COMP 9334 assignment

Question 1：

~~(a)~~ $T = 60$ min

CPU busy time : $B(cpu) = 2929s$

disk busy time: $B(disk) = 2765s$.

complete jobs : $C = 1267$

(a) $D(j) = \frac{U(j)}{X(0)} = \frac{B(j)/T}{C/T} = \frac{B(j)}{C}.$

$$D(cpu) = \frac{B(cpu)}{C} = \frac{2929}{1267} \approx 2.312$$

$$D(disk) = \frac{B(disk)}{C} = \frac{2765}{1267} \approx 2.182 .$$

(b)

$$X(0) \le \min\left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{k} D_i}\right]$$

bound 1: $\frac{1}{\max D_i} = \frac{1}{D(cpu)} \approx 0.433$

bound 2: ~~N = 40 20~~ assume t_0 is the crosspoint

$$\frac{\cancel{N}\, t_0}{\sum_{i=1}^{k} D_i} = \frac{\cancel{40}\ \cancel{20}\ t_0}{D(cpu) + D(disk) + \text{think time}} = \cancel{X}\ 0.433$$

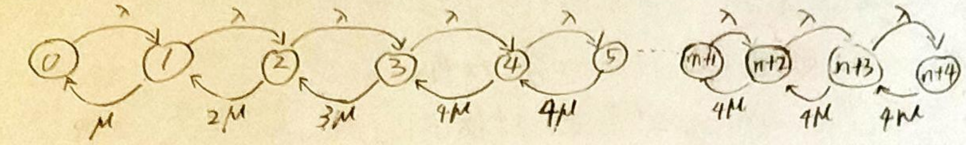
$$\Rightarrow t_0 \approx 8.01 < 20.$$

$$= \frac{\cancel{40}\ 20}{\cancel{2.312 + 2.182 + 14}} \approx \cancel{2.163}\ \cancel{1.081} .$$

So the asymptictic bound should be 0.433

Question2：

(a)



state 0 : all staffs are idle

state 1 : only 1 staff is busy

state 2 : only 2 staffs are busy

state 3 : only 3 staffs are busy

state 4 : all staffs are busy.

state 5 : all staffs are busy. 1 call in queue

The max number of queue is $n=2$, the max number of queries in centre is $n+4$

(b)

$$\lambda P_0 = \mu P_1 \Rightarrow P_1 = \left(\frac{\lambda}{\mu}\right) P_0$$

$$\lambda P_1 = 2\mu P_2 \Rightarrow P_2 = \frac{1}{2} \times \left(\frac{\lambda}{\mu}\right) P_1 = \frac{1}{2} \times \left(\frac{\lambda}{\mu}\right)^2 P_0$$

$$\lambda P_2 = 3\mu P_3 \Rightarrow P_3 = \frac{1}{2} \times \frac{1}{3} \times \left(\frac{\lambda}{\mu}\right)^3 P_0$$

$$\lambda P_3 = 4\mu P_4 \Rightarrow P_4 = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \left(\frac{\lambda}{\mu}\right)^4 P_0$$

$$\lambda P_4 = 4\mu P_5 \Rightarrow P_5 = \frac{1}{2} \times \frac{1}{3} \times \left(\frac{1}{4}\right)^2 \times \left(\frac{\lambda}{\mu}\right)^5 P_0$$

$$\cdots$$

$$\Rightarrow P_k = \frac{1}{6} \times \left(\frac{1}{4}\right)^{k-3} \times \left(\frac{\lambda}{\mu}\right)^k P_0.$$

(c) The probability of all states' sum is 1

$$P_0 + P_1 + P_2 + P_3 + \cdots P_{n+4} = 1$$

since $\rho = \frac{\lambda}{\mu}$

So $$P_0 + \rho P_0 + \frac{1}{2}\rho^2 P_0 + \frac{1}{6}\rho^3 P_0 + \sum_{k=4}^{n+4} \frac{1}{6} \times \left(\frac{1}{4}\right)^{k-3} \times \rho^k P_0 = 1$$

$$\Leftrightarrow P_0 = \frac{1}{1+\rho+\frac{1}{2}\rho^2+\frac{1}{6}\rho^3+\sum_{k=4}^{n+4} \frac{1}{6} \times \left(\frac{1}{4}\right)^{k-3} \times \rho^k}$$

(d)

i) the call is rejected = the centre is full

$n=2$. $\rho = \frac{\lambda}{\mu} = 5$

$$P_0 = \frac{1}{1+\rho+\frac{1}{2}\rho^2+\frac{1}{6}\rho^3+\sum_{k=4}^{6}\frac{1}{6}\times\left(\frac{1}{4}\right)^{k-3}\times\rho^k}$$

$$= \frac{1}{1+5+\frac{25}{2}+\frac{125}{6}+\frac{625}{24}+\frac{3125}{96}+\frac{15625}{384}}$$

$= 0.007214$

$\Rightarrow P_6 = 0.29354$

So the probability is 0.29354

(e) $P_k = \frac{1}{6}\times\left(\frac{1}{4}\right)^{k-3}\times\left(\frac{\lambda}{\mu}\right)^k P_0$.

Add 5, 10, 15, 20 slots waiting time:

① $n=7$

$P(11) = 0.22332$

② $n=12$

$P(16) = 0.2071$

③ $n=17$

$P(21) = 0.2023$

④ $n=22$

$P(26) = 0.20073$

(f) because only 4 staffs with 3 queries per hour. If after 12, it means the centre need more waiting slots, t

Question3：

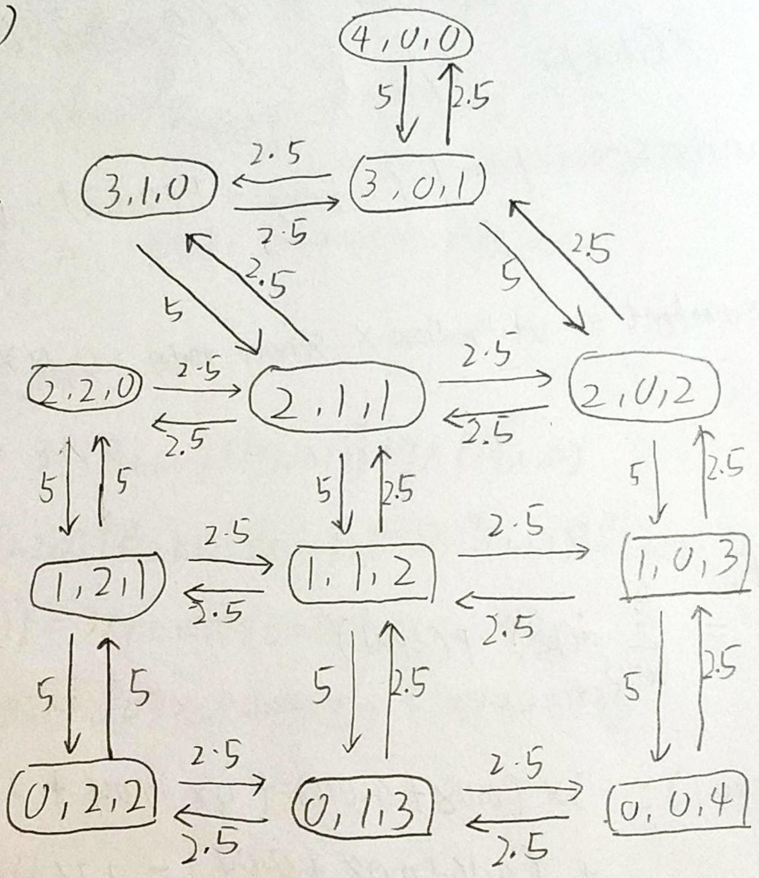
(a) 3-tuple state: (#CPU1, #CPU2, # Disk)

(4, 0, 0)
(3, 1, 0)
(3, 0, 1)
(2, 2, 0)
(2, 0, 2)
(2, 1, 1)
~~(1, 3, 0)~~
(1, 0, 3)
(1, 2, 1)
(1, 1, 2)
~~(0, 4, 0)~~
(0, 0, 4)
~~(0, 3, 1)~~
(0, 1, 3)
(0, 2, 2)

$\mu(cpu1) = 1/0.2 = 5/s$

$\mu(cpu2) = 1/0.4 = 2.5/s$

$\mu(disk) = 1/0.2 = 5/s$



(b)

A=

P(4,0,0)	P(3,1,0)	P(3,0,1)	P(2,2,0)	P(2,1,1)	P(2,0,2)	P(1,2,1)	P(1,1,2)	P(1,0,3)	P(0,2,2)	P(0,1,3)	P(0,0,4)
5	0	-2.5	0	0	0	0	0	0	0	0	0
0	7.5	-2.5	0	-2.5	0	0	0	0	0	0	0
-5	-2.5	10	0	0	-2.5	0	0	0	0	0	0
0	0	0	7.5	-2.5	0	-5	0	0	0	0	0
0	-5	0	-2.5	12.5	-2.5	0	-2.5	0	0	0	0
0	0	-5	0	-2.5	10	0	0	-2.5	0	0	0
0	0	0	-5	0	0	12.5	-2.5	0	-5	0	0
0	0	0	0	-5	0	-2.5	12.5	-2.5	0	-2.5	0
0	0	0	0	0	-5	0	-2.5	10	0	0	-2.5
0	0	0	0	0	0	-5	0	0	7.5	-2.5	0
0	0	0	0	0	0	0	-5	0	-2.5	7.5	-2.5
1	1	1	1	1	1	1	1	1	1	1	1

(c)

b= [0,0,0,0,0,0,0,0,0,0,0,1]

X=A\b

So

(c) P(4,0,0) = 0.0130   P(2,2,0) = 0.0871   P(1,2,1) = 0.1021   P(0,2,2) = 0.1213

P(3,1,0) = 0.0277   P(2,1,1) = 0.0572   P(1,1,2) = 0.0935   P(0,1,3) = 0.1598

P(3,0,1) = 0.0259   P(2,0,2) = 0.0501   P(1,0,3) = 0.0912   P(0,0,4) = 0.1711

(it depends on the matlab file: dataserver_z5103407.m)

(d) since (4,0,0), (3,1,0) (2,2,0) state, disk is the only idle state

So $U(disk) = 1 - P(4,0,0) - P(3,1,0) - P(2,2,0)$

$= 1 - 0.013 - 0.0277 - 0.0871$

$= 0.8722$

So $X(disk) = U(disk)/S(disk) = 0.8722/0.2$

$= 4.361$ transactions/seconds

So throughput is 4.361 transactions/seconds.

(e) mean number of jobs:

$N(cpu1) = 3 \times (P(3,1,0) + P(3,0,1)] + 4 \times P(4,0,0)$

$+ 2 \times (P(2,2,0) + P(2,1,1) + P(2,0,2)) + 1 \times (P(1,2,1) + P(1,1,2)$

$+ P(1,0,3)) = 3 \times (0.0277 + 0.0259) + 4 \times 0.013$

$+ 2 \times (0.0871 + 0.0572 + 0.0501) + 0.1021 + 0.0935$

$+ 0.0912$

$= 0.8884$

(f) response time of CPU1:

$$R = \frac{N(cpu1)}{X(cpu1)} = \frac{0.8884}{(1 - P(0,2,2) - P(0,1,3) - P(0,0,4))/0.2}$$

$$= \frac{0.8884}{2.739} = 0.3243s.$$