All-optical Machine Learning using Diffractive Deep Neural Networks

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PH 202
Waves

“Motivation”

Even the most powerful computers are still no match for the human brain when it comes to pattern recognition, risk management, and other similarly complex tasks. Recent advances in optical neural networks, however, are closing that gap by simulating the way neurons respond in the human brain. In a key step toward making large-scale optical neural networks practical, researchers have demonstrated a first-of-its-kind multilayer all-optical artificial neural network. Generally, this type of artificial intelligence can tackle complex problems that are impossible with traditional computational approaches, but current designs require extensive computational resources that are both time-consuming and energy intensive. For this reason, there is great interest developing practical optical artificial neural networks, which are faster and consume less power than those based on traditional computers.

“Introduction”

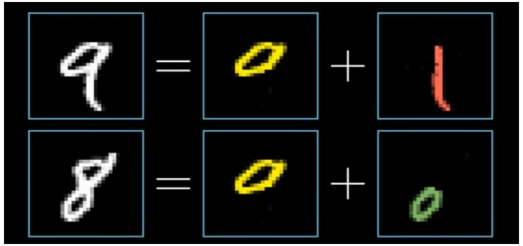
- All-optical diffractive deep neural network (D^2 NN) architecture can implement various functions following the deep learning–based design of passive diffractive layers that work collectively.

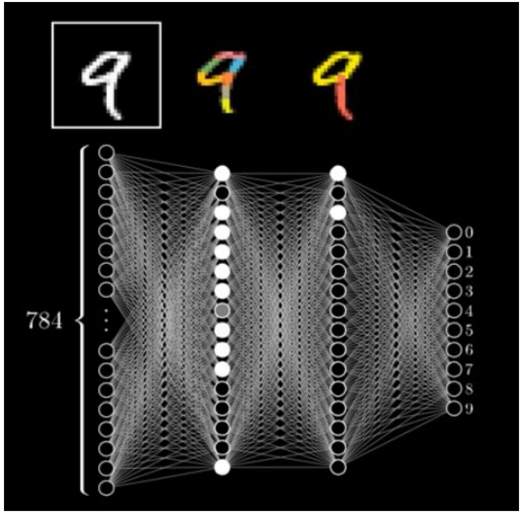
- The 3D-printed D^2 NNs can implement classification of images of handwritten digits and fashion products, as well as the function of an imaging lens at a terahertz spectrum.

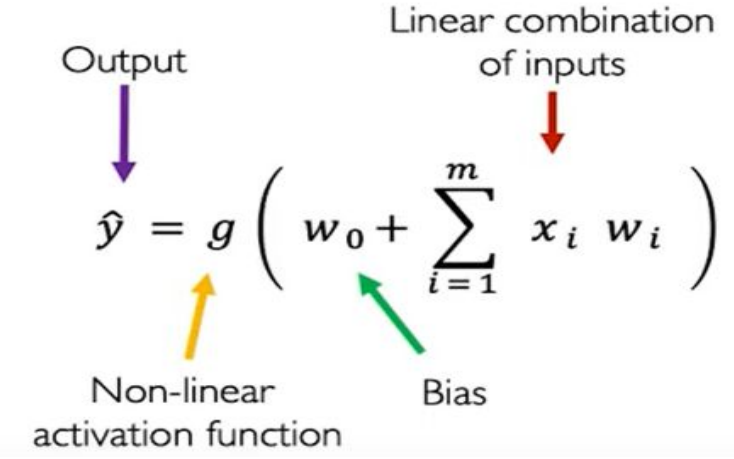
- This all-optical deep learning framework will find applications in all-optical image analysis, feature detection, and object classification.

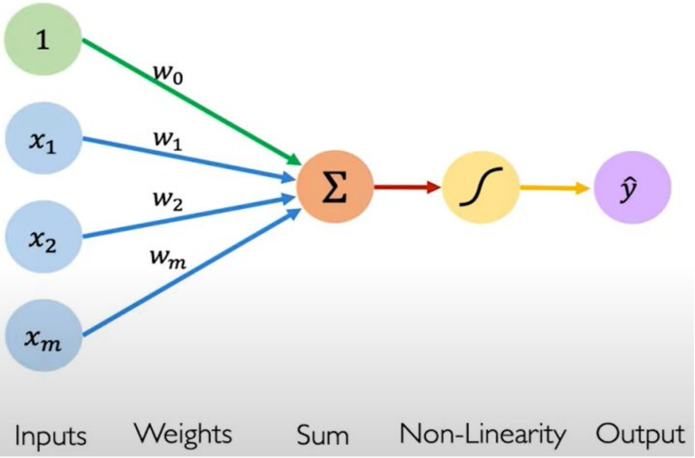
- The inference and prediction mechanism of the physical network is all optical, the learning part (training) that leads to its design is done through a computer.

“Neural Networks in Deep Learning”







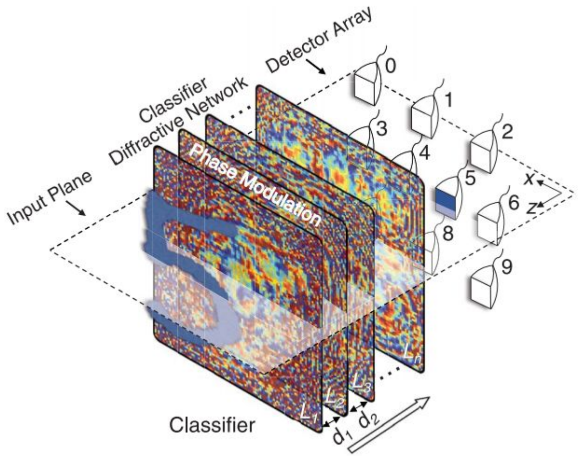


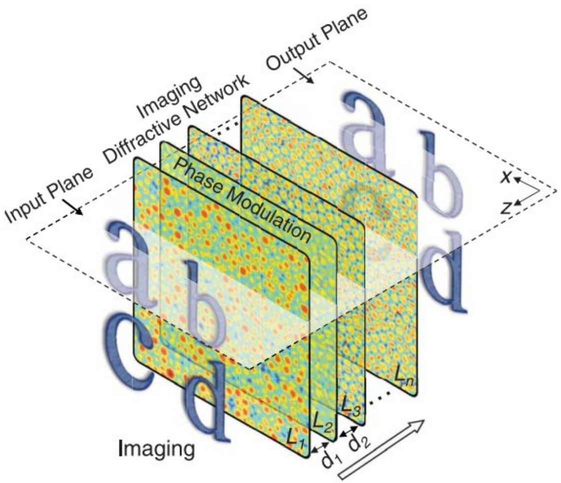
"Setup"

- A D^2 NN comprises multiple transmissive (or reflective) layers, where each point on a given layer acts as a neuron, with a complex-valued transmission (or reflection) coefficient.

- The transmission or reflection coefficients of each layer can be trained by using deep learning to perform a function between the input and output planes of the network.

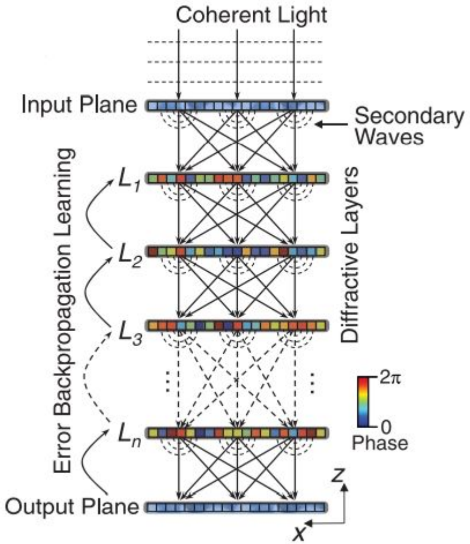
1. Passive Layers
2. Calculates at the speed of light
3. Scalable

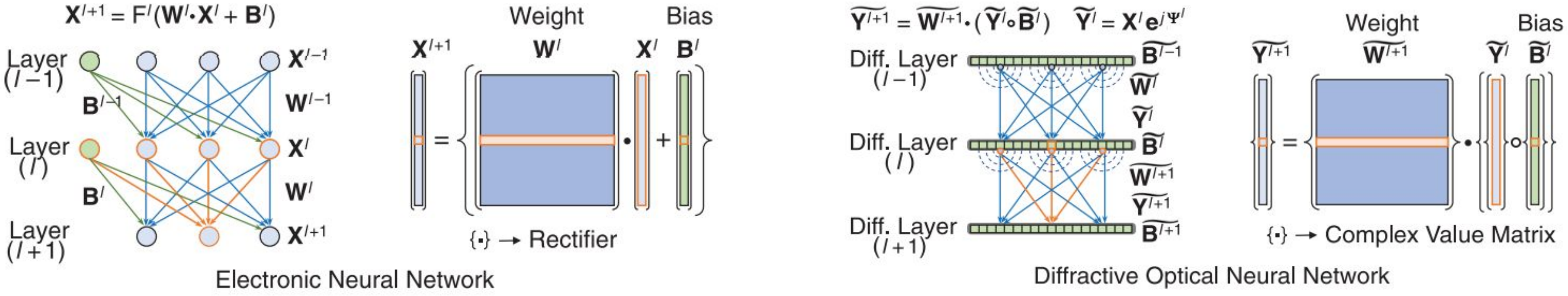




"Diffractive Deep Neural Networks"

- **Input:** Coherent Light Source (Laser)
- **Layers:** Transmissive/Reflected 3D printed filters
- **Output:** Intensity profile of filtered light

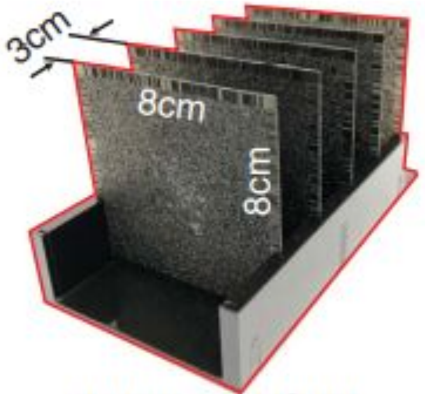




"∘" denotes a Hadamard product operation; **Y**, optical field at a given layer; **Ψ**, phase of the optical field; **X**, amplitude of the optical field; **F,** nonlinear rectifier function

“Experiment : MNIST”
(Modified National Institute of Standards and Technology)

- Trained to map input digits to ten detector regions
- Digits encoded into amplitude of input field
- Five layers 8cm X 8cm
- Separated by 3cm
- Neurons 400µm X 400µm
- Numerical Simulations (done in paper): **91.75%**
- Adding two more layers ‘transfer learning’ (done in paper) : **93.39%**



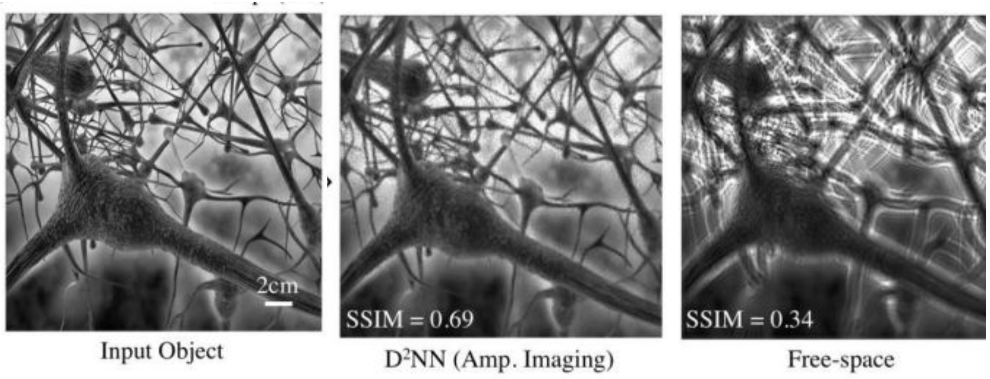
3D Printed D^2NN
(Classifier)

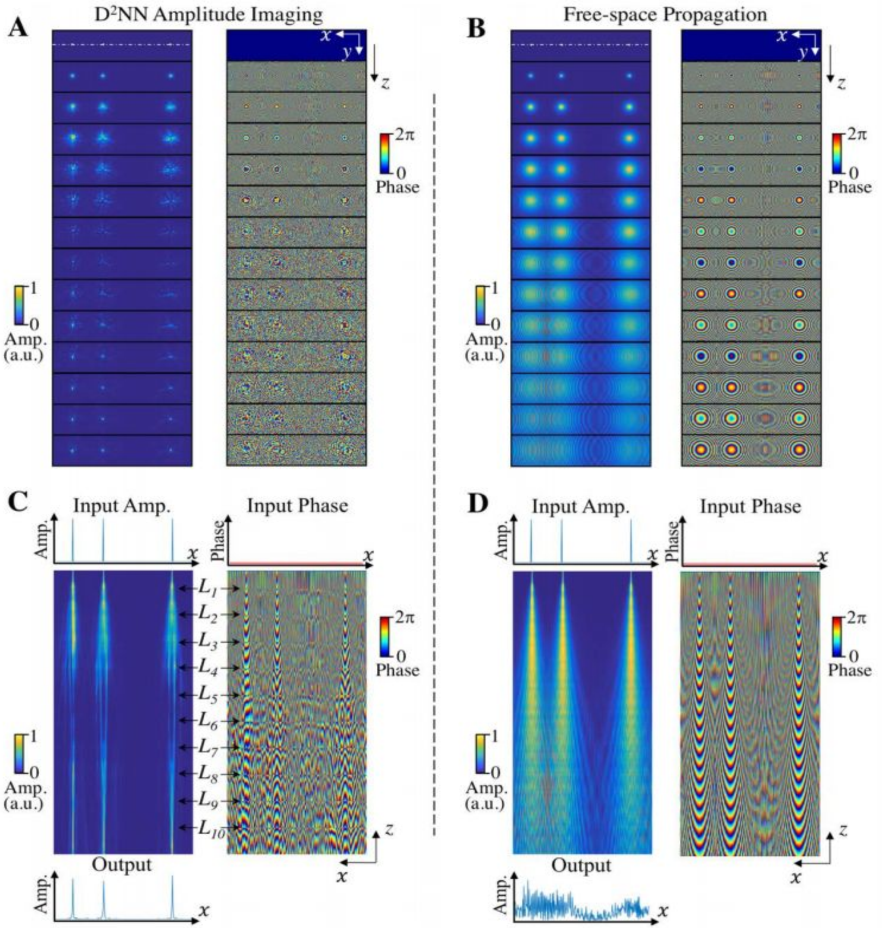
“Experiment : Fashion MNIST”

- Image encoded into phase of input field
- Ten classes: (t-shirts, trousers, pullovers, dresses, coats, sandals, shirts, sneakers, bags, and ankle boots)
- Simulation:
  5 layers (done in paper): **81.13%**
  10 layers (done in paper): **86.60%**
- Experiment (done in paper): 90% match with simulation

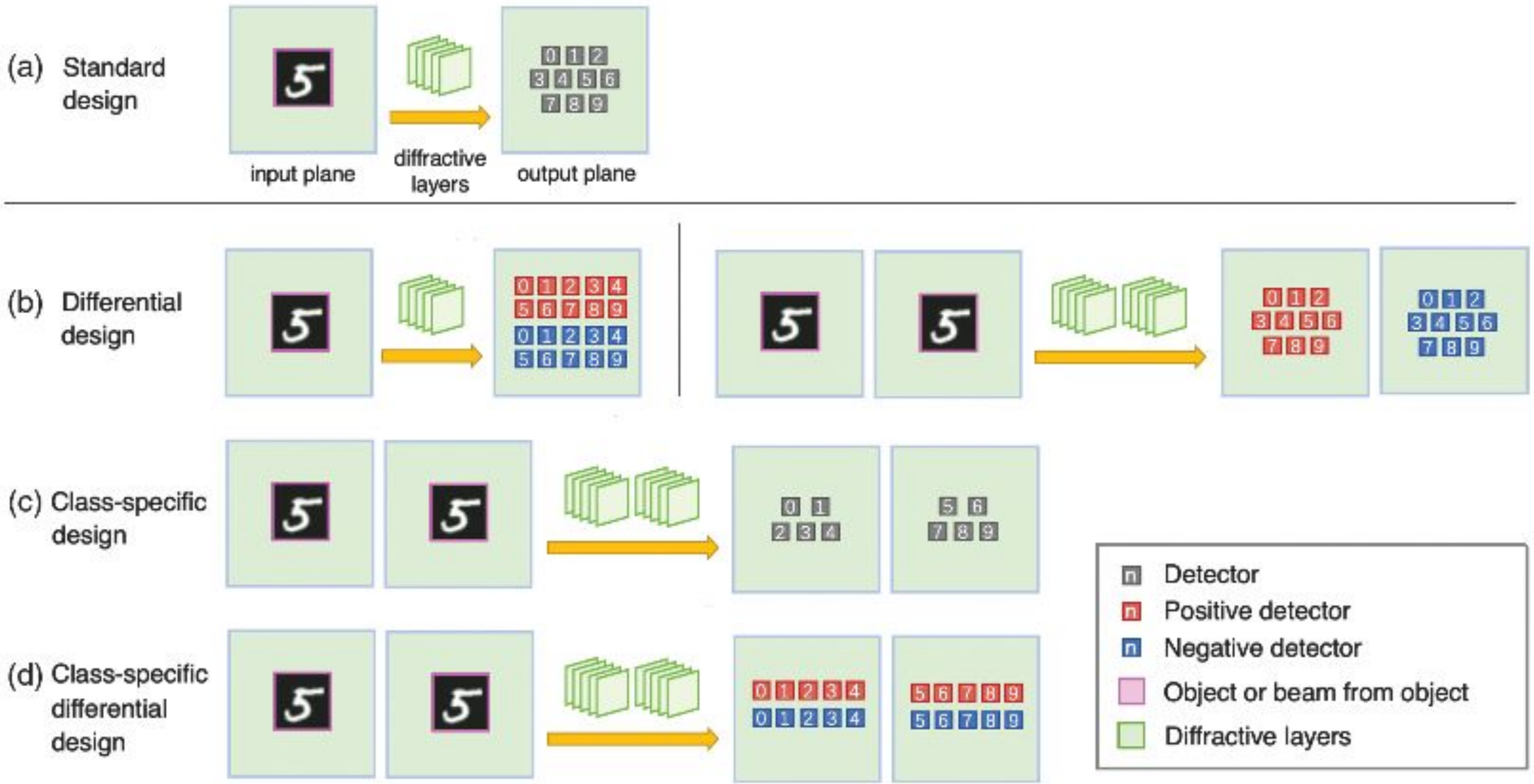
Imaging

- ImageNet datasets were used to train the D^2 NNs for imaging lens tasks
- Five layers 9cm X 9cm
- Separated by 4mm
- SSIM (structural similarity index) was used in this analysis

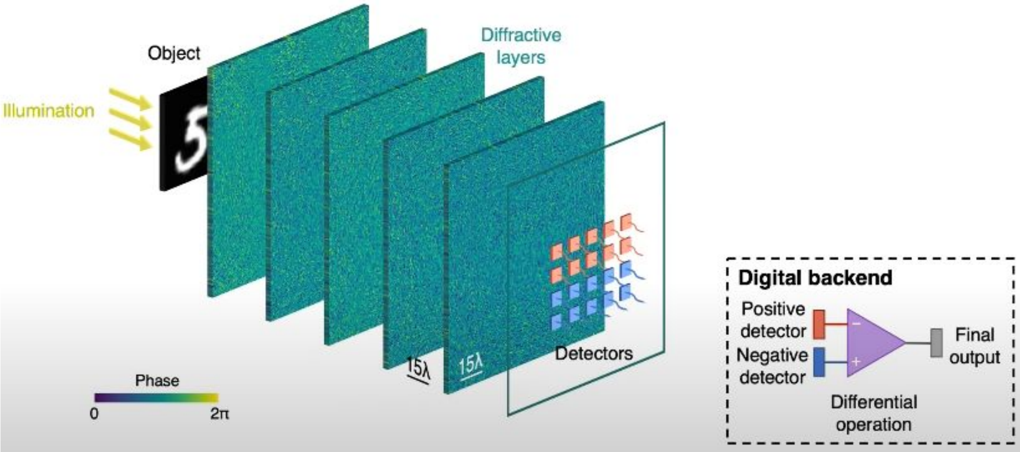


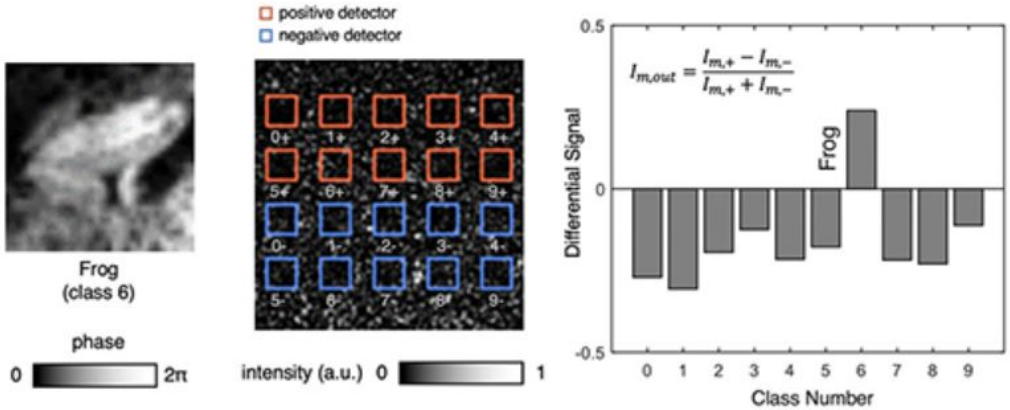


“Different Designs of Optical Neural Networks”



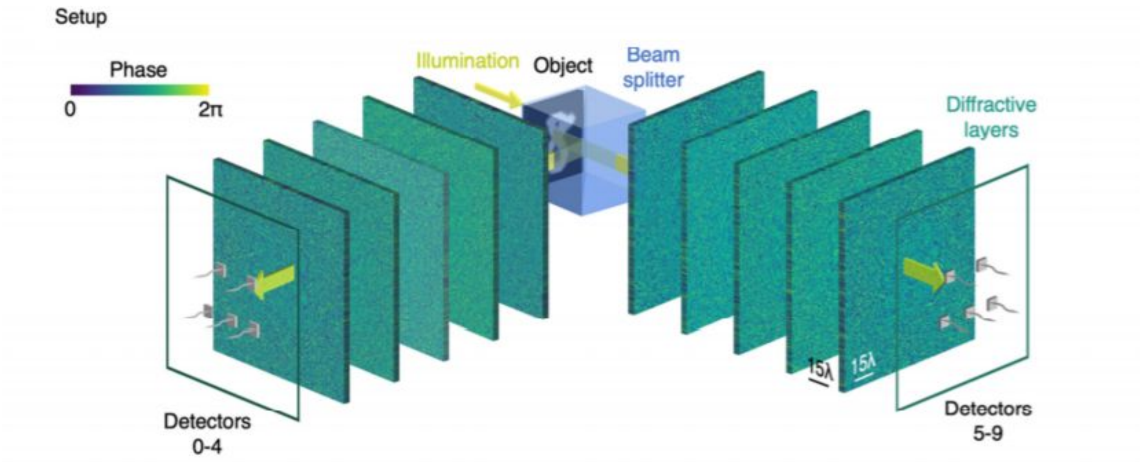
Differential diffractive optical neural network

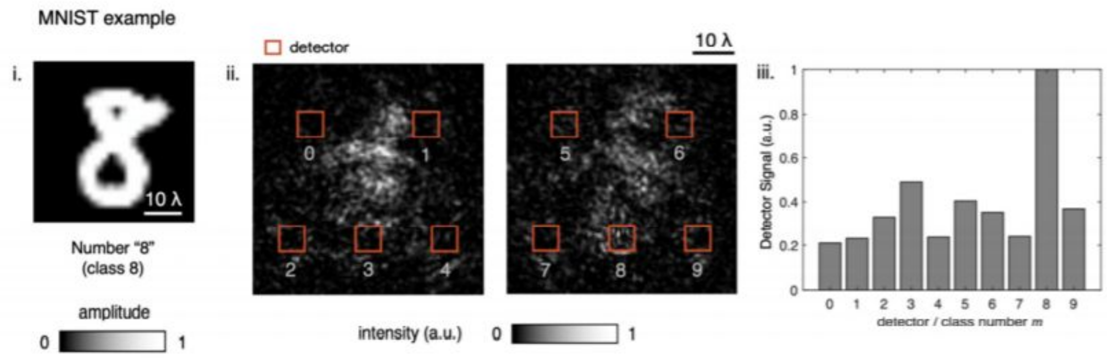




- Each class score at the optical network's output plane is calculated using two different detectors.
- One detector represents positive numbers and the other represents negative numbers.
- The correct object class is inferred by the detector pair that has the largest normalized difference between the positive and the negative detectors.

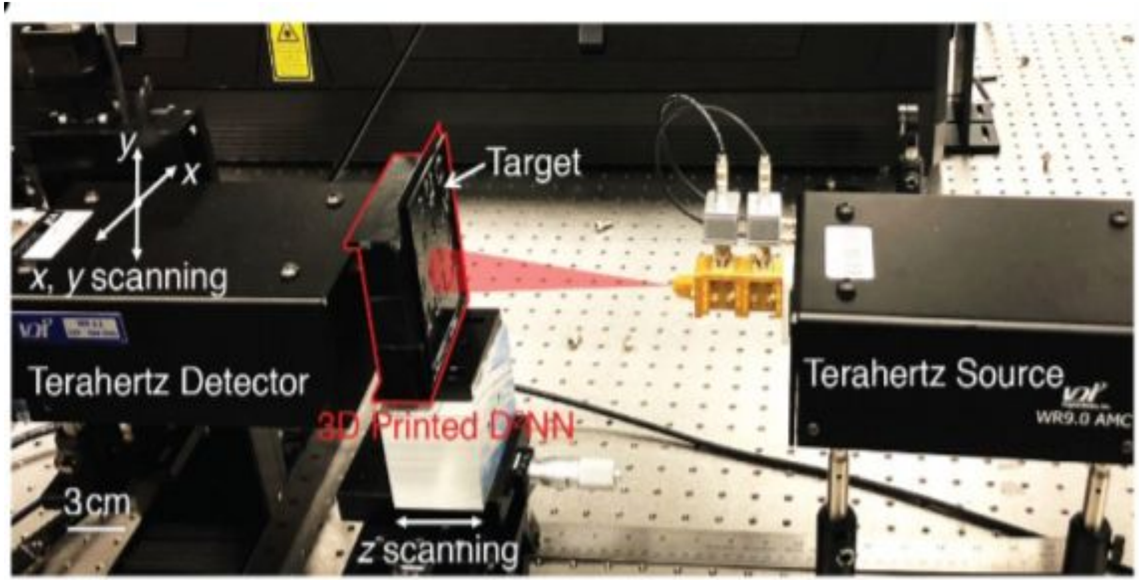
Class-specific diffractive optical neural network





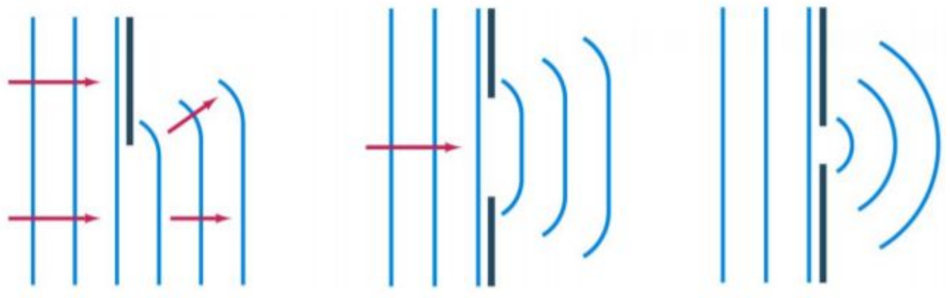
- Each class is assigned to a separate pair of detectors, behind a diffractive optical network
- Individual class detectors are split into separate networks based on their classes

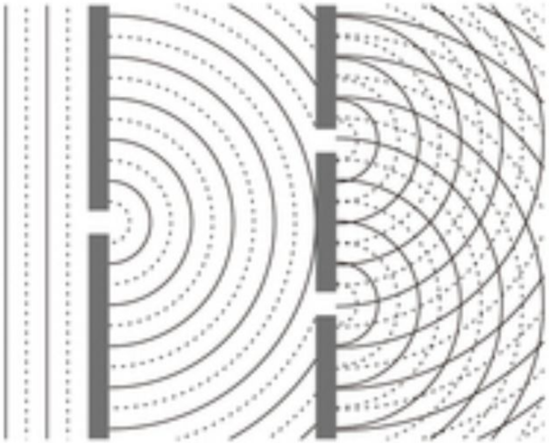
Experimental setup

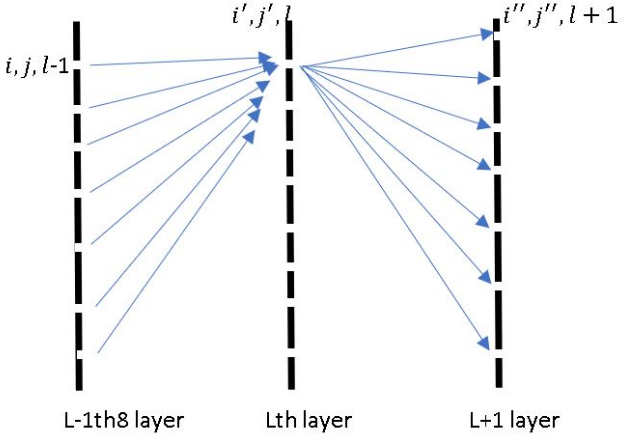


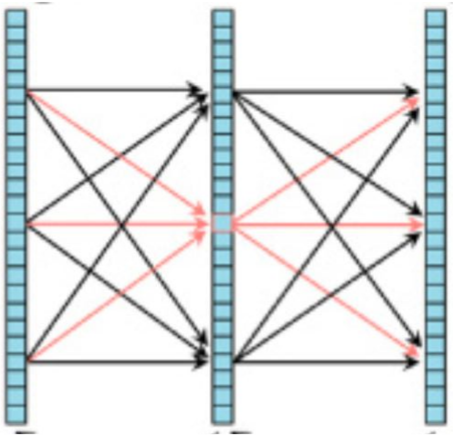
“Diffraction “

- Bending of light into the shadow reason due to interference with obstacles







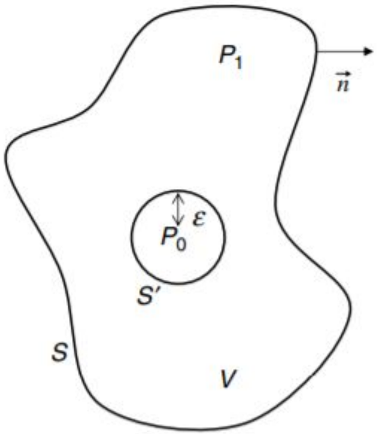


Green's Wave Function

$$\nabla^2 G(r) + k^2 G(r) = -\delta(r)$$

$$\widehat{G} = \frac{1}{2\pi r} e^{\frac{j2\pi r}{\lambda}} \hat{r}$$

G represents a spherical expanding wave
j=√(-1)
λ=wave length   k=wave no.
r is radius



The surfaces of integration for Green's theorem $S = S' + S_\varepsilon$

Rayleigh-Sommerfeld's Formulations

Derivation of wave trailing w.r.t to normal of surface

$$\frac{\partial G}{\partial n}\hat{n} = \frac{\partial G}{\partial r}\frac{\partial r}{\partial n}\hat{n}$$
$$= \frac{\partial}{\partial r}\left(\frac{1}{2\pi r}e^{\frac{j2\pi r}{\lambda}}\right)\frac{\partial r}{\partial n}\hat{n}$$
$$= \left(\frac{1}{2\pi r}\frac{\partial}{\partial r}\left(e^{\frac{j2\pi r}{\lambda}}\right) + e^{\frac{j2\pi r}{\lambda}}\frac{\partial}{\partial \mathrm{r}}\left(\frac{1}{2\pi r}\right)\right)\frac{\partial r}{\partial n}\hat{n}$$
$$= \left(\frac{1}{2\pi r}\frac{j2\pi}{\lambda}\left(e^{\frac{j2\pi r}{\lambda}}\right) - \frac{1}{2\pi r^2}e^{\frac{j2\pi r}{\lambda}}\right).1.\hat{n}$$
$$= \frac{1}{r}e^{\frac{j2\pi r}{\lambda}}\left(\frac{j}{\lambda} - \frac{1}{2\pi r}\right)\hat{n}$$

Wave analysis in a D2NN

Following the Rayleigh-Sommerfeld diffraction, one can consider every single neuron of a given D2 NN layer as a secondary source of a wave that is composed of the following optical mode:

$$w_i^l(x,y,z) = \frac{z-z_i}{r^2}\left(\frac{1}{2\pi r} + \frac{1}{j\lambda}\right) exp\left(\frac{j2\pi r}{\lambda}\right)$$

l represents the l-th layer of the network
i represents the i-th neuron located at (xi,yi,zi) of layer L

The amplitude and relative phase of this secondary wave are

$$n_i^l(x,y,z) = w_i^l(x,y,z) \cdot t_i^l(x_i,y_i,z_i) \cdot \sum_k n_k^{l-1}(x_i,y_i,z_i) = w_i^l(x,y,z) \cdot |A| \cdot e^{j\Delta\theta}$$

t- transmission coefficient
n- is the output function from a neuron

|A|- amplitude of secondary wave
$\Delta\theta$ refers to the additional phase delay

The transmission coefficient of a neuron is composed of-

$$t_i^l(x_i, y_i, z_i) = a_i^l(x_i, y_i, z_i) exp(j\phi_i^l(x_i, y_i, z_i))$$

- a is amplitude(or root of ratio of energy loss), ideally 1 if zero optical loses
- Φ is phase delay

Forward Propagation Model

Let we define m as input wave to neuron from layer l

$$n_{i,p}^l = w_{i,p}^l \cdot t_i^l \cdot m_i^l$$
$$m_i^l = \sum_k n_{k,i}^{l-1}$$
$$t_i^l = a_i^l exp(j\phi_i^l)$$

Error Backpropagation by mean square error

Let define E as loss function

mean square error between the output plane intensity s_i^{M+1} and the target, g_i^{M+1}

$$E(\phi_i^l) = \frac{1}{K}\sum_k (s_k^{M+1} - g_k^{M+1})^2$$

$$\min_{\phi_i^l} E\,(\phi_i^l),\ \ s.t. \qquad 0 \le \phi_i^l < 2\pi.$$

$$\frac{\partial E(\phi_i^l)}{\partial \phi_i^l} = \frac{4}{K}\sum_k (s_k^{M+1} - g_k^{M+1}) \cdot Real\{(m_k^{M+1})^* \cdot \frac{\partial m_k^{M+1}}{\partial \phi_i^l}\}$$

$$\frac{\partial m_k^{M+1}}{\partial \phi_i^{l=M}} = j \cdot t_i^M \cdot m_i^M \cdot w_{i,k}^M$$

$$\frac{\partial m_k^{M+1}}{\partial \phi_i^{l=M-1}} = j \cdot t_i^{M-1} \cdot m_i^{M-1} \cdot \sum_{k_1} w_{k_1,k}^M \cdot t_{k_1}^M \cdot w_{i,k_1}^{M-1}$$

$$\frac{\partial m_k^{M+1}}{\partial \phi_i^{l=M-2}} = j \cdot t_i^{M-2} \cdot m_i^{M-2} \cdot \sum_{k_1} w_{k_1,k}^M \cdot t_{k_1}^M \cdot \sum_{k_2} w_{k_2,k_1}^{M-1} \cdot t_{k_2}^{M-1} \cdot w_{i,k_2}^{M-2}$$

$$\frac{\partial m_k^{M+1}}{\partial \phi_i^{l=M-L}} = j \cdot t_i^{M-L} \cdot m_i^{M-L} \cdot \sum_{k_1} w_{k_1,k}^M \cdot t_{k_1}^M \cdots\cdots \sum_{k_L} w_{k_L,k_{L-1}}^{M-L+1} \cdot t_{k_L}^{M-L+1} \cdot w_{i,k_L}^{M-L}$$

D2NN	Deep neural network (Machine Learning)
this work considers a coherent diffractive network modelled by physical wave propagation to connect various layers through the phase and amplitude of interfering waves, controlled with multiplicative bias terms and physical distances	the inputs for neurons are complex-valued, determined by wave interference and a multiplicative bias, i.e., the transmission/reflection coefficient
optical nonlinearity can also be incorporated into a diffractive neural network in various ways(//explained in next slide)	individual function of a neuron is the phase and amplitude modulation of its input to output a secondary wave
neuron’s output is coupled to the neurons of the next layer through wave propagation and coherent (or partially-coherent) interference, providing a unique form of interconnectivity within the network, and not depends “spacing and alignment consecutive layers”	for a given spacing between the successive layers, the intensity of the wave from a neuron will decay below the detection noise floor after a certain propagation distance the radius of this propagation distance at the next layer practically sets the receptive field of a diffractive neural network and can be physically adjusted by changing the spacing between the network layers, the intensity of the input optical beam

optical backpropagation training of diffractive optical neural networks

1. Maths

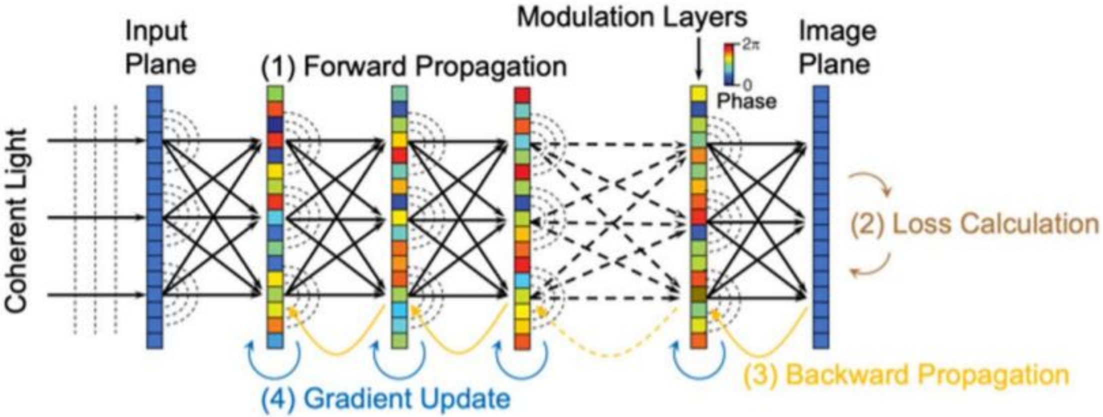
$$\mathbf{U}_k = \mathbf{M}_k\mathbf{W}_k\mathbf{U}_{k-1}$$

$\mathbf{U}_k$ represents the vectorized output optical field of a k-th layer of the network

$\mathbf{W}_k$ diffractive weight matrix with the forward light propagation from the k − 1-th to the k-th layer

$\mathbf{M}_k = \mathrm{diag}(e^{j\boldsymbol{\phi}_k})$ represents the diagonalization of a vectorized diffractive modulation at the k-th layer with a

$\boldsymbol{\phi}_k$ phase coefficient of



$$\mathbf{U}_{N+1} = \mathbf{W}_{N+1}\left(\prod_{k=N}^{1}\mathbf{M}_k\mathbf{W}_k\right)\mathbf{U}_0$$

Defining O the intensity distribution of the resulting optical field which is going to detector at the imaging plane

$$\mathbf{O} = |\mathbf{U}_{N+1}|^2 = \left|\mathbf{W}_{N+1}\left(\prod_{k=N}^{1}\mathbf{M}_k\mathbf{W}_k\right)\mathbf{U}_0\right|^2.$$

Let L(O,T) represent the loss function of diffractive ONN that measures the differentials between network outputs O and ground truth labels T

Then gradient can be measure as

$$\begin{aligned}\frac{\partial L}{\partial \boldsymbol{\phi}_k} &= \frac{\partial L}{\partial \mathbf{U}_{N+1}}\frac{\partial \mathbf{U}_{N+1}}{\partial \boldsymbol{\phi}_k} + \frac{\partial L}{\partial \mathbf{U}^*_{N+1}}\frac{\partial \mathbf{U}^*_{N+1}}{\partial \boldsymbol{\phi}_k}\\ &= 2\mathrm{Re}\left\{\left(\frac{\partial L}{\partial \mathbf{O}} \odot \mathbf{U}^*_{N+1}\right)^T \frac{\partial \mathbf{U}_{N+1}}{\partial \boldsymbol{\phi}_k}\right\},\end{aligned}$$

And gradient w.r.t to phase we can calculate as

$$\partial \mathbf{U}_{N+1}/\partial \boldsymbol{\phi}_k = j\mathbf{W}_{N+1}\left(\prod_{i=N}^{k+1}\mathbf{M}_i\mathbf{W}_i\right)\mathrm{diag}\left(\left(\prod_{i=k}^{1}\mathbf{M}_i\mathbf{W}_i\right)\mathbf{U}_0\right)$$

$$= \left(j\cdot\mathrm{diag}\left(\left(\prod_{i=k}^{1}\mathbf{M}_i\mathbf{W}_i\right)\mathbf{U}_0\right)^T\left(\left(\prod_{i=k+1}^{N}\mathbf{W}_i^T\mathbf{M}_i\right)\mathbf{W}_{N+1}^T\right)\right)^T$$

letting $\mathbf{E} = \frac{\partial L}{\partial \mathbf{O}} \odot \mathbf{U}_{N+1}^*$

$$\frac{\partial \mathrm{L}}{\partial \boldsymbol{\phi}_k} = 2\mathrm{Re}\{\mathbf{E}^T\partial \mathbf{U}_{N+1}/\partial \boldsymbol{\phi}_k\}$$

$$= 2\mathrm{Re}\left\{j\left(\left(\prod_{i=k}^{1}\mathbf{M}_i\mathbf{W}_i)\mathbf{U}_0\right)\odot\left(\left(\prod_{i=k+1}^{N}\mathbf{W}_i^T\mathbf{M}_i\right)\mathbf{W}_{N+1}^T\mathbf{E}\right)\right\}^T$$

$$= 2\mathrm{Re}\{(j\mathbf{P}_k^f \odot \mathbf{P}_k^b)\}^T$$

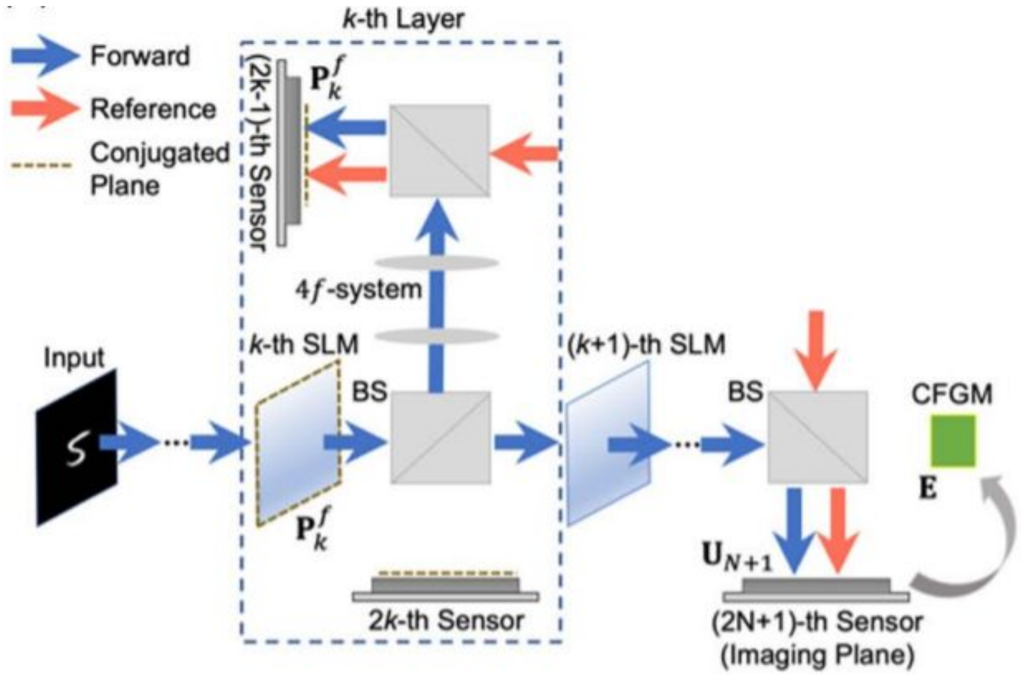
Where

$$\begin{cases} \mathbf{P}_k^f = \left(\prod_{i=k}^{1} \mathbf{M}_i \mathbf{W}_i\right) \mathbf{U}_0, \\ \mathbf{P}_k^b = \left(\prod_{i=k+1}^{N} \mathbf{W}_i^T \mathbf{M}_i\right) \mathbf{W}_{N+1}^T E. \end{cases}$$

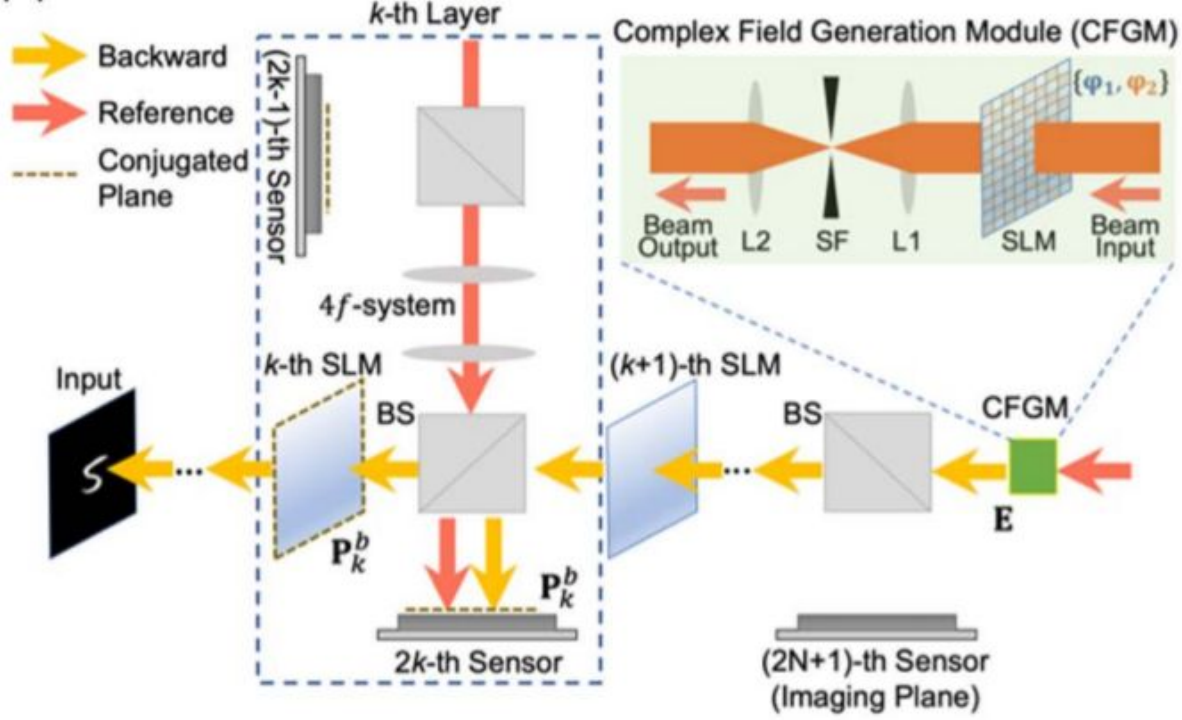
$\mathbf{P}_k^f$ is the output optical field of a k-th layer by propagating the optical field of an object from the input layer to layer k

$\mathbf{W}_k^T$ represents the diffractive weight matrix between the k − 1-th layer and the k-th layer with backward light propagation in the opposite direction of **Wk**

$\mathbf{P}_k^b$ is the optical field corresponding to the backward propagation of the error optical field E from the output plane of the network to the k-th layer



The forward propagated optical field is modulated by the phase coefficients of multilayer SLMs and measured by the image sensors with phase shifted reference beams at the output image plane as well as at the individual layers. The image sensor is set to be conjugated to the diffractive layer relayed by a 1:1 beam splitter (BS) and a 4f system



The backward propagated optical field is formed by propagating the error optical field from the output image plane back to the input plane with the modulation of multilayer SLMs. The error optical field is generated from the complex field generation module (CFGM) by calculating the residual errors between the network output optical field and the ground truth label. With the measured forward and backward propagated optical fields, the gradients of the diffractive layers are calculated, and the modulation coefficients of SLMs are successively updated from the last to first layer.

Optical Training of Nonlinear Diffractive ONN

We extend the proposed architecture for in situ optical training of the nonlinear diffractive ONN by incorporating the optical nonlinearity layers in between the diffractive layers considering the physical implementation

$\mathbf{W}_{k-\frac{1}{2}}$ represents the diffractive propagation between the (k − 1)-th nonlinear layer and k-th diffractive layer

$$\mathbf{O} = |\mathbf{U}_{N+1}|^2 = \left| \mathbf{W}_{N+1} \left(\prod_{k=N}^{1} f_k \mathbf{W}_k \mathbf{M}_k \mathbf{W}_{k-\frac{1}{2}} \right) \mathbf{U}_0 \right|^2$$

f is the activation function of each layer

$$\partial \mathbf{U}_{N+1} / \partial \phi_k = \begin{pmatrix} j\mathrm{diag}\left(\mathbf{M}_k \mathbf{W}_{k-\frac{1}{2}} \left(\prod_{i=k-1}^{1} f_i \mathbf{W}_i \mathbf{M}_i \mathbf{W}_{i-\frac{1}{2}} \right) \mathbf{U}_0 \right)^T \\ \left(\mathbf{W}_k^T \mathrm{diag}(\mathbf{g}_k) \left(\prod_{i=k+1}^{N} \mathbf{W}_{i-\frac{1}{2}}^T \mathbf{M}_i \mathbf{W}_i^T \mathrm{diag}(\mathbf{g}_i) \right) \mathbf{W}_{N+1}^T \right) \end{pmatrix}^T$$

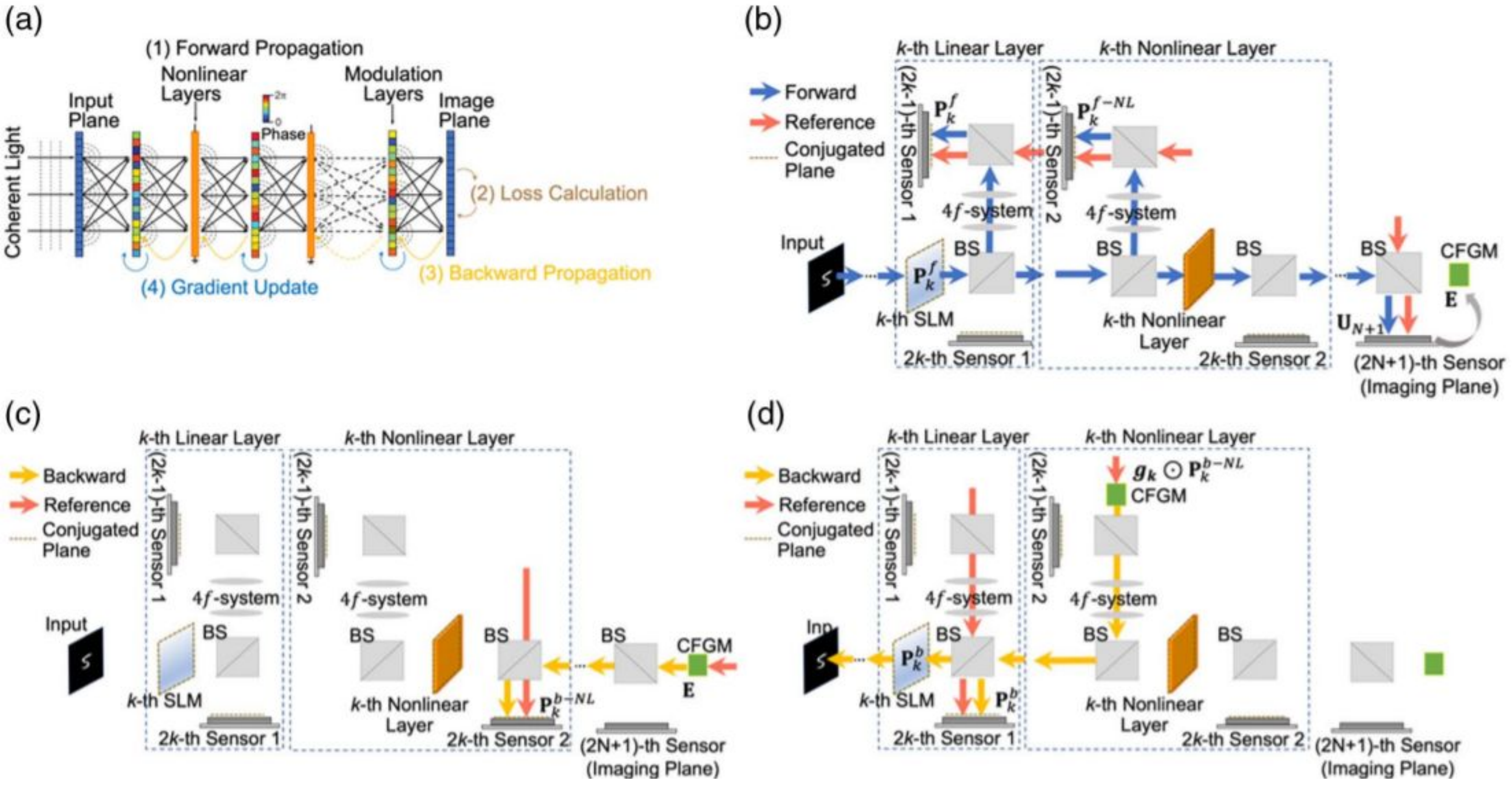
$$\mathbf{g}_k = f'_k (\mathbf{W}_k \mathbf{M}_k \mathbf{W}_{k-\frac{1}{2}} (\prod_{i=k-1}^{1} f_i \mathbf{W}_i \mathbf{M}_i \mathbf{W}_{i-\frac{1}{2}}) \mathbf{U}_0)$$

Output field of forward and backward propagation will be

$$
\begin{cases}
\mathbf{P}_k^{f} = \mathbf{M}_k \mathbf{W}_{k-\frac{1}{2}} \left(\prod_{i=k-1}^{1} \boldsymbol{f}_i \mathbf{W}_i \mathbf{M}_i \mathbf{W}_{i-\frac{1}{2}} \right) \mathbf{U}_0, \\
\mathbf{P}_k^{b} = \mathbf{W}_k^T \operatorname{diag}(\boldsymbol{g}_k) \left(\prod_{i=k+1}^{N} \mathbf{W}_{i-\frac{1}{2}}^T \mathbf{M}_i \mathbf{W}_i^T \operatorname{diag}(\boldsymbol{g}_i) \right) \mathbf{W}_{N+1}^T E
\end{cases}
$$

the optical field of the nonlinear layer in the forward and backward propagations as $\mathbf{P}_k^{f\text{-}NL}$ and $\mathbf{P}_k^{b\text{-}NL}$, which can be formulated as

$$
\begin{cases}
\mathbf{P}_k^{f\text{-}NL} = \mathbf{W}_k \mathbf{M}_k \mathbf{W}_{k-\frac{1}{2}} \left(\prod_{i=k-1}^{1} \boldsymbol{f}_i \mathbf{W}_i \mathbf{M}_i \mathbf{W}_{i-\frac{1}{2}} \right) \mathbf{U}_0, \\
\mathbf{P}_k^{b\text{-}NL} = \left(\prod_{i=k+1}^{N} \mathbf{W}_{i-\frac{1}{2}}^T \mathbf{M}_i \mathbf{W}_i^T \operatorname{diag}(\boldsymbol{g}_i) \right) \mathbf{W}_{N+1}^T E.
\end{cases}
$$



(a)
(1) Forward Propagation
Coherent Light
Input Plane
Nonlinear Layers
Phase
Modulation Layers
Image Plane
(2) Loss Calculation
(3) Backward Propagation
(4) Gradient Update
(b)
k-th Linear Layer
k-th Nonlinear Layer
Forward
Reference
Conjugated Plane
(2k-1)-th Sensor 1
(2k-1)-th Sensor 2
4f-system
Input
BS
k-th SLM
k-th Nonlinear Layer
CFGM
2k-th Sensor 1
2k-th Sensor 2
(2N+1)-th Sensor (Imaging Plane)
(c)
k-th Linear Layer
k-th Nonlinear Layer
Backward
Reference
Conjugated Plane
(2k-1)-th Sensor 1
(2k-1)-th Sensor 2
4f-system
Input
BS
k-th SLM
k-th Nonlinear Layer
CFGM
2k-th Sensor 1
2k-th Sensor 2
(2N+1)-th Sensor (Imaging Plane)
(d)
k-th Linear Layer
k-th Nonlinear Layer
Backward
Reference
Conjugated Plane
CFGM
(2k-1)-th Sensor 1
(2k-1)-th Sensor 2
4f-system
BS
k-th SLM
k-th Nonlinear Layer
2k-th Sensor 1
2k-th Sensor 2
(2N+1)-th Sensor (Imaging Plane)

The differences between the nonlinear and linear *in situ* optical training are as follows: (1) in forward propagation, the optical field of the nonlinear layer $\mathbf{P}_k^{f\text{-}NL}$ should be additionally measured; (2) the backpropagation of each layer is divided into two steps, i.e., measuring the optical field $\mathbf{P}_k^{b\text{-}NL}$ first and then using $\mathbf{P}_k^{f\text{-}NL}$ and $\mathbf{P}_k^{b\text{-}NL}$ to generate the modulation field $\boldsymbol{g_k} \odot \mathbf{P}_k^{b\text{-}NL}$ for performing the second step of backpropagation.

To demonstrate the optical training of diffractive ONN with nonlinearity, ferroelectric thin films are adopted for the diffractive ONN, which are placed between the successive diffractive layers. The nonlinear activation function can be formulated as

$$\boldsymbol{f}(\mathbf{E}_{\text{in}}) = \mathbf{E}_{\text{in}} e^{j\pi \frac{\langle |\mathbf{E}_{\text{in}}|^2 \rangle}{1+\langle |\mathbf{E}_{\text{in}}|^2 \rangle}},$$

Measuring the Network Optical Field

- Assume $\mathbf{P}_k = \mathbf{A}_k e^{j\boldsymbol{\theta}_k}$

Represents the forward propagated optical field $\mathbf{P}_k^f$ or the backward propagated optical field $\mathbf{P}_k^b$ at the k-th layer, where $\mathbf{A}_k$ refers to its amplitude, and $\boldsymbol{\theta}_k$ refers to its phase

- The network optical field was interfered with a four-step phase-shifted reference beam

$$U_R = A_R e^{j\theta_R} \quad \theta_R = 0, \pi/2, \pi, 3\pi/2$$

- The corresponding intensity distributions of the interference results

$$\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3 = |\mathbf{U}_R + \mathbf{P}_k|^2$$

$$\begin{cases} \boldsymbol{\theta}_k = \arctan((\mathbf{I}_3 - \mathbf{I}_1)/(\mathbf{I}_0 - \mathbf{I}_2)), \\ \mathbf{A}_k = \alpha((\mathbf{I}_1 - \mathbf{I}_0)/2(\sin\boldsymbol{\theta}_k - \cos\boldsymbol{\theta}_k)) \end{cases} \qquad \alpha = 1/A_R$$

Generating the Error Optical Field

The error optical field was generated with a complex field generation module (CFGM) that acts as the source of a backward propagated optical field.

To generate the complex optical field with a phase-only SLM, the error optical field-

$\mathbf{E} = \mathbf{A}_\xi e^{j\theta_\xi}$ was decomposed into two optical fields with a constant amplitude

$\mathbf{E} = \beta(e^{j\varphi_1} + e^{j\varphi_2})$ two phase patterns **φ1** and **φ2** can be interleaved multiplexed on the SLM

M1 and M2 represent a pair of a complementary pixel-wise binary checkerboard pattern, i.e

$\mathbf{M}_1 + \mathbf{M}_2 = 1$ multiplexed phase pattern for the SLM can be formulated

$\varphi = \mathbf{M}_1(\varphi_1)\uparrow_2 + \mathbf{M}_2(\varphi_2)\uparrow_2$ ↑2 represents the 2× nearest-neighbor spatial upsampling of two phase patterns

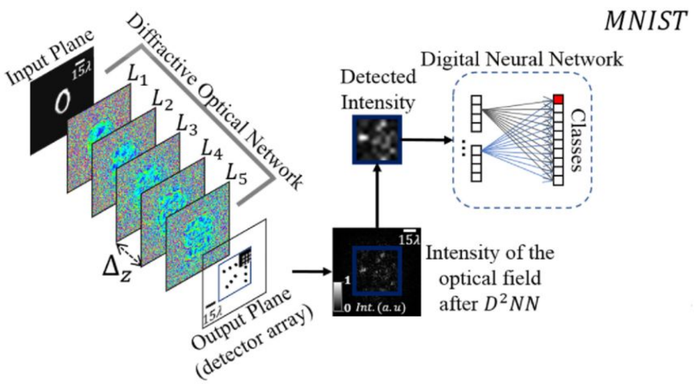
the gradient at each diffractive layer is successively calculated, and the phase coefficients of SLMs are iteratively updated by adding the phase increment, is a constant network parameter determining the learning rate of the training

$$\Delta\phi_k = -\eta(\partial L/\partial\phi_k) \qquad \phi_k = \phi_k - \eta\frac{\partial L}{\partial\phi_k}$$

Integration with the Electronic Neural Network

We are integrating the D2NN's to create hybrid machine learning and computer vision systems.

Where the D2NN will be used at the front end of the system before the electronic neural network.



Advantages of using a hybrid a approach

1. Reduction in the number of pixels (detectors) that are going to be digitised for an electronic neural network to act on .
2. Due to low numbers of pixels, we will achieve a high frame rate for the entire system
3. Also , it will reduce the complexity and the power consumption of the entire system.

Using the Hybrid Approach we can potentially create a low-power and less complex machine learning systems, with the help of simple and complex imagers.

In D2NN framework , each neuron has a complex transmission coefficient given by

Where

$$t_i^l(x_i, y_i, z_i) = a_i^l(x_i, y_i, z_i) exp(j\phi_i^l(x_i, y_i, z_i))$$

$$a_i^l = sigmoid(\alpha_i^l),$$

$$\phi_i^l = 2\pi \times sigmoid(\beta_i^l),$$

Th ... inable parameters of a diffractive optical network.

The sigmoid will bound the range of the alpha and phi function, which will prevent the network to utilize the available dynamic range for amplitude and phase term for each neuron.

To prevent these issues we introduce the **Rectified Linear Unit(ReLU),**and **M** (number of neurons per layer).

$$a_i^l = \frac{ReLU(\alpha_i^l)}{max_{0<i\leq M}\{ReLU(\alpha_i^l)\}},$$
$$\phi_i^l = 2\pi \times \beta_i^l,$$

From the above equation we can now clearly see the phase part is now unbounded ,and the anmplituceterm is still lying between (0,1)

By just doing this change for the All-optical D2NN design (5-layer ,phase only (complex valued))a significant amount of accuracy is achieved in the Fashion-MNIST case (with a axial length of 40x wavelength)

But we still have the

$$exp(j\phi_i^l(x_i, y_i, z_i))$$

Term as periodic and bounded with respect to the phase.

Effect of Loss functions on the performance of all-optical diffractive neural network

For this part of D2NN design we are using the **cross-entropy loss.**

Where minimizing the cross-entropy loss is equivalent to minimizing the negative log-likelihood of a an underlying probability distribution.

Cross-entropy acts on probability measures, which take values from in the interval (0,1), but the signals coming from the detectors (one from each class) at the output layer of a D2NN are not necessarily in this range.

To compensate the previous point, a **softmax** layer is introduced for the cross-entropy loss.

Softmax is used during the training process.

By including this outline a very significant improvement in the classification performance of an all -optical diffractive neural network is achieved .

Performance trade-offs in D2NN design

Despite the significant increase observed in the testing accuracy of D2NN design, the use of **softmax-cross-entropy (SCE)** also presents some trade offs in terms of practical system parameters.

What is Optical Backpropagation ?

Backpropagation error through nonlinear neurons is impressive challenge to the field of optical neural networks here , each neuron is required to exhibit a directionally dependent response to propagating optical signals, with the backwards response conditioned on the forward signal, which is highly non-trivial to implement optically

Approach to Demonstrate the Project

Here we will do simulations with readily obtainable optical depths , such that our approach can achieve equivalent performance to state-of-the-art computational networks on Handwritten MNIST image classification using D2NN

Looking at the simplicity and effectiveness of the method– it required optical operations for both forwards and backwards propagation realised using the same physical elements

Simulations done here with the physically realistic parameters will show that the proposed approach can train networks to performance levels equivalent to state of-the-art Artificial Neural Network when combined with D2NN calculation of the error term at the output layer via interference

Implementing Optical Backpropagation

Seeded with data at input layer a_0, forward propagation maps the neuron activations from layer l-1 to the neuron inputs at layer l as

$$z_j^{(l)} = \sum_i w_{ji}^{(l)} a_i^{(l-1)}$$

Via a weight matrix (w),before applying a non linear activation function individually to each neuron.

After that at the output layer we evaluate the loss function, (**L**) and its calculate its gradient with respect to the weights.

$$\frac{\partial \mathcal{L}}{\partial w_{ji}^{(l)}} = \frac{\partial \mathcal{L}}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial w_{ji}^{(l)}} = \delta_j^{(l)} a_i^{(l-1)},$$

where

$$\delta_j^{(l)} \equiv \partial\mathcal{L}/\partial z_j^{(l)}$$

Which is commonly referred to as the 'error' at the j-th neuron in the l-th layer. (by applying chain rule further we have)

$$\delta_j^{(l)} = \sum_k \frac{\partial \mathcal{L}}{\partial z_k^{(l+1)}} \frac{\partial z_k^{(l+1)}}{\partial z_j^{(l)}} = g'(z_j^{(l)})\rho_j^{(l+1)},$$

The above equation is the backpropagation equation

where $\rho_j^{(l+1)} = \sum_k \delta_k^{(l+1)} w_{kj}^{(l+1)}$

And here the error at the error at the output layer (the delta function) is calculated directly using the loss function, for each layer by the help of above equation.

We can one find the gradients of the error functions with respect to all the weights using the errors and activations of all neurons and hence we can apply gradient descent.

The transformation for l(input layer) is readily implemented as a linear optical(interferometric) operation, with the neurons represented by real-valued field amplitudes in different spatial nodes.

Also, the left hand side of the backpropagation equation involves the same weight matrix, which suggests that it can be implemented by physical backward propagation of an optical signal through the same linear optical arrangement

However , multiplying this signal by the derivative of the activation function is a challenge while using digital electronics.

To overcome this challenge , we are using the optical implementation of the activation function with following features:

1. Nonlinear response for the forward input
2. Linear response for the backward input
3. Modulation of backward input with the derivative of non linear function .

Here we are using saturable absorption (in pump-probe configuration) to solve this system.

What is Saturable Absorber response ?

Consider passing a strong pump, $E_{\_p}$ and a weak probe $E_{\_pr}$ through a two-level medium(eg. atomic vapour). The pump transmission is then a nonlinear function of the input

$$E_{\mathrm{P,out}} = g(E_{\mathrm{P,in}}) = \exp\left(-\frac{\alpha_0/2}{1+E_{\mathrm{P,in}}^2}\right)E_{\mathrm{P,in}},$$

Where α_0 is the resonant optical depth and all the fields are assumed to be normalised by the saturation threshold. High optical depth induces strong nonlinearity in the unsaturated region, and a sufficiently strong pump renders the medium nearly transparent in the saturated region.
A suitably weak probe , on the other hand , does not modify the transmissivity of the atomic media, and hence experiences linear absorption with the absorption coefficient determined by the pump

$$\frac{E_{\mathrm{pr,out}}}{E_{\mathrm{pr,in}}} = \exp\left(-\frac{\alpha_0/2}{1+E_{\mathrm{P,in}}^2}\right).$$

We are assuming both beams to be resonant with the atomic transition , while the phase of the electric field is unchanged.

So, with the pump and probe taking the roles of forward-propagating signal and backwards-propagating error in an ONN, with required features as stated above.

The derivative of the pump transmission is

$$g'(E_{\mathrm{P,in}}) = \left[1+\frac{\alpha_0 E_{\mathrm{P,in}}^2}{\left(1+E_{\mathrm{P,in}}^2\right)^2}\right] \exp\left(-\frac{\alpha_0/2}{1+E_{\mathrm{P,in}}^2}\right)$$

Our key insight is that in many instances the square-bracketed factor in above eqn. Can be considered as a constant.

Also, because of the constant scaling of the network gradients can be absorbed into the learning rate.

The proposed scheme can be implemented on either integrated or free-space platforms .

In the integrated settings, optical interference units that combine integrated phase-shifters and attenuators to realise intralayer weights.

A free space implementation of the required matrix multiplication can be achieved using a spatial light modulator (**SLM**) with the non-linear unit provided by a standard atomic vapour cell.

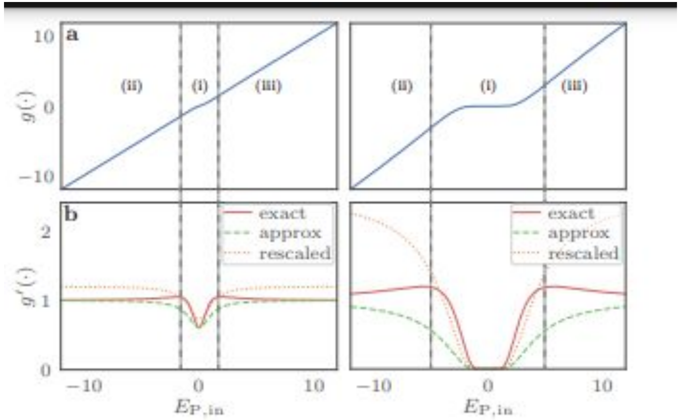


Fig. 2. **Saturable absorber response.** The transmission **(a)** and the transmission derivative **(b)** of a SA unit with optical depths of 1 (left) and 30 (right), as defined by Eqs. (4) and (6), respectively. Also shown in **(b)** are the actual probe transmissions given by Eq. (5) which approximate the derivatives, with and without the rescaling. The scaling factors are 1.2 (left) and 2.5 (right). Region (i) is the unsaturated (nonlinear) region exhibiting strong nonlinearity, and region (ii) is the saturated (linear) region.

Model framework for D2NN

Considering the canonical deep neural network task of image classification. Our experiment is to classify images of handwritten digits from 0 to 9.

Using MNIST dataset that contains greyscale bitmaps of size 28×28, which are fed into the input layer of the ONN.

WHat is Happening while Training ?

The fully-connected network we train to classify MNIST first unrolls each image into a 784-dimensional input vector, before two 128- neuron hidden layers and a 10-neuron output layer.

The convolutional network has two convolutional layers of 32-channel and 64-channels, respectively.

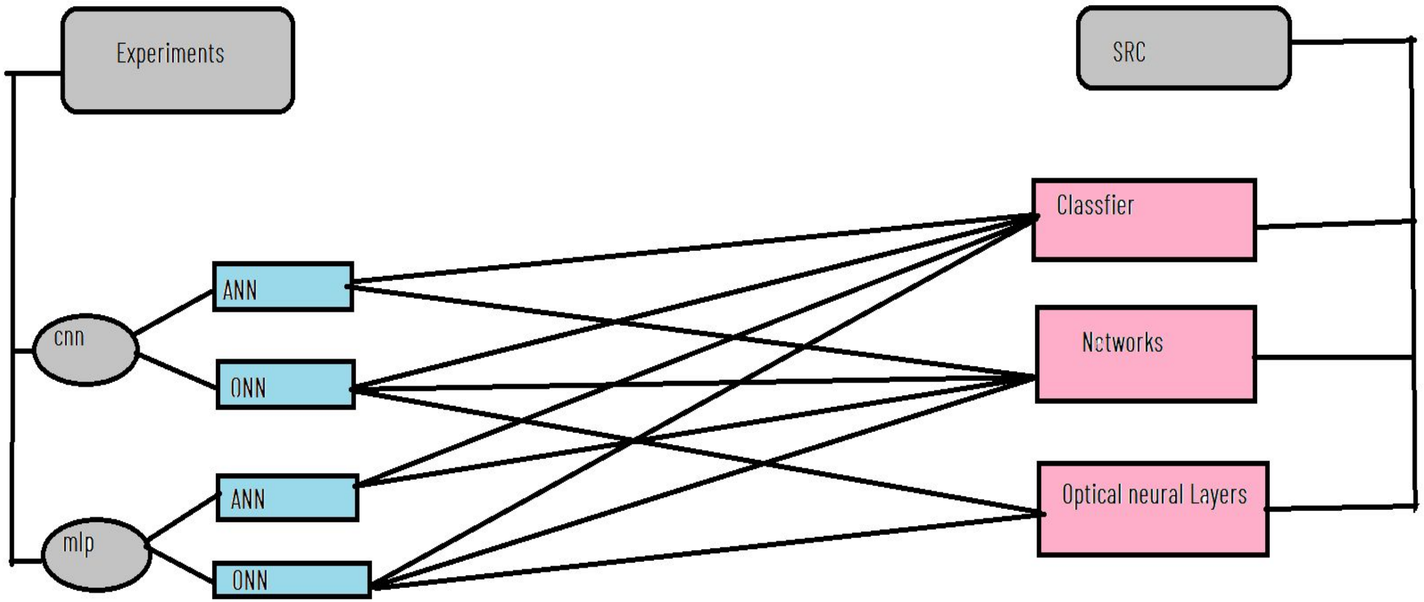
Each layer convolves the input with 5×5 filters (with a stride of 1 and no padding),

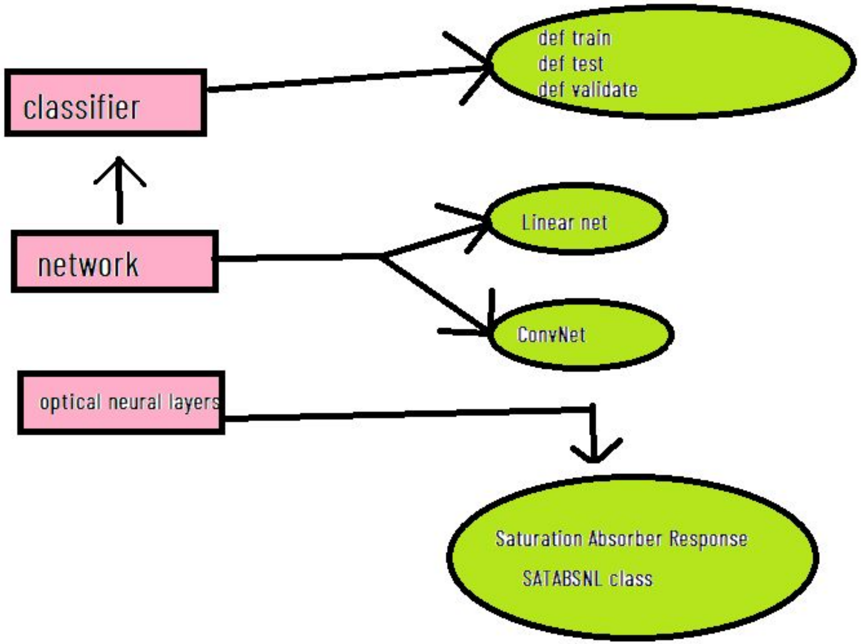
followed by a nonlinear activation function and finally a pooling operation (with both kernel size and stride of 2). After the convolutional network, classification is carried out by a fully-connected network with a single 128-neuron hidden layer and NC-neuron output layer, where NC is the number of classes in the target dataset.

Multilayer D2NN are assumed to have the same optical depth of their saturable absorbers in all layers.

Code Flow

In Bigger picture , Code flow is like this :




classifier
def train
def test
def validate
network
Linear net
ConvNet
optical neural layers
Saturation Absorber Response
SATABSNL class

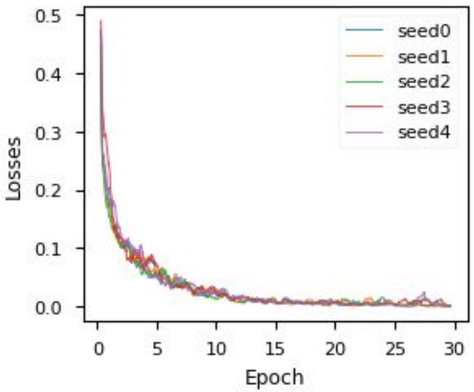
Github Link for Code :

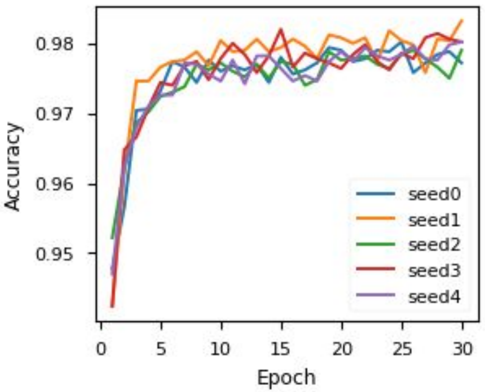
https://github.com/Prashant-Tomar/PH202-Code

Result

For Multilayer Perceptron Model :

Deep Neural Network :



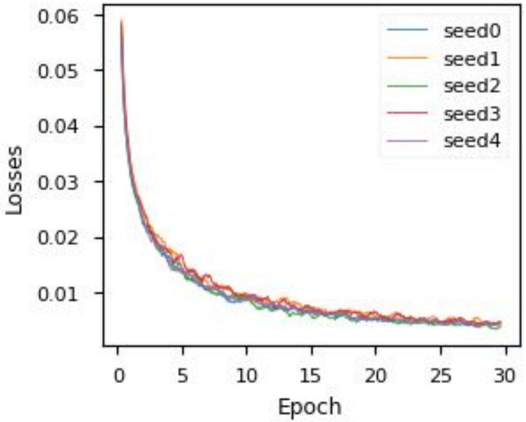


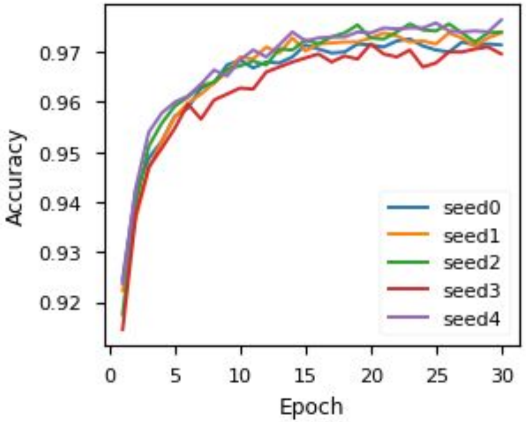
On testing data we had avg . accuracy : 97.5 % ; avg. loss : 0.1366

Result

For Multilayer Perceptron Model   :

Optical Neural Network :



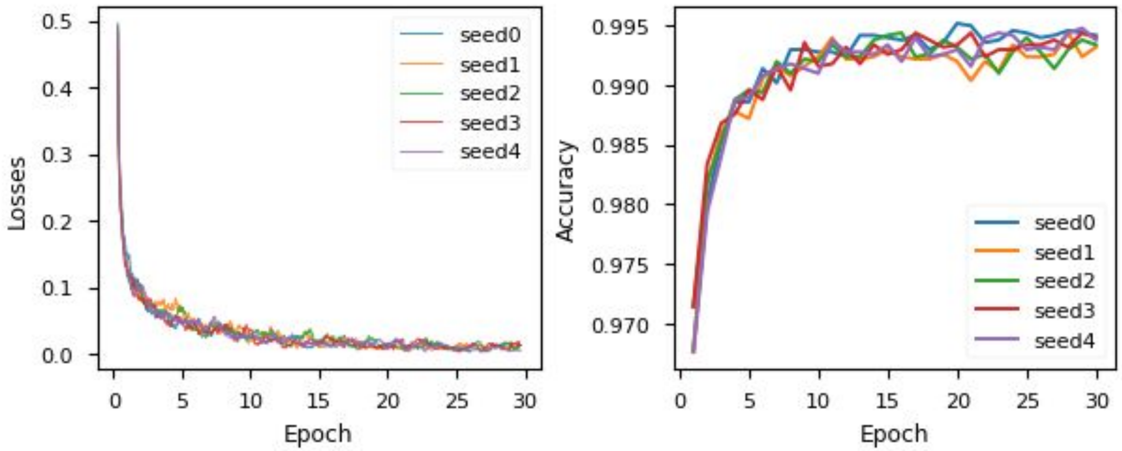


On testing data we had avg . accuracy : 97.34 % ; avg. loss : 0.0732

Result

For CNN Model :

Deep Neural Network :

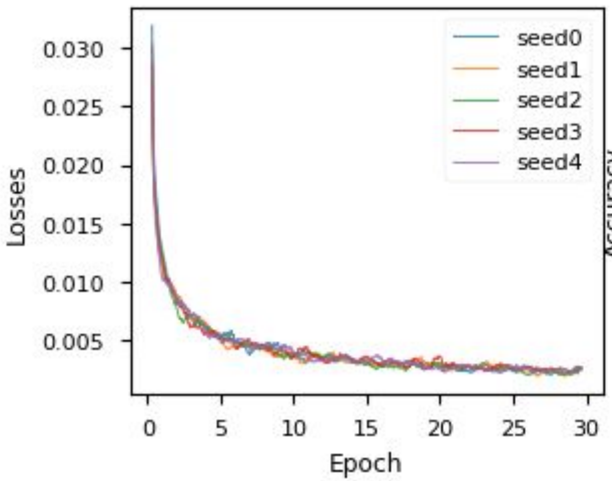


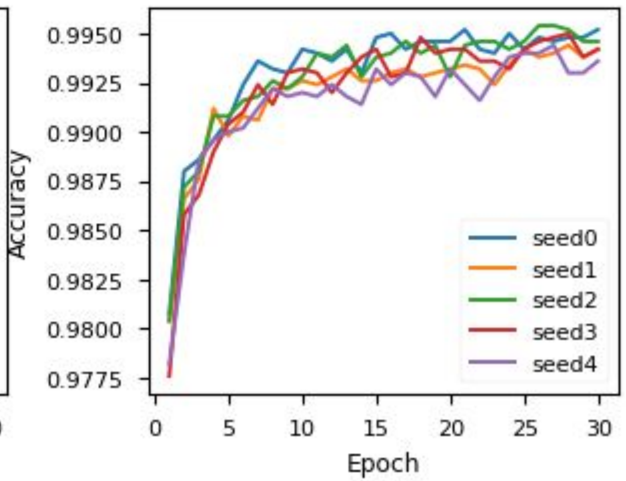
On testing data we had avg . accuracy : 99.38 % ; avg. loss : 0.0213

Result

For CNN Model :

Optical Neural Network :





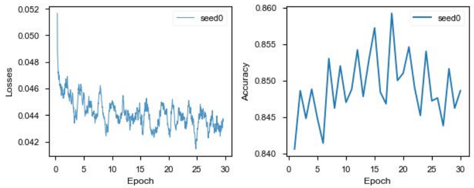
On testing data we had avg . accuracy : 99.12 % ; avg. loss : 0.0425

Conclusion drawn from above Simulation :

When it came down to accuracy which was produced by optical neural network wrt deep neural network it was always slightly lesser , due to the fact that physical device implementation of saturation absorber (as a nonlinear activation function) would always induce the error because it was real life optical model approximating behaviorally , the traditional nonlinear activation function.

We also observed that as α_0 (optical density) tends to $\rightarrow 0$, our network can only learn linear functions of the input which restricts the classification accuracy to (84.86) %

On the other hand if α_0 large enough we realised that approximation error was increasing wrt α_0



(When α_0 tend to zero)

Where we finally reached from this project:

On every seed we got different plots of (Accuracy vs Epoch and Loss vs Epoch) following same trendline in every case showing consistency of our model

We saw that we were able to learn and build prototype all-optical neural networks, **with information encoded in the intensity/amplitude of different electric field modes** by implementing Saturation absorber response as nonlinear activation function and successfully simulating the performance of these Optical neural network which were found to produce equivalent performance to computationally trained Deep neural network

Future of D^2NN (ONN)?

Optical neural network(ONN) is a novel machine learning framework on the physical principles of optics, which is still in its infancy and shows great potential it will surely open new doors for the machine learning.

Large-scale D2 NNs may be transformative for various applications, including image analysis, feature detection, and object classification, and may also enable new microscope or camera designs that can perform specific imaging tasks using D2 NNs.

With the gradual maturity of nanotechnology and the rapid advancement of silicon photonic integrated circuits.The construction of optical neural network on the future integrated photonic platform has potential application value

Just Some Hypothetical Creative Insight

Solar panels can use optical neural network using sunlight where every physical parameter will be learn about the environment on its own and then further it can give suitable temperature to us for bathing/cleansing purpose considering all the conditions

Scope of using light and automata is literally the next generation thing that has a high chances to replace "only" electronic computational automata machine

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Project Contribution

Divya (190260017) : Introduction, Experimental examples, Design varieties and Theory of the model

Divyansh (190100047) : Theoretical/physics background, Derivation of formula, Relation b/w design and formula, Understanding ONN

Hemant (190260023) : New design implementation to the existing model both in theory and code. Helped in brainstorming \ideas for codes.

Prashant (170260039) : Coding implementation and interpretations and Future Insights