Introduction to Computer Graphics

6. Rasterization

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Textbook: E.Angel, D. Shreiner Interactive Computer Graphics, 6th Ed., Pearson
Ref: D.D. Hearn, M. P. Baker, W. Carithers, Computer Graphics with OpenGL, 4th Ed., Pearson

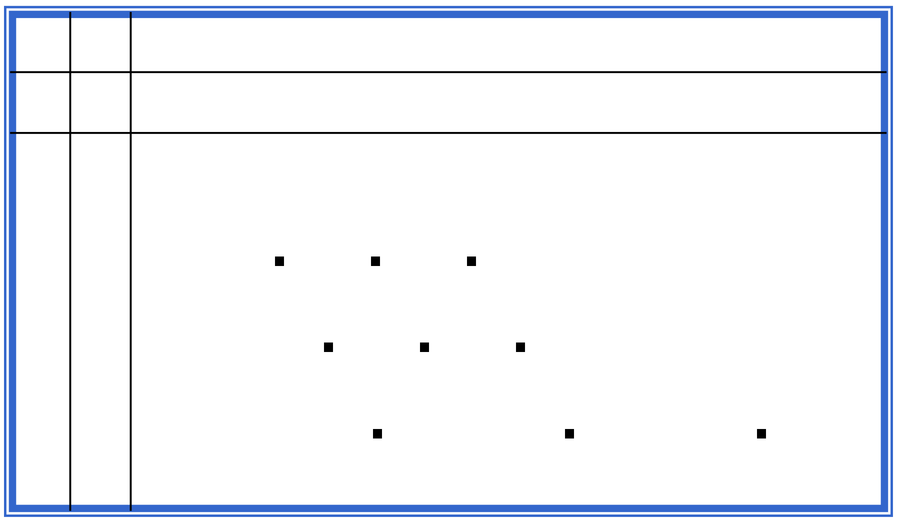
Outline

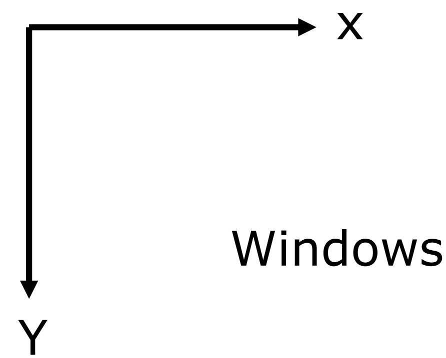
- Draw primitives in discrete screen space.

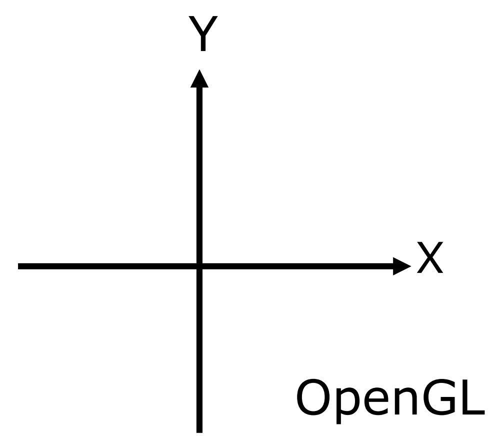
- 2D graphics primitives
  - Line drawing
  - Circle drawing

- Area filling
  - Polygons

Discrete Video Screen







- Assigning pixel values by
  - Functions:
    - e.g. SetPixel(x, y, color)
  - Buffer or arrary:
    - e.g. FrameBuf[x][y] = color

How to Draw Primitives ?

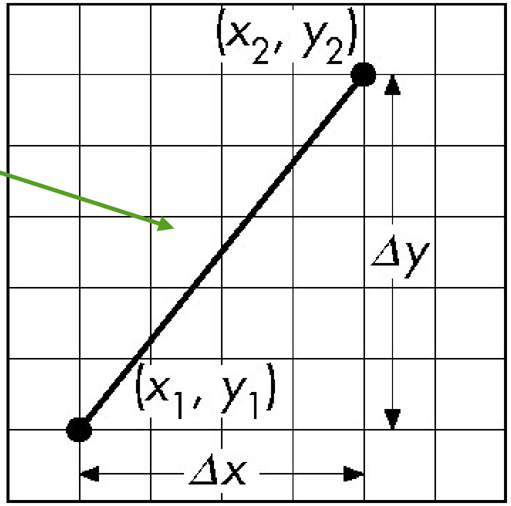
- From math representation to screen.
- In addition to “brute-force”, how to improve the efficiency of computation or memory usage.
- Primitives
  - Lines
  - Circles
  - Curves
  - ……

Line-Drawing Algorithms

- Start with line segment in window coordinates with integer values for endpoints.

$$y = mx + h$$

$$m = \frac{\Delta y}{\Delta x}$$



DDA Algorithm

- Digital Differential Analyzer
  - Line $y=mx+h$ satisfies differential equation.

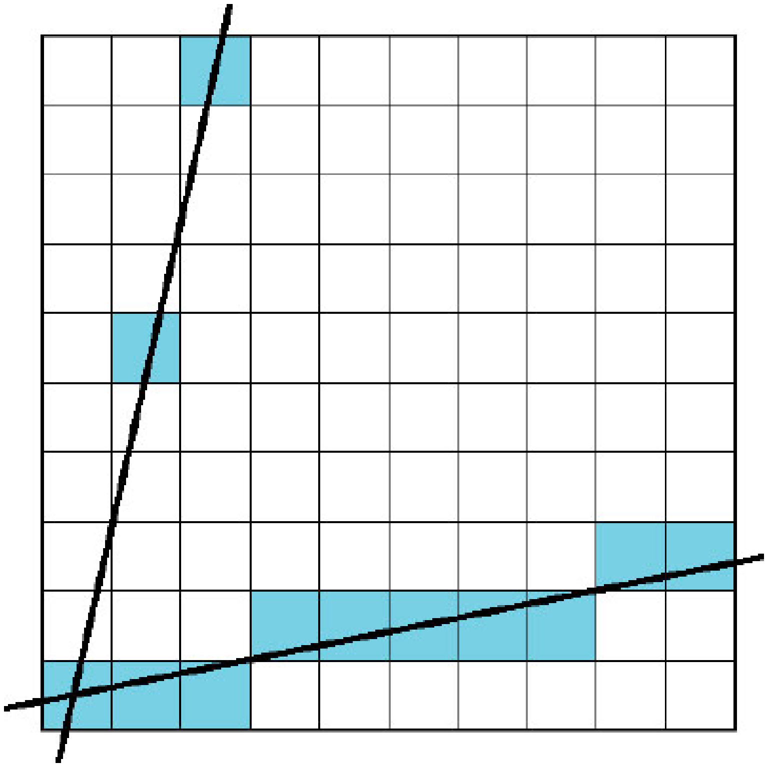
$$\frac{dy}{dx} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- Along scan line $\Delta x = 1$

```
For(x=x1; x<=x2,ix++) {
    y+=m;
    write_pixel(x, round(y), line_color)
}
```

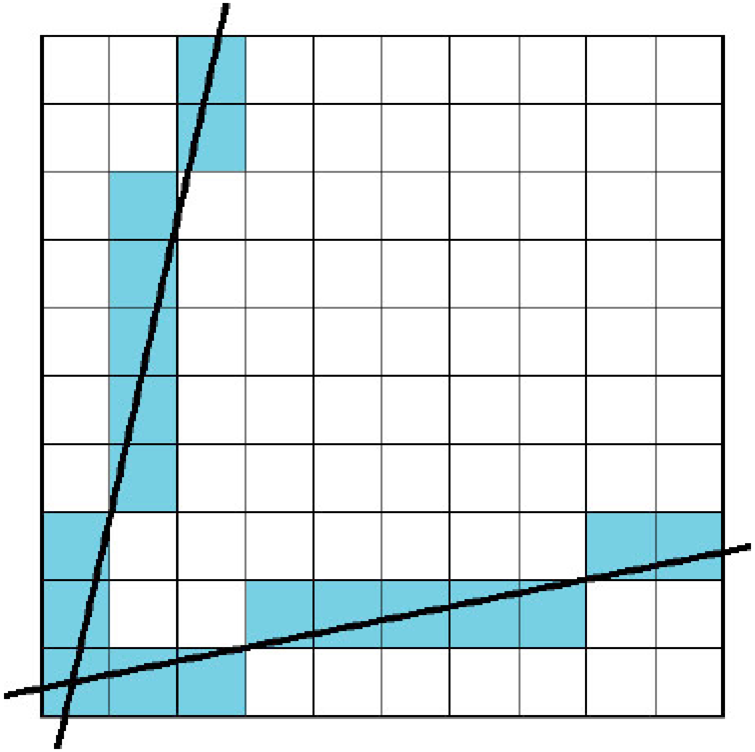
Problem

- DDA = for each *x* plot pixel at closest *y*.
  - Problems for steep lines



Using Symmetry

- Use for $1 \geq m \geq 0$
- For $m > 1$, swap roles of x and y
  - For each y, plot closest x



Bresenham's Algorithm

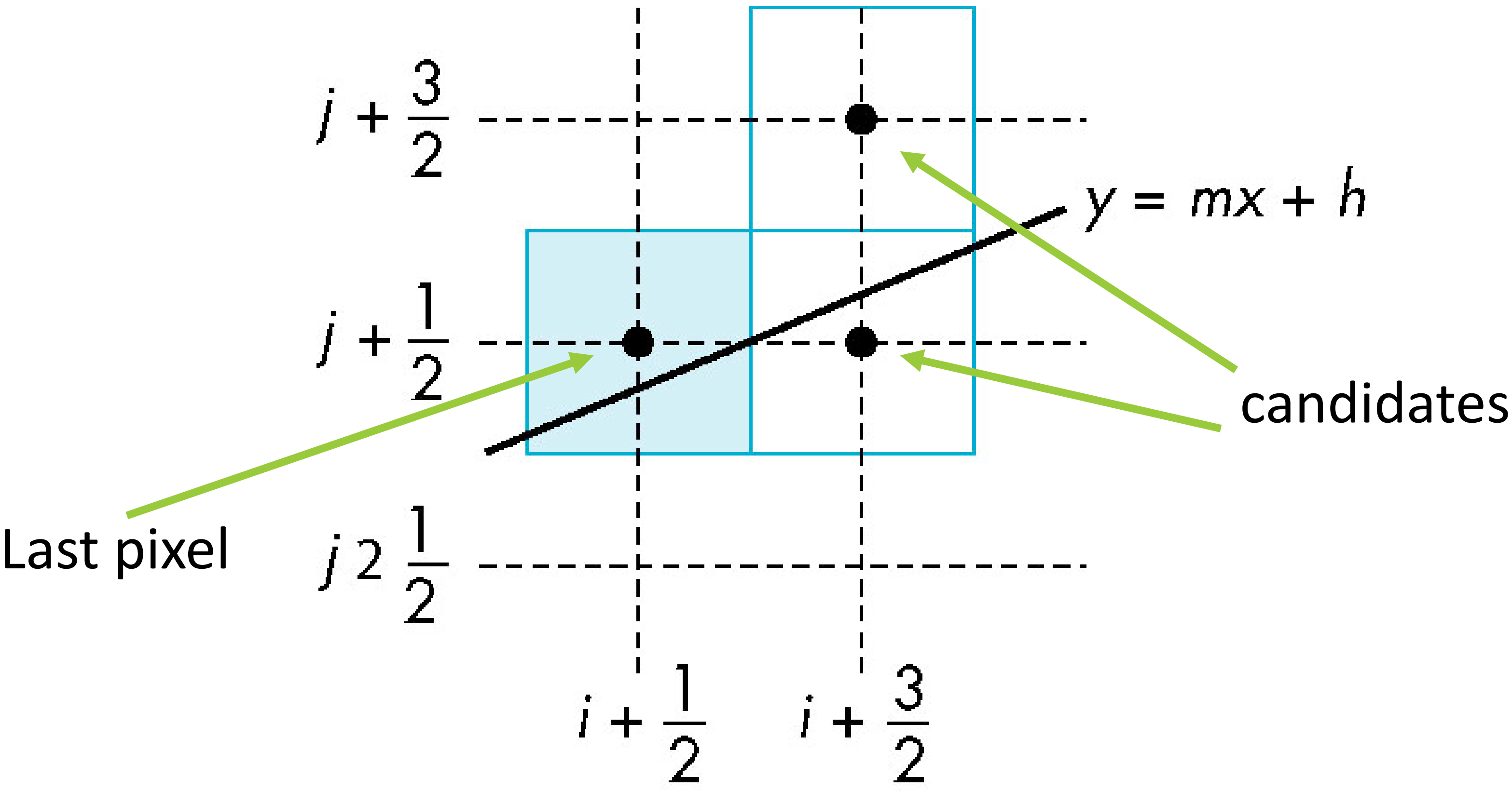
- DDA requires one floating point addition per step.
- Bresenham's algorithm eliminates all fp.

- Consider only $1 \geq m \geq 0$
  - Handing other cases by symmetry
- Assume pixel centers are at half integers.

- Characteristics:
  - If we start at a pixel that has been written, there are only two candidates for the next pixel

Candidate Pixels

- $1 \geq m \geq 0$

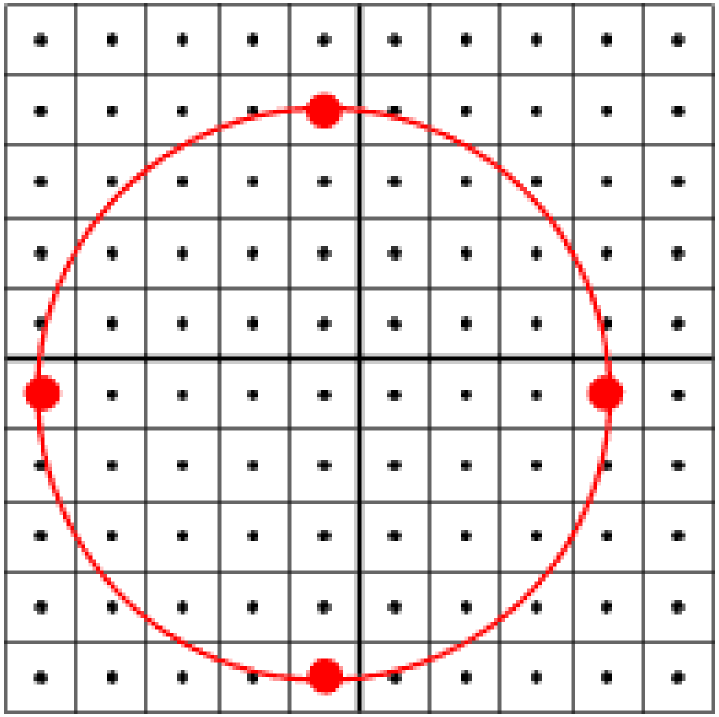


Bresenham's Algorithm

```
function line(x0, x1, y0, y1)
    int deltax := abs(x1 - x0)
    int deltay := abs(y1 - y0)
    real error := 0
    real deltaerr := deltay ÷ deltax
    int y := y0
    for x from x0 to x1
      plot(x,y)
      error := error + deltaerr
      if error ≥ 0.5
        y := y + 1
        error := error - 1.0
```

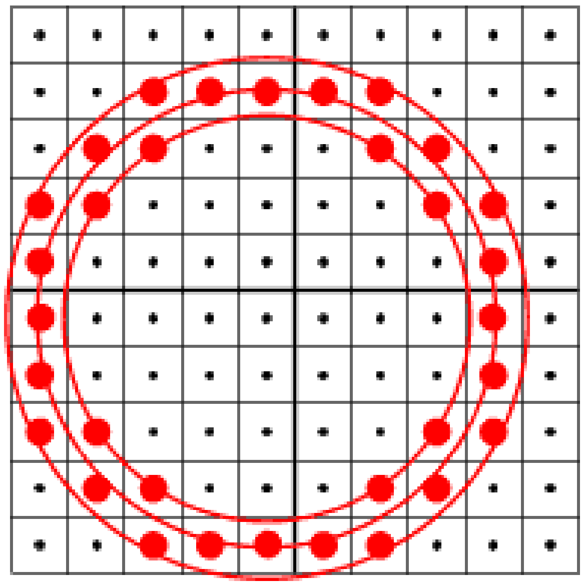
```
function line(x0, x1,y0, y1)
int deltax := abs(x1 - x0)
int deltay := abs(y1 - y0)
int error := 0
int deltaerr := deltay
int y := y0
for x from x0 to x1
      plot(x,y)
      error := error + deltaerr
      if 2×error ≥ deltax
        y := y + 1
        error := error - deltax
```

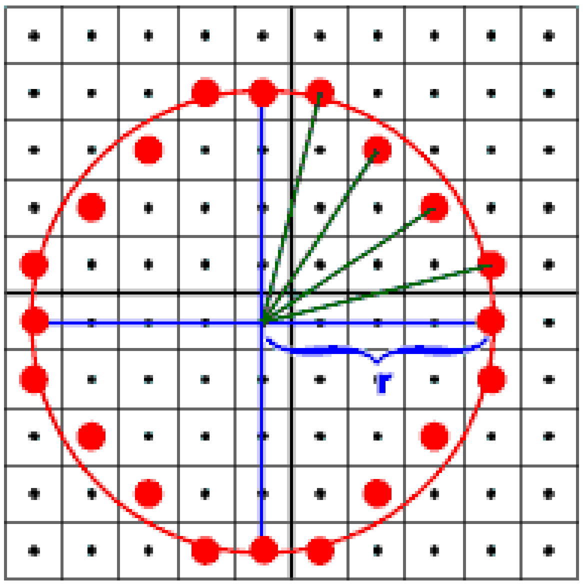
Circle-drawing Algorithms



Ref: http://www.cs.umbc.edu/~rheingan/435/index.html

Circle-drawing Algorithms



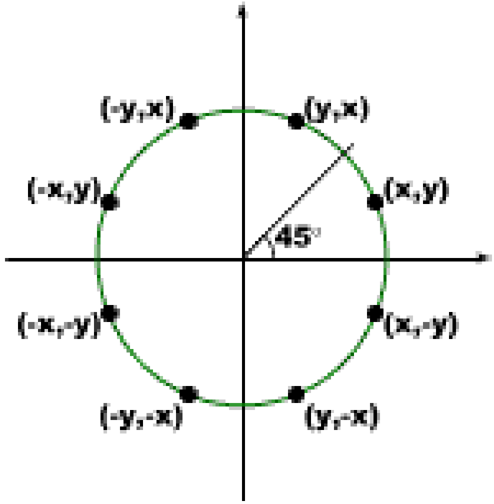


for each x, y
  if $| x^2 + y^2 - r^2 | <= \varepsilon$
  SetPixel (x, y)

for θ in [0~360 degree]
  x = r cos(θ)
  y = r sin(θ)
  SetPixel (x, y)

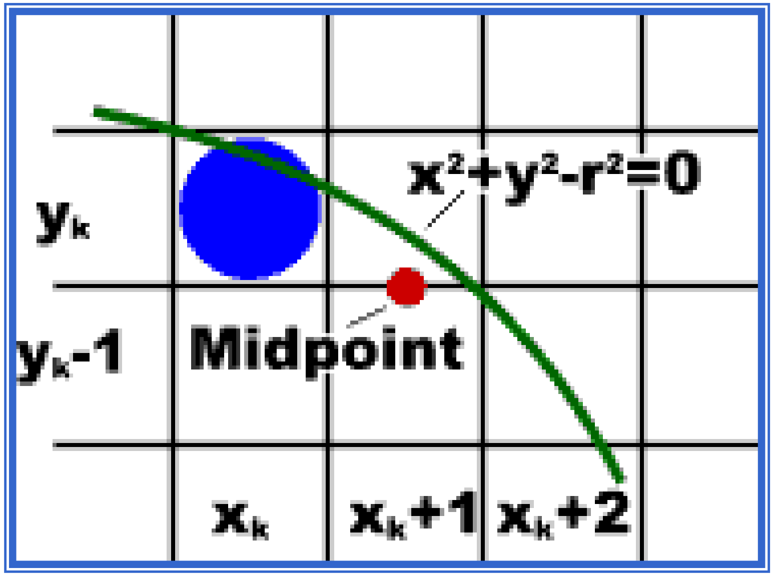
Midpoint Circle Algorithm

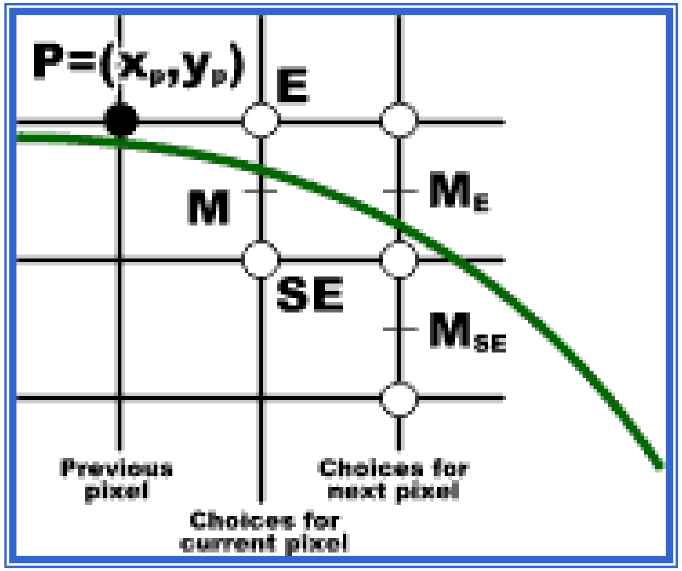
- Can we utilize the similar idea in Bresenham's line-drawing algorithm ?
  - Check only the next candidates.
  - Use symmetry and simple decision rules.



Symmetry of a Circle

Midpoint Circle Algorithm (cont.)



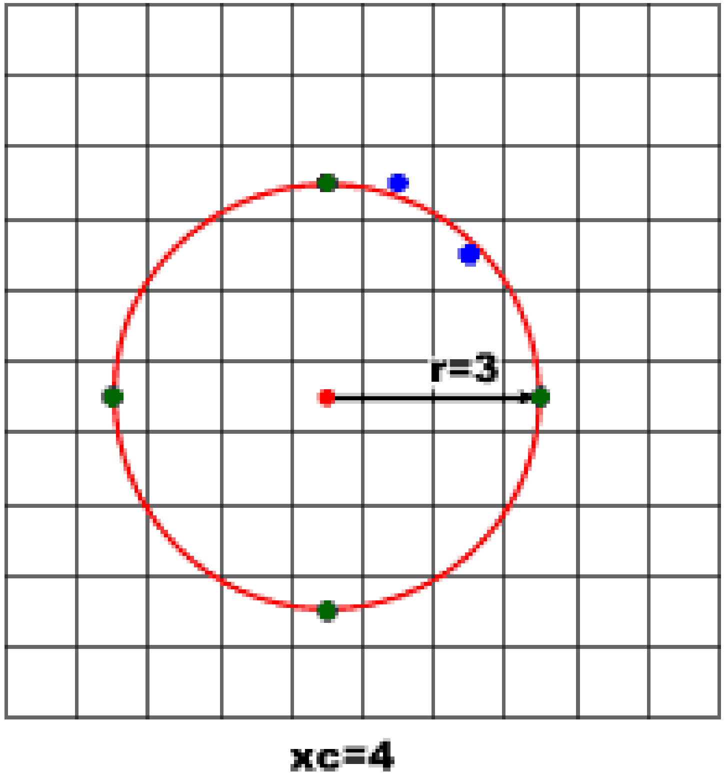


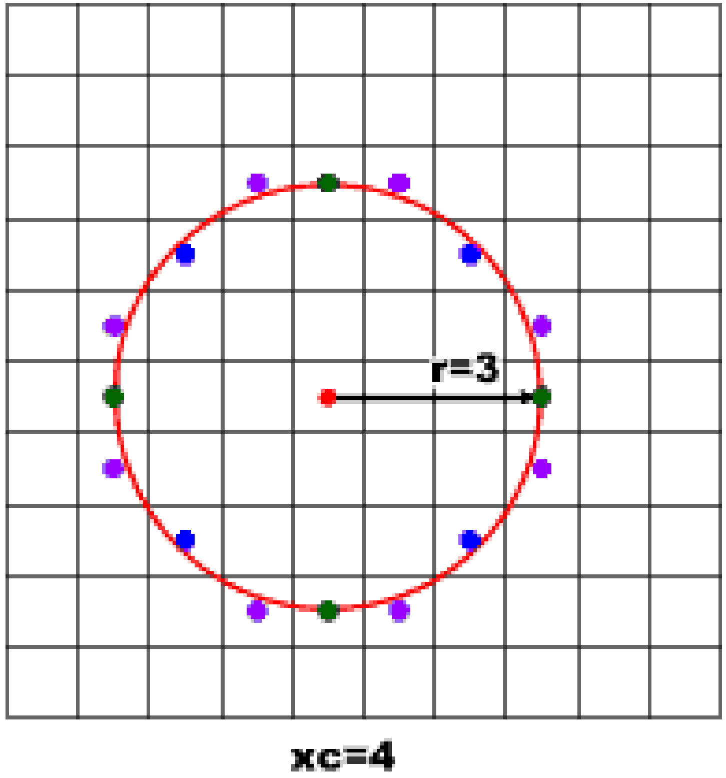
f(x,y) = $x^2 + y^2 - R^2$
f(x,y) > 0 => point outside circle
f(x,y) < 0 => point inside circle

f(M) > 0 => choose SE
f(M) < 0 => choose E

$$P_k = f_{circ}(x_k + 1, y_k - ½)$$

Midpoint Circle Algorithm (cont.)





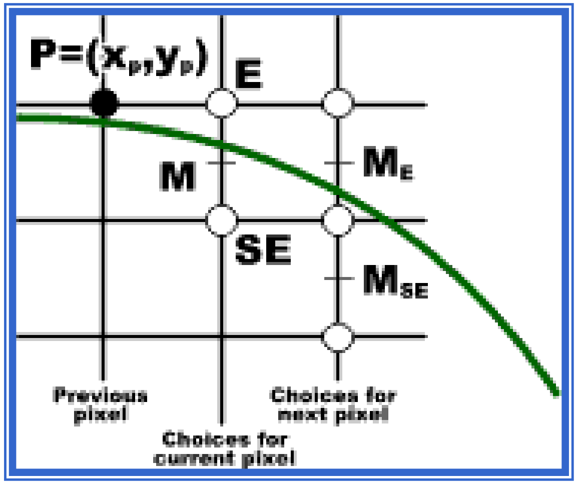
Midpoint Circle Algorithm

- Given the starting point (0,r), the computation is more efficient.

  $p_0 = f_{circle}(1, r\text{-}1/2)$

  $= 1 + (r-1/2)^2 - r^2$

  $= 5/4 - r$



- For each x position,

  $p_k = f_{circle}(x_k+1, y_k-1/2) = (x_k+1)^2 + (y_k-1/2)^2 - r^2,$

  If $p_k < 0$, choose E, $(x_{k+1} = x_k+1,\ y_{k+1} = y_k)$

  $p_{k+1} = f_{circle}(x_{k+1}+1, y_{k+1}-1/2) = [(x_k+1)+1]^2 + (y_k-1/2)^2 - r^2$

  $= p_k + 2x_k + 3 = p_k + 2x_{k+1} + 1$

  If $p_k > 0$, choose SE, $(x_{k+1} = x_k+1,\ y_{k+1} = y_k-1)$

  $p_{k+1} = f_{circle}(x_{k+1}+1, y_{k+1}-1/2) = [(x_k+1)+1]^2 + (y_k-1/2-1)^2 - r^2$

  $= p_k + 2x_k - 2y_k + 5 = p_k + 2x_{k+1} - 2y_{k+1} + 1$

Midpoint Circle Algorithm (cont.)

Summary of the algorithm:

- Given the starting point (0,r),

  Initialization,

  $P_0 = 5/4 - r$

  At each x position,

  if(pk < 0)

  the next point is (x_{k+1}, y_k)

  $p_{k+1} = p_k + 2x_{k+1} + 1$

  else

  the next point is (x_{k+1}, y_k-1)

  $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

Other Primitives

- The same concept can be extended to other primitives.
  - Ellipse, polynomials, splines, etc.

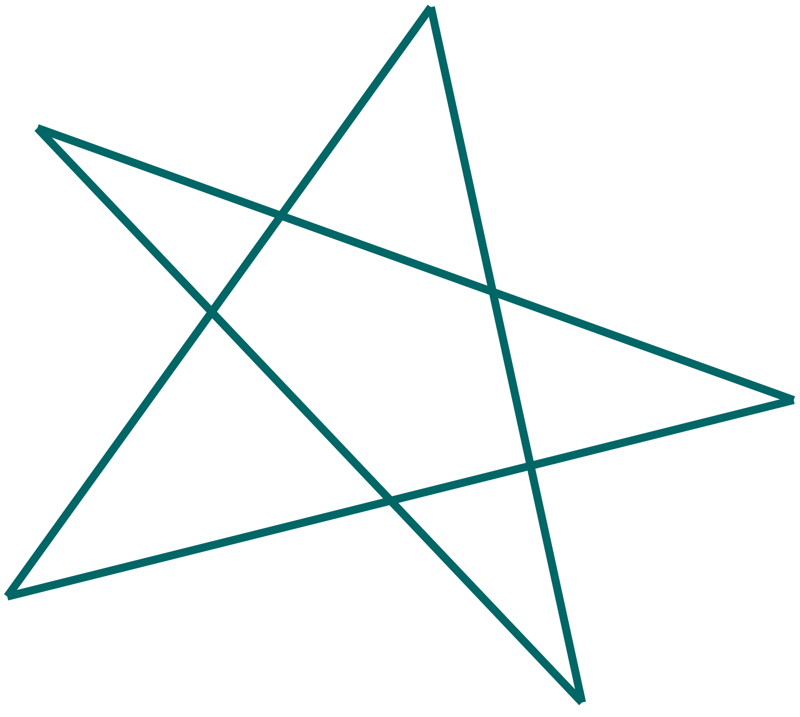
2D Polygon Filling

- Recall:
  - In computer graphics, we usually use polygons to approximate complex surfaces.

- Let’s focus on the polygon filling !

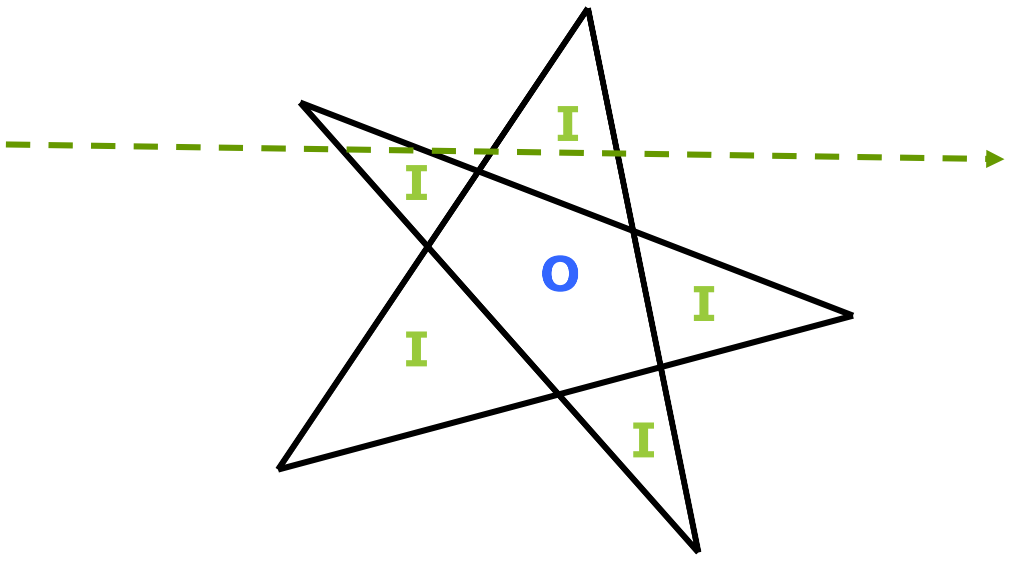
General Polygons

- Inside or Outside are not obvious
  - It’s not obvious when the polygon intersects itself.



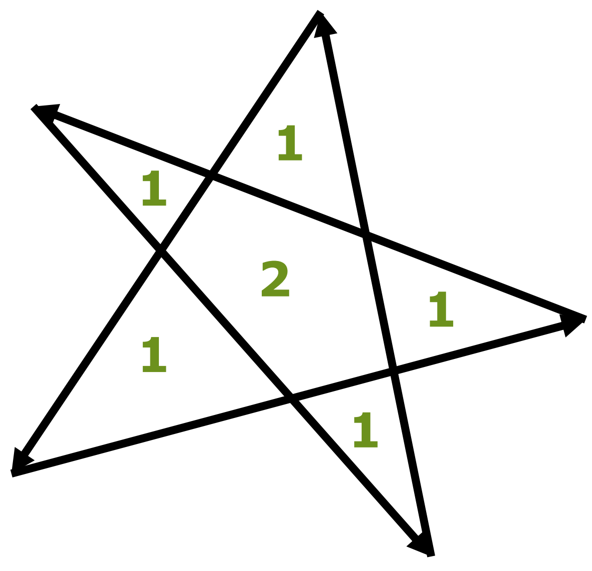
Inside or Outside

- Odd-even rule :
  - Draw a ray to infinity and count the number of edges that cross it.
  - Even → outside; odd → inside
  - usually used for polygon rasterization



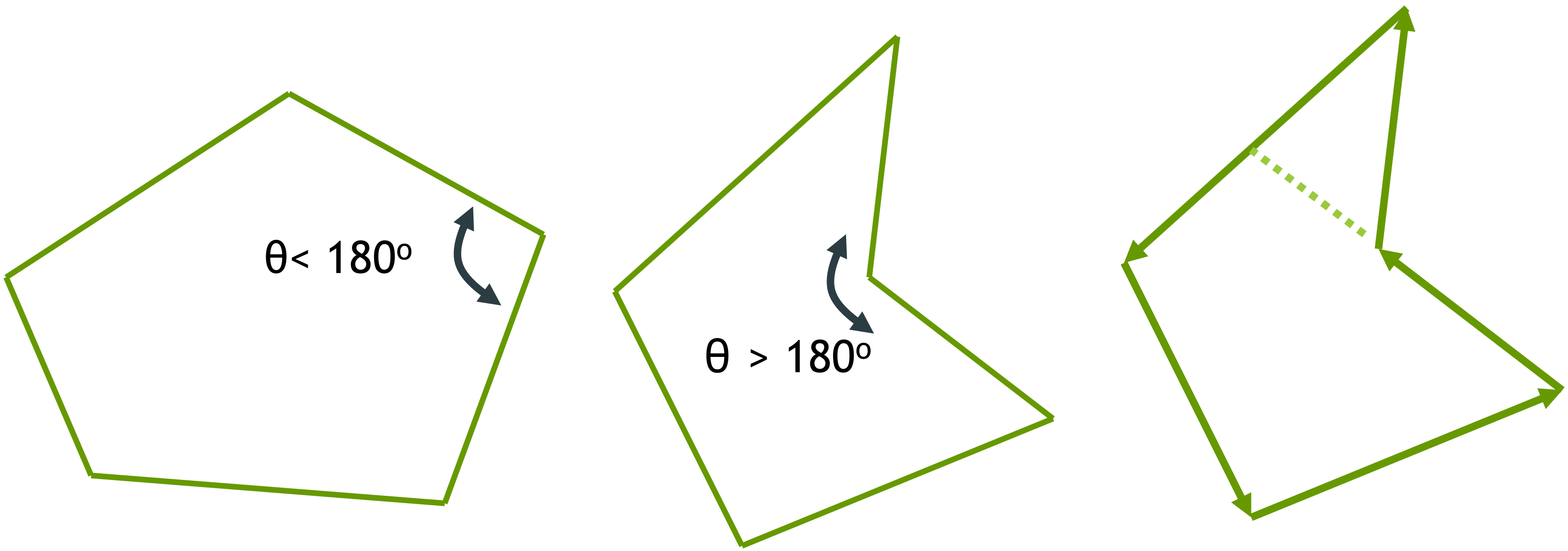
Inside or Outside

- Non-zero winding rule
  - trace around the polygon, count the number of times the point is circled (+1 for clockwise, -1 for counter clockwise).
  - Non-zero winding counts = inside



Concave vs. Convex

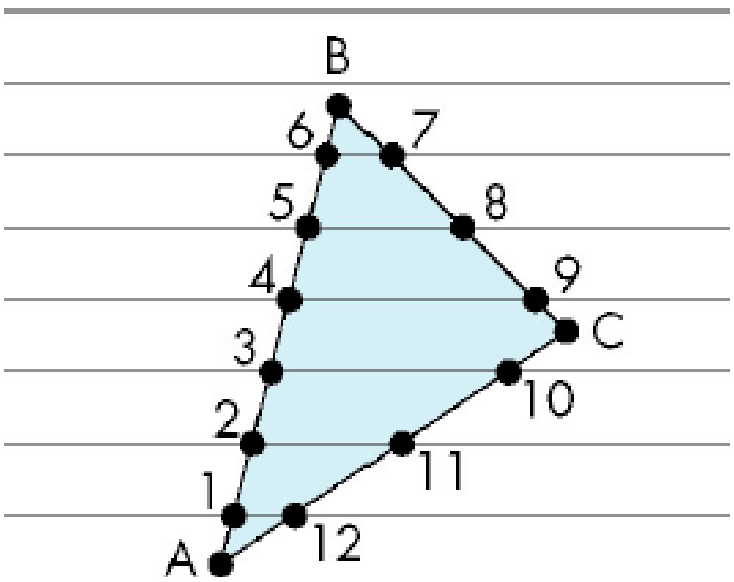
- We prefer dealing with "simpler" polygons.
- Convex (easy to break into triangles)



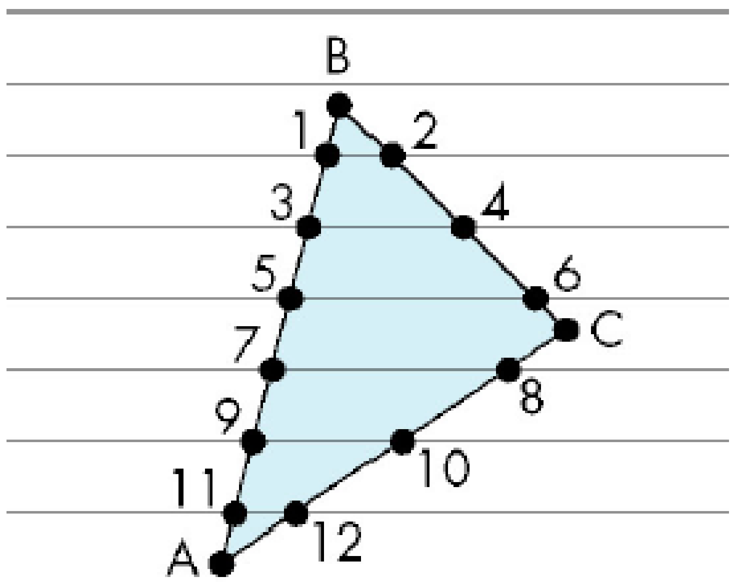
convex　　　　concave

Polygon Filling by Scan Lines

- Fill by maintaining a data structure of all intersections of polygons with scan lines
  - Sort the scan lines
  - Fill each span

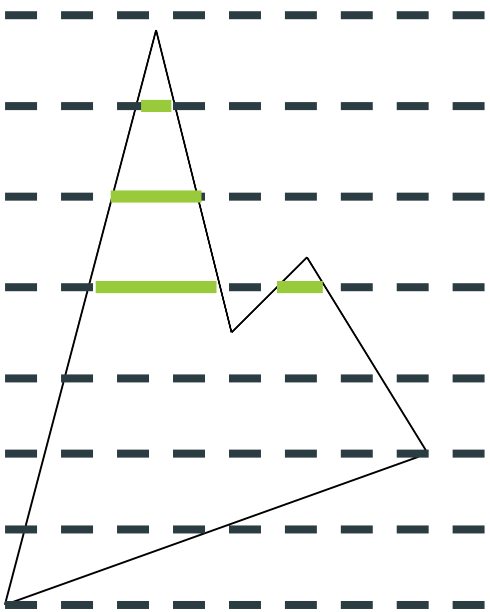


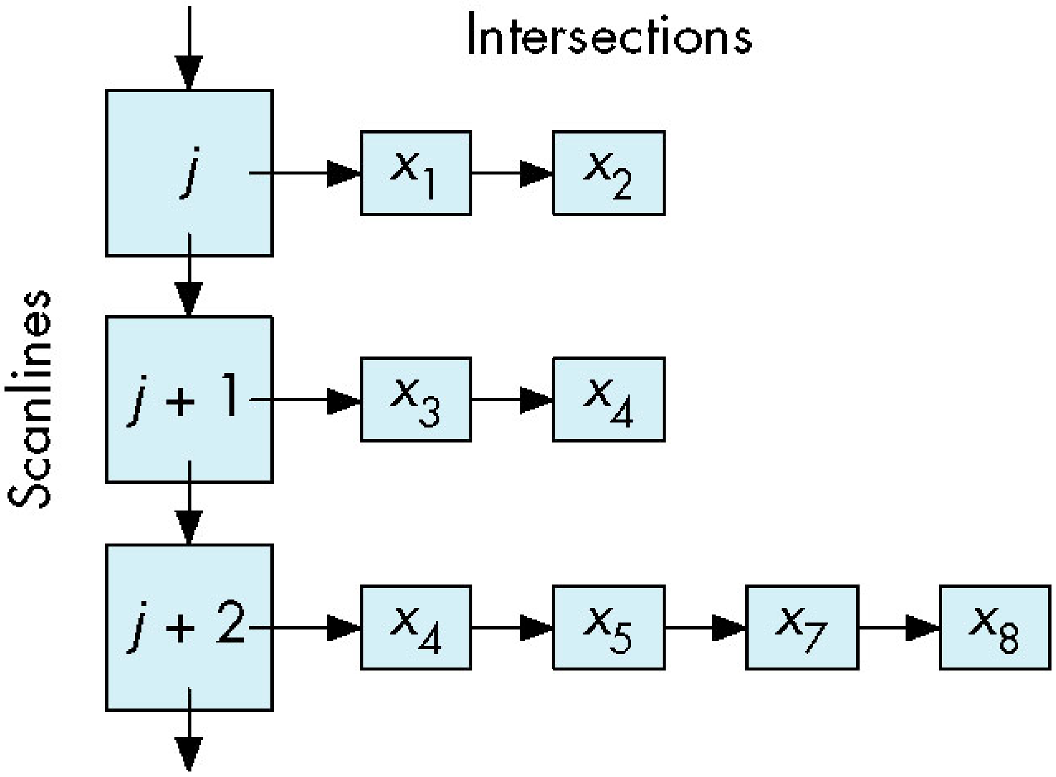
vertex order generated by vertex list



desired order

Data Structure for General Cases





Applying the odd-even rule