1. 1C-1

   (a) We have $S = \pi r^2$, S is area and r is radius.

   $$f(r) = \pi r^2$$

   $g(r)$ is the function represent the change.

   $$\begin{aligned} g(r) &= \lim_{\Delta r \to 0} \frac{f(r+\Delta r) - f(r)}{\Delta r} \\ &= \lim_{\Delta r \to 0} \frac{\pi(r+\Delta r)^2 - \pi r^2}{\Delta r} \\ &= \lim_{\Delta r \to 0} \pi \frac{\Delta r^2 + 2r\Delta r}{\Delta r} \\ &= \lim_{\Delta r \to 0} 2\pi r + \pi \Delta r \\ &= 2\pi r \end{aligned}$$

   (b) we have $V = 4/3\pi r^3$, V is volume and r is radius.

   $$f(r) = 4/3\pi r^3$$

   $g(r)$ is the function represent the change.

   $$\begin{aligned} g(r) &= \lim_{\Delta x \to 0} \frac{f(r+\Delta r) - f(r)}{\Delta r} \\ &= \lim_{\Delta x \to 0} \frac{4}{3}\pi \frac{(r+\Delta r)^3 - r^3}{\Delta r^3} \\ &= \lim_{\Delta x \to 0} \frac{4}{3}\pi \frac{r^3 + \Delta r^3 + 3r^2\Delta r + 3\Delta r^2 r - r^3}{\Delta r} \\ &= \lim_{\Delta x \to 0} \frac{4}{3}\pi(\Delta r^2 + 3r^2 + 3r\Delta r) \\ &= 4\pi r^2 \end{aligned}$$

2. 1C-2

   (a)

   $$f(x) = (x-a)g(x)$$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x+\Delta x - a)g(x+\Delta x) - (x-a)g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{g(x+\Delta x)\Delta x + (x-a)g(x+\Delta x) - (x-a)g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} g(x+\Delta x) + (x-a)\frac{g(x+\Delta x) - g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} g(x+\Delta x) + (x-a)g'(x) \\
&= g(x) + (x-a)g'(x)
\end{aligned}
$$

if $x = a$:

$$
\begin{aligned}
f'(a) &= g(a) + (a-a)g'(a) \\
&= g(a)
\end{aligned}
$$

3. 1c-3

(a)

$$f(x) = \frac{1}{2x+1}$$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{\frac{1}{2(x+\Delta x)+1} - \frac{1}{2x+1}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{2x+1-2(x+\Delta x)-1}{(2(x+\Delta x)+1)(2x+1)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{-2\Delta x}{(2(x+\Delta x)+1)(2x+1)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-2}{(2(x+\Delta)+1)(2x+1)} \\
&= \frac{-2}{4x^2+8x+1}
\end{aligned}
$$

(b)

$$f(x) = 2x^2 + 5x + 4$$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 + 5(x+\Delta x) + 4 - (2x^2+5x+4)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 + 5x + 5\Delta x + 4 - 2x^2 - 5x - 4}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 + 5\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} 4x + 5 + 2\Delta x \\
&= 4x + 5
\end{aligned}
$$

(c)

$$
f(x) = \frac{1}{x^2+1}
$$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)^2+1} - \frac{1}{x^2+1}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{x^2+1}{[(x+\Delta x)^2+1](x^2+1)} - \frac{(x+\Delta x)^2+1}{[(x+\Delta x)^2+1](x^2+1)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{-2x\Delta x - \Delta x^2}{[(x+\Delta x)^2+1](x^2+1)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{[(x+\Delta x)^2+1](x^2+1)} \\
&= \frac{-2x}{(x^2+1)^2}
\end{aligned}
$$

(d)

$$
f(x) = \frac{1}{\sqrt{x}}
$$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{x^{\frac{1}{2}} - (x+\Delta x)^{\frac{1}{2}}}{x^{\frac{1}{2}}(x+\Delta x)^{\frac{1}{2}}}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{x - x - \Delta x}{[x^{\frac{1}{2}} + (x+\Delta x)^{\frac{1}{2}}][x^{\frac{1}{2}}(x+\Delta x)^{\frac{1}{2}}]}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-1}{[x^{\frac{1}{2}} + (x+\Delta x)^{\frac{1}{2}}][x^{\frac{1}{2}}(x+\Delta x)^{\frac{1}{2}}]} \\
&= \frac{-1}{2x^{\frac{1}{2}}x} \\
&= \frac{-1}{2x^{\frac{3}{2}}}
\end{aligned}
$$

4. 1c-4

   (a)

$$f'(1) = 11$$

   (b)

$$f'(a) = 4a + 5$$

   (c)

$$f'(0) = 0$$

   (d)

$$f'(a) = -\frac{1}{2a^{\frac{3}{2}}}$$

5. 1c-5

$$f(x) = 1 + (x-1)^2$$

$$f'(x) = 2x - 2$$

   define tangent line : y and it cross point(a, f(a))

$$
\begin{aligned}
\frac{y - f(a)}{x - a} &= 2(a-1) \\
y &= 2(a-1)(x-a) + (a-1)^2 + 1 \\
0 &= -2a^2 + 2a + a^2 - 2a + 2 \\
0 &= a^2 + 2
\end{aligned}
$$

   when $x = 0$, $y = 0$

$$a = \pm\sqrt{2}$$

$$y = 2(\sqrt{2} - 1)(x - \sqrt{2}) + (\sqrt{2} - 1)^2 + 1$$

$$y = 2(-\sqrt{2} - 1)(x + \sqrt{2}) + (\sqrt{2} + 1)^2 + 1$$