ME 44206 - Assignment Q1

October 15, 2024

Sets and Indices

- M: Set of months, indexed by m (e.g., Jan, Feb, etc.)
- S: Set of suppliers, indexed by s (A, B, C, D, E)
- P: Set of products, indexed by p (18/10, 18/8, 18/0)

Parameters

- $d_{p,m}$: Demand for product p in month m (kg)
- C_s: Cost per kilogram of material from supplier s (euro/kg)
- k_s: Maximum supply available from supplier s per month (kg)
- h_p: Holding cost per kilogram of product p (euro/kg)
- chromium_s: Percentage of chromium in material from supplier s
- nickel_s: Percentage of nickel in material from supplier s
- c_p: Required percentage of chromium for product p
- n_p: Required percentage of nickel for product p
- $P_{\max}$: Maximum total production capacity per month (100 kg)

Decision Variables

- $x_{p,m}$: Amount of product p produced in month m (kg)
- $y_{s,m}$: Amount of material purchased from supplier s in month m (kg)
- $z_{p,m}$: Amount of product p held in inventory at the end of month m (kg)

Objective Function

Minimize the total cost, which includes the cost of purchasing material and the cost of holding inventory:

$$\text{Minimize} \sum_{m \in M} \left(\sum_{s \in S} C_s y_{s,m} + \sum_{p \in P} h_p z_{p,m} \right)$$

Constraints

1. Demand Satisfaction

For each month, the production and inventory must meet the demand:

$$x_{p,m} + z_{p,m-1} - z_{p,m} = d_{p,m} \quad \forall p \in P, m \in M$$

For the first month:

$$x_{p,1} = d_{p,1} \quad \forall p \in P$$

2. Production Capacity

The total production in any month cannot exceed the production capacity:

$$\sum_{p \in P} x_{p,m} \leq P_{\max} \quad \forall m \in M$$

3. Supplier Maximum Supply

The material purchased from any supplier in a month cannot exceed the supplier's maximum supply:

$$y_{s,m} \leq k_s \quad \forall s \in S, m \in M$$

4. Composition Constraints

The percentage of chromium and nickel in the materials purchased must match the required percentages for each product:

$$\frac{\sum_{s \in S} y_{s,m} \cdot \text{chromium}_s}{\sum_{s \in S} y_{s,m}} = c_p \quad \forall p \in P, m \in M$$

$$\frac{\sum_{s \in S} y_{s,m} \cdot \text{nickel}_s}{\sum_{s \in S} y_{s,m}} = n_p \quad \forall p \in P, m \in M$$

5. Non-negativity

All decision variables must be non-negative:

$$x_{p,m} \geq 0, y_{s,m} \geq 0, z_{p,m} \geq 0 \quad \forall p \in P, s \in S, m \in M$$

After solving the optimization in Python Gurobi, the following optimal decisions are observed.

- The model meets the demand for all three products (18/10, 18/8, and 18/0) for every month.
- Supplier A is the dominant supplier, and supplier D and B serve as complements. No purchase from Suppliers C and E
- Some level of inventory holding is observed for product 18/0.

The detailed solutions are

Month	**BuyA**	**BuyB**	**BuyD**	**18/10**	**18/8**	**18/0**
January	23.96	0.00	20.62	25.00	10.00	5.00
February	18.96	0.00	20.62	25.00	10.00	0.00
March	90.00	7.20	0.00	0.00	10.00	90.00
April	90.00	7.20	0.00	0.00	10.00	90.00
May	90.00	7.20	0.00	0.00	10.00	90.00
June	71.81	0.00	36.25	50.00	10.00	40.00
July	90.00	0.42	12.11	12.00	10.00	78.00
August	86.11	0.00	5.00	0.00	10.00	80.00
September	73.25	0.00	11.25	10.00	10.00	62.00
October	90.00	1.89	9.47	10.00	10.00	80.00
November	70.74	0.00	37.62	45.00	19.00	36.00
December	51.49	0.00	62.38	99.00	1.00	0.00

Table 1: Monthly Purchases and Production Quantities

For verification test, we checked the constraints are satisfied, and checked that the results intuitively make sense, such as no purchasing