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PROJECT PLAN

What is Dimensional Analysis?

Dimensional analysis (DA) is the technique to evaluate forms of equations that describe natural phenomena. Using just basic algebra, the practical application of DA can make significant contribution to not just furthering the understanding of the phenomena but also examine and investigate its mathematical expression.

What is a Dimension?

Dimensions are the labels given to those characteristics describing the nature of a physical quantity. These "characteristics" are granted to us through our observation of the world around us. On the surface of our experience lies the raw sensory materials of knowledge. Careful selection and precise definition are a critical antecedent in the scientific method. This is done through measurement. As a consequence, the qualitative experience which had only sensory or esthetic value gets assigned a numerical value making the experience objective and communicable.

Examining the various numerical values of measurement, relationships between them can be sought out. This is expressed in formulas. However, readings on gauges do not automatically produce laws. One must search for pattern in them. Searching for patterns among measured numerical values will be like looking for desert foot prints.

Standards make the measured numerical values more purposeful. The International Bureau of Weights and Measures[1] (BIPM) has defined seven *base units*. Thus, suggesting that there are seven fundamental dimensions of the physical world. They are called the *base quantities* in reference to the fundamental dimensions for the physical quantities. All other physical quantities can be represented by derived units, expressed in terms of some combination of the base quantities.

The Seven Base Quantities and Base Units.

Below shows the table of the base quantities and base units defined by BIPM[2].

Base quantity		**Base unit**	
Name	**Symbol**	**Name**	**Symbol**
time	T	second	s
length	l, x, r, etc.	meter	m
Mass	M	kilogram	kg
electric current	I, i	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	N	mole	mol
luminous intensity	I_v	candela	cd

BIPM (2020) gives a detailed definition for each base quantity.

[1] https://www.bipm.org/en/about-us/
[2] https://www.bipm.org/en/measurement-units/base-units.html

Mathematical Definition of Dimensions.

Let us denote the base quantities with square brackets [ ]. Thus,

Dimension name	Notation
time	[T]
length	[L]
mass	[M]
electric current	[A]
thermodynamic temperature	[K]
amount of substance	[mol]
luminous intensity	[cd]

Notice that square brackets around the quantity means we are dealing with **the dimensions of** the quantity.

Since all physical quantities can be derived from the base quantities the dimension for volume is equal to $[L] \times [L] \times [L] = [L]^3 = [L^3]$. Note that this is the reduced form of

$$[T]^0\,[L]^3\,[M]^0\,[A]^0\,[K]^0\,[mol]^0\,[cd]^0 = [T^0\,L^3\,M^0\,A^0\,K^0\,mol^0\,cd^0].$$

Therefore, omitting all the base quantities with zero exponent the dimension for volume is represented by the length base quantity and its non-zero exponent.

Hence, **the dimension of a physical quantity is the base quantities and its exponent, raised to represent the quantity**.

Plan of Attack.

- Ability to create dimensional formulae.
- Ability to create dimensional equations.
- Application of the principle of dimensional homogeneity.
- Application of Buckingham's theorem.
  - Create dimensional matrix.
  - Implement taking determinant of the matrix, rank of the matrix and linear dependence.
  - Set up homogeneous system.
  - Compute complete set of pi.
  - Compute dimensionless products.

Bibliography

- BIPM (2020). Base unit definitions. Retrieved from https://www.bipm.org/en/measurement-units/base-units.html
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