MIE1622

Assignment 4

Ziyang Zeng
1001226577

1. Introduction

This assignment investigated several option pricing strategies: the Black Scholes formula for European option pricing, and the Monte Carlo Simulation for European and Barrier option pricing. The effect of volatility change to the option pricing will be investigate, and the optimal number of paths and number of time steps will be justified.

2. Option Pricing

2.1 Matlab Output

Black-Scholes price of an European call option is 8.0214
Black-Scholes price of an European put option is 7.9004
One-step MC price of an European call option is 8.0074
One-step MC price of an European put option is 7.9193
Multi-step MC price of an European call option is 7.9647
Multi-step MC price of an European put option is 7.9385
One-step MC price of an Barrier call option is 7.7949
One-step MC price of an Barrier put option is 0
Multi-step MC price of an Barrier call option is 7.9413
Multi-step MC price of an Barrier put option is 1.9925

2.2 Choices of Number of paths and number of time steps

The number of paths was chosen to be 100000 paths. When using 10000 paths or smaller number of paths, the results for 10 tested runs of the program have shown large fluctuations, whereas using 100000 paths gives the most stable results. The number of time steps was chosen to be 252 because there were 252 trading days a year. The stock prices evolve everyday so that 252 number of steps mostly reflects the reality in financial markets.

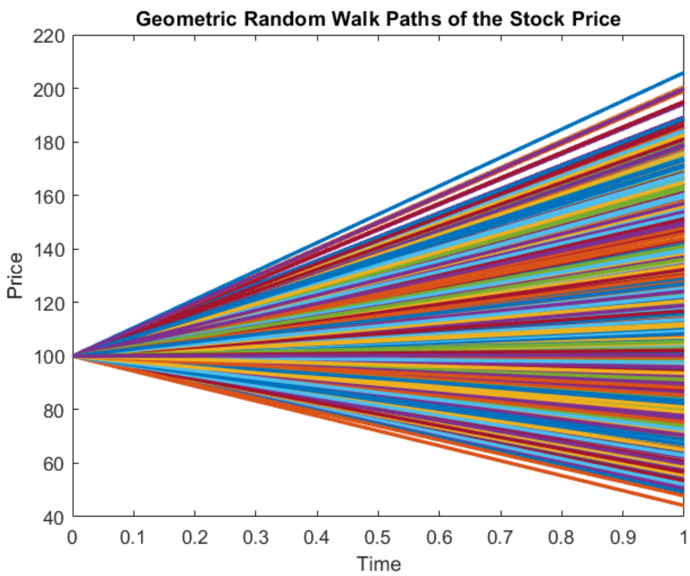

Figure1. Geometric Random Walk Paths for 1-time step

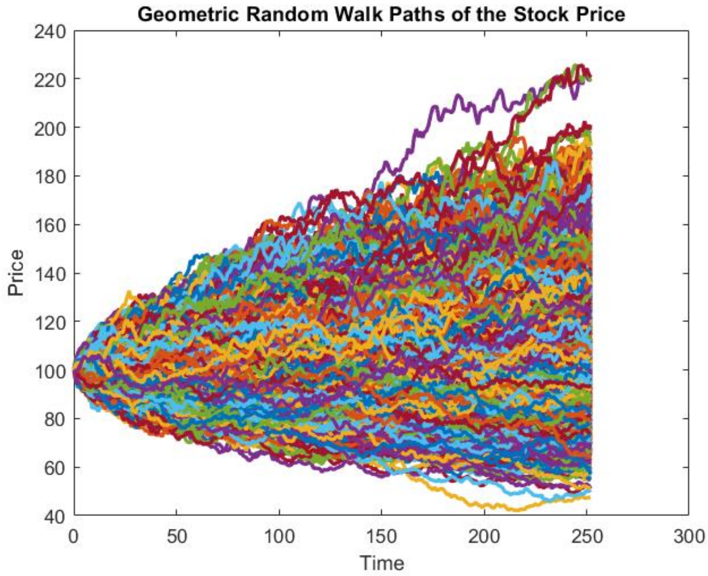

Figure2. Geometric Random Walk Paths for 252-time step

2.3 Strategy Comparison

As we can see from the Matlab outputs, the results of the 1 step Monte Carlo simulation are closer to the results of the Black Scholes than for 252 step simulation. Therefore, when comparing the results to Black Scholes, one step Monte Carlo simulation is better than multi step ones. However, Black Scholes may not be closer to the actual case because it has a lot of assumptions. Usually, the price will be taken from the market and compute the implied volatility using Black Scholes formula. Then the implied volatility will be used to do the simulations.

2.4 European option vs Barrier option

Overall, the price for Barrier call options was slightly smaller than European call options while the price for Barrier put options was much smaller than those of European put options. For call options, it is mainly because the barrier option adds more conditions on the payoff. When K = 105 which is smaller than the barrier 110, the stock price has to go to 110 to get the payoff max(S-K,0) which has smaller probability than the original European call option. For example, for European option, the stock prices that satisfy the range between 105 and 110 at the maturity but does not hit the 110 will get the payoff. However, for barrier options, those stocks will not get the payoffs. Therefore, the barrier option will be cheaper. For put options, the barrier option is much cheaper than the European put option with 1 step barrier put option being 0$. This is because the increase in price has already made the put options being in a bad position. The lower the price of the stock, the higher the put options will be. However, with respect to barrier put options, the price has to be higher than the strike price to get the payoff choice activated. This makes a lot of the barrier put options to be zero.

2.5 Change of Volatility

Increase in volatility by 10%

One-step MC price of an Barrier call option is 8.517

One-step MC price of an Barrier put option is 0

Multi-step MC price of an Barrier call option is 8.7852

Multi-step MC price of an Barrier put option is 2.4962

Decrease in volatility by 10%

One-step MC price of an Barrier call option is 7.0056

One-step MC price of an Barrier put option is 0

Multi-step MC price of an Barrier call option is 7.1916

Multi-step MC price of an Barrier put option is 1.5314

When increasing the volatility, the Barrier option prices increase. This is because the volatility increase has also increased the chance of hitting the barrier because the magnitude of fluctuation will be higher, which leads to more hits. The maturity prices range becomes larger while the mean price doesn't change that much. This leads to higher price for call option and put options. However, when decreasing the volatility, the price has lower chance to hit the barrier. The range for maturity price becomes narrower. Therefore, the payoff for both end gets smaller.

3. Optimal Step Numbers

To reduce the computational time, the number of paths is limited at 10000. The algorithm to find the optimal step numbers is to calculate the difference between call and put option prices of Monte Carlo and Black Scholes every 4 number of steps increments. The plot is shown below. As we can see, the lowest pricing error is 0.0197 and the number of steps is 150.

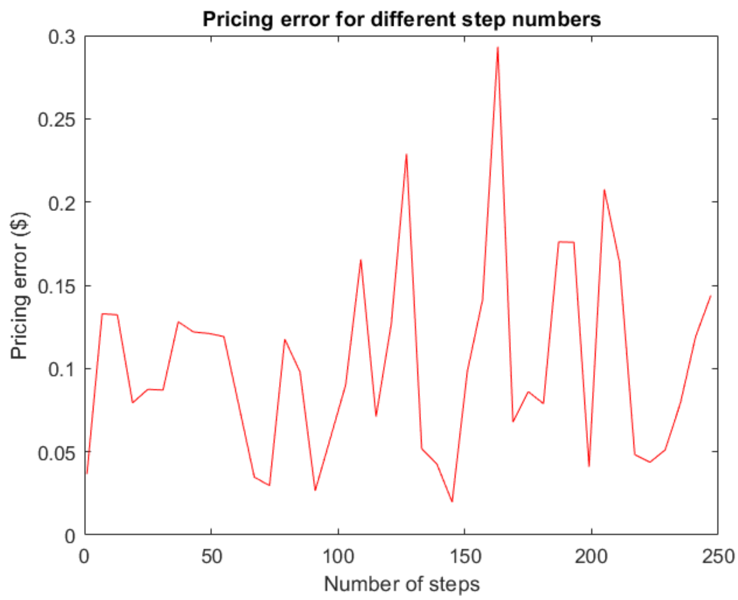

Figure 3. Pricing error vs Number of steps.