HOMEWORK PROBLEMS 02, ANLY 561, FALL 2018
DUE 09/22/18

Readings: Continue reading Lecture 01 Notes; Goodfellow and Bengio, Chapter 2; and Chapter 2 from `https://jakevdp.github.io/PythonDataScienceHandbook/`

Exercises:

1. Compute (by hand and showing your work) $x^{(1)}$ and $x^{(2)}$ of backtracking with steepest descent increments for $f(x) = x^2$ with $\alpha = \beta = 1/2$ and starting at $x^{(0)} = 1$. You may use the Python implementation of backtracking to check your work and that inequalities hold, but your answers should be exact.

2. For $f \in C^2(\mathbb{R})$, show that the Newton update $x^{(k+1)} = x^{(k)} - f'(x^{(k)})/f''(x^{(k)})$ coincides with the critical point of the second order Taylor approximation to f when $f''(x^{(k)}) \neq 0$:

$$g(x) = f(x^{(k)}) + f'(x^{(k)})(x - x^{(k)}) + \frac{1}{2} f''(x^{(k)})(x - x^{(k)})^2$$

   If f is also convex, what more can be said about this critical point?

3. Using the Python implementations of backtracking and accelerated backtracking provided in class, compute, display, and comment on the convergence of the function values and the iterates when minimizing the following functions using increments from both steepest descent and Newton's method:

   (a) $f(x) = e^x/x$

   (b) $f(x) = \log(x^2+1) + \log((x-1)^2+1) + \log\left(\left(x - \frac{3}{2}\right)^2 + 1\right)$

   (c) $f(x) = -\frac{1}{x^2+1} - \frac{1}{(x-1)^2+1} - \frac{1}{\left(x-\frac{3}{2}\right)^2+1}$

   Use three different starting points to initialize the different iterations for comparison: $x^{(0)} = 10, 100, 1000$, and run 50 iterations in every case.