

HOMEWORK PROBLEMS 10, ONLY 561, FALL 2018

DUE 11/18/18

Exercises:

1. Use the code from Lecture 03 to generate a dataset $\{\mathbf{x}^{(i)}\}_{i=0}^{19} \subset \mathbb{R}^2$ with labels $\{y^{(i)}\}_{i=0}^{19} \in \{-1, 1\}$ such that $\mathbf{x}^{(i)}$ is drawn uniformly from the unit disc centered at $(-2, -2)$ and $y^{(i)} = -1$ for $i = 0, \dots, 9$, and $x^{(i)}$ is drawn uniformly from the unit disc centered at $(2, 2)$ and $y^{(i)} = +1$ for $i = 10, \dots, 19$.

- (a) Code up a Phase I method to find an interior point for the (3D) program

$$\min_{\mathbf{v} \in \mathbb{R}^2, b \in \mathbb{R}} \frac{1}{2} \|\mathbf{v}\|^2 \text{ subject to } h_i(\mathbf{v}, b) \leq 0$$

where $h_i(\mathbf{v}, b) = 1 - y^{(i)}(\mathbf{v}^T \mathbf{x}^{(i)} - b)$. That is, use the log barrier method to solve the 4D program

$$\min_{\mathbf{v} \in \mathbb{R}^2, b \in \mathbb{R}, z \in \mathbb{R}} \tilde{f}(\mathbf{v}, b, z) \text{ subject to } \tilde{h}_i(\mathbf{v}, b, z) \leq 0$$

where $\tilde{f}(\mathbf{v}, b, z) = z$ and $\tilde{h}_i(\mathbf{v}, b, z) = 1 - y^{(i)}(\mathbf{v}^T \mathbf{x}^{(i)} - b) - z$, but stop as soon as you find an iterate $(\mathbf{v}^{(k)}, b^{(k)}, z^{(k)})$ such that $z^{(k)} < 0$ (which then implies that $(\mathbf{v}^{(k)}, b^{(k)})$ is an interior point). Initialize the Phase I method with the data $\mathbf{v}^{(0)} = (-20, 20)$, $b^{(0)} = 10$, and determine $z^{(0)}$ (in your code) such that $(\mathbf{v}^{(0)}, b^{(0)}, z^{(0)})$ is an interior point of the 4D convex program.

- (b) Use the $(\mathbf{v}^{(k)}, b^{(k)})$ from part (a) to initialize Phase II for your generated data. That is, use this interior point to initialize the log barrier method for the 3D program. Use 3 centering steps, $M = 10$, and 5 iterations in the outer loop with 2 inner loop iterations each. For each backtracking step, use Newton search directions, $\alpha = 0.1$, and $\beta = 0.5$.
- (c) Use the $(\mathbf{v}^{(k)}, b^{(k)})$ from part (a) to initialize the primal-dual algorithm. Take 10 steps with $\nu = 10$.
- (d) Compare the answers you get from parts (b) and (c), and provide a simultaneous plot of your data, the separating line corresponding to the interior point $(\mathbf{v}^{(k)}, b^{(k)})$ from part (a), and the two separating lines from parts (b) and (c).