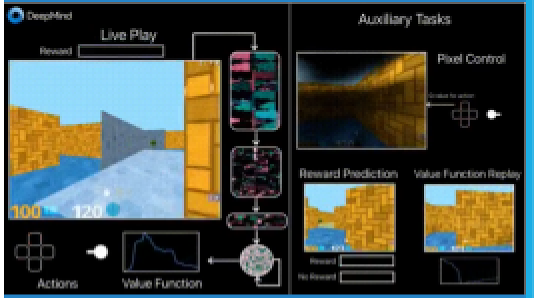
What is Reinforcement Learning?

Pablo Maldonado.

Purpose of this talk

- Provide a good motivation/survey of the main ideas.
- Demistify the hype.
- Present RL as a **framework** for decision-making under uncertainty.

Games and reinforcement learning



Auctions and mechanism design

Network games

Bandit algorithms

Reinforcement Learning fundamentals

and more...

Subscribe to our new course **MI-GLR** starting next semester (February 2017)

The aim of this course is, on one hand, to introduce you to **algorithmic methods** in **game theory**, and on the other hand, to serve as an introduction to more **advanced topics in reinforcement learning**.

About the instructor: Pablo Maldonado, Ph.D. is an applied mathematician and data scientist consultant. He has collaborated with global organizations and small companies alike, and is a regular invited speaker in academic conferences in both sides of the Atlantic. His professional interests are around strategic decision making under uncertainty, both in theory and applications.

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Vanity slide

- Freelance consultant (math/"data science").
- Lecturer:
    - CERGE-EI Economics Discovery Hub.
    - Czech Technical University (FIT).
- PhD math (mathematical game theory), Universite Paris VI.
- **First attendant to the first Machine Learning Meetup in Prague.**

The Reinforcement Learning Problem

- An **agent** learns by interaction with the **environment** to perform a **goal**.
- More *realistic* and *ambitious* than other kinds of machine learning (supervised learning).

Demo: treasure_on_right.py

Where does it come from?

- Learning in animal psychology (Edward Thorndike, 1911).
- Optimal Control and Dynamic Programming (1950's, but core ideas from 18th century).
- Operations Research / Economics.
- Robotics.

Some RL success stories

- Learned the world's best player of Backgammon (Tesauro, 1995).
- **Acrobatic helicopter autopilots** (Ng, Abbeel, Coates et al 2006+).
- Used for **strategic decision-making** in *Jeopardy!* (IBM's Watson, 2011).
- **Human-level performance** on Atari games from pixel input, borrowing deep learning methods (Google Deepmind, 2015).
- **40% reduction** of Google's Data Center Cooling Bill

The two faces of RL

- Model-Based RL:
  - A description of the environment is available (will be made precise soon).
- Model-Free RL:
  - No need of having a model.

Wait, what's a model anyway?

- A **model** is a representation of the environment on which the agent lives.

Markov Decision Processes

A **Markov decision process (MDP)** is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ where

- $\mathcal{S}$ is a finite set of states.
- $\mathcal{A}$ is a finite set of actions.
- $\mathcal{P}$ is a state-action transition probability matrix,
  - $\mathcal{P}^a_{s,s'} = \mathbb{P}(S_{t+1} = s \mid S_t = s, A_t = a)$.
- $\mathcal{R}$ is a reward function
  - $\mathcal{R}^a_s = \mathbb{E}(R_{t+1} \mid S_t = s, A_t = a)$
- γ is a **discount factor**, $\gamma \in [0, 1]$.

Payoff

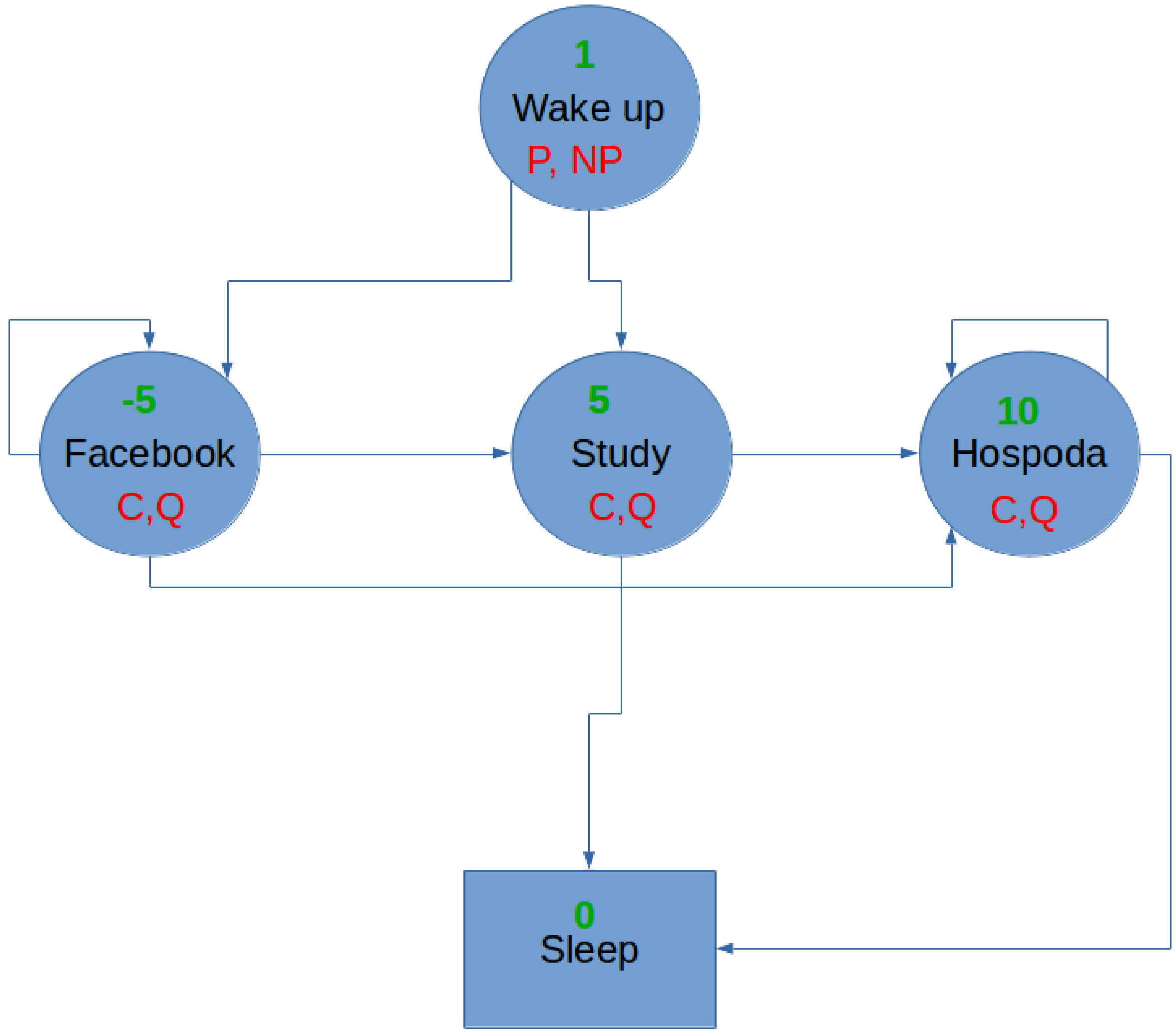
Associated to a **history**

$$(s_1, a_1, r_1, s_2, a_2, r_2, \ldots, s_t, a_t, r_t, S_{t+1}, A_{t+1}, R_{t+1}, \ldots)$$

there is a to-go payoff:

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

Example: Student MDP



Policies

A (stationary) **policy** π is a distribution over actions given states,

$$\pi(a \mid s) := \mathbb{P}(A_t = a \mid S_t = s).$$

State-value function

- The **state-value function** $V_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π:

$$V_\pi(s) = \mathbb{E}(G_t \mid S_t = s)$$

State-action value function

- The **state-action value function** $Q_\pi(s, a)$ is the expected return starting from state s, taking action a and then following policy π,

$$Q_\pi(s, a) = \mathbb{E}_\pi(G_t \mid S_t = s, A_t = a).$$

Solve an MDP

- **Solve** means answering the question *what shall I do in each state to maximize my expected discounted reward?*.
- We use the knowledge of the environment for **prediction** of a policy, and **control** (optimize such policy).
  - **Prediction** means evaluation of the policy (how would the trajectories look like).
  - **Control** means finding the best policy.

Optimal value functions

- The **optimal state-value function** $V_*(s)$ is the maximum over all policies:

$$V_*(s) = \max_\pi V_\pi(s).$$

- The **optimal state-action-value function** $Q_*(s, a)$ is the maximum over all policies:

$$Q_*(s, a) = \max_\pi Q_\pi(s, a).$$

Finding optimal policies

An optimal policy can be found maximizing over Q_*:

$$\pi_*(a \mid s) = 1 \text{ if } a \in \operatorname{argmax} Q_*(s, a) \text{ else } 0.$$

Bellman Equation

$$V_*(s) = \max_{a \in \mathcal{A}} \left\{ \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a V_*(s') \right\}$$

A similar relation holds for the Q_* function since

$$V_*(s) = \max_{a \in \mathcal{A}} Q_*(s, a).$$

Model-Based Control: Value Iteration.

Idea:

Use the optimality equation:

$$V_*(s) = \max_{a \in \mathcal{A}} \left\{ \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a V_*(s') \right\}$$

to get a sequence:

$$V_{k+1} \leftarrow \max_{a \in \mathcal{A}} \left\{ \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a V_k(s') \right\}$$

Model-Based Control: Policy Iteration.

- Given policy π :
  - **Evaluate** π, that is, calculate Q_π.
  - **Improve** the policy by being ϵ-**greedy** with respect to v_π and/or Q_π.
  - Update and repeat.

Why ϵ-greedy?

- *Greedy in the Limite with Infinite Exploration* (GLIE).
- Exploration vs Exploitation dilemma.
- Having a positive chance to visit infinitely often the state-action combinations rules out the chance of missing out anything important.

Planning by Learning the model

What's a model?

- A model is a representation of the environment on which the agent lives.
- $\mathcal{M} = \langle \mathcal{P}, \mathcal{R} \rangle$
- Learn the model first, optimize accordingly later.
- **Planning** means optimizing policy choice; **learning** means understanding the environment.

How can we do that?

- Learning $s, a \mapsto r$ is a **supervised learning problem**.
- Learning $s, a \mapsto s'$ is a **density estimation problem**.

Table lookup model

- Count visits $N(s,a)$ to each state-action pair.
- $\hat{\mathcal{P}}^{a}_{s,s'} = \frac{1}{N(s,a)} \sum 1_{S_t=s, A_t=a, S_{t+1}=s'}$
- $\hat{\mathcal{R}}^{a}_{s} = \frac{1}{N(s,a)} \sum 1_{S_t=s, A_t=a} R_t$

Planning with a model

- Get an approximation of the model $\hat{\mathcal{M}}$.
- Use your favorite planning algorithm:
  - Value iteration
  - Policy iteration
  - Tree search

What could possibly go wrong?

- Continuous state/action spaces.
- Large problems become quickly intractable.
- It's not hopeless, though... there are ways around in many cases. Usually requires domain knowledge to narrow down the problem from the theory side.

Model-Free RL

GLIE Monte-Carlo Control

- Sample an episode using policy π
- For each state S_t and action A_t in the episode,

$$
\begin{aligned}
N(S_t, A_t) &\leftarrow N(S_t, A_t) + 1 \\
Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)}(G_t - Q(S_t, A_t))
\end{aligned}
$$

- Improve the policy based on the new state-action-value function
  - $\epsilon \leftarrow 1/k$
  - $\pi \leftarrow \epsilon - \text{greedy}(Q)$

Demo:

https://gym.openai.com/evaluations/eval_13s9iRXQcG0oaZ1DkN7g

Temporal Difference Learning

- Drawback of MC: you wait until you die to update your experience.
- Break up the episode and learn directly from the experience, not wait until the end.

On-Policy Learning: Sarsa

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Demo:

https://gym.openai.com/evaluations/eval_HmYHTSY5QYOkl77yacPLkg

Off-Policy Learning: Q-Learning

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$;
  until S is terminal

Demo: Frozen Lake

https://gym.openai.com/evaluations/eval_j8yzfQ4BQ9O6dI5Pr7SrtQ

On and Off-Policy Learning

- **On-policy learning:**
  - "Learn on the job"
  - Learn about π by sampling experience from π.
- **Off-policy learning:**
  - "Shadow" someone.
  - Learn about policy π from experience sampled from policy π'.
  - Learn about *optimal* policy while following an *exploratory* policy.

What's the difference here?

- Sarsa can learn from the experience several steps ahead.
  - Sarsa(λ): record the future reward from a bigger part of the future path.
- Q-Learning learns from the experience just one step ahead.

What could possibly go wrong this time?

- Not really useful to solve really **really large** problems:
  - Backgammon: 10^{20} states.
  - Computer Go: 10^{170} states.
  - Chess: 10^{120} states.
  - Helicopter flying: continuous state space.
  - Atoms in the observable universe: 10^{81}.

Function Approximation

How can we solve large problems effectively?

- **Function approximation**
- The goal is to find a parameter vector θ such that

$$\hat{Q}(s, a, \theta) \approx Q(s, a)$$

- θ might be, for instance:
  - Weights in a neural network.
  - Coefficients for a linear regression model.

Motivation: Cart-Pole

- https://gym.openai.com/envs/CartPole-v0
- https://openai.com/requests-for-research/#cartpole
- State: 4 parameters
- Action: move left or right.nt.
- Reward: +1 for not bending the pole too much, and not moving far from the center.
- An episode is solved if you get 200 points.

Random search

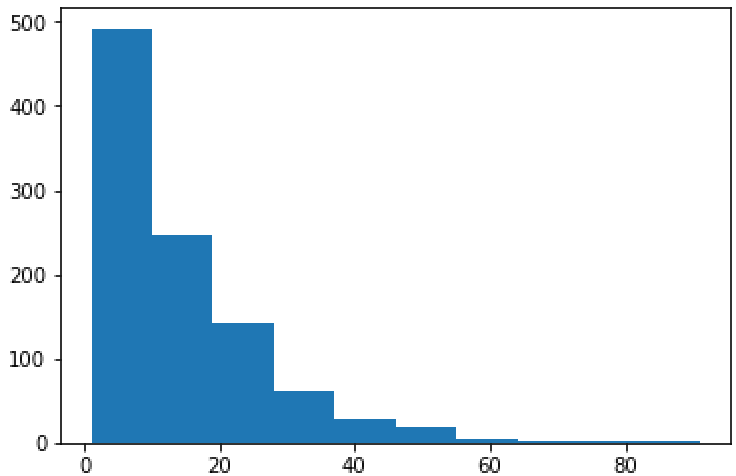
```
# Random search
for _ in range(n_episodes):
    counter += 1
    weights = 2*np.random.rand(4)-1
    ep_reward = run_episode(env, weights)

    if ep_reward > best_ep_reward:
        best_weights = weights
        best_ep_reward = ep_reward

        if ep_reward == 200:
            break
```

Demo: cart_pole_random.py

Random search: Iterations needed to win



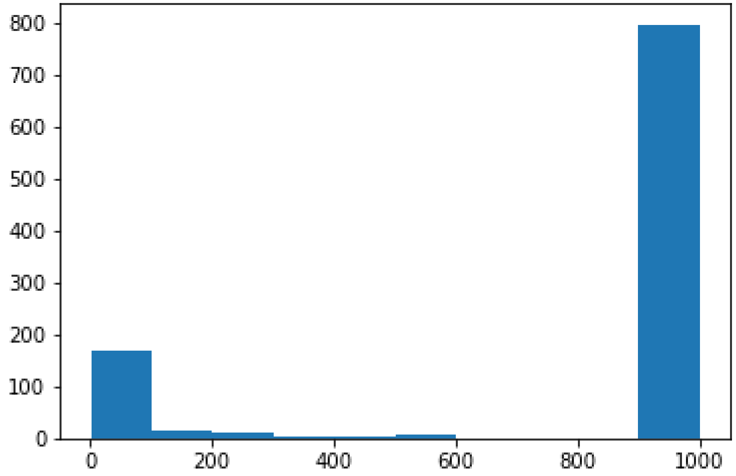
Hill climbing

```
'''Hill-climbing algorithm for training'''

    for _ in range(n_episodes):
        counter+=1
        weights_update = best_weights+ \
                                alpha*(2*np.random.rand(4)-1)
        ep_reward = run_episode(env,weights_update)
        if ep_reward > best_ep_reward:
            best_ep_reward = ep_reward
            best_weights = weights_update
        if ep_reward == 200:
            break
```

Demo: cart-pole-hill-climb.py

Hill climbing: Iterations needed to win



Why so bad?

- Random search jumps all over the place.
- Hill-climbing can get stuck if you are in a region where the improvement is insignificant.

Neural networks: Iterations needed to win

- Around 100 episodes to reach 200 points, 1 hidden layer with 20 nodes.
  (no picture, I'm impatient...)

Function backups

- DP: $s \mapsto \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \cdot \max_{a' \in \mathcal{A}} \hat{q}(s', a', \theta)$
- Monte-Carlo: $s \mapsto G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$
- TD: $s \mapsto R_{t+1} + \gamma \hat{Q}(S_{t+1}, A_{t+1} \theta)$

- In general, we have something of the form $s \mapsto g$, where g is some target value.
  - Up to now, trivial updates: move the estimated value "a bit more" towards g.
  - Viewing each backup as a *training example* we can use any **supervised learning** method to estimate the value function.

Error functional

- **Goal**: Find a parameter θ that minimizes:

$$J(\theta) := \frac{1}{2} \sum_{s \in \mathcal{S}, a \in \mathcal{A}} \left[\left(Q(s,a) - \hat{Q}(s,a,\theta) \right)^2 \right]$$

- **How?** Update the parameters in the direction of the gradient:

$$\theta_{t+1} \leftarrow \theta_t - \alpha \nabla_\theta J(\theta)$$

Batch Reinforcement Learning

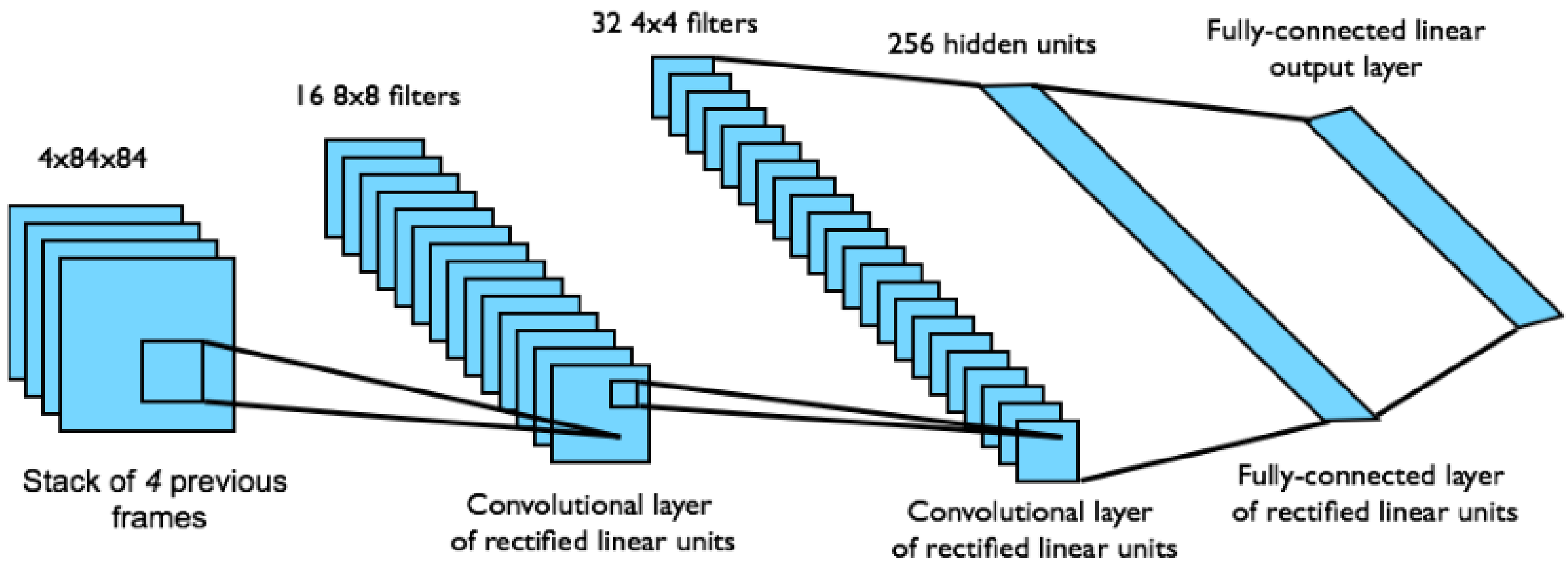
- **Idea**: Find the best fitting value function given the agent's experience.
- An **experience** or **replay memory** $\mathcal{D}$ is a collection of tuples (s, a, r).
- The agent chooses randomly a minibatch from $\mathcal{D}$, to replay his experience and update θ.
  - Why? Break correlations from the data.

Experience Replay in Deep Q-Networks (DQN)

- Off-policy learning (Q-learning).
- Take action a according to ϵ-greedy policy.
- Store transition s, a, r, s' in replay memory $\mathcal{D}$
- Choose a random sample from $\mathcal{D}$ (minibatch).
- Compute Q-learning target with old, fixed parameters w^-.
- Choose the new parameter w that minimizes the error

$$\sum_{s,a,r,s'} \left(r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2$$

DQN in Atari games



- Input: stack of raw pixels from last 4 frames.
- Reward is change of score for the step.
- Output: 18 joystick/button positions.
- Training time: **2 weeks** on GPU to reach human-level performance.

Demo: cart-pole.py

Conclusion

By no means the full story

- Policy gradient
- Actor-critic
- Dyna
- Deep belief networks

  ...

Many more applications!

- Inspecting passengers at airports.
- Multi-skill call centers.
- Planning manufacturing of flu vaccines.
- Fleet planning (logistics, charter operations, car rentals).
- Workforce planning.
- Fraud detection.
- Portfolio Optimization.
- Recommendation Systems.
- NLP.

Multiple agents?

- Advances in computing power (i.e. making it cheaper) have helped MDP/RL to bridge the gap between intractable applications to problems that can be solved with relatively little money.
- **Game Theory** has lots of the same problems! (curse of dimensionality, untractable value functions, etc).

Gracias!

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