$$
{}_{org}^{0}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(q_0) & -\sin(q_0) & L_0 \cos(q_0) \\ 0 & \sin(q_0) & \cos(q_0) & L_0 \sin(q_0) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

$$
{}_{0}^{1}T = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 & -L_1 \sin(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & L_1 \cos(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

$$
{}_{1}^{2}T = \begin{bmatrix} \cos(q_2) & -\sin(q_2) & 0 & -L_2 \sin(q_2) \\ \sin(q_2) & \cos(q_2) & 0 & L_2 \cos(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
$$

$$
{}_{2}^{3}T = \begin{bmatrix} \cos(q_3) & -\sin(q_3) & 0 & -L_3 \sin(q_3) \\ \sin(q_3) & \cos(q_3) & 0 & L_3 \cos(q_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

$$
com_0 = \begin{bmatrix} 0 \\ l_0 \cos(q_0) \\ l_0 \sin(q_0) \\ 1 \end{bmatrix} \tag{5}
$$

$$
com_1 = \begin{bmatrix} -l_1 \sin(q_1) \\ l_1 \cos(q_1) \\ 0 \\ 1 \end{bmatrix} \tag{6}
$$

$$
com_2 = \begin{bmatrix} -l_2 \sin(q_2) \\ l_2 \cos(q_2) \\ 0 \\ 1 \end{bmatrix} \tag{7}
$$

$$
com_3 = \begin{bmatrix} -l_3 \sin(q_3) \\ l_3 \cos(q_3) \\ 0 \\ 1 \end{bmatrix} \tag{8}
$$

$$
x_{ee} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}
$$

$$
{}_{org}^{1}T = {}_{org}^{0}T \, {}_{0}^{1}T \tag{10}
$$

$$
{}_{org}^{2}T = {}_{org}^{0}T \, {}_{0}^{1}T \, {}_{1}^{2}T \tag{11}
$$

$$
{}_{org}^{3}T = {}_{org}^{0}T \, {}_{0}^{1}T \, {}_{1}^{2}T \, {}_{2}^{3}T \tag{12}
$$

$$
Jacobian = \begin{bmatrix} \frac{\partial x}{\partial q_0} & \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_0} & \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_0} & \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \\ \frac{\partial \omega_x}{\partial q_0} & \frac{\partial \omega_x}{\partial q_1} & \frac{\partial \omega_x}{\partial q_2} & \frac{\partial \omega_x}{\partial q_3} \\ \frac{\partial \omega_y}{\partial q_0} & \frac{\partial \omega_y}{\partial q_1} & \frac{\partial \omega_y}{\partial q_2} & \frac{\partial \omega_y}{\partial q_3} \\ \frac{\partial \omega_z}{\partial q_0} & \frac{\partial \omega_z}{\partial q_1} & \frac{\partial \omega_z}{\partial q_2} & \frac{\partial \omega_z}{\partial q_3} \end{bmatrix} \tag{13}
$$

$$
J_0 = Jacobian(com_0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -l_0 \sin(q_0) & 0 & 0 & 0 \\ l_0 \cos(q_0) & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}
$$

$$J_1 = Jacobian(^{0}_{org}T\, com_1) = \begin{bmatrix} 0 & -l_1 \cos(q_1) & 0 & 0 \\ -(L_0 + l_1 \cos(q_1)) \sin(q_0) & -l_1 \sin(q_1) \cos(q_0) & 0 & 0 \\ (L_0 + l_1 \cos(q_1)) \cos(q_0) & -l_1 \sin(q_0) \sin(q_1) & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{15}$$

$$J_2 = Jacobian(^{1}_{org}T\, com_2) \tag{16}$$

$$J_3 = Jacobian(^{2}_{org}T\, com_3) \tag{17}$$

$$J_{EE} = Jacobian(^{3}_{org}T\, x_{ee}) \tag{18}$$

$$x = \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ p_x \\ p_y \\ p_z \\ \omega_x \\ \omega_y \\ \omega_z \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} \tag{19}$$

$$f = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1z} \\ f_{2x} \\ f_{2y} \\ f_{2z} \end{bmatrix} \tag{20}$$

$$g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ gravity \end{bmatrix} \tag{21}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & dtrz_{11} & dtrz_{12} & dtrz_{13} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & dtrz_{21} & dtrz_{22} & dtrz_{23} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & dtrz_{31} & dtrz_{32} & dtrz_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & dt & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
dt\,(i_{12}r1z - i_{13}r1y) & dt\,(-i_{11}r1z + i_{13}r1x) & dt\,(i_{11}r1y - i_{12}r1x) & dt\,(i_{12}r2z - i_{13}r2y) & dt\,(-i_{11}r2z + i_{13}r2x) & dt\,(i_{11}r2y - i_1 \\
dt\,(i_{22}r1z - i_{23}r1y) & dt\,(-i_{21}r1z + i_{23}r1x) & dt\,(i_{21}r1y - i_{22}r1x) & dt\,(i_{22}r2z - i_{23}r2y) & dt\,(-i_{21}r2z + i_{23}r2x) & dt\,(i_{21}r2y - i_2 \\
dt\,(i_{32}r1z - i_{33}r1y) & dt\,(-i_{31}r1z + i_{33}r1x) & dt\,(i_{31}r1y - i_{32}r1x) & dt\,(i_{32}r2z - i_{33}r2y) & dt\,(-i_{31}r2z + i_{33}r2x) & dt\,(i_{31}r2y - i_3 \\
\frac{dt}{mass} & 0 & 0 & \frac{dt}{mass} & 0 & 0 \\
0 & \frac{dt}{mass} & 0 & 0 & \frac{dt}{mass} & 0 \\
0 & 0 & \frac{dt}{mass} & 0 & 0 & \frac{dt}{mass}
\end{bmatrix} \tag{23}
$$

$$
x(k+1) = \begin{bmatrix}
dt\omega_x rz_{11} + dt\omega_y rz_{12} + dt\omega_z rz_{13} + \theta_x \\
dt\omega_x rz_{21} + dt\omega_y rz_{22} + dt\omega_z rz_{23} + \theta_y \\
dt\omega_x rz_{31} + dt\omega_y rz_{32} + dt\omega_z rz_{33} + \theta_z \\
dt\dot{p}_x + p_x \\
dt\dot{p}_y + p_y \\
dt\dot{p}_z + p_z \\
dtf_{1x}\,(i_{12}r1z - i_{13}r1y) + dtf_{1y}\,(-i_{11}r1z + i_{13}r1x) + dtf_{1z}\,(i_{11}r1y - i_{12}r1x) + dtf_{2x}\,(i_{12}r2z - i_{13}r2y) + dtf_{2y}\,(-i_{11}r2z + \\
dtf_{1x}\,(i_{22}r1z - i_{23}r1y) + dtf_{1y}\,(-i_{21}r1z + i_{23}r1x) + dtf_{1z}\,(i_{21}r1y - i_{22}r1x) + dtf_{2x}\,(i_{22}r2z - i_{23}r2y) + dtf_{2y}\,(-i_{21}r2z + \\
dtf_{1x}\,(i_{32}r1z - i_{33}r1y) + dtf_{1y}\,(-i_{31}r1z + i_{33}r1x) + dtf_{1z}\,(i_{31}r1y - i_{32}r1x) + dtf_{2x}\,(i_{32}r2z - i_{33}r2y) + dtf_{2y}\,(-i_{31}r2z + \\
\frac{dtf_{1x}}{mass} + \frac{dtf_{2x}}{mass} + \dot{p}_x \\
\frac{dtf_{1y}}{mass} + \frac{dtf_{2y}}{mass} + \dot{p}_y \\
\frac{dtf_{1z}}{mass} + \frac{dtf_{2z}}{mass} + gravity + \dot{p}_z
\end{bmatrix} \tag{24}
$$

$$
\|f(k)\|_R = \sqrt{f(k)_R} = \sqrt{f(k)^T R f(k)} \tag{25}
$$