

Note:  $\mu$  is the observed expectation value, and  $\sigma$  is the standard error of  $\mu$ .  
For  $\mu > 0$  condition:

$$\begin{aligned}
P(x \geq 0 | \mu_{opt}) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_{opt})^2}{2\sigma^2}} = P(x \geq 0 | \mu_{opt} = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} \\
P(x < 0 | \mu_{opt}) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_{opt})^2}{2\sigma^2}} = P(x < 0 | \mu_{opt} = 0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-0)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \\
R(x \geq 0) &= P(x \geq 0 | \mu) / P(x \geq 0 | \mu_{opt}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} / \frac{1}{\sigma\sqrt{2\pi}} = e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
R(x < 0) &= P(x < 0 | \mu) / P(x < 0 | \mu_{opt}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} / \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} = e^{\frac{2x\mu - \mu^2}{2\sigma^2}}
\end{aligned}$$

$$\begin{aligned}
-x_u^2 + 2x_u\mu - \mu^2 &= 2x_l\mu - \mu^2 \\
\Rightarrow -x_u^2 + 2x_u\mu &= 2x_l\mu \\
\Rightarrow x_l &= x_u - \frac{x_u^2}{2\mu}
\end{aligned}$$

For  $\mu < b$  condition:

$$\begin{aligned}
P(x \leq b | \mu_{opt}) &= P(x \leq b | \mu_{opt} = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} \\
P(x > b | \mu_{opt}) &= P(x > b | \mu_{opt} = b) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma^2}} \\
R(x \leq b) &= P(x \leq b | \mu) / P(x \geq 0 | \mu_{opt}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} / \frac{1}{\sigma\sqrt{2\pi}} = e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
R(x > b) &= P(x > b | \mu) / P(x < 0 | \mu_{opt}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} / \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma^2}} = e^{\frac{(2x - (\mu+b))(\mu-b)}{2\sigma^2}}
\end{aligned}$$

$$\begin{aligned}
-x_u^2 + 2x_u\mu - \mu^2 &= 2x_l(\mu - b) - \mu^2 + b^2 \\
\Rightarrow -x_u^2 + 2x_u\mu - b^2 &= 2x_l(\mu - b) \\
\Rightarrow x_l &= \frac{2x_u\mu - x_u^2 - b^2}{2(\mu - b)}
\end{aligned}$$