

★ WEEK 2

Binary Classification.

- we're doing cat classification
- final answer will be whether input image is of cat or no.

Notation.

input consists of
given data consists of (x, y)

$$(x, y) \rightarrow x \in \mathbb{R}^n, \quad y \in \{0, 1\}$$

\downarrow
vector in n dimensions

m training examples : $\{ (x^{(1)}, y^{(1)}) , (x^{(2)}, y^{(2)}) , \dots , (x^{(m)}, y^{(m)}) \}$

\downarrow m_{train} testing data = m_{test}

we combine all the input data (x) into a single matrix

$$X = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

\downarrow
 $n \times m$
matrix

how each input is represented as a column

At some places it can also be represented as a row of matrix. But "column representation" is better.

similarly

$$Y = [y^{(1)}, y^{(2)}, \dots, y^{(m)}]$$

\downarrow
 $1 \times m$
matrix

Logistic Regression - (used for binary classification problems)

Given x , we want $\hat{y} = P(y=1|x)$

given n total possible of rat.

$$x \in \mathbb{R}^n$$

parameters : $w \in \mathbb{R}^n$, $b \in \mathbb{R}$

$$\text{output } \hat{y} = \sigma(w^T x + b)$$

(sigmoid bcz $w^T x + b$ can be outside of $[0,1]$)

$$w = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

parameter coming from x_1, x_2, \dots, x_n to θ_1

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

center of universe

Logistic Regression, Cost function

$$\text{we can do } \text{Loss}^n = \frac{1}{2} (\hat{y} - y)^2$$

(predicted value - actual value) squared

but in logistic regression we usually don't do this bcz it has multiple local minima, so Logistic regression or gradient descent usually doesn't find the principal minimum.

$$\text{Loss}^n = - (y \log \hat{y} + (1-y) \log(1-\hat{y}))$$

we want to minimize

$$\text{eg:- } y=1$$

$$\text{loss}^n = -\log \hat{y}$$

minimize

maximize $\log \hat{y}$

maximize \hat{y}

since we take sigmoid max \hat{y} can be 1 only

If $y=0$

$$\text{loss} = -\log(1-y)$$

↓
minimizing $y \rightarrow$ maximizing $(1-y)$

↓
minimizing $\hat{y} \rightarrow$ min value of y
is 0 $\hat{y} \geq$ sigmoid

Loss $f^h \rightarrow$ measures how well your're doing on on training examples

Cost $f^h \rightarrow$ measures how well your're doing on entire training set

$$\text{Cost } f^h = J(w, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)})$$

$$= \frac{1}{m} \sum_{i=1}^m \left[\hat{y}^{(i)} + (1 - \hat{y}^{(i)}) \log(1 - \hat{y}^{(i)}) \right]$$