A General Transmission Scheme for Bi-Directional Communication by Using Eigenmode Sharing

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***Abstract*—In this paper, we develop a general transmission scheme for bi-directional communications by using eigenmode sharing, where existing two-way relaying protocols can be viewed as a special case of the proposed transmission scheme. In addition, the proposed bi-directional scheme can also be applied to more challenging scenarios with more than one source pair. Asymptotical behavior of the outage probability achieved by the proposed transmission protocol is studied in order to obtain insightful understandings for the fundamental limits of the proposed scheme. Our developed results show that the proposed bi-directional transmission scheme can realize larger system throughput than time sharing based approaches, and serving more than one pair at the same time is more beneficial than simple two-way relaying in terms of multiplexing gains.**

***Index Terms*—Two-way relaying, multi-way relaying, precoding, diversity, network coding.**

I. Introduction

BI-DIRECTIONAL transmission is one of the fundamental building blocks in modern communications systems. Cellular networks are a typical example for such bi-directional communications, where a voice call is originated from a mobile user and the user is also expecting to receive the same amount of voice data from its partner at the same time. Similarly such a bi-directional patten can also be found in many data communication networks. For example, during a point-to-point protocol based data session, two nodes are sharing their files with each other and data streams are flowing to the both directions simultaneously. Traditionally time sharing approaches have been used for such bi-directional communications, where orthogonal channels, such as time slots, frequency channels, or pseudo-noise codes, are allocated to transmissions at different directions. Such time sharing approaches suffer loss of system throughput and reception reliability, particularly for the cases with multiple pairs of source nodes and multi-hop transmissions.

A recent breakthrough to improve the spectral efficiency for bi-directional transmissions has emerged from the extensive studies of two-way relaying channel, which is a special case of bi-directional communications with one pair of sources exchanging information via a relay [1]–[4]. The idea of network coding, a concept originally developed for routing problems, has been applied to two-way relaying channel. Specifically the bi-directional transmission can be divided into two phases. During the first phase, both two sources transmit their messages to the relay, and during the second phase the relay broadcasts its observation which is the mixture of the two source messages. Each destination can first subtract its own message from the mixture and then detect the message from its partner. As a result, only two time slots are required, whereas time sharing approaches require four time slots. In [1], [3], it has been considered that nodes are equipped with a single antenna, where the impact of relay selection has been studied in [4]. In [5], [6] the scenarios with multiple-antenna nodes have been studied and the achievable diversity-multiplexing tradeoff has been developed. Differential modulation has been applied to two-way relaying channel to avoid the strong assumption of channel state information (CSI) [7], [8]. In [9]–[11] the optimal design of distributed beamforming has been studied in the context of bi-directional communication scenarios with a single pair of sources, where the sum rate has been the focus. On the other hand, there are few works to study bi-directional communications in a more general setup. In [12] multi-way relaying has been studied and the achievable rates have been developed. In [13] uni-directional interference relay channel has been studied, where the design of precoding matrices has been investigated. Note that multi-way relaying is more challenging than two-way relaying due to the existence of co-channel interference.

The main contribution of this paper is to develop a general scheme for bi-directional communications, where both two-way relaying and multi-way relaying can be fit into such a general framework. Specifically consider that there are M pairs of source nodes exchanging information with each other via a relay. Note that each source only has one corresponding destination, whereas the existing work [12] considers a different scenario where each source communicates multiple destinations. It is assumed that each source node is equipped with N antennas and the relay has L antennas. The key idea of the proposed transmission protocol is to apply carefully designed precoding at the sources, which ensures that the resultant virtual channels associated with the same pair share the same eigenmode. Furthermore, inter-pair interference, which is the co-channel interference caused by the transmissions from different pairs, can be efficiently mitigated by using

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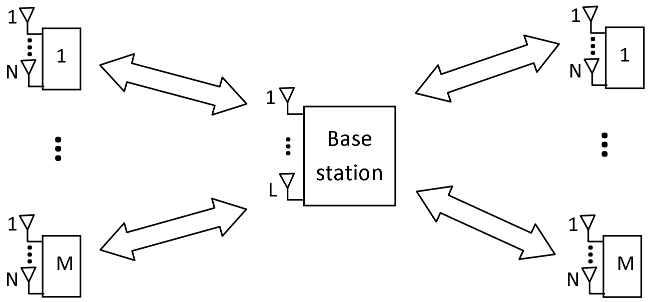

Fig. 1. A diagram for the addressed two-way communication scenario.

beamforming at the relay. It can be easily shown that the original physical layer network coding scheme proposed in [1] can be viewed as a special case of our proposed transmission protocol when $M = N = L = 1$.

The second contribution of this paper is to provide a few case studies for different choices of the system parameters. The first scenario studied in the paper is two-way relaying channel, and then we move to the scenario with multiple pairs of source nodes. Analytical results, such as outage probability and diversity-multiplexing tradeoff, have been developed for the proposed transmission protocols. Recall time sharing approaches will require four time slots to exchange information between the two sources and $4M$ time slots for the case with M pairs. The proposed transmission protocol can always realize the information exchange within two time slots, no matter how many pairs are involved. Our developed analytical results have also shown that it is beneficial to serve more than one user pair in most cases. For example, most existing two-way relaying protocols, including the one proposed in this paper, can only achieve the multiplexing gain N, when the number of the relay antennas is larger than the number of source antennas $N \leq L$. On the other hand, multi-way relaying channel invites more than one user pair for cooperation and can achieve the multiplexing gain L, the maximum value as pointed out by the max-flow min-cut theorem. In addition, multi-way relaying is also helpful to reduce the delay experienced by each node pair, whereas two-way relaying protocols can only serve one pair at each time and therefore result in severe unfairness among the multiple pairs.

II. Signal Model by Using Eigen-mode Sharing

Consider a bi-directional communication scenario as shown in Fig. 1 where M pair of nodes exchange information with their partners via a relay. The relay is equipped with L antennas and all the other nodes are equipped with N antennas. It is assumed $N \leq L < 2N$ which is clarified as the following

- For two-way relaying channel, $L \geq 2N$ means that the relay has enough degrees of freedom to support all incoming $2N$ messages from the two sources, where traditional beamforming can be straightforwardly used [14], [15]. In this paper, we only consider the more challenging scenarios with $L < 2N$ where traditional beamforming cannot be used directly.
- The assumption of $N \leq L$ is made only to simplify the notations, where the dimensions of the matrices from the eigenvalue decomposition can be expressed more simply. In practice, it is reasonable to assume $N \leq L$ since the supernode, e.g. the relay, might have better capabilities than the others.

It is important to point out that the proposed protocol can realize multi-way relaying with a moderate assumption for the relay and user antennas, whereas many existing works, such as interference alignment [16], requires much more demanding assumptions in order to have sufficient degrees of freedom. The half-duplexing constraint is applied to all nodes. All wireless channels are considered to be identically and independent quasi-static Rayleigh fading. It is assumed that the relay, i.e the base station in a cellar network or a controller in a sensor network, has access to the global CSI. Each source only has access to the intra-pair CSI, i.e the channels between itself and the relay and the one between its partner and the relay. The knowledge of inter-pair CSI, i.e channels associated to other pairs, is not required, which reduces the system overhead significantly. Furthermore, time division duplexing will be used, where the symmetry between incoming and outgoing channels will also be helpful to reduce the system overhead.

A. Description of the proposed protocol

1) Multiple access phase: Prior to transmission, each source will choose an appropriate precoder, $\mathbf{P}_m$, and the design of such precoding will be discussed later. During the first time slot, all sources are invited to transmit their messages to the relay simultaneously. Specifically, the two transmitters within the m-th pair will send $\mathbf{P}_m\mathbf{s}_m$ and $\mathbf{P}_{m'}\mathbf{s}_{m'}$, where the two sources within one pair are denoted as m and m', $\mathbf{P}_m$ is a $N \times x_m$ precoding matrix and $\mathbf{s}_m$ is a $x_m \times 1$ information bearing vector. To make the proposed transmission protocol more general, we keep the number of symbols transmitted by each source, x_m, flexible since some sources may transmit more than one stream particularly for the case $M < L$.

At the end of the multiple access phase, the relay receives

$$\mathbf{y}_r = \sum_{m=1}^{M} \left(\mathbf{H}_m\mathbf{P}_m\mathbf{s}_m + \mathbf{H}_{m'}\mathbf{P}_{m'}\mathbf{s}_{m'}\right) + \mathbf{n}_r, \qquad (1)$$

where $\mathbf{y}_r$ is the $L \times 1$ observation vector and, $\mathbf{H}_m$ is the channel matrix between the m-th user and the relay, $\mathbf{n}_r$ is the $L \times 1$ additive white Gaussian noise vector. An observation from (1) is that the information from the m-th pair, i.e. $\mathbf{s}_m$ and $\mathbf{s}_{m'}$, is viewed as interference by other pairs. Due to such cross-pair interference, traditional two-way relaying approaches, such as the ones in [14], [15], require that the number of the relay antennas needs to be at least larger than $\sum_{m=1}^{M}(x_m + x_{m'})$, so the relay can have sufficient degrees of freedom to accommodate these incoming signals. In the following we propose a more efficient approach which avoids such a demanding requirement.

The key idea for eigenmode sharing is to first convert the two virtual channels within the same pair, $\mathbf{H}_m$ and $\mathbf{H}_{m'}$, into a single one and then apply singular value decomposition to the virtual channel matrix. So first define the $N \times (2N - L)$ matrices $\mathbf{Q}_m$ and $\mathbf{Q}_{m'}$ which ensure eigenmode sharing between the two channels associated to the same pair

$$\mathbf{H}_m\mathbf{Q}_m = \mathbf{H}_{m'}\mathbf{Q}_{m'}. \tag{2}$$

The matrices $\mathbf{Q}_m$ can be obtained from the null space of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$, and for simplicity the normalized null vectors are used, which means $\mathbf{Q}_{m'}^H\mathbf{Q}_{m'} + \mathbf{Q}_{m'}^H\mathbf{Q}_{m'} = \mathbf{I}_{2N-L}$. Or in other words, the columns of $\begin{bmatrix}\mathbf{Q}_m^T & \mathbf{Q}_{m'}^T\end{bmatrix}^T$ are the base vectors of the null space of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$. Since the dimension of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$ is $L\times 2N$, the dimension of its null space is $2N-L$. By using $\mathbf{Q}_m$ the eigenmode sharing precoding matrices $\mathbf{P}_m$ can be designed as

$$\mathbf{P}_m = \mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m, \quad \&, \quad \mathbf{P}_{m'} = \mathbf{Q}_{m'}\mathbf{V}_m\mathbf{D}_m, \tag{3}$$

where the $(2N-L)\times(2N-L)$ matrices $\mathbf{V}_m$ are from the singular value decomposition $\mathbf{H}_m\mathbf{Q}_m = \mathbf{H}_{m'}\mathbf{Q}_{m'} = \mathbf{U}_m\tilde{\mathbf{\Lambda}}_m\mathbf{V}_m^H$,, the dimensions of $\tilde{\mathbf{\Lambda}}_m$ and $\mathbf{U}_m$ are $L\times(2N-L)$ and $L\times L$ respectively, the $(2N-L)\times x_m$ matrix $\mathbf{D}_m$ is to combine or select the best x_m eigenvalues, as discussed in the following.

By using such a precoding design, the original signal model at the relay can be simplified as

$$\mathbf{y}_r = \sum_{m=1}^{M}\mathbf{U}_m\mathbf{\Lambda}_m(\mathbf{s}_m + \mathbf{s}_{m'}) + \mathbf{n}_r \tag{4}$$

where $\mathbf{\Lambda}_m = \tilde{\mathbf{\Lambda}}_m\mathbf{D}_m$ is a $L\times x_m$ matrix. As can be observed from the new signal model in (4), the messages from the users within one pair have been grouped together due to the use of eigenmode sharing. The idea of eigen mode sharing is quite similar to interference alignment, a technology mainly proposed for uni-directional transmissions [16]. From physical layer network coding [1], we learn that the relay can treat the mixture $(\mathbf{s}_m + \mathbf{s}_{m'})$ as a single message and such intra-pair interference can be perfectly cancelled at the destinations. As a result, the dimension of the signals to be detected at the relay has been reduced from $\sum_{m=1}^{M}(x_m + x_{m'})$ to $\sum_{m=1}^{M}x_m$. Here we assume a symmetrical setup, i.e. $x_m = x_{m'}$. Hence the requirement for the number of relay antennas according to the new signal model is only $L \geq \sum_{m=1}^{M}x_m$, whereas the approach without eigenmode sharing requires twice relay antennas as shown in (1). It is important to point out that the choice of $\mathbf{D}_m$ should ensure that the transmission power at the sources constrainted, which will be discussed in the next sections.

2) Broadcasting phase: According to the max-flow min-cut theorem, the total number of the data streams for the addressed scenario is constrained as the following

$$x_1 + \cdots + x_M \leq \min\{L, M(2N-L)\}.$$

Prior to broadcasting, the relay tries to manipulate its observations, where there are a few different forwarding strategies. Due to its simplicity, the amplify-forward strategy will be used at the relay.

Specifically, during the second time slot, the relay applies a $L\times L$ precoding matrix, $\mathbf{X}$, to its observation and broadcasts the output of the precoder, $\mathbf{X}\mathbf{y}_r$. Such a precoding matrix $\mathbf{X}$ is to realize two functions, avoiding co-channel interference from other pairs and ensuring the transmission power constrained. The details about the design of the precoding matrix $\mathbf{X}$ will be discussed in the next section, and it is important to note that such a precoder needs to ensure that the transmission power constraint at the relay is met

$$\mathcal{E}\left\{\text{tr}\left\{\mathbf{X}\mathbf{y}_r\mathbf{y}_r^H\mathbf{X}^H\right\}\right\} \leq 1.$$

So at the m-th destination, the signal model can be written as

$$\mathbf{y}_m = \mathbf{H}_m^H\mathbf{X}\sum_{i=1}^{M}\mathbf{U}_i\mathbf{\Lambda}_i(\mathbf{s}_i + \mathbf{s}_{i'}) + \mathbf{H}_m^H\mathbf{X}\mathbf{n}_r + \mathbf{n}_m. \tag{5}$$

For simplicity, we apply the same matrices, $\mathbf{Q}_m$, $\mathbf{V}_m$ and $\mathbf{D}_m$ at the receiver and obtain

$$\begin{aligned}\tilde{\mathbf{y}}_m &= \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{H}_m^H\mathbf{X}\sum_{i=1}^{M}\mathbf{U}_i\mathbf{\Lambda}_i(\mathbf{s}_i + \mathbf{s}_{i'}) \\ &\quad + \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\left(\mathbf{H}_m^H\mathbf{X}\mathbf{n}_r + \mathbf{n}_m\right),\end{aligned} \tag{6}$$

where $\tilde{\mathbf{y}}_m = \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{y}_m$. After some algebraic manipulations, we can eventually express the signal model at the m-th receiver as (7). As can be seen from the above equation, the receiver observes a mixture of desirable signals, interference from other pairs and amplified noise. To achieve interference-free communications, the precoding matrices, $\mathbf{X}$ and $\mathbf{D}_m$, should be carefully designed, as shown in the next sections.

III. Two-Way Relaying Communications

Among M source pairs, it is considered in this section that only one pair will be served during each time slot. The scheduling of these multiple pairs can be completed by using round robin approaches which achieve perfect fairness but poor throughput, or using opportunistic protocols which improve system throughput but reduce fairness. The impact of such two approaches on the system performance will be investigated in the following two subsections respectively.

A. Random choices of a user pair

Without loss of generality, consider the m-th pair to be served. Since only one pair will be served during each time slot, there is no co-channel interference. So the signal model in (7) can be simplified as

$$\begin{aligned}\tilde{\mathbf{y}}_m = \mathbf{\Lambda}_m^H\mathbf{U}_m^H\mathbf{X}\mathbf{U}_m\mathbf{\Lambda}_m(\mathbf{s}_m + \mathbf{s}_{m'}) + \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H \\ \times\left(\mathbf{H}_m^H\mathbf{X}\mathbf{n}_r + \mathbf{n}_m\right).\end{aligned} \tag{8}$$

For such a simple case, the precoding matrix at the source can be chosen as $\mathbf{D}_m = \frac{1}{\sqrt{x_m}}\begin{bmatrix}\mathbf{I}_{x_m}^T & \mathbf{0}_{(2N-L-x_m)\times x_m}^T\end{bmatrix}^T$, so only the x_m largest eigenvalues of $\mathbf{H}_m\mathbf{Q}_m$ will be used, $x_m \leq \min\{L, 2N-L\}$. Note that $\min\{L, 2N-L\} = 2N-L$ since $N \leq L$ is assumed in this paper. The use of such a precoding matrix can ensure the overall transmission power of one user pair is constrained as

$$\begin{aligned}P_{ow}^s = \text{tr}\left\{\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H + \mathbf{Q}_{m'}\mathbf{V}_m\mathbf{D}_m \right. \\ \left. \times\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_{m'}^H\right\} = \text{tr}\left\{\mathbf{D}_m^H\mathbf{D}_m\right\} = 1.\end{aligned}$$

Given the fact that there is only a single pair of sources, there is no cross-pair interference and hence the precoding matrix $\mathbf{X}$ only needs to realize the relay transmission power constrained,

$$\tilde{\mathbf{y}}_m = \underbrace{\mathbf{\Lambda}_m^H \mathbf{U}_m^H \mathbf{X} \mathbf{U}_\mathbf{m} \mathbf{\Lambda}_m \left(\mathbf{s}_m + \mathbf{s}_{m'}\right)}_{desirable \quad signals} + \underbrace{\mathbf{\Lambda}_m^H \mathbf{U}_m^H \mathbf{X} \sum_{i=1, i\neq m}^{M} \mathbf{U}_i \mathbf{\Lambda}_i \left(\mathbf{s}_i + \mathbf{s}_{i'}\right)}_{interference} + \underbrace{\mathbf{D}_m^H \mathbf{V}_m^H \mathbf{Q}_m^H \left(\mathbf{H}_m^H \mathbf{X} \mathbf{n}_r + \mathbf{n}_m\right)}_{noise}. \quad (7)$$

which means the precoding matrix $\mathbf{X}$ can be designed simply as $\mathbf{X} = \frac{\mathbf{I}_L}{\sqrt{\alpha}}$, where α is the power of the observations

$$\alpha = \text{tr}\left\{\mathbf{y}_r \mathbf{y}_r^H\right\} \approx 2 \text{ tr}\left\{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m\right\}, \quad (9)$$

where the approximation is obtained at high SNR. By using such precoding matrices, we can express the signal model at the m-th receiver as

$$\tilde{\mathbf{y}}_m = \frac{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m}{\sqrt{\alpha}} \left(\mathbf{s}_m + \mathbf{s}_{m'}\right) + \mathbf{D}_m^H \mathbf{V}_m^H \mathbf{Q}_m^H \left(\frac{\mathbf{H}_m^H \mathbf{n}_r}{\sqrt{\alpha}} + \mathbf{n}_m\right).$$

So the mutual information from the m'-th source to the m-th node can be expressed as

$$\mathcal{I}_{TW,m} = \log\det\left(\mathbf{I}_{x_m} + \rho\frac{1}{\alpha}(\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m)^2 \left(\frac{1}{\alpha}\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m + \mathbf{D}_m^H \mathbf{V}_m^H \mathbf{Q}_m^H \mathbf{Q}_m \mathbf{V}_m \mathbf{D}_m\right)^{-1}\right), \quad (10)$$

where ρ denotes the signal-to-noise ratio (SNR). It is interesting to observe that the mutual information from the m-th source to the m'-th node is exactly the same as the above expression, $\mathcal{I}_{TW,m} = \mathcal{I}_{TW,m'}$. By using the fact $\mathbf{Q}_m \mathbf{Q}_m^H + \mathbf{Q}_{m'} \mathbf{Q}_{m'}^H = \mathbf{I}_{2N-L}$ we can easily obtain the upper and lower bounds of the mutual information as

$$\log\det\left(\mathbf{I}_{x_m} + \rho\frac{1}{\alpha}(\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m)^2 \left(\frac{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m}{\alpha} + \frac{\mathbf{I}_{2N-L}}{x_m}\right)^{-1}\right) \leq \mathcal{I}_{TW,m} \leq \log\det\left(\mathbf{I}_{x_m} + \rho \mathbf{\Lambda}_m^H \mathbf{\Lambda}_m\right). \quad (11)$$

By using the fact that $\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m$ is a diagonal matrix, the above expression can be simplified as

$$\log\prod_{i=1}^{x_m}\left(1 + \rho\frac{\lambda_{(i)}^2}{\frac{\alpha}{x_m} + \lambda_{(i)}}\right) \leq \mathcal{I}_{TW,m} \leq \log\prod_{i=1}^{x_m}\left(1 + \rho\lambda_{(i)}\right),$$

where $\lambda_{(i)}$ denotes the i-th largest eigenvalue of $\mathbf{H}_m \mathbf{Q}_m \mathbf{Q}_m^H \mathbf{H}_m^H$. If the joint distribution of these eigenvalues is known, a theorem about the outage probability for the m-th user pair can be obtained straightforwardly as the following.

Theorem *1:* With a random pair selection, the outage probability for the m-th user pair can be bounded as (12), where $g(\lambda_{(1)}, \cdots, \lambda_{(x_m)})$ is the joint probability density function of $\lambda_{(i)}$.

Recall that $\lambda_{(i)}$ are the ordered eigenvalues of $\mathbf{H}_m \mathbf{Q}_m$. If $\mathbf{H}_m \mathbf{Q}_m$ is complex Gaussian distributed, the distribution of these eigenvalues can be easily obtained. Surprisingly we find out that $\mathbf{H}_m \mathbf{Q}_m$ is indeed a classical complex Gaussian matrix as shown in the following proposition.

Proposition 2: [17] Consider the two $L \times N$ complex Gaussian matrices, $\mathbf{H}_m$ and $\mathbf{H}_{m'}$, and $2N - L \geq 1$. $\mathbf{Q}_m$ is obtained as in (2). The matrix $\mathbf{H}_m \mathbf{Q}_m$ is a complex Gaussian matrix with zero mean and variance $\frac{1}{4}$, i.e $\mathbf{H}_m \mathbf{Q}_m \sim CN(0, \frac{1}{4}\mathbf{I}_L)$.

By using such a proposition, the joint probability density function (pdf) function in Theorem 1 can be replaced with the joint density function of x_m largest eigenvalues of a Wishart matrix. As a result, more explicit closed-form results can be obtained as the following.

1) The power normalization factor α: Recall that the power normalization factor α at the relay is expressed as $\alpha = 2\mathcal{E}\left\{\text{tr}\left\{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m\right\}\right\}$. And such a factor is bounded by a constant as

$$\mathcal{E}\left\{\text{tr}\left\{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m\right\}\right\} \leq \mathcal{E}\left\{\text{tr}\left\{\mathbf{H}_m \mathbf{Q}_m \mathbf{Q}_m^H \mathbf{H}^H\right\}\right\} = L(2N - L),$$

where the first inequality follows from the fact that all eigenvalues are non-negative and the last one is obtained from the trace property of a Wishart matrix [18].

2) When $x_m = 1$: In such a case, only one data stream will be transmitted by each source. As a result, the bounds for the mutual information can be simplified as

$$\log\left(1 + \rho\frac{\lambda_{(1)}^2}{\alpha + \lambda_{(1)}}\right) \leq \mathcal{I}_{TW,m} \leq \log\left(1 + \rho\lambda_{(1)}\right). \quad (13)$$

Furthermore the power normalization factor can be simplified as $\alpha \approx 2 \text{ tr}\left\{\mathbf{\Lambda}_m^H \mathbf{\Lambda}_m\right\} = 2\lambda_{(1)}$. By using the distribution of the largest eigenvalue of a Wishart matrix, we can obtain the following lemma.

Lemma *3:* When each source only transmits one data stream, i.e $x_m = 1$, the outage probability for the proposed transmission protocol can be approximated as

$$P(\mathcal{I}_{TW,m} < R) \doteq \frac{1}{\rho^{L(2N-L)}}.$$

Proof: Please refer to the appendix. ■

From the lemma, we observe that the diversity gain achievable for the proposed transmission protocol will be $L(2N-L)$, and this is in line with our intuition since the simplified signal model after applying eigenmode sharing is analog to a $L \times (2N - L)$ MIMO system.

3) The maximum multiplexing gain: Recall that the mutual information for the m-th user can be bounded as the following

$$\mathcal{I}_{TW,m} \leq \log\prod_{i=1}^{x_m}\left(1 + \rho\lambda_{(i)}\right). \quad (14)$$

Note that $\lambda_{(i)}$ is the i-th largest eigenvalue from the matrix $\mathbf{H}_m \mathbf{Q}_m$ which is a $L \times (2N - L)$ classical complex Gaussian matrix, as stated in Proposition 2. Therefore, the upper bound for the achievable multiplexing gain is $\min\{L, 2N - L\} = 2N - L$ according to the results in [19].

B. Opportunistic Scheduling of User Pairs

For two-way relaying channel, only a single user pair will be selected for transmission at each time among all M pairs. So it is interesting to study which user should be scheduled

$$\int\cdots\int_{\prod_{i=1}^{x_m}\left(1+\rho\frac{\lambda_{(i)}^4}{1+\lambda_{(i)}^2}\right)<2^{2x_mR}} \log\prod_{i=1}^{x_m}\left(1+\rho\frac{\lambda_{(i)}^2}{\frac{\alpha}{x_m}+\lambda_{(i)}}\right)g(\lambda_{(1)},\cdots,\lambda_{(x_m)})d\lambda_1\cdots d\lambda_{x_m} \geq P\left(\mathcal{I}_{TW}<2x_mR\right) \tag{12}$$
$$\geq \int\cdots\int_{\prod_{i=1}^{x_m}\left(1+\rho\lambda_{(i)}^2\right)<2^{2x_mR}} \log\log\prod_{i=1}^{x_m}\left(1+\rho\lambda_{(i)}\right)g(\lambda_{(1)},\cdots,\lambda_{x_m})d\lambda_1\cdots d\lambda_{(x_m)},$$

first. The simplest scheduling method is based on round robin approaches, where each pair will be scheduled equally. Such a round robin method can achieve the best fairness among all pairs, but result in poor system throughput and reliability. On the other hand, opportunistic scheduling can utilize multi-user diversity gains and yield significant improvement in throughput and reliability. The criterion for the opportunistic group selection can be simply based on the mutual information of each pair and the scheduling problem can be formulated as the following optimization problem

$$\arg\max_m \quad \log\det\left(\mathbf{I}_{x_m}+\rho\frac{1}{\alpha}(\mathbf{\Lambda}_m^H\mathbf{\Lambda}_m)^2\left(\frac{1}{\alpha}\mathbf{\Lambda}_m^H\mathbf{\Lambda}_m +\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\right)^{-1}\right). \tag{15}$$

In general, it is difficult to obtain closed form analytic results for the performance achieved by the above scheduling algorithm. The main difficulty is due to the fact that $\mathbf{\Lambda}_m$, $\mathbf{V}_m$ and $\mathbf{Q}_m$ are not independent. Furthermore, the inverse of the matrix in the determinant expression makes the problem more challenging. However, for a special case when each source only transmits one data stream, i.e. $x_m=1$, the above optimization problem can be simplified as the following

$$\arg\max_m \quad \lambda_{(1),m}, \tag{16}$$

where $\lambda_{(1),m}$ is the largest eigenvalue of $\mathbf{H}_m\mathbf{Q}_m$. In addition to the reception reliability, it is also of interest to study the averaged delay experienced by each user. We particularly consider the channels to be constant for one frame and independently changing between different frames. One pair will be scheduled for transmission by using the proposed selection method. We are interested in the number of the frames which is sufficient to ensure that each of M node pairs is scheduled at least once, denoted as $d(M)$. And the following corollary shows the reliability and the delay performance achieved by the scheduling protocol using such a criterion.

Corollary *4:* When each source only transmits one data stream, i.e $x_m=1$, opportunistic scheduling using the criterion in (16) can achieve the outage probability as

$$P(\mathcal{I}_{TW,m^*}<R)\doteq\frac{1}{\rho^{ML(2N-L)}},$$

and the averaged delay experienced by each pair is

$$d(M)=M\sum_{m=1}^{M}\frac{1}{m}\sim\mathcal{O}(M\ln M).$$

Proof: The first part of the corollary can be obtained from Lemma 3 by using the fact that all channels are identically and independent Rayleigh distributed between different frames, and the second part can be obtained by applying the coupon collector problem [20]. ■

As can be observed from Corollary 4, the use of opportunistic scheduling can further improve the achievable diversity gain from $L(2N-L)$ to $ML(2N-L)$, but causes unfairness among the M user pairs. Some pairs have to wait for a long time to be served and such a delay can be further increased by increasing the number of the nodes.

IV. Multi-way Relaying Channel

In this section, we consider the case that more than one node pair will be transmitting at the same time. According to the proposed transmission protocol, each pair will send x_m data streams. When multiple users transmit at the same time, co-channel interference exists, and it is important to study how to cope with co-channel interference as shown in (7). To realize interference-free communications, we use the following for the precoding matrix at the relay

$$\mathbf{X}=\left(\left[\mathbf{U}_1\mathbf{\Lambda}_1\cdots\mathbf{U}_M\mathbf{\Lambda}_M\right]^H\right)^{-1}\mathbf{D}_R\left(\left[\mathbf{U}_1\mathbf{\Lambda}_1\cdots\mathbf{U}_M\mathbf{\Lambda}_M\right]\right)^{-1} =\left(\mathbf{U}^H\right)^{-1}\mathbf{D}_R\left(\mathbf{U}\right)^{-1},$$

where the $L\times L$ matrix $\mathbf{U}$, denoted as the matrix $\left[\mathbf{U}_1\mathbf{\Lambda}_1\cdots\mathbf{U}_M\mathbf{\Lambda}_M\right]$, is to cancel cross-pair interference and $\mathbf{D}_R$ is to meet the transmission power constraint at the relay, i.e.

$$\mathbf{D}_R=\frac{1}{\sqrt{2L}}\text{diag}\left[\left(\left[\mathbf{U}^H\mathbf{U}\right]_{11}^{-1}\right)^{-\frac{1}{2}}\quad\cdots\quad\left(\left[\mathbf{U}^H\mathbf{U}\right]_{LL}^{-1}\right)^{-\frac{1}{2}}\right],$$

and $[\mathbf{A}]_{ii}$ denotes the i-th element at the diagonal of $\mathbf{A}$. Given such a precoding matrix, the power consumption at the relay can be expressed as

$$\begin{aligned} P_{ow}^R &= \mathcal{E}\left\{\text{tr}\left\{2\mathbf{X}\mathbf{y}_r\mathbf{y}_r^H\mathbf{X}^H\right\}\right\} \\ &\approx 2\left\{\text{tr}\left\{\mathbf{U}^{-1}\left(\mathbf{U}^H\right)^{-1}\mathbf{D}_R^2\right\}\right\}=1, \end{aligned} \tag{17}$$

where the approximation follows from the high SNR assumption. With such a precoder, the signal model at the m-th receiver can now be written as

$$\tilde{\mathbf{y}}_m=\mathbf{D}_{r,m}\left(\mathbf{s}_m+\mathbf{s}_{m'}\right)+\mathbf{D}_{r,m}\tilde{\mathbf{I}}_m\mathbf{U}^{-1}\mathbf{n}_r+\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{n}_m,$$

where $\tilde{\mathbf{I}}_m=\left[\mathbf{0}_{x_m\times\sum_{i=1}^{m-1}x_i}\quad\mathbf{I}_{x_m}\quad\mathbf{0}_{x_m\times\sum_{i=m+1}^{L}x_i}\right]$ and $\mathbf{D}_{r,m}$ is a diagonal matrix formed by using the $(\sum_{i=1}^{m-1}x_i+1)$-th to $(\sum_{i=1}^{m}x_i)$-th diagonal elements of $\mathbf{D}_R$. Compared to the original signal model in (7), it is important to point out that the use of the proposed precoding matrix can remove the interference completely. As a result, the mutual information from the source m' to the source m can be written as

$$\mathcal{I}_m = \log\det\left(\mathbf{I}_{x_m} + \rho\mathbf{D}_{r,m}^2\Big(\mathbf{D}_{r,m}\tilde{\mathbf{I}}_m(\mathbf{U}^H\mathbf{U})^{-1}\tilde{\mathbf{I}}_m^H\mathbf{D}_{r,m}^H + \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\Big)^{-1}\right). \quad (18)$$

Similar to the symmetry observed in the previous section, it can be verified that the mutual information from the source m to the source m' is exactly the same as $\mathcal{I}_m$.

A. When $M \le L$

When the number of the user pairs is less than the relay antennas, the multiplexing gain supported by the proposed eigenmodel sharing protocol is larger than the user pairs $L \ge M$ if $L \le M(2N-L)$. So some sources will transmit more than one data stream, which results in the problem of how to allocate the available degrees of freedom to M user pairs. Such a problem can be formulated as an optimization problem shown in (19). Note that the minimal value of x_m has been replaced with 1 in order to ensure that each pair can transmit at least one data stream. It is difficult to obtain the optimal solutions for x_M and $\mathbf{D}_m$ by directly solving the above optimization problem. A useful observation from the signal model at the relay in (4) is that the use of a larger eigenvalue is helpful to improve the signal-to-noise ratio. In such a case, a reasonable choice for the precoding matrices $\mathbf{D}_m$ can be expressed as

$$\mathbf{D}_m = \frac{1}{\sqrt{x_m}}\begin{bmatrix}\mathbf{I}_{x_m}^T & \mathbf{0}_{(2N-L-x_m)\times x_m}^T\end{bmatrix}^T,$$

which is only to use the x_m largest eigenvalues of $\mathbf{H}_m\mathbf{Q}_m$. Note that the use of such a precoding matrix can ensure that the total transmission power of the two users within one pair is constrained as

$$\begin{aligned}&\text{tr}\left\{\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H + \mathbf{Q}_{m'}\mathbf{V}_m\mathbf{D}_m\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_{m'}^H\right\}\\ &\quad = \text{tr}\left\{\mathbf{D}_m^H\mathbf{V}_m^H(\mathbf{Q}_m^H\mathbf{Q}_m + \mathbf{Q}_{m'}^H\mathbf{Q}_{m'})\mathbf{V}_m\mathbf{D}_m\right\} = 1,\end{aligned}$$

where we have used the fact that $\mathbf{Q}_m^H\mathbf{Q}_m + \mathbf{Q}_{m'}^H\mathbf{Q}_{m'} = \mathbf{I}_{2N-L}$. By applying the exhaustive search, the optimal solution for x_m can be obtained. Unfortunately such an exhaustive search method does not offer any insight for the achievable performance gain, and also causes a lot of system overhead for the coordination among the multiple transmitters. Fortunately for some special cases, the closed form of the optimal solution, as well as the achievable outage probability, can be obtained as shown in the following.

B. When $M = L$

Consider a case that $M = L$ and each user will send just one data stream, i.e. $x_m = 1$. In such a case, all the matrices in (18) can be reduced to scales and the expression of the mutual information can be simplified as

$$\mathcal{I}_m = \log\left(1 + \frac{\rho\left(\frac{1}{2L} + \mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{d}_m\right)^{-1}}{2L[\mathbf{U}^H\mathbf{U}]_{mm}^{-1}}\right) \quad (20)$$

where we use the fact $\tilde{\mathbf{I}}_m(\mathbf{U}^H\mathbf{U})^{-1}\tilde{\mathbf{I}}_m^H = \left[\mathbf{U}^H\mathbf{U}\right]_{mm}^{-1}$ and $\mathbf{D}_m$ has been replaced with $\mathbf{d}_m$ to emphasize the fact that $\mathbf{D}_m$ is now a vector since $x_m = 1$. The optimal solution of $\mathbf{d}_m$ to ensure the best reception reliability can be found from the following optimization problem

$$\begin{aligned}\min_{\mathbf{d}_1,\cdots,\mathbf{d}_M} \quad & \max\left\{P(\mathcal{I}_1 < 2R), \cdots, P(\mathcal{I}_M < 2R)\right\} \quad (21)\\ s.t. \quad & \mathbf{d}_m^H\mathbf{d}_m = 1, \quad \forall m \in \{1, \cdots, M\}.\end{aligned}$$

To simplify the development of analytical results, we consider that the vector $\mathbf{d}_m$ is normalized, i.e $\mathbf{d}_m^H\mathbf{d}_m = 1$. Note that such a choice of $\mathbf{d}_m$ can ensure the transmission power at the sources constrained as follows

$$\begin{aligned}P_{ow}^s &= \text{tr}\left\{\mathbf{Q}_m\mathbf{V}_m\mathbf{d}_m\mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H + \mathbf{Q}_{m'}\mathbf{V}_m\mathbf{d}_m\mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_{m'}^H\right\}\\ &= \text{tr}\left\{\mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{d}_m + \mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_{m'}^H\mathbf{Q}_{m'}\mathbf{V}_m\mathbf{d}_m\right\} = 1.\end{aligned}$$

Note that changing $\mathbf{d}_m$ for one pair will affect the mutual information of other pairs since each column of $\mathbf{U}$ equals to $\mathbf{U}_m\tilde{\mathbf{\Lambda}}_m\mathbf{d}_m$. And a joint optimization of all $\mathbf{d}_m$ is difficult to solve since the above optimization problem is not convex. So in the following we first try to find the upper and lower bounds for the mutual information. Recall that $\begin{bmatrix}\mathbf{Q}_m^H & \mathbf{Q}_{m'}^H\end{bmatrix}\begin{bmatrix}\mathbf{Q}_m^T & \mathbf{Q}_{m'}^T\end{bmatrix}^T = \mathbf{I}_L$, which results in the following inequality

$$0 \le \mathbf{d}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{d}_m \le \mathbf{d}_m^H\mathbf{d}_m = 1.$$

By using such an inequality, the denominator inside of the expression of the mutual information can be simplified and the mutual information can be bounded as

$$\log\left(1 + \frac{\frac{\rho}{2L+1}}{[\mathbf{U}^H\mathbf{U}]_{mm}^{-1}}\right) \le \mathcal{I}_m \le \log\left(1 + \frac{\rho}{[\mathbf{U}^H\mathbf{U}]_{mm}^{-1}}\right). \quad (22)$$

We can observe that both the upper and lower bounds share a similar structure, where $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$ has a significant impact on the value of the mutual information. Therefore the original optimization can be relaxed as in the following

$$\begin{aligned}\underset{\mathbf{d}_1,\cdots,\mathbf{d}_M}{max} \quad & \min\left\{\frac{1}{[\mathbf{U}^H\mathbf{U}]_{11}^{-1}}\cdots, \frac{1}{[\mathbf{U}^H\mathbf{U}]_{MM}^{-1}}\right\} \quad (23)\\ s.t. \quad & \mathbf{d}_m^H\mathbf{d}_m = 1, \quad \forall m \in \{1, \cdots, M\}.\end{aligned}$$

The following lemma provides us some insight to the optimal solution of the above optimization problem.

Lemma *5:* Provided that all other $\mathbf{d}_k$, $k \ne m$, are fixed, the optimal solution of $\mathbf{d}_m$ for the optimization problem in (23) is given as follows

$$\mathbf{d}_m^* = \tilde{\mathbf{V}}_m^H\tilde{\mathbf{d}}_m,$$

where $\tilde{\mathbf{d}}_m = \frac{\tilde{\mathbf{h}}_m}{\sqrt{\tilde{\mathbf{h}}_m^H\tilde{\mathbf{h}}_m}}$, $\tilde{\mathbf{h}}_m^H = \mathbf{p}_m^H\mathbf{H}_m\mathbf{Q}_m$, and $\mathbf{p}_m$ is the eigenvector of $\left(\mathbf{I}_L - \tilde{\mathbf{U}}_m\left(\tilde{\mathbf{U}}_m^H\tilde{\mathbf{U}}_m\right)^{-1}\tilde{\mathbf{U}}_m^H\right)$ corresponding to its non-zero eigenvalue. And the use of such a vector can ensure that the outage probability for the m-th pair can be approximated at high SNR as

$$P(\mathcal{I}_m < 2R) \approx \frac{1}{\rho^{2N-L}}. \quad (24)$$

And the achievable multiplexing-diversity tradeoff is $d_m(r) = (2N-L)(1-2r)$.

Proof: Please refer to the appendix. ■

$$\begin{aligned} \underset{x_m,\mathbf{D}_m}{max} \quad & \sum_{m=1}^{M} \log\det\left(\mathbf{I}_{x_m} + \rho\mathbf{D}_{r,m}^2\left(\mathbf{D}_{r,m}\tilde{\mathbf{I}}_m(\mathbf{U}^H\mathbf{U})^{-1}\tilde{\mathbf{I}}_m^H\mathbf{D}_{r,m}^H + \mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\right)^{-1}\right). \\ s.t. \quad & \mathrm{tr}\left\{\mathbf{Q}_m\mathbf{V}_m\mathbf{D}_m\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_m^H + \mathbf{Q}_{m'}\mathbf{V}_m\mathbf{D}_m\mathbf{D}_m^H\mathbf{V}_m^H\mathbf{Q}_{m'}^H\right\} \leq 1 \\ s.t. \quad & \sum_{m=1}^{M} x_m \leq \min\{L, M(2N-L)\} \\ s.t. \quad & 1 \leq x_m \leq \min\{(2N-L), L\}. \end{aligned} \tag{19}$$

From Lemma 5, we learn that the maximum diversity gain for the addressed scenario is $(2N-L)$, which is conditioned on the assumption that the parameters of the other $(M-1)$ source pairs have been fixed. So the solution in Lemma 5 can be viewed as a local minimal for $\mathbf{d}_m$ given a set of fixed $\mathbf{d}_k$, $k \neq m$. However, due to the non-convexity of the addressed optimization problem, it is not clear whether the global optimal solution exists and therefore it is still difficult to solve the optimization problem for $\mathbf{d}_m$, $m \in \{1, \cdots, M\}$, jointly.

1) A suboptimal solution with the maximum diversity gain: In the following we propose a simple algorithm and obtain a suboptimal solution for the problem in (23). Such a suboptimal solution can be obtained with only $(2N-L)$ iterations, and the maximum diversity gain pointed out in Lemma 5 can be achieved by such a suboptimal approach. Specifically during the k-th iteration, all source pairs use the same precoding vector as the following

$$\mathbf{d}_{m,k} = \tilde{\mathbf{V}}_m^H\tilde{\mathbf{1}}_k,$$

where $\tilde{\mathbf{1}}_k$ is an $(2N-L)\times 1$ vector whose k-th element is equal to one and all other elements are zero. At the end of the k-th iteration, we find out the pair with the worst outage performance, denoted as P_k. After $(2N-L)$ iterations, we find out the iteration which gives the best worst-pair performance, denoted as k^*, i.e $k^* = \arg\min_k\{P_1, \cdots, P_{2N-L}\}$. Therefore the chosen precoding vector will be $\mathbf{d}_{m,k^*}$. The following lemma provides the outage performance achieved by such an approach.

Lemma *6:* The outage probability for the m-th pair by using the selected precoding vector $\mathbf{d}_{m,k^*}$ can be approximated at high SNR as

$$P_{\mathbf{d}_{m,k^*}}(\mathcal{I}_m < 2R) \doteq \frac{1}{\rho^{(2N-L)(1-2r)}}. \tag{25}$$

Proof: Please refer to the appendix. ■

As can be seen from the lemma, the use of the proposed suboptimal method to choose the precoding vectors $\mathbf{d}_m$ can achieve a diversity gain $(2N-L)$. Recall that such a diversity gain is the maximum we can get for the addressed scenario as pointed out in Lemma 5. So the proposed approach is optimal in terms of the outage performance. And it is important to point out that such an optimal reliability performance is achieved with only $(2N-L)$ iterations.

C. When $M > L$

When the number of the user pairs is larger than the supportable degrees of freedom, only a fraction of the M node pairs will be served at each time. Particularly given L relay antennas, it is preferable to schedule L streams, which can explore the available degrees of freedom and maximize the achievable multiplexing gain. The system throughput and reliability are influenced by which group of the users is scheduled. The optimal selection strategy is to exhaustively search a group of L pairs yielding the best performance, which could result in significant system overhead since at least $\binom{M}{L}$ iterations are required. Inspired by the suboptimal approach proposed for the special case of $M = L$, a simple scheduling protocol with low system overhead can be constructed as follows. Priori to transmission, the M node pairs are randomly allocated into $\lfloor \frac{M}{L} \rfloor$ groups with the group size of L, where the marginal effect for the remaining $(M - L\lfloor \frac{M}{L} \rfloor)$ pairs will be ignorable if $M \gg L$. Within each group, the precoding matrices will be formed as discussed in Section IV-B1, and each group reports their worst pair performance to the relay. Among these $\lfloor \frac{M}{L} \rfloor$ groups, the group which has the best worst-pair performance will be scheduled. The following lemma describes the performance achieved by such an opportunistic scheme.

Lemma *7:* By using the proposed group scheduling protocol, the outage probability for each pair within the selected group can be approximated as

$$P_{\mathbf{d}_{m,k^*}}(\mathcal{I}_m < 2R) \approx \frac{1}{\rho^{\lfloor \frac{M}{L} \rfloor (2N-L)(1-2r)}}. \tag{26}$$

and the averaged delay experienced by each pair is

$$d(M,L) = \binom{M}{L}\sum_{j=0}^{M-1}(-1)^{M-j+1}\binom{M}{j}\left[\binom{M}{L} - \binom{j}{L}\right]^{-1}.$$

where we adopt the convention that $\binom{n}{k}$ will be zero if $n < k$.

Proof: The first part of the lemma can be proven by applying Lemma 6 and also using the fact that wireless channels of different groups are independent to each other, so order statistics can be applied in a straightforward way. The second part of the lemma can be obtained by treating the addressed problem as a coupon collector problem with a fixed sample size L. By applying the result from [20], the second part of the lemma can be obtained easily. ■

Consider a special case with $M = L$, the delay in the lemma can be simplified as

$$d(M,M) = \sum_{j=0}^{M-1}(-1)^{M-j+1}\binom{M}{j} = 1,$$

since $\sum_{j=0}^{m}(-1)^j\binom{n}{j} = (-1)^m\binom{n-1}{m}$ as shown in [21]. Using simulations, it can be observed that $d(M,L)$ is a decreasing

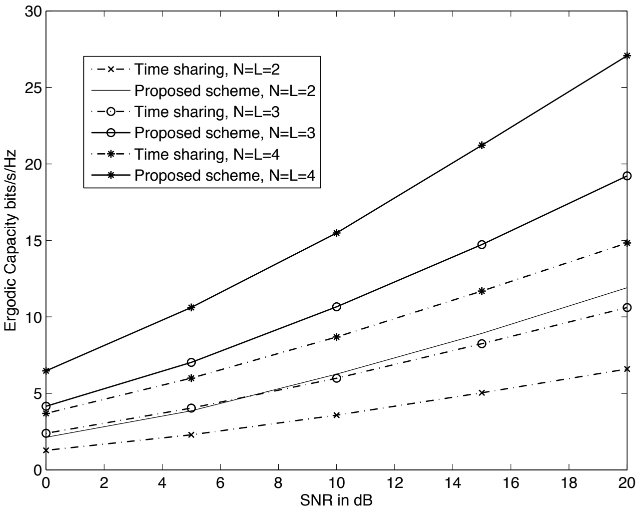

Fig. 2. The ergodic capacity of the proposed two-way relaying protocol vs SNR.

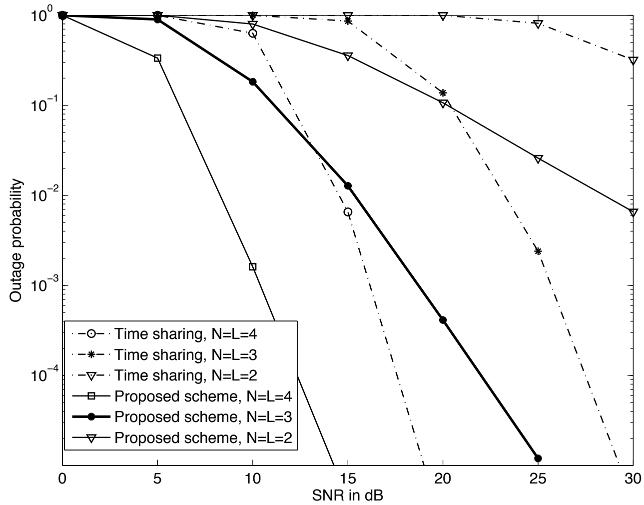

Fig. 3. The outage probability of the proposed two-way relaying protocol vs SNR.

function of L. Since the delay performance in Lemma 3 is a special case $d(M, L)$ with $L = 1$, we find out that the use of multi-way relaying is able to reduce delay and improve the fairness among the multiple source pairs.

V. Numerical Results

In this section, Monte-Carlo simulations will be carried out to evaluate the performance of the proposed bidirectional transmission protocol by using eigenmode sharing. Two-way and multi-way relaying channels are studied in the two following sections respectively.

A. Two-way relaying channel

In Fig. 2 and 3, the performance of the proposed two-way relaying protocol has been studied. To clearly demonstrate the performance of the proposed transmission protocol, we adopted a MIMO version of the classical AF relay scheme developed in [22]. Specifically during the first two time slots, one source is transmitting messages to its destination via the relay, and during the next two time slots, its partner sends information back to it again via the relay. The ergodic capacity is used as the criterion for performance evaluation in Fig. 2 and the outage probability is shown as a function of SNR in Fig. 3, where x_m has been set as $\min\{L, 2N - L\} = L$. The traditional time-sharing based transmission protocol has been used as a comparable scheme. As can be seen from Fig. 2, the proposed eigenmode sharing protocol can achieve larger ergodic capacity than the traditional scheme. Such a performance gain can be explained as the following. Recall that the proposed bidirectional transmission scheme only requires two time slots to accomplish information exchange between the two source nodes, whereas the time sharing approach requires four time slots. As a result, the bandwidth resource consumed by the time sharing based scheme is twice than the proposed protocol.

In Fig. 3, the outage probability has been used as the criterion for the performance comparison. For both two transmission protocols, the targeted data rate has been set as $R = 8$ bits/s/Hz. As can be observed from the figure, the proposed eigenmode sharing protocol can achieve better outage performance than the time sharing based scheme. Note that the performance gap between the two transmission schemes can be significantly large when the number of the relay and source antennas is small. The reason for such a performance gain is because of the spectral efficiency of the proposed relaying protocol. Particularly one relay transmission for the proposed scheme can help both two destinations, whereas the use of the time sharing based scheme can only serve destinations one by one. Due to space limit, the numerical results for pair scheduling have been omitted, however, the channel randomness among multiple source pairs implies that the outage probability can be further reduced as indicated by Lemma 3.

B. Multi-way relaying channel

We first focus on the scenario where the number of the user pairs is the same as the relay antennas $M = L$ and each source only transmits one data stream. Again time sharing approaches are used, where each pair uses the classical AF strategy for information exchanging within four time slots and the M pairs of users take turn to transmit, which means that $4M$ time slots are required. As pointed out in Lemma 6, the overall multiplexing gain achieved by the proposed multi-way relaying protocol is L, which is also the maximum achievable multiplexing gain as pointed out by the max-flow min-cut theorem for such a communication scenario. On the other hand, the time sharing based approach can only achieve a multiplexing gain of $0.5 \min\{L, N\}$ as discussed before. Therefore the proposed eigenmode sharing approach can achieve larger ergodic capacity than the time sharing scheme, as observed from Fig. 4. Note that when $L \geq N$, most existing two-way relaying protocols can only achieve a multiplexing gain N, and hence the proposed multi-way relaying scheme can also achieve larger multiplexing gains than these two-way relaying schemes. So by allowing more than one pair transmitting at the same time, the use of the proposed eigenmode sharing protocol does not only ensure interference free communications among multiple pairs, but also pushes the achievable multiplexing gain to the maximum, whereas both two-way relaying protocols and time sharing

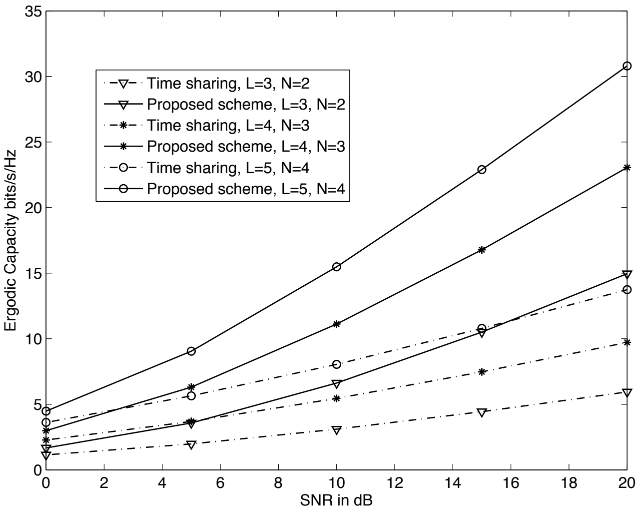

Fig. 4. The ergodic capacity of the proposed multi-way relaying protocol vs SNR when $M = L$.

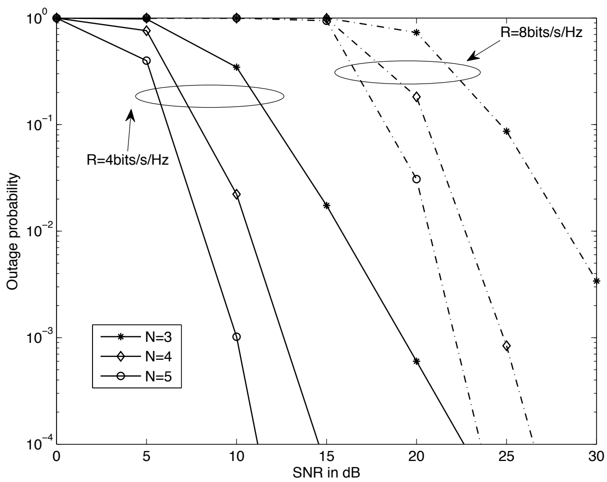

Fig. 5. The outage probability of the proposed multi-way relaying protocol vs SNR when $M = L = 5$.

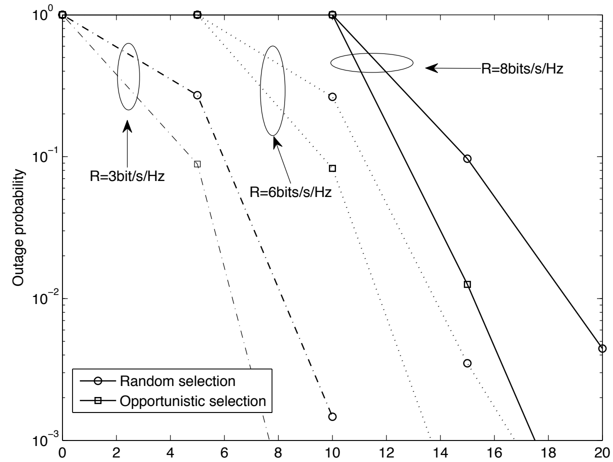

Fig. 6. The outage probability of the proposed multi-way relaying protocol vs SNR when $M < L$.

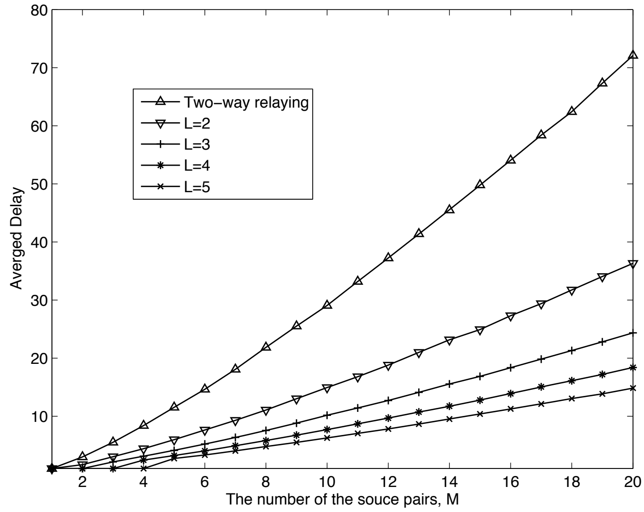

Fig. 7. The averaged delay experienced by each pair vs the number of the source pairs.

schemes suffer some loss of multiplexing gains. Another fact pointed out in Lemma 6 is that the diversity gain achieved by the proposed eigenmode sharing approach can be increased by increasing the number of the source antennas. This result is demonstrated in Fig. 5, where the outage performance of the proposed multi-way relaying transmission protocol is studied with different choices of the source antennas. Particularly the numbers of the user pairs and relay antennas are fixed, $M = L = 5$, and the targeted data rate has been set as $R = 4$ bits/s/Hz and $R = 8$ bits/s/Hz.

In Fig. 6, we focus on the scenario where the number of the user pairs is less than the relay antennas $M < L$. Particularly we set $M = 2$, $L = 3$ and $N = 3$. As discussed in Section IV-A, some suers can transmit more than one data stream since the supportable degrees of freedom are larger than the number of the user pairs for such a scenario. In Fig. 6, the performance of the opportunistic selection algorithm proposed in Section IV-A is compared to a scheme with random pair selection. As can be observed from the figure, the use of opportunistic selection can improve the outage performance, however, it is worthy to point out that the scheme with random selection requires less system overhead. Finally in Fig. 7 we study the impact of different transmission protocols on the delay performance. Corollary 4 has pointed out that the use of two-way relaying can cause severe unfairness among multiple pairs and one pair could wait for a long time to be served, which has been confirmed by Fig. 7. On the other hand, the use of multi-way relaying can improve fairness experienced by each user, as pointed out by Lemma 7. As can be observed from the figure, the number of time slots during which each user waits to be served becomes proportional to the number of the pairs, whereas two-way relaying causes a delay in the order of $M \log M$.

VI. Conclusion

In this paper, we have proposed a general transmission scheme for bi-directional communications, where two-way relaying and multi-way relaying can be viewed as a special case of the proposed transmission scheme. Detailed case studies for the special cases have been provided, where analytical results have been developed for a better performance evaluation. Our developed results show that the proposed bi-directional transmission scheme can always outperform time sharing based approaches, and that serving more than one pair at the same time is more beneficial than simple two-way relaying in terms of multiplexing gains.

Appendix

Proof for Lemma 3: The proof for the lemma can be completed provided both the upper and lower bounds of the outage probability have the same high SNR approximation. Without loss of generality, we only focus on the upper bound and the result can be obtained similarly for the lower bound. From Proposition 2, we can find out that the matrix $2\mathbf{H}_m\mathbf{Q}_m$ is a $L \times (2N - L)$ complex Gaussian matrix with zero mean and variance one. As a result, $\mathbf{Q}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{Q}_m$ is a classical Wishart matrix. Define $\tilde{L} = \min\{L, 2N - L\}$ and $\tilde{N} = \max\{L, 2N - L\}$. As shown in [23], the cumulative distribution function of the largest eigenvalue of the $(2N - L) \times (2N - L)$ $\mathbf{Q}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{Q}_m$ is

$$F(x) = \frac{(\tilde{L})!}{\prod_{i=1}^{L}(\tilde{L}-i)!(\tilde{N}-i)!}\det(\mathbf{S}(x)),$$

where the element at its k-th row and l-th column is

$$[\mathbf{S}(x)]_{k,l} = (\phi+k+l-2)!\left[1 - e^{-x}\sum_{m=0}^{\phi+k+l-2}\frac{u^m}{m!}\right],$$

$\forall k, l \in [1, \cdots, \tilde{L}]$, where $\phi = \tilde{N} - \tilde{L}$. So the addressed outage probability can be expressed as

$$\begin{aligned} P(\mathcal{I}_{TW,m} < 2R) &= P(\lambda_m \leq \epsilon) \\ &= \frac{(\tilde{L})!}{\prod_{i=1}^{L}(\tilde{L}-i)!(\tilde{N}-i)!}\det(\mathbf{S}(\epsilon)), \end{aligned}$$

where $\epsilon = \frac{3\cdot 4\cdot\alpha(2^{2R}-1)}{\rho}$ and the factor 4 is due to the fact that $2\mathbf{H}_m\mathbf{Q}_m$, not $\mathbf{H}_m\mathbf{Q}_m$, is complex Gaussian distributed with zero mean and variance one. By using the high SNR assumption, we have

$$\begin{aligned} (\phi+k+l-2)!\left(1 - e^{-x}\sum_{m=0}^{\phi+k+l-2}\frac{u^m}{m!}\right) &= (\phi+k+l-2)! \\ \sum_{m=\phi+k+l-1}^{\infty}\frac{u^m}{m!} &\approx \frac{u^{\phi+k+l-1}}{(\phi+k+l-1)}. \end{aligned}$$

By using such an approximation, we can find the matrix $\mathbf{S}$ simplified as follows

$$\mathbf{S} \approx \begin{bmatrix} \frac{\epsilon^{\phi+1}}{\phi+1} & \cdots & \frac{\epsilon^{\phi+\tilde{L}}}{\phi+\tilde{L}} \\ \vdots & \cdots & \vdots \\ \frac{\epsilon^{\phi+\tilde{L}}}{\phi+\tilde{L}} & \cdots & \frac{\epsilon^{\phi+2\tilde{L}-1}}{\phi+2\tilde{L}-1}. \end{bmatrix}.$$

To find the closed form expression of the determinant of the matrix, we first rewrite the matrix as

$$\mathbf{S} \approx \begin{bmatrix} 1 & \frac{\epsilon^{-1}}{\phi+2} & \cdots & \frac{\epsilon^{-(\tilde{L}-1)}}{\phi+\tilde{L}} \\ \frac{\epsilon}{\phi+2} & \frac{1}{\phi+3} & \cdots & \frac{\epsilon^{-(\tilde{L}-2)}}{\phi+\tilde{L}} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\epsilon^{\tilde{L}-1}}{\phi+\tilde{L}} & \frac{\epsilon^{-\tilde{L}}}{\phi+\tilde{L}+1} & \cdots & \frac{1}{\phi+2\tilde{L}-1} \end{bmatrix}\mathbf{D}_\epsilon,$$

where $\mathbf{D}_\epsilon = \mathrm{diag}\begin{bmatrix}\epsilon^{\phi+1} & \epsilon^{\phi+3} & \cdots & \epsilon^{\phi+2\tilde{L}-1}\end{bmatrix}$. Because of the high SNR approximation, the first matrix is nearly an upper tri-diagonal matrix. So the determinant can be easily written as

$$\det\mathbf{S} \approx \prod_{m=1}^{\tilde{L}}\frac{\epsilon^{\phi+2m-1}}{k+2m-1} = \prod_{m=1}^{\tilde{L}}\frac{\epsilon^{\sum_{k=1}^{\tilde{L}}(\phi+2k-1)}}{k+2m-1}.$$

The exponent of ϵ can be expressed as

$$\sum_{k=1}^{\tilde{L}}(\phi+2k-1) = \tilde{L}\phi + \tilde{L}^2 = \tilde{L}\tilde{N},$$

where we use the fact that $\phi = \tilde{N} - \tilde{L}$. So the outage probability can be expressed as

$$P(\mathcal{I}_{TW,m} < 2R) \approx \frac{\epsilon^{L(2N-L)}(\tilde{L})!}{\prod_{i=1}^{L}(\tilde{L}-i)!(\tilde{N}-i)!}\prod_{m=1}^{\tilde{L}}\frac{1}{\phi+2m-1}.$$

The expression for the lower bound can be obtained similarly and the lemma is proven. ■

Proof for Lemma 5: The first step of the proof is to simplify the the factor $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$. Following similar steps in [24], the i-th element at the diagonal of the inverse of a matrix can be expressed as

$$\mathbf{\Lambda}_m^H\mathbf{U}_m^H\left(\mathbf{I}_L - \tilde{\mathbf{U}}_m\left(\tilde{\mathbf{U}}_m^H\tilde{\mathbf{U}}_m\right)^{-1}\tilde{\mathbf{U}}_m^H\right)\mathbf{U}_m\mathbf{\Lambda}_m, \tag{27}$$

where $\tilde{\mathbf{U}}_m$ is a $L \times (L-1)$ submatrix of $\mathbf{U}$ by removing the m-th column of $\tilde{\mathbf{U}}$. It can be easily verified that $\left(\mathbf{I}_L - \tilde{\mathbf{U}}_m\left(\tilde{\mathbf{U}}_m^H\tilde{\mathbf{U}}_m\right)^{-1}\tilde{\mathbf{U}}_m^H\right)$ is an idempotent matrix, which means its eigenvalues are either ones or zeros. Given that its rank is 1, the matrix $\left(\mathbf{I}_L - \tilde{\mathbf{U}}_m\left(\tilde{\mathbf{U}}_m^H\tilde{\mathbf{U}}_m\right)^{-1}\tilde{\mathbf{U}}_m^H\right)$ has only one non-zero eigenvalue which equals to one. So the eigenvalue decomposition of the matrix is

$$\left(\mathbf{I}_L - \tilde{\mathbf{U}}_m\left(\tilde{\mathbf{U}}_m^H\tilde{\mathbf{U}}_m\right)^{-1}\tilde{\mathbf{U}}_m^H\right) = \mathbf{p}_m\mathbf{p}_m^H.$$

By using such eigenvalue decomposition, the factor $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$ can be written as

$$\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}} = \mathbf{\Lambda}_m^H\mathbf{U}_m^H\mathbf{p}_m\mathbf{p}_m^H\mathbf{U}_m\mathbf{\Lambda}_m. \tag{28}$$

Recall that the matrices $\mathbf{\Lambda}_m$ and $\mathbf{\Lambda}_m$ are obtained from the SVD of $\mathbf{H}_m\mathbf{Q}_m$, which means that

$$\mathbf{U}_m\mathbf{\Lambda}_m = \mathbf{H}_m\mathbf{Q}_m\tilde{\mathbf{V}}_m\mathbf{d}_m.$$

Substituting the above equation into the expression of $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$, we can obtain

$$\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}} = \mathbf{d}_m^H\tilde{\mathbf{V}}_m^H\mathbf{Q}_m^H\mathbf{H}_m^H\mathbf{p}_m\mathbf{p}_m^H\mathbf{H}_m\mathbf{Q}_m\tilde{\mathbf{V}}_m\mathbf{d}_m. \tag{29}$$

Note that the normalized vector $\mathbf{p}_m$ is obtained from the channels not associated with $\mathbf{H}_m$, which means $\mathbf{p}_m$ is independent to $\mathbf{H}_m$. Also note that $\mathbf{H}_m\mathbf{Q}_m$ is a $L \times (2N - L)$ complex Gaussian matrix. Since unitary transformation of a Gaussian matrix does not change its statistical property, it is obvious that each element of the vector $\tilde{\mathbf{h}}_m^H = \mathbf{p}_m^H \mathbf{H}_m \mathbf{Q}_m$ is still i.i.d complex Gaussian distributed. So we can obtain

$$\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}} = \mathbf{d}_m^H \tilde{\mathbf{V}}_m^H \tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m \tilde{\mathbf{V}}_m \mathbf{d}_m = \tilde{\mathbf{d}}_m^H \tilde{\mathbf{h}}_m \tilde{\mathbf{h}}_m^H \tilde{\mathbf{d}}_m,$$

where $\tilde{\mathbf{d}}_m = \tilde{\mathbf{V}}_m \mathbf{d}_m$. Provided that the precoding vectors $\mathbf{d}_k$, $k \neq m$, are fixed, $\mathbf{p}_m$ is also fixed. And hence the optimal choice of the precoding matrix $\tilde{\mathbf{d}}_m$ to maximize $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$ is $\tilde{\mathbf{d}}_m = \frac{\tilde{\mathbf{h}}_m}{\sqrt{\tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m}}$ which can be validated from the following eigenvalue decomposition

$$\tilde{\mathbf{h}}_m \tilde{\mathbf{h}}_m^H = \frac{\tilde{\mathbf{h}}_m}{\sqrt{\tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m}} \tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m \frac{\tilde{\mathbf{h}}_m^H}{\sqrt{\tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m}}.$$

As a result, the maximum of $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$ is $\tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m$. Given $\tilde{\mathbf{h}}_m$ is a complex Gaussian vector, the distribution of $\tilde{\mathbf{h}}_m^H \tilde{\mathbf{h}}_m$ can be obtained from the Chi-square distribution as follows

$$F_{\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}}(x) = \frac{y^{2N-L-1}}{(2N-L)!} e^{-x}.$$

Therefore the outage probability in the lemma can be obtained in a straightforward way. ■

Proof for Lemma 6: According to the proposed sub-optimal approach, $\mathbf{d}_{m,k}$ will be used as the precoding vector during the k-th iteration. By using such a vector, the precoding matrix at the relay during the k-th iteration, denoted as $\mathbf{U}^k$, can be expressed as

$$\mathbf{U}^k = \left[\mathbf{U}_1 \tilde{\mathbf{\Lambda}}_1 \mathbf{d}_{1,k} \cdots \mathbf{U}_M \tilde{\mathbf{\Lambda}}_M \mathbf{d}_{M,k}\right] = \left[\bar{\mathbf{h}}_{1,k} \quad \cdots \quad \bar{\mathbf{h}}_{M,k}\right],$$

and the parameter $\frac{1}{[\mathbf{U}^{-1}\mathbf{U}^{-H}]_{m,m}}$ during the k-th iteration, denoted as $\beta_{m,k}$, can be obtained from (29) as

$$\begin{aligned} \beta_{m,k} &= \tilde{\mathbf{1}}_k^H \mathbf{Q}_m^H \mathbf{H}_m^H \mathbf{p}_{m,k} \mathbf{p}_{m,k}^H \mathbf{H}_m \mathbf{Q}_m \tilde{\mathbf{1}}_k \\ &= \bar{\mathbf{h}}_{m,k}^H \mathbf{p}_{m,k} \mathbf{p}_{m,k}^H \bar{\mathbf{h}}_{m,k}, \end{aligned} \tag{30}$$

where $\bar{\mathbf{h}}_{m,k}$ is the k-th column of $\mathbf{H}_m\mathbf{Q}_m$ and $\mathbf{p}_{m,k}$ is generated accordingly from $\mathbf{U}^k$ as described in Lemma 5. According to Proposition 2, $\mathbf{H}_m\mathbf{Q}_m$ is a complex Gaussian matrix, and hence hence the vectors $\bar{\mathbf{h}}_{m,k}$, $k \in \{1, \cdots, M\}$, are identically and independent complex Gaussian vectors. Recall that unitary transformation of a complex Gaussian vector is a still Gaussian vector, so $\mathbf{p}_{m,k}^H \bar{\mathbf{h}}_{m,k}$ is complex Gaussian distributed. Furthermore by using the upper and lower bounds of the outage probability in (22), the outage probability at the m-th pair during the k-th iteration can be expressed as

$$P_{\mathbf{d}_{1,k}}(\mathcal{I}_m < 2R) \doteq \frac{1}{\rho^{1-2r}}.$$

By using such a result, the outage probability for the pair with the worst performance during the k-th iteration can be expressed as

$$\begin{aligned} P_k &= \max\{P_{\mathbf{d}_{1,k}}(\mathcal{I}_1 < 2R), \cdots, P_{\mathbf{d}_{M,k}}(\mathcal{I}_M < 2R)\} \\ &\leq MP_{\mathbf{d}_{1,k}}(\mathcal{I}_m < 2R). \end{aligned}$$

On the other hand, the worst outage performance during each iteration is lower bounded as $P_k \geq P_{\mathbf{d}_{1,k}}(\mathcal{I}_m < 2R)$. As a result, we can conclude that $P_k \doteq \frac{1}{\rho^{1-2r}}$. After $(2N-L)$ iterations, the k^*-th iteration is selected since

$$k^* = \arg\min_k \{P_1, \cdots, P_{2N-L}\}.$$

Denote $P_{k^*} = \min\{P_1, \cdots, P_{2N-L}\}$. Therefore, we can have $P_{k^*} \doteq \frac{1}{\rho^{(2N-L)(1-2r)}}$ following the fact that identically and independent $\bar{\mathbf{h}}_{m,k}$ have been used during different iterations. And the lemma is proven. ■

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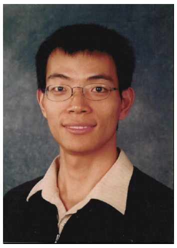

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