

# Q1 (40 points)

Q1(40 points) We collected a sample grade of midterm and final exam. See data below

<b><i>i</i></b>	<b>midterm, <math>x_i</math></b>	<b>final, <math>y_i</math></b>	$x_i^2$	$y_i^2$	$x_i y_i$	$\hat{y}_i$	$e_i^2$
1	1.5	2.5					
2	4	3.5					
3	3	4					
4	3	2.5					
5	4.5	5					
6	2	1.5					
<b>sum</b>							
<b>mean</b>							

Manually solve the following, do NOT use SAS ( $\alpha = 0.1$ ). Keep four digits decimals for any calculation.

1. (5 points) Fill the table.
2. (4 points) Fit the data to a simple linear regression model, calculate the least squares point estimates for  $\beta_0, \beta_1$ . And interpret the model (slop, y-intercept).
3. (2 points) Calculate the error sum of squares and the residual mean square.
4. (9 points) Calculate the 90% confidence intervals for  $\beta_0, \beta_1$ . And interpret the confidence interval of the slop.
5. (3 points) Apply a *t*-test to test  $H_0 : \beta_0 = 0$ ,  $H_a : \beta_0 \neq 0$ . Solve using reject point.
6. (4 points) Apply a *t*-test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Solve using *P*-value.
7. (1 points) Calculate the prediction value of  $y$ , given  $x = 3$ .
8. (3 points) Calculate the 90% confidence interval for the expected value of  $y$ , given  $x = 3$ .
9. (2 points) Calculate the 90% prediction interval for  $y$ , given  $x = 3$ .
10. (4 points) Calculate the simple coefficient of determination and the simple correlation coefficient based on the regression model. And use the simple correlation coefficient to interpret the relations between midterm grade and final grade.
11. (3 points) Apply a *F*-test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Note  $F_{0.1}^{(1,4)} = 4.5448$ .

Q1(40 points) We collected a sample grade of midterm and final exam. See data below

$i$	midterm, $x_i$	final, $y_i$	$x_i^2$	$y_i^2$	$x_i y_i$	$\hat{y}_i$	$e_i^2$
1	1.5	2.5	2.25	6.25	3.75	1.8397	0.4399
2	4	3.5	16	12.25	14	4.0513	0.3039
3	3	4	9	16	12	3.1667	0.6944
4	3	2.5	9	6.25	7.5	3.1667	0.4444
5	4.5	5	20.25	25	22.5	4.4936	0.2565
6	2	1.5	4	2.25	3	2.2821	0.6116
<b>sum</b>	$\sum x_i = 18$	$\sum y_i = 19$	60.5	68	62.75	19	2.7468 $SS_{Res}$
<b>mean</b>	$\bar{x} = 3$	$\frac{19}{6} \approx 3.1667$	$\frac{60.5}{6} \approx 10.0833$	$\frac{68}{6} \approx 11.3333$	$\frac{62.75}{6} \approx 10.4583$	$\bar{y} \approx 3.1667$	$0.4578$

1. (5 points) Fill the table.

2. (4 points) Fit the data to a simple linear regression model, calculate the least squares point estimates for  $\beta_0, \beta_1$ . And interpret the model (slop, y-intercept).

$$\hat{\beta}_1 = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{6(62.75) - 18(19)}{6(60.5) - (18)^2}$$

$$\approx 0.8846$$

$$\begin{aligned}\hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \\ &= \frac{19}{6} - \left[ \frac{6(62.75) - 18(19)}{6(60.5) - (18)^2} \right] (3) \\ &\approx 0.5128\end{aligned}$$

fitted simple linear regression:

$$\hat{y}_i = 0.5128 + 0.8846 x_i$$

$$y\text{-intercept: } \hat{\beta}_0 = 0.5128$$

$$\text{slope: } \hat{\beta}_1 = 0.8846$$

#### Interpretation:

Every 1 mark increase for midterm, the final grade will increase 0.8846 mark.

#### Simple linear regression model

$$\begin{aligned}y_i &= \beta_0 + \beta_1 x_i + \varepsilon \\ &= \mu_i + \varepsilon\end{aligned}$$

3. (2 points) Calculate the error sum of squares and the residual mean square.

$$\begin{aligned}SS_{Res} &= \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \\ &= \sum_{i=1}^6 e_i^2\end{aligned}$$

$$= 0.4399 + 0.3039 + 0.6944 + 0.4444 + 0.2565 + 0.6116$$

$$\approx 2.7468$$

residual mean square:

$$\hat{\sigma}^2 = MS_{Res} = \frac{SS_{Res}}{n-2},$$

$$\begin{aligned}&= \frac{2.7468}{6-2} \\ &= \frac{2.7468}{4}\end{aligned}$$

$$\approx 0.6867$$

$e_i$

0.6603

-0.5513

0.8333

-0.6667

0.5064

-0.7821

-0.0001

-0.000017

sum

mean

$\ell_i = y_i - \hat{y}_i$

4. (9 points) Calculate the 90% confidence intervals for  $\beta_0, \beta_1$ . And interpret the confidence interval of the slope.

$$\hat{y}_i = 0.5128 + 0.8846 x_i$$

$$\bar{x} = 3$$

$$\sum x_i^2 = 60.5$$

$$C_{11} = \frac{1}{\sum (x_i - \bar{x})^2}$$

$$= \frac{1}{\sum x_i^2 - n \bar{x}^2}$$

$$= \frac{1}{60.5 - 6(3)^2}$$

$$\approx 0.1538$$

$$SS_{\text{Res}} = 2.7468$$

$$\text{Given } \alpha = 0.1$$

$$t_{\alpha/2(n-2)} = t_{0.05}(4) = 2.1318$$

$$\hat{\sigma} = \sqrt{MS_{\text{Res}}} = \sqrt{\frac{SS_{\text{Res}}}{n-2}} = \sqrt{0.6867} \approx 0.8287$$

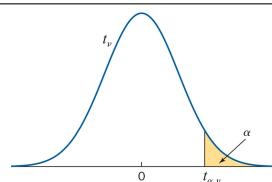
$$\begin{aligned} \therefore 90\% \text{ CI for } \beta_1 \text{ is } & \hat{\beta}_1 \pm t_{\alpha/2(n-2)} \hat{\sigma} \sqrt{C_{11}} \\ & = 0.8846 \pm 2.1318 (0.8287) \sqrt{0.1538} \\ & = (0.1917, 1.5775) \end{aligned}$$

#### Interpretation of the CI of the slope:

- We are 90% confident that if the midterm grade increases by 1 mark, the final grade will increase by at least 0.1917 marks and at most 1.5775 marks.
- Zero doesn't fall within the CI, so there is evidence that the midterm grade and the final grade has a linear relation.

**Table V** Critical Values for the t Distribution

This table contains critical values associated with the  $t$  distribution,  $t_{\alpha, \nu}$ , defined by  $\alpha$  and the degrees of freedom,  $\nu$ .



$\nu$	0.20	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.3764	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192	3183.0988
2	1.0607	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991	70.7001
3	0.9785	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240	22.2037
4	0.9410	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103	13.0337

$$\begin{aligned} C_{11} &= \frac{1}{n} + \frac{\bar{x}^2}{\sum (x_i - \bar{x})^2} \\ &= \frac{1}{n} + \frac{\bar{x}^2}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{1}{6} + \frac{3^2}{60.5 - 6(3)^2} \end{aligned}$$

$$\approx 1.5513$$

$\therefore 90\% \text{ CI for } \beta_0 \text{ is}$

$$\begin{aligned} \hat{\beta}_0 &\pm t_{\alpha/2(n-2)} \hat{\sigma} \sqrt{C_{00}} \\ &= 0.5128 \pm 2.1318 (0.8287) \sqrt{1.5513} \\ &= (-1.6874, 2.7131) \end{aligned}$$

5. (3 points) Apply a  $t$ -test to test  $H_0 : \beta_0 = 0$ ,  $H_a : \beta_0 \neq 0$ . Solve using reject point.

6. (4 points) Apply a  $t$ -test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Solve using  $P$ -value.

} Given  
 $\alpha = 0.1$

Q5

$$H_0 : \beta_0 = 0, H_a : \beta_0 \neq 0$$

$$\hat{y}_i = 0.5128 + 0.8846 x_i$$

$$C_{00} \approx 1.5513$$

$$\hat{\sigma} \approx 0.8287$$

$$\begin{aligned} \text{Test statistics : } t &= \frac{\hat{\beta}_0 - \beta_{00}}{\hat{\sigma} \sqrt{C_{00}}} \\ &= \frac{0.5128 - 0}{0.8287 \sqrt{1.5513}} \\ &\approx 0.4969 \end{aligned}$$

$$\text{reject point : } t_{\alpha/2}(n-2) = t_{0.05}(4) = 2.1318$$

$|t| = 0.4969 < 2.1318$  doesn't fall in the reject region, so fail to reject  $H_0$ .

Q6

$$H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$$

$$\hat{y}_i = 0.5128 + 0.8846 x_i$$

$$C_{11} \approx 0.1538$$

$$\hat{\sigma} \approx 0.8287$$

$$\begin{aligned} t &= \frac{\hat{\beta}_1 - \beta_{10}}{\hat{\sigma} \sqrt{C_{11}}} \\ &= \frac{0.8846 - 0}{0.8287 \sqrt{0.1538}} \\ &\approx 2.7216 \end{aligned}$$

$$P\text{-value} = 2 \cdot P(T > |t|)$$

$$= 2 \cdot P(T > 2.7216)$$

$$P(T > 2.7216) < P(T > 2.1318) = P(T > t_{0.05}(4))$$

$$\therefore P(T > 2.7216) < \alpha = 0.05$$

$$P\text{-value} < 2(0.05) = 0.1 = \alpha$$

So, reject  $H_0$ ,  $x$  has a significant effect on  $y$ .

### Table V Critical Values for the $t$ Distribution

This table contains critical values associated with the  $t$  distribution,  $t_{\alpha, \nu}$ , defined by  $\alpha$  and the degrees of freedom,  $\nu$ .

$\nu$	0.20	0.10	0.05	0.025	0.01	$\alpha$
1	1.3764	3.0777	6.3138	12.7062	31.8205	
2	1.0607	1.8856	2.9200	4.3027	6.9646	
3	0.9785	1.6377	2.3534	3.1824	4.5407	
4	0.9410	1.5332	2.1318	2.7764	3.7469	
5	0.9195	1.4759	2.0150	2.5706	3.3649	

7. (1 points) Calculate the prediction value of  $y$ , given  $x = 3$ .

$$\hat{y}_i = 0.5128 + 0.8846 x_i$$

$$\hat{y}_3 = 0.5128 + 0.8846(3)$$

$$\approx 3.1667$$

8. (3 points) Calculate the 90% confidence interval for the expected value of  $y$ , given  $x = 3$ .

$$\text{Given } x = 3, \hat{y}_3 = 0.5128 + 0.8846(3) \approx 3.1667$$

$$t_{\alpha/2}(n-2) = t_{0.05}(4) = 2.1318$$

$$\hat{\sigma} = \sqrt{MS_{\text{Res}}} = \sqrt{\frac{SS_{\text{Res}}}{n-2}} = \sqrt{0.6867} \approx 0.8287$$

$$h_{\infty} = \frac{1}{n} + \frac{(x_3 - \bar{x})^2}{\sum(x_i - \bar{x})^2}$$

$$= \frac{1}{6} + \frac{(3 - 3)^2}{\sum x_i^2 - n\bar{x}^2}$$

$$= \frac{1}{6} + \frac{(3 - 3)^2}{60.5 - 6(3)^2}$$

$$\approx 0.1667$$

90% CI for the given  $x = 3$

$$\hat{y}_3 \pm t_{\alpha/2}(n-2) \hat{\sigma} \sqrt{h_{\infty}}$$

$$= 3.1667 \pm 2.1318 (0.8287) \sqrt{0.1667}$$

$$= (2.4455, 3.8879)$$

9. (2 points) Calculate the 90% prediction interval for  $y$ , given  $x = 3$ .

$$\text{Given } x=3, \hat{y}_3 = 0.5128 + 0.8846(3) \approx 3.1667, h_{\infty} \approx 0.1667$$

$$t_{\alpha/2}(n-2) = t_{0.05}(4) = 2.1318, \hat{\sigma} \approx 0.8287$$

$$90\% \text{ P.I. for } \hat{y}_3 : 3.1667 \pm 2.1318(0.8287)\sqrt{1+0.1667} = (1.2586, 5.0748)$$

10. (4 points) Calculate the simple coefficient of determination and the simple correlation coefficient based on the regression model. And use the simple correlation coefficient to interpret the relations between midterm grade and final grade.

Simple coefficient of determination  $R^2$  :

$$\begin{aligned} \hat{y}_i &= 0.5128 + 0.8846x_i \\ \sum y_i &= 19, \quad \sum y_i^2 = 68 \\ \bar{y} &= \frac{19}{6} \approx 3.1667 \\ \sum x_i y_i &= 62.75 \end{aligned}$$

$$\left. \begin{aligned} &\hat{\beta}_0 \sum y_i + \hat{\beta}_1 \sum x_i y_i \\ &= 0.5128(19) + 0.8846(62.75) \\ &= 65.2532 \end{aligned} \right\}$$

$$SS_{\text{Res}} = 2.7468$$

$$\begin{aligned} SS_T &= \sum y_i^2 - \bar{y}^2 \\ &= 68 - 6\left(\frac{19}{6}\right)^2 \\ &= 7.8333 \end{aligned}$$

$$\begin{aligned} SS_R &= SS_T - SS_{\text{Res}} \\ &= 7.8333 - 2.7468 \\ &\approx 5.0865 \end{aligned}$$

$$R^2 = \frac{SS_R}{SS_T} = \frac{5.0865}{7.8333} \approx 0.6493$$

Simple correlation coefficient r :

$$R^2 \approx 0.6493$$

$$r = \sqrt{R^2} = \sqrt{0.6493} \approx 0.8058$$

Interpretation:

Since  $r = 0.8058$  is close to 1, the midterm grade and the final grade are highly positively correlated.

11. (3 points) Apply a F-test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Note  $F_{0.1}^{(1,4)} = 4.5448$ .

$$H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$$

F-test :

$$\text{T.S. } F(\text{model}) = \frac{SS_R / 1}{SS_{\text{res}} / (n - 2)}$$

$$= \frac{5.0865 / 1}{2.7468 / 4}$$

$$\approx 7.4072$$

$$\text{R.P.} : F_{\alpha}(1, n-2) = F_{0.1}(1, 4) = 4.5448$$

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$$\frac{4.5448}{F_{\alpha}(1, n-2)} \quad 7.4072 \quad \text{T.S.} \in R.R \rightarrow \text{reject } H_0 \rightarrow \text{significant} \rightarrow \text{good fit}$$

## Q2 (24 points)

Q2(24 points). Apply SAS to work on the following sample data, both SAS code and output are required.

	$x_i$	$y_i$
1	8.3	1.3
2	3.4	18.3
3	4.5	14.4
4	6.0	9.0
5	6.1	9.5
6	6.4	7.9
7	5.8	10.2
8	4.8	13.9
9	7.1	5.5
10	9.1	-1.1
11	8.7	0.0
12	5.0	12.9
13	8.2	2.2
14	3.6	17.6
15	8.9	-0.2
16	5.7	10.7
17	6.1	9.4
18	5.7	10.5
19	4.2	15.1
20	4.2	15.9

$\alpha = 0.01$  Find the following information from the SAS output.

- (2 points) Fit the data to a simple linear regression model, find the least squares point estimates for  $\beta_0, \beta_1$ .
- (1 points) Plot of the data with the regression line.
- (4 points) Find  $SS_T, SS_R, SS_{Res}, \hat{\sigma}^2$ .
- (2 points) Find the 99% confidence intervals for  $\beta_0, \beta_1$ .
- (3 points) Apply a  $t$ -test to test  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$ . Specify the test statistic's value and  $P$ -value, and make conclusion.
- (3 points) Apply a  $t$ -test to test  $H_0 : \beta_0 = 0, H_a : \beta_0 \neq 0$ . Specify the test statistic's value and  $P$ -value, and make conclusion.
- (3 points) Find the prediction value, 99% confidence interval, and 99% prediction interval for  $y$ , given  $x = 7.1$ .
- (1 points) Find the simple coefficient of determination.
- (3 points) Apply a  $F$ -test to test  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$ . Specify the test statistic's value and  $P$ -value, and make conclusion.
- (2 points) Find the value of  $\hat{\sigma}\sqrt{c_{00}}, \hat{\sigma}\sqrt{c_{11}}$ .
- (1 points) Find  $\hat{\sigma}\sqrt{h_{00}}$  and residual for  $x = 7.1$ .
- (1 points) Compare the test statistics' value in 5 and 9, what is the relation between these two values?

```
data A1_Q2; /* Assigns name A1_Q2 to file*/
input x y; /* x; y */
cards; /* start to input data */
8.3 1.3
3.4 18.3
4.5 14.4
6.0 9.0
6.1 9.5
6.4 7.9
5.8 10.2
4.8 13.9
7.1 5.5
9.1 -1.1
8.7 0.0
5.0 12.9
8.2 2.2
3.6 17.6
8.9 -0.2
5.7 10.7
6.1 9.4
5.7 10.5
4.2 15.1
4.2 15.9
; /* inputs data, .= missing value*/
proc reg data = A1_Q2; /* specifies regression procedure */
model y = x /alpha=0.01 p clm cli clb;
run;
proc reg data = A1_Q2; /* specifies regression procedure */
model x = y ;
output out = A1_Q2new residual = e p = yhat; /*output residual and prediction value to a new data object, "e" and "yhat"
are two new columns names, ddnew is the new data object name*/
run;
```

## The REG Procedure

Model: MODEL1

Dependent Variable: y

Number of Observations Read	20
Number of Observations Used	20

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model $SS_R$	1	705.49198	705.49198	9490.75	<.0001
Error $SS_E$	18	1.33802	0.07433	$\hat{\sigma}^2 = MS_{Error}$	
Corrected Total $SS_T$	19	706.83000			

Root MSE	0.27264	R-Square	0.9981
Dependent Mean	9.15000	Adj R-Sq	0.9980
Coeff Var	2.97971		

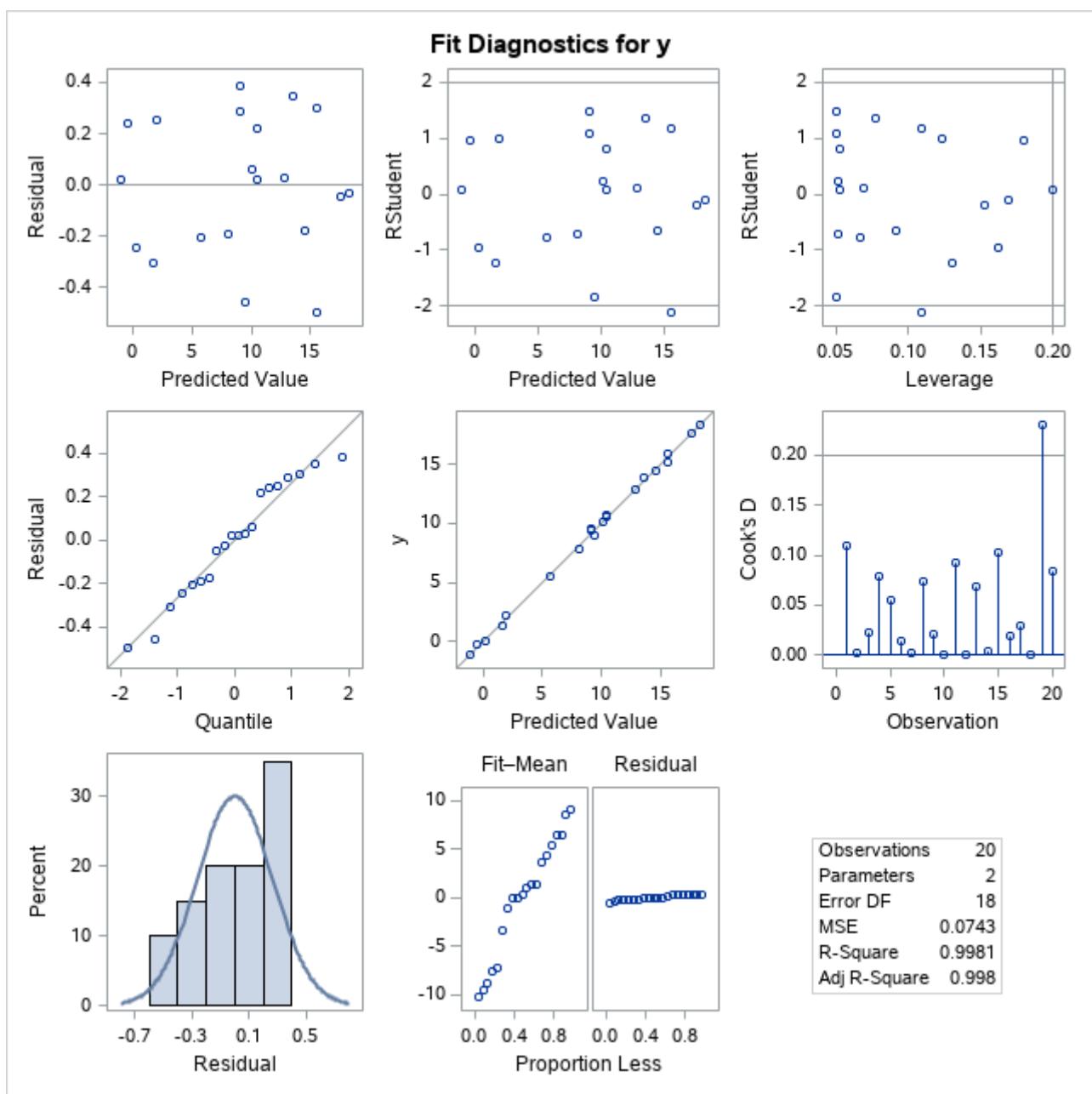
Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t	99% Confidence Limits
Intercept	1	29.93289	0.22187	134.91	<.0001	29.29424 30.57153
x	1	-3.41263	0.03503	-97.42	<.0001	-3.51346 -3.31179

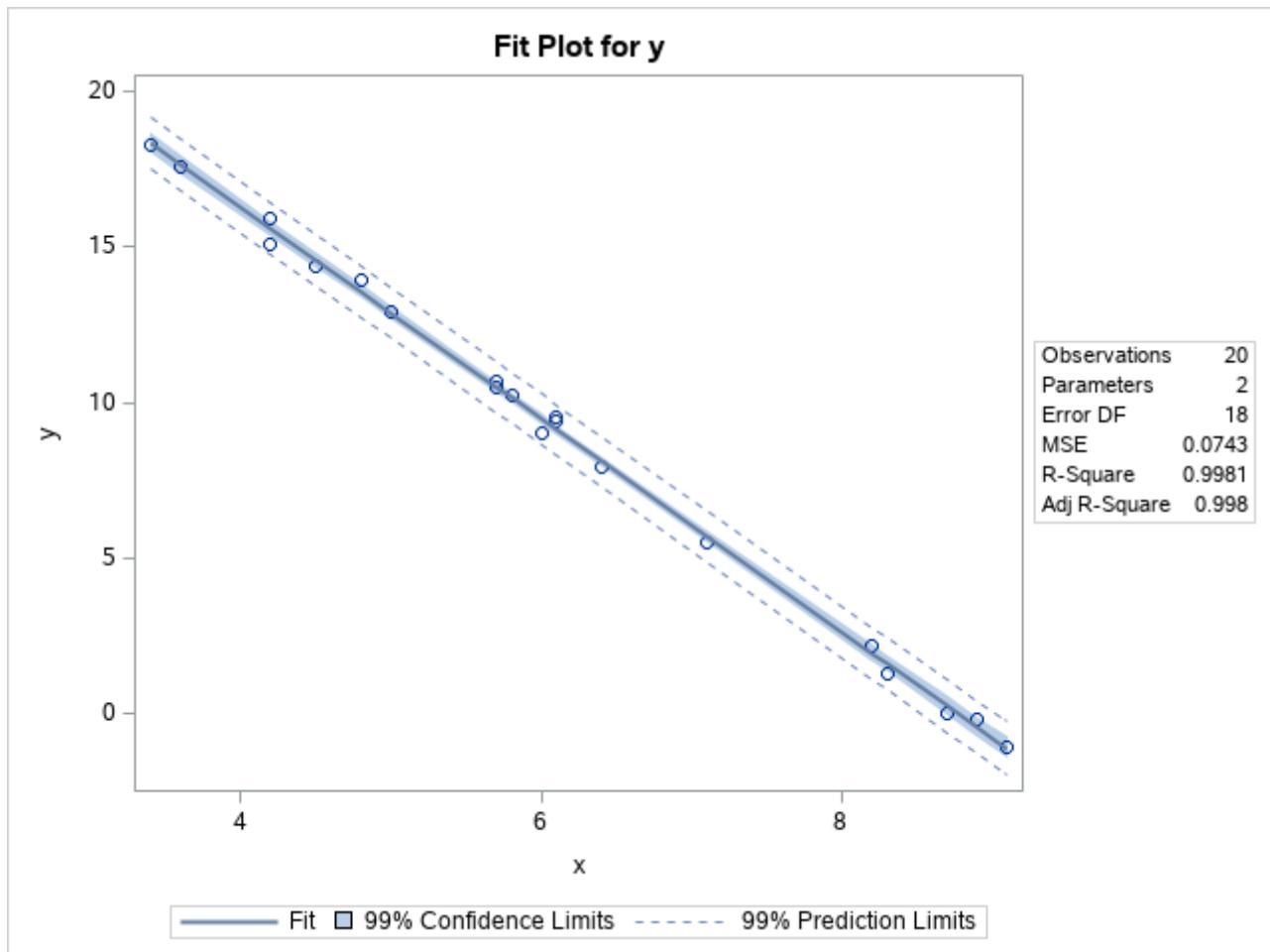
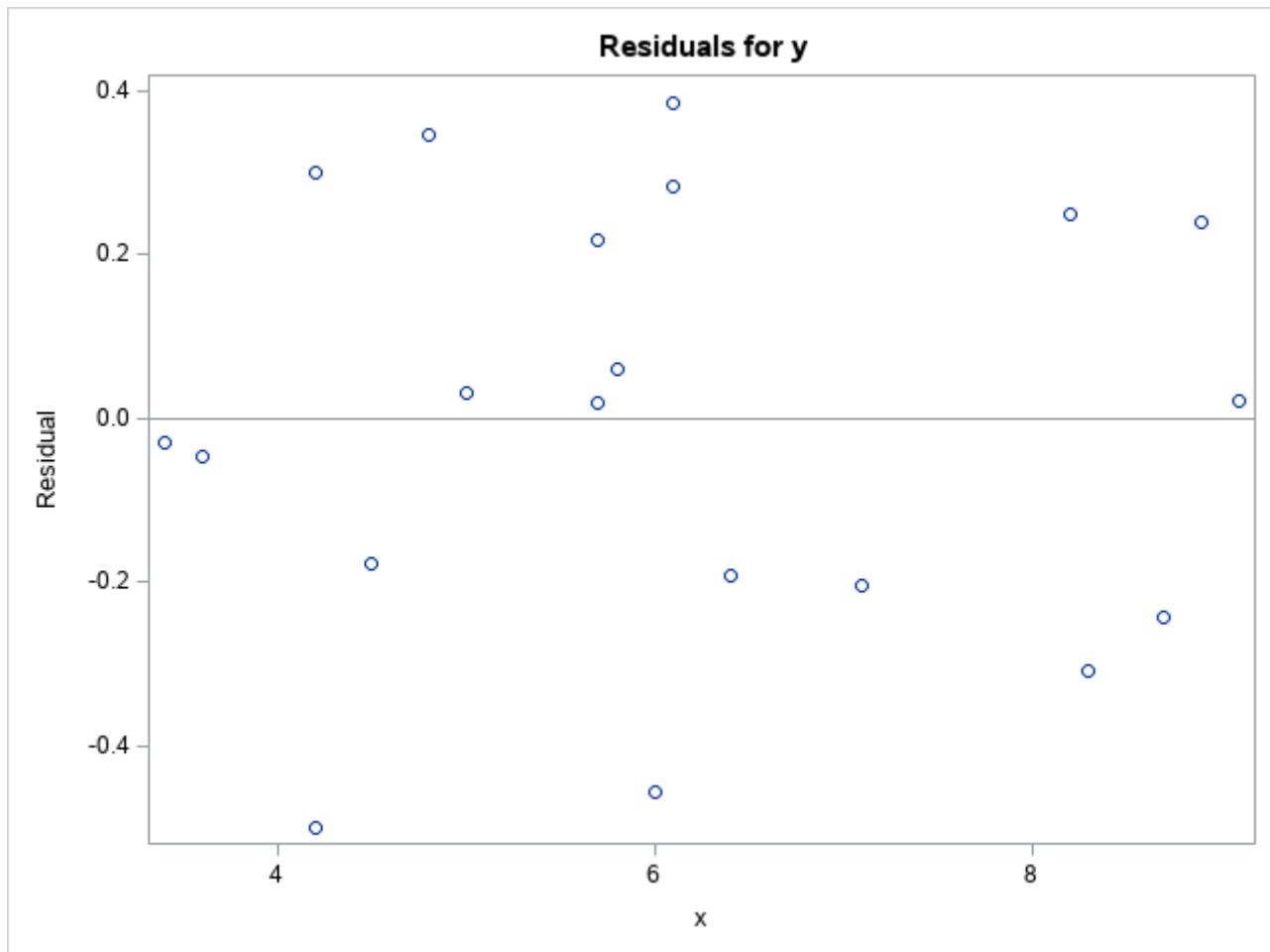
$$\text{① } \text{② } \hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

The REG Procedure  
Model: MODEL1  
Dependent Variable: y

Output Statistics								
Obs	Dependent Variable	Predicted Value	Std Error Mean Predict	CI $\mu_0$		PI $y$		$e_i$ Residual
				99% CL Mean	99% CL Predict			
1	1.3	1.6081	0.0985	1.3245	1.8917	0.7736	2.4426	-0.3081
2	18.3	18.3300	0.1122	18.0069	18.6530	17.4813	19.1786	-0.0300
3	14.4	14.5761	0.0826	14.3384	14.8138	13.7561	15.3961	-0.1761
4	9.0	9.4571	0.0610	9.2814	9.6329	8.6529	10.2614	-0.4571
5	9.5	9.1159	0.0610	8.9404	9.2914	8.3117	9.9200	0.3841
6	7.9	8.0921	0.0619	7.9138	8.2703	7.2873	8.8969	-0.1921
7	10.2	10.1397	0.0618	9.9618	10.3176	9.3350	10.9444	0.0603
8	13.9	13.5523	0.0759	13.3339	13.7707	12.7377	14.3669	0.3477
9	5.5	5.7032	0.0705	5.5004	5.9061	4.8927	6.5138	-0.2032
10	-1.1	-1.1220	0.1218	-1.4726	-0.7714	-1.9815	-0.2625	0.0220
11	0.0	0.2430	0.1099	-0.0733	0.5594	-0.6031	1.0892	-0.2430
12	12.9	12.8698	0.0719	12.6627	13.0768	12.0581	13.6814	0.0302
13	2.2	1.9494	0.0958	1.6736	2.2251	1.1175	2.7812	0.2506
14	17.6	17.6474	0.1064	17.3411	17.9538	16.8050	18.4899	-0.0474
15	-0.2	-0.4395	0.1158	-0.7728	-0.1062	-1.2921	0.4131	0.2395
16	10.7	10.4809	0.0625	10.3011	10.6608	9.6758	11.2861	0.2191
17	9.4	9.1159	0.0610	8.9404	9.2914	8.3117	9.9200	0.2841
18	10.5	10.4809	0.0625	10.3011	10.6608	9.6758	11.2861	0.0191
19	15.1	15.5999	0.0900	15.3408	15.8589	14.7734	16.4263	-0.4999
20	15.9	15.5999	0.0900	15.3408	15.8589	14.7734	16.4263	0.3001

Sum of Residuals	0
Sum of Squared Residuals	1.33802





**The REG Procedure**  
**Model: MODEL1**  
**Dependent Variable: x**

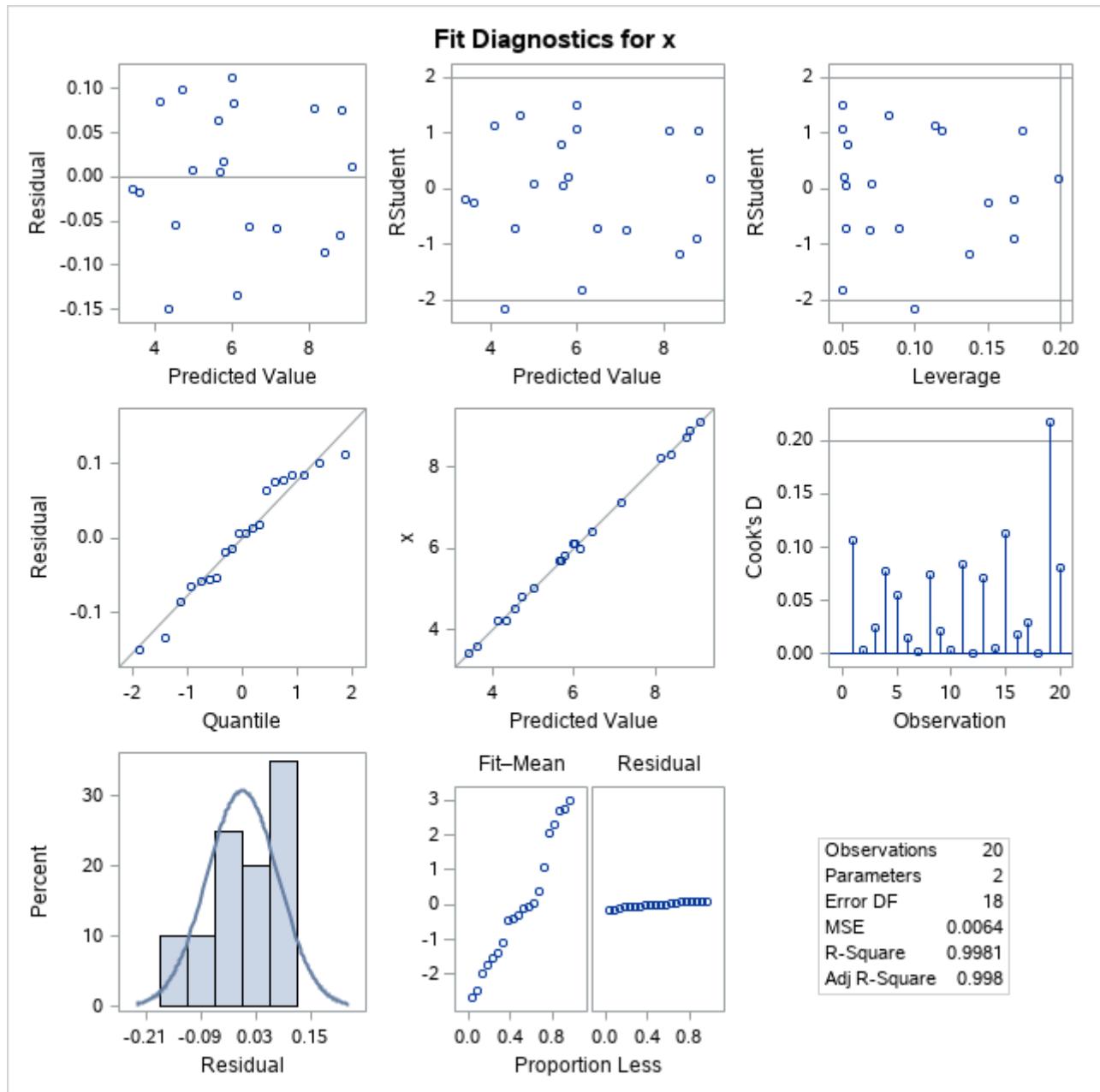
Number of Observations Read	20
Number of Observations Used	20

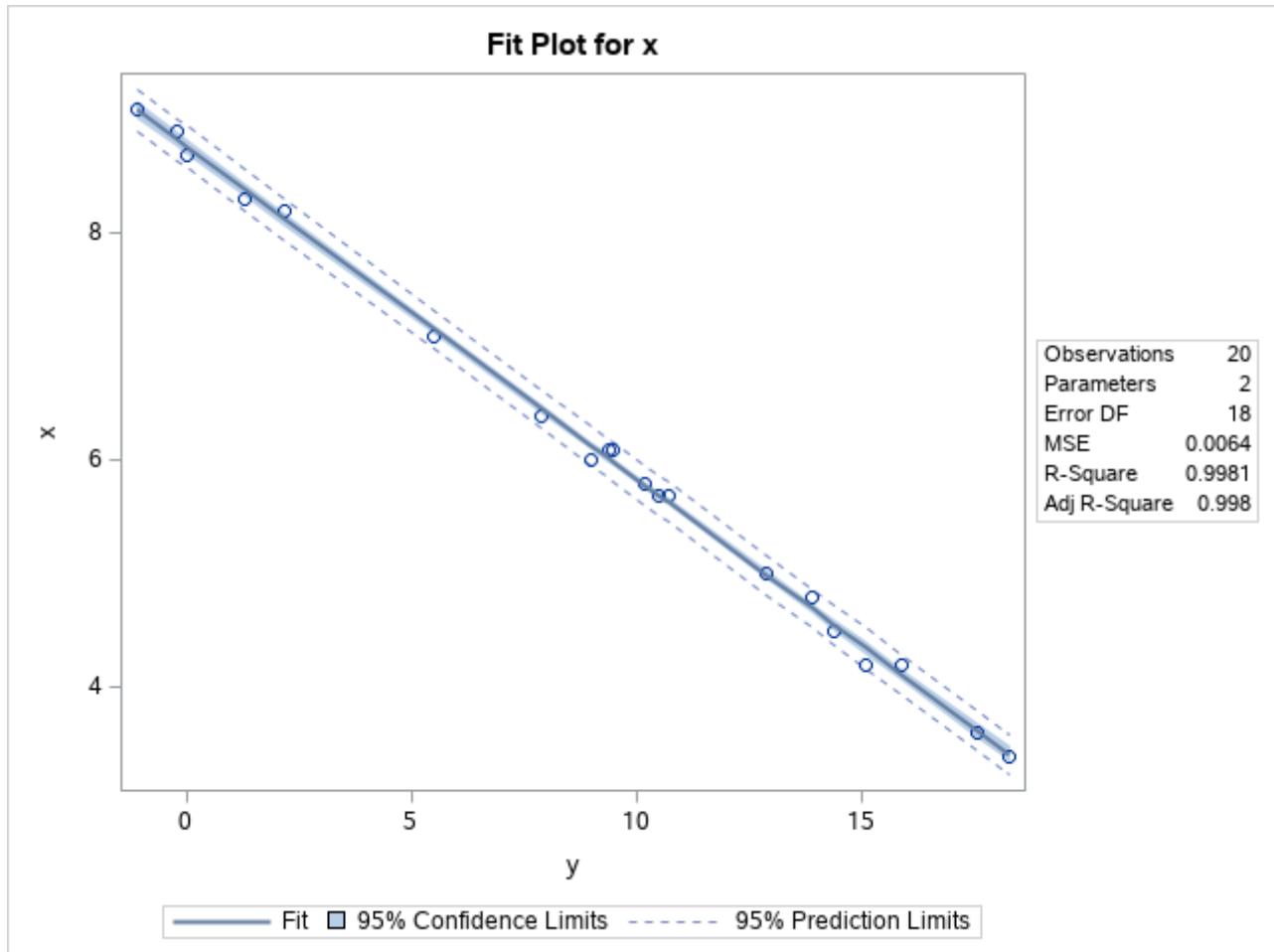
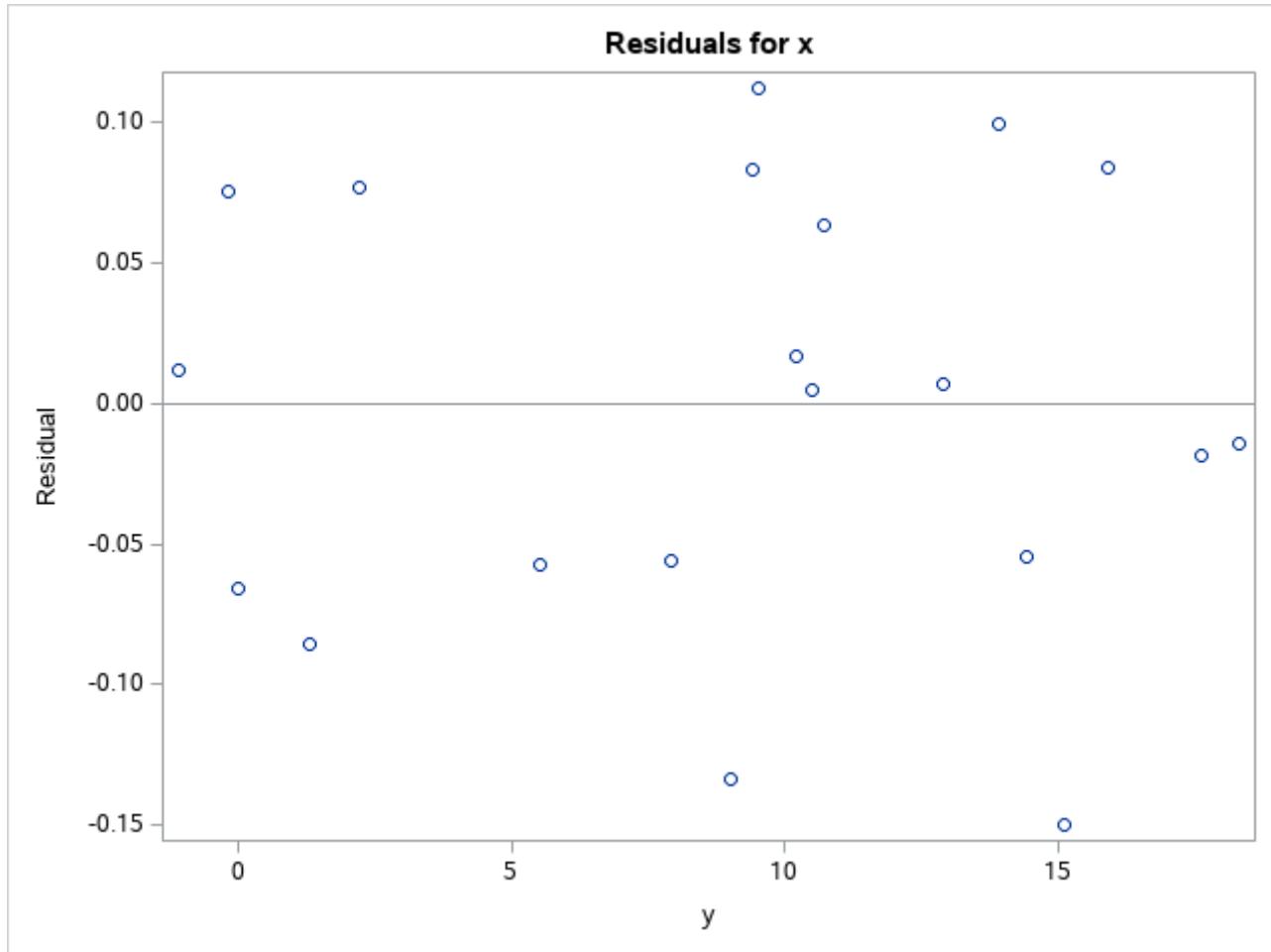
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	60.46333	60.46333	9490.75	<.0001
Error	18	0.11467	0.00637		
Corrected Total	19	60.57800			

Root MSE	0.07982	R-Square	0.9981
Dependent Mean	6.09000	Adj R-Sq	0.9980
Coeff Var	1.31062		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	8.76614	0.03276	267.60	<.0001
y	1	-0.29247	0.00300	-97.42	<.0001

**The REG Procedure**  
**Model: MODEL1**  
**Dependent Variable: x**

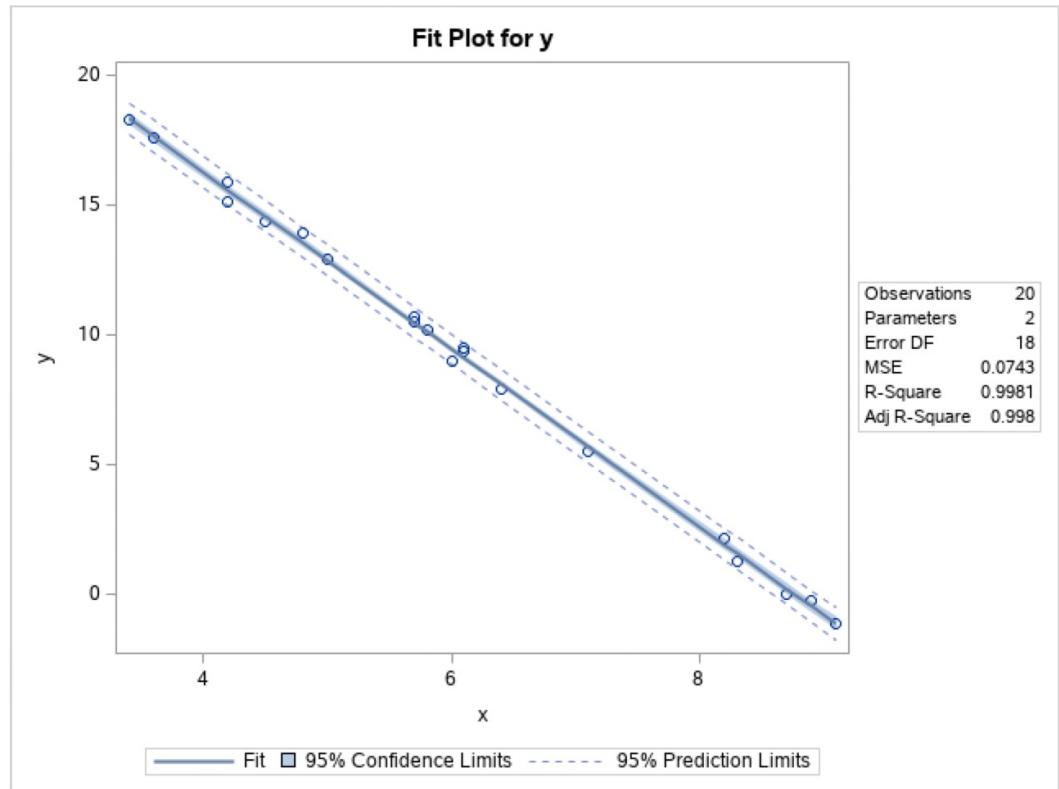




1. (2 points) Fit the data to a simple linear regression model, find the least squares point estimates for  $\beta_0, \beta_1$ .  
 $\beta_0 = 29.9329$ ,  $\beta_1 = -3.4126$

$$\hat{y}_i = 29.9329 - 3.4126x_i$$

2. (1 points) Plot of the data with the regression line.



3. (4 points) Find  $SS_T, SS_R, SS_{Res}, \hat{\sigma}^2$ .

$$SS_T = 706.8300$$

$$SS_R = 705.4920$$

$$SS_{Res} = 1.33802$$

$$\hat{\sigma}^2 = MS_{Res} = 0.0743$$

4. (2 points) Find the 99% confidence intervals for  $\beta_0, \beta_1$ .

$$\beta_0: (29.2942, 30.5715)$$

$$\beta_1: (-3.5135, -3.3118)$$

5. (3 points) Apply a  $t$ -test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Specify the test statistic's value and  $P$ -value, and make conclusion.
6. (3 points) Apply a  $t$ -test to test  $H_0 : \beta_0 = 0$ ,  $H_a : \beta_0 \neq 0$ . Specify the test statistic's value and  $P$ -value, and make conclusion.

Q5

$$H_0 : \beta_0 = 0, H_a : \beta_0 \neq 0$$

$$\text{Test statistics : } t = 134.91$$

$$P\text{-value} < 0.0001$$

$P$ -value  $< 0.0001 < \alpha = 0.01$  (given), so we reject the null hypothesis, and conclude that  $x$  and  $y$  has a significant linear relations.

Q6

$$H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0$$

$$t = -97.42$$

$$P\text{-value} < 0.0001$$

$P$ -value  $< 0.0001 < \alpha = 0.01$  (given), so we reject the null hypothesis, and conclude that  $x$  and  $y$  has a significant linear relations.

7. (3 points) Find the prediction value, 99% confidence interval, and 99% prediction interval for  $y$ , given

$$x = 7.1$$

$$x_g = 7.1, y_g = 5.5$$

$$\text{Predicted value} = 5.7032$$

$$99\% \text{ CI Mean} = (5.5004, 5.9061)$$

$$99\% \text{ PI} = (4.8927, 6.5138)$$

8. (1 points) Find the simple coefficient of determination.

$$R\text{-square} = 0.9981$$

9. (3 points) Apply a  $F$ -test to test  $H_0 : \beta_1 = 0$ ,  $H_a : \beta_1 \neq 0$ . Specify the test statistic's value and  $P$ -value and make conclusion.

$F$  Value = 9490.75

$\Pr > F = < 0.0001$

Since the  $P$ -value is less than alpha = 0.01, we reject the null hypothesis and conclude that the model is a good fit, there is a significant linear relation between  $x$  and  $y$ .

standard error for  $\beta_0$

10. (2 points) Find the value of  $\hat{\sigma} \sqrt{c_{00}}$ ,  $\hat{\sigma} \sqrt{c_{11}}$ .

$$\hat{\sigma} \sqrt{c_{00}} = 0.22187$$

$$\hat{\sigma} \sqrt{c_{11}} = 0.03503$$

11. (1 points) Find  $\hat{\sigma} \sqrt{h_{00}}$  and residual for  $x = 7.1$ .

std Error Mean Predict

$e_i$

$$\hat{\sigma} \sqrt{h_{00}} = 0.0705, e_9 = -0.2032$$

12. (1 points) Compare the test statistics' value in 5 and 9, what is the relation between these two values?

$$t = -97.42, F(\text{model}) = 9490$$

$$t^2 = (-97.42)^2 = 9490 = F(\text{model})$$

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