

Solution to COMP9334 Revision Questions for Week 5

Question 1

First note that one counter is not sufficient to serve all the customers. If we consider all the customers together, each customer carries on average 20.5 items, which takes $\frac{20.5}{15}$ to complete. Since the customer arrival rate is 1, the utilisation will be above 1 if only one counter is used.

Let us refer to the two counters as Counter 1 and Counter 2. Let us assume that Counter 1 serves customers with x items or less and Counter 2 serves customers with more than x items, where $1 \leq x \leq 39$.

Let $\lambda(=1)$ denote the overall arrival rate and $\mu = 15$ be the service rate of each counter.

Let us consider Counter 1 first. Since only customers with x items or less go to Counter 1, the arrival rate at Counter 1 is $\lambda \frac{x}{40}$. The customers arriving at Counter 1 bring with them 1, 2, ..., x items uniformly distributed. Let S_1 denote the service time at Counter 1. We have

$$E[S_1] = \sum_{i=1}^x \frac{i}{\mu} \frac{1}{x} \quad (1)$$

$$E[S_1^2] = \sum_{i=1}^x \left(\frac{i}{\mu}\right)^2 \frac{1}{x} \quad (2)$$

Let $\rho_1 = \lambda \frac{x}{40} E[S_1]$, by the P-K formula, the mean waiting time at Counter 1 is

$$W_1 = \begin{cases} \lambda \frac{x}{40} \frac{E[S_1^2]}{2(1-\rho_1)} & \text{if } \rho_1 < 1 \\ \infty & \text{if } \rho_1 \geq 1 \end{cases} \quad (3)$$

Similarly, the arrival rate to Counter 2 is $\lambda_2 = \lambda \frac{40-x}{40}$. Let S_2 denote the service time at Counter 2, then

$$E[S_2] = \sum_{i=x+1}^{40} \frac{i}{\mu} \frac{1}{(40-x)} \quad (4)$$

$$E[S_2^2] = \sum_{i=x+1}^{40} \left(\frac{i}{\mu}\right)^2 \frac{1}{(40-x)} \quad (5)$$

Let $\rho_2 = \lambda \frac{40-x}{40} E[S_2]$, by the P-K formula, the mean waiting time at Counter 2 is

$$W_2 = \begin{cases} \lambda \frac{40-x}{40} \frac{E[S_2^2]}{2(1-\rho_2)} & \text{if } \rho_2 < 1 \\ \infty & \text{if } \rho_2 \geq 1 \end{cases} \quad (6)$$

The mean waiting time of the customers is

$$W = \frac{x}{40} W_1 + \frac{40-x}{40} W_2 \quad (7)$$

Note that W is a function of x . We write a computer program (Matlab file *week05.q1.m*) to calculate how W varies with x . Figure 1 shows how W varies with x . It can be seen that the minimum value of W is achieved at $x = 28$.

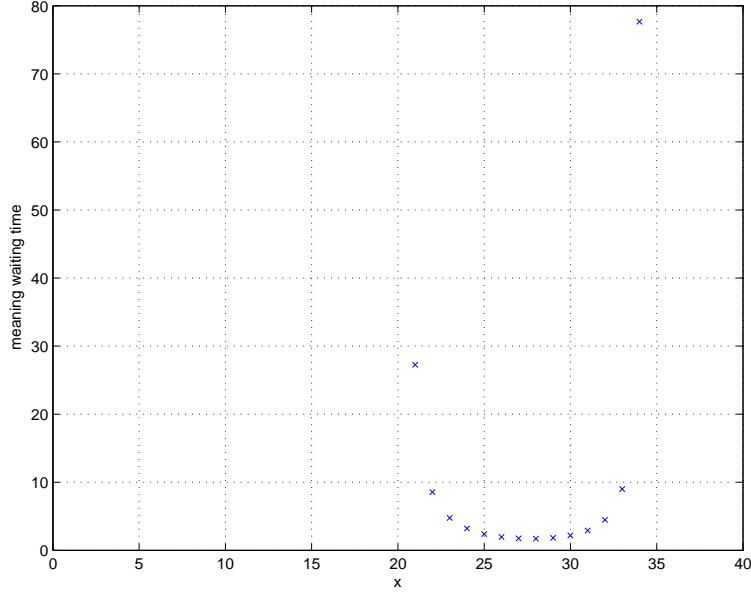


Figure 1: For Question 1

Question 2

Let us first compute the second moment of each customer type. Let us use C_a , σ_a , $E[S_a]$ and $E[S_a^2]$ to denote, respectively, the coefficient of variation, standard deviation, mean and second moment of the service time of customer of type a . Recall that the coefficient of variation of a random variable is its standard deviation divided by mean, i.e $C_a = \frac{\sigma_a}{E[S_a]}$. By using the relation:

$$E[S_a^2] = E[S_a]^2 + \sigma_a^2, \quad (8)$$

it can be showed that

$$E[S_a^2] = E[S_a]^2(1 + C_a^2) \quad (9)$$

Since we know $E[S_a] = 0.1$ and $C_a = 1.5$, we can compute $E[S_a^2]$ using the above equation.

Similarly, let C_b , σ_b , $E[S_b]$ and $E[S_b^2]$ to denote, respectively, the coefficient of variation, standard deviation, mean and second moment of the service time of customer of type b . We have

$$E[S_b^2] = E[S_b]^2(1 + C_b^2) \quad (10)$$

With $E[S_b] = 0.08$ and $C_b = 1.2$, we can compute $E[S_b^2]$ using the above equation.

Requests of type a and b have equal priorities

This is an M/G/1 queue without priority. The arrival rate is 10 requests per second ($= \lambda$). Since 30% of the request are type a and the remaining are type b, we have the mean service

time $E[S]$ and second moment $E[S^2]$ of the aggregate are, respectively,

$$E[S] = 0.3E[S_a] + 0.7E[S_b] \quad (11)$$

$$E[S^2] = 0.3E[S_a^2] + 0.7E[S_b^2] \quad (12)$$

The mean response time is therefore $E[S] + \frac{\lambda E[S^2]}{2(1-\rho)}$ where $\rho = \lambda E[S]$.

Requests of type b have non-preemptive priority over type a

Let

$$R = \frac{1}{2}(0.3\lambda E[S_a^2] + 0.7\lambda E[S_b^2]) \quad (13)$$

$$\rho_a = 0.3\lambda E[S_a] \quad (14)$$

$$\rho_b = 0.7\lambda E[S_b] \quad (15)$$

Response time of type b is

$$E[S_b] + \frac{R}{1 - \rho_b} \quad (16)$$

Response time of type a is

$$E[S_a] + \frac{R}{(1 - \rho_b)(1 - \rho_a - \rho_b)} \quad (17)$$

Requests of type b have preemptive priority over type a

Let

$$R_b = \frac{1}{2}(0.7\lambda E[S_b^2]) \quad (18)$$

$$R_a = \frac{1}{2}(0.3\lambda E[S_a^2] + 0.7\lambda E[S_b^2]) \quad (19)$$

$$\rho_a = 0.3\lambda E[S_a] \quad (20)$$

$$\rho_b = 0.7\lambda E[S_b] \quad (21)$$

Response time of type b is

$$E[S_b] + \frac{R_b}{1 - \rho_b} \quad (22)$$

Response time of type a is

$$E[S_a] \frac{1}{1 - \rho_b} + \frac{R_a}{(1 - \rho_b)(1 - \rho_a - \rho_b)} \quad (23)$$

The numerical answers are summarised in the following table.

Mean response time	Part 1	Part 2	Part 3
type a	0.8246	1.7787	1.9059
type b	0.8246	0.3150	0.2042

Observe that the response time for type b customers have become better because it has a higher priority, this is of course at the expense of type a customers which have a lower priority.

Question 3

The system behaves as an M/G/1 queueing system.

Since there are 10 sessions each generating Poisson traffic at a rate of 150 packets/minute, the packet arrival rate to the communication line is 1500 packets/minute or 25 packets/s ($= \lambda$).

With a transmission rate of 50 kbits/s, a 100-bit packet requires a transmission time (= service time in queueing theory terminology) 0.002s and a 1000-bit packet requires a transmission time of 0.02s. (Recall that transmission time is packet size divided by transmission rate.)

Given that 10% of the packets are 100 bits long and the rest are 1000 bits long, the mean service time $E[S]$ (where S denotes the service time random variable)

$$E[S] = 0.1 * 0.002 + 0.9 * 0.02 = 0.0182s \quad (24)$$

and the second moment of the service time is

$$E[S^2] = 0.1 * 0.002^2 + 0.9 * 0.02^2 = 3.6040 \times 10^{-4}s^2 \quad (25)$$

The mean waiting time W , according to the P-K formula, which applies to M/G/1 queueing system, is

$$W = \frac{\lambda E[S^2]}{2(1 - \lambda E[S])} = 8.3ms \quad (26)$$

By Little's Law, the mean queue length is given by the product of the throughput of the queue and the mean waiting time,

$$\lambda * W = 0.21 \text{ packets} \quad (27)$$