(p_2)^{-\frac{1}{n}+\frac{2n-1}{n}} = (p_1)^{\frac{n-1}{n}} (p_3)^{\frac{n-1}{n}}$$

or $$(p_2)^{\frac{2(n-1)}{n}} = (p_1 p_3)^{\frac{n-1}{n}}$$

Therefore,

$$p_2^2 = p_1 p_3 \qquad ...(25.38)$$

or $$\frac{p_2}{p_1} = \frac{p_3}{p_2} \qquad ...(25.39)$$

It is proved that for minimum compression work, the conditions required are the following

1. The pressure ratio of each stage should be the same.
2. The pressure ratio of any stage is the square root of overall pressure ratio, for a two stage compressor.
3. Air after compression in each stage should be cooled to intial temperature of air intake.
4. The work input to each stage is same.

Consider multistage compression with z stages. Then

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \ldots$$

$$= \frac{p_{(z+1)}}{p_z} = X \text{ (say)} \qquad ...(25.40)$$

Then $p_2 = Xp_1$; $p_3 = Xp_2 = X^2 p_1$;

$p_4 = Xp_3 = X^2 p_2 = X^3 p_1$

and $p_{(z+1)} = Xp_z = \ldots = X^z p_1$

or $$X^z = \frac{p_{(z+1)}}{p_1}$$

or $$X = \sqrt[z]{\frac{p_{(z+1)}}{p_1}}$$

$$= \sqrt[z]{\text{(Pressure ratio through compressor)}} \qquad ...(25.41)$$

25.12.6 Minimum Compression Work Input for Two-Stage Compression

Inserting Eq. (25.39) in Eq. (25.34), we get total minimum power,

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \quad ...(25.42)$$

= 2 × power required for one stage

In terms of overall pressure ratio

$$\frac{p_2}{p_1} = \frac{\sqrt{p_1 p_3}}{p_1} = \sqrt{\frac{p_3}{p_1}} = \left(\frac{p_3}{p_1}\right)^{\frac{1}{2}}$$

Total minimum power;

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right] \quad ...(25.43)$$

This expression can be extended to z stages of compression. Total minimum power;

$$IP_{min} = z \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_{(z+1)}}{p_1}\right)^{\frac{n-1}{zn}} - 1\right] \quad ...(25.44)$$

where the pressure ratio in each stage $= \left(\frac{p_{z+1}}{p_1}\right)^{\frac{1}{z}}$

Example 25.19 *Calculate the power required to compress 25 m³/min atmospheric air at 101.3 kPa, 20°C to a pressure ratio of 7 in an LP cylinder. Air is then cooled at constant pressure to 25°C in an intercooler, before entering HP cylinder, where air is again compressed to a pressure ratio of 6. Assume polytropic compression with n = 1.3 and R = 0.287 kJ/kg·K.*

Solution

Given A two-stage reciprocating air compressor with imperfect inercooler;

$\dot{V}_1 = 25$ m³/min $p_1 = 101.3$ kPa

$T_1 = 20°C = 293$ K $T_3 = 25°C = 298$ K

$\frac{p_2}{p_1} = 7.0$ $\frac{p_3}{p_2} = 6.0$

$n = 1.3$ $R = 0.287$ kJ/kg·K

To find Power input to compressor.

Analyis The mass of air compressed per minute, using perfect gas equation

$$\dot{m}_a = \frac{p_1 \dot{V}_1}{R T_1} = \frac{(101.3 \text{ kPa}) \times (25 \text{ m}^3/\text{min})}{(0.287 \text{ kJ/kg·K}) \times (293 \text{ K})} = 30.11 \text{ kg/min}$$

Temperature of air after first-stage compression;

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (293 \text{K}) \times \left(\frac{7}{1}\right)^{\frac{1.3-1}{1.3}} = 459.08 \text{ K}$$

The work input in LP cylinder compressor,

$$IP_{LP} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1) = \frac{1.3}{1.3-1} \times 30.11 \times 0.287 \times (459.08 - 300)$$

$= 6219.24$ kJ/min or **103.65 kW**

Temperature of air after second-stage compression;

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = (298 \text{K}) \times \left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} = 450.59 \text{ K}$$

The work input in HP cylinder of compressor,

$$IP_{HP} = \frac{n}{n-1} \dot{m}_a R (T_4 - T_3)$$

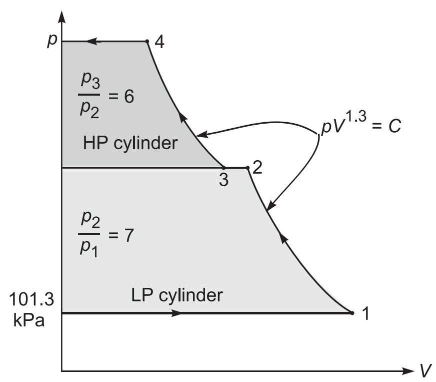


Fig. 25.21 *Two-stage compression*

$$= \frac{1.3}{1.3-1} \times 30.11 \times 0.287 \times (450.59 - 298)$$

$= 5714$ kJ/min or **95.23 kW**

Total power input to the compressor

$$IP = IP_{LP} + IP_{HP}$$
$$= 103.65 \text{ kW} + 95.23 \text{ kW} = \mathbf{198.88\ kW}$$

Example 25.20 *The LP cylinder of a two-stage double-acting reciprocating air compressor running at 120 rpm has a 50-cm diameter and 75-cm stroke. It draws air at a pressure of 1 bar and 20°C and compresses it adiabatically to a pressure of 3 bar. The air is then delivered to the intercooler, where it is cooled at constant pressure to 35°C and is then further compressed polytropically (index n = 1.3) to 10 bar in HP cylinder. Determine the power required to drive the compressor. The mechanical efficiency of the compressor is 90% and motor effieincy is 86%.*

Solution

Given A two-stage, double-acting reciprocating air compressor with imperfect intercooling;

$N = 120$ rpm	$d_1 = 50$ cm $= 0.5$ m
$L = 75$ cm $= 0.75$ m	$p_1 = 1$ bar
$T_1 = 20°C = 293$ K	$p_2 = 3$ bar
$p_1V_1^{\gamma} = p_2V_2^{\gamma}$	$p_2 = p_3 = 3$ bar
$T_3 = 35°C = 308$ K	$p_4 = 10$ bar
$p_3V_3^{n} = p_4V_4^{n}$ with $n = 1.3$	$k = 2$
$\eta_{mech} = 0.9$	$\eta_{Motor} = 0.86$

To find Motor power input to drive the compressor.

Assumptions

(i) The effect of the piston rod is negligible on the cylinder volume.

(ii) For air: $R = 0.287$ kJ/kg·K, $C_p = 1.005$ kJ/kg·K and $\gamma = 1.4$.

Analysis The volume-flow rate of air to LP cylinder

$$\dot{V}_1 = \frac{\pi}{4} d_{LP}^2 L \frac{N}{60} k$$

$$= \frac{\pi}{4} \times (0.5 \text{ m})^2 \times (0.7\,5\text{m}) \times \left(\frac{120}{60}\text{ rps}\right) \times 2$$

$= 0.589$ m^3/s

The density of incoming air

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(100 \text{ kPa})}{(0.287 \text{ kJ/kg}\cdot\text{K}) \times (293 \text{ K})}$$

$= 1.189$ kg/m^3

The mass-flow rate of air into LP cylinder

$$\dot{m}_a = \rho_1 \dot{V}_1 = 1.189 \times 0.589 = 0.7 \text{ kg/s}$$

Temperature of air after first-stage compression;

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (293\text{K}) \times \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}}$$

$= 401.04$ K

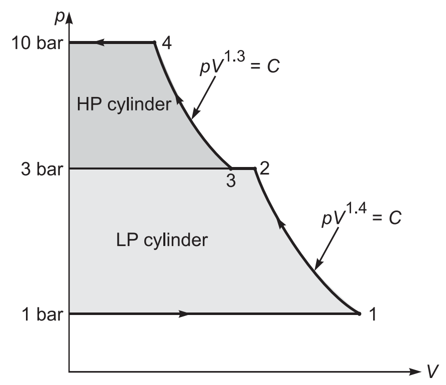


Fig. 25.22 *Two stage compression*

The *IP* input to LP cylinder,

$$IP_{LP} = \frac{\gamma}{\gamma-1} \dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1} \times 0.7 \times 0.287 \times (401.04 - 293)$$

$= 75.97$ kW

Temperature of air after second-stage compression;

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = (308\text{K}) \times \left(\frac{10}{3}\right)^{\frac{1.3-1}{1.3}}$$

$= 406.64$ K

The work input in HP cylinder of compressor,

$$IP_{HP} = \frac{n}{n-1} \dot{m}_a R(T_4 - T_3)$$

$$= \frac{1.3}{1.3-1} \times 0.7 \times 0.287 \times (406.64 - 308)$$

$= 85.87$ kW

Total power input to the compressor

$$IP = IP_{LP} + IP_{HP}$$
$$= 75.97 \text{ kW} + 85.87 \text{ kW} = 161.85 \text{ kW}$$

Motor power input

$$= \frac{IP}{\eta_{mech}\,\eta_{Motor}} = \frac{161.85}{0.9 \times 0.86}$$
$$= \mathbf{209.1\ kW}$$

Example 25.21 *Find the percentage saving in work input by compressing air in two stages from 1 bar to 7 bar instead of one stage. Assume a compression index of 1.35 in both the cases and optimum pressure and complete intercooling in a two-stage compressor.*

Solution

Given A two-stage air compressor with perfect intercooling

$p_1 =$ 1 r ba $\qquad p_3 = 7$ bar $\qquad n = 1.35$

To find Saving in work in comparison with single-stage compression.

Assumptions

(i) Single-acting reciprocating air compressor.

(ii) Compressions and expansions are reversible processes.

(iii) No effect of valve opening and closing on induction and delivery processes.

Analysis *Power required to drive compressor*

The minimum power input for two-stage compression with perfect intercooling;

$$IP_{multi} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a RT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

$$= 2 \times \left(\frac{1.35}{1.35-1}\right) \times \dot{m}_a RT \times \left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{2\times 1.35}} - 1\right]$$

$$= 2.213\ \dot{m}RT$$

Power input with single-stage compression from 1 bar to 7 bar;

$$IP_{single} = \frac{n}{n-1} \dot{m}_a RT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.35}{1.35-1} \times \dot{m}_a RT \times \left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}} - 1\right]$$

$$= 2.53\ \dot{m}RT$$

Percentage saving in power due to multistage compression

$$= \frac{IP_{single} - IP_{multi}}{IP_{single}} \times 100 = \frac{2.53 - 2.213}{2.53} \times 100$$

$$= \mathbf{12.56\%}$$

Example 25.22 *2 kg/s of air enters the LP cylinder of a two-stage, reciprocating air compressor. The overall pressure ratio is 9. The air at inlet to compressor is at 100 kPa and 35°C. The index of compression in each cylinder is 1.3. Find the intercooler pressure for perfect intercooling. Also, find the minimum power required for compression, and percentage saving over single-stage compression.*

Solution

Given A two-stage, single-acting, reciprocating air compressor with perfect intercooling

$\dot{m}_a = 2$ kg/s $\qquad p_1 = 100$ kPa

$T_1 = 35°\text{C} = 308$ K $\qquad p_3 = 9$ bar

$n = 1.3$

To find

(i) Intermediate pressure,

(ii) Power required to drive the compressor, and

(iii) Percentage saving in work in comparison with single-stage compression.

Assumptions

(i) For air; $R = 0.287$ kJ/kg·K and $C_p = 1$ kJ/kg·K.

(ii) No effect of valve opening and closing on induction and delivery processes.

Analysis

(i) *For perfect intercooling*

or $\quad p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 9} = 3$ bar

or $\quad \dfrac{p_2}{p_1} = 3$

(ii) *Power required to drive the compressor*

The minimum power input for two-stage compression with perfect intercooling;

$$IP_{multi} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}}-1\right]$$

$$= 2\times\left(\frac{1.3}{1.3-1}\right)\times 2\times 0.287\times 308\times\left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{2\times 1.3}}-1\right]$$

$$= \mathbf{442.1\ kW}$$

(iii) *Percentage saving in work of comparison with single-stage compression*

Power input with single stage compression from 1 bar to 9 bar;

$$IP_{single} = \frac{n}{n-1}\dot{m}_a RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$= \frac{1.3}{1.3-1}\times 2\times 0.287\times 308\times\left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= 505.92\ \text{kW}$$

Saving in power due to multistage compression

$$= IP_{single} - IP_{multi} = 505.92 - 442.1$$
$$= 63.82\ \text{kW}$$

Per cent saving

$$= \frac{IP_{multi} - IP_{single}}{IP_{single}}\times 100 = \frac{63.82}{505.92}\times 100$$

$$= \mathbf{12.61\%}$$

Example 25.23 *A two-stage, single-acting, reciprocating air compressor takes in air at 1 bar and 300 K. Air is discharged at 10 bar. The intermediate pressure is ideal for minimum work and perfect intercooling. The law of compression is $pV^{1.3}$ = constant. The rate of discharge is 0.1 kg/s. Calculate*

(a) power required to drive the compressor,
(b) saving in work in comparison with single stage compression,
(c) isothermal efficiency,
(d) heat transferred in intercooler.

Take $R = 0.287$ kJ/kg·K and $C_p = 1$ kJ/kg·K.

Solution

Given A two-stage, single-acting, reciprocating air compressor with perfect intercooling

$\dot{m}_a = 0.1$ kg/s	$p_1 = 1$ bar $= 100$ kPa
$T_1 = 300$ K	$p_3 = 10$ bar
$n = 1.3$	$R = 0.287$ kJ/kg·K
$C_p = 1$ kJ/kg·K	$k = 1$

To find

(i) Power required to drive the compressor,
(ii) Saving in work in comparison with single-stage compression,
(iii) Isothermal efficiency, and
(iv) Heat transferred in intercooler.

Assumptions

(i) Given conditions leads to perfect intercooling.
(ii) Compressions and expansions are reversible processes.
(iii) No effect of valve opening and closing on induction and delivery processes.

Analysis For perfect intercooling

or $$p_2 = \sqrt{p_1\times p_3} = \sqrt{1\times 10} = 3.162\ \text{bar}$$

or $$\frac{p_2}{p_1} = 3.162$$

(i) *Power required to drive compressor*

The minimum power input for two-stage compression with perfect intercooling;

$$IP_{multi} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}}-1\right]$$

$$= 2\times\left(\frac{1.3}{1.3-1}\right)\times 0.1\times 0.287\times 300 \times\left[\left(\frac{10}{1}\right)^{\frac{1.3-1}{2\times 1.3}}-1\right]$$

$$= \mathbf{22.7\ kW}$$

Power input with single-stage compression from 1 bar to 10 bar;

$$IP_{single} = \frac{n}{n-1}\dot{m}_a RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$= \frac{1.3}{1.3-1}\times 0.1\times 0.287\times 300 \times\left[\left(\frac{10}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= 26.16\ \text{kW}$$

(ii) Saving in power due to multistage compression

$= IP_{single} - IP_{multi} = 26.16 - 22.7$

$= \mathbf{3.46\,kW}$

Isothermal power input,

$$IP_{iso} = \dot{m}_a R T_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.1 \times 0.287 \times 300 \times \ln\left(\frac{10}{1}\right)$$

$$= 19.825\,\text{kW}$$

(iii) Isothermal efficiency;

$$\eta_{iso} = \frac{IP_{iso}}{IP_{act}} = \frac{19.825}{22.7}$$

$= 0.8733$ or $\mathbf{87.33\%}$

Temperature after first-stage compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (300\,\text{K}) \times \left(\frac{3.162}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$= 391.3\,\text{K}$$

(iv) Heat rejection rate in the intercooler

$$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$$

$$= 0.1 \times 1.0 \times (391.3 - 300)$$

$$= \mathbf{9.13\,kW}$$

Example 25.24 *In a three-stage compressor, air is compressed from 98 kPa to 20 bar. Calculate for 1 m^3 of air per second,*

(a) Work under ideal condition for n = 1.3,
(b) Isothermal work,
(c) Saving in work due to multi staging,
(d) Isothermal efficiency.

Solution

Given An ideal three-stage reciprocating air compressor

$p_1 = 98$ kPa $\quad p_2 = 20$ bar $= 2000$ kPa

$n = 1.3 \quad \dot{V}_1 = 1$ m^3/s

To find

(i) Power input to compressor,
(ii) Isothermal work,
(iii) Saving in work due to multi-staging, and
(iv) Isothermal efficiency.

Assumptions

(i) Neglecting clearance volume.
(ii) Perfect inetercooling.

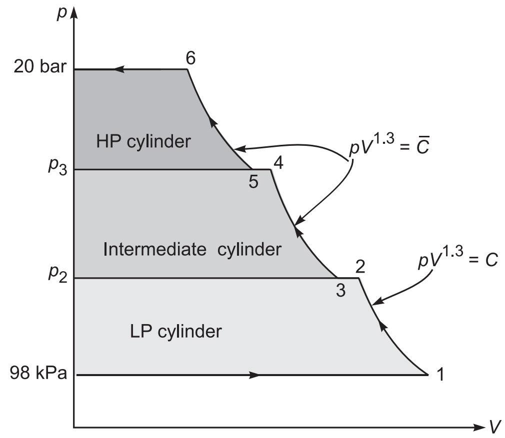


Fig. 25.23

Analysis For perfect intercooling

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \left(\frac{p_4}{p_1}\right)^{\frac{1}{3}} = \left(\frac{2000}{98}\right)^{\frac{1}{3}} = 2.732$$

(i) *Power input to compressor*

$$IP_{3,\,stage} = 3 \times \left(\frac{n}{n-1}\right) p_1 \dot{V}_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{3n}} - 1\right]$$

$$= 3 \times \left(\frac{1.3}{1.3-1}\right) \times (98\,\text{kPa})$$

$$\times (1\,\text{m}^3/\text{s}) \times \left[\left(\frac{2000}{98}\right)^{\frac{1.3-1}{3\times 1.3}} - 1\right]$$

$$= \mathbf{332.62\,kW}$$

(ii) *Isothermal work*

$$IP_{iso} = p_1 \dot{V}_1 \ln\left(\frac{p_4}{p_1}\right)$$

$$= (98\,\text{kPa}) \times (1\,\text{m}^3/\text{s}) \times \ln\left(\frac{2000}{98}\right)$$

$$= \mathbf{295.56\,kW}$$

Power required in single-stage compression,

$$IP_{1,\,stage} = \left(\frac{n}{n-1}\right) p_1 \dot{V}_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{1.3}{1.3-1}\right) \times 98 \times 1 \times \left[\left(\frac{2000}{98}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$= 427.08$ kW

(iii) *Saving in work due to multistaging*

$= IP_{1,stage} - IP_{3,stage}$

$= 427.08$ kW $- 332.62$ kW $=$ **94.42 kW**

Percentage saving;

$$= \frac{\text{Saving}}{IP_{1,stage}} \times 100 = \frac{94.42}{427.08} \times 100$$

= **22.1%**

(iv) *Isothermal efficiency*

$$\eta_{iso} = \frac{IP_{iso}}{IP_{act}} \times 100 = \frac{295.56}{332.62} \times 100$$

= **88.85%**

Example 25.25 *A two-stage, single-acting reciprocating air compressor has an LP cylinder bore and stroke of 250 mm each. The clearance volume of a low-pressure cylinder is 5% of the stroke volume of the cylinder. The intake pressure and temperature are 1 bar and 17°C, respectively. Delivery pressure is 9 bar and the compressor runs at 300 rpm. The polytropic index is 1.3 throughout. The intercooling is complete and intermediate pressure is 3 bar. The overall efficiency of the plant including electric driving motor is 70%. Calculate*

(a) the mass-flow rate through the compressor, and

(b) energy input to electric motor.

Solution

Given Two-stage, single-acting reciprocating air compressor

LP cylinder: $d_1 = 250$ mm $= 0.25$ m

$L_1 = 250$ mm $= 0.25$ m

$c_1 = 0.05\ V_s$ or $c = 0.05$

$p_1 = 1$ bar $= 100$ kPa

$T_1 = 17°C = 290$ K

$p_2 = 3$ bar $\quad p_3 = 9$ bar

$n = 1.3 \quad k = 1$

$N = 300$ rpm $\quad \eta_{Overall} = 0.7$

To find

(i) The mass-flow rate of air through the compressor,

(ii) Power input to electric motor.

Assumptions

(i) Air as an ideal gas, with $R = 0.287$ kJ/kg·K.

(ii) Compression and expansion are reversible polytropic.

Analysis The stroke (swept) volume of LP cylinder

$$V_s = \left(\frac{\pi}{4}\right) d_1^2 L_1 = \left(\frac{\pi}{4}\right) \times (0.25\,\text{m})^2 \times (0.25\,\text{m})$$

$= 0.01227\ \text{m}^3$

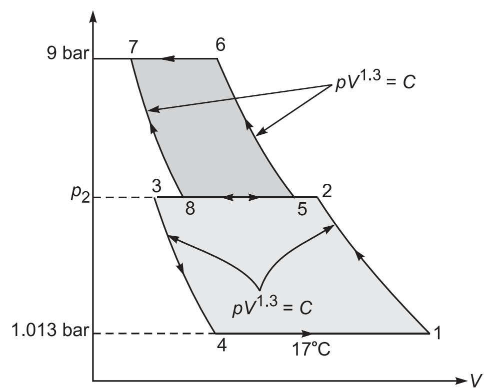


Fig. 25.24 *Two-stage compression with clearance*

The volumetric efficiency of LP cylinder is given by

$$\eta_{vol,\,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.05 - 0.05 \times \left(\frac{3}{1}\right)^{\frac{1}{1.3}} = 0.933$$

It is also expressed as

$$\eta_{vol,\,LP} = \frac{V_1 - V_4}{V_s}$$

or $\quad V_1 - V_4 = \eta_{vol,\,LP}\, V_s = 0.933 \times 0.01227$

$= 0.01145\ \text{m}^3$

The mass of air inducted per cycle into LP cylinder

$$m_a = \frac{p_1(V_1 - V_4)}{RT_1}$$

or $$m_a = \frac{(100\ \text{kPa}) \times (0.01145\ \text{m}^3)}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (290\ \text{K})}$$

$= 0.0137$ kg/cycle

(i) *Mass-flow rate of air*

The mass flow rate of air per minute

$\dot{m}_a = m_a N k$

$= 0.0137 \times 300 \times 1 = 4.13\,\text{kg/min}$
$= \mathbf{0.069\,kg/s}$

(ii) *Motor power input*

The indicated power input to the compressor with

$$\frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2}$$

$$IP = \frac{2n}{n-1}\,\dot{m}_a\,RT\left[\left(\frac{p_2}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{2\times 1.3}{1.3-1}\times 0.069\times 0.287\times 290 \times\left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= 14.32\text{ kW}$$

Motor power

$$= \frac{IP}{\eta_{Overall}} = \frac{14.32}{0.7} = \mathbf{20.46\,kW}$$

25.13 CYLINDER DIMENSIONS OF A MULTISTAGE COMPRESSOR

In an air compressor, the mass of air inducted through each cylinder is same. Therefore,

$$m_1 = m_2 = m_3 = \ldots$$

or $$\rho_1(V_1 - V_4) = \rho_2(V_5 - V_8) = \rho_3(V_9 - V_{12}) = \ldots \quad \ldots(25.45)$$

where $(V_1 - V_4)$, $(V_5 - V_8)$ and $(V_9 - V_{12})$ represent effective suction volume per cycle taken in LP, intermediate, and HP cylinders, respectively and ρ_1, ρ_2, ρ_3 are corresponding densities of air. The density of air can be expressed as

$$\rho = \frac{p}{RT}$$

$$\therefore \quad \frac{p_1}{RT_1}(V_1 - V_4) = \frac{p_2}{RT_3}(V_5 - V_8) = \frac{p_3}{RT_5}(V_9 - V_{12}) = \ldots$$

For perfect intercooling, isothermal conditions prevail, i.e.,

$$T_1 = T_3 = T_5 = \ldots$$

$$\therefore \quad p_1(V_1 - V_4) = p_2(V_5 - V_8) = p_3(V_9 - V_{12}) = \ldots \quad \ldots(25.46)$$

Introducing volumetric efficiency of respective cylinders;

$$p_1\eta_{vol_1}V_{s_1} = p_2\eta_{vol_2}V_{s_2} = p_3\eta_{vol_3}V_{s_3} = \ldots \quad \ldots(25.47)$$

Piston displacement volume as

$$V_s = \left(\frac{\pi}{4}\right)d^2 L\text{, then}$$

$$p_1\eta_{vol_1}\left(\frac{\pi}{4}\right)d_1^2 L_1 = p_2\eta_{vol_2}\left(\frac{\pi}{4}\right)d_2^2 L_2 = p_3\,\eta_{vol_3}\left(\frac{\pi}{4}\right)d_3^2 L_3 = \ldots \quad \ldots(25.48)$$

Usually, the stroke length for all cylinders is same, i.e.,

$$L_1 = L_2 = L_3 = \ldots$$

$$\therefore \quad p_1\eta_{vol_1}d_1^2 = p_2\eta_{vol_2}d_2^2 = p_3\eta_{vol_3}d_3^2 = \ldots \quad \ldots(25.49)$$

If all cylinders have same clearance ratio, then

$$\eta_{vol_1} = \eta_{vol_2} = \eta_{vol_3} = \ldots$$

and $$p_1 d_1^2 = p_2 d_2^2 = p_3 d_3^2 = \ldots \quad \ldots(25.50)$$

Equations (25.49) and (25.50) are used to calculate the cylinder dimension for multi-stage compressors.

Example 25.26 *A two-stage, single-acting, reciprocating air compressor with complete intercooling receives atmospheric air at 1 bar and 15°C, compresses it polytropically (n = 1.3) to 30 bar. If both cylinders have the same stroke, calculate the diameter of the HP cylinder, if the diameter of the LP cylinder is 300 mm.*

Solution

Given Two-stage, single-acting reciprocating air compressor

LP cylinder: $d_1 = 300$ mm $\quad p_1 = 1$ bar
$T_1 = 15°\text{C} = 288\,\text{K} \quad T_3 = 15°\text{C} = 288\,\text{K}$
$p_3 = 30$ bar $\quad n = 1.3 \quad k = 1$

To find Diameter of HP cylinder.

Assumptions

(i) Compression in both cylinders is reversible.
(ii) Negligible clearance in both cylinders.
(iii) No effect of valve opening and closing on induction and delivery processes.

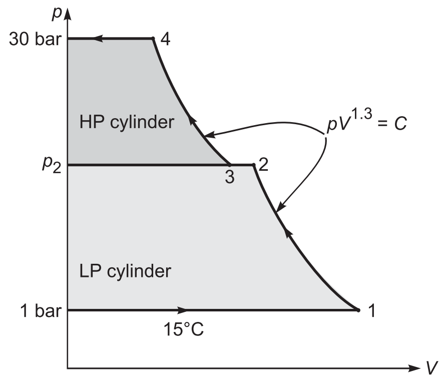


Fig. 25.25

Analysis For perfect intercooling, the pressure ratio per stage

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \sqrt{p_1 \times p_3} = \sqrt{1 \times 30} = 5.477$$

$$p_2 = 5.477\, p_1 = 5.477 \times 1 = 5.477\,\text{bar}$$

The total volume of LP cylinder

$$V_1 = V_{s,LP} = \left(\frac{\pi}{4}\right) d_1^2\, L$$

The total volume of HP cylinder

$$V_3 = V_{s,HP} = \left(\frac{\pi}{4}\right) d_2^2\, L$$

Equation (25.46), for perfect intercooling, without clearance leads to

$$p_1 V_1 = p_2 V_3$$

or $$V_{s,HP} = V_{s,LP}\left(\frac{p_1}{p_2}\right) = V_{s,LP} \times \frac{1}{5.477}$$

$$= 0.1825\, V_{s,LP}$$

Without clearance, with same pressure ratio and with perfect intercooling, the volumetric efficiency will be same. Expressing the above equation in terms of cylinder diameters

$$\left(\frac{\pi}{4}\right) d_{HP}^2\, L = 0.1825 \times \left(\frac{\pi}{4}\right) \times (300\text{ mm})^2 \times L$$

or $$d_{HP} = \sqrt{0.1825 \times (300\text{ mm}^2)}$$

$$= \mathbf{128.18\,mm}$$

Example 25.27 *A two-stage, single-acting, reciprocating air compressor receives atmospheric air at 15°C, compresses it isentropically in the LP cylinder to intermediate pressure, where air cools to its initial temperature and then again compresses polytropically (n = 1.3) in the HP cylinder. The clearance volume and pressure ratio in both cylinders are 5% of the swept volume and 2, respectively. Determine the stroke volume of HP cylinder for 60 litre swept volume of LP cylinder.*

Solution

Given Two-stage, single-acting reciprocating air compressor

$V_{s,LP} = 60$ litres $\qquad V_3 = 0.05\, V_{s,LP} = 3$ litres

$T_1 = 15°\text{C} = 288\,\text{K}$

$T_3 = 15°\text{C} = 288\,\text{K}$

$\frac{p_2}{p_1} = 2.0 \qquad \frac{p_3}{p_2} = 2.0$

$V_7 = 0.05\, V_{s,HP} \qquad n = 1.3 \qquad k = 1$

To find Stroke volume ($V_5 - V_7$) of HP cylinder.

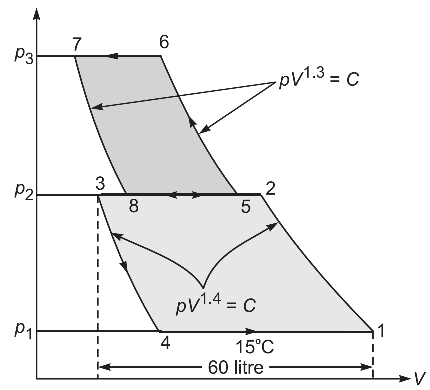


Fig. 25.26

Assumptions

(i) For air $\gamma = 1.4$.

(ii) For LP cylinder, the re-expansion of air is isentropic.

(iii) For HP cylinder, the re-expansion of air is polytropic.

Analysis The total volume of LP cylinder

$$V_1 = V_{s,LP} + V_3 = 60\text{ lit} + 3\text{ lit} = 63\text{ litres}$$

The volume of air after first-stage compression

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

$$= (63 \text{ lit}) \times \left(\frac{1}{2}\right)^{\frac{1}{1.4}} = 38.4 \text{ litres}$$

The temperature of air after first-stage compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (288\,\text{K}) \times \left(\frac{2}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 351 \text{ K}$$

The total volume of HP cylinder

$$V_5 = V_{c,HP} + V_{s,HP} = V_{c,HP} + 20\, V_{c,HP}$$

$$= V_7 + 20\, V_7 = 21\, V_7$$

The volume of air after re-expansion in HP cylinder

$$V_8 = V_7\left(\frac{p_7}{p_8}\right)^{\frac{1}{n}} = V_7 \times \left(\frac{2}{1}\right)^{\frac{1}{1.3}} = 1.704\, V_7$$

In intercooling, air is cooled to initial temperature, and volume of air reduces from $(V_2 - V_3)$ to $(V_5 - V_8)$ at constant pressure. Therefore,

$$\frac{V_2 - V_3}{T_2} = \frac{V_5 - V_8}{T_1}$$

or $$V_5 - V_8 = (V_2 - V_3)\frac{T_1}{T_2} = (38.4 - 3) \times \frac{288}{351}$$

$$= 29.046 \text{ litres}$$

or $21\, V_7 - 1.704\, V_7 = 29.046$ litres

or $$V_7 = \frac{29.046}{21 - 1.704} = 1.505$$

or $$V_5 = 21 \times 1.505 = 31.61 \text{ litres}$$

Stroke volume of HP cylinder

$$V_5 - V_7 = 31.61 - 1.505 = \mathbf{30.10 \text{ litres}}$$

Example 25.28 *In a trial on a two-stage, single-acting, reciprocating air compressor, following data were recorded:*

Free air delivery per min = *6 m^3*
Free air conditions = *1 bar, 27°C*
Delivery pressure = *30 bar*
Compressor speed = *300 rpm*
Intermediate pressure = *6 bar*
Temperature at the inlet of HP cylinder = *27°C*
Law of compression = *$pV^{1.3}$*
Mechanical efficiency = *85%*
Stroke to bore ratio for LP cylinder = *1:2*
Stroke of HP cylinder = *Stroke of LP cylinder*

Calculate

(a) Cylinder diameters,
(b) Power input, neglecting clearance volume.

Solution

Given Two-stage, single-acting reciprocating air compressor $k = 1$

$FAD = \dot{V}_1 = 6\,\text{m}^3/\text{min}$	$p_1 = 1 \text{ bar} = 100 \text{ kPa}$
$T_1 = 27°\text{C} = 300\,\text{K}$	$p_2 = 6 \text{ bar}$
$p_3 = 30 \text{ bar}$	$T_3 = 27°\text{C} = 300\,\text{K}$
$n = 1.3$	$N = 300 \text{ rpm}$
$\eta_{mech} = 0.85$	

LP cylinder: $L_1/d_1 = 1.2$
and $L_2 = L_1$
and Negligible clearance

To find

(i) Cylinder dimensions,
(ii) Power input to compressor.

Assumptions

(i) Air as an ideal gas, with $R = 0.287$ kJ/kg·K.
(ii) Compression and expansion are reversible polytropic.
(iii) Volumetric efficiency of both cylinders as 100%.

Analysis

(i) *Cylinder diameters*

The stroke (swept) volume rate of LP cylinder with $\eta_{vol} = 1.0$

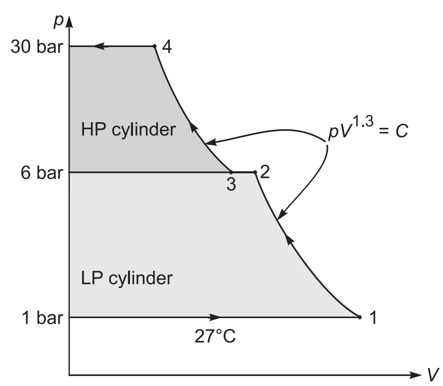


Fig. 25.27

$$\dot{V}_1 = FAD = \left(\frac{\pi}{4}\right) d_1^2 L_1 N k$$

or $$6 \text{ m}^3/\text{min} = \left(\frac{\pi}{4}\right) \times d_1^2 \times (1.2 d_1) \times 300 \times 1$$

or $\quad d_1 = 0.276$ m or **276 mm**

For same stroke length and with $\eta_{vol} = 1.0$, the diameters of *LP* and *HP* cylinders are related as

$$p_1 d_1^2 = p_2 d_2^2$$

$$1 \times (276)^2 = 6 \times d_2^2$$

or $\quad d_2 = \mathbf{112.67\ mm}$

(ii) *Power input to compressor*

The mass-flow rate of air into compressor

$$\dot{m}_a = \frac{p_1 \dot{V}_1}{R T_1} = \frac{(100 \text{ kPa}) \times (6 \text{ m}^3/\text{min})}{(0.287 \text{ kJ/kg}\cdot\text{K}) \times (300 \text{ K})}$$

$$= 6.9686 \text{ kg/min} \quad \text{or} \quad 0.116 \text{ kg/s}$$

The indicated power input to two-stage, single-acting air compressor

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right]$$

$$= \frac{1.3}{1.3-1} \times 0.116 \times 0.287 \times 300 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} + \left(\frac{30}{6}\right)^{\frac{1.3-1}{1.3}} - 2 \right]$$

$$= 84.9 \text{ kW}$$

Brake power input

$$= \frac{IP}{\eta_{mech}} = \frac{84.9 \text{ kW}}{0.85} = \mathbf{99.88\ kW}$$

Example 25.29 *In a single-acting, two-stage reciprocating air compressor handles 4.5 kg of air per minute, and compresses it from 1.013 bar 17°C through a pressure ratio of 9. The index of compression and expansion in both stages is 1.3. If the intercooling is complete, find the minimum indicated power and cylinder swept volumes required. Assume the clearance volume of both the stages are 5% of their respective stroke volumes and the compressor runs at 300 rpm. Take R = 0.287 kJ/kg·K.*

Solution

Given A single-acting, two-stage reciprocating air compressor with perfect intercooling

$\dot{m}_a = 4.5$ kg/min $= 0.075$ kg/s

$k = 1$

$p_1 = 1.013$ bar $= 101.3$ kPa

$T_1 = 17°\text{C} = 290$ K

$\dfrac{p_3}{p_1} = 9 \qquad n = 1.3$

$c_1 = c_2 = 0.05\, V_s$

$N = 300$ rpm $\qquad R = 0.287$ kJ/kg·K

To find

(i) Minimum indicated power, and

(ii) Swept volumes of LP and HP cylinders.

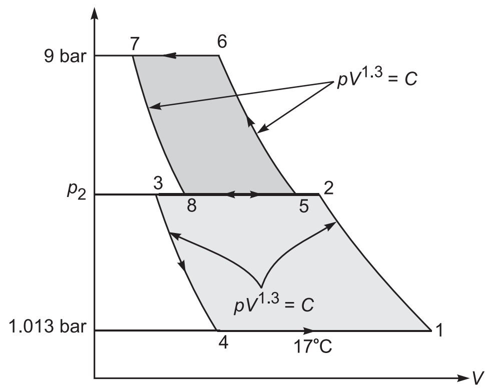


Fig. 25.28

Analysis

(i) *Minimum indicated power*

For perfect intercooling

$$p_2 = \sqrt{p_1 \times p_3} = \sqrt{p_1 \times 9 p_1} = 3 p_1$$

or $$\frac{p_2}{p_1} = 3$$

The minimum power input, Eq. (25.42)

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= 2 \times \left(\frac{1.3}{1.3-1}\right) \times 0.075 \times 0.287 \times 290 \times \left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1 \right] = \mathbf{15.61\ kW}$$

(ii) *The stroke (swept) volume of LP cylinder*

The mass of air inducted per cycle into LP cylinder

$$m_a = \frac{\dot{m}_a}{N\,k} = \frac{4.5 \text{ kg/min}}{(300 \text{ rotation/min}) \times 1}$$

$= 0.015$ kg/cycle

Efffective swept volume of LP cylinder;

$$(V_1 - V_4) = \frac{m_a R T_1}{p_1} = \frac{0.015 \times 0.287 \times 290}{101.3}$$

$= 0.0123 \text{ m}^3$/cycle

The volumetric efficiency of LP and HP cylinders

$$\eta_{vol} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.05 - 0.05 \times \left(\frac{3}{1}\right)^{\frac{1}{1.3}} = 0.933$$

It is also expressed as

$$\eta_{vol} = \frac{V_1 - V_4}{V_{s,\,LP}}$$

or $$V_{s,LP} = \frac{V_1 - V_4}{\eta_{vol}} = \frac{0.0123}{0.933}$$

$= \mathbf{0.0131 \text{ m}^3\text{/cycle}}$

The swept volume of LP cylinder is 0.0131 m^3/cycle.

Using Eq. (25.46) for ratio of LP and HP cylinders,

$$p_1(V_1 - V_4) = p_2(V_5 - V_8)$$

$$V_5 - V_8 = \frac{p_1}{p_2}(V_1 - V_4) = \frac{1}{3} \times (0.0123)$$

$= 0.0041 \text{ m}^3$/cycle

Then swept volume of HP cylinder

$$V_{s,HP} = \frac{V_5 - V_8}{\eta_{vol}} = \frac{0.0041}{0.933} = \mathbf{0.00440 \text{ m}^3}$$

Example 25.30 *A single-acting, two-stage reciprocating air compressor with complete intercooling delivers 10.5 kg/min of air at 16 bar. The compressor takes in air at 1 bar and 27°C. The compression and expansion follow the law $pV^{1.3}$ = Const. Calculate*

(a) Power required to drive the compressor

(b) Isothermal efficiency

(c) Free air delivery

(d) Heat transferred in intercooler

If the compressor runs at 440 rpm, the clearance ratios for LP and HP cylinders are 0.04 and 0.06, respectively, calculate the swept and clearance volumes for each cylinder.

Solution

Given A single-acting, two-stage reciprocating air compressor with perfect intercooling

$\dot{m}_a$ = 10.5 kg/min = 0.175 kg/s

p_1 = 1 bar = 100 kPa T_1 = 27°C = 300 K

p_3 = 16 bar n = 1.3

c_1 = 0.04 c_2 = 0.06

N = 440 rpm k = 1

To find

(i) Indicated power,

(ii) Isothermal efficiency,

(iii) FAD,

(iv) Heat transfer in intercooler, and

(v) Swept volumes of LP and HP cylinders.

Assumptions

(i) Air an ideal gas, $R = 0.287$ kJ/kg·K,

(ii) Compressions and expansions are reversible processes, and

(iii) Suction takes place at free air conditions.

Analysis For perfect intercooling, the pressure ratio per stage

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \sqrt{p_1 \times p_3} = \sqrt{1 \times 16} = 4$$

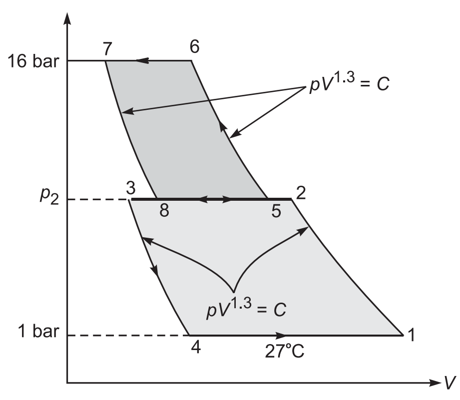


Fig. 25.29

(i) *Indicated power input for two-stage compressor*

$$IP_{min} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$= 2\times\left(\frac{1.3}{1.3-1}\right)\times 0.175\times 0.287 \times 300\times\left[\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= \mathbf{49.23\ kW}$$

(ii) *Isothermal efficiency*

Power input for isothermal compression from 1 bar to 16 bar pressure

$$IP_{iso} = \dot{m}_a RT_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.175\times 0.287\times 300\times \ln\left(\frac{16}{1}\right)$$

$$= 41.77\ \text{kW}$$

$$\eta_{iso} = \frac{\text{Isothermal power}}{\text{Actual power}} = \frac{41.77\ \text{kW}}{49.23\ \text{kW}}$$

$$= 0.848 \text{ or } \mathbf{84.8\%}$$

(iii) *Free air delivery* (*FAD*)

$$\dot{V}_f = \dot{V}_1 - \dot{V}_4 = \frac{\dot{m}_a RT_1}{p_1}$$

$$= \frac{(10.5\ \text{kg/min})\times(0.287\ \text{kJ/kg}\cdot\text{K})\times(300\ \text{K})}{(100\ \text{kPa})}$$

$$= \mathbf{9.04\ m^3/min}$$

(iv) *Heat transferred in intercooler*

Temperature after compression in each stage

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300\times\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$= 413.1\ \text{K}$$

Heat transfer rate in intercooler

$$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$$

$$= 10.5\times 1.005\times(413.1-300)$$

$$= 1193.5\ \text{kJ/min or} = \mathbf{19.89\ kW}$$

(v) *Swept volumes*

The volumetric efficiency of *LP* cylinder

$$\eta_{vol,LP} = 1 + c_1 - c_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04\times\left(\frac{4}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.923 \text{ or } 92.3\%$$

The volume handled per cycle by *LP* cylinder

$(V_1 - V_4)$

$$= \frac{FAD}{\text{No. of cycle per minute}} = \frac{\dot{V}_f}{Nk} = \frac{9.04}{440\times 1}$$

$$= 0.0205\ \text{m}^3/\text{cycle}$$

Further, it can also be expressed as

$$\eta_{vol,LP} = \frac{V_1 - V_4}{V_{s,LP}}$$

or $$V_{s,LP} = \frac{V_1 - V_4}{\eta_{vol,LP}} = \frac{0.0205}{0.923}$$

$$= \mathbf{0.0222\ m^3/cycle}$$

For *HP* cylinder

$$\eta_{vol,HP} = 1 + c_2 - c_2\left(\frac{p_3}{p_2}\right)^{\frac{1}{n}}$$

$$= 1 + 0.06 - 0.06\times\left(\frac{4}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.885 \text{ or } 85.5\%$$

The effective swept volume handled by *HP* cylinder with perfect intercooling

$$p_2(V_5 - V_8) = p_1(V_1 - V_4)$$

or $$(V_5 - V_8) = \frac{p_1}{p_2}\times(V_1 - V_4)$$

$$= \frac{1}{4}\times 0.0205 = 0.005125\ \text{m}^3/\text{cycle}$$

The piston displacement volume of *HP* cylinder;

$$V_{s,HP} = \frac{V_5 - V_8}{\eta_{vol,HP}} = \frac{0.005125}{0.885}$$

$$= \mathbf{0.0058\ m^3/cycle}$$

Example 25.31 *A two-stage, double-acting, reciprocating air compressor operating at 300 rpm, receives air at 1 bar and 27°C. The bore of LP cylinder is 360 mm and its stroke is 400 mm. Both cylinders have equal stroke and equal clearance of 4% . The LP cylinder discharges air at a pressure of 5 bar. The air then passes through an intercooler to cool air to its initial temperature. Pressure drops in intercooler to 4.75 kPa. Finally air is discharged from HP cylinder at 15 bar. The index* *of compression and expansion in both cylinders is 1.3. C_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.*

Calculate

(a) Heat rejected in the intercooler,
(b) Diameter of HP cylinder,
(c) Power required to drive HP cylinder.

Solution

Given Two-stage, double-acting reciprocating air compressor

$k = 2$	$N = 300$ rpm
$p_1 = 1$ bar $= 100$ kPa	$T_1 = 27°\text{C} = 300$ K
$p_2 = 5$ bar	$p_5 = 4.75$ bar
$T_3 = 27°\text{C} = 300$ K	$p_6 = 15$ bar
$n = 1.3$	$d_1 = 360$ mm
$L_1 = 400$ mm	$c_1 = c_2 = 0.04$
$C_p = 1.005$ kJ/kg·K	$R = 0.287$ kJ/kg·K
$L_2 = L_1$	

To find

(i) Heat rejected in the intercooler,
(ii) Diameter of HP cylinder, and
(iii) Power required to drive HP cylinder.

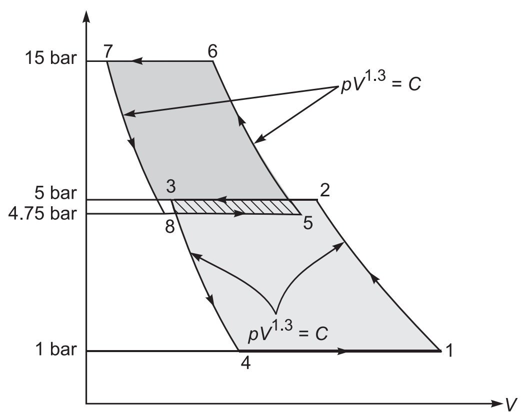


Fig. 25.30

Analysis The stroke (swept) volume per minute of *LP* cylinder

$$\dot{V}_{s,LP} = \dot{V}_1 - \dot{V}_3 = \left(\frac{\pi}{4}\right) d_1^2 L_1 N k$$

$$= \left(\frac{\pi}{4}\right) \times (0.3\ 6\text{m})^2 \times (0.4\text{ m}) \times 300 \times 2$$

$$= 24.42 \text{ m}^3/\text{min}$$

The pressure ratio for *LP* cylinder;

$$\frac{p_2}{p_1} = 5$$

The volumetric efficiency of *LP* cylinder;

$$\eta_{vol,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{5}{1}\right)^{\frac{1}{1.3}} = 0.902$$

It is also expressed using effective stroke volume rate $(\dot{V}_1 - \dot{V}_4)$ of *LP* cylinder as

$$\eta_{vol,LP} = \frac{\dot{V}_1 - \dot{V}_4}{\dot{V}_{s,LP}}$$

or $\dot{V}_1 - \dot{V}_4 = \dot{V}_{s,LP} \times \eta_{vol,LP}$

$$= (24.42\text{ kg/min}) \times 0.902 = 22.02 \text{ m}^3/\text{min}$$

The mass flow rate of air into the *LP* cylinder;

$$\dot{m}_a = \frac{p_1(\dot{V}_1 - \dot{V}_4)}{RT_1} = \frac{(100\text{ kPa}) \times (22.02\text{ m}^3/\text{min})}{(0.287\text{ kJ/kg}\cdot\text{K}) \times (300\text{ K})}$$

$$= 25.584 \text{ kg/min} \quad \text{or} \quad 0.426 \text{ kg/s}$$

Temperature after compression in each stage

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300 \times \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$= 434.93 \text{ K}$$

(i) *Heat transfer rate in the intercooler*

$$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$$

$$= 25.584 \times 1.005 \times (434.93 - 300)$$

$$= 3469.33 \text{ kJ/min} \quad \text{or} \quad \mathbf{57.82\ kW}$$

(ii) *Diameter of HP cylinder*

The effective swept volume rate of *HP* cylinder

$$\dot{V}_5 - \dot{V}_8 = \frac{\dot{m}_a R T_1}{p_5}$$

$$= \frac{25.584 \times 0.287 \times 300}{475}$$

$$= 4.637 \text{ m}^3/\text{min}$$

The volumetric efficiency of *HP* cylinder;

$$\eta_{vol,HP} = 1 + c_2 - c_2 \left(\frac{p_3}{p_5}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{15}{4.75}\right)^{\frac{1}{1.3}} = 0.943$$

Further, the volumetric efficiency can be expressed in terms of piston displacement volume rate of *HP* cylinder as

$$\eta_{vol,\,HP} = \frac{\dot{V}_5 - \dot{V}_8}{\dot{V}_{s,\,HP}}$$

$$\text{or}\quad \dot{V}_{s,HP} = \frac{\dot{V}_5 - \dot{V}_8}{\eta_{vol,HP}} = \frac{4.637}{0.943} = 4.917\ \text{m}^3/\text{min}$$

which can be further, expressed as

$$\dot{V}_{s,HP} = \left(\frac{\pi}{4}\right) d_2^{\,2} L_1 N k$$

$$\text{or}\quad 4.917 = \left(\frac{\pi}{4}\right) \times d_2^2 \times 0.4 \times 300 \times 2$$

$$\text{or}\quad d_2 = 0.1615\ \text{m} \quad \text{or} \quad \mathbf{161.5\ mm}$$

(iii) *Power required to drive HP cylinder*

$$IP = \frac{n}{n-1} p_5 (\dot{V}_5 - \dot{V}_8)\left[\left(\frac{p_3}{p_5}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 4.75 \times 10^2 \times \left(\frac{4.637}{60}\right) \times \left[\left(\frac{15}{4.75}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{48.34\ kW}$$

Example 25.32 *A three-stage, double-acting, reciprocating air compressor operating at 300 rpm, receives air at 1 bar and 27°C. The bore of LP cylinder is 360 mm and its stroke is 400 mm. Intermediate cylinder and HP cylinder have same stroke as LP cylinder. The clearance volume in each cylinder is 4 % of the stroke volume . The LP cylinder dischrges air at a pressure of 5 bar, the intermediate cylinder discharges at 20 bar and air is finally discharged by the HP cylinder at 75 bar. The air is cooled in intercoolers to initial temperature after each stage of compression. A Pressure drop of 0.2 bar takes place in intercooler after each stage. The index of compression and expansion for an LP cylinder is 1.3, for intermediate cylinder is 1.32 and for HP cylinder is 1.35. Neglect the effect of piston rod and assume C_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.*

Calculate

(a) Heat rejected in each stages in intercooler and during compression,

(b) Heat rejected in after-cooler, if delivered air is cooled to initial temperature,

(c) Diameter of intermediate and HP cylinders,

(d) Power required to drive compressor, if its mechanical efficiency is 85%.

Solution

Given Three-stage, double-acting reciprocating air compressor

$k = 2$	$N = 300$ rpm
$p_1 = 1$ bar = 100 kPa	$T_1 = 27°\text{C} = 300$ K
$p_2 = 5$ bar	$p_5 = 4.8$ bar
$T_3 = 27°\text{C} = 300$ K	$p_6 = 20$ bar
$p_9 = 19.8$ bar	$p_{10} = 75$ bar
$n_1 = 1.3$	$n_2 = 1.32$
$n_3 = 1.35$	$d_1 = 360$ mm
$L_1 = 400$ mm	$c_1 = c_2 = c_3 = 0.04$
$C_p = 1.005$ kJ/kg·K	$R = 0.287$ kJ/kg·K
$L_3 = L_2 = L_1$	$\eta_{mech} = 0.85$

To find

(i) Heat rejected in each stage in intercooler and during compression,

(ii) Heat rejected in after-cooler,

(iii) Diameter of intermediate and HP cylinders,

(iv) Power required to drive compressor.

Analysis The stroke (swept) volume per minute of *LP* cylinder

$$\dot{V}_{s,LP} = \dot{V}_1 - \dot{V}_3 = \left(\frac{\pi}{4}\right) d_1^{\,2} L_1 N k$$

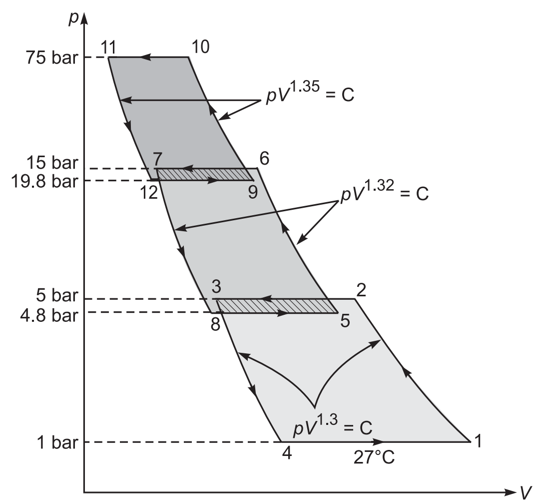


Fig. 25.31

$$= \left(\frac{\pi}{4}\right) \times (0.3\,6\text{m})^2 \times (0.4\text{ m}) \times 300 \times 2$$

$$= 24.42\text{ m}^3/\text{min}$$

The volumetric efficiencies of *LP*, intermediate and *HP* cylinders;

$$\eta_{vol,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_1}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{5}{1}\right)^{\frac{1}{1.3}} = 0.902$$

For *IP* cylinder

$$\eta_{vol,IP} = 1 + c_2 - c_2 \left(\frac{p_6}{p_5}\right)^{\frac{1}{n_2}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{20}{4.8}\right)^{\frac{1}{1.32}} = 0.922$$

For *HP* cylinder

$$\eta_{vol,IP} = 1 + c_3 - c_3 \left(\frac{p_{10}}{p_9}\right)^{\frac{1}{n_3}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{75}{19.8}\right)^{\frac{1}{1.35}} = 0.9327$$

The effective stroke volume $(\dot{V}_1 - \dot{V}_4)$ of air per minute in *LP* cylinder;

$$\dot{V}_1 - \dot{V}_4 = \dot{V}_{s,LP} \times \eta_{vol,LP}$$

$$= (24.42\text{ m}^3/\text{min}) \times 0.902 = 22.02\text{ m}^3/\text{min}$$

The mass flow rate of air in the compressor;

$$\dot{m}_a = \frac{p_1(\dot{V}_1 - \dot{V}_4)}{RT_1} = \frac{(100\text{ kPa}) \times (22.02\text{ m}^3/\text{min})}{(0.287\text{ kJ/kg}\cdot\text{K}) \times (300\text{ K})}$$

$$= 25.584\text{ kg/min} \quad \text{or} \quad 0.426\text{ kg/s}$$

Temperature after compression in each stage

After first stage,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n_1 - 1}{n_1}}$$

$$= 300 \times \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = 434.93\text{ K}$$

After second stage,

$$T_6 = T_5 \left(\frac{p_6}{p_5}\right)^{\frac{n_2 - 1}{n_2}}$$

$$= 300 \times \left(\frac{20}{4.8}\right)^{\frac{1.32-1}{1.32}} = 424.00\text{ K}$$

After third stage

$$T_{10} = T_9 \left(\frac{p_{10}}{p_9}\right)^{\frac{n_3 - 1}{n_3}}$$

$$= 300 \times \left(\frac{75}{19.8}\right)^{\frac{1.35-1}{1.35}} = 423.71\text{ K}$$

Specific heat at constant volume;

$$C_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717\text{ kJ/kg}\cdot\text{K}$$

(i) *Heat rejection rate in each stage*

$$\dot{Q}_{comp} = \dot{m}_a \frac{(\gamma - n)}{n - 1} C_v (T_2 - T_1)$$

$$\dot{Q}_{intercooler} = \dot{m}_a C_p (T_2 - T_1)$$

Heat rejcetion rate in **first** stage (during compression and in intercooler);

$$\dot{Q}_1 = \dot{m}_a \left[\frac{(\gamma - n_1)}{n_1 - 1} C_v + C_p\right] (T_2 - T_1)$$

$$= 25.584 \times \left[\frac{(1.4 - 1.3)}{1.3 - 1} \times 0.717 + 1.005\right] \times (434.93 - 300)$$

$$\mathbf{= 4295\text{ kJ/min}}$$

Heat rejcetion rate in **second** stage (during compression and in intercooler);

$$\dot{Q}_2 = \dot{m}_a \left[\frac{(\gamma - n_2)}{n_2 - 1} C_v + C_p\right] (T_6 - T_5)$$

$$= 25.584 \times \left[\frac{(1.4 - 1.32)}{1.32 - 1} \times 0.717 + 1.005\right] \times (424 - 300)$$

$$\mathbf{= 3757\text{ kJ/min}}$$

Heat rejction rate in **third** stage compression;

$$\dot{Q}_3 = \dot{m}_a \left(\frac{\gamma - n_3}{n_3 - 1}\right) C_v (T_{10} - T_9)$$

$$= 25.584 \times \left(\frac{1.4 - 1.35}{1.35 - 1}\right) \times 0.717 \times (423.71 - 300)$$

$$= \mathbf{324.18\ kJ/min}$$

Heat rejection rate in after-cooler

$$\dot{Q}_3 = \dot{m}_a C_p (T_{10} - T_1)$$
$$= 25.584 \times 1.005 \times (423.71 - 300)$$
$$= \mathbf{3180.82\ kJ/min}$$

(ii) *Diameter of IP and HP cylinders*

The effective swept volume rate of *IP* cylinder

$$\dot{V}_5 - \dot{V}_8 = \frac{\dot{m}_a R T_1}{p_5}$$
$$= \frac{25.584 \times 0.287 \times 300\ \text{K}}{480}$$
$$= 4.588\ \text{m}^3/\text{min}$$

Further, the volumetric efficiency can be expressed using piston displacement volume rate of *IP* cylinder as

$$\eta_{vol,IP} = \frac{\dot{V}_5 - \dot{V}_8}{\dot{V}_{s,\,IP}}$$

$$\therefore \dot{V}_{s,IP} = \frac{\dot{V}_5 - \dot{V}_8}{\eta_{vol,IP}} = \frac{4.588}{0.922} = 4.976\ \text{m}^3/\text{min}$$

which can be further, expressed as

$$\dot{V}_{s,IP} = \left(\frac{\pi}{4}\right) d_2{}^2 L_1 N k$$

$$\text{or } 4.976 = \left(\frac{\pi}{4}\right) \times d_2^2 \times 0.4 \times 300 \times 2$$

or $d_2 = 0.1624$ m or **162.4 mm**

Similarly, for *HP* cylinder;

The effective swept volume rate of *HP* cylinder

$$\dot{V}_9 - \dot{V}_{12} = \frac{\dot{m}_a R T_9}{p_9}$$
$$= \frac{25.584 \times 0.287 \times 300\ \text{K}}{1980}$$
$$= 1.1125\ \text{m}^3/\text{min}$$

$$\dot{V}_{s,HP} = \frac{\dot{V}_9 - \dot{V}_{12}}{\eta_{vol,HP}} = \frac{1.1125}{0.9327} = 1.192\ \text{m}^3/\text{min}$$

which can be further, expressed as

$$1.192 = \left(\frac{\pi}{4}\right) \times d_3^2 \times 0.4 \times 300 \times 2$$

or $d_3 = 0.07954$ m or **79.54 mm**

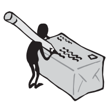
The power input to compressor

Since $T_1 = T_5 = T_9$,

$$IP = \dot{m}_a R T_1 \left[\frac{n_1}{n_1 - 1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n_1 - 1}{n_1}} - 1\right\} + \frac{n_2}{n_2 - 1}\left\{\left(\frac{p_6}{p_5}\right)^{\frac{n_2 - 1}{n_2}} - 1\right\} + \frac{n_3}{n_3 - 1}\left\{\left(\frac{p_{10}}{p_9}\right)^{\frac{n_3 - 1}{n_3}} - 1\right\}\right]$$

$$= 25.584 \times 0.287 \times 300 \times \left[\frac{1.3}{1.3 - 1}\left\{\left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right\} + \frac{1.32}{1.32 - 1}\left\{\left(\frac{20}{4.8}\right)^{\frac{1.32-1}{1.32}} - 1\right\} + \frac{1.35}{1.35 - 1}\left\{\left(\frac{75}{19.8}\right)^{\frac{1.35-1}{1.35}} - 1\right\}\right]$$

$$= 2202.78 \times (1.949 + 1.705 + 1.59)$$
$$= 11{,}551.4\ \text{kJ/min} \quad \text{or} \quad \mathbf{192.34\ kW}$$



Summary

- An air compressor is a machine that decreases the volume and increases the pressure of a quantity of air by mechanical means. The reciprocating compressor handles a small quantity of gas and produces very high pressure, while rotary compressors are used to handle a large volume of gas and to produce low and medium pressures.
- The volume flow rate to a single-acting, single-cylinder reciprocating compressor is
$$\dot{V} = \frac{\pi}{4} d^2 L \frac{N}{60}$$
- For the *double-acting reciprocating compressor*, the induction takes place on both sides of the piston for each revolution. Thus

$$\dot{V} = \frac{\pi}{4} d^2 L \left(\frac{2N}{60} \right)$$

where N is the speed of compressor in rotations per minute.

- *The capacity of a compressor* is the actual quantity of air delivered per unit time at atmospheric conditions. *Free Air delivery* (*FAD*) is the discharge volume of the compressor corresponding to ambient conditions.
- Piston speed is the linear speed of the piston measured in m/min. It is expressed as

$$\mathcal{V}_{piston} = 2LN$$

- The indicated work input to a single-stage, single-acting reciprocating compressor without clearance is

$$W_{in} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{(kJ/cycle)}$$

- The clearance volume is provided in the cylinder to accomodate valves. The clearance ratio c is the ratio of clearance volume to the swept volume. The clearance ratio for a reciprocating air compressor is usually 2 to 10%. The work input to the compressor with clearance ratio c is

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c-1}{n_c}} - 1 \right] - \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e-1}{n_e}} - 1 \right]$$

where n_c = index of compression and n_e = index of expansion.

If both indices are same, i.e., $n_c = n_e$ then

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

- Isothermal efficiency of a compressor is defined as the ratio of isothermal work input to actual work input.
- The indicated power required by an air compressor is given as

$$IP = W_{in} \text{ per cycle} \times \text{No of compression per unit time} = W_{in} \left(\frac{Nk}{60} \right)$$

- From an indicated diagram, the indicated power is obtained in terms of indicated mean effective pressure, p_m as

$$IP = \frac{p_m L A N k}{60} \text{(kW)}$$

where $k = 1$ for single acting and $k = 2$ for double acting reciprocating compressor.

- The mechanical efficiency of the compressor is given by

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Brake power}}$$

- The brake power is derived from a driving motor or engine. The input of the driving motor can be expressed as

Motor power

$$= \frac{\text{Shaft power (or Brake power)}}{\text{Mechanical efficiency of motor and drive}}$$

- The volumetric efficiency of a reciprocating air compressor is

$$\eta_{vol} = \frac{\text{Actual Mass sucked}}{\text{Mass corresponds to swept volume at atmospheric pressure and temperature}} = 1 + c - c \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}}$$

- Multistage compression with intercooling reduces the compression work.
- For a two-stage reciprocating air compressor with intercooler, the work input per cycle is

$$W_{in} = \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

where p_2 and T_2 are intermediate pressure and temperature, respectively.

- The heat rejected with intercooler is

$$Q_{cooling} = m_a C_p (T_2 - T_1) \text{ (kJ)}$$

and heat rejected during polytropic compression is

$$Q_{comp} = m_a \frac{(\gamma - n)}{n - 1} C_v (T_2 - T_1)$$

- The compression work in a reciprocating compressor would be minimum when a stage pressure ratio is

$$\frac{p_2}{p_1} = \frac{p_3}{p_2}$$

and the minimum compression power for a two-stage compressor

$$IP_{min} = 2 \times \left(\frac{n}{n-1} \right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Glossary

Reciprocating compressor A reciprocating machine, used to compress the air during each stroke of piston

Rotary compressor A machine which compresses the air by dynamic action

Single-acting compressor A compressor in which all actions take place only one side of the piston during a cycle

Double-acting compressor A compressor in which suction, compression and delivery of gas take place on both sides of the piston

Single-stage compressor A compressor in which the compression of gas to final delivery pressure is carried out in one cylinder only

Multistage compressor A compressor which compresses the gas to the final pressure in more than one cylinder in series

Pressure ratio The ratio of absolute discharge pressure to absolute suction pressure

Free air Air that exists under atmospheric condition

Free Air delivery (FAD) Discharge volume of compressor corresponding to ambient conditions

Compressor Capacity Quantity of air delivered per unit time at atmospheric conditions

Inter-stage coolers Used to cool the air in between stages of compression

After coolers Used to remove the moisture in the air by cooling it

Air-dryers Removes traces of moisture after after-cooler is used

Moisture drain traps used for removal of moisture in the compressed air

Air receiver Storage tank used to store the compressed air

Review Questions

1. What is an air compressor? Why is it an important machine?
2. Write the uses of compressed air.
3. Classify the air compressors.
4. How do the suction and delivery valve activate in reciprocating air compressor?
5. State the main parts of reciprocating air compressor.
6. Differentiate between
   (i) Single-acting and double-acting compressors
   (ii) Single-stage and multistage compressors.
7. Why is a cooling arrangement provided with all compressors?
8. Define swept volume, and deduce it for single-cylinder, single-acting and double-acting compressor having bore *d,* stroke *L*, and speed *N* rpm.
9. Write the construction of a single-acting, single-stage reciprocating air compressor.
10. Explain the working of a single-acting reciprocating air compressor.
11. Explain the working of double-acting reciprocating air compressor.
12. Derive an expression for indicated work of a reciprocating air compressor by neglecting clearance.
13. Why is the clearance volume provided in each reciprocating compressor? Is it desirable to have a high clearance volume in a compressor?
14. What is clearance ratio? Write the effect of clearance volume on performance of a reciprocating compressor.
15. Derive an expression for indicated work of a reciprocating air compressor by considering its clearance.
16. Define volumetric efficiency and prove that

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}}$$

where each term has its usual meaning.

17. Define volumetric efficiency. How is it affected by (i) pressure ratio, (ii) speed of compressor, and

(iii) throttling across the valves? Explain in brief.

18. What are the advantages of multistage compression over single-stage compression?
19. Why is the intercooler provided between stages?
20. Prove that in a reciprocating air compressor, with perfect intercooling, the work done for compressing the air is equal to heat rejected by the air.
21. What is an after-cooler? Why is it provided with an air compressor.
22. Prove that for complete intercooling between two stages, the compression work would be minimum when intermediate pressure

$$p_2 = \sqrt{p_1 \times p_3}$$

23. Define overall volumetric efficiency. Discuss the parameters in brief, which affect it.
24. Show the effect of increase in compression ratio in single-stage reciprocating compressor on a p–V diagram and give its physical explanation.
25. Draw the indicator diagram for single-stage, double-acting reciprocating air compressor on a p–V diagram.
26. What are the advantages of using an after-cooler with an air compressor, when air under pressure has to be stored over long periods?
27. What is the effect of intake temperature and pressure on output of an air compressor?

Problems

1. Calculate the bore of the cylinder for a double-acting, single-stage reciprocating air compressor runs at 100 rpm with average piston speed of 150 m/min. The indicated power input is 50 kW. It receives air at 1 bar and 15°C and compresses it according to $pV^{1.2}$ = constant to 6 bar.

[349 mm]

2. A single-acting, single-cylinder reciprocating air compressor has a cylinder diameter of 300 mm and a stroke of 400 mm. It runs at 100 rpm. Air enters the cylinder at 1 bar; 20°C. It is then compressed to 5 bar. Calculate the mean effective pressure and indicated power input to compressor, when compression is

(a) isothermal,
(b) according to the law $pV^{1.2}$ = constant,
(c) adiabatic.

Calculate isothermal efficiency for each case. Neglect clearance.

[(a) 1.61 bar, 7.58 kW, 100% (b) 1.85 bar, 8.7 kW, 87.2% (c) 2.043 bar, 9.63 kW, 78.8%]

3. An air compressor takes in air at 100 kPa, 300 K. The air delivers at 400 kPa, 200°C at the rate of 2 kg/s. Determine minimum compressor work input. *[312.7 kW]*

4. A single-acting, single-cylinder reciprocating air compressor receives 30 m^3 of atmopheric air per hour at 1 bar and 15°C . It runs at 450 rpm and discharges air at 6.5 bar. It has a mechanical efficiency of 80% and a clearance ratio of 8.9%. Calculate

(a) the volumetric efficiency,
(b) mean effcetive pressure,
(c) brake power.

[(a) 75% (b) 1.06 bar (c) 1.48 kW]

5. Calculate the power required to drive a single-stage, single-acting reciprocating air compressor to compress 8 m^3/min of air, receiving at 1 bar, 20°C to 7 bar. The index of compression is 1.3.

Also, calculate the percentage saving in indicated power by compressing the same mass of air

(a) in two stages with optimum intercooler pressure and perfect intercooling,
(b) in two stages with imperfect intercooling to 27°C, intercooler pressure remaining the same as in case in (a),
(c) in three stages with optimum intercooler and perfect cooling.

[32.8 kW (a) 11.3% (b) 10.2% (c) 14.63%]

6. A power cylinder of 0.5 m^3 capacity is charged with compressed air without after-cooling it at 170 bar from a four-stage compressor with perfect intercooling between stages and working in best conditions. What are the most economical intermediate pressure?

[3.611bar 13.04 bar and 47.08 bar]

7. The free air delivered by a single-stage, double-acting reciprocating air compressor, measured at 1 bar and 15°C is 16 m^3/min. The suction takes place at 96 kPa and 30°C and delivery pressure is 6 bar. The clearance volume is 4% of swept volume and mean piston speed is limited to 300 m/min. D etermine

(a) power input to compressor, if mechanical efficiency is 90% and compression efficiency is 85%

(b) Bore and stroke if compressor runs at 500 rpm

Assume index of compression and expansion = 1.3.

[(a) 83.6 kW (b) 290 mm and 300 mm]

8. A double-acting, single-stage reciprocating air compressor has a bore of 330 mm, stroke of 350 mm, clearance of 5%, and runs at 300 rpm. It receives air at 95 kPa and 25°C . The delivery pressure is 4.5 bar and the index of compression is 1.25. The free air conditions are 1.013 bar and 20°C. Determine

(a) FAD,

(b) heat rejected during the compression, and

(c) power input to compressor, if its mechanical efficiency is 80%.

[(a) 14.51 m^3/min (b) 817.4 kJ/min. (c) 56.82 kW]

9. A Four cylinder, double acting reciprocating air compressor is used to compress 30 m^3/min of air at 1 bar and 27°C to a pressure of 16 bar. Calculate the size of motor required and cylinder dimensions for the following data:

speed of compressor = 320 rpm, clearance ratio 4%, stroke to bore ratio 1.2, η_{mech} = 82%, index of compression and expansion, n = 1.3.

Assume air gets healed by 12°C during suction.

[241 kW, 263 mm, 315.6 mm]

10. A single-acting, single-cylinder air compressor running at 300 rev/min is driven by an electric motor. Using the data given below, and assuming that the bore is equal to the stroke, calculate

(a) free air delivery,

(b) volumetric efficiency,

(c) bore and stroke.

Data: Air inlet conditions = 1.013 bar and 15°C; delivery pressure = 8 bar; clearance volume = 7% of swept volume; index of compression and re-expansion = 1.3; mechanical efficiency of the drive between motor and compressor = 87%; motor power output = 23 kW

(4.47 m^3/min;72.7% 297 mm)

11. The LP cylinder of a two-stage, double-acting reciprocating air compressor running at 120 rpm has a 50-cm diameter and 75-mm stroke. It receives air at 1 bar and 20°C and compresses it adiabatically to 3 bar. Air is then delivered to an intercooler, where it is cooled at constant pressure to 35°C and then furter compressed to 10 bar in HP cylinder. Determine the power required of an electric motor to drive a compressor. Assume the mechanical efficiency of the compressor as 90% and of the motor as 86%. *[212.9 kW]*

12. A reciprocating air compressor takes in air at 40°C and 1.013 bar in the daytime.

(a) Calculate the percentage increase of mass output in the night, if the night temperature is 10°C.

(b) If the compressor is shifted to a hill station, where the barometric pressure is 0.92 bar, calculate percentage decrease in output, assuming suction temperature to be same at two places.

(c) Calculate the pressure ratio of the compressor at two places, if the law of compression is $pV^{1.25}$ = constant, if delivery gauge pressure is 7 bar at both places.

[(a) 10.61% (b) 9.18% (c) at first place 4.81% and second place 5.24]

13. A three-stage single-acting reciprocating air compressor has perfect intercooling. The pressure and temperature at the end of the suction stroke in an LP cylinder are 1.013 bar and 15°C, respetively. If 8.4 m^3 of free air is delivered by the compressor at 70 bar per minute and work done is minimum, calculate

(a) LP and IP cylinder delivery pressures,

(b) ratio of cylinder volumes,

(c) total indicated power.

Neglect clearance and assume n = 1.2.

[(a)4.16 bar, 17.05 bar (b) 2.02 (c) 676.8 kW]

Objective Questions

1. For isothermal compression in a compressor, the compressor should run at
   (a) very high speed (b) very slow speed
   (c) constant speed (d) none of above
2. A reciprocating compressor handles
   (a) large volume for high pressure ratio
   (b) large volume for low pressure ratio
   (c) small volume for high pressure ratio
   (d) small volume for low pressure ratio
3. Usually, the index of actual compression is
   (a) near to 1 (b) 1.3 to 1.4
   (c) 1.1 to 1.3 (d) 1.4 to 1.6
4. Which of the following process takes place in an air compressor?
   (a) Specific volume of air decreases
   (b) Pressure of air increases
   (c) Mechanical energy is supplied
   (d) All of above
5. For which one of the following applications, the compressed air is not used?
   (a) Driving air motors
   (b) Oil and gas transmission
   (c) Starting of I.C. engines
   (d) Transmission of electrical energy
6. Reciprocating compressor is
   (a) a positive displacement machine
   (b) a negative displacement machine
   (c) a dynamic action machine
   (d) none of above
7. Air dryers are used in an air compressor
   (a) before air entry into cylinder
   (b) before entering air receiver
   (c) between two stages
   (d) after leaving air receiver
8. Air receiver used in an air compressor is used to
   (a) cool the air after compression
   (b) eliminate the pulsation
   (c) supply the air to utility
   (d) to separate the moisture
9. In a reciprocating air compressor, inlet and delivery valves actuate
   (a) by separate cam mechanism
   (b) by pressure difference
   (c) by use of compressed air
   (d) none of the above
10. In a reciprocating air compressor, the work input is minimum when compression is
   (a) isentropic (b) polytropic
   (c) isothermal (d) isobaric
11. What is the sequence of processes in a reciprocating air compressor?
   (a) Compression, expansion, and constant volume discharge
   (b) Induction, compression and constant pressure discharge
   (c) Induction, expansion and constant pressure discharge
   (d) Induction, compression and constant volume discharge
12. Work input in a reciprocating air compressor is given by
   (a) $\frac{n-1}{n} p_1 v_1 \left[1 + \left(\frac{p_2}{p_1}\right)^{\frac{n}{n-1}}\right]$
   (b) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n}{n-1}} - 1\right]$
   (c) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$
   (d) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + 1\right]$
13. The isothermal efficiency of a reciprocating air compressor is given by

(a) $\dfrac{\text{Indicated power}}{\text{Isothermal power}}$

(b) $\dfrac{\text{Isothermal power}}{\text{Indicated power}}$

(c) $\dfrac{\text{Isothermal power}}{\text{Brake power}}$

(d) $\dfrac{\text{Brake power}}{\text{Isothermal power}}$

14. The compressor efficiency of a reciprocating air compressor is given by

(a) $\dfrac{\text{Indicated power}}{\text{Isothermal power}}$

(b) $\dfrac{\text{Isothermal power}}{\text{Indicated power}}$

(c) $\dfrac{\text{Isothermal power}}{\text{Brake power}}$

(d) $\dfrac{\text{Brake power}}{\text{Isothermal power}}$

15. The isothermal efficiency of a reciprocating air compressor can be improved by use of
    (a) water jacketing (b) external fins
    (c) intercooler (d) all of the above
16. The clearance volume in a reciprocating air compressor
    (a) reduces work input
    (b) reduces suction capacity
    (c) reduces discharge pressure
    (d) all of the above
17. The volumetric efficiency of a reciprocating air compressor is defined as

(a) $1 + c - c\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

(b) $1 - c + c\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

(c) $1 + c - c\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}}$

(d) $1 - c + c\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}}$

18. The maximum pressure ratio in a single-stage reciprocating air compressor is limited to
    (a) 2 (b) 4
    (c) 7 (d) 10
19. Multistage compression in a reciprocating air compressor improves
    (a) isothermal efficiency
    (b) volumetric efficiency
    (c) mechanical balance
    (d) all of above
20. Ideal intermediate pressure p_2 for two-stage reciprocating air compressor is given by

(a) $p_1 \times p_3$ (b) $\sqrt{p_1 \times p_3}$

(c) $\sqrt{\dfrac{p_3}{p_1}}$ (d) $\sqrt{\dfrac{p_3}{p_1}}$

21. Heat rejection rate per stage of air with perfect intercooler is given by

(a) $\dot{m}_a\left[\dfrac{(\gamma - n)}{n-1}C_v + C_p\right](T_2 - T_1)$

(b) $\dot{m}_a\left[C_p - \dfrac{(\gamma - n)}{n-1}C_v\right](T_2 - T_1)$

(c) $\dot{m}_a\left[C_p - \dfrac{(n-1)}{\gamma - 1}C_v\right](T_2 - T_1)$

(d) $\dot{m}_a\left[C_p + \dfrac{(n-1)}{\gamma - 1}C_v\right](T_2 - T_1)$

Answers

1. (b) 2. (c) 3. (c) 4. (d) 5. (d) 6. (a) 7. (b) 8. (b)
9. (b) 10. (c) 11. (b) 12. (c) 13. (c) 14. (b) 15. (d) 16. (b)
17. (c) 18. (c) 19. (d) 20. (b) 21. (a)