ANLY512_HW3

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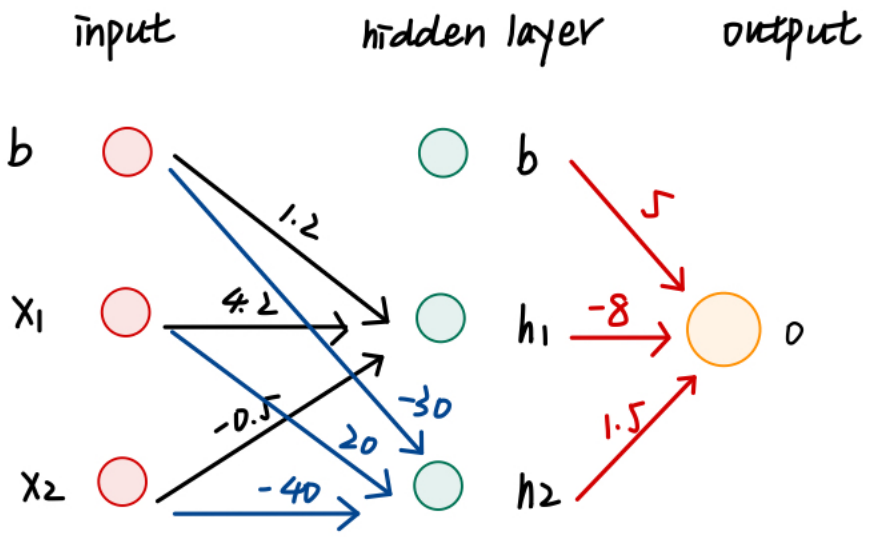
2019/2/18

Problem #28

a) Diagram for neural network in Problem 28



b)

$$h_1 = tanh(1.2 + 4.2 * x_1 - 0.5 * x_2)$$
$$h_2 = tanh(-30 + 20 * x_1 - 40 * x_2)$$

c)

$$z = tanh(5 - 8 * h_1 + 1.5 * h_2)$$

d)

$$z = tanh(5 - 8 * tanh(1.2 + 4.2 * x_1 - 0.5 * x_2) + 1.5 * tanh(-30 + 20 * x_1 - 40 * x_2))$$

Problem #30

From the graph, we can see that the responses(probability) from network1 and network2 are close, which means, if we set Y=1 when P>some threshold and Y=0 otherwise, the output of Y will be very similar for network1 and network2. When we use these predictions to draw ROC curve, the curve will be same.

Problem #34

```
set.seed(20305)
# Generate the dataframe
df1=data.frame(matrix(rnorm(1100,0,1),nrow=100))
names(df1)[11]='z'
```

a)

```
# Fit a linear model
model1=lm(z~., data=df1)
summary(model1)
```

```
##
## Call:
## lm(formula = z ~ ., data = df1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.35816 -0.67302  0.04407  0.74991  2.70681
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.17797    0.11162  -1.594   0.1144
## X1           0.20702    0.11184   1.851   0.0675 .
## X2          -0.02103    0.12071  -0.174   0.8621
## X3           0.08535    0.11437   0.746   0.4575
## X4           0.03585    0.10940   0.328   0.7439
## X5          -0.01923    0.10478  -0.184   0.8548
## X6           0.13443    0.10667   1.260   0.2108
## X7           0.02458    0.11090   0.222   0.8251
## X8          -0.03366    0.11685  -0.288   0.7740
## X9          -0.15123    0.10347  -1.462   0.1474
## X10         -0.08584    0.10481  -0.819   0.4150
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.054 on 89 degrees of freedom
## Multiple R-squared:  0.09073,    Adjusted R-squared:  -0.01143
## F-statistic: 0.8881 on 10 and 89 DF,  p-value: 0.5475
```

```
# Function to calculate root mean squared error
RMSE<-function(model)
{
  sqrt(mean(residuals(model)^2))
}

# RMSE for model1
RMSE(model1)
```

```
## [1] 0.9940173
```

The RMSE for this linear model is about 0.99, which is very high. Therefore, this linear model performs not good.

b)

```
library(nnet)
# Scalar the data
df1$z01 = (df1$z-min(df1$z))/(max(df1$z)-min(df1$z))

# Fit a neural network with 2 hidden units
model2=nnet(z01~.-z, data=df1, size=2, decay=0.01, maxit=2000)
```

```
## # weights:  25
## initial  value 4.061706
## iter  10 value 3.176917
## iter  20 value 2.715157
## iter  30 value 2.640194
## iter  40 value 2.639013
## iter  50 value 2.638485
## iter  60 value 2.638178
## iter  70 value 2.638165
## final  value 2.638165
## converged
```

```
# Function to calculate RMSE
error_rmse <- function (model)
{
  predict.1=predict(model, type='r')
  predict.2=predict.1*(max(df1$z)-min(df1$z))+min(df1$z)

  # Calculate RMSE for the new model
  error1=sqrt(mean((predict.2-df1$z)**2))
  return(error1)
}

error_rmse(model2)
```

```
## [1] 0.7946809
```

This model performs better than the one in a) since the RMSE decreases from 0.99 to about 0.80

c)

```
# Fit a neural network with 5 hidden units
model3=nnet(z01~.-z, data=df1, size=5, decay=0.01, maxit=2000)
```

```
## # weights:  61
## initial  value 5.359081
## iter  10 value 3.445173
## iter  20 value 3.053240
## iter  30 value 2.488940
## iter  40 value 2.101915
## iter  50 value 1.948961
## iter  60 value 1.844793
## iter  70 value 1.817040
## iter  80 value 1.797868
## iter  90 value 1.791175
## iter 100 value 1.790779
## iter 110 value 1.790739
```

```
## iter 120 value 1.790737
## final  value 1.790737
## converged
```

```
error_rmse(model3)
```

```
## [1] 0.5245953
```

```
# Fit a neural network with 10 hidden units
model4=nnet(z01~.-z, data=df1, size=10, decay=0.01, maxit=2000)
```

```
## # weights:  121
## initial  value 5.316215
## iter  10 value 3.209332
## iter  20 value 2.255210
## iter  30 value 1.809307
## iter  40 value 1.535723
## iter  50 value 1.451185
## iter  60 value 1.416817
## iter  70 value 1.395414
## iter  80 value 1.381510
## iter  90 value 1.371985
## iter 100 value 1.363439
## iter 110 value 1.359665
## iter 120 value 1.358592
## iter 130 value 1.357888
## iter 140 value 1.356886
## iter 150 value 1.353535
## iter 160 value 1.350372
## iter 170 value 1.347048
## iter 180 value 1.345292
## iter 190 value 1.343638
## iter 200 value 1.340420
## iter 210 value 1.338535
## iter 220 value 1.336664
## iter 230 value 1.332425
## iter 240 value 1.329449
## iter 250 value 1.327727
## iter 260 value 1.322688
## iter 270 value 1.301703
## iter 280 value 1.283095
## iter 290 value 1.273627
## iter 300 value 1.263593
## iter 310 value 1.261787
## iter 320 value 1.261618
## iter 330 value 1.261579
## iter 340 value 1.261566
## iter 350 value 1.261564
## final  value 1.261564
## converged
```

```
error_rmse(model4)
```

```
## [1] 0.2808335
```

Both neural network converged and the final value for the one with 10 hidden nodes is lower than the one with 5 hidden nodes, which means the last one performs better. Compared the RMSE, we got that the last

one has the lowest RMSE which is equal to about 0.30 (5 hidden nodes having rmse about 0.50). With the number of hidden nodes inceasing, the neural network performs better and better (higher accuracy) when we fit a model using the same variables.

Problem #33

a)

```
MNIST=load('~/Desktop/other/Data/mnist_all.RData')

# Generate dataframe
# Extract data for digit4 and digit7
y.nist = train$y
index <- (y.nist == 4 | y.nist == 7)
x.nist <- train$x[index,]
x.train <- as.data.frame(x.nist)

y.nist1 = test$y
index1 <- (y.nist1 == 4 | y.nist1 == 7)
x.nist1 <- test$x[index1,]
x.test <- as.data.frame(x.nist1)

# y=1 if it is digit4, y=0 if it is digit7
x.train$y <- 0
x.train$y[y.nist[index] == 4] <- 1
x.test$y <- 0
x.test$y[y.nist1[index1] == 4] <- 1
```

```
# Find a variable with large variance
# Calculate all variance for V1 to V784
variance=rep(0,784)
for (i in 1:784)
{
  variance[i]=var(x.train[,i])
}
variance=as.data.frame(variance)
variance$index=seq(1:784)

# Sort from biggest to smallest
variance_sort=variance[order(variance[,1], decreasing=TRUE),]

# Print the top 10 largest variance index
print(variance_sort[1:20,2])
```

```
##  [1] 430 402 240 241 239 267 266 401 431 268 374 238 429 574 269 346 242
## [18] 602 262 373
```

```
# Calculate correlation for some random combination
cor(x.train$V241, x.train$V266)
```

```
## [1] 0.3920093
```

```
cor(x.train$V267, x.train$V373)
```

```
## [1] -0.3162502
```

```r
cor(x.train$V602, x.train$V268)
```

```
## [1] 0.1551939
```

```r
cor(x.train$V602, x.train$V373)
```

```
## [1] -0.1304914
```

```r
cor(x.train$V574, x.train$V374)
```

```
## [1] -0.09565478
```

```r
cor(x.train$V574, x.train$V269)
```

```
## [1] 0.08884313
```

```r
cor(x.train$V584, x.train$V364)
```

```
## [1] -0.0005790498
```

```r
# Find that V584 and V364 have a very low correlation -0.0005790498
library("pROC")
```

```
## Type 'citation("pROC")' for a citation.
```

```
##
## Attaching package: 'pROC'
```

```
## The following objects are masked from 'package:stats':
##
##     cov, smooth, var
```

```r
fit1=glm(y~V584+V364, data=x.train, family = 'binomial')
# AUC score
pred.fit1=predict(fit1, data=x.train, type = 'response')
auc(roc(x.train$y, pred.fit1), col=4)
```

```
## Area under the curve: 0.5043
```

b)

```r
# Using neural network
fit2=nnet(y~V584+V364, data=x.train, size=1, decay=0.01, maxit=2000)
```

```
## # weights:  5
## initial  value 3251.666402
## iter  10 value 3021.701406
## iter  20 value 3014.940583
## iter  30 value 3013.215141
## iter  40 value 3012.963177
## iter  50 value 3012.896790
## iter  60 value 3012.860669
## iter  70 value 3012.846405
## final  value 3012.840371
## converged
```

```r
# AUC score
pred.fit2=as.vector(predict(fit2, data=x.train, type = "raw"))
auc(roc(x.train$y, pred.fit2), col=4)
```

```
## Area under the curve: 0.5043
```

The AUC score is same for the two models, because when we use neural network, if we set the number of hidden units equal to 1, then it is same as the logistic model. Therefore, the AUC score is the same.

c)

```
# Using neural network with 3 hidden units
fit3=nnet(y~V584+V364, data=x.train, size=3, decay=0.01, maxit=2000)
```

```
## # weights:  13
## initial  value 3226.978837
## iter  10 value 3013.246511
## iter  20 value 3012.686334
## iter  30 value 3012.646760
## iter  40 value 3012.599072
## final  value 3012.591180
## converged
```

```
pred.fit3=as.vector(predict(fit3, data=x.train, type = "raw"))
auc(roc(x.train$y, pred.fit3), col=4)
```

```
## Area under the curve: 0.5043
```

```
# Using neural network with 5 hidden units
fit4=nnet(y~V584+V364, data=x.train, size=5, decay=0.01, maxit=2000)
```

```
## # weights:  21
## initial  value 3028.328214
## iter  10 value 3013.061644
## iter  20 value 3012.687224
## iter  30 value 3012.461757
## iter  40 value 3012.370619
## iter  50 value 3012.338743
## final  value 3012.329276
## converged
```

```
pred.fit4=as.vector(predict(fit4, data=x.train, type = "raw"))
auc(roc(x.train$y, pred.fit4), col=4)
```

```
## Area under the curve: 0.5046
```

```
# Using neural network with 10 hidden units
fit5=nnet(y~V584+V364, data=x.train, size=8, decay=0.01, maxit=2000)
```

```
## # weights:  33
## initial  value 4289.336107
## iter  10 value 3013.019414
## iter  20 value 3012.712531
## iter  30 value 3012.574235
## iter  40 value 3012.498639
## iter  50 value 3012.453524
## iter  60 value 3012.439025
## iter  70 value 3012.420456
## iter  80 value 3012.353431
## iter  90 value 3012.341020
## iter 100 value 3012.337669
## iter 110 value 3012.335261
```

```
## iter 120 value 3012.330703
## iter 130 value 3012.327791
## final  value 3012.327427
## converged
```

```
pred.fit5=as.vector(predict(fit5, data=x.train, type = "raw"))
auc(roc(x.train$y, pred.fit5), col=4)
```

```
## Area under the curve: 0.5046
```

The AUC score goes up and fixes at the same score. The model with 5 or 10 hidden units performs better.

d)

```
# Using test data for 3 hidden units
fit3=nnet(y~V584+V364, data=x.train, size=3, decay=0.01, maxit=2000)
```

```
## # weights:  13
## initial  value 3035.780494
## iter  10 value 3013.960210
## iter  20 value 3012.984793
## iter  30 value 3012.826814
## iter  40 value 3012.661554
## iter  50 value 3012.467669
## iter  60 value 3012.413412
## final  value 3012.405363
## converged
```

```
pred.fit3.test=as.vector(predict(fit3, newdata=x.test, type = "raw"))
auc(x.test$y, pred.fit3.test)
```

```
## Area under the curve: 0.5042
```

```
# Using test data for 5 hidden units
fit4=nnet(y~V584+V364, data=x.train, size=5, decay=0.01, maxit=2000)
```

```
## # weights:  21
## initial  value 3053.871639
## iter  10 value 3013.225629
## iter  20 value 3012.797508
## iter  30 value 3012.595045
## iter  40 value 3012.445164
## iter  50 value 3012.361086
## iter  60 value 3012.342931
## final  value 3012.341088
## converged
```

```
pred.fit4.test=as.vector(predict(fit4, newdata=x.test, type = "raw"))
auc(x.test$y, pred.fit4.test)
```

```
## Area under the curve: 0.5042
```

```
# Using test data for 10 hidden units
fit5=nnet(y~V584+V364, data=x.train, size=10, decay=0.01, maxit=2000)
```

```
## # weights:  41
## initial  value 3628.768988
## iter  10 value 3013.135659
```

```
## iter  20 value 3012.724230
## iter  30 value 3012.618539
## iter  40 value 3012.531640
## iter  50 value 3012.466897
## iter  60 value 3012.400715
## iter  70 value 3012.363042
## iter  80 value 3012.349749
## final  value 3012.347202
## converged
```

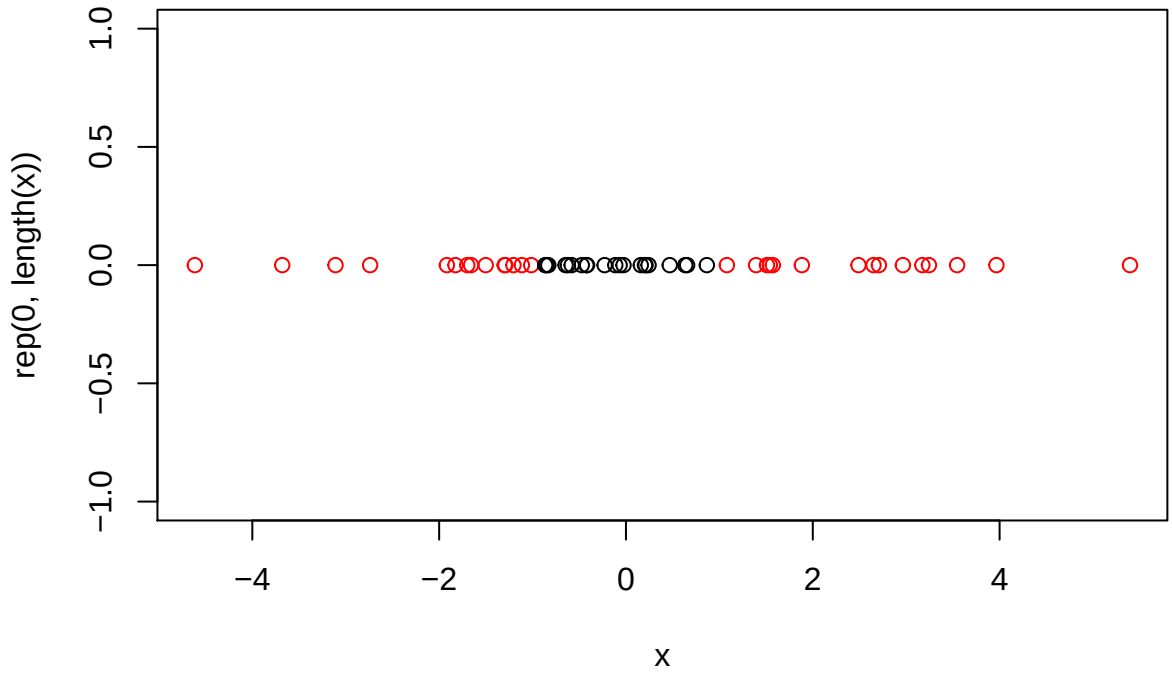
```
pred.fit5.test=as.vector(predict(fit5, newdata=x.test, type = "raw"))
auc(x.test$y, pred.fit5.test)
```

```
## Area under the curve: 0.5042
```

The AUC score for the test data does not increase when I add more units into the hidden layer. Compared with increased AUC score for training data (although the change is very small), I think the model is overfitting. The performance of model does not increase with more hidden nodes.

Problem #36

```
# Generate dataset
x = rnorm(50, 0 ,2)
y<-rep(1, length(x))
y[abs(x) < 1] = 0
plot(x,rep(0,length(x)),col=y+1)
```



a)

```
# Fit a logistic model
log.model=glm(y~x, family='binomial')
```

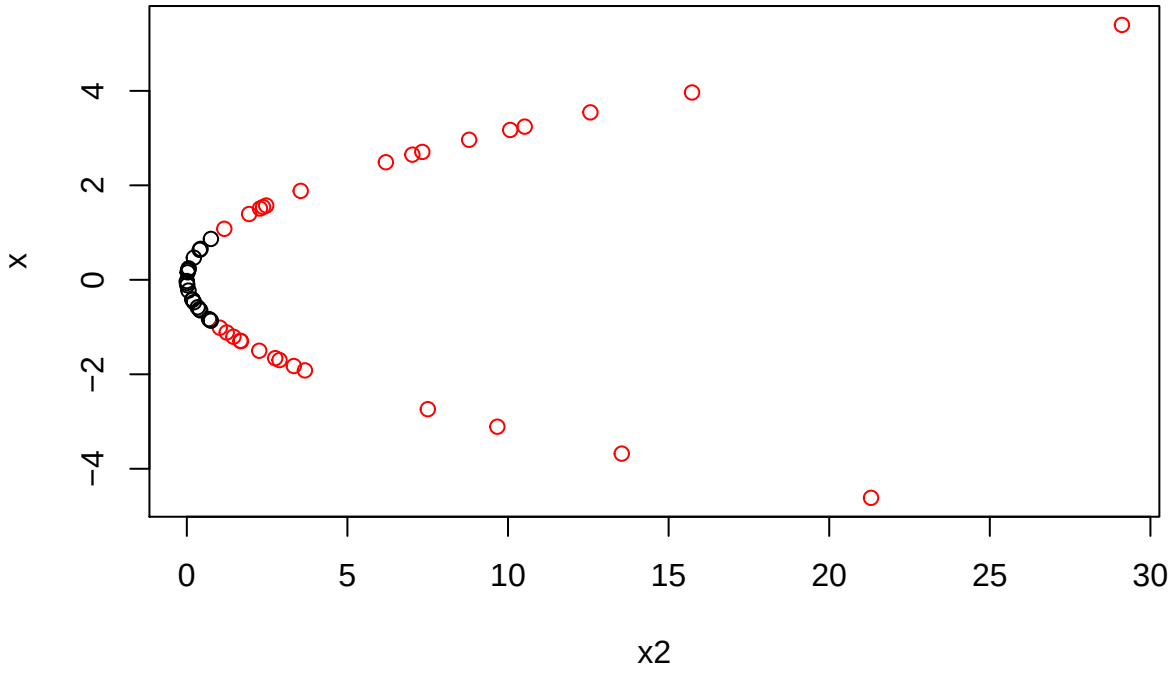
```
pred.log.model=predict(log.model, type = 'r')
auc(roc(y, pred.log.model), col=4)
```

```
## Area under the curve: 0.5172
```

The auc score for this logistic model is about 0.52, which is a low score. Therefore, the performance of this model is not good.

b)

```
# New feature
x2=(x)^2
# Plot for x and x2
plot(x2, x, col=y+1)
```




```
# Fit a new logistic model
log.model.new=glm(y~x+x2, family = 'binomial')
```

```
## Warning: glm.fit: algorithm did not converge
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
pred.log.new=predict(log.model.new, type='r')
auc(roc(y, pred.log.new), col=4)
```

```
## Area under the curve: 1
```

The auc curve for this new regression model is 1, which is a perfect score. This new model performs very good.

c)

```
# Fit a nnet model
nnet.model=nnet(y~x, size=2, decay=0.01, maxit=2000)
```

```
## # weights:  7
## initial  value 12.209999
## iter  10 value 4.834954
## iter  20 value 2.570733
## iter  30 value 2.568255
## final  value 2.568249
## converged
```

```
pred.nnet=as.vector(predict(nnet.model, type = 'r'))
auc(roc(y, pred.nnet), col=4)
```

```
## Area under the curve: 1
```

The auc score for this nnet model is 1, which is a perfect score.

d)

As the graph shows below, when we add two nodes in the hidden layer, we will have three values(including a bias) as input to calculate the final output. If we use a logistic model, we only have two input values which are x and the bias to calculate the final output. Therefore, by this method, we increase the number of features used to build the model and increase the dimension of the space.

Problem #37

a)

```
# Load the data and change the names
library(readxl)
Concrete_Data <- read_excel("~/Desktop/HW/Concrete_Data.xls")
names(Concrete_Data)=c('Cement', 'Slag', 'Fly', 'Water', 'Super', 'Coarse', 'Fine', 'Age', 'Strength')
# Add one column for regularization of Strength
Concrete_Data$Strength01 = (Concrete_Data$Strength-min(Concrete_Data$Strength))/(max(Concrete_Data$Stren

# Train set and test set
n=nrow(Concrete_Data)
index=sample(n, n*0.7, replace = FALSE)
train=Concrete_Data[index, ]
test=Concrete_Data[-index, ]
```

b)

```
# Vector to store RMSE
RMSE.train=rep(NA, 19)
# Calculate RMSE for different number of nodes
for (a in seq(2,20))
{
  model=nnet(Strength01~.-Strength, data=train, size=a, decay=0.01, maxit=2000)
  train.predict.1 <- predict(model)
  train.predict = train.predict.1*(max(Concrete_Data$Strength)-min(Concrete_Data$Strength))+min(Concrete
  RMSE.train[a-1] <- sqrt(mean((train.predict - train$Strength)^2))
}
```

```
## # weights:  21
## initial  value 32.952292
```

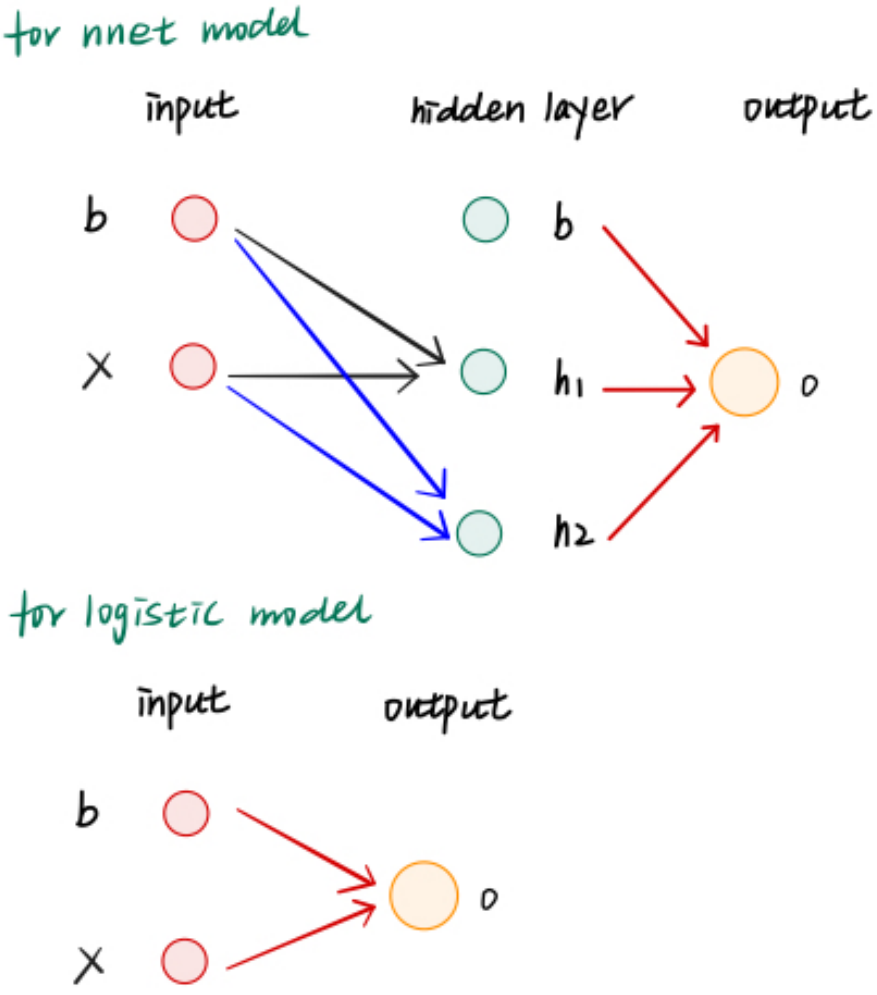


Figure 1: Diagram for Problem 36

```
## iter  10 value 31.409670
## iter  20 value 31.406829
## iter  30 value 30.689658
## iter  40 value 25.443542
## iter  50 value 11.665835
## iter  60 value 10.647204
## iter  70 value 10.042275
## iter  80 value 9.764137
## iter  90 value 9.755379
## final  value 9.755356
## converged
## # weights:  31
## initial  value 110.245617
## iter  10 value 31.382874
## iter  20 value 25.468046
## iter  30 value 17.807124
## iter  40 value 13.892110
## iter  50 value 12.087437
## iter  60 value 10.544511
## iter  70 value 9.998406
## iter  80 value 9.156610
## iter  90 value 7.774839
## iter 100 value 6.665796
## iter 110 value 6.183105
## iter 120 value 5.703482
## iter 130 value 5.173531
## iter 140 value 4.852011
## iter 150 value 4.748092
## iter 160 value 4.741062
## iter 170 value 4.695901
## iter 180 value 4.460738
## iter 190 value 4.299754
## iter 200 value 4.267060
## iter 210 value 4.257492
## iter 220 value 4.257186
## iter 230 value 4.257124
## iter 240 value 4.252537
## iter 250 value 4.252120
## final  value 4.252119
## converged
## # weights:  41
## initial  value 42.484157
## iter  10 value 31.269869
## iter  20 value 21.434197
## iter  30 value 18.959343
## iter  40 value 16.853657
## iter  50 value 15.683815
## iter  60 value 15.628093
## iter  70 value 14.056775
## iter  80 value 13.557442
## iter  90 value 12.888320
## iter 100 value 10.188560
## iter 110 value 9.775449
## iter 120 value 9.748578
```

```
## iter 130 value 9.723416
## iter 140 value 9.650534
## iter 150 value 9.305647
## iter 160 value 9.232302
## iter 170 value 8.445291
## iter 180 value 7.796730
## iter 190 value 5.885680
## iter 200 value 5.499770
## iter 210 value 5.338955
## iter 220 value 5.289526
## iter 230 value 5.164940
## iter 240 value 4.825337
## iter 250 value 4.696033
## iter 260 value 4.647484
## iter 270 value 4.625249
## iter 280 value 4.608012
## iter 290 value 4.602752
## iter 300 value 4.587956
## iter 310 value 4.569639
## iter 320 value 4.562326
## iter 330 value 4.562214
## final  value 4.562212
## converged
## # weights:  51
## initial  value 169.962348
## iter  10 value 31.330041
## iter  20 value 18.848901
## iter  30 value 13.819988
## iter  40 value 12.362233
## iter  50 value 11.163639
## iter  60 value 10.950454
## iter  70 value 9.839679
## iter  80 value 8.924812
## iter  90 value 8.389126
## iter 100 value 7.598948
## iter 110 value 6.952674
## iter 120 value 6.384376
## iter 130 value 5.941111
## iter 140 value 5.497645
## iter 150 value 5.256414
## iter 160 value 5.094197
## iter 170 value 4.774339
## iter 180 value 4.463306
## iter 190 value 4.394231
## iter 200 value 4.339564
## iter 210 value 4.269223
## iter 220 value 4.236006
## iter 230 value 4.214760
## iter 240 value 4.189423
## iter 250 value 4.153244
## iter 260 value 4.055974
## iter 270 value 3.873505
## iter 280 value 3.800472
## iter 290 value 3.796765
```

```
## iter 300 value 3.796017
## iter 310 value 3.795671
## iter 320 value 3.795647
## final  value 3.795645
## converged
## # weights:  61
## initial  value 31.689736
## iter  10 value 21.992386
## iter  20 value 19.213801
## iter  30 value 17.084246
## iter  40 value 14.664594
## iter  50 value 13.657929
## iter  60 value 13.194256
## iter  70 value 13.038436
## iter  80 value 12.909355
## iter  90 value 12.831810
## iter 100 value 12.260715
## iter 110 value 11.583829
## iter 120 value 9.891460
## iter 130 value 8.348940
## iter 140 value 6.537036
## iter 150 value 5.648887
## iter 160 value 5.302929
## iter 170 value 5.222640
## iter 180 value 5.212935
## iter 190 value 5.136763
## iter 200 value 4.894482
## iter 210 value 4.814697
## iter 220 value 4.814230
## iter 230 value 4.802214
## iter 240 value 4.688354
## iter 250 value 4.473172
## iter 260 value 4.453289
## iter 270 value 4.442618
## iter 280 value 4.429695
## iter 290 value 4.427617
## iter 300 value 4.392514
## iter 310 value 4.281536
## iter 320 value 4.221012
## iter 330 value 4.170175
## iter 340 value 4.160795
## iter 350 value 4.154069
## iter 360 value 4.153584
## iter 370 value 4.152446
## iter 380 value 4.148723
## iter 390 value 4.111408
## iter 400 value 3.919809
## iter 410 value 3.868390
## iter 420 value 3.861444
## iter 430 value 3.860199
## iter 440 value 3.860028
## iter 450 value 3.860006
## iter 460 value 3.859963
## iter 470 value 3.859325
```

```
## iter 480 value 3.856596
## iter 490 value 3.856244
## final  value 3.856235
## converged
## # weights:  71
## initial  value 218.670722
## iter  10 value 32.393386
## iter  20 value 25.771681
## iter  30 value 23.591638
## iter  40 value 20.626182
## iter  50 value 19.993759
## iter  60 value 19.598548
## iter  70 value 17.998956
## iter  80 value 15.106357
## iter  90 value 14.405356
## iter 100 value 13.408994
## iter 110 value 13.326830
## iter 120 value 13.260654
## iter 130 value 13.128531
## iter 140 value 12.547373
## iter 150 value 11.937346
## iter 160 value 10.978002
## iter 170 value 10.317041
## iter 180 value 8.121265
## iter 190 value 5.565316
## iter 200 value 5.272086
## iter 210 value 5.165274
## iter 220 value 4.956215
## iter 230 value 4.926344
## iter 240 value 4.923628
## iter 250 value 4.918290
## iter 260 value 4.914943
## iter 270 value 4.908290
## iter 280 value 4.890763
## iter 290 value 4.836513
## iter 300 value 4.735582
## iter 310 value 4.460644
## iter 320 value 4.306951
## iter 330 value 4.167643
## iter 340 value 4.106047
## iter 350 value 3.996307
## iter 360 value 3.962005
## iter 370 value 3.915284
## iter 380 value 3.911634
## iter 390 value 3.901845
## iter 400 value 3.894913
## iter 410 value 3.889040
## iter 420 value 3.828397
## iter 430 value 3.611953
## iter 440 value 3.465123
## iter 450 value 3.438085
## iter 460 value 3.425694
## iter 470 value 3.398502
## iter 480 value 3.393674
```

```
## iter 490 value 3.392480
## iter 500 value 3.392458
## iter 510 value 3.392457
## iter 520 value 3.392456
## final  value 3.392456
## converged
## # weights:  81
## initial  value 88.934736
## iter  10 value 24.545056
## iter  20 value 23.959789
## iter  30 value 22.971176
## iter  40 value 22.749098
## iter  50 value 22.343364
## iter  60 value 22.070710
## iter  70 value 17.027859
## iter  80 value 14.176851
## iter  90 value 11.757369
## iter 100 value 11.564399
## iter 110 value 10.731141
## iter 120 value 9.596581
## iter 130 value 8.709324
## iter 140 value 7.639581
## iter 150 value 6.528171
## iter 160 value 5.565087
## iter 170 value 5.516203
## iter 180 value 5.330834
## iter 190 value 5.035406
## iter 200 value 4.632213
## iter 210 value 4.543256
## iter 220 value 4.487051
## iter 230 value 4.378913
## iter 240 value 3.787692
## iter 250 value 3.674988
## iter 260 value 3.643650
## iter 270 value 3.632180
## iter 280 value 3.620883
## iter 290 value 3.619488
## iter 300 value 3.617916
## iter 310 value 3.613626
## iter 320 value 3.605708
## iter 330 value 3.599085
## iter 340 value 3.597276
## iter 350 value 3.595115
## iter 360 value 3.592635
## iter 370 value 3.570258
## iter 380 value 3.554042
## iter 390 value 3.545667
## iter 400 value 3.527436
## iter 410 value 3.511079
## iter 420 value 3.506464
## iter 430 value 3.505815
## final  value 3.505810
## converged
## # weights:  91
```

```
## initial  value 62.381932
## iter  10 value 30.433209
## iter  20 value 29.403414
## iter  30 value 22.615131
## iter  40 value 20.140347
## iter  50 value 19.341571
## iter  60 value 17.452228
## iter  70 value 15.374199
## iter  80 value 14.852146
## iter  90 value 14.042896
## iter 100 value 13.397417
## iter 110 value 10.453616
## iter 120 value 9.845052
## iter 130 value 9.793311
## iter 140 value 8.770668
## iter 150 value 7.734725
## iter 160 value 7.037926
## iter 170 value 6.808090
## iter 180 value 6.613457
## iter 190 value 6.589110
## iter 200 value 6.567384
## iter 210 value 6.300626
## iter 220 value 6.135182
## iter 230 value 5.506758
## iter 240 value 5.220026
## iter 250 value 5.098031
## iter 260 value 5.014297
## iter 270 value 4.921511
## iter 280 value 4.791828
## iter 290 value 4.397844
## iter 300 value 3.982002
## iter 310 value 3.822768
## iter 320 value 3.716575
## iter 330 value 3.675279
## iter 340 value 3.661933
## iter 350 value 3.641532
## iter 360 value 3.608578
## iter 370 value 3.534722
## iter 380 value 3.436756
## iter 390 value 3.287855
## iter 400 value 3.186758
## iter 410 value 3.037794
## iter 420 value 3.001703
## iter 430 value 2.987125
## iter 440 value 2.980593
## iter 450 value 2.978776
## iter 460 value 2.977798
## iter 470 value 2.976856
## iter 480 value 2.975992
## iter 490 value 2.974643
## iter 500 value 2.971052
## iter 510 value 2.957835
## iter 520 value 2.956905
## iter 530 value 2.956887
```

```
## final  value 2.956886
## converged
## # weights:  101
## initial  value 56.230080
## iter  10 value 29.545879
## iter  20 value 17.580088
## iter  30 value 14.715571
## iter  40 value 13.320460
## iter  50 value 11.624257
## iter  60 value 10.795802
## iter  70 value 9.875103
## iter  80 value 8.676766
## iter  90 value 7.297430
## iter 100 value 6.056270
## iter 110 value 5.599133
## iter 120 value 4.793558
## iter 130 value 4.405357
## iter 140 value 4.306999
## iter 150 value 4.240864
## iter 160 value 4.190113
## iter 170 value 4.152855
## iter 180 value 4.134389
## iter 190 value 4.021839
## iter 200 value 3.833600
## iter 210 value 3.784710
## iter 220 value 3.727518
## iter 230 value 3.710527
## iter 240 value 3.688066
## iter 250 value 3.630180
## iter 260 value 3.555076
## iter 270 value 3.443811
## iter 280 value 3.235744
## iter 290 value 3.062670
## iter 300 value 3.024997
## iter 310 value 2.990833
## iter 320 value 2.967995
## iter 330 value 2.955289
## iter 340 value 2.947221
## iter 350 value 2.944583
## iter 360 value 2.941357
## iter 370 value 2.939062
## iter 380 value 2.938654
## iter 390 value 2.937812
## iter 400 value 2.933808
## iter 410 value 2.926015
## iter 420 value 2.920610
## iter 430 value 2.894736
## iter 440 value 2.885131
## iter 450 value 2.883646
## iter 460 value 2.883590
## final  value 2.883588
## converged
## # weights:  111
## initial  value 99.137723
```

```
## iter  10 value 31.423945
## iter  20 value 30.320810
## iter  30 value 18.021490
## iter  40 value 17.841187
## iter  50 value 16.435578
## iter  60 value 13.676025
## iter  70 value 12.124856
## iter  80 value 11.226332
## iter  90 value 10.700833
## iter 100 value 10.410322
## iter 110 value 10.261863
## iter 120 value 9.931121
## iter 130 value 9.607910
## iter 140 value 9.536326
## iter 150 value 9.025724
## iter 160 value 8.782533
## iter 170 value 8.070721
## iter 180 value 7.614964
## iter 190 value 6.728715
## iter 200 value 5.917264
## iter 210 value 4.993010
## iter 220 value 4.483711
## iter 230 value 4.332989
## iter 240 value 4.259256
## iter 250 value 4.207752
## iter 260 value 3.959431
## iter 270 value 3.712038
## iter 280 value 3.495527
## iter 290 value 3.445904
## iter 300 value 3.337981
## iter 310 value 3.279219
## iter 320 value 3.248308
## iter 330 value 3.215947
## iter 340 value 3.210604
## iter 350 value 3.208987
## iter 360 value 3.208181
## iter 370 value 3.207733
## iter 380 value 3.204774
## iter 390 value 3.203119
## iter 400 value 3.202227
## iter 410 value 3.202131
## iter 420 value 3.202107
## iter 430 value 3.202019
## iter 440 value 3.201785
## iter 450 value 3.201668
## iter 460 value 3.200004
## iter 470 value 3.189318
## iter 480 value 3.170445
## iter 490 value 3.162950
## iter 500 value 3.159665
## iter 510 value 3.159092
## iter 520 value 3.159067
## final  value 3.159065
## converged
```

```
## # weights:  121
## initial  value 50.738835
## iter  10 value 28.848270
## iter  20 value 23.501576
## iter  30 value 22.778944
## iter  40 value 20.536409
## iter  50 value 20.453393
## iter  60 value 20.262613
## iter  70 value 20.130051
## iter  80 value 19.933842
## iter  90 value 19.852195
## iter 100 value 19.607296
## iter 110 value 19.266570
## iter 120 value 19.123907
## iter 130 value 19.010002
## iter 140 value 18.758250
## iter 150 value 18.414113
## iter 160 value 18.041965
## iter 170 value 13.860616
## iter 180 value 11.919188
## iter 190 value 11.155008
## iter 200 value 10.793763
## iter 210 value 10.417962
## iter 220 value 10.006128
## iter 230 value 8.754717
## iter 240 value 8.139619
## iter 250 value 7.639562
## iter 260 value 6.842280
## iter 270 value 5.672749
## iter 280 value 5.249546
## iter 290 value 4.784636
## iter 300 value 4.581452
## iter 310 value 4.504385
## iter 320 value 4.487316
## iter 330 value 4.482834
## iter 340 value 4.441425
## iter 350 value 4.422739
## iter 360 value 4.388332
## iter 370 value 4.136481
## iter 380 value 3.953182
## iter 390 value 3.822044
## iter 400 value 3.730928
## iter 410 value 3.658437
## iter 420 value 3.639153
## iter 430 value 3.636030
## iter 440 value 3.634797
## iter 450 value 3.633190
## iter 460 value 3.616859
## iter 470 value 3.553411
## iter 480 value 3.430108
## iter 490 value 3.357730
## iter 500 value 3.344235
## iter 510 value 3.340807
## iter 520 value 3.334721
```

```
## iter 530 value 3.307484
## iter 540 value 3.255694
## iter 550 value 3.187868
## iter 560 value 3.139812
## iter 570 value 3.129531
## iter 580 value 3.120106
## iter 590 value 3.103220
## iter 600 value 3.086641
## iter 610 value 3.051745
## iter 620 value 2.997934
## iter 630 value 2.921468
## iter 640 value 2.879031
## iter 650 value 2.863909
## iter 660 value 2.860460
## iter 670 value 2.860052
## iter 680 value 2.859639
## iter 690 value 2.856529
## iter 700 value 2.835710
## iter 710 value 2.790690
## iter 720 value 2.747196
## iter 730 value 2.727826
## iter 740 value 2.705381
## iter 750 value 2.697903
## iter 760 value 2.692854
## iter 770 value 2.692231
## iter 780 value 2.691709
## iter 790 value 2.691685
## final  value 2.691684
## converged
## # weights:  131
## initial  value 179.978987
## iter  10 value 31.451849
## iter  20 value 24.152380
## iter  30 value 12.848511
## iter  40 value 10.310016
## iter  50 value 8.550970
## iter  60 value 6.806909
## iter  70 value 6.298921
## iter  80 value 6.211018
## iter  90 value 6.092251
## iter 100 value 5.936824
## iter 110 value 5.825624
## iter 120 value 5.699039
## iter 130 value 5.589504
## iter 140 value 5.452271
## iter 150 value 5.024846
## iter 160 value 4.937513
## iter 170 value 4.808243
## iter 180 value 4.629323
## iter 190 value 4.478531
## iter 200 value 4.346419
## iter 210 value 4.310852
## iter 220 value 4.291627
## iter 230 value 4.287209
```

```
## iter 240 value 4.276459
## iter 250 value 4.202629
## iter 260 value 4.004225
## iter 270 value 3.867891
## iter 280 value 3.710820
## iter 290 value 3.503385
## iter 300 value 3.313069
## iter 310 value 3.222865
## iter 320 value 3.086087
## iter 330 value 2.969815
## iter 340 value 2.904263
## iter 350 value 2.852062
## iter 360 value 2.804615
## iter 370 value 2.762999
## iter 380 value 2.759504
## iter 390 value 2.749121
## iter 400 value 2.724096
## iter 410 value 2.694432
## iter 420 value 2.647382
## iter 430 value 2.602529
## iter 440 value 2.577584
## iter 450 value 2.558083
## iter 460 value 2.551926
## iter 470 value 2.548967
## iter 480 value 2.548119
## iter 490 value 2.547090
## iter 500 value 2.543363
## iter 510 value 2.536313
## iter 520 value 2.520928
## iter 530 value 2.468865
## iter 540 value 2.437841
## iter 550 value 2.420533
## iter 560 value 2.410967
## iter 570 value 2.403653
## iter 580 value 2.400237
## iter 590 value 2.399565
## iter 600 value 2.393976
## iter 610 value 2.390193
## iter 620 value 2.388975
## iter 630 value 2.388916
## final  value 2.388914
## converged
## # weights:  141
## initial  value 244.763113
## iter  10 value 29.720563
## iter  20 value 21.151245
## iter  30 value 19.345540
## iter  40 value 15.341993
## iter  50 value 12.112730
## iter  60 value 10.700199
## iter  70 value 10.344602
## iter  80 value 9.133756
## iter  90 value 7.602276
## iter 100 value 7.386342
```

```
## iter 110 value 6.901555
## iter 120 value 6.795916
## iter 130 value 6.745959
## iter 140 value 6.643875
## iter 150 value 6.579394
## iter 160 value 6.441442
## iter 170 value 6.295533
## iter 180 value 6.258711
## iter 190 value 6.236329
## iter 200 value 5.889136
## iter 210 value 5.662600
## iter 220 value 5.243146
## iter 230 value 4.985564
## iter 240 value 4.768100
## iter 250 value 4.515311
## iter 260 value 4.277266
## iter 270 value 4.048413
## iter 280 value 3.827022
## iter 290 value 3.638352
## iter 300 value 3.496794
## iter 310 value 3.384986
## iter 320 value 3.277816
## iter 330 value 3.223735
## iter 340 value 3.154421
## iter 350 value 3.062500
## iter 360 value 3.024406
## iter 370 value 2.995766
## iter 380 value 2.889933
## iter 390 value 2.827758
## iter 400 value 2.815325
## iter 410 value 2.813572
## iter 420 value 2.807186
## iter 430 value 2.784623
## iter 440 value 2.753507
## iter 450 value 2.714245
## iter 460 value 2.683776
## iter 470 value 2.667451
## iter 480 value 2.654713
## iter 490 value 2.644749
## iter 500 value 2.631412
## iter 510 value 2.630238
## iter 520 value 2.630009
## iter 530 value 2.628207
## iter 540 value 2.624529
## iter 550 value 2.615135
## iter 560 value 2.596358
## iter 570 value 2.573885
## iter 580 value 2.558631
## iter 590 value 2.543786
## iter 600 value 2.527614
## iter 610 value 2.514685
## iter 620 value 2.512024
## iter 630 value 2.511884
## iter 640 value 2.511869
```

```
## final  value 2.511869
## converged
## # weights:  151
## initial  value 130.219773
## iter  10 value 30.464638
## iter  20 value 23.029494
## iter  30 value 21.760960
## iter  40 value 21.481370
## iter  50 value 16.100589
## iter  60 value 14.250689
## iter  70 value 13.293149
## iter  80 value 12.405227
## iter  90 value 11.155164
## iter 100 value 10.370941
## iter 110 value 9.998634
## iter 120 value 9.753662
## iter 130 value 9.237820
## iter 140 value 8.976659
## iter 150 value 8.853776
## iter 160 value 8.791703
## iter 170 value 8.657624
## iter 180 value 8.187387
## iter 190 value 7.788297
## iter 200 value 7.123175
## iter 210 value 6.651375
## iter 220 value 5.916122
## iter 230 value 4.873340
## iter 240 value 4.579913
## iter 250 value 4.552610
## iter 260 value 4.537117
## iter 270 value 4.521112
## iter 280 value 4.484868
## iter 290 value 4.411251
## iter 300 value 4.342647
## iter 310 value 4.138691
## iter 320 value 3.940894
## iter 330 value 3.706760
## iter 340 value 3.617027
## iter 350 value 3.568351
## iter 360 value 3.555211
## iter 370 value 3.548487
## iter 380 value 3.527731
## iter 390 value 3.486080
## iter 400 value 3.429447
## iter 410 value 3.399788
## iter 420 value 3.395387
## iter 430 value 3.385555
## iter 440 value 3.369812
## iter 450 value 3.339244
## iter 460 value 3.250137
## iter 470 value 3.190009
## iter 480 value 3.147935
## iter 490 value 3.122334
## iter 500 value 3.118063
```

```
## iter 510 value 3.112955
## iter 520 value 3.103922
## iter 530 value 3.094295
## iter 540 value 3.083784
## iter 550 value 3.080420
## iter 560 value 3.079836
## iter 570 value 3.077625
## iter 580 value 3.068384
## iter 590 value 3.047890
## iter 600 value 2.935534
## iter 610 value 2.854120
## iter 620 value 2.815878
## iter 630 value 2.753126
## iter 640 value 2.687121
## iter 650 value 2.632713
## iter 660 value 2.625762
## iter 670 value 2.612313
## iter 680 value 2.599331
## iter 690 value 2.562395
## iter 700 value 2.518886
## iter 710 value 2.481586
## iter 720 value 2.467336
## iter 730 value 2.438931
## iter 740 value 2.387257
## iter 750 value 2.365395
## iter 760 value 2.354924
## iter 770 value 2.351261
## iter 780 value 2.349697
## iter 790 value 2.348111
## iter 800 value 2.347260
## iter 810 value 2.345877
## iter 820 value 2.344785
## iter 830 value 2.343202
## iter 840 value 2.341880
## iter 850 value 2.341673
## iter 860 value 2.341670
## final  value 2.341670
## converged
## # weights:  161
## initial  value 105.104907
## iter  10 value 23.644989
## iter  20 value 19.124346
## iter  30 value 18.275324
## iter  40 value 17.578507
## iter  50 value 15.870057
## iter  60 value 12.370316
## iter  70 value 11.278249
## iter  80 value 9.813364
## iter  90 value 9.187762
## iter 100 value 8.721694
## iter 110 value 7.870743
## iter 120 value 7.122803
## iter 130 value 5.921862
## iter 140 value 5.696324
```

```
## iter 150 value 5.556763
## iter 160 value 5.369103
## iter 170 value 5.039727
## iter 180 value 4.803819
## iter 190 value 4.465215
## iter 200 value 4.327496
## iter 210 value 4.228465
## iter 220 value 4.205286
## iter 230 value 4.185437
## iter 240 value 4.175377
## iter 250 value 4.147965
## iter 260 value 4.033037
## iter 270 value 3.959904
## iter 280 value 3.849591
## iter 290 value 3.703407
## iter 300 value 3.602749
## iter 310 value 3.579848
## iter 320 value 3.577086
## iter 330 value 3.565940
## iter 340 value 3.544756
## iter 350 value 3.494154
## iter 360 value 3.439060
## iter 370 value 3.382289
## iter 380 value 3.352489
## iter 390 value 3.346609
## iter 400 value 3.341275
## iter 410 value 3.317681
## iter 420 value 3.257453
## iter 430 value 3.196073
## iter 440 value 3.118992
## iter 450 value 3.039978
## iter 460 value 2.996503
## iter 470 value 2.966327
## iter 480 value 2.925154
## iter 490 value 2.823987
## iter 500 value 2.775622
## iter 510 value 2.761574
## iter 520 value 2.725984
## iter 530 value 2.714763
## iter 540 value 2.683446
## iter 550 value 2.652275
## iter 560 value 2.579720
## iter 570 value 2.576133
## iter 580 value 2.572582
## iter 590 value 2.564479
## iter 600 value 2.542569
## iter 610 value 2.511912
## iter 620 value 2.480867
## iter 630 value 2.445043
## iter 640 value 2.422859
## iter 650 value 2.413675
## iter 660 value 2.405068
## iter 670 value 2.398075
## iter 680 value 2.390990
```

```
## iter 690 value 2.382673
## iter 700 value 2.377190
## iter 710 value 2.376763
## iter 720 value 2.376154
## iter 730 value 2.375848
## iter 740 value 2.372283
## iter 750 value 2.360837
## iter 760 value 2.342727
## iter 770 value 2.323589
## iter 780 value 2.310229
## iter 790 value 2.299683
## iter 800 value 2.289177
## iter 810 value 2.281091
## iter 820 value 2.275552
## iter 830 value 2.244552
## iter 840 value 2.215105
## iter 850 value 2.151322
## iter 860 value 2.132154
## iter 870 value 2.124511
## iter 880 value 2.121311
## iter 890 value 2.116523
## iter 900 value 2.099145
## iter 910 value 2.071961
## iter 920 value 2.038531
## iter 930 value 2.025778
## iter 940 value 2.022194
## iter 950 value 2.021850
## iter 960 value 2.021254
## iter 970 value 2.020117
## iter 980 value 2.018353
## iter 990 value 2.014824
## iter1000 value 2.012576
## iter1010 value 2.011386
## iter1020 value 2.010109
## iter1030 value 2.007993
## iter1040 value 2.005714
## iter1050 value 2.005582
## iter1060 value 2.005532
## iter1070 value 2.005527
## final  value 2.005527
## converged
## # weights:  171
## initial  value 48.097445
## iter  10 value 24.395300
## iter  20 value 24.047220
## iter  30 value 23.733720
## iter  40 value 23.414558
## iter  50 value 23.314747
## iter  60 value 23.288993
## iter  70 value 18.904972
## iter  80 value 16.513080
## iter  90 value 14.440393
## iter 100 value 11.260670
## iter 110 value 9.785409
```

```
## iter 120 value 9.106666
## iter 130 value 7.553755
## iter 140 value 5.942528
## iter 150 value 5.568670
## iter 160 value 5.435518
## iter 170 value 5.404819
## iter 180 value 5.361237
## iter 190 value 5.168411
## iter 200 value 4.901029
## iter 210 value 4.576249
## iter 220 value 4.398745
## iter 230 value 4.370151
## iter 240 value 4.359783
## iter 250 value 4.349095
## iter 260 value 4.312413
## iter 270 value 4.270605
## iter 280 value 4.244254
## iter 290 value 4.215247
## iter 300 value 4.139394
## iter 310 value 4.009293
## iter 320 value 3.800799
## iter 330 value 3.731223
## iter 340 value 3.727514
## iter 350 value 3.712632
## iter 360 value 3.692252
## iter 370 value 3.642007
## iter 380 value 3.498462
## iter 390 value 3.405901
## iter 400 value 3.327982
## iter 410 value 3.249478
## iter 420 value 3.200577
## iter 430 value 3.192431
## iter 440 value 3.190731
## iter 450 value 3.190646
## iter 460 value 3.190165
## iter 470 value 3.188944
## iter 480 value 3.182925
## iter 490 value 3.154317
## iter 500 value 3.087884
## iter 510 value 3.027643
## iter 520 value 3.002841
## iter 530 value 2.997084
## iter 540 value 2.986162
## iter 550 value 2.982197
## iter 560 value 2.973409
## iter 570 value 2.959514
## iter 580 value 2.927705
## iter 590 value 2.899847
## iter 600 value 2.878350
## iter 610 value 2.868075
## iter 620 value 2.857358
## iter 630 value 2.851710
## iter 640 value 2.847116
## iter 650 value 2.820330
```

```
## iter 660 value 2.802824
## iter 670 value 2.794765
## iter 680 value 2.794604
## iter 690 value 2.794599
## final  value 2.794599
## converged
## # weights:  181
## initial  value 203.442842
## iter  10 value 33.654717
## iter  20 value 26.026958
## iter  30 value 23.545081
## iter  40 value 21.938879
## iter  50 value 20.904392
## iter  60 value 18.930580
## iter  70 value 17.594581
## iter  80 value 15.791223
## iter  90 value 14.773468
## iter 100 value 14.201613
## iter 110 value 13.900424
## iter 120 value 13.404170
## iter 130 value 12.960100
## iter 140 value 11.308425
## iter 150 value 9.834074
## iter 160 value 8.783885
## iter 170 value 7.860293
## iter 180 value 7.539266
## iter 190 value 6.893737
## iter 200 value 6.190936
## iter 210 value 6.031700
## iter 220 value 5.604963
## iter 230 value 5.491140
## iter 240 value 5.285567
## iter 250 value 4.958206
## iter 260 value 4.860870
## iter 270 value 4.857462
## iter 280 value 4.829764
## iter 290 value 4.824236
## iter 300 value 4.803304
## iter 310 value 4.753714
## iter 320 value 4.692868
## iter 330 value 4.647588
## iter 340 value 4.539034
## iter 350 value 4.485566
## iter 360 value 4.467105
## iter 370 value 4.461142
## iter 380 value 4.401356
## iter 390 value 4.240556
## iter 400 value 4.071706
## iter 410 value 4.029755
## iter 420 value 4.003402
## iter 430 value 3.979695
## iter 440 value 3.967896
## iter 450 value 3.947972
## iter 460 value 3.871303
```

```
## iter 470 value 3.800708
## iter 480 value 3.752659
## iter 490 value 3.723877
## iter 500 value 3.690047
## iter 510 value 3.545912
## iter 520 value 3.435203
## iter 530 value 3.358411
## iter 540 value 3.342626
## iter 550 value 3.338239
## iter 560 value 3.332978
## iter 570 value 3.322114
## iter 580 value 3.281344
## iter 590 value 3.206285
## iter 600 value 3.116285
## iter 610 value 2.996487
## iter 620 value 2.907206
## iter 630 value 2.897665
## iter 640 value 2.893307
## iter 650 value 2.887523
## iter 660 value 2.885037
## iter 670 value 2.875535
## iter 680 value 2.863034
## iter 690 value 2.842968
## iter 700 value 2.731653
## iter 710 value 2.681048
## iter 720 value 2.670278
## iter 730 value 2.661897
## iter 740 value 2.648507
## iter 750 value 2.629141
## iter 760 value 2.604026
## iter 770 value 2.578759
## iter 780 value 2.537853
## iter 790 value 2.526598
## iter 800 value 2.518776
## iter 810 value 2.511388
## iter 820 value 2.496517
## iter 830 value 2.468985
## iter 840 value 2.424596
## iter 850 value 2.352822
## iter 860 value 2.282531
## iter 870 value 2.230023
## iter 880 value 2.198791
## iter 890 value 2.182763
## iter 900 value 2.165057
## iter 910 value 2.123150
## iter 920 value 2.095797
## iter 930 value 2.068899
## iter 940 value 2.044879
## iter 950 value 2.020995
## iter 960 value 1.999133
## iter 970 value 1.983290
## iter 980 value 1.970996
## iter 990 value 1.963477
## iter1000 value 1.962409
```

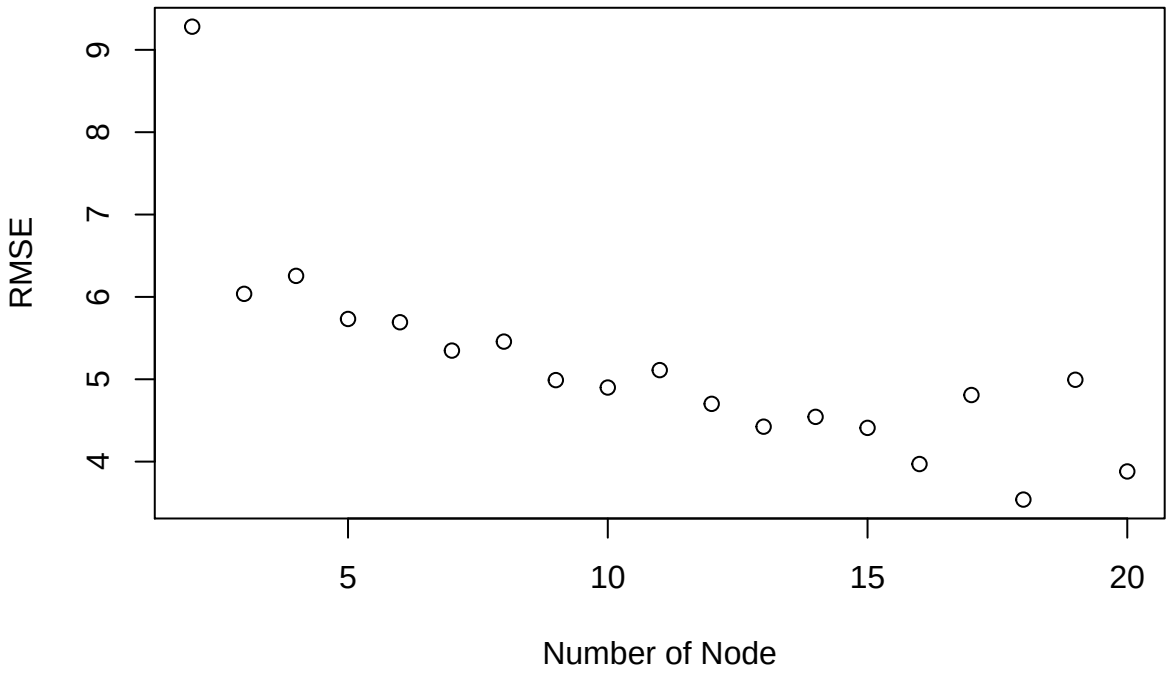
```
## iter1010 value 1.961832
## iter1020 value 1.960911
## iter1030 value 1.959712
## iter1040 value 1.957292
## iter1050 value 1.951610
## iter1060 value 1.945571
## iter1070 value 1.931799
## iter1080 value 1.909391
## iter1090 value 1.879051
## iter1100 value 1.834955
## iter1110 value 1.799649
## iter1120 value 1.779512
## iter1130 value 1.753488
## iter1140 value 1.735871
## iter1150 value 1.731433
## iter1160 value 1.730488
## iter1170 value 1.729927
## iter1180 value 1.728848
## iter1190 value 1.726303
## iter1200 value 1.721213
## iter1210 value 1.711095
## iter1220 value 1.702841
## iter1230 value 1.691636
## iter1240 value 1.677061
## iter1250 value 1.664785
## iter1260 value 1.657688
## iter1270 value 1.653847
## iter1280 value 1.652636
## iter1290 value 1.652229
## iter1300 value 1.652145
## iter1310 value 1.652125
## final  value 1.652124
## converged
## # weights:  191
## initial  value 33.721291
## iter  10 value 27.176370
## iter  20 value 19.387370
## iter  30 value 18.111909
## iter  40 value 16.592221
## iter  50 value 15.713143
## iter  60 value 15.442493
## iter  70 value 15.108451
## iter  80 value 14.961014
## iter  90 value 14.786619
## iter 100 value 14.632969
## iter 110 value 14.536422
## iter 120 value 14.394339
## iter 130 value 14.132586
## iter 140 value 12.819454
## iter 150 value 11.636273
## iter 160 value 11.316576
## iter 170 value 11.058570
## iter 180 value 10.999922
## iter 190 value 10.945437
```

```
## iter 200 value 10.796090
## iter 210 value 10.306131
## iter 220 value 9.750069
## iter 230 value 9.210982
## iter 240 value 9.095730
## iter 250 value 8.772471
## iter 260 value 7.889453
## iter 270 value 7.605688
## iter 280 value 7.030835
## iter 290 value 6.256144
## iter 300 value 5.856407
## iter 310 value 5.783960
## iter 320 value 5.765662
## iter 330 value 5.730535
## iter 340 value 5.700135
## iter 350 value 5.627832
## iter 360 value 5.438523
## iter 370 value 5.048495
## iter 380 value 4.532860
## iter 390 value 4.307629
## iter 400 value 4.242173
## iter 410 value 4.079514
## iter 420 value 3.975646
## iter 430 value 3.949090
## iter 440 value 3.856783
## iter 450 value 3.759277
## iter 460 value 3.670055
## iter 470 value 3.533586
## iter 480 value 3.354494
## iter 490 value 3.279683
## iter 500 value 3.260452
## iter 510 value 3.259479
## iter 520 value 3.255185
## iter 530 value 3.245756
## iter 540 value 3.201269
## iter 550 value 3.111814
## iter 560 value 3.067880
## iter 570 value 3.049535
## iter 580 value 3.037898
## iter 590 value 3.033851
## iter 600 value 3.031339
## iter 610 value 3.031190
## iter 620 value 3.031183
## iter 630 value 3.031180
## iter 640 value 3.031173
## iter 650 value 3.031148
## iter 660 value 3.031146
## final  value 3.031146
## converged
## # weights:  201
## initial  value 121.926950
## iter  10 value 26.048374
## iter  20 value 20.260280
## iter  30 value 16.726912
```

```
## iter  40 value 15.257905
## iter  50 value 14.769897
## iter  60 value 14.270848
## iter  70 value 13.423955
## iter  80 value 12.417196
## iter  90 value 11.182917
## iter 100 value 8.621570
## iter 110 value 7.906124
## iter 120 value 6.934224
## iter 130 value 6.466243
## iter 140 value 5.745886
## iter 150 value 5.431673
## iter 160 value 5.393063
## iter 170 value 5.262325
## iter 180 value 5.038749
## iter 190 value 4.931614
## iter 200 value 4.916653
## iter 210 value 4.913217
## iter 220 value 4.857190
## iter 230 value 4.798478
## iter 240 value 4.762093
## iter 250 value 4.608880
## iter 260 value 4.559526
## iter 270 value 4.436326
## iter 280 value 4.170685
## iter 290 value 3.920462
## iter 300 value 3.885702
## iter 310 value 3.882416
## iter 320 value 3.881762
## iter 330 value 3.879556
## iter 340 value 3.871801
## iter 350 value 3.824762
## iter 360 value 3.568159
## iter 370 value 3.459828
## iter 380 value 3.406069
## iter 390 value 3.388780
## iter 400 value 3.377884
## iter 410 value 3.327272
## iter 420 value 3.167525
## iter 430 value 3.038750
## iter 440 value 2.973549
## iter 450 value 2.934305
## iter 460 value 2.914446
## iter 470 value 2.910058
## iter 480 value 2.904795
## iter 490 value 2.891464
## iter 500 value 2.867821
## iter 510 value 2.822820
## iter 520 value 2.772659
## iter 530 value 2.729694
## iter 540 value 2.702334
## iter 550 value 2.684497
## iter 560 value 2.647163
## iter 570 value 2.539075
```

```
## iter 580 value 2.513725
## iter 590 value 2.476315
## iter 600 value 2.399727
## iter 610 value 2.305935
## iter 620 value 2.242803
## iter 630 value 2.237305
## iter 640 value 2.231645
## iter 650 value 2.225613
## iter 660 value 2.205419
## iter 670 value 2.187870
## iter 680 value 2.165876
## iter 690 value 2.149608
## iter 700 value 2.128118
## iter 710 value 2.101933
## iter 720 value 2.078420
## iter 730 value 2.066579
## iter 740 value 2.054928
## iter 750 value 2.041781
## iter 760 value 2.031377
## iter 770 value 1.992905
## iter 780 value 1.963399
## iter 790 value 1.954360
## iter 800 value 1.949073
## iter 810 value 1.945712
## iter 820 value 1.943790
## iter 830 value 1.943398
## iter 840 value 1.943371
## iter 850 value 1.943365
## iter 860 value 1.943361
## final  value 1.943360
## converged
```

```
# Make a plot
plot(seq(2,20), RMSE.train, xlab='Number of Node', ylab='RMSE')
```



c)

```
# Vector to store RMSE
RMSE.test=rep(NA, 19)
# Calculate RMSE for different number of nodes
for (a in seq(2,20))
{
  model=nnet(Strength01~.-Strength, data=train, size=a, decay=0.01, maxit=2000)
  test.predict.1 <- predict(model, newdata = test)
  test.predict = test.predict.1*(max(Concrete_Data$Strength)-min(Concrete_Data$Strength))+min(Concrete_
  RMSE.test[a-1] <- sqrt(mean((test.predict - test$Strength)^2))
}
```

```
## # weights:  21
## initial  value 69.667976
## iter  10 value 31.452636
## iter  20 value 30.964124
## iter  30 value 23.035713
## iter  40 value 10.983698
## iter  50 value 10.297745
## iter  60 value 8.099682
## iter  70 value 7.115070
## iter  80 value 6.473516
## iter  90 value 5.607001
## iter 100 value 5.565474
## iter 110 value 5.564615
## iter 110 value 5.564615
## final  value 5.564615
## converged
## # weights:  31
## initial  value 36.553855
## iter  10 value 31.505463
```

```
## iter  20 value 27.168058
## iter  30 value 20.607162
## iter  40 value 17.981395
## iter  50 value 17.004727
## iter  60 value 15.628901
## iter  70 value 14.988443
## iter  80 value 13.975811
## iter  90 value 12.229541
## iter 100 value 10.333283
## iter 110 value 9.861987
## iter 120 value 9.771232
## iter 130 value 9.647056
## iter 140 value 8.658617
## iter 150 value 7.962983
## iter 160 value 5.692368
## iter 170 value 5.006279
## iter 180 value 4.823753
## iter 190 value 4.695165
## iter 200 value 4.683783
## iter 210 value 4.675394
## iter 220 value 4.632135
## iter 230 value 4.544488
## iter 240 value 4.507888
## iter 250 value 4.360041
## iter 260 value 4.305211
## iter 270 value 4.302557
## iter 280 value 4.298377
## iter 290 value 4.282325
## final  value 4.282249
## converged
## # weights:  41
## initial  value 45.534243
## iter  10 value 25.916446
## iter  20 value 17.131456
## iter  30 value 14.265019
## iter  40 value 13.108209
## iter  50 value 12.922609
## iter  60 value 12.879018
## iter  70 value 12.087014
## iter  80 value 10.919829
## iter  90 value 9.410150
## iter 100 value 8.328410
## iter 110 value 7.294477
## iter 120 value 6.464440
## iter 130 value 6.228437
## iter 140 value 6.049932
## iter 150 value 5.688461
## iter 160 value 5.193018
## iter 170 value 5.066238
## iter 180 value 4.932755
## iter 190 value 4.888998
## iter 200 value 4.816523
## iter 210 value 4.447418
## iter 220 value 4.352011
```

```
## iter 230 value 4.332468
## iter 240 value 4.329063
## iter 250 value 4.328607
## iter 260 value 4.322361
## iter 270 value 4.306680
## iter 280 value 4.291398
## iter 290 value 4.267614
## iter 300 value 4.265944
## iter 310 value 4.265882
## final  value 4.265880
## converged
## # weights:  51
## initial  value 85.136026
## iter  10 value 31.566810
## iter  20 value 24.466167
## iter  30 value 21.623960
## iter  40 value 20.821567
## iter  50 value 20.484065
## iter  60 value 19.309165
## iter  70 value 14.551002
## iter  80 value 12.649142
## iter  90 value 10.295056
## iter 100 value 9.804865
## iter 110 value 9.766459
## iter 120 value 9.714237
## iter 130 value 7.563581
## iter 140 value 6.835132
## iter 150 value 6.576567
## iter 160 value 5.932278
## iter 170 value 5.323142
## iter 180 value 5.186757
## iter 190 value 5.116884
## iter 200 value 5.003178
## iter 210 value 4.384278
## iter 220 value 4.142839
## iter 230 value 4.078543
## iter 240 value 4.055227
## iter 250 value 4.042122
## iter 260 value 4.036067
## iter 270 value 4.034657
## iter 280 value 4.033428
## iter 290 value 4.032245
## iter 300 value 4.029944
## iter 310 value 4.004828
## iter 320 value 3.857693
## iter 330 value 3.839492
## iter 340 value 3.837343
## iter 350 value 3.819015
## iter 360 value 3.816351
## iter 370 value 3.815986
## iter 380 value 3.815786
## final  value 3.815760
## converged
## # weights:  61
```

```
## initial  value 37.723692
## iter  10 value 31.425147
## iter  20 value 30.169507
## iter  30 value 22.619946
## iter  40 value 20.768674
## iter  50 value 18.234968
## iter  60 value 12.611789
## iter  70 value 11.119317
## iter  80 value 10.064230
## iter  90 value 8.087419
## iter 100 value 6.345479
## iter 110 value 5.517744
## iter 120 value 5.189781
## iter 130 value 4.999184
## iter 140 value 4.977955
## iter 150 value 4.967429
## iter 160 value 4.906292
## iter 170 value 4.481727
## iter 180 value 4.071350
## iter 190 value 3.963816
## iter 200 value 3.922851
## iter 210 value 3.920925
## iter 220 value 3.915190
## iter 230 value 3.904697
## iter 240 value 3.878910
## iter 250 value 3.836947
## iter 260 value 3.661623
## iter 270 value 3.583009
## iter 280 value 3.541173
## iter 290 value 3.431776
## iter 300 value 3.399091
## iter 310 value 3.387608
## iter 320 value 3.386006
## iter 330 value 3.385921
## iter 340 value 3.385900
## iter 350 value 3.385897
## final  value 3.385896
## converged
## # weights:  71
## initial  value 37.052398
## iter  10 value 23.369020
## iter  20 value 16.373374
## iter  30 value 11.300625
## iter  40 value 8.937122
## iter  50 value 8.150724
## iter  60 value 7.463172
## iter  70 value 7.173095
## iter  80 value 6.882360
## iter  90 value 6.378204
## iter 100 value 6.215516
## iter 110 value 5.837012
## iter 120 value 5.557757
## iter 130 value 5.487355
## iter 140 value 5.077537
```

```
## iter 150 value 4.904339
## iter 160 value 4.805491
## iter 170 value 4.788968
## iter 180 value 4.782346
## iter 190 value 4.722746
## iter 200 value 4.428277
## iter 210 value 4.226721
## iter 220 value 4.210080
## iter 230 value 4.208352
## iter 240 value 4.203197
## iter 250 value 4.192313
## iter 260 value 4.093663
## iter 270 value 3.931841
## iter 280 value 3.715630
## iter 290 value 3.613543
## iter 300 value 3.568650
## iter 310 value 3.552838
## iter 320 value 3.480427
## iter 330 value 3.443827
## iter 340 value 3.425446
## iter 350 value 3.419864
## iter 360 value 3.415632
## iter 370 value 3.404485
## iter 380 value 3.346713
## iter 390 value 3.328007
## iter 400 value 3.323614
## iter 410 value 3.323559
## iter 420 value 3.322616
## iter 430 value 3.322320
## iter 440 value 3.321481
## iter 450 value 3.320215
## iter 460 value 3.316716
## iter 470 value 3.313969
## iter 480 value 3.308907
## iter 490 value 3.306596
## iter 500 value 3.306494
## final  value 3.306493
## converged
## # weights:  81
## initial  value 44.334905
## iter  10 value 31.420784
## iter  20 value 28.468542
## iter  30 value 20.743557
## iter  40 value 12.656725
## iter  50 value 11.650757
## iter  60 value 10.900669
## iter  70 value 10.495262
## iter  80 value 10.007560
## iter  90 value 9.328615
## iter 100 value 8.566890
## iter 110 value 7.766783
## iter 120 value 6.569241
## iter 130 value 5.442923
## iter 140 value 4.856151
```

```
## iter 150 value 4.745486
## iter 160 value 4.711117
## iter 170 value 4.696514
## iter 180 value 4.670080
## iter 190 value 4.567279
## iter 200 value 4.198270
## iter 210 value 3.965009
## iter 220 value 3.901036
## iter 230 value 3.889008
## iter 240 value 3.887318
## iter 250 value 3.881333
## iter 260 value 3.823579
## iter 270 value 3.713244
## iter 280 value 3.651310
## iter 290 value 3.630200
## iter 300 value 3.618862
## iter 310 value 3.569747
## iter 320 value 3.450870
## iter 330 value 3.435197
## iter 340 value 3.432357
## iter 350 value 3.428359
## iter 360 value 3.387438
## iter 370 value 3.347073
## iter 380 value 3.319517
## iter 390 value 3.313522
## iter 400 value 3.313262
## iter 410 value 3.313253
## final  value 3.313253
## converged
## # weights:  91
## initial  value 60.906658
## iter  10 value 25.188186
## iter  20 value 20.925072
## iter  30 value 19.355017
## iter  40 value 18.908037
## iter  50 value 18.740347
## iter  60 value 18.685643
## iter  70 value 14.907589
## iter  80 value 11.073784
## iter  90 value 9.847585
## iter 100 value 9.669800
## iter 110 value 9.255602
## iter 120 value 7.306543
## iter 130 value 6.299547
## iter 140 value 6.134411
## iter 150 value 6.131195
## iter 160 value 6.073433
## iter 170 value 5.992779
## iter 180 value 5.680636
## iter 190 value 5.236511
## iter 200 value 5.185524
## iter 210 value 5.183001
## iter 220 value 5.178483
## iter 230 value 5.086922
```

```
## iter 240 value 4.872098
## iter 250 value 4.649844
## iter 260 value 4.502130
## iter 270 value 4.434463
## iter 280 value 4.422094
## iter 290 value 4.417806
## iter 300 value 4.406164
## iter 310 value 4.397686
## iter 320 value 4.383702
## iter 330 value 4.193225
## iter 340 value 4.129633
## iter 350 value 4.099382
## iter 360 value 4.075733
## iter 370 value 4.033716
## iter 380 value 3.923679
## iter 390 value 3.855176
## iter 400 value 3.816871
## iter 410 value 3.796897
## iter 420 value 3.793502
## iter 430 value 3.793442
## iter 440 value 3.792676
## iter 450 value 3.788190
## iter 460 value 3.776044
## iter 470 value 3.746485
## iter 480 value 3.638492
## iter 490 value 3.564281
## iter 500 value 3.534143
## iter 510 value 3.513055
## iter 520 value 3.500993
## iter 530 value 3.499564
## iter 540 value 3.494715
## iter 550 value 3.489464
## iter 560 value 3.484672
## iter 570 value 3.482789
## iter 580 value 3.475161
## iter 590 value 3.473577
## iter 600 value 3.466984
## iter 610 value 3.463442
## iter 620 value 3.462848
## iter 630 value 3.462688
## iter 640 value 3.460743
## iter 650 value 3.460430
## iter 660 value 3.454541
## iter 670 value 3.452826
## iter 680 value 3.446122
## iter 690 value 3.429394
## iter 700 value 3.418831
## iter 710 value 3.413206
## iter 720 value 3.411665
## iter 730 value 3.411228
## iter 740 value 3.411196
## iter 750 value 3.410983
## iter 760 value 3.410641
## iter 770 value 3.410400
```

```
## iter 780 value 3.409993
## iter 790 value 3.409917
## iter 800 value 3.409911
## final  value 3.409911
## converged
## # weights:  101
## initial  value 31.478410
## iter  10 value 21.263292
## iter  20 value 20.638162
## iter  30 value 20.547171
## iter  40 value 20.368043
## iter  50 value 20.240109
## iter  60 value 20.133593
## iter  70 value 20.069945
## iter  80 value 17.729830
## iter  90 value 16.094487
## iter 100 value 15.638704
## iter 110 value 15.551815
## iter 120 value 15.511866
## iter 130 value 15.171016
## iter 140 value 15.110922
## iter 150 value 13.752099
## iter 160 value 12.494725
## iter 170 value 10.661637
## iter 180 value 9.578778
## iter 190 value 9.169106
## iter 200 value 8.248801
## iter 210 value 8.015325
## iter 220 value 6.964336
## iter 230 value 6.277047
## iter 240 value 5.723541
## iter 250 value 5.413229
## iter 260 value 5.198794
## iter 270 value 4.924677
## iter 280 value 4.656698
## iter 290 value 4.291951
## iter 300 value 4.059337
## iter 310 value 4.005627
## iter 320 value 3.938450
## iter 330 value 3.769334
## iter 340 value 3.603049
## iter 350 value 3.444119
## iter 360 value 3.327212
## iter 370 value 3.259190
## iter 380 value 3.188557
## iter 390 value 3.162941
## iter 400 value 3.154793
## iter 410 value 3.150622
## iter 420 value 3.149657
## iter 430 value 3.149257
## iter 440 value 3.149114
## iter 450 value 3.148639
## iter 460 value 3.148635
## iter 470 value 3.148627
```

```
## iter 480 value 3.148616
## iter 490 value 3.148585
## iter 500 value 3.148546
## iter 510 value 3.148445
## iter 520 value 3.148391
## final  value 3.148384
## converged
## # weights:  111
## initial  value 31.682211
## iter  10 value 30.048477
## iter  20 value 29.377903
## iter  30 value 29.134419
## iter  40 value 28.454193
## iter  50 value 16.880237
## iter  60 value 15.997568
## iter  70 value 15.467612
## iter  80 value 14.980750
## iter  90 value 13.478900
## iter 100 value 12.613956
## iter 110 value 10.940108
## iter 120 value 9.818793
## iter 130 value 9.760300
## iter 140 value 9.758783
## iter 150 value 9.733392
## iter 160 value 9.729024
## iter 170 value 9.708679
## iter 180 value 8.994102
## iter 190 value 7.819311
## iter 200 value 7.470625
## iter 210 value 7.079706
## iter 220 value 6.014087
## iter 230 value 5.864002
## iter 240 value 5.712689
## iter 250 value 5.580093
## iter 260 value 5.159917
## iter 270 value 4.594158
## iter 280 value 4.270858
## iter 290 value 4.186236
## iter 300 value 4.154694
## iter 310 value 4.119704
## iter 320 value 4.086693
## iter 330 value 4.020131
## iter 340 value 3.912762
## iter 350 value 3.792980
## iter 360 value 3.704825
## iter 370 value 3.602523
## iter 380 value 3.535595
## iter 390 value 3.402677
## iter 400 value 3.388947
## iter 410 value 3.363503
## iter 420 value 3.339187
## iter 430 value 3.320049
## iter 440 value 3.307455
## iter 450 value 3.299664
```

```
## iter 460 value 3.294740
## iter 470 value 3.283627
## iter 480 value 3.078584
## iter 490 value 2.969415
## iter 500 value 2.932845
## iter 510 value 2.921619
## iter 520 value 2.906949
## iter 530 value 2.903582
## iter 540 value 2.903046
## iter 550 value 2.902940
## final  value 2.902937
## converged
## # weights:  121
## initial  value 96.862793
## iter  10 value 31.497590
## iter  20 value 21.718098
## iter  30 value 20.965836
## iter  40 value 17.877590
## iter  50 value 14.848869
## iter  60 value 12.132508
## iter  70 value 10.985009
## iter  80 value 10.096513
## iter  90 value 8.182261
## iter 100 value 6.934332
## iter 110 value 6.264867
## iter 120 value 5.863540
## iter 130 value 5.638351
## iter 140 value 5.390163
## iter 150 value 5.181647
## iter 160 value 4.872326
## iter 170 value 4.484752
## iter 180 value 4.374744
## iter 190 value 4.290202
## iter 200 value 4.181903
## iter 210 value 3.898019
## iter 220 value 3.710759
## iter 230 value 3.559046
## iter 240 value 3.339499
## iter 250 value 3.168528
## iter 260 value 3.086051
## iter 270 value 3.075191
## iter 280 value 3.065400
## iter 290 value 3.054691
## iter 300 value 3.020475
## iter 310 value 2.955421
## iter 320 value 2.815745
## iter 330 value 2.708263
## iter 340 value 2.677898
## iter 350 value 2.665710
## iter 360 value 2.663833
## iter 370 value 2.661005
## iter 380 value 2.658407
## iter 390 value 2.653095
## iter 400 value 2.626713
```

```
## iter 410 value 2.603224
## iter 420 value 2.569680
## iter 430 value 2.515883
## iter 440 value 2.472712
## iter 450 value 2.441454
## iter 460 value 2.436701
## iter 470 value 2.433991
## iter 480 value 2.431994
## iter 490 value 2.416463
## iter 500 value 2.398028
## iter 510 value 2.359301
## iter 520 value 2.337855
## iter 530 value 2.329553
## iter 540 value 2.320774
## iter 550 value 2.312037
## iter 560 value 2.303680
## iter 570 value 2.283908
## iter 580 value 2.226264
## iter 590 value 2.192119
## iter 600 value 2.181047
## iter 610 value 2.176810
## iter 620 value 2.169837
## iter 630 value 2.168770
## iter 640 value 2.168691
## iter 650 value 2.168686
## final  value 2.168686
## converged
## # weights:  131
## initial  value 37.529864
## iter  10 value 23.586710
## iter  20 value 20.416409
## iter  30 value 17.318891
## iter  40 value 15.514952
## iter  50 value 13.166775
## iter  60 value 12.857605
## iter  70 value 12.256701
## iter  80 value 12.108954
## iter  90 value 11.026650
## iter 100 value 8.288022
## iter 110 value 8.057729
## iter 120 value 7.544979
## iter 130 value 6.217857
## iter 140 value 5.398703
## iter 150 value 4.879001
## iter 160 value 4.780622
## iter 170 value 4.684397
## iter 180 value 4.573743
## iter 190 value 4.312507
## iter 200 value 4.197879
## iter 210 value 3.992302
## iter 220 value 3.868681
## iter 230 value 3.747889
## iter 240 value 3.572990
## iter 250 value 3.353907
```

```
## iter 260 value 3.340914
## iter 270 value 3.319436
## iter 280 value 3.300343
## iter 290 value 3.257534
## iter 300 value 3.129942
## iter 310 value 3.046107
## iter 320 value 3.005502
## iter 330 value 2.974807
## iter 340 value 2.881479
## iter 350 value 2.781692
## iter 360 value 2.745437
## iter 370 value 2.733145
## iter 380 value 2.731115
## iter 390 value 2.727151
## iter 400 value 2.720631
## iter 410 value 2.714828
## iter 420 value 2.705064
## iter 430 value 2.697275
## iter 440 value 2.685564
## iter 450 value 2.681556
## iter 460 value 2.680214
## final  value 2.680170
## converged
## # weights:  141
## initial  value 46.156222
## iter  10 value 24.820777
## iter  20 value 16.720595
## iter  30 value 14.963309
## iter  40 value 13.275218
## iter  50 value 10.612602
## iter  60 value 8.057512
## iter  70 value 6.818419
## iter  80 value 6.320239
## iter  90 value 5.979272
## iter 100 value 5.758253
## iter 110 value 5.491711
## iter 120 value 4.739521
## iter 130 value 4.565935
## iter 140 value 4.516044
## iter 150 value 4.471305
## iter 160 value 4.123824
## iter 170 value 3.875906
## iter 180 value 3.820980
## iter 190 value 3.799350
## iter 200 value 3.791477
## iter 210 value 3.765665
## iter 220 value 3.645100
## iter 230 value 3.538530
## iter 240 value 3.492075
## iter 250 value 3.474163
## iter 260 value 3.457137
## iter 270 value 3.412021
## iter 280 value 3.309045
## iter 290 value 3.174541
```

```
## iter 300 value 3.114204
## iter 310 value 3.097236
## iter 320 value 3.082725
## iter 330 value 3.067314
## iter 340 value 2.981230
## iter 350 value 2.914920
## iter 360 value 2.832922
## iter 370 value 2.803403
## iter 380 value 2.787955
## iter 390 value 2.762491
## iter 400 value 2.755835
## iter 410 value 2.746471
## iter 420 value 2.728755
## iter 430 value 2.680033
## iter 440 value 2.632436
## iter 450 value 2.580853
## iter 460 value 2.561849
## iter 470 value 2.537492
## iter 480 value 2.524527
## iter 490 value 2.523768
## iter 500 value 2.523389
## iter 510 value 2.521862
## iter 520 value 2.520425
## iter 530 value 2.512420
## iter 540 value 2.486912
## iter 550 value 2.423941
## iter 560 value 2.389344
## iter 570 value 2.366406
## iter 580 value 2.350448
## iter 590 value 2.347447
## iter 600 value 2.347240
## iter 610 value 2.347135
## iter 620 value 2.347027
## iter 630 value 2.346116
## iter 640 value 2.343070
## iter 650 value 2.336780
## iter 660 value 2.332329
## iter 670 value 2.325810
## iter 680 value 2.302738
## iter 690 value 2.273972
## iter 700 value 2.258202
## iter 710 value 2.241141
## iter 720 value 2.226977
## iter 730 value 2.214926
## iter 740 value 2.210252
## iter 750 value 2.194582
## iter 760 value 2.178176
## iter 770 value 2.170035
## iter 780 value 2.167319
## iter 790 value 2.166348
## iter 800 value 2.166271
## iter 810 value 2.166265
## final  value 2.166264
## converged
```

```
## # weights:  151
## initial  value 37.172618
## iter  10 value 29.661110
## iter  20 value 21.940534
## iter  30 value 21.241441
## iter  40 value 20.715204
## iter  50 value 16.486033
## iter  60 value 12.962213
## iter  70 value 12.270024
## iter  80 value 11.078858
## iter  90 value 10.598051
## iter 100 value 9.292263
## iter 110 value 8.381466
## iter 120 value 7.803120
## iter 130 value 6.318636
## iter 140 value 5.546076
## iter 150 value 5.264574
## iter 160 value 5.187368
## iter 170 value 5.183158
## iter 180 value 5.120170
## iter 190 value 4.931917
## iter 200 value 4.873348
## iter 210 value 4.816200
## iter 220 value 4.801460
## iter 230 value 4.704372
## iter 240 value 4.676857
## iter 250 value 4.649772
## iter 260 value 4.623241
## iter 270 value 4.477019
## iter 280 value 4.378212
## iter 290 value 4.108723
## iter 300 value 3.846638
## iter 310 value 3.836436
## iter 320 value 3.815890
## iter 330 value 3.745128
## iter 340 value 3.636984
## iter 350 value 3.558361
## iter 360 value 3.501012
## iter 370 value 3.496094
## iter 380 value 3.489645
## iter 390 value 3.459684
## iter 400 value 3.412428
## iter 410 value 3.408743
## iter 420 value 3.402054
## iter 430 value 3.395594
## iter 440 value 3.374669
## iter 450 value 3.299490
## iter 460 value 3.219631
## iter 470 value 3.139575
## iter 480 value 3.116120
## iter 490 value 3.115862
## iter 500 value 3.115424
## iter 510 value 3.110912
## iter 520 value 3.092596
```

```
## iter 530 value 3.015043
## iter 540 value 2.969360
## iter 550 value 2.959117
## iter 560 value 2.954088
## iter 570 value 2.952264
## iter 580 value 2.943342
## iter 590 value 2.941409
## iter 600 value 2.932434
## iter 610 value 2.885959
## iter 620 value 2.847080
## iter 630 value 2.812294
## iter 640 value 2.801044
## iter 650 value 2.792271
## iter 660 value 2.782055
## iter 670 value 2.779945
## iter 680 value 2.750939
## iter 690 value 2.712307
## iter 700 value 2.711879
## iter 710 value 2.711394
## iter 720 value 2.710270
## iter 730 value 2.705683
## iter 740 value 2.697103
## iter 750 value 2.686486
## iter 760 value 2.679614
## iter 770 value 2.672599
## iter 780 value 2.666218
## iter 790 value 2.653150
## iter 800 value 2.645312
## iter 810 value 2.642171
## iter 820 value 2.641806
## iter 830 value 2.641753
## iter 840 value 2.641735
## iter 850 value 2.641693
## iter 860 value 2.641672
## final  value 2.641671
## converged
## # weights:  161
## initial  value 50.552446
## iter  10 value 31.423442
## iter  20 value 24.772332
## iter  30 value 23.304711
## iter  40 value 19.881669
## iter  50 value 18.431210
## iter  60 value 17.185744
## iter  70 value 13.922304
## iter  80 value 12.452470
## iter  90 value 11.624999
## iter 100 value 10.426296
## iter 110 value 9.985792
## iter 120 value 9.355511
## iter 130 value 8.799190
## iter 140 value 8.199019
## iter 150 value 6.995948
## iter 160 value 6.281608
```

```
## iter 170 value 5.840606
## iter 180 value 5.501869
## iter 190 value 5.114873
## iter 200 value 5.088435
## iter 210 value 5.002271
## iter 220 value 4.877869
## iter 230 value 4.718378
## iter 240 value 4.515906
## iter 250 value 4.336627
## iter 260 value 4.294989
## iter 270 value 4.265502
## iter 280 value 4.262081
## iter 290 value 4.248597
## iter 300 value 4.237754
## iter 310 value 4.225425
## iter 320 value 4.215807
## iter 330 value 4.213264
## iter 340 value 4.212705
## iter 350 value 4.204930
## iter 360 value 4.155772
## iter 370 value 4.056609
## iter 380 value 3.909626
## iter 390 value 3.837299
## iter 400 value 3.799416
## iter 410 value 3.731823
## iter 420 value 3.705305
## iter 430 value 3.674983
## iter 440 value 3.549640
## iter 450 value 3.337458
## iter 460 value 3.264347
## iter 470 value 3.129349
## iter 480 value 3.001087
## iter 490 value 2.851805
## iter 500 value 2.760125
## iter 510 value 2.682823
## iter 520 value 2.646216
## iter 530 value 2.621843
## iter 540 value 2.619641
## iter 550 value 2.614668
## iter 560 value 2.612487
## iter 570 value 2.608016
## iter 580 value 2.604822
## iter 590 value 2.602339
## iter 600 value 2.599187
## iter 610 value 2.591830
## iter 620 value 2.589746
## iter 630 value 2.587348
## iter 640 value 2.573690
## iter 650 value 2.568875
## iter 660 value 2.566559
## iter 670 value 2.555996
## iter 680 value 2.540573
## iter 690 value 2.536420
## iter 700 value 2.534374
```

```
## iter 710 value 2.533505
## iter 720 value 2.532479
## iter 730 value 2.531899
## iter 740 value 2.529878
## iter 750 value 2.523408
## iter 760 value 2.518331
## iter 770 value 2.515759
## iter 780 value 2.514117
## iter 790 value 2.505317
## iter 800 value 2.483601
## iter 810 value 2.463054
## iter 820 value 2.430511
## iter 830 value 2.420053
## iter 840 value 2.419069
## iter 850 value 2.419021
## iter 860 value 2.419020
## final  value 2.419020
## converged
## # weights:  171
## initial  value 146.195466
## iter  10 value 26.410658
## iter  20 value 24.961301
## iter  30 value 23.835261
## iter  40 value 19.627557
## iter  50 value 16.098133
## iter  60 value 15.873241
## iter  70 value 15.581062
## iter  80 value 14.291345
## iter  90 value 14.071093
## iter 100 value 13.846225
## iter 110 value 13.350082
## iter 120 value 11.958218
## iter 130 value 10.083953
## iter 140 value 8.705608
## iter 150 value 8.484901
## iter 160 value 8.436612
## iter 170 value 8.291662
## iter 180 value 8.253386
## iter 190 value 8.201939
## iter 200 value 8.139213
## iter 210 value 7.565086
## iter 220 value 6.933421
## iter 230 value 6.222452
## iter 240 value 6.129913
## iter 250 value 6.044067
## iter 260 value 5.787762
## iter 270 value 5.586783
## iter 280 value 5.258950
## iter 290 value 4.847511
## iter 300 value 4.453483
## iter 310 value 4.164855
## iter 320 value 3.923541
## iter 330 value 3.865740
## iter 340 value 3.828620
```

```
## iter 350 value 3.779735
## iter 360 value 3.602738
## iter 370 value 3.499521
## iter 380 value 3.495070
## iter 390 value 3.478138
## iter 400 value 3.426056
## iter 410 value 3.336440
## iter 420 value 3.224132
## iter 430 value 3.045504
## iter 440 value 2.828104
## iter 450 value 2.762511
## iter 460 value 2.719970
## iter 470 value 2.675753
## iter 480 value 2.619638
## iter 490 value 2.558296
## iter 500 value 2.524532
## iter 510 value 2.485680
## iter 520 value 2.450968
## iter 530 value 2.433659
## iter 540 value 2.430715
## iter 550 value 2.427739
## iter 560 value 2.427320
## iter 570 value 2.426220
## iter 580 value 2.412455
## iter 590 value 2.393259
## iter 600 value 2.351394
## iter 610 value 2.338400
## iter 620 value 2.325802
## iter 630 value 2.319090
## iter 640 value 2.305689
## iter 650 value 2.295081
## iter 660 value 2.286564
## iter 670 value 2.266858
## iter 680 value 2.227454
## iter 690 value 2.195780
## iter 700 value 2.190017
## iter 710 value 2.189256
## iter 720 value 2.189160
## iter 730 value 2.189155
## iter 740 value 2.189141
## iter 750 value 2.189041
## iter 760 value 2.187959
## iter 770 value 2.185572
## iter 780 value 2.177770
## iter 790 value 2.161067
## iter 800 value 2.150265
## iter 810 value 2.145032
## iter 820 value 2.140994
## iter 830 value 2.138785
## iter 840 value 2.137577
## iter 850 value 2.131726
## iter 860 value 2.092595
## iter 870 value 2.073381
## iter 880 value 2.067267
```

```
## iter 890 value 2.062032
## iter 900 value 2.061392
## iter 910 value 2.061307
## iter 920 value 2.061294
## final  value 2.061292
## converged
## # weights:  181
## initial  value 43.950760
## iter  10 value 31.186173
## iter  20 value 27.406196
## iter  30 value 21.430284
## iter  40 value 15.392620
## iter  50 value 14.918568
## iter  60 value 14.561504
## iter  70 value 11.357119
## iter  80 value 10.590981
## iter  90 value 9.668266
## iter 100 value 8.335527
## iter 110 value 7.133956
## iter 120 value 6.215047
## iter 130 value 5.883998
## iter 140 value 5.728765
## iter 150 value 5.599015
## iter 160 value 5.402997
## iter 170 value 4.985851
## iter 180 value 4.738902
## iter 190 value 4.616285
## iter 200 value 4.578070
## iter 210 value 4.564633
## iter 220 value 4.564163
## iter 230 value 4.559628
## iter 240 value 4.530682
## iter 250 value 4.507242
## iter 260 value 4.491856
## iter 270 value 4.345973
## iter 280 value 4.277486
## iter 290 value 4.113492
## iter 300 value 3.772873
## iter 310 value 3.553154
## iter 320 value 3.464716
## iter 330 value 3.456080
## iter 340 value 3.449390
## iter 350 value 3.435834
## iter 360 value 3.368348
## iter 370 value 3.171870
## iter 380 value 3.022850
## iter 390 value 2.948545
## iter 400 value 2.879961
## iter 410 value 2.839069
## iter 420 value 2.771331
## iter 430 value 2.667921
## iter 440 value 2.619678
## iter 450 value 2.498136
## iter 460 value 2.417268
```

```
## iter 470 value 2.389226
## iter 480 value 2.388160
## iter 490 value 2.386837
## iter 500 value 2.384524
## iter 510 value 2.379100
## iter 520 value 2.371340
## iter 530 value 2.360055
## iter 540 value 2.332382
## iter 550 value 2.295500
## iter 560 value 2.264170
## iter 570 value 2.242146
## iter 580 value 2.207929
## iter 590 value 2.190900
## iter 600 value 2.163046
## iter 610 value 2.114172
## iter 620 value 2.083597
## iter 630 value 2.068548
## iter 640 value 2.029693
## iter 650 value 2.012869
## iter 660 value 2.000426
## iter 670 value 1.993709
## iter 680 value 1.992357
## iter 690 value 1.991923
## iter 700 value 1.991779
## iter 710 value 1.991710
## iter 720 value 1.991697
## iter 730 value 1.991688
## iter 740 value 1.991686
## final  value 1.991686
## converged
## # weights:  191
## initial  value 49.988141
## iter  10 value 33.457627
## iter  20 value 31.904962
## iter  30 value 29.782955
## iter  40 value 21.420354
## iter  50 value 15.101430
## iter  60 value 11.633240
## iter  70 value 10.841488
## iter  80 value 10.410759
## iter  90 value 10.218485
## iter 100 value 10.205674
## iter 110 value 10.171838
## iter 120 value 9.499698
## iter 130 value 8.578709
## iter 140 value 8.343142
## iter 150 value 8.124658
## iter 160 value 7.824084
## iter 170 value 7.520510
## iter 180 value 7.280098
## iter 190 value 6.103504
## iter 200 value 4.978416
## iter 210 value 4.464336
## iter 220 value 4.197695
```

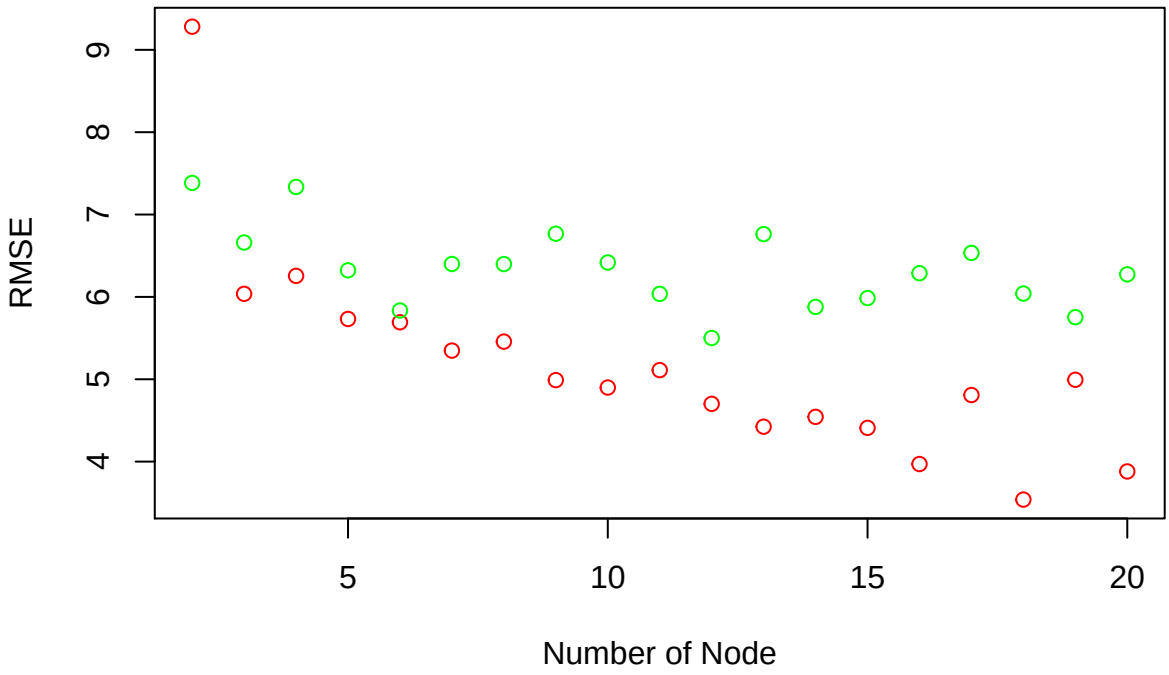
```
## iter 230 value 4.148779
## iter 240 value 4.082248
## iter 250 value 3.962558
## iter 260 value 3.745964
## iter 270 value 3.512584
## iter 280 value 3.353273
## iter 290 value 3.318771
## iter 300 value 3.290037
## iter 310 value 3.261674
## iter 320 value 3.227276
## iter 330 value 3.155467
## iter 340 value 3.049067
## iter 350 value 2.943119
## iter 360 value 2.883540
## iter 370 value 2.855099
## iter 380 value 2.774844
## iter 390 value 2.759403
## iter 400 value 2.746485
## iter 410 value 2.718415
## iter 420 value 2.664726
## iter 430 value 2.609444
## iter 440 value 2.526288
## iter 450 value 2.461017
## iter 460 value 2.369341
## iter 470 value 2.303564
## iter 480 value 2.277444
## iter 490 value 2.265784
## iter 500 value 2.262927
## iter 510 value 2.258444
## iter 520 value 2.232707
## iter 530 value 2.207351
## iter 540 value 2.185017
## iter 550 value 2.150638
## iter 560 value 2.121670
## iter 570 value 2.093849
## iter 580 value 2.071651
## iter 590 value 2.054914
## iter 600 value 2.045181
## iter 610 value 2.038931
## iter 620 value 2.038002
## iter 630 value 2.036243
## iter 640 value 2.028606
## iter 650 value 2.021864
## iter 660 value 2.012249
## iter 670 value 2.001628
## iter 680 value 1.993587
## iter 690 value 1.975943
## iter 700 value 1.966699
## iter 710 value 1.958928
## iter 720 value 1.918829
## iter 730 value 1.886388
## iter 740 value 1.881264
## iter 750 value 1.875390
## iter 760 value 1.868397
```

```
## iter 770 value 1.857275
## iter 780 value 1.847416
## iter 790 value 1.812931
## iter 800 value 1.789908
## iter 810 value 1.768539
## iter 820 value 1.742521
## iter 830 value 1.717560
## iter 840 value 1.699887
## iter 850 value 1.674664
## iter 860 value 1.662043
## iter 870 value 1.642237
## iter 880 value 1.632747
## iter 890 value 1.627536
## iter 900 value 1.625886
## iter 910 value 1.623546
## iter 920 value 1.613779
## iter 930 value 1.612979
## iter 940 value 1.612654
## iter 950 value 1.612537
## iter 960 value 1.612533
## iter 970 value 1.612522
## iter 980 value 1.612507
## iter 990 value 1.612484
## iter1000 value 1.612267
## iter1010 value 1.612034
## iter1020 value 1.611659
## iter1030 value 1.610899
## iter1040 value 1.610194
## iter1050 value 1.609199
## iter1060 value 1.607200
## iter1070 value 1.605725
## iter1080 value 1.604330
## iter1090 value 1.603868
## iter1100 value 1.603681
## iter1110 value 1.603662
## final  value 1.603662
## converged
## # weights:  201
## initial  value 50.972794
## iter  10 value 22.390413
## iter  20 value 17.629430
## iter  30 value 14.582005
## iter  40 value 13.620711
## iter  50 value 12.638221
## iter  60 value 12.304630
## iter  70 value 11.957113
## iter  80 value 11.546467
## iter  90 value 11.205793
## iter 100 value 10.812933
## iter 110 value 9.018339
## iter 120 value 8.368632
## iter 130 value 7.359567
## iter 140 value 6.693813
## iter 150 value 5.611403
```

```
## iter 160 value 5.530818
## iter 170 value 5.470679
## iter 180 value 5.352822
## iter 190 value 5.201194
## iter 200 value 5.158041
## iter 210 value 5.149355
## iter 220 value 5.146623
## iter 230 value 5.138637
## iter 240 value 5.044851
## iter 250 value 4.772164
## iter 260 value 4.633341
## iter 270 value 4.479187
## iter 280 value 4.316873
## iter 290 value 4.182275
## iter 300 value 4.055169
## iter 310 value 3.935984
## iter 320 value 3.795563
## iter 330 value 3.755054
## iter 340 value 3.745721
## iter 350 value 3.661741
## iter 360 value 3.543569
## iter 370 value 3.445951
## iter 380 value 3.389178
## iter 390 value 3.357591
## iter 400 value 3.279350
## iter 410 value 3.242608
## iter 420 value 3.225197
## iter 430 value 3.213879
## iter 440 value 3.172155
## iter 450 value 3.091789
## iter 460 value 3.018682
## iter 470 value 2.972536
## iter 480 value 2.936419
## iter 490 value 2.894831
## iter 500 value 2.857218
## iter 510 value 2.826649
## iter 520 value 2.808256
## iter 530 value 2.797241
## iter 540 value 2.780580
## iter 550 value 2.693415
## iter 560 value 2.595687
## iter 570 value 2.553533
## iter 580 value 2.538867
## iter 590 value 2.525139
## iter 600 value 2.517234
## iter 610 value 2.516452
## iter 620 value 2.515854
## iter 630 value 2.515175
## iter 640 value 2.511499
## iter 650 value 2.495326
## iter 660 value 2.452897
## iter 670 value 2.404656
## iter 680 value 2.369692
## iter 690 value 2.359602
```

```
## iter 700 value 2.352513
## iter 710 value 2.339551
## iter 720 value 2.265875
## iter 730 value 2.094471
## iter 740 value 1.996896
## iter 750 value 1.968258
## iter 760 value 1.952684
## iter 770 value 1.934401
## iter 780 value 1.929993
## iter 790 value 1.928398
## iter 800 value 1.923464
## iter 810 value 1.915281
## iter 820 value 1.907505
## iter 830 value 1.906311
## iter 840 value 1.906102
## iter 850 value 1.905931
## iter 860 value 1.905787
## iter 870 value 1.905472
## iter 880 value 1.905266
## iter 890 value 1.905220
## iter 900 value 1.905217
## final  value 1.905216
## converged
```

```r
# Make a plot
plot(2:20, RMSE.train, xlab='Number of Node', ylab='RMSE',col='red')
points(2:20, RMSE.test, col='green')
```



d)

I think there is no obvious overfitting. We can tell that the difference between the RMSE for training data and the RMSE for test data is not very big on average, and there is no tendency that the test RMSE increases from certain low point, it just goes up and down.