Ejercicios de Cálculo de Límites y Continuidad

Recordatorio...

1. $\lim\limits_{n\to\infty} \frac{1}{n} = 0$
2. $\lim\limits_{n\to\infty} \frac{\sin n}{n} = 0$
3. $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$
4. $\lim\limits_{n\to\infty} a^n = \begin{cases} 0, & \text{si } 0<a<1 \\ \infty, & \text{si } a>1 \end{cases}$
5. $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$
6. $\lim\limits_{x\to 0} \frac{\cos x - 1}{x} = 0$
7. $\lim\limits_{x\to 0} \frac{\arctan x}{x} = 1$
8. $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}} = e$
9. $\lim\limits_{x\to 0} \frac{e^x-1}{x} = 1$
10. $\lim\limits_{x\to 0} \frac{\ln(x+1)}{x} = 1$
11. **Stolz.** $\{x_n\}$ arbitraria, $\{y_n\}$ estrictamente creciente y divergente. Entonces $\lim\limits_{n\to\infty} \frac{x_n}{y_n} = \lim\limits_{n\to\infty} \frac{x_{n+1}-x_n}{y_{n+1}-y_n}$.

Calcule los siguientes límites

1. $\lim\limits_{n\to\infty} 3^n \sin\left(3^{-n}\pi\right) = \pi$
2. $\lim\limits_{x\to 1} \frac{\ln x}{\sin(x-1)} = 1$
3. $\lim\limits_{x\to 0} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}} = \sqrt{e}$
4. $\lim\limits_{n\to\infty} \frac{\frac{\sqrt{1}}{2}+\frac{\sqrt{2}}{3}+\ldots+\frac{\sqrt{n}}{n+1}}{\sqrt{n}} = 2$
5. $\lim\limits_{x\to 3} (7-2x)^{\tan\left(\frac{\pi x}{6}\right)} = e^{\frac{12}{\pi}}$
6. $\lim\limits_{x\to 0} \frac{1-\sqrt{1-x^3}}{\tan x - \sin x} = 1$
7. $\lim\limits_{x\to 0} \frac{\ln(\cos x + a\sin x)}{\sin x} = a$
8. $\lim\limits_{n\to\infty} \frac{1+2\sqrt{2}+3\sqrt{3}+\ldots+n\sqrt{n}}{n^2\sqrt{n}} = \frac{2}{5}$
9. $\lim\limits_{x\to\infty} x\left(\sqrt{x^2+\sqrt{x^4+1}} - x\sqrt{2}\right) = 0$
10. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\sqrt[3]{(1-\sin x)^2}} = \infty$
11. $\lim\limits_{x\to\infty} \frac{(2x+3)^5(3x+2)^6}{2+x^{11}} = 2^5\cdot 3^6$
12. $\lim\limits_{x\to 1} \frac{\sqrt{x+3}-\sqrt[3]{9-x}}{\sqrt{x^3+15}-2} = 0$
13. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}} = -1$
14. $\lim\limits_{n\to\infty} (2n+3)^{n+2}(2n+1)^{-n-2} = e$
15. $\lim\limits_{n\to\infty} \sqrt{n^2+an+1} - \sqrt{n^2+bn+1} = \frac{a-b}{2}$
16. $\lim\limits_{x\to 1} \frac{\sin 2(x-1)}{x^3-1} = \frac{2}{3}$
17. $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = 0$
18. $\lim\limits_{x\to 0} \frac{\tan 2x}{\sin 3x} = \frac{2}{3}$
19. $\lim\limits_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right) = 0$
20. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x} = \frac{\sqrt{2}}{2}$
21. $\lim\limits_{x\to 2} \left(x^2-4\right)\sin\left(\frac{1}{x-2}\right) = 0$
22. $\lim\limits_{x\to 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x} = \frac{\sqrt{2}}{8}$
23. $\lim\limits_{x\to\infty} \left(\frac{\ln ax}{\ln bx}\right)^{\ln x} = \frac{a}{b}$
24. $\lim\limits_{x\to 0} \sqrt[x]{\frac{1+x}{1-x}} = e^2$
25. $\lim\limits_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\sin\left(x-\frac{\pi}{3}\right)} = \sqrt{3}$
26. $\lim\limits_{x\to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$

27. $\lim\limits_{x\to\infty} x\left(\ln\left(1+x\right)-\ln x\right)=1$

28. $\lim\limits_{x\to 0}\frac{x+x^2}{|x|}=\nexists$

29. $\lim\limits_{n\to\infty}\frac{1^2+3^2+5^2+\ldots+(2n-1)^2}{1^2+2^2+\ldots+n^2}=4$

30. $\lim\limits_{n\to\infty}\frac{1^2\cdot 2^1+2^2\cdot 2^2+3^2\cdot 2^3+\ldots+n^2\cdot 2^n}{n^2\cdot 2^n}=2$

31. $\lim\limits_{x\to 1}\frac{1-x^2}{\sin\pi x}=\frac{2}{\pi}$

Estudie la continuidad de las siguientes funciones.

1. $f\left(x\right)=\begin{cases} x+1 & \text{si } x<2,\\ 5-x & \text{si } 2\leq x\leq 4,\\ x-3 & \text{si } x>4.\end{cases}$

2. $f\left(x\right)=\frac{1}{x-2}-3x.$

Encuentre el valor de las constantes para que la función sea contínua.

1. $f\left(x\right)=\begin{cases} x+1 & \text{si } 1<x<3,\\ x^2+4x+M & \text{si } |x-2|\geq 1.\end{cases}$

2. $f\left(x\right)=\begin{cases} \frac{A\left(x^3+1\right)}{x+1}+B & \text{si } x<2,\\ 2Ax-3 & \text{si } 2\leq x\leq 4,\\ \frac{B\left(x^2+3x-10\right)}{x-2} & \text{si } x>4.\end{cases}$