HOMEWORK PROBLEMS 08, ANLY 561, FALL 2018
DUE 11/03/18

Exercises:

1. This example illustrates the need for constraint qualifications. Consider the maximization program

$$\max_{x \in \mathbb{R}^2} x_1 \text{ subject to } x_2 + (x_1 - 1)^3 \le 0,\ x_1 \ge 0,\ x_2 \ge 0.$$

   (a) Show that the Linear Independence Constraint Qualification fails at $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (b) Write down the KKT conditions for this problem.

   (c) Show that $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ does not satisfy the KKT conditions.

   (d) Show that $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the solution to this program.

2. Consider the minimization program

$$\min_{x \in \mathbb{R}^2} 3x_1^2 - 2x_1x_2 + 3x_2^2 - 2x_2x_3 + 3x_3^2 \text{ subject to } 0 \le x_1 \le 1,\ 0 \le x_2 \le 1,\ 0 \le x_3 \le 1.$$

   (a) By verifying some constraint qualification, show that the solution to this program must satisfy the KKT conditions.

   (b) Write down the KKT conditions for this problem.

   (c) Find all points satisfying the KKT conditions.

   (d) Determine the solution to the minimization program.