British Math Olympiad Round 1 2011 — P2/6

Jonathan Kasongo

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Consider the numbers $1, 2, \ldots, n$. Find, in terms of n, the largest integer t such that these numbers can be arranged in a row so that all consecutive terms differ by at least t.

Solution

The answer is $t = \left\lfloor \frac{n}{2} \right\rfloor$ and we split the proof into 2 separate cases.

Case 1: n is even. Write $n = 2m$ for some positive integer m. First we will show that $t = \left\lfloor \frac{n}{2} \right\rfloor = m$ works by construction. Look at the sequence

$$m, 2m, m-1, 2m-1, \ldots, 1, m+1$$

The differences are always alternating between m and $m+1$ which are both at least t so this construction works. Now we show that any $t > \left\lfloor \frac{n}{2} \right\rfloor$ doesn't work. The number $\left\lfloor \frac{n}{2} \right\rfloor$ appears in our sequence, the difference with the consecutive number is at most $\max\left(\left\lfloor \frac{n}{2} \right\rfloor - 1, 2n - \left\lfloor \frac{n}{2} \right\rfloor\right) = \max(m-1, m)$ and thus any $t > \left\lfloor \frac{n}{2} \right\rfloor = m$ won't work since there will be at least one pair of consecutive terms that differ by at most m.

Case 2: n is odd. Write $n = 2m+1$ for some non-negative integer m. First we show that $t = \left\lfloor \frac{n}{2} \right\rfloor = m$ works by construction. Take the construction we made for even n, then add $2m+1$ onto the end of that sequence. That last difference is m and all the other differences are m or $m+1$ so this value of t works. Now we show that any $t > \left\lfloor \frac{n}{2} \right\rfloor = m$ doesn't work. $m+1$ is in this sequence and the difference with the consecutive number will be at most $\max(m+1-1, 2m-(m+1)) = \max(m, m-1)$ and thus any $t > \left\lfloor \frac{n}{2} \right\rfloor = m$ won't work because there is at least one pair of consecutive terms that differ by at most m. ■