Introduction to Matrix Algebra

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Why should I learn matrix algebra?

For at least two reasons, one

> [Matrix algebra (MA)] furnishes a compact language that unifies importan aspects of univariate and mutivariate statistics. In this way, MA allows one to understand mutivariable techniques using their uniariate analogues and counterpoints (Raykov and Marcoulides 2011, 21 *passim*).

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1p} \\ a_{21} & a_{22} & ... & a_{2p} \\ . & . & . & . \\ a_{n1} & a_{n2} & ... & a_{np} \end{bmatrix},$$

which is referred to as a $n \times p$ matrix.

A vector is a special kind of matrix used often in statistics. a row vector is a matrix with a single row and two or more columns, while a column vector is a vector with one column and two or more rows:

row vector:

$$x = \begin{bmatrix} 1, 3, 5, 7 \end{bmatrix},$$

column vector:

$$y = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}.$$

Note that x is a 1×4 row vector and y is a 4×1 column matrix. The convention is that when the type of vector is not specified, it is a column vector. Think of variables in a data frame as iconic vectors, which are column vectors.

A scalar can be thought of as a one dimensional matrix.

Matrix Operations

```
R version 3.2.2 (2015-08-14)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 14.04.3 LTS

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
 [3] LC_TIME=en_US.UTF-8        LC_COLLATE=en_US.UTF-8
 [5] LC_MONETARY=en_US.UTF-8    LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C
 [9] LC_ADDRESS=C               LC_TELEPHONE=C
```

```
[11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
[1] knitr_1.11        mosaic_0.13.0     mosaicData_0.13.0 car_2.1-1
[5] ggplot2_2.0.0     lattice_0.20-33   dplyr_0.4.3

loaded via a namespace (and not attached):
 [1] Rcpp_0.12.2         formatR_1.2.1       nloptr_1.0.4
 [4] plyr_1.8.3          tools_3.2.2         digest_0.6.8
 [7] lme4_1.1-10         evaluate_0.8        gtable_0.1.2
[10] nlme_3.1-122        mgcv_1.8-10         Matrix_1.2-3
[13] DBI_0.3.1           yaml_2.1.13         parallel_3.2.2
[16] SparseM_1.7         ggdendro_0.1-17     gridExtra_2.0.0
[19] stringr_1.0.0       MatrixModels_0.4-1 grid_3.2.2
[22] nnet_7.3-11         R6_2.1.1            rmarkdown_0.9.2
[25] minqa_1.2.4         reshape2_1.4.1      magrittr_1.5
[28] scales_0.3.0        htmltools_0.3       MASS_7.3-45
[31] splines_3.2.2       assertthat_0.1      pbkrtest_0.4-4
[34] colorspace_1.2-6    quantreg_5.19       stringi_1.0-1
[37] munsell_0.4.2
```

References

Raykov, Tenko, and George A Marcoulides. 2011. *Introduction to Psychometric Theory*. Routledge.