Cyprus IMO Team Selection Test 2018 — P1/5

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July 3, 2025

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Determine all integers $n \geq 2$ for which the number 11111 in base n is a perfect square.

Solution

Recalling the definition of base-n numbers the problem is asking us to find solutions to

$$(11111)_n = n^4 + n^3 + n^2 + n + 1 = m^2$$

for some integer m. Reducing that equation modulo n yields $m^2 \equiv 1 \pmod{n}$. But by Euler's theorem $\varphi(n) \mid 2$, where $\varphi(n)$ is Euler's totient function. That means $\varphi(n) \in \{1, 2\}$, so $n \in \{2, 3\}$. Now simply checking both cases shows that only $n = 3$ works where $(11111)_3 = 121 = 11^2$.