1 泰勒展开式

$e^x = 1 + x + \frac{1}{2!}x^2$

$\ln(1+x) = x - \frac{1}{2!}x^2$

$\sin x = x - \frac{1}{3!}x^3$

$\arcsin x = x + \frac{1}{3!}x^3$

$\tan x = x + \frac{1}{3}x^3$

$\arctan x = x - \frac{1}{3}x^3$

$\cos x = 1 - \frac{1}{2!}x^2$

$\frac{1}{1-x} = 1 + x + x^2$

$\frac{1}{1+x} = 1 - x + x^2$

$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2$

2 基本求导公式

$(x^k)' = kx^{k-1}$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

$(e^x)' = e^x$

$(a^x)' = a^x \ln a \quad (a > 0, a \neq 1)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x \cot x$

$(\ln|\cos x|)' = -\tan x$

$(\ln|\sec x + \tan x|)' = \sec x$

$(\ln|\csc x - \cot x|)' = -\csc x$

$(\arcsin x)' = \frac{1}{\sqrt{1+x^2}}$

$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{1}{1+x^2}$

$(\operatorname{arccot} x)' = \frac{-1}{1+x^2}$

$[\ln(x + \sqrt{x^2 + a^2})]' = \frac{1}{\sqrt{x^2+a^2}}$

$[\ln(x + \sqrt{x^2 - a^2})]' = \frac{1}{\sqrt{x^2-a^2}}$

3 基本积分公式

$\int x^k \mathrm{d}x = \frac{1}{k+1}x^{k+1} + C \quad k \neq 1$

$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$

$\int \frac{1}{x^2} = -\frac{1}{x} + C$

$\int \sin x \mathrm{d}x = -\cos x + C$

$\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$

$\int \cos x \mathrm{d}x = \sin x + C$

$\int \frac{1}{x} = \ln|x| + C$

$\int \tan x \mathrm{d}x = -\ln|\cos x| + C$

$\int e^x \mathrm{d}x = e^x + C$

$\int \cot x \mathrm{d}x = \ln|\sin x| + C$

$\int \frac{\mathrm{d}x}{\cos x} = \int \sec x \mathrm{d}x = \ln|\sec x + \tan x| + C$

$\int \frac{\mathrm{d}x}{\sin x} = \int \csc x \mathrm{d}x = \ln|\csc x - \cot x| + C$

$\int \sec^2 x \mathrm{d}x = \tan x + C$

$\int \frac{1}{1+x^2}\mathrm{d}x = \arctan x + C$

$\int \csc^2 x \mathrm{d}x = -\cot x + C$

$\int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\operatorname{arccot}\frac{x}{a} + C$

$\int \sec x \tan x \mathrm{d}x = \sec x + C$

$\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \arcsin x + C$

$\int \csc x \cot x \mathrm{d}x = -\csc x + C$

$\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \arcsin\frac{x}{a} + C$

$\int \frac{1}{\sqrt{x^2+a^2}}\mathrm{d}x = \ln(x + \sqrt{x^2+a^2}) + C$

$\int \frac{1}{\sqrt{x^2-a^2}}\mathrm{d}x = \ln\left|(x + \sqrt{x^2-a^2}\right| + C$

$\int \frac{1}{x^2-a^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

$\int \frac{1}{a^2-x^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$

$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C$

$\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{\sin 2x}{4} + C$

$\int \cot^2 x \mathrm{d}x = -\cot x + x + C$

$\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{\sin 2x}{4} + C$

$\int u \mathrm{d}v = uv - \int v \mathrm{d}u + C$

$\int \tan^2 x \mathrm{d}x = \tan x - x + C$

$\int_a^b f(x)\mathrm{d}x = \int_a^b f(a+b-x)\mathrm{d}x$

$\int_a^b f(x)\mathrm{d}x = \frac{1}{2}\int_a^b [f(x) + f(a+b-x)]\mathrm{d}x$

$\int_a^b f(x)\mathrm{d}x = \int_a^{\frac{a+b}{2}} [f(x) + f(a+b-x)]\mathrm{d}x$

$$\int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \cdots \cdot \frac{2}{3} * 1 \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \cdots \cdot \frac{1}{2} * \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \sin^n x \mathrm{d}x = \begin{cases} 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} * 1 \\ 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} * \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \cos^n x \mathrm{d}x = \begin{cases} 0 \\ 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} * \frac{\pi}{2} \end{cases}$$

$$\int_0^{2\pi} \sin^n x \mathrm{d}x = \int_0^{2\pi} \sin^n x \mathrm{d}x = \begin{cases} 0 \\ 4 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} * \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} x f(\sin x) \mathrm{d}x = \frac{\pi}{2} \int_0^{\pi} f(\sin x) \mathrm{d}x \qquad \int_0^{\frac{\pi}{2}} f(\sin x) \mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\cos x) \mathrm{d}x$$

$$\int_0^{\pi} x f(\sin x) \mathrm{d}x = \pi \int_0^{\frac{\pi}{2}} f(\sin x) \mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x, \cos x) \mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\sin x, \cos x) \mathrm{d}x$$

$$\int_a^b f(x) \mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\tfrac{a+b}{2} + \tfrac{b-a}{2} \sin t) \cdot \tfrac{b-a}{2} \cos x \mathrm{d}x$$

$$\int_a^b f(x) \mathrm{d}x = \int_0^1 (b-a) f[a + (b-a)t] \mathrm{d}t$$

$$\int_{-a}^{a} f(x) \mathrm{d}x = \int_0^a [f(x) + f(-x)] \mathrm{d}x (a > 0)$$

4 微分方程

$y' = f(x) \cdot g(x)$ 型： $\Rightarrow \frac{\mathrm{d}x}{g(y)} = f(x)\mathrm{d}x \Rightarrow \int \frac{\mathrm{d}x}{g(y)} = \int f(x)\mathrm{d}x$

$y' = f(ax+by+c)$ 型： 令 $u = ax+by+c \Rightarrow u' = a + bf'(u) \Rightarrow \frac{\mathrm{d}x}{a+bf(u)} = \mathrm{d}x \Rightarrow \int \frac{\mathrm{d}x}{a+bf(u)} = \int \mathrm{d}x$

$y' = f(\frac{y}{x})$ 型： 令 $\frac{y}{x} = u \Rightarrow y = ux \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u + \frac{\mathrm{d}u}{\mathrm{d}x}$ 原方程 $y\frac{\mathrm{d}u}{\mathrm{d}y} + u = f(u) \Rightarrow \frac{\mathrm{d}u}{f(u)-u} = \frac{\mathrm{d}x}{x} \Rightarrow \int \frac{\mathrm{d}u}{f(u)-u} = \int \frac{\mathrm{d}x}{x}$

$\frac{1}{y'}=f(\frac{x}{y})$ 型：令 $\frac{x}{y}=u\Rightarrow x=uy\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=u+\frac{\mathrm{d}u}{\mathrm{d}y}$ 原方程 $y\frac{\mathrm{d}u}{\mathrm{d}y}+u=f(u)\Rightarrow\frac{\mathrm{d}u}{f(u)-u}=\frac{\mathrm{d}y}{y}\Rightarrow\int\frac{\mathrm{d}u}{f(u)-u}=\int\frac{\mathrm{d}y}{y}$

$y'+p(x)y=q(x)$ 型：方程两边同时乘上 $e^{\int p(x)\mathrm{d}x}\Rightarrow e^{\int p(x)\mathrm{d}x}\cdot y'+e^{\int p(x)\mathrm{d}x}p(x)\cdot y=e^{\int p(x)\mathrm{d}x}\cdot q(x)\Rightarrow\left[e^{\int p(x)\mathrm{d}x}\cdot y\right]'=e^{\int p(x)\mathrm{d}x}\cdot q(x)\Rightarrow e^{\int p(x)\mathrm{d}x}\cdot y=\int e^{\int p(x)\mathrm{d}x}\cdot q(x)\mathrm{d}x+C$

得 $y=e^{-\int p(x)\mathrm{d}x}\left[\int e^{\int p(x)\mathrm{d}x}\cdot p(x)+C\right]$

$y'+p(x)y=q(x)y^n$ 型：先变形到 $y^{-n}\cdot y'+p(x)y^{1-n}=q(x)\overset{z=y^{1-n}}{\Longrightarrow}$ 得 $\frac{\mathrm{d}z}{\mathrm{d}x}=(1-n)y^{-n}\frac{\mathrm{d}y}{\mathrm{d}x}$，则 $\frac{1}{1-n}\frac{\mathrm{d}z}{\mathrm{d}x}+p(x)z=q(x)$

$y''=f(x,y')$ 型：令 $y'=p\Rightarrow y''=p'\Rightarrow\frac{\mathrm{d}p}{\mathrm{d}x}=f(x,p)$ 若解得 $p=\varphi(x,C_1)$ 即 $y'=\varphi(x,C_1)$ 则通解为 $y=\int\varphi(x,C_1)\mathrm{d}x+C_2$

$y''=f(y',y'')$ 型：令 $y'=p\Rightarrow y''=\frac{dp}{\mathrm{d}x}=\frac{\mathrm{d}p}{\mathrm{d}y}\cdot\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}p}{\mathrm{d}y}p$ 得 $p\frac{\mathrm{d}p}{\mathrm{d}y}=f(y,p)$ 若解得 $p=\varphi(y,C_1)$ 则由 $p=\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\varphi(y,C_1)$ 分离变量得 $\frac{\mathrm{d}y}{\varphi(y,C_1)}=\mathrm{d}x\Rightarrow\int\frac{\mathrm{d}y}{\varphi(y,C_1)}=\int\mathrm{d}x$

$y''+py'+qy=f(x)$ 型：

$$\begin{cases}\lambda^2+p\lambda+q=0\Rightarrow\lambda_1,\lambda_2\text{写出齐次方程的通解}\\ \text{设特解}y''\Rightarrow\text{回代，求待定系数}\Rightarrow\text{特解}\end{cases}\Rightarrow\text{写出通解}$$

$y''+py'+qy=f_1(x)+f_2(x)$ 型：

$$\begin{cases} 写\lambda^2+px+q=0 \Rightarrow 齐次方程的通解 \\ \\ y''+py'+q=f_1(x)写特解y_1' \\ \\ y''+py'+qy=f_2(x)写特解y_2' \end{cases}$$