RoboJackets RoboNav

URC Manipulation - From an Electrical Viewpoint

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1 Introduction

As RoboNav moves away from IGVC and towards URC, understanding of the underlying principles for manipulation would be important for the electrical team. The study of manipulation has a long history in ECE and is arguably one of the most well-studied types of robot due to the ability to model the robot and the environment it operates in. Whether the task of planning (and / or) control of the manipulators eventually falls on the electrical team or not, a good grasp of the underlying principles of manipulation would always be a handy tool at hand for any unexpected happenings.

This guide borrowed much from the book "A Mathematical Introduction to Robotic Manipulation" by Dr. Richard Murray, Dr. Zexiang Li, and Dr. S. Shankar Sastry. It is meant to provide a simplified introduction to manipulation that freshmen wouldn't need to spent more than a semester understanding the basis of manipulation. We will start from rigid body motion, the right representation for rigid body motion and how it applies to the representation and planning of a manipulator.

2 Rigid Body Motions

The study of robot kinematics and controls has its core in the study of rigid body motions [1] , and we will attempt to approach rigid body motion using linear algebra and screw theory.

Michel Chases proved that a rigid body can be moved from any position to any other by a movement consisting of:

- A movement consisting of rotation about a straight line
- followed by translation parallel to that line.

One such motion is called a **screw motion**. The time derivative version of screw motion is called a **Twist**. Screw motion and twist play the central roles in the formulation of robot kinematics.

2.1 Rotational Motion in $\mathbb{R}^3$

2.1.1 Representation

Rotation Matrices

We begin the study by considering only the rotation aspect of rigid body motion. One of the most common method to describe the orientation of coordinate frame $\mathcal{B}$ relative to inertial frame $\mathcal{A}$ is to sequentially rotate about the z-axis of $\mathcal{B}$ by α, then y-axis of $\mathcal{B}$ by β, and finally along z-axis by γ. This yields a net rotation of $R(\alpha, \beta, \gamma)$ and α, γ, β are called the ZYZ Euler angles.

We use rotation matrix to represent a, well, rotation of coordinates.

[1] think that the position of each joint on the manipulator is a movement away from the last joint

$$R_{\mathbf{x}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & -\sin \\ 0 & \sin & \cos \end{bmatrix}, R_{\mathbf{y}} = \begin{bmatrix} \cos & 0 & \sin \\ 0 & 1 & 0 \\ -\sin & 0 & \cos \end{bmatrix}, R_{\mathbf{z}} = \begin{bmatrix} \cos & -\sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A ZYZ Euler angle rotation would result in

$$R_{ab} = R_{\mathbf{z}}(\alpha)R_{\mathbf{y}}(\beta)R_{\mathbf{z}}(\gamma)$$

We use R_{ab} in the sense that it can transform coorindate points such that for point q currently attached on coordinate frame $\mathcal{A}$, and represented as $\mathbf{q}_a = [x, y, z]$. When coordinate frame $\mathcal{B}$ rotate to frame $\mathcal{B}$, point q's relative position in the frame $\mathcal{B}$ is still $\mathbf{q}_b = [x, y, z]$, since the point moved with the reference frame. However the current location of $\mathbf{q}_a = R_{ab}\mathbf{q}_b$

If point q rotates with frame $\mathcal{B}$ to a new frame $\mathcal{C}$, then

$$\mathbf{q}_a = R_{ab}\mathbf{q}_b = R_{ab}R_{bc}\mathbf{q}_c$$

There exists other types of euler angle parameterizations by using different ordered sets of rotation axes, including ZYX[2] and YZX. They avoided singularity at identity orientation, however do contain singularity at other orientations. We do not cover the details of singularity at this point, but more info at gimbal lock[3].

Quaternion

Quaternion works in a similar way that complex number works on the unit circle to represent planar rotations. They give a global parameterization of $SO(3)$ at the cost of using 4 numbers.

$$Q = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

w is the scalar component and $\mathbf{q} = (x, y, z)$ being vector component, a convenient shorthand notation being $Q = (w, \mathbf{q})$. Vector component satisfies the following relationship

$$\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = -1$$

$$\mathbf{i} \cdot \mathbf{j} = -\mathbf{j} \cdot \mathbf{i} = \mathbf{k}, \mathbf{j} \cdot \mathbf{k} = -\mathbf{k} \cdot \mathbf{j} = \mathbf{i}, \mathbf{k} \cdot \mathbf{i} = -\mathbf{i} \cdot \mathbf{k} = \mathbf{j}$$

2.1.2 Understanding

Figure here

Consider that we have a inertial frame $\mathcal{A}$, and a body frame $\mathcal{B}$ that has undergone a rotation about the origin point in inertial frame and no translation. $\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}$ are the coordinates of the principle axes of $\mathcal{B}$ in the inertial frame. Three vectors be vertical vector and the concatenation coordinate vectors obtain

$$R_{ab} = [\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}]$$

[2] roll pitch yaw

[3] https://en.wikipedia.org/wiki/Gimbal_lock

Since the column vectors of R_{ab} are from principle axes, dot product of the column vector with itself is 1, otherwise 0, yielding

$$R^T R = RR^T = 1 \Rightarrow \det R = \pm 1$$

Since $\det R = \mathbf{x}^T(\mathbf{y} \times \mathbf{z})$ and that $\mathbf{x} = \mathbf{y} \times \mathbf{z}$ under right-handed coordinate system, $\det R = 1$.

Therefore **right-handed coordinate frame are represented by orthogonal matrices with determinant 1**. The set of all matrices in $n \times n$ dimension are denoted by $SO(n)$, as special orthogonal. $SO(3) \in \mathbb{R}^{3\times 3}$ is a **group** under matrix multiplication. A set G, with a binary operation $\circ$ defined on elements of G is considered a group if it satisfies the following axioms:

- Closure: $g_1 \circ g_2 \in G$ if $g_1, g_2 \in G$
- Identity: exists e for every $g \in G$ that $g \circ e = e \circ g = g$
- Inverse: for each $g \in G$ there exists a unique inverse, $g^{-1} \in G$ that $g \circ g^{-1} = g^{-1} \circ g = e$
- Associativity.

In the case of $SO(3)$, it satiefies the above criteria in that

- $AB = C$ where $A, B \in \mathbb{R}^3$ gives $C \in \mathbb{R}^3$ by definition.
- $IR = RI = R$.
- $R^T R = RR^T = I$.
- Associativity from matrix multiplication.

Special orthogonal group satisfies the above criterion
Rodrigue's formula.
Relate back to Rotation matrix.

2.1.3 Application

2.2 Rigid Motion in $\mathbb{R}^3$

2.3 Velocity of a Rigid Body

3 Manipulator Kinematics

3.1 Forward Kinematics

3.2 Inverse Kinematics

3.3 Manipulator Jacobian

4 Building A Manipulator in MATLAB