Stock Trading Strategy based on Dynamic Bayesian Network

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Abstract

The pricing earnings (PE) ratio is a widely used index for constructing investment strategy in a security market. There exist hidden patterns in PE ratio. In this project, we implement an advanced dynamic Bayesian network (DBN) methodology for value investment through fundamental PE estimations. Two algorithms are adopted to estimate the parameters in RStan. Diagnostics and posterior predictive distribution show the validity of the model. The estimated PE ratio from our model can be used either as a information support for an expert to make investment decisions, or as an automatic trading system illustrated in experiments. Different from existing works in literature, the economic interpretation of our DBN model is well-justified by behavioral finance evidences of volatility.

1 Introduction

Our work builds on the paper "Bayesian Probabilistic Inference on Firm-Level Stock Price Dynamics"[1]. Since machine learning is becoming a ubiquitous tool in finance, investors in security markets are leaning more towards including machine algorithms in their trading systems. The general philosophy of applying machine learning to trading is based on the assumption that there are hidden patterns in the data that can be uncovered and used to trade accordingly. However, machine learning models tend to be very complicated and sometimes lack adequate financial explanation. This type of investment strategy pertains to the so called technical analysts group. At the other end of the spectrum, there is another kind of investors group called fundamental analysts who make trading decisions based on concrete knowledge of the company's financial and accounting information. One of the main tools that fundamental analysts use in their evaluation of a company is the so called "PE Ratio" (Price Earnings ratio). This index can be thought of as the premium of an individual firm in the sense that given two companies with the same earnings, the one with the higher PE ratio maybe more valuable hence more attractive. However, a higher PE ratio may also suggest the company is over-priced, and a lower PE ratio may suggest the company is under-priced. Growth-based investors are attracted to stocks with high PE ratios while value-based investors favor stock with low PE ratios. It is not clear which investment style should be chosen, so instead, we intend to get pass this issue by estimating a company's true PE ratio. Unfortunately, the current available literature does not provide effective ways to estimate a company's PE ratio beyond mere expert opinion. Instead of solely relying on subjective expert opinion to estimate a company's PE Ratio, the goal of this work is to use Bayesian analysis to combine expert opinion with the available data in order to make a more informed inference about the fundamental PE ratio of a company.

2 Investment Problem Background and Motivation for Statistical Modeling

Price-earning ratio equals to market value per share divided by annual earning per share. Therefore, it varies on a daily basis. However, we believe a firm's appropriate price to earning ratio should be stable over a short-time period[3]. The appropriate PE ratio is usually determined by experts using business and financial accounting factors such as debt burden, cash flow, growth rate, business risk, etc[1]. Once we have the appropriate PE ratio, we can short sell a company's stock when the firm's observed PE ratio is higher than the appropriate one, and buy the stock if the observed PE ratio is lower than the appropriate PE ratio.

Our motivation for statistical modeling comes from behavior finance. The observed PE ratio may deviate from the fundamental PE ratio because traders sometimes make irrational behaviors so that stock price deviates from its fundamental true price. For example, the investor may over-react to new information. What is worse, the new information may not be important at all. Also, the advice that investors obtained from experts are not always helpful, and because of limit of arbitrage, even the rational investor cannot eliminate this irrational pricing[1,3]. Hence, it is meaningful to use statistical method to infer true underlying price/PE-ratio.

3 Data Description

We used the stock price and EPS(earning per companies' share) collected from FactSet financial terminal to fit and test our model. Our panel data are time series of stock prices and corresponding companies' earnings per share.

1. Stock price: Adjusted backward for dividends and spin-off. Including 19 stocks with 4855 trading days (from Jan. 2000 to April. 2019).

2. Corresponding companies' earnings per share: Including 19 stocks with 4855 trading days (from Jan. 2000 to April. 2019). Daily data are generated by forward fill from quarterly financial reports in order to track the newest information.

There is no missing data.
The Figure 1 shows the Price plot and EPS plot.

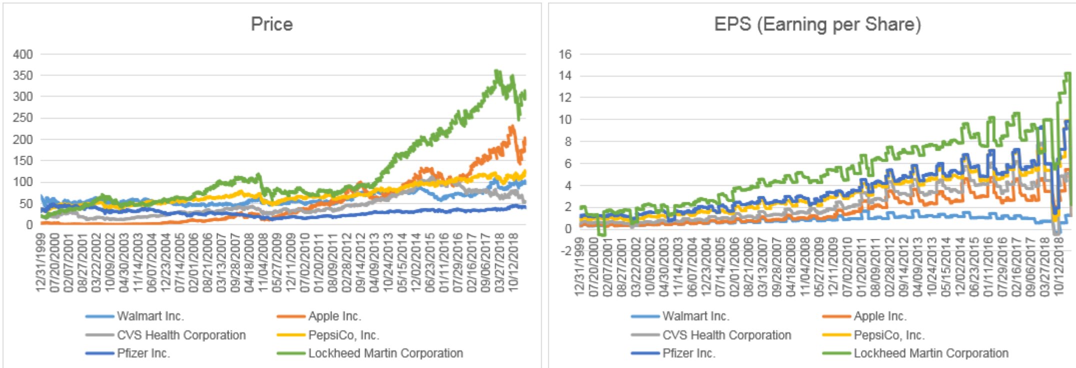

Figure 1: Price (left) and EPS (right)

As we can see, most companies' stock prices have exponential growth trend while there exist much volatility. The EPS remains the same for a quarter until the company's the next earning announcement.

The PE ratio is the Stock price divided by earnings per share. The following plot shows the PE for all companies in our research, including Alphabet, Walmart, Apple, CVS Health, PepsiCo, Pfizer, Lockheed Martin, Caterpillar and Goldman Sachs. One interesting thing we can see from this plot is that the shape of PE is somehow like a staircase curve, remaining at one stage with small volatility and then jumping to another stage. Our model proposed in section 4 is convenient to model staircase shape.

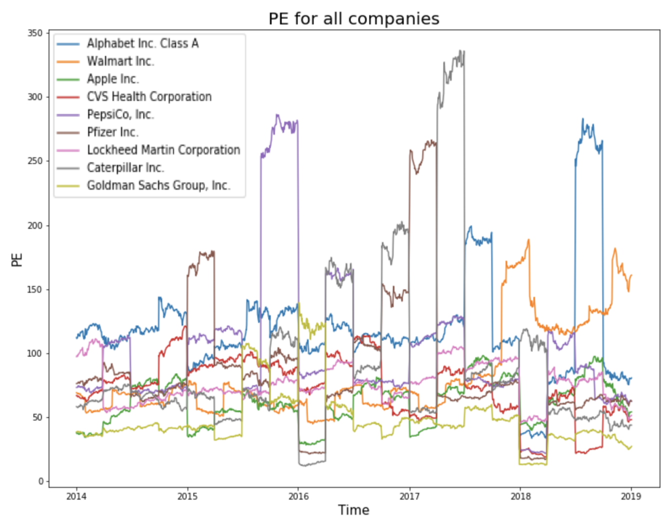

Figure 2: PE for All Companies

The following table shows the basic statistics of PE of different companies.

	count	mean	std	min	25%	50%	75%	max
Alphabet Inc. Class A	1259.0	119.265558	43.870148	33.588628	104.048338	112.500345	125.922757	283.149780
Walmart Inc.	1259.0	82.483720	34.631024	44.839161	58.962513	70.242718	84.098958	188.879310
Apple Inc.	1259.0	61.819187	17.630224	28.481707	46.136658	62.329730	75.255188	98.233533
CVS Health Corporation	1259.0	73.572597	23.782859	18.819876	58.279206	74.726415	90.778812	121.296296
PepsiCo, Inc.	1259.0	101.972510	53.876934	21.953136	73.263393	81.089041	113.960955	286.333333
Pfizer Inc.	1259.0	87.653715	50.983347	16.648515	64.245098	75.106383	92.934211	266.384615
Lockheed Martin Corporation	1259.0	75.685140	14.893227	45.248445	66.523551	73.047445	86.164452	111.226667
Caterpillar Inc.	1259.0	90.406178	65.470452	12.014523	54.889535	68.732484	100.874747	336.250000
Goldman Sachs Group, Inc.	1259.0	49.026727	23.210568	12.773958	36.891297	42.824945	52.494403	139.480315

Figure 3: PE statistics

4 Mathematical Modelling

As described in section 2, because of irrationality of investors, observed PE ratio sometimes deviate from its fundamental true value. In addition to relying on fundamental analysis, it is reasonable to propose a statistical model to estimate the true P-E ratio.
We propose a Markov process $(z_t)_t$ to model traders' irrationality briefly discussed in section 2. We divide irrationality into two types according to its persistence[1]. (1) Less persistent irrational behaviors: for example, the irrational behaviors caused by noise trading or overreaction to newly released market news. (2) Persistent irrational behaviors: for example, behaviors caused by incorrect predictions made by experts or unreliable information which may take time to confirm.

Let P_t/E_t be the observed P-E ratio, and PE^* be the fundamental true P-E ratio during a short-time period. Taking into account traders' irrational behaviors and subtle random effects in the market, we used the following equation to model the observed P-E ratio:

$$P_t/E_t = PE^*(1 + z_t)(1 + \epsilon_t) \tag{1}$$

Notice the above equation allows the P_t/E_t to have a staircase shape which conforms with our data visualization in Section3. Also:
(a) z_t is a Markov chain modeling the irrational behaviors.
(b) ϵ_t is a Gaussian random noise modeling subtle random effects, $\epsilon_t \sim N(0, \sigma^2)$.
Now, impose log transformation to y_t, and assume ϵ is small, we have emission probability for $log(P_t/E_t)$

$$y_t = ln(P_t/E_t) = ln(PE^*(1 + z_t)) + \epsilon_t, \tag{2}$$

$$p(y_t|z_t = a_m, PE^* = b_n, \sigma) = N(y; ln(b_n(1 + a_m)), \sigma^2) = \Phi_t^{mn} \tag{3}$$

where $z_t \in (a_1, ..., a_M)$ $PE^* \in (b_1, ..., b_N)$.

Two types of irrational behaviors can be shown by the diagonal components of learned transition probability matrix W associated with z_t The figure below is an illustration of the Dynamic Bayesian Network we used[1].

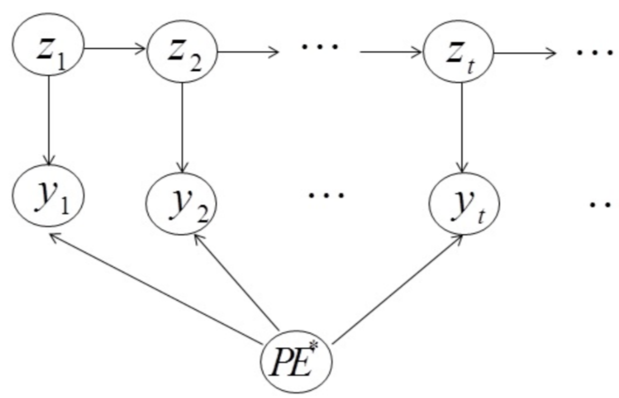

Figure 4: DBN

One of the advantages of the above model is that it encodes conditional independent properties, which also helped simplify likelihood derivation. In addition, we can plug in expert knowledge into this model by proposing suitable prior which will be discussed later.

We clarify notations for transition matrix, initial probability and parameter space by the following equations:

(a)The transition probability matrix:
Let $i, m \in \{1, ..., M\}, t \in \{2, 3, ...\}$

$$p(z_t = a_i|z_{t-1} = a_m) = w_{mi}. \tag{4}$$

Note that $0 \leq w_{mi} \leq 1$. The matrix $\mathbf{W} = (w_{mi})_{M\times M}$ is a transition matrix of the Markov chain z_t.

(b) The initial probability distribution:
For each $m \in \{1, ..., M\}$

$$u_m = p(z_1 = a_m|\phi) \tag{5}$$

Where $0 \leq u_m \leq 1$ and $\sigma_{m=1}^{M} u_m = 1$
For each $n \in \{1, ..., N\}$

$$v_n = p(PE^* = b_n|\phi) \tag{6}$$

Where $0 \leq v_n \leq 1$ and $\sigma_{n=1}^{N} v_n = 1$

The vectors $\mathbf{u} = (u_m)_M$ and $\mathbf{v} = (v_n)_N$ are called initial vectors.

Therefore, the set of model parameters is $\theta = \{\mathbf{W}, \mathbf{u}, \mathbf{v}, \sigma^2\}$, and the parameters space is

$$\Theta = \{\theta|0 \leq u_m \leq 1, \sum_{m=1}^{M} u_m = 1, 0 \leq v_n \leq 1, \sum_{n=1}^{N} v_n = 1, 0 \leq w_{im} \leq 1, \sum_{m=1}^{M} w_{im} = 1, \sigma > 0\}. \tag{7}$$

Notice that PE^* can be modeled continuously instead of using discreet hidden states. In that case, our model is the same with standard hidden Markov model. However, discreet modeling is more suitable in our case.

5 Likelihood Derivation and Strategy Construction

Before moving to the likelihood derivation, it is helpful to keep in mind that z_t is independent from PE^* without observing y_t. Also, future observations are independent from the past if current hidden states are blocked. Forward probability is defined as follow

$$P(\mathbf{y}_{1:t}, z_t = a_m, PE^* = b_n|\theta) := F_t^{mn}$$

, where

$$F_1^{mn} = \Phi_1^{mn}(\sigma)u_m v_n$$

.

$$\begin{aligned}
F_t^{mn} &= \sum_{z_1 \ldots z_{t-1}} P(\mathbf{y}_{1:t-1}, z_1, ..., z_{t-1}, z_t = a_m, PE^* = b_n|\theta)\Phi_t^{mn} \\
&= \sum_{z_{t-1}=a_k} \sum_{z_1 \ldots z_{t-2}} P(\mathbf{y}_{1:t-1}, z_1 ... z_{t-1} = a_k, PE^* = b_n|\theta)W_{km} \\
&= \Phi_t^{mn} \sum_{k=1}^{M} F_{t-1}^{kn} W_{km}
\end{aligned}$$

$$P(\mathbf{y}_{1:T}|\theta) = \sum_{mn} F_T^{mn}$$

The above program was coded up in RStan and PyStan for generating likelihood[2]. In addition to generating likelihood, we also use the forward probability to make inference on $_t$ and also PE^* by the following relations:

$$F_t^{mn} \propto P(z_t = a_m, PE^* = b_n|y_{1:t}, \theta) \tag{8}$$

The normalization constant only depends on (y,θ) and thus we can directly use F_t^{mn} for inference purpose:

$$z_t|I(t) = \underset{am}{\text{argmax}} \sum_{n=1}^{n=N} F_t^{mn}, \tag{9}$$

$$PE^*|I(t) = \underset{bn}{\text{argmax}} \sum_{m=1}^{m=M} F_t^{mn} \tag{10}$$

, where I(t) represent filtration generated by $y_{1:t}$. Smooth probability is not useful for our objective, since when deriving and back-testing strategy, we can only use information up to current time for inference. States inferred from smooth probability may generate better performance, but the performance is meaningless.

Before using forward algorithm, we used a self-developed algorithm to construct the likelihood. The statistic and computational efficiency of two methods is compared in the next section.

$$P(\mathbf{y}_{1:T}|\theta) = P(\mathbf{y}_{2:T}|y_1, \theta)P(y_1|\theta)$$

, and

$$P(y_1|\theta) = \sum_{nm} \Phi_1^{mn} u_m v_n$$

$$P(\mathbf{y}_{2:T}|y_1, \theta) = \sum_{mn} P(z_1 = a_m, PE^* = b_n|y_1, \theta)P(\mathbf{y}_{2:T}|z_1 = a_m, PE^* = b_n, \theta)$$

Based on the above equation, define

$$\beta_{t-1}^{mn} = P(\mathbf{y}_{t:T}|y_{t-1}, z_t = a_m, PE^* = b_n; \theta)$$

Similar with the construction of forward algorithm, we use the same trick by adding hidden state z_{t+1} to form the joint distribution:

$$
\begin{aligned}
\beta_t^{mn} &= \sum_{i=1}^{M} P(\mathbf{y}_{t+1:T}, z_{t+1} = a_i | z_t = a_m, PE^* = b_n, y_t) \\
&= \sum_{i=1}^{M} P(\mathbf{y}_{t+1:T} | z_{t+1} = a_i, PE^* = b_n) W_{mi} \\
&= \sum_{i=1}^{M} p(y_{t+1}, \mathbf{y}_{t+2:T} | z_{t+1} = a_i, PE^* = b_n) W_{mi} \\
&= \sum_{i=1}^{M} p(\mathbf{y}_{t+2:T} | y_{t+1}, z_{t+1} = a_i, PE^* = b_n) W_{mi} p(y_{t+1} | z_{t+1} = a_i, PE^* = b_n) \\
&= \sum_{i=1}^{M} \beta_{t+1}^{mn} \phi_{t+1}^{in} W_{mi}
\end{aligned} \tag{11}
$$

With the base case of

$$
\beta_{T-1}^{mn} = \sum_{i=1}^{M} \phi_T^{in} W_{mi}
$$

.

Thus the conditional likelihood is given by

$$
P(\mathbf{y}_{1:T} | \theta) = \sum_{mn} \phi_1^{mn} u_m v_n \frac{1}{C_1} \beta_1^{mn}
$$

Now we are ready to set up our strategies. We tested our strategies during the most recent time period: 01/02/2014 - 01/02/2019. The first 3 years is used for learning parameters and states. The last 2 years is used for back-testing. After we trained our model during the first 3 years, we plug learned parameters into forward-probability scheme to infer path of hidden state z.
Our mid-term strategy is based on forecasting underlying P-E ratio. Define probability measure Q such that in the back-test window, we use the forward probability to infer states of z_t and PE^*. According to equation 1, our forecast on P-E ratio is given by:

$$
E[P_t/E_t | PE^*, z_t] = PE^*(1 + z_t).
$$

Thus, if

$$
P_t/E_t(1 - Tr) > PE^*(1 + z_t),
$$

we sell all of stocks we hold. If

$$
P_t/E_t(1 + Tr) < PE^*(1 + z_t),
$$

use all of our available free cash to buy the stocks assuming that we can buy fraction of one share. Tr is a threshold, e.g 0.02.
The other type of the strategy is the long-term strategy. We only change our position based on inferred underlying P-E ratio. So if the observed P-E ratio is higher than the underlying P-E ratio by certain threshold, we sell all the stocks, and otherwise we buy stocks with all available cash.

6 Algorithm Testing on Synthetic Data

To test our algorithms accuracy, we generated synthetic data through our model under known parameters. If we generated over 500 data and our both algorithms are correct, the learned parameters should be fairly close to the true parameter values. We ran the above two algorithms on synthetic data. Since the two algorithms provide us with quite similar results, we show the following output of the forward method.

	True Value	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff
u [1]	●	0.51	0.01	0.22	0.13	0.34	0.51	0.68	0.91	1107
u [2]		0.25	0.01	0.18	0.01	0.09	0.21	0.35	0.69	695
u [3]		0.24	0.01	0.18	0.01	0.09	0.21	0.36	0.67	591
v [1]		0.25	0.01	0.19	0.01	0.09	0.20	0.37	0.70	959
v [2]	●	0.51	0.01	0.22	0.09	0.35	0.51	0.69	0.91	821
v [3]		0.24	0.01	0.19	0.01	0.09	0.20	0.36	0.67	1077
w[1,1]	0.95	0.88	0.00	0.02	0.83	0.87	0.89	0.90	0.92	618
w[1,2]	0.03	0.03	0.00	0.01	0.01	0.02	0.03	0.04	0.06	921
w[1,3]	0.02	0.08	0.00	0.02	0.04	0.07	0.08	0.10	0.13	670
w[2,1]	0.04	0.06	0.00	0.02	0.04	0.05	0.06	0.08	0.10	831
w[2,2]	0.9	0.87	0.00	0.02	0.82	0.85	0.87	0.88	0.91	1058
w[2,3]	0.06	0.07	0.00	0.02	0.04	0.06	0.07	0.08	0.11	1112
w[3,1]	0.05	0.04	0.00	0.02	0.01	0.03	0.04	0.05	0.08	755
w[3,2]	0.02	0.16	0.00	0.03	0.10	0.13	0.16	0.18	0.23	965
w[3,3]	0.93	0.80	0.00	0.04	0.73	0.78	0.80	0.82	0.86	925
sigma	0.03	0.05	0.00	0.00	0.04	0.04	0.05	0.05	0.05	1100

Figure 5: The true values fall exactly in the confidence intervals. Our algorithm works very well in estimating parameters.

We can see that the estimates are close to true values. The synthetic data parameter of $W[1,2]$ is 0.03 while the estimated $W[1,2]$ is 0.03. The 95% interval is from 0.01 to 0.06. Our algorithm works very well in estimating parameters but may under estimate the persistence of states. For example, synthetic data parameter of $W[1,1]$ is 0.95 while the estimated $W[1,1]$ is 0.88. The 95% interval is from 0.88 to 0.92. We only fit on 500 data points, if we plugged in more data close 1000, then we expect the estimation results to be more close to true values. It is valid to check on the algorithms using more data, since our training window contains nearly 800 data points.

7 Prior selection

As discussed in the previous section, our model should allow financial analyst to plug in their knowledge about fundamental P-E ratio. They should be able to give suitable candidates for appropriate P-E states and how much observed P-E usually deviates from the appropriate one. Therefore, we would like the user to propose states of z and PE^*. After setting up states of z and PE^*, we add prior distribution to model initial probability of PE^*, and also the transition probability W

$$p(v) = \frac{\tau(\kappa_1 + \ldots + \kappa_N)}{\tau(\kappa_1)\tau(\kappa_2)\ldots\tau(\kappa_N)} \prod_{n=1}^{N} v_n^{\kappa_n - 1} \tag{12}$$

κ_n is the degree of prior belief that PE^* is at state b_n. The user can assign higher values to κ_n if she believes PE^* should be at b_n. Dirichlet distribution is used to model this simplex. Prior distribution of the transition probability is used for the user to adjust the persistence degree of the irrational behavior effect.

$$p(w_m) = \frac{\tau(\sum_{i=1}^{M} \kappa_{mi})}{\prod_{i=1}^{M}(\kappa_{mi})} \prod_{i=1}^{M} w_{mi}^{\kappa_{mi} - 1} \tag{13}$$

Similarly, if the user thinks certain levels of irrational effects are more persistent, she can simply plug in her prior belief by giving more weights to the κs that correspond to stable states. w_m is a row of transition probability. It is a simplex, and thus naturally modeled using Dirichlet distribution. In general, we do expect z to be persistent, because recall that the early plot of observed P-E in section 3 shows observed P-E has a form of stairs. Those "stairs" in the plots seem to be persistent. Different for W and v do make a difference in terms of strategies' out of sample performance, thus a uniform prior may not be desired. The difference of various choices of κ and other hyper-parameters will be illustrated later.

8 Efficiency of the Computational Algorithms

We run forward and backward algorithms for Walmart observed log(PE) during 2004 and 2005 with 1000 iterations, 4 chains and compare the statistical efficiency and computational efficiency. The convergence can be checked in the result section with $\hat{r} = 1$, which is shown in section Results and Visualization Figure 10. Following is the table for comparison of two algorithms.

	Statistical Efficiency		Computational Efficiency
Method	Effective Sample Size in 500 Samples	Effective Samples per Second	Time to Finish 1000 Iterations (in second, burnin half)
Forward Method	500	2.01	249.19
Backward Method	500	1.07	468.89

Figure 6: As for statistical efficiency, the forward algorithm and backward algorithm are the same. They both have 500 effective samples out of 500 samples. The forward algorithm is faster than the backward algorithm in running the same number of iterations. The forward algorithm also generates more effective samples within per unit of time.

Since the forward and backward method all have 500 out of 500 effective samples, we choose the one with better computational efficiency. The forward algorithm can generate around 2 times samples per unit time than the backward algorithm can.

9 Diagnostics

Based on this DBN model, we derive the Posterior Predictive Distribution (PPD) of log(PE), The red lines are moments from the observations.: First, we consider the log(PE) mean and log(PE) variance, and compare them with the observed log(PE) mean and log(PE) variance. The following histogram plot shows the PPD of log(PE) mean and the PE variance and the vertical line represents the observed value. From the plot, the PPD of mean looks pretty good since the observed log(PE) mean lies near the center of the Posterior Predictive distribution. The PPD of variance shows that the variance characteristic is captured but not as well as the mean was.

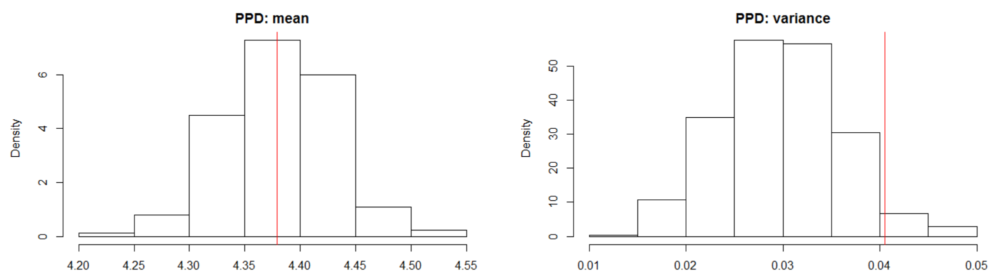

Figure 7: PPD for mean (left) and variance (right)

Secondly, we consider the auto-correlation function of log(PE). The following histogram plot shows the PPD of ACF of log(PE) with lag 1 and lag 2. The red lines are value from the observations.

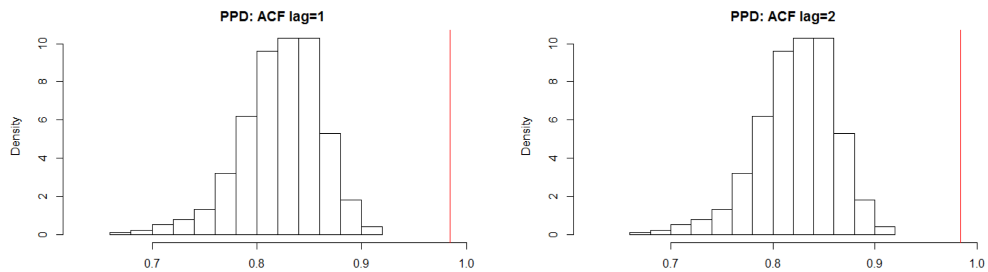

Figure 8: PPD for ACF with lag1 (left) and lag2 (right)

In order to reveal the autocorrelation characteristic of our simulated data, we also compare the posterior distribution of autocorrelation with lag=1 and lag=2 with those of the observations. From the graph we can see that our model can not capture these two characteristics well. Possible reasons include: Markov property of hidden states z might not hold; the process might not be stationary. These enlightened us to come up with methods of future improvements.

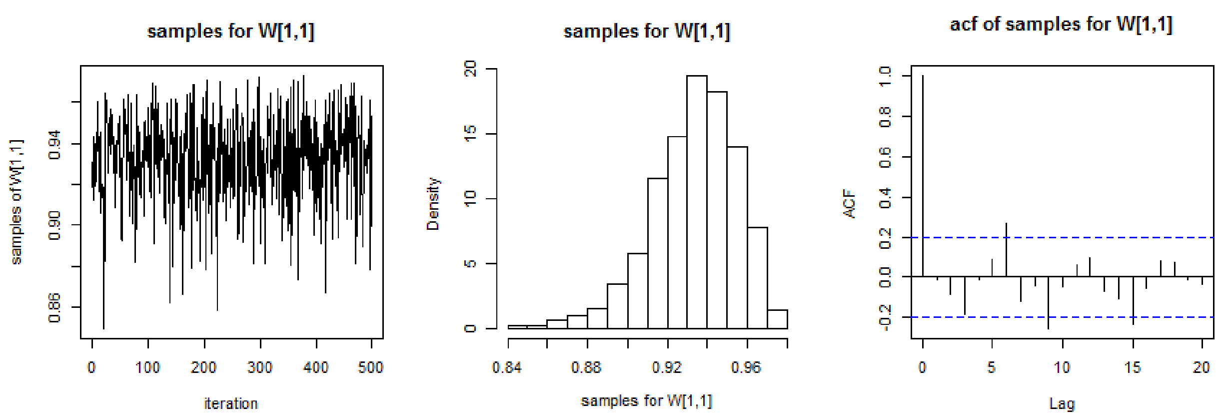

Figure 9: Trace plot for parameters W_{11} and W_{12}

Moreover, we make diagnostics on parameter W_{11}, including the iteration plot, distribution plot and ACF plot.

The iteration series looks quite stationary. From the plot we conclude that the algorithm converges within 1000 iterations. The distribution plot shows that the W_{11} is highly left-skewed, and the ACF plot shows that the auto-correlation for most lag is in the 95% confidence interval and is not significant. Therefore, we draw the conclusion that the algorithm is converging quite well.

10 MCMC Convergence Result

This tables covers parameters in our model and provide $\hat{r}$ exactly 1 which means the convergence is quite well. When $\hat{r} = 1$, a potential reduction of 0% in posterior interval width, given infinite iterations.[4]

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
W[1,1]	0.93	0.00	0.02	0.89	0.92	0.93	0.95	0.97	4034	1
W[1,2]	0.03	0.00	0.01	0.01	0.02	0.03	0.04	0.06	3891	1
W[1,3]	0.01	0.00	0.01	0.00	0.01	0.01	0.02	0.04	2467	1
W[1,4]	0.01	0.00	0.01	0.00	0.00	0.01	0.01	0.03	1760	1
W[1,5]	0.01	0.00	0.01	0.00	0.00	0.00	0.01	0.03	2624	1
W[1,6]	0.01	0.00	0.01	0.00	0.00	0.01	0.01	0.03	1883	1
v[1]	0.20	0.00	0.16	0.01	0.07	0.16	0.28	0.60	2439	1
v[2]	0.60	0.00	0.19	0.21	0.46	0.62	0.75	0.92	2961	1
v[3]	0.20	0.00	0.16	0.01	0.07	0.16	0.29	0.58	3028	1
u[1]	0.14	0.00	0.12	0.00	0.04	0.11	0.20	0.46	3204	1
u[2]	0.14	0.00	0.12	0.00	0.05	0.11	0.21	0.45	2632	1
u[3]	0.14	0.00	0.12	0.00	0.05	0.11	0.21	0.44	3248	1
u[4]	0.14	0.00	0.12	0.00	0.04	0.11	0.21	0.46	3390	1
u[5]	0.14	0.00	0.12	0.00	0.05	0.11	0.21	0.48	2720	1
u[6]	0.29	0.00	0.17	0.04	0.15	0.27	0.39	0.67	2677	1
sigma	0.06	0.00	0.00	0.06	0.06	0.06	0.06	0.06	4236	1

Figure 10: RStan results printing

We can see that the estimates of $W[1,1]$ is 0.93 and the 95% interval is from 0.89 to 0.97. The estimate of parameter $W[1,2]$ is 0.03 and the 95% interval is from 0.01 to 0.06. The estimate of parameter $v[1]$ is 0.20 and the 95% interval is from 0.01 to 0.60. Other estimates of the transition matrix can be shown in the similar way and there are 36 elements in the transition matrix in total and results of others are omitted to save space. The estimate of parameter $u[1]$ is 0.14 and the 95% interval is from 0.00 to 0.46. The estimate of parameter $sigma$ is 0.06 and the 95% interval is from 0.06 to 0.06. The likelihood of $u[1]$ and $v[2]$ are greatest among u and v, which is also corresponding to that we started from those two initial states. Our algorithm works very well in estimating parameters especially in transition probabilities and volatility $sigma$.

These inferences will be used for out-of-sample trading strategies constructing. There are two major ways. The long term trading strategy involves using all parameters and in-sample observations to give a most likely state of PE^*. The medium term trading strategy will use all parameters and out-of-sample F_t measurable observations (which means not using any future information from future observations) to derive the most likely $PE^* * (1 + z_t)$ and build the medium trading strategy.

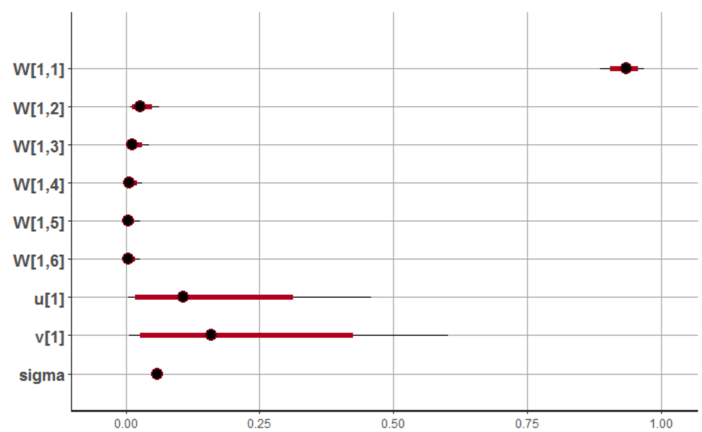

Figure 11: Point estimates, 80% intervals, 95% intervals of parameters

We can see from the graph that the intervals for Ws and $sigma$ is narrow while the intervals for $u[1]$ and $v[1]$ is wide. We got quite accurate estimates for the transition matrix elements and volatility and less accurate estimates for initial probabilities of the Markov chain and PE^*.

11 Out of Sample Performance Results

In this section, we will will out of sample performances change among different choices of hyper-parameters. We have three strategies
1. hold only strategy: the benchmark
2. long term strategy
3. mid term strategy

For each strategy and a given stock, we programmed back-test framework(taking into account a synthetic commission fee) of section 5 in python for testing different sets of hyper-parameters. Various numbers of hidden states, values of hidden states and κs of prior distributions are combined to form 36 sets of hyper-parameters. We ran this hyper-parameter tuning on 8 stocks from SP500 companies list and we only present 2 stocks in this report. The figure below illustrates out of sample performance results of Pfizer.Inc under 36 sets of hyper-parameters. Vertical blue and orange bars represent mid-term and long-term strategy, while horizontal bar represents bench-mark strategy.

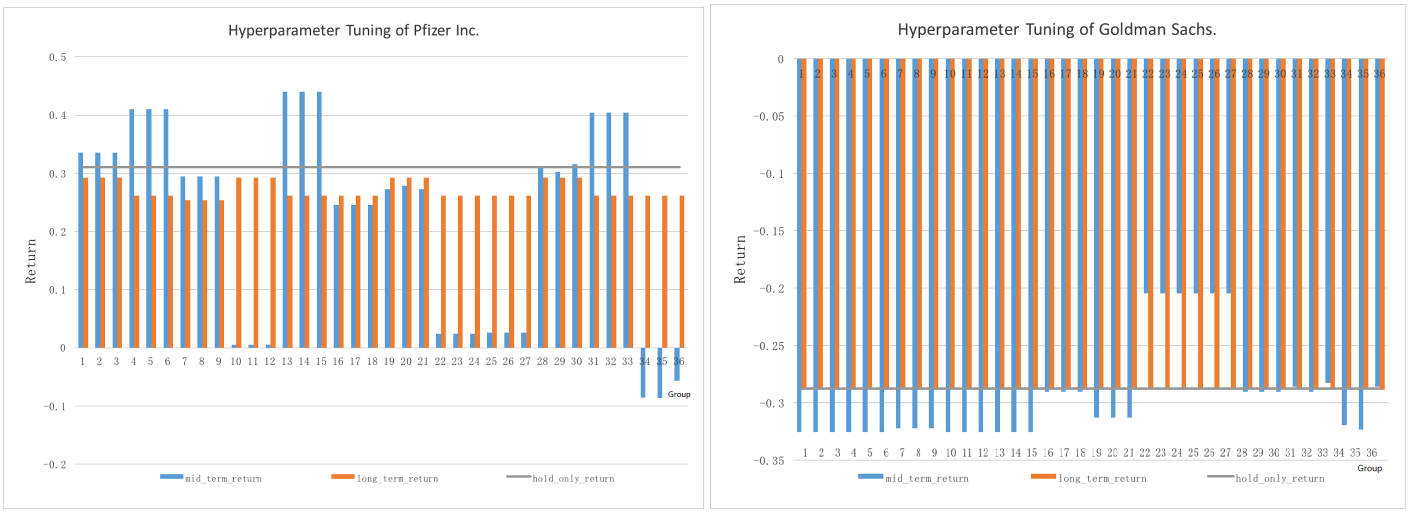

Figure 12: Hyper parameters tuning for Pfizer and Goldman

For Pfizer, we can see that our benchmark hold-only strategy gives a 31.06% return while several choices of hyper-parameters deliver much better returns. The highest return is 44.04%, which is promising. In addition, our position was successfully closed. However, the long-term strategy does not seem to do well compared with the bench mark. It might because the states of PE_* was determined by a K-Means method; however those should be determined according to financial reports or expert's advises. Besides, a long-term strategy might be too short to implement within a two-year period. When z has four states and number of states for PE* equals to 3, our strategy has best performance. The corresponding tuned κ for the prior distribution of transition matrix W is: $\kappa_{ii} = 3$, $\kappa_{ij} = 1$ where $i \neq j$. κ for the prior distribution of v is $\kappa_1 = 1$,$\kappa_2 = 2$, $\kappa_3 = 1$.

Unlike Pfizer Inc., Goldman Sachs has been performing very badly during our test window. It might give interesting insight when analyzing our strategies under this scenario. Stock price falls by 28.79%, while our best performance gives 20.45% loss. So we lose a smaller amount of money than the benchmark strategy. Let's take a closer look at those three strategies:

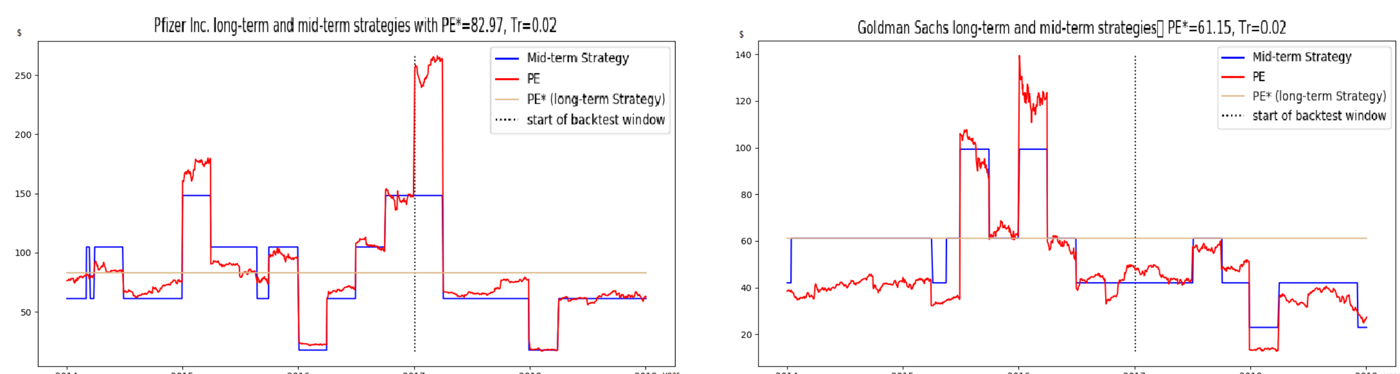

Figure 13: Strategy Path

The above plots illustrate values of P-E we used for mid-term and long-term strategies. Our model correctly identified time frames when the Pfizer2.Inc was undervalued and overvalued, when compared with the real-time price movement. However, our model also seems to work fine on the Goldman stock, then why do we still have a significant negative return? One of the reasons might be our back-test frame work is limited. We did not allow short sell, or a short position on future/option. When the test window starts, we have a sell signal. However, since we did not hold any stocks, we were not allowed to sell. That might significantly influence the profitability of our strategy.

Besides, one should notice that our back-test framework can only demonstrate the potential of this model. In reality, one should not tune the hyper-parameters in the way that we did. The investor should directly plug in appropriate knowledge into the model by analyzing financial reports and consulting experts.

12 Future Development

Our model did not capture the variance very well. One of the future development may be let σ varies on different states of z, so the emission probability becomes:

$$p(y_t|z_t = a_m, PE^* = b_m; \sigma) = N(y_t; ln(b_n(1 + a_m)), \sigma^2_{z_t}) = \phi_t^{mn} \tag{14}$$

Another possible gap between the real-world scenario and the model might be that Markov property of irrational behaviors(z) does not hold or at least it is not only lag=1. This can be partially fixed by adding more lag terms but the state space will grow exponentially and cause curse of dimension in parameter estimating. We can also relax the other HMM hypothesis, the stationary hypothesis, makes use of time information and proposes non-stationary HMM (NSHMM)[5].

In addition, it is reasonable to apply macro risk signals to complement with our model. For example, if we have a risk signal telling us the shift of current macro regime, or the US equity class is becoming more turbulent(stocks we discussed are from US equity class), we should not use the most recent 3 years as training window because the fundamental P-E ratio may change.

13 Conclusion

The two algorithms can estimate the parameters pretty well. As we illustrated above, the our algorithm works very well for synthetic data which proves the correctness of the algorithm. When fitting the real-world data, our convergence result is also good. Because R-hat values from Gelman's diagnostic seem to be promising and the posterior intervals are narrow, except for initial probabilities. However, since initial probabilities are inferred without observed information, the posterior confidence interval should be wider. The diagnostics of convergence show that simulated values converge quickly within 1000 iterations in both these algorithms. In addition, diagnostic test shows that the model can capture the mean of the stochastic process quite well as the observed mean lies exactly around the median of the posterior values. The variance is also captured by this model. However, as for the auto-correlation of the time series, our model parameters underestimate both auto-correlation in lag=1 and lag=2. This leads to previously mentioned future improvement.

14 Reference

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