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INTRODUCTION

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The VSOP2013 files contain the series of the elliptic elements for the 8 planets Mercury, Venus, Earth-Moon barycenter, Mars, Jupiter, Saturn, Uranus, Neptune and for the dwarf planet Pluto of the solution VSOP2013.

List of the files:

VSOP2013p1.dat : Mercury VSOP2013p2.dat : Venus

VSOP2013p3.dat : Earth-Moon Barycenter

VSOP2013p4.dat : Mars VSOP2013p5.dat : Jupiter VSOP2013p6.dat : Saturn VSOP2013p7.dat : Uranus VSOP2013p8.dat : Neptune VSOP2013p9.dat : Pluto

The planetary solution VSOP2013 is fitted to the numerical integration INPOP10a built at IMCCE, Paris Observatory (http://www.imcce.fr/inpop/).

FILES DESCRIPTION

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Each VSOP2013 file corresponds to a planet and contains trigonometric series, functions of time (Periodic series and Poisson series), that represent the 6 elliptic elements of the planet:

Variable 1 : a = semi-major axis (ua) Variable 2 : λ = mean longitude (radian)

Variable 3 : $k = e \cos \varpi$ Variable 4 : $h = e \sin \varpi$

Variable 5 : $q = \sin(i/2) \cos \Omega$ Variable 6 : $p = \sin(i/2) \sin \Omega$

with:

e : eccentricity

 ϖ : perihelion longitude

i : inclination

 Ω : ascending node longitude

VSOP2013 series are characterized by 3 parameters:

- the planet index 1-9 from Mercury to Pluto,
- the variable index 1-6 for a, λ , k, h, q, p,
- the time power α .

TERMS OF SERIES

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The terms of series are given under the form: \operatorname{\mathtt{T}}^{\alpha} ( S sin \Phi + C cos \Phi )
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- T is the time (TDB) from J2000 (JD2451545.0) expressed in Thousand of Julian Years (tcy = 365250 days)
- α is the time power of the series (0 <= α <= 20).
- S, C are the coefficients for the variable a (au), the variable λ (radian) and the variables, k, h, q, p (without unit).
- Φ is equal to the sum of the products $a(i).\lambda l(i)$ with i=1,17.
- a(i) are integers, numerical coefficients of the quantities $\lambda l(i)$.

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\lambda l \, (1,13) : linear part of the mean longitudes of the planets (radian).
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- λ l(14) : argument μ derived from TOP2013 and used for Pluto (radian).
- λl (15,17) : linear part of Delaunay lunar arguments D, F, ℓ (radian).

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\lambda l(1) = 4.402608631669 + 26087.90314068555 * T Mercury 
 <math>\lambda l(2) = 3.176134461576 + 10213.28554743445 * T Venus
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- $\lambda 1(3) = 1.753470369433 + 6283.075850353215 * T Earth-Moon$
- λ 1(4) = 6.203500014141 + 3340.612434145457 * T Mars
- $\lambda 1(7) = 5.598641292287 + 1428.948917844273 * T Bamberga$
- $\lambda 1(8) = 2.805136360408 + 1364.756513629990 * T Ceres$
- $\lambda 1(9) = 2.326989734620 + 1361.923207632842 * T Pallas$
- $\lambda l(10) = 0.599546107035 + 529.6909615623250 * T Jupiter$
- $\lambda l(11) = 0.874018510107 + 213.2990861084880 * T Saturn \\ \lambda l(12) = 5.481225395663 + 74.78165903077800 * T Uranus$
- $\lambda 1(13) = 5.311897933164 + 38.13297222612500 * T Neptune$
- $\lambda l(14) = 0.3595362285049309 * T \mu Pluto$
- $\lambda 1(15) = 5.198466400630 + 77713.7714481804 * T D Moon$
- $\lambda 1(16) = 1.627905136020 + 84334.6615717837 * T F Moon$
- $\lambda 1(17) = 2.355555638750 + 83286.9142477147 * T & Moon$

VSOP2013 files contain the numerical values of α , S, C and a(i) (i=1,17).

RECORDS ORGANIZATION

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There are two types of records:

The "HEADER" record for the characteristics of the series (planet, variable, time power).

The "TERMS" records for the quantities a, S and C in each term of the series.

In a VSOP2013 file, series are put in order of the variables $<a,\lambda,k,h,q,p>$ and, for each variable, in order of the time power (α) .

HEADER RECORD

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```
Fortran Format:
```

read (ifile,1001) ip,iv,it,nt

1001 format (9x,3i3,i7)

Specifications:

 $\begin{array}{lll} \mbox{ip : planete index} & \mbox{(integer)} \\ \mbox{iv : variable index} & \mbox{(integer)} \\ \mbox{it : time power } \alpha & \mbox{(integer)} \\ \mbox{nt : number of terms in series} & \mbox{(integer)} \end{array}$

Planet index (ip):

1 : Mercury

2 : Venus

3 : Earth-Moon Barycenter

4 : Mars
5 : Jupiter
6 : Saturn
7 : Uranus
8 : Neptune
9 : Pluto

Variable index (iv):

1 : a = semi-major axis (ua) 2 : λ = mean longitude (radian)

3 : $k = e \cos \omega$ 4 : $h = e \sin \omega$

5 : $q = \sin(i/2) \cos \Omega$ 6 : $p = \sin(i/2) \sin \Omega$

Time power α (it): it=0 : Periodic terms it>0 : Poisson terms

TERMS RECORDS

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The "TERMS" records are put in the file according to the decreasing values of the sum of absolute values of the coefficients S and C: |S| + |C|.

Each "TERMS" record contains respectively: the rank of the term in the series and the quantities a(i) (i=1,17), S and C.

Fortran format:

```
read (ifile,1002) num,(iphi(i),i=1,17),c1,ie1,c2,ie2
1002 format (i5,1x,4i3,1x,5i3,1x,4i4,1x,i6,1x,3i3,2(f20.16,1x,i3))
```

Specifications:

c1, ie1: coefficient S, mantissa and exponent (real*8 and integer) c2, ie2: coefficient C, mantissa and exponent (real*8 and integer) Units of the coefficients: au for a, radian for λ , without unit for the other variables k, h, q, p.

TIME SCALE AND REFERENCE SYSTEM

The time used in VSOP2013 series is TDB (Barycentrc Dynamical Time). This time can be considered equal to TAI + 32.184 s with an approximation less than 0.002s (TAI: International Atomic Time).

The solution VSOP2013 is fitted to the numerical integration INPOP10a over the time interval [1890-2000].

The VSOP2013 coordinates are referred to the inertial frame defined by the dynamical equinox and ecliptic J2000 (JD 2451545.0).

The planetary coordinates of INPOP10a are referred in ICRF.

If X_E , Y_E , Z_E are the rectangular coordinates of a planet computed from VSOP2013, the rectangular coordinates of the planet in equatorial frame of the ICRF, X_Q , Y_Q , Z_Q , may be obtained by the following rotation:

$$\begin{bmatrix} X_{\mathcal{Q}} \\ Y_{\mathcal{Q}} \\ Z_{\mathcal{Q}} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\sin \varphi \cos \varepsilon & \sin \varphi \sin \varepsilon \\ \sin \varphi & \cos \varphi \cos \varepsilon & -\cos \varphi \sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{bmatrix} \begin{bmatrix} X_{E} \\ Y_{E} \\ Z_{E} \end{bmatrix}$$

with: $\mathcal{E} = 23^{\circ} \ 26' \ 21.41136''$ et $\varphi = -0.05188''$

PRECISION OF THE SERIES VOP2013

An estimation of the precision of the series VSOP2013 is given by the largest differences between VSOP2013 and INPOP10a

TIME INTERVAL [1890; 2000]

Planet	a	λ	k	h	q	р	L	В	R
Mercury	0.003	0.03	1.0	1.2	0.1	0.1	0.06	0.01	0.008
Venus	0.002	0.02	0.3	0.2	0.1	1.1	0.02	0.05	0.002
EMB	0.003	0.01	0.8	0.5	0.1	1.9	0.02	0.08	0.011
Mars	0.078	0.74	4.6	5.4	0.3	1.1	0.93	0.06	0.162
Jupiter	0.099	0.19	2.9	3.3	0.5	0.4	0.20	0.02	0.277
Saturn	0.173	0.10	4.8	4.3	0.9	0.9	0.24	0.05	0.592
Uranus	15.120	0.76	40.0	39.1	3.0	2.3	2.19	0.13	5.962
Neptune	3.432	0.08	9.1	3.8	1.5	1.0	0.38	0.05	2.764
Pluto	124.710	2.85	198.1	185.8	56.9	23.3	10.83	3.19	118.419

Unîts:

Elliptic elements: a (km), λ (mas), k, h, q, p (10⁻¹⁰). Heliocentric longitude L and latitude B (mas). Distance Sun-Planet R (km).

TIME INTERVAL [-4000; +8000]

λ	L	В	R
0.12	0.20	0.02	9
0.15	0.15	0.01	4
0.98	1.01	0.01	19
1.49	1.74	0.06	153
4.80	4.47	0.16	3393
8.92	11.73	0.52	21911
1.23	2.45	0.07	13007
1.28	1.27	0.04	7761
	0.12 0.15 0.98 1.49 4.80 8.92 1.23	0.12 0.20 0.15 0.15 0.98 1.01 1.49 1.74 4.80 4.47 8.92 11.73 1.23 2.45	0.12 0.20 0.02 0.15 0.15 0.01 0.98 1.01 0.01 1.49 1.74 0.06 4.80 4.47 0.16 8.92 11.73 0.52 1.23 2.45 0.07

Unîts:

mean longitude λ (arcsecond).

Heliocentric longitude L and latitude B (arcsecond).

Distance Sun-Planet R (km).

COMPUTATION

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The program VSOP2013.f (Fortran) computes the planetary elliptic elements between 1890 and 2000 using the files VSOP2013.

This program computes also the planetary rectangular coordinates referred to the dynamical ecliptic frame of J2000 and referred to the equatorial frame of ICRS.

Starting date : 1890 June 6 (DJ 2411544.5)

Number of dates : 11

Step : 4000 days.

The results of this computation are given in the file: VSOP2013.out

The file VSOP2013.ctl is given for checking these results.

The mean elements of the elliptic variables of the series VSOP2013 are given in the file: VSOP2013-secular.dat.

The astronomical constants consistent with the solution VSOP2013 are the constants used in the numerical integration INPOP10a, especially:

AU CLIGHT GM_Mer GM_Ven GM_EMB GM_Mar GM_Ves GM_Iri GM_Bam GM_Cer GM_Pal GM_Jup GM_Jup GM_Sat GM_Ura GM_Nep GM_Plu	0.1495978706910000D+09 0.2997924580000000D+06 0.4912547451450812D-10 0.7243452486162703D-09 0.8997011603631609D-09 0.9549535105779258D-10 0.3939673413269574D-13 0.2299798032187260D-14 0.1388563508297703D-14 0.1408056343979966D-12 0.3296275038741825D-13 0.2825345842083778D-06 0.8459715185680659D-07 0.1292024916781969D-07 0.1524358900784276D-07 0.2188699765425970D-11	au ³ /day ² au ³ /day ² au ³ /day ² au ³ /day ²
_		
_		
GM_Bam	0.1388563508297703D-14	_
GM_Cer	0.1408056343979966D-12	
GM_Pal	0.3296275038741825D-13	au³/day²
GM_Jup	0.2825345842083778D-06	au³/day²
GM_Sat	0.8459715185680659D-07	au³/day²
GM_Ura	0.1292024916781969D-07	
GM_Nep	0.1524358900784276D-07	au³/day²
GM_Plu	0.2188699765425970D-11	_
GM_Sun	0.2959122083684144D-03	au³/day²
EMRAT	0.8130056999999999D+02	
RSUN	0.696000000000000D+06	km
RMOON	0.173800000000000D+04	km
REARTH	0.637813660000000D+04	km
J2SUN	0.240000000000000D-06	

TRUNCATION OF THE SERIES

The series VSOP2013 can be reduced with the elimination of terms less than a level of truncation $\boldsymbol{\rho}$ such as:

$$(C^2+S^2)^{1/2} < \rho$$

with the following units: au for a, radian for $\lambda,$ without unit for the other variables k, h, q, p.

This table gives the largest differences, on the planetary heliocentric longitudes (L) in arcseconds, between a computation using the complete series and a computation with reduced series for 4 levels of truncation and 4 different time intervals.

	TIME INTERVALS	+1890 +2000	+0900 +3100	+0000 +4000	-4000 +8000
====== MERCURY	======================================	0.67721"10 ⁻⁰¹	0.75801"10 ⁻⁰¹	0.17037"10 ⁰⁰	0.97520"10 ⁺⁰¹
	Truncation 10 ⁻¹⁰	0.15743"10 ⁻⁰²	0.17826"10 ⁻⁰²	0.38273"10 ⁻⁰²	0.69811"10 00
	Truncation 10^{-12}	0.48047"10 ⁻⁰⁴	0.44625"10 ⁻⁰⁴	0.14062"10 ⁻⁰³	0.81597"10 ⁻⁰¹
	Truncation 10 ⁻¹⁴	0.81680"10 ⁻⁰⁶	0.33789"10 ⁻⁰⁵	0.10122"10 ⁻⁰⁴	0.31690"10 ⁻⁰¹
/ENUS	Truncation 10^{-08}	0.53251"10 ⁻⁰¹	0.70522"10 ⁻⁰¹	0.12720"10 00	0.65818"10 ⁺⁰¹
	Truncation 10 ⁻¹⁰	0.14642"10 ⁻⁰²	0.16925"10 ⁻⁰²	0.46000"10 ⁻⁰²	0.13975"10 ⁺⁰¹
	Truncation 10 ⁻¹²	0.41051"10 ⁻⁰⁴	0.41504"10 ⁻⁰⁴	0.16087"10 ⁻⁰³	0.17887"10 00
	Truncation 10 ⁻¹⁴	0.19316"10 ⁻⁰⁶	0.20458"10 ⁻⁰⁵	0.79408"10 ⁻⁰⁵	0.67518"10 ⁻⁰¹
EMB	Truncation 10^{-08}	0.65292"10 ⁻⁰¹	0.70898"10 ⁻⁰¹	0.15865"10 ⁰⁰	0.29256"10 ⁺⁰²
	Truncation 10 ⁻¹⁰	0.19588"10 ⁻⁰²	0.19376"10 ⁻⁰²	0.62822"10 ⁻⁰²	0.36086"10 ⁺⁰²
	Truncation 10 ⁻¹²	0.56497"10 ⁻⁰⁴	0.63936"10 ⁻⁰⁴	0.23802"10 ⁻⁰³	0.24040"10 00
	Truncation 10 ⁻¹⁴	0.24503"10 ⁻⁰⁷	0.34754"10 ⁻⁰⁶	0.97589"10 ⁻⁰⁵	0.47410"10 ⁻⁰¹
MARS	Truncation 10 ⁻⁰⁸	0.97685"10 ⁻⁰¹	0.14296"10 ⁰⁰	0.27820"10 00	0.75319"10 ⁺⁰²
	Truncation 10 ⁻¹⁰	0.24870"10 ⁻⁰²	0.35107"10 ⁻⁰²	0.11166"10 ⁻⁰¹	0.31709"10 ⁺⁰²
	Truncation 10 ⁻¹²	0.85882"10 ⁻⁰⁴	0.89968"10 ⁻⁰⁴	0.43410"10 ⁻⁰³	0.78464"10 00
	Truncation 10 ⁻¹⁴	0.27480"10 ⁻⁰⁹	0.94669"10 ⁻⁰⁷	0.36979"10 ⁻⁰⁶	0.24848"10 ⁻⁰²
JUPITER	Truncation 10 ⁻⁰⁸	0.72111"10 ⁻⁰¹	0.98043"10 ⁻⁰¹	0.43798"10 ⁰⁰	0.19935"10 ⁺⁰³
	Truncation 10 ⁻¹⁰	0.23149"10 ⁻⁰²	0.41402"10 ⁻⁰²	0.18410"10 ⁻⁰¹	0.78025"10 ⁺⁰²
	Truncation 10 ⁻¹²	0.52419"10 ⁻⁰⁴	0.77432"10 ⁻⁰⁴	0.95349"10 ⁻⁰³	0.39318"10 ⁺⁰²
	Truncation 10 ⁻¹⁴	0.18091"10 ⁻⁰⁸	0.67189"10 ⁻⁰⁷	0.12293"10 ⁻⁰⁴	0.10433"10 ⁺⁰²
SATURN	Truncation 10 ⁻⁰⁸	0.85180"10 ⁻⁰¹	0.13503"10 00	0.83459"10 00	0.59209"10 ⁺⁰³
	Truncation 10 ⁻¹⁰	0.44443"10 ⁻⁰²	0.40293"10 ⁻⁰²	0.36953"10 ⁻⁰¹	0.14434"10 ⁺⁰³
	Truncation 10 ⁻¹²	0.58705"10 ⁻⁰⁴	0.11242"10 ⁻⁰³	0.99838"10 ⁻⁰³	0.66771"10 ⁺⁰²
	Truncation 10 ⁻¹⁴	0.68700"10 ⁻⁰⁹	0.30549"10 ⁻⁰⁷	0.15439"10 ⁻⁰⁴	0.15582"10 ⁺⁰²
JRANUS	Truncation 10 ⁻⁰⁸	0.12999"10 00	0.16070"10 ⁰⁰	0.41629"10 ⁰⁰	0.21533"10 ⁺⁰³
	Truncation 10 ⁻¹⁰	0.28179"10 ⁻⁰²	0.46278"10 ⁻⁰²	0.15507"10 ⁻⁰¹	0.28820"10 ⁺⁰²
	Truncation 10 ⁻¹²	0.56458"10 ⁻⁰⁴	0.86700"10 ⁻⁰⁴	0.54348"10 ⁻⁰³	0.91941"10 00
	Truncation 10^{-14}	0.15780"10 ⁻⁰⁸	0.30961"10 ⁻⁰⁷	0.29137"10 ⁻⁰⁵	0.38149"10 ⁻⁰¹
NEPTUNE	Truncation 10 ⁻⁰⁸	0.11042"10 00	0.12789"10 ⁰⁰	0.24913"10 00	0.29114"10 ⁺⁰²
	Truncation 10 ⁻¹⁰	0.24840"10 ⁻⁰²	0.32288"10 ⁻⁰²	0.91847"10 ⁻⁰²	0.34946"10 ⁺⁰¹
	Truncation 10 ⁻¹²	0.60694"10 ⁻⁰⁴	0.69311"10 ⁻⁰⁴	0.44163"10 ⁻⁰³	0.57928"10 00
	Truncation 10^{-14}	0.86562"10 ⁻⁰⁸	0.22378"10 ⁻⁰⁶	0.99586"10 ⁻⁰⁵	0.15162"10 00
PLUTO	Truncation 10^{-08}	0.25573"10 00	0.29364"10 ⁰⁰	0.51874"10 ⁺⁰¹	0.93747"10 ^{+0!}
	Truncation 10 ⁻¹⁰	0.26349"10 ⁻⁰²	0.40748"10 ⁻⁰²	0.27407"10 00	0.14000"10 ^{+0!}
	Truncation 10^{-12}	0.12252"10 ⁻⁰⁴	0.30712"10 ⁻⁰⁴	0.78821"10 ⁻⁰²	0.36128"10 ⁺⁰
	Truncation 10 ⁻¹⁴	0.45800"10 ⁻⁰⁹	0.15325"10 ⁻⁰⁶	0.19191"10 ⁻⁰³	0.14328"10 ⁺⁰³

Unit: arcsecond