

# Environments

# Class outline:

- Multiple environments
- Environments for HOFs
- Local names
- Function composition
- Self-referencing functions
- Currying

# Multiple Environments

# Life cycle of a function

## What happens?

### Def statement

```
def square ( x ) :  
    return x * x
```

- A new function is created!
- Name bound to that function in the current frame.

### Call expression

```
square ( 2 + 2 )
```

- Operator & operands evaluated
- Function (value of operator) called on arguments (values of operands)

### Calling/applying

```
def square( x )
```

▶ 16

4 ▶

- A new frame is created!
- Parameters bound to arguments
- Body is executed in that new environment

# A nested call expression

- 1.
- 2.
- 3.

```
def square(x):  
    return x * x  
square(square(3))
```



# A nested call expression

- 1.
- 2.
- 3.

next

```
def square(x):  
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square(square(3))
```





# A nested call expression

1.

prev

2.

3.

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```
def square(x):  
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```

Global frame

square | • ----> func square(x) [parent=Global]



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```
square( square(3) )
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# A nested call expression

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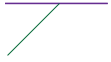
next

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def square(x):  
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Global frame

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```
square( square(3) )
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```
func square(x)
```

# A nested call expression

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- 2.
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def square(x):  
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square(square(3))
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Global frame

square | • ----> func square(x) [parent=Global]

`square( square(3) )`

`func square(x)`

`square(3)`

```
graph TD; A[square( square(3) )] --- B[func square(x)]; A -.- C[square(3)];
```



# A nested call expression

1.

prev

2.

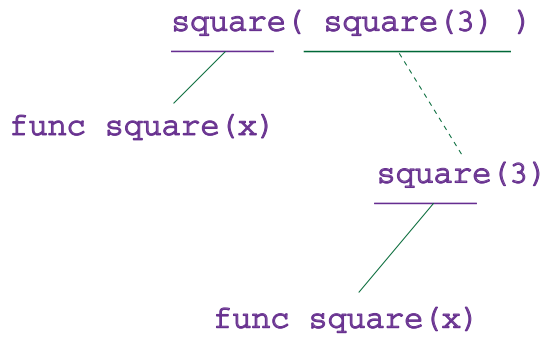
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# A nested call expression

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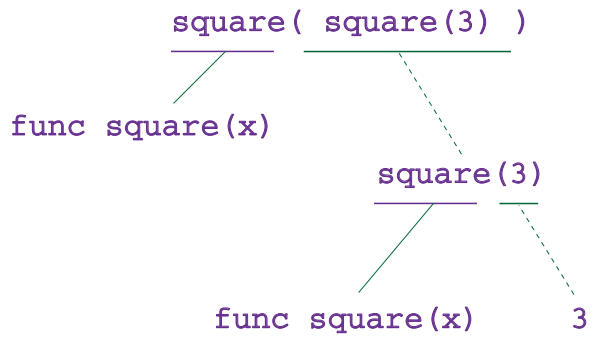
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# A nested call expression

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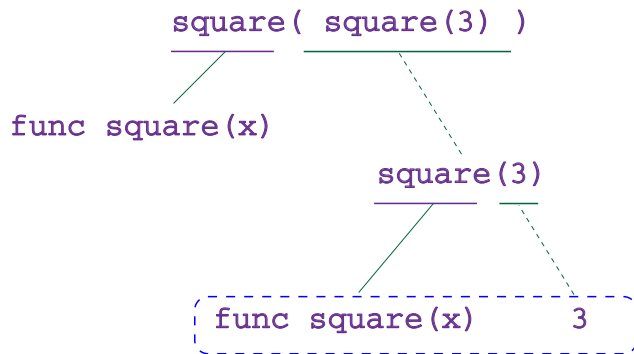
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# A nested call expression

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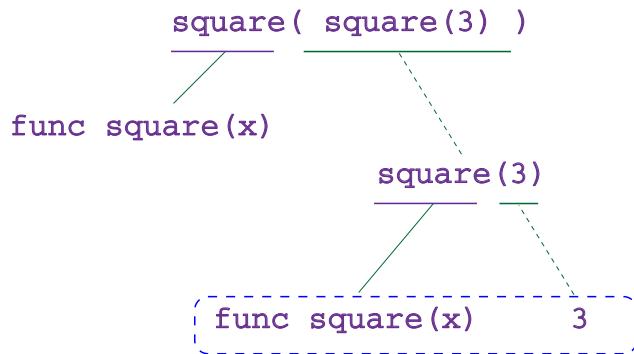
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f1: square [parent=Global]

x   3





# A nested call expression

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- 2.
- 3.

[prev](#)

[next](#)

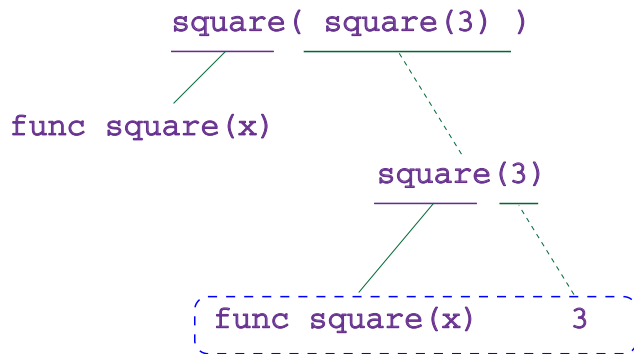
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    return x * x  
  
square(square(3))
```

Global frame

square    ----> func square(x) [parent=Global]

f1: square [parent=Global]

x 3



# A nested call expression

1.

next

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prev

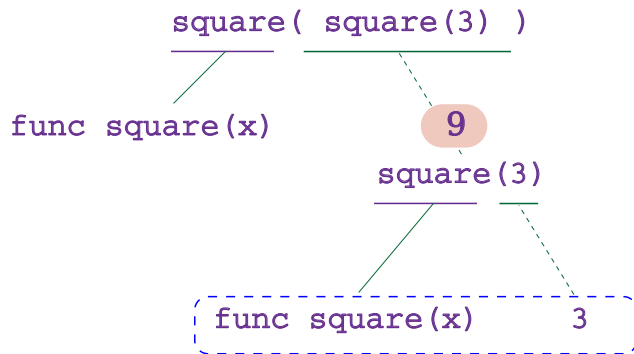
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square(square(3))
```

Global frame

square | • ----> func square(x) [parent=Global]

f1: square [parent=Global]

x	3
Return value	9



# A nested call expression

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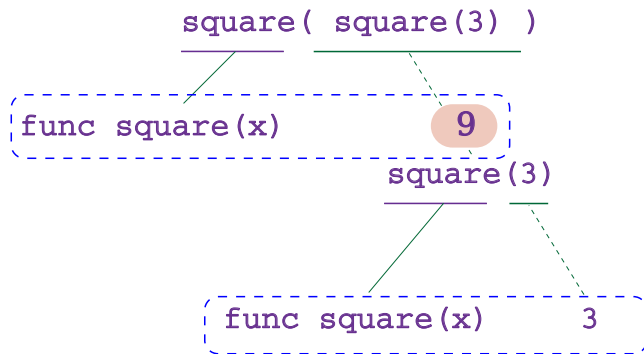
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Global frame

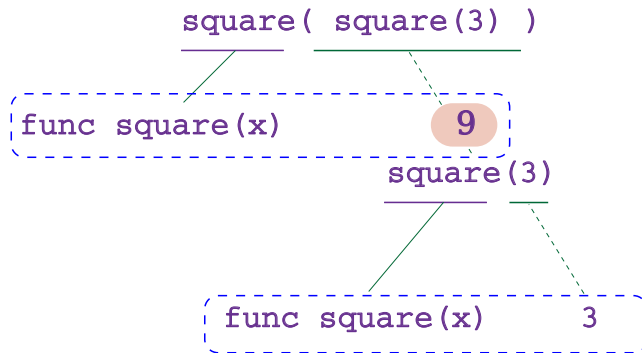
square | • ----> func square(x) [parent=Global]

f1: square [parent=Global]

x	3
Return value	9

f2: square [parent=Global]

|





# A nested call expression

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```

Global frame

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f1: square [parent=Global]

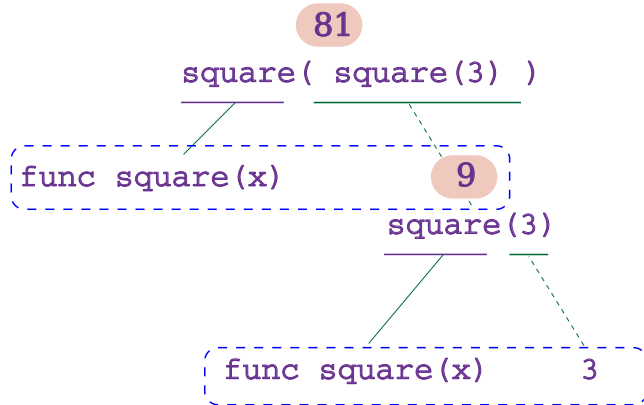
x	3
Return value	9

f2: square [parent=Global]

|

Return value


x	9
	81



# Multiple environments in one diagram!

```
def square(x):  
    return x * x  
  
square(square(3))
```

Global frame

square  ----> func square(x) [parent=Global]

f1: square [parent=Global]

x	3
Return value	9

f2: square [parent=Global]

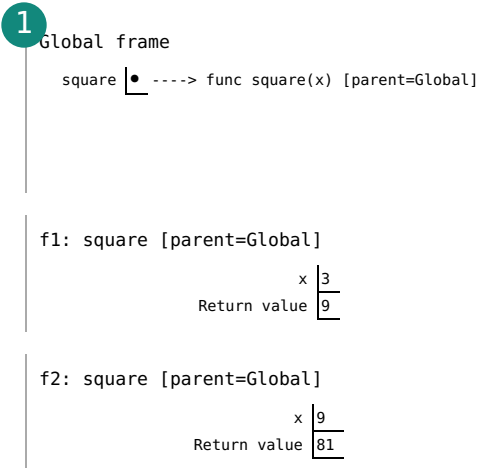
x	9
Return value	81

An environment is a sequence of frames.



# Multiple environments in one diagram!

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```

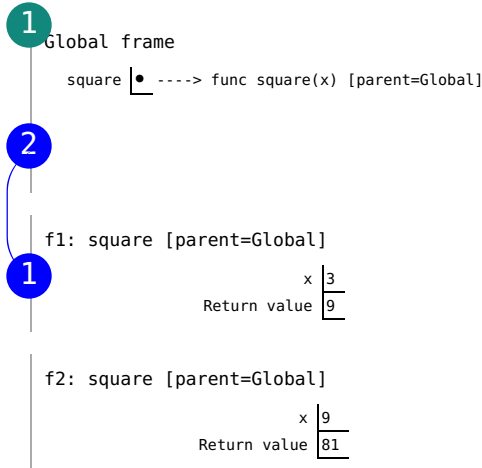


An environment is a sequence of frames.

- Environment: Global frame

# Multiple environments in one diagram!

```
def square(x):  
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```



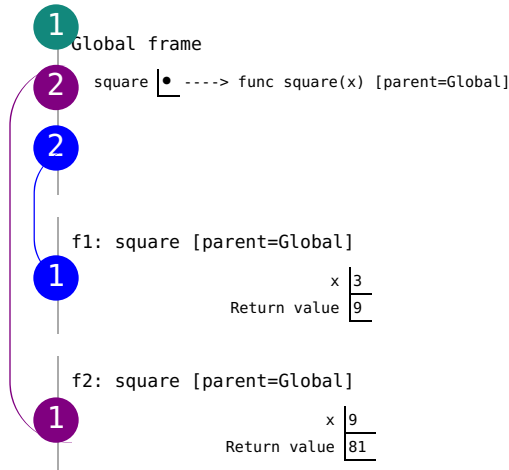
An environment is a sequence of frames.

- Environment: Global frame
- Environment: Local frame (f1), then global frame



# Multiple environments in one diagram!

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def square(x):  
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```

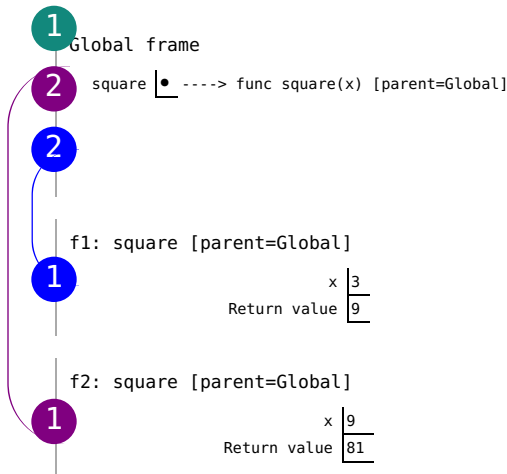


An environment is a sequence of frames.

- Environment: Global frame
- Environment: Local frame (f1), then global frame
- Environment: Local frame (f2), then global frame

# Names have no meanings without environments

```
def square(x):  
    return x * x  
  
square(square(3))
```



Every expression is evaluated in the context of an environment.

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

# Names have different meanings in different environments

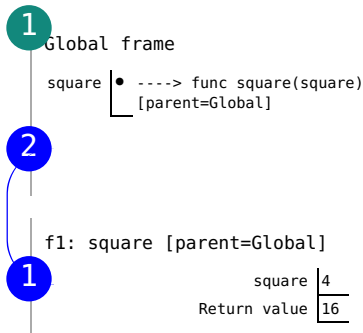
```
def square(square) :  
    return square * square  
  
square(4)
```

Every expression is evaluated in the context of an environment.

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# Names have different meanings in different environments

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# Environments for higher-order functions

# Review: Higher-order functions

A higher-order function is either...

- A function that takes a function as an argument value  
`summation(5, lambda x: x**2)`
- A function that returns a function as a return value  
`make_adder(3)(1)`

**Functions are first class:** Functions are values in Python.



# Example: Apply twice

```
def apply_twice(f, x):  
    return f(f(x))  
  
def square(x):  
    return x ** 2  
  
apply_twice(square, 3)
```



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# Arguments bound to functions



# Arguments bound to functions

?

?

# Arguments bound to functions

?

?



# Environments for nested definitions

# Example: Make texter

```
def make_texter(emoji):  
    def texter(text):  
        return emoji + text + emoji  
    return texter  
  
happy_text = make_texter("😊")  
result = happy_text("lets go to the beach!")
```



View in PythonTutor

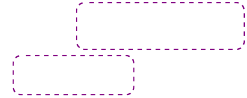
# Environments for nested def statements



# Environments for nested def statements



- Every user-defined **function** has a parent frame
- The parent of a **function** is the **frame in which it was defined**





# Environments for nested def statements



- Every user-defined **function** has a parent frame
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- The parent of a **frame** is the **parent of the called function**

# Environments for nested def statements



- Every user-defined **function** has a parent frame
- The parent of a **function** is the **frame in which it was defined**
- Every local **frame** has a parent frame
- The parent of a **frame** is the **parent of the called function**
- An environment is a **sequence of frames**.

# How to draw an environment diagram

## When a function is defined:

1. Create a function value:  
`func <name>(<formal parameters>) [parent=<label>]`
2. Its parent is the current frame.
3. Bind `<name>` to the function value in the current frame

## When a function is called:

1. Add a local frame, titled with the `<name>` of the function being called.
2. Copy the parent of the function to the local frame:  
`[parent=>label<]`
3. Bind the `<formal parameters>` to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

# Local names

# Example: Thingy Bobber

```
def thingy(x, y):  
    return bobber(y)  
  
def bobber(a):  
    return a + y  
  
result = thingy("ma", "jig")
```

What do you think will happen?

# Example: Thingy Bobber

```
def thingy(x, y):  
    return bobber(y)  
  
def bobber(a):  
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result = thingy("ma", "jig")
```

What do you think will happen?



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# Local name visibility

Local names are not visible to other (non-nested) functions.



- An environment is a sequence of frames.
- The environment created by calling a top-level function consists of one local frame followed by the global frame.

1

# Function Composition



# Example: Composer

```
def happy(text):  
    return "😊" + text + "😊"  
  
def sad(text):  
    return "😞" + text + "😞"  
  
def composer(f, g):  
    def composed(x):  
        return f(g(x))  
    return composed  
  
msg1 = composer(sad, happy)("cs61a!")  
msg2 = composer(happy, sad)("eecs16a!")
```

What do you think will happen?

# Example: Composer (Part 2)

One of the composed functions could itself be an HOF...

```
def happy(text):  
    return "😊" + text + "😊"  
  
def sad(text):  
    return "😞" + text + "😞"  
  
def make_texter(emoji):  
    def texter(text):  
        return emoji + text + emoji  
    return texter  
  
def composer(f, g):  
    def composed(x):  
        return f(g(x))  
    return composed  
  
composer(happy, make_texter("😊"))('snow day!')
```



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# Composer 2 expression tree

```
composer(happy, make_texter("🌨"))("snow day!")
```

# Composer 2 expression tree

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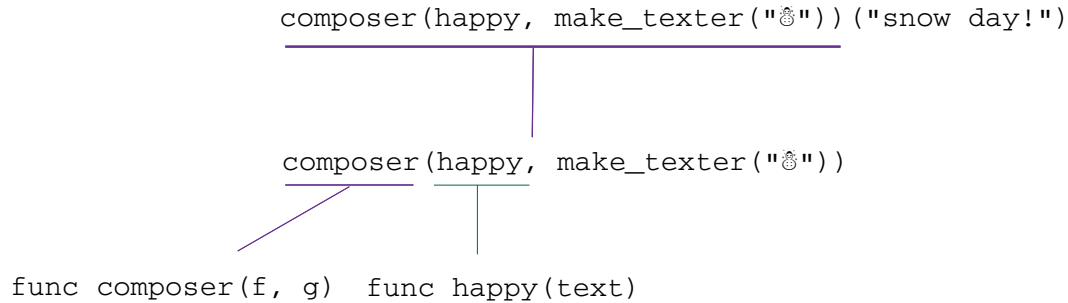
# Composer 2 expression tree

`composer(happy, make_texter("🌨"))("snow day!")`

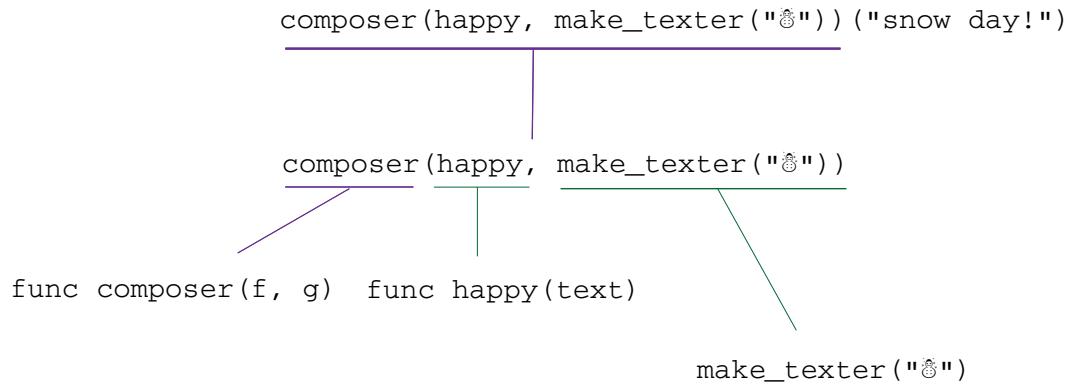
`composer(happy, make_texter("🌨"))`

`func composer(f, g)`

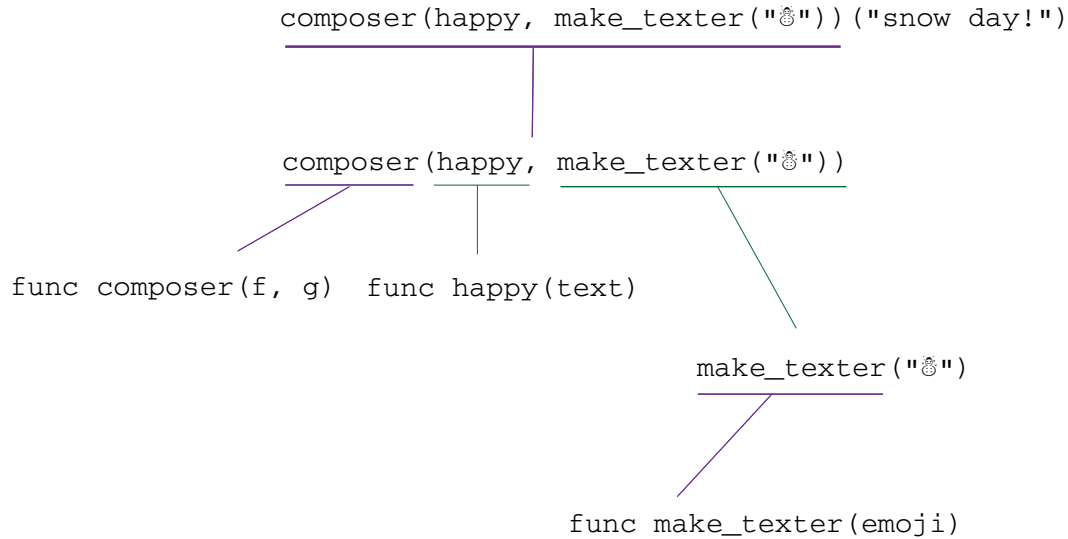
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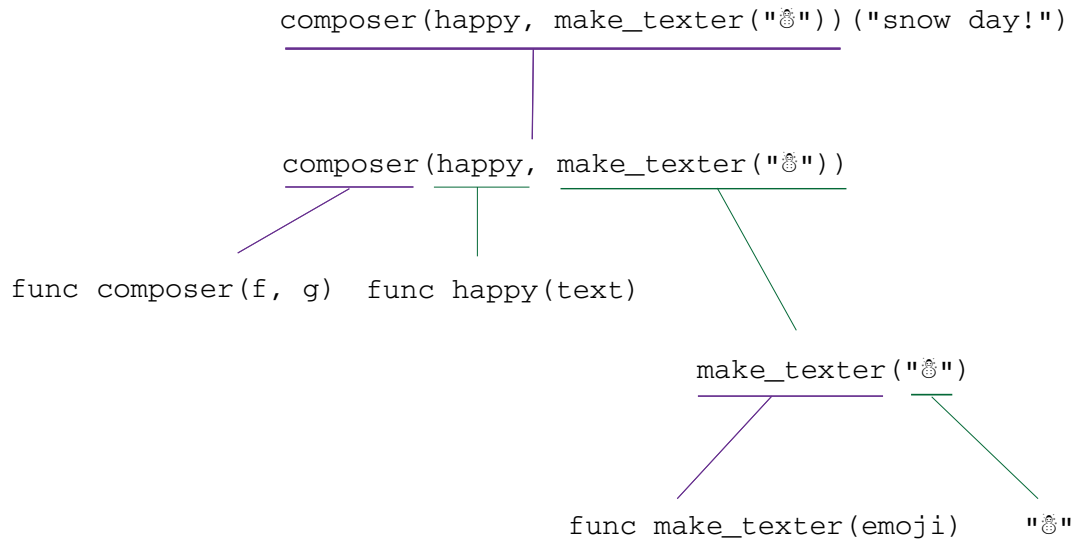


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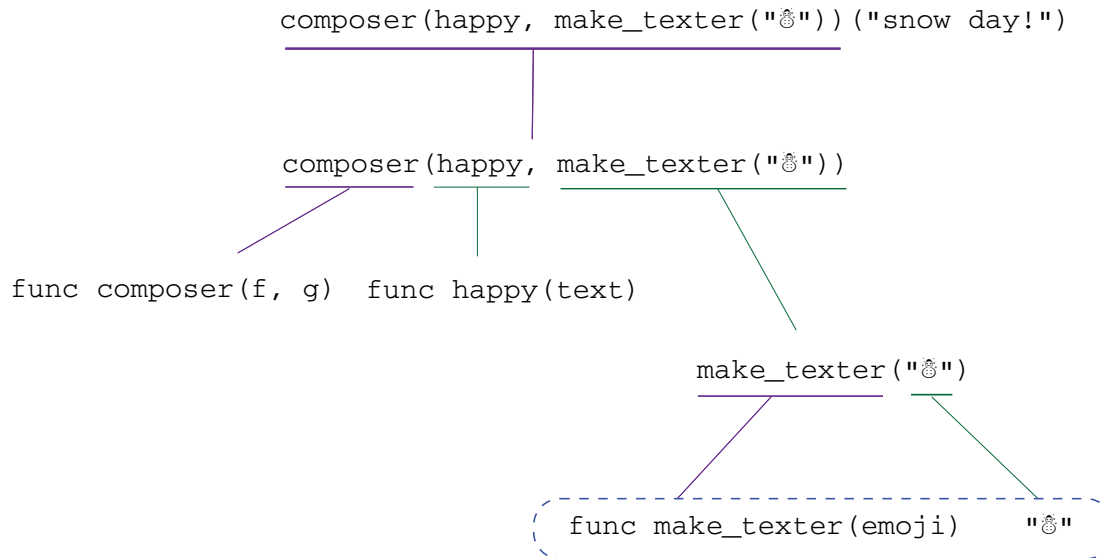




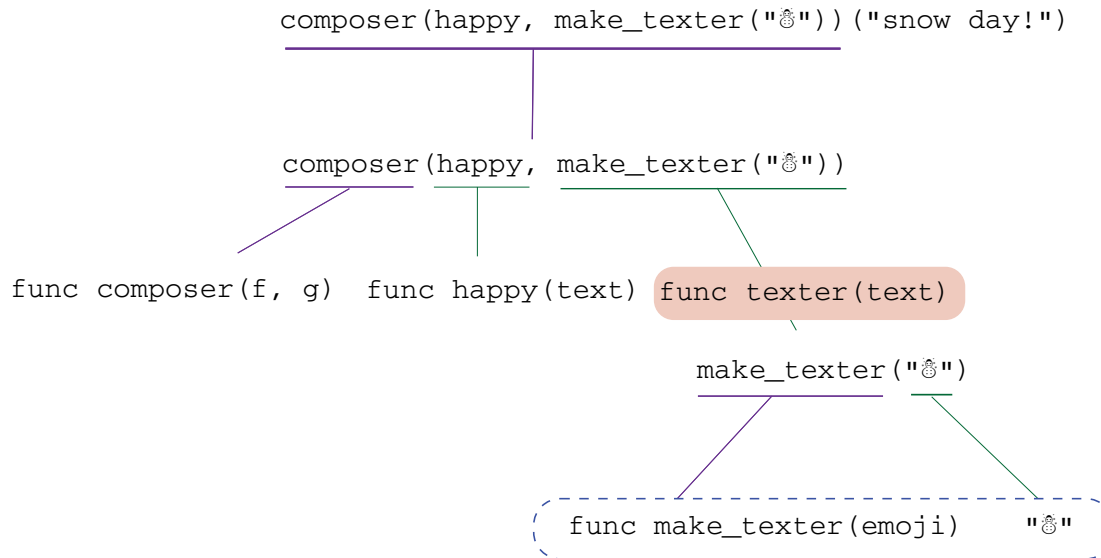
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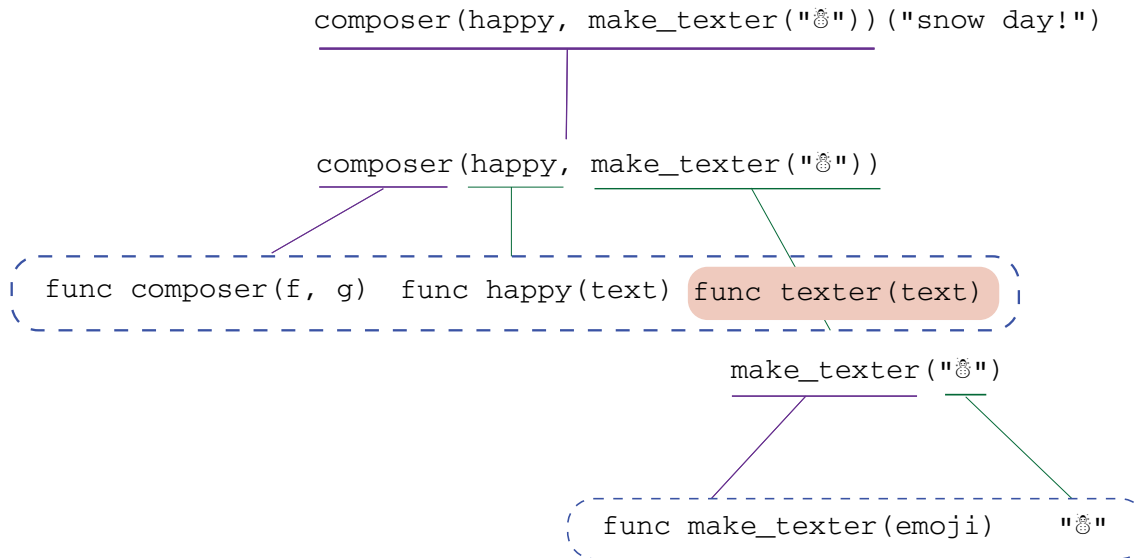
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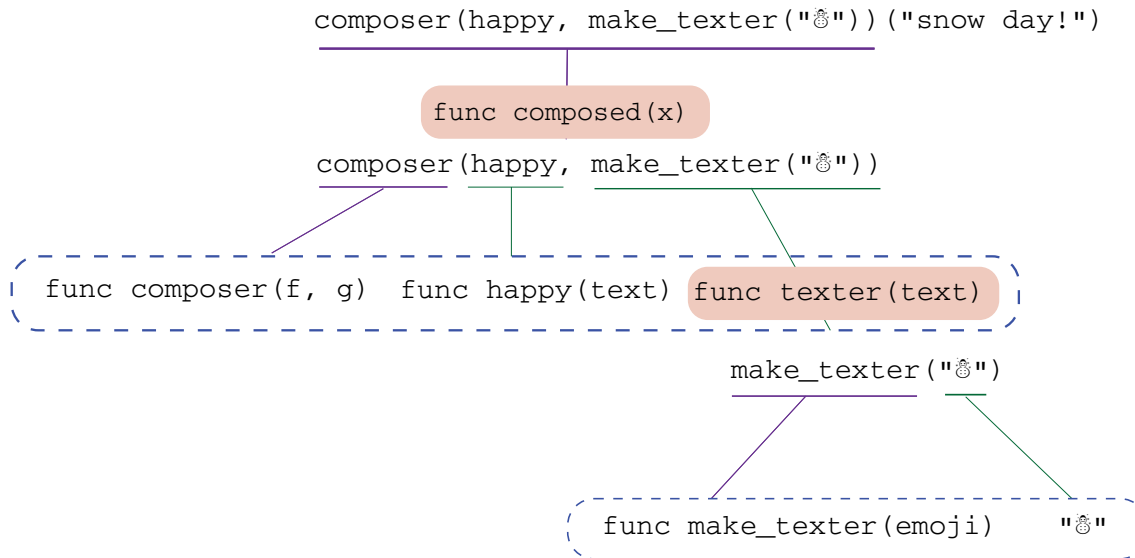
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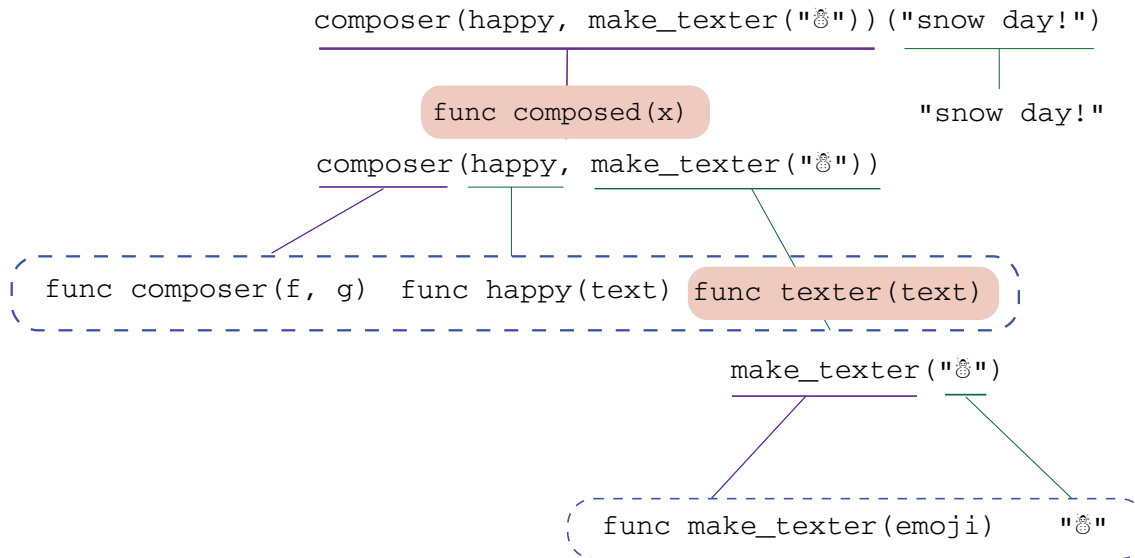
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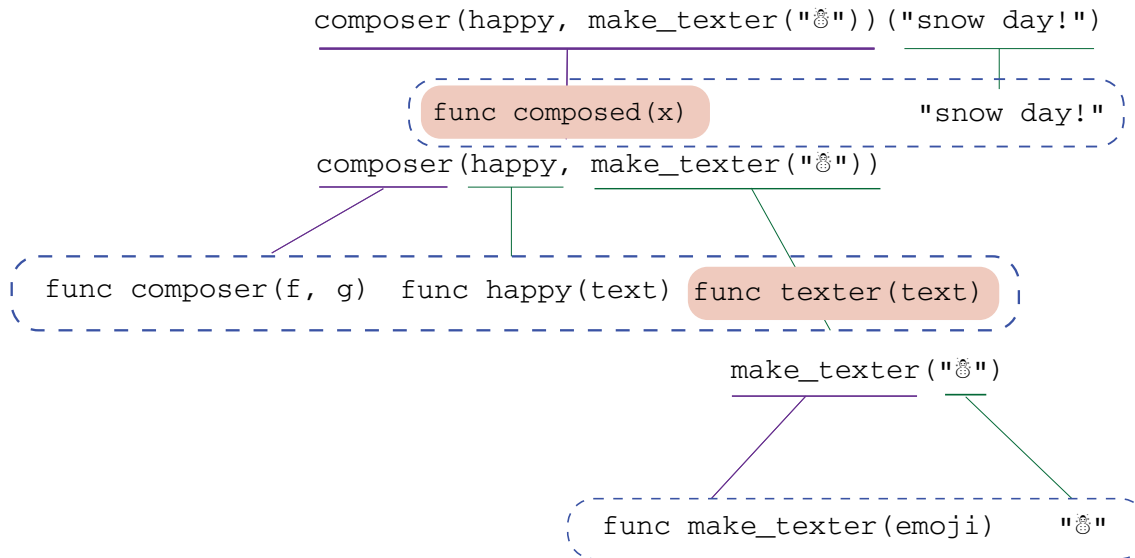
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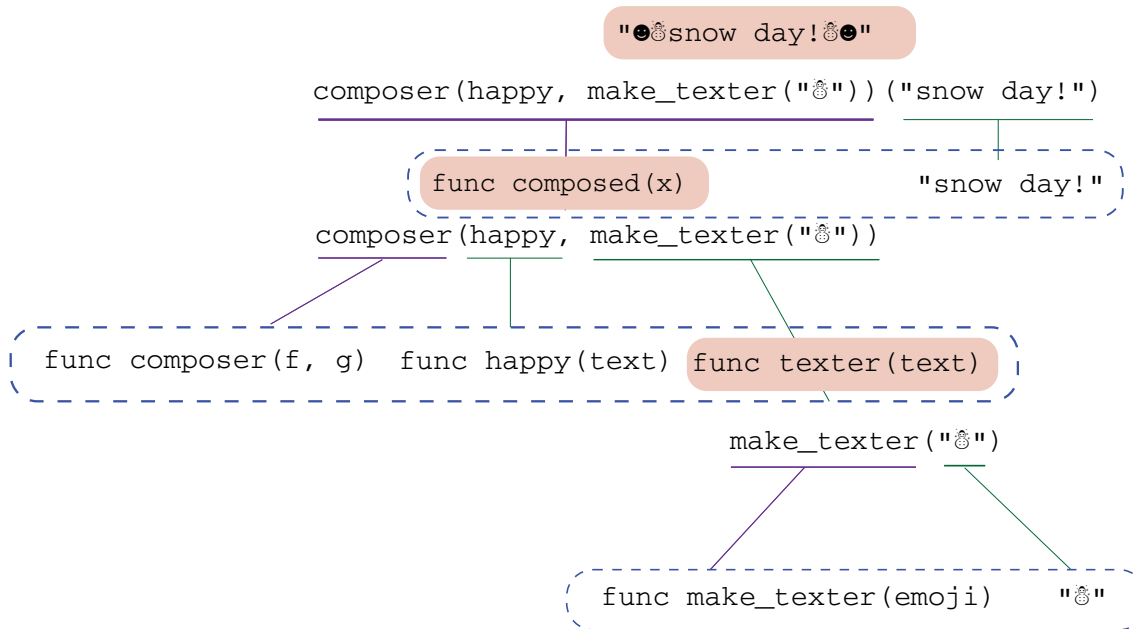
# Composer 2 expression tree



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# Composer 2 expression tree





# Self-reference

# A self-referencing function

A higher-order function could return a function that references its own name.

```
def print_sums(n) :  
    print(n)  
    def next_sum(k) :  
        return print_sums(n + k)  
    return next_sum  
  
print_sums(1) (3) (5)
```



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# Environment for print\_sums



# Understanding print\_sums

The call:

```
print sums(1)(3)(5)
```

produces the same result as:

```
g1 = print sums(1)
g2 = g1(3)
g2(5)
```

A call to `print sums(x)` returns a function that:

- Prints `x` as a side-effect, and
- Returns a function that, when called with argument `y`, will do the same thing, but with `x+y` instead of `x`.

So these calls will...

- First print 1 and return `g1`,
- which when called with 3, will print 4 (= 1+3) and return `g2`,
- which when called with 5, will print 9 (= 4+5), and return. . .

# Currying

# add vs. make\_adder

Compare...

```
from operator import add  
  
add(2, 3)
```

```
def make_adder(n):  
    return lambda x: n + x  
  
make_adder(2)(3)
```

What's the relationship between `add(2, 3)` and `make_adder(2)(3)`?

# Function currying

**Currying:** Converting a function that takes multiple arguments into a single-argument higher-order function.

A function that currys any two-argument function:

```
def curry2(f):  
    def g(x):  
        def h(y):  
            return f(x, y)  
        return h  
    return g
```

# Function currying

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```
make_adder = curry2(add)  
make_adder(2)(3)
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        return h  
    return g
```

```
make_adder = curry2(add)  
make_adder(2)(3)
```

```
curry2 = lambda f: lambda x: lambda y: f(x, y)
```

# Why "currying"?

It's not food! ✖ ✖

Named after American logician Haskell Curry, but actually published first by Russian Moses Schönfinkel, based on principles by German Gottlob Frege.

See also: [Stigler's law of eponymy](#)