Final Cheatsheet

December 8, 2018

EE127

- Sion's minimax theorem:
  Let $X \subseteq \mathbb{R}^n$ be convex, and let $Y \subseteq \mathbb{R}^m$ be a compact set. Let $F : X \times Y \to \mathbb{R}$ be a function such that $F(\dot{;}y)$ is convex and continuous over X, $\forall y \in Y$, and $F(x, \dot{)}$ is concave and continuous over Y, $\forall x \in X$. Then
  $$\max_{y \in Y} \min_{x \in X} F(x, y) = \min_{x \in X} \max_{y \in Y} F(x, y)$$

- Convex sets:
  The intersection of convex sets is convex.
  The affine transformation of a convex set is convex.

- Convex functions:
  A function is convex if and only if its epigraph is.
  The pointwise maximum of a family of functions is convex.
  The composition of a convex function with an affine map is convex.
  The non-negatively weighted sum of convex functions is convex.
  A twice-differentiable function is convex if and only if its Hessian is PSD everywhere.

- $$\|A\|_F = \sqrt{\operatorname{tr}(AA^T)} = \sqrt{\sum_i \sum_j |A_{ij}|^2} = \sqrt{\sum_i \lambda_i(AA^T)} = \sqrt{\sum_i \sigma_i^2(A)}$$

- $\|Ax\|_2 \leq \|A\|_F \|x\|_2, \|AB\|_F \leq \|A\|_F \|B\|_F$.

- Define operator norms:
  $$\|A\|_p = \max_{u \neq 0} \frac{\|Au\|_p}{\|u\|_p} = \max_{\|u\|_p = 1} \|Au\|_p$$