Copenhagen masterclass highlight

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Abstract

Notes.

1 Sander Kupers: operadic embedding calculus.

Notes on https://www.utsc.utoronto.ca/people/kupers/seminars/.

♠Setting up Goodwille–Weiss calculus operadically.

Let Man_d^c be the ∞-category of d-dimensional manifolds and $c \in \{o, t\}$ -embeddings (o for smooth, t for continuous). Then $M \in Man_d^c$ vis Yoneda embedding gives a presheaf

$$E_M : (Man_d^c)^{op} \to Space,\ P \mapsto Emb^c(P, M).$$

Let $Disc_d^c$ be the full subcategory on $S \times \mathbb{R}^d$ for finite sets S and $Disc_{d,\le k}^c$ be its full subcategory for $|S| \le k$. There is

$$Disc_{d,\le 1}^c \subset Disc_{d,\le 2}^c \subset \cdots \subset Disc_d^c \overset{i}{\subset} Man_d^c,$$

which yields

$$Psh(Disc_{d,\le 1}^c) \leftarrow Psh(Disc_{d,\le 2}^c) \leftarrow \cdots \leftarrow Psh(Disc_d^c) \overset{i^*}{\leftarrow} Psh(Man_d^c). \tag{1.1}$$

For $M, N \in Man_d^c$, define $T_k Emb^c(M, N) = Map_{Psh(Disc_{d,\le k}^c)}(E_M, E_N)$. (1.1) gives the embedding calculus tower

$$T_1 Emb^c(M, N) \leftarrow T_2 Emb^c(M, N) \leftarrow \cdots T_\infty Emb^c(M, N) \leftarrow Emb^c(M, N)$$

Definition 1.2. *An embedding $P \hookrightarrow Q$ is called an equivalence on tangential k-type if there exists a space B and factorization of the tangent bundle/micro-bundle*

$$\begin{tikzcd} P \arrow[r] \arrow[dr] & Q \arrow[r] \arrow[d] & BO \text{ or } BTop \\ & B \arrow[ur] & \end{tikzcd}$$

such that both $P \to B$ and $Q \to B$ are k-connected maps.

Theorem 1.3. *(Krannich–Kupers, improvement of Goodwille-Klein–Weiss) $d \ge 5$, M^d compact, $\partial M \to M$ is an equivalence on tangential 2-type. Then $Emb^o(M, N) \xrightarrow{\sim} T_\infty Emb^o(M, N)$.*

First layer:

$$T_1 Emb^c(M, N) \simeq \begin{cases} Map^{/BO(d)}(M, N) & c = 0; \\ Map^{/BTop(d)}(M, N) & c = t. \end{cases}$$

♠The particle tangential structure.

Definition 1.4. *Let $\mathscr{O}$ be an operad with a map $\theta : B \to BAut_{Opd}(\mathscr{O})$. Define a new operad (the "semidirect product operad")*

$$\mathscr{O}^\theta = \operatorname{colim}_B(B \to BAut_{Opd}(\mathscr{O}) \hookrightarrow Opd).$$

The spaces of $\mathscr{O}^\theta$ are $\mathscr{O}^\theta(k) \simeq \mathscr{O}(k) \times (\Omega B)^k$. Let E_d be the little d-disk operad.

Definition 1.5. $\mathrm{E}_d^p = (E_d)^{id: BAut(\mathrm{E}_d) \to BAut(\mathrm{E}_d)}$.

Remark 1.6. E_d^p is "maximal" in the sense that $Aut(\mathrm{E}_d^p) \simeq *$.