Solution to COMP9334 Revision Questions for Week 2 — Part 2

Question 1

(a) Since the mean arrival rate is 20 requests per second. The mean inter-arrival time is $\frac{1}{20}$ = 50ms.

(b) The mean number of requests arriving in 1 minute = 20 requests per seconds $\times$ 60 seconds / minute = 1200 requests per minute.

(c) and (d) Recalling that for Poisson arrivals with mean arrival rate λ and time interval t, the probability of n arrivals is

$$\frac{(\lambda t)^n \exp(-\lambda t)}{n!}. \tag{1}$$

For this question, $\lambda = 20$ and $t = 60$, so $\lambda t = 1200$.

In order to calculate the probability of no arrivals in a minute, we put $n = 0$ to obtain

$$\exp(-\lambda t) = \exp(-1200) \tag{2}$$

In order to calculate the probability of 10 arrivals in a minute, we put $n = 10$ to obtain

$$\frac{(1200)^{10} \exp(-1200)}{10!} \tag{3}$$

Question 2

In order to refer to the two Poisson processes in a convenient way, I call them P_1 and P_2. The Poisson processes P_1 and P_2, have rates r_1 and r_2, respectively.

Consider a time interval T. Since P_1 is a Poisson process with rate r_1, we know that the probability that there are k arrivals in time interval T is

$$\frac{e^{-r_1 T}(r_1 T)^k}{k!} \tag{4}$$

Similarly, the probability that there are j arrivals in time interval T from P_2 is

$$\frac{e^{-r_2 T}(r_2 T)^j}{j!} \tag{5}$$

Let us consider the aggregation of the two Poisson processes P_1 and P_2 over the time interval T. The arrivals can come from P_1 or P_2. Let us find the probability that there are n arrivals in T. If there are n arrivals from P_1 and P_2 together, this can be resulted from

- 0 arrivals from P_1 and n arrivals from P_2
- 1 arrivals from P_1 and $(n-1)$ arrivals from P_2

- 2 arrivals from P_1 and $(n-2)$ arrivals from P_2
  ...
- $(n-1)$ arrivals from P_1 and 1 arrivals from P_2
- n arrivals from P_1 and 0 arrivals from P_2

Therefore

$$
\begin{aligned}
& \text{Probability that there are } n \text{ arrivals over time } T \text{ from } P_1 \text{ and } P_2 \text{ together} \\
= & \sum_{i=0}^{n} \text{Probability of } i \text{ arrivals over time } T \text{ from } P_1 \times \text{Probability of } (n-i) \text{ arrivals over time } T \text{ from } P_2 \\
= & \sum_{i=0}^{n} \frac{e^{-r_1 T}(r_1 T)^i}{i!} \frac{e^{-r_2 T}(r_2 T)^{n-i}}{(n-i)!} \\
= & \frac{1}{n!} e^{-(r_1+r_2)T} \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} (r_1 T)^i (r_2 T)^{(n-i)} \\
= & \frac{1}{n!} e^{-(r_1+r_2)T} ((r_1+r_2)T)^n
\end{aligned}
$$

This shows that the aggregation of P_1 and P_2 is a Poisson process with rate $r_1 + r_2$.